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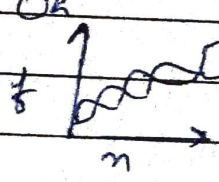
SE
32
Assignment : 01
Design AND Analysis of Algorithm

Ans 1 Asymptotic notations to analyse an algo. running time identifying its behaviour as the input size for the algorithm increases. These notations are used to tell the complexity of an algo. when input is larg.

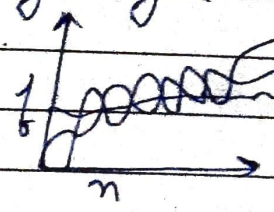
Type of Asymptotic notations

1) Big-Oh



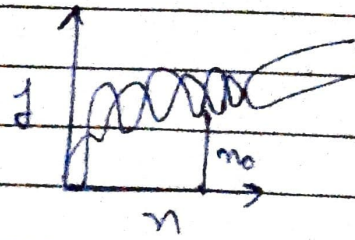
$f(n) = o(g(n))$ if & only if $f(n) \leq C \cdot g(n)$ $\forall\ n \geq n_0$

2) Big omega (Ω) Theta



$f(n) = \Theta(g(n))$ if & only if $C_1 g(n) \leq f(n) \leq C_2 g(n)$ $\forall\ n \geq \max(n_1, n_2)$

3) Big omega (Ω)



$f(n) = \Omega g(n)$ if & only if $f(n) \geq C g(n)$ $\forall\ n \geq n_0$

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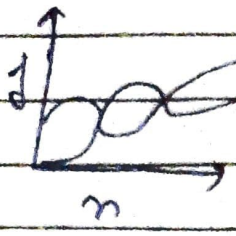
4) Small-Oh (o)



$f(n) = o(g(n))$

$f(n) < C \cdot g(n) \; \forall \; n > n_0$

Ans 2 $i = 1, 2, 3, 4, 8 \ldots n$

$2^0, 2^1, 2^2, 2^3 \ldots 2^k \rightarrow$ G.P

$a = 1, \; r = 2$

$t_k = ar^{k-1}$

$= 1 \times 2^{k-1}$

$n = \frac{2^k}{2}$

$2^k = 2n$ $\qquad K = \log_2(2n)$

$K = \log_2(n) + \log_2(2)$

$= \log n + 1$

$\therefore TC = O(\log n + 1)$

$= O(\log n)$

Ans 3 $T(n) = 3T(n-1) \quad -(1) \qquad n > 0$

$T(1) = 1$

but $n = n-1$ in Eq^n (1)

$T(n-1) = 3T(n-2) \quad -(2)$

put $T(n-1)$ in Eq^n (1)

$T(n) = 3(3T(n-2))$

$T(n) = 9T(n-2) \quad -(3)$

put $n = n-2$ in Eq^n (1)

$T(n-2) = 3T(n-3) \quad -(4)$

put $T(n-2)$ in Eq^n (3)

$T(n) = 9(3T(n-3))$

$T(n) = 27T(n-3)$

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$T(n) = 3^k T(n-k)$ — (5)

$\because T(1) = 1$

$n-k = 1$

$k = n-1$ — (6)

from (5) & (6)

$T(n) = 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3} \times 1$

$\Rightarrow \boxed{T.C = O(3^n)}$

Ans 4 $T(n) = 2T(n-1) - 1$ — (1)

$T(1) = 1$

put $n = n-1$ in Eqn (1)

$T(n-1) = 2T(n-2) - 1$

put $T(n-1)$ in Eqn (1)

$T(n) = 2(2T(n-2)-1)-1$

$T(n) = 4T(n-2) - 2 - 1$ — (2)

put $n = n-2$ in Eqn (1)

$T(n-2) = 2T(n-3) - 1$

put $T(n-2)$ in Eqn (2)

$T(n) = 4(2T(n-3)-1) - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$ — (3)

⋮

$T(n) = 2^k [T(n-k)] - 2^{k-1} - 2^{k-2} \cdots 2^1 - 2^0$ — (4)

$T(1) = (1)$

$n-k = 1$

$k = n-1$ — (5)

from (4) & (5)

$T(n) = 2^{n-1} [T(n-(n-1))] - 2^{n-2} - 2^{n-3} \cdots 2^0$

$= 2^{n-2} - 2^{n-1} - 2^{n-3} - 1$

$= \frac{1}{2}[2^n - (2^n - 1)]$

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$= \frac{1}{2} \times 1 = \frac{1}{2}$ $\boxed{T.C = O(1)}$

Ans 5 $n = 1, 3, 6 \dots K \longrightarrow AP$

$T.C = \frac{K(K+1)}{2}$

$\frac{O(K^2+K)}{2}$

$O(K^2) \Rightarrow \boxed{T.C = O(n^2)}$

Ans 6
```
Void function (int n) {
    int i, count = 0;          → 1
    for (i=1; i*i <= n; i++) {
        count++; }             (n+1)^2
    }                          n
```

$\Rightarrow 1 + 1 + (n+1)^2 + n + n$

$\Rightarrow 2 + 2n + n^2 + 1 + 2n$

$\Rightarrow O(n^2 + 4n + 3)$

$O(n^2)$ //

Ans 7
```
Void function (int n) {
    int i, j, K, count = 0;
    for (i = n/2; i <= n; i++)          — O(n)
        for (j = 1; j <= n; j = j*2)    — log(n)
            for (K = 1; K <= n; K = K*2)
                count++; }              — log(n)
```

$T.C = \log(n) * \log(n)$

$= \log^2(n)$

$= O(\log^2 n)$ //

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Ans 8

```
function (int n) {
    if (n==1) return;          → 1
    for (i=1 to n)             ]
      for (j=1 to n)           ] → n*n
        printf("*");           → 1
    }
}
function (n-3) → n * n^2
}
```

$1 + n^2 + 1 + n^3$

$n^3 + n^2 + 2 \Rightarrow O(n^3)$

Ans 9

```
for (i=1 to n)
    for (j=1; j<=n; j=j+1)
        printf("*");
    }
```

i	j	times
1	1→n	(n+1)/2
2	1→n	(n+1)/2
⋮		
n	1→n	(n+1)/2

$T.C = \log n \left(\frac{n+1}{2}\right)$

$= O\left(\frac{n+1}{2} \log n\right)$

$= O(n \log n)$

Ans 10

$n^k \leq c\,a^n$

$a^n + n^k \leq c \cdot a^n \rightarrow a^n$

$a^n + n^k \leq a^n (c-1)$

$\frac{a^n + n^k}{a^n} \leq (c-1)$

$c \geq 1 + \frac{n^k}{a^{n_0}} + 1$

$c \geq 2 + \frac{n_0^k}{a^n}$

$c \geq 2 + \frac{n_0^1}{1.5^n}$

$n_0 = 1$

$c \geq 2 + \frac{1}{1.5}$

$c \geq 3.0 + 1$

$c \geq 4$

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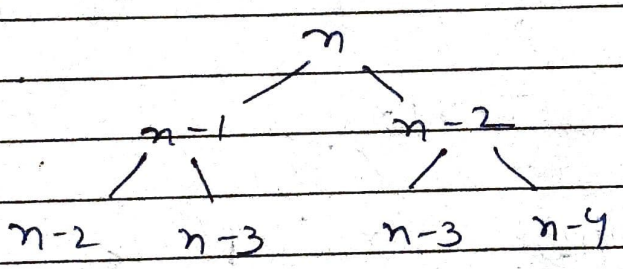
Ans 11 Time Complexity = O(n).
The Execution of different code lines here are
1) while ⇒ (n-1)
2) i=i+j ⇒ (n)
3) j++ ⇒ (n)

T.C = n+n+n-1
= 3n-1
T.C = O(3n-1)
T.C = O(n)

Ans 12 The main working of fibonacci series is
f(n) = f(n-1) + f(n-2)



T(n) = 1+2+4+ -- 2^n
a = 1, r = 2, $\frac{a(r-1)}{r-1} = \frac{1(2^{n+1}-1)}{2-1}$
$= 2^{n+1}$

$T(n) = O(2^{n+1}) = O(2^n \cdot 2^1) = O(2^n)$ //

Ans 13 O(n(log n))

```
int n;
for (int i=0; i<n; i++){
    for (int j=n; j>0; j/=2){
        printf("*");
    }
}
```

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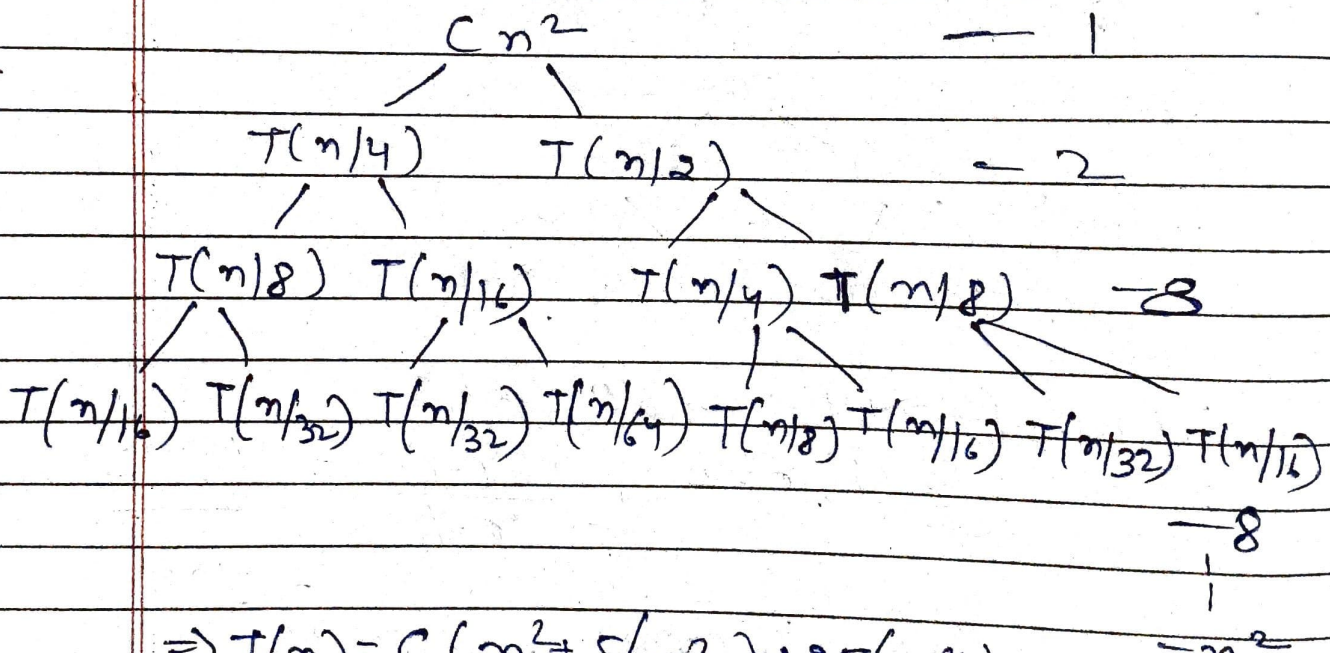
Ans 13 $O(n^3)$

```
int i, j, k;
for (i=1; i<=n; i++) {
    for (j=1; j<=n; j++)
    {
        for (k=1; k<=n; k++) {
            printf("*");
        }
    }
}
```

Ans 14 $T(n) = T(n/4) + T(n/2) + cn^2$



$\Rightarrow T(n) = c\left(n^2 + 5\left(\frac{n^2}{16}\right) + 25\left(\frac{n^2}{256}\right) + \dots\right)$

ratio = 5/16.

$= \frac{n^2}{1 - 5/16} \Rightarrow O(n^2)$ //

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Ans 15

```
int fun(int n){
    for(int i=1; i<=n; i++)
        for(int j=1; j<=n; j=j+i){
            O(1);}
    }
```

i	j	times
1	1→n	n
2	1→n	n/2
.	.	.
n	1→n	n/n

$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{n}$

$= n(1 + 1/2 + 1/3 + \cdots + 1/n)$

$= T(n) = n \log(n)$

Ans 16. $i = 2, 2^c, 2^{c^2}, 2^{c^3} \cdots 2^{c \log_c(\log n)}$

The last term has to be $<= n$

$2^{c \log_c(\log n)} = 2^{\log n} = n$

There are in total $\log_c(\log(n))$ iterations

Each takes a constant time to run

$TC = O(\log(\log n))$

Ans 18 a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n = \log(n!)$
$< n^2 < 2^n < 2^2 < 4^n < n!$

b) $1 < \log\log(n) < \sqrt{\log(n)} < \log n < 2n < 4n$
$< 2(2^n) < \log(2^n) < 2\log(n) < n < n\log n$
$= \log(n!) < n < n^2$

c) $96 < \log_2(n) = \log_8(n) < n\log_8(n) =$
$n\log_2(n^8) = \log(n!) < 5n < 8n^2 < 7n^3$
$< 8^{2n}$

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Ans 19
```
for (int i=0 to n-1){
    if (Array[i]=key){
        return i;}
    }
    return -1;
}
```

Ans 20 a) Iterative Insertion sort

```
Void Insertion sort (int arr[], int n)
{
int i, temp, j;
for (int i=1; i<=n-1; i++)
    temp=arr[i];
    j=i-1;
while (j>=0 && arr[j]>temp){
    arr[j+1]=arr[j];
    j=j-1;}
    arr[j+1]=temp;
```

b) Recursive Insertion Sort

```
Void Insertion sort (int arr[], int n){
    if (n<2)
        return;
    Insertion Sort (arr, n-1);
    last=arr[n-1], j=n-2;
while (j>=0 && arr[j]>temp){
    arr[j+1]=arr[j];
    j=j-1;}
    arr[j+1]=last;
}
```

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Ans21

Algorithm	Best Case	Avg. case	worst case
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n^2)$	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Heap	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Ans22

Algorithm	In place	stable	Online
Bubble	✓	✓	✗
Selection	✓	✗	✗
Insertion	✓	✓	✓
Merge	✗	✓	✗
Quick	✗	✗	✗
Heap	✓	✗	✗

Ans23. Iterative Binary Search

```
Int Binary search (int arr[], int l, int r, int x)
    while ( l <= r){
        int m=(l+r)/2;
        if (arr[m]=x;
            return m;
        else if (arr[m] <x)
            l = m+1;
        else r = m-1; }
        return -1;
    }
```

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Recursive Binary Search

```
int Binary search (int arr[], int l, int r,
                   int n)
  if (l > r)
     return -1;
  int m = (l+r)/2;
   if (arr[m] = n)
      return m;
  else if (arr[m] < n)
  return Binary search (arr, m+1, r, n);
  else
  return Binary search (arr, l, m-1, n);
  }
```

Time Complexity,

Linear (Recursive) → O(n)
Binary (") → O(n)
Linear (Iterative) → O(1)
Binary (") → O(1)

Space Complexity

Linear (Recursive) → O(1)
Binary (") → O(log n)
Linear (Iterative) → O(1)
Binary (") → O(1).

Ans 24. Recurrence Relation for binary search
$T(n) = T(n/2) + 1$.