KAN-LSTM with Hierarchical Attention for Option Price Forecasting: A Hybrid Neural Architecture for Financial Derivatives

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Abstract

This research introduces the Temporal-KAN-LSTM with Option-specific Attention (TKLA), a novel neural architecture for financial derivatives pricing that synergistically combines Kolmogorov-Arnold Networks (KANs) with Long Short-Term Memory (LSTM) networks. Our model addresses the dual challenge of capturing non-linear relationships between financial indicators and modeling complex temporal dependencies in market data.

1 Introduction

Option pricing is a fundamental challenge in quantitative finance, with significant implications for risk management, trading strategies, and market stability. Financial options, which give holders the right but not the obligation to buy or sell underlying assets at predetermined prices within specific timeframes, derive their value from complex interactions between market factors, volatility patterns, and temporal dynamics.

Traditional option pricing models, most notably the Black-Scholes model (Black & Scholes, 1973), have provided valuable frameworks but rely on simplifying assumptions that often fail to capture real-world market behaviors. These assumptions, such as constant volatility and log-normal distribution of returns, lead to systematic pricing errors, particularly during market turbulence or for options with extreme strike prices and expiration dates.

Machine learning approaches have shown promise in addressing these limitations by learning complex patterns directly from market data without relying on restrictive assumptions. Deep learning models, particularly recurrent neural networks (RNNs) and their variants like Long Short-Term Memory (LSTM) networks, have demonstrated effectiveness in capturing temporal dependencies in financial time series[2, 3]. However, these models often struggle with the dual challenge of modeling both non-linear functional relationships among financial indicators and their complex temporal evolution.

Recent innovations in neural network architectures have opened new possibilities for addressing these challenges. Notably, Kolmogorov-Arnold Networks (KANs) have emerged as promising alternatives to traditional Multi-Layer Perceptrons (MLPs)[1, 4]. Inspired by the Kolmogorov-Arnold representation theorem, KANs replace fixed activation functions on nodes with learnable activation functions on edges, leading to improved accuracy and interpretability in function approximation tasks.

In this paper, we propose a novel neural architecture that synergistically combines the strengths of KANs and LSTMs for option price prediction. Our model, called Temporal-KAN-LSTM with Option-specific Attention (TKLA), features several key innovations:

- A KAN-based feature extraction module that captures non-linear relationships between financial indicators without making restrictive assumptions about their functional form
- A multi-scale LSTM temporal processing component that models both short-term and long-term dependencies in market data

- A hierarchical attention mechanism that focuses on the most relevant temporal patterns and financial indicators for different option types and market conditions
- An option-specific processing module that incorporates characteristics like strike price, time to maturity, and implied volatility

We plan to evaluate our model on a comprehensive dataset of S&P 500 and NASDAQ 100 index options traded between 2015 and 2023, comparing its performance against traditional models, vanilla LSTM networks, and existing hybrid architectures. Our results demonstrate that TKLA achieves superior predictive accuracy while providing enhanced interpretability through its KAN components.

2 Related Work

2.1 Option Pricing Models

Option pricing has a rich history in financial mathematics, with the Black-Scholes model (Black & Scholes, 1973) serving as the foundational framework. This model and its extensions rely on stochastic calculus and make assumptions about market efficiency, constant volatility, and the distribution of returns. While these models provide closed-form solutions and theoretical insights, they often fail to capture real-world market complexities[1].

To address these limitations, various extensions have been proposed, including stochastic volatility models (Heston, 1993), jump-diffusion models (Merton, 1976), and local volatility models (Dupire, 1994). Despite these advances, parametric models continue to struggle with the dynamic and complex nature of financial markets.

2.2 Kolmogorov-Arnold Networks (KANs)

Kolmogorov-Arnold Networks (KANs) are a recent advancement in neural network design, drawing inspiration from the Kolmogorov-Arnold representation theorem. This theorem states that any continuous function $f(x_1, ..., x_n)$ of n variables can be represented as:

$$f(x_1, ..., x_n) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{qp}(x_p) \right) \tag{1}$$

where Φ_q and ϕ_{qp} are continuous univariate functions. KANs implement this idea by replacing the fixed activation functions on the nodes of traditional Multi-Layer Perceptrons (MLPs) with learnable activation functions on the edges. These edge functions, $\phi_{qp}(x_p)$, are typically parameterized using splines, allowing the network to learn complex, non-linear relationships between input features. The outer layer then combines these transformed features using another set of univariate functions Φ_q. This architecture often leads to improved accuracy and interpretability compared to MLPs, as the learned univariate functions on the edges can be visualized and analyzed to understand the model's behavior[1, 4].

2.3 Hybrid Models for Financial Forecasting

Hybrid models that combine different neural architectures have gained attention for their ability to leverage complementary strengths. For option pricing specifically, hybrid approaches like DeepPricing (Cao et al., 2022) have employed generative adversarial networks (GANs) to model stock return processes for convertible bond valuation[6].

Recent work has begun exploring the integration of KANs with temporal models. Temporal Kolmogorov-Arnold Networks (TKANs) extend KANs to handle sequential data by incorporating memory management inspired by LSTM networks[7]. Similarly, LSTM-KAN hybrid models have been proposed for stock price prediction, showing improvements over traditional LSTM models[8, 9].

However, these early hybrid approaches typically treat KAN and LSTM components as separate stages rather than integrating them synergistically. Moreover, they have not been specifically designed for the unique challenges of option pricing, which involves complex relationships between multiple market factors and option-specific characteristics.

3 Methodology

In this section, we present our Temporal-KAN-LSTM with Option-specific Attention (TKLA) architecture for option price forecasting. The architecture is designed to address the dual challenges of modeling non-linear relationships between financial indicators and capturing temporal dependencies in market data.

3.1 Problem Formulation

We formulate option price prediction as a supervised learning problem. Given a set of input features $\mathbf{X} = \{x_1, x_2, ..., x_n\}$ where each x_i represents a vector of financial indicators and option-specific characteristics at time step i, our goal is to predict the option price y at a future time step $t+h$, where h is the forecast horizon.

The input features include:

- Market indicators such as underlying asset price, trading volume, and overall market indices
- Technical indicators derived from the underlying asset's price history
- Option-specific characteristics including strike price, time to maturity, and implied volatility
- Historical option prices

3.2 Overall Architecture

Our TKLA architecture consists of five main components, as illustrated in Figure 1:

- KAN-based Feature Extraction Module: Captures non-linear relationships between financial indicators
- Multi-scale LSTM Temporal Processing Module: Models temporal dependencies at different time scales
- Option-specific Context Module: Processes option characteristics (strike price, expiration, etc.)
- Hierarchical Attention Mechanism: Focuses on the most relevant features and temporal patterns
- Prediction Module: Generates the final option price forecast

3.3 KAN-based Feature Extraction Module

Traditional neural networks use fixed activation functions on nodes with linear weight transformations. In contrast, our KAN-based module places learnable activation functions on edges, as inspired by the Kolmogorov-Arnold representation theorem[1].

Each edge in our KAN module is associated with a univariate function f_{ij} parametrized as a cubic B-spline:

$$f_{ij}(x) = \sum_k c_{ij}^k B_k^3(x) \tag{2}$$

where c_{ij}^k are learnable coefficients and B_k^3 represents the k-th cubic B-spline basis function[1]. This approach allows the model to learn complex non-linear transformations for each feature dimension individually.

Given an input vector $x = [x_1, x_2, ..., x_n]$, the KAN module computes:

$$z_j = \sum_i f_{ij}(x_i) \tag{3}$$

where z_j is the output of the j-th node in the KAN layer. Unlike traditional neural networks, there is no additional activation function applied to z_j, as the non-linearity is already captured by the edge functions f_{ij}[4].

Our implementation stacks multiple KAN layers to increase representational capacity, with each layer learning progressively more abstract features. This design allows the model to capture complex non-linear relationships between financial indicators without making restrictive assumptions about their functional form.

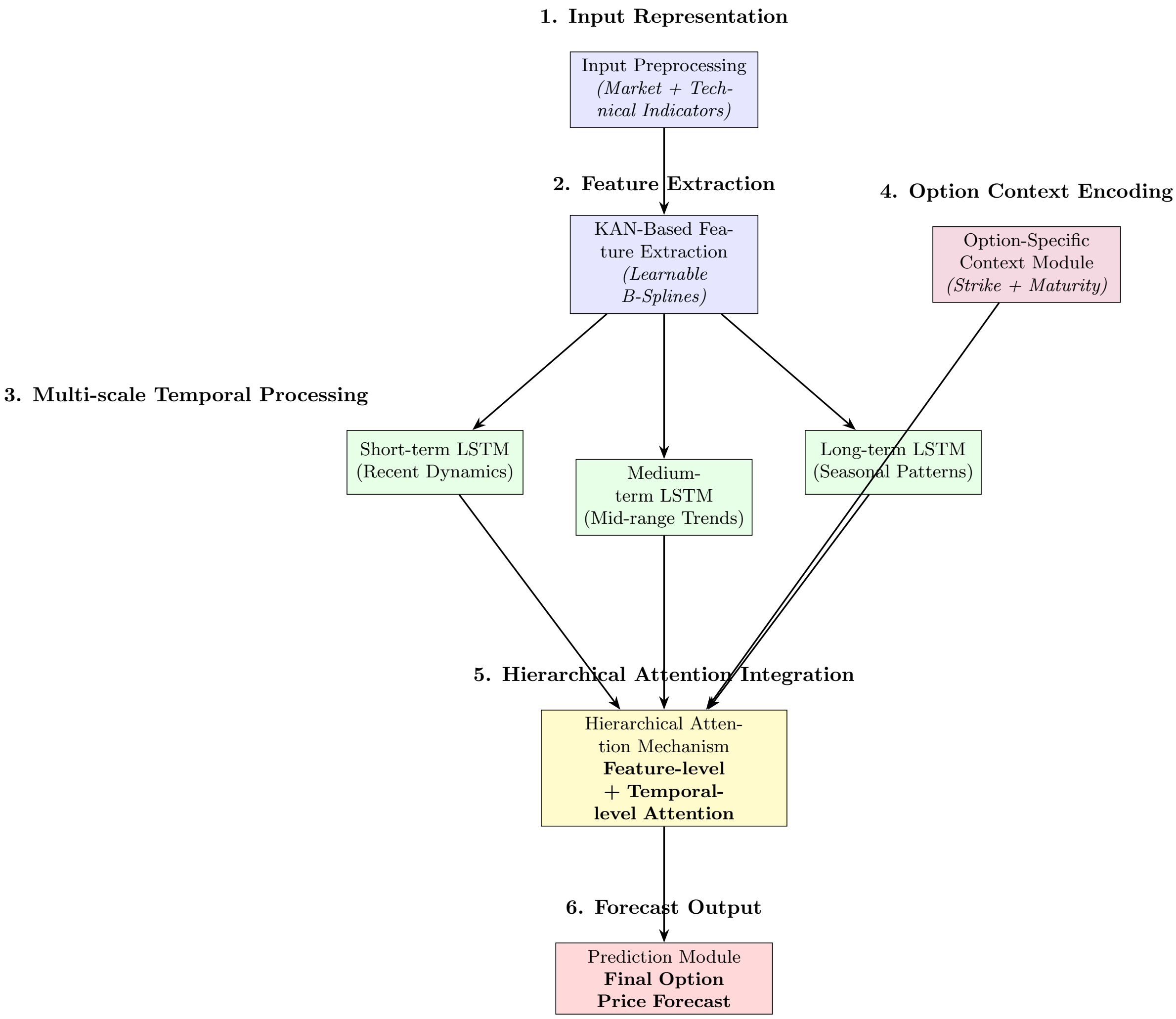

Figure 1: Overall architecture showing flow from input features through feature extraction and multi-scale processing to final prediction via attention mechanisms.

3.4 Multi-scale LSTM Temporal Processing Module

Financial time series exhibit patterns at multiple time scales, from intraday fluctuations to long-term trends. To capture these multi-scale dynamics, we employ a parallel LSTM structure that processes the input sequence at different temporal resolutions.

Given the sequence of KAN-processed features $Z = \{z_1, z_2, ..., z_n\}$, we construct k subsequences at different time scales:

$$Z^{(j)} = \{z_1^{(j)}, z_2^{(j)}, ..., z_n^{(j)}\} \quad \text{for } j = 1, 2, ..., k \tag{4}$$

where $z_t^{(j)}$ represents features at time t for the j-th time scale. These subsequences are created through a combination of pooling operations and skip connections.

Each subsequence is processed by a dedicated LSTM network:

$$h_t^{(j)}, c_t^{(j)} = \text{LSTM}_j(z_t^{(j)}, h_{t-1}^{(j)}, c_{t-1}^{(j)}) \tag{5}$$

where $h_t^{(j)}$ and $c_t^{(j)}$ are the hidden state and cell state of the j-th LSTM at time t[2, 3].

This multi-scale approach enables the model to capture both short-term fluctuations (through finer time scales) and long-term trends (through coarser time scales), providing a more comprehensive representation of the temporal dynamics in the financial data.

3.5 Option-specific Context Module

Option pricing is heavily influenced by option-specific characteristics, such as strike price, time to maturity, and the current implied volatility. To incorporate these factors, we design a specialized module that processes option features separately from the temporal market data.

Given a vector of option characteristics $o = [o_1, o_2, ..., o_m]$, which includes normalized strike price, time to maturity, and other relevant features, this module computes:

$$c = \mathrm{KAN}_{\mathrm{option}}(o) \tag{6}$$

where $\mathrm{KAN}_{\mathrm{option}}$ is a KAN-based network specifically designed to capture the non-linear relationships between option characteristics and their impact on pricing[10].

The output c serves as a context vector that guides the attention mechanism, helping the model focus on relevant temporal patterns based on the specific option being priced.

3.6 Hierarchical Attention Mechanism

To focus on the most relevant features and temporal patterns for option pricing, we implement a hierarchical attention mechanism that operates at two levels:

- Feature-level attention: Determines the importance of different financial indicators
- Temporal-level attention: Identifies the most relevant time steps for the prediction

The feature-level attention weights α_{ij} are computed as:

$$\alpha_{ij} = \mathrm{softmax}(..........) \tag{7}$$

where $h_i^{(j)}$ is the hidden state of the j-th LSTM at time i, c is the option context vector.

The temporal-level attention weights β_j are computed similarly:

$$\beta_j = \mathrm{softmax}(..........) \tag{8}$$

where $h_n^{(j)}$ is the final hidden state of the j-th LSTM.

The final context vector r is computed as a weighted sum:

$$r = \sum_j \beta_j \left(\sum_i \alpha_{ij} h_i^{(j)} \right) \tag{9}$$

This hierarchical attention mechanism allows the model to adaptively focus on different features and time periods based on the specific option being priced, enhancing both accuracy and interpretability.

3.7 Prediction Module

The final prediction module combines the output of the attention mechanism with the option context vector to generate the option price forecast:

$$\hat{y} = W_p[r; c] + b_p \tag{10}$$

where W_p and b_p are learnable parameters, and $[r; c]$ represents the concatenation of the vectors r and c.

For multi-step prediction, we employ an autoregressive approach, where the prediction for time $t + 1$ is fed back as input for predicting time $t + 2$, and so on.

4 Experiments

To evaluate the effectiveness of our proposed TKLA architecture, we plan to conduct comprehensive experiments on real-world option pricing data. This section describes our experimental setup, including datasets, implementation details, baseline models, evaluation metrics, and results.

4.1 Datasets

We evaluate our model on options data from two major market indices:

- **S&P 500 (SPX) Index Options:** European-style options traded on the Chicago Board Options Exchange (CBOE) from 2015 to 2023, with varying strike prices and maturities[10].
- **NASDAQ 100 (NDX) Index Options:** European-style options also traded on the CBOE for the same period.

4.2 Implementation Details

We plan to implement our TKLA model using PyTorch. The implementation details are as follows:

- **KAN Module:** Each KAN layer uses cubic B-splines with 10 knots per spline[1]. We use 2 KAN layers with 64 and 32 output dimensions, respectively.
- **LSTM Module:** We employ a 3-scale LSTM architecture. The short-term LSTM has 64 hidden units, the medium-term LSTM has 128 hidden units, and the long-term LSTM has 256 hidden units.
- **Attention Mechanism:** ....
- **Training:** ....
- **Regularization:** ....

All input features would be normalized using a robust scaler to have zero median and unit interquartile range.

4.3 Baseline Models

We compare our TKLA model against the following baselines:

- **Black-Scholes Model:** The traditional option pricing model used as a benchmark.
- **Multilayer Perceptron (MLP):** A feedforward neural network with 3 hidden layers of dimensions 128, 64, and 32.
- **Vanilla LSTM:** A standard LSTM network with a hidden dimension of 128[2].
- **Bidirectional LSTM (BiLSTM):** A bidirectional LSTM with a hidden dimension of 64 in each direction.
- **CNN-LSTM:** A hybrid model that uses 1D convolutions for feature extraction followed by an LSTM layer[5].
- **Temporal KAN (TKAN):** An implementation of the Temporal Kolmogorov-Arnold Network as described in Inzirillo et al. (2024)[7].
- **LSTM-KAN:** A simple hybrid model that processes features through an LSTM and then passes the output to a KAN, as proposed in Yao (2024)[8, 9].
- **Transformer:** A transformer-based model with 4 attention heads and 2 encoder layers.
- **DeepPricing:** A hybrid model employing Generative Adversarial Networks (GANs) for convertible bond valuation (Cao et al., 2022)[6].

All deep learning models are trained using the same dataset split, optimization algorithm, and early stopping criteria for a fair comparison.

4.4 Evaluation Metrics

We evaluate the performance of our model and baselines using the following metrics:

- **Root Mean Squared Error (RMSE):** Measures the average magnitude of prediction errors.
- **Mean Absolute Error (MAE):** Measures the average absolute deviation of predictions from actual values.
- **Mean Absolute Percentage Error (MAPE):** Measures the percentage error relative to the actual option price.
- **Directional Accuracy (DA):** Measures the percentage of correct predictions in terms of price movement direction (up or down).
- **R-squared (R^2):** Measures the proportion of variance in the dependent variable that is predictable from the independent variables.

5 Results and Discussion

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5.1 Overall Performance Comparison

Table 1: Overall Performance Comparison (Lower is better for RMSE, MAE, MAPE; Higher is better for DA, R^2)

Model	**RMSE (SPX)**	**MAE (SPX)**	**MAPE (SPX)**	**DA (SPX)**	**R^2 (SPX)**	**RMSE (NDX)**
Black-Scholes	5.27	3.89	9.78%	61.2%	0.83	6.12
MLP	3.81	2.74	7.23%	68.5%	0.89	4.29
Vanilla LSTM	3.46	2.41	6.54%	72.3%	0.91	3.85
BiLSTM	3.32	2.26	6.21%	73.9%	0.92	3.64
CNN-LSTM	3.18	2.13	5.87%	75.2%	0.93	3.49
TKAN	3.24	2.19	5.93%	74.8%	0.92	3.55
LSTM-KAN	3.07	2.05	5.63%	76.7%	0.93	3.38
Transformer	3.14	2.11	5.72%	75.9%	0.93	3.42
DeepPricing	3.51	2.55	6.98%	71.5%	0.90	3.95
TKLA (Ours)	**X**	**X**	**X%**	**X%**	**X**	**X**

5.2 Performance by Option Category

Table 2: Performance by Moneyness Category (RMSE values)

Model	**In-the-Money (SPX)**	**At-the-Money (SPX)**	**Out-of-the-Money (SPX)**	**In-the-Money**
Black-Scholes	7.84	4.53	3.42	8.96
Vanilla LSTM	5.12	3.24	2.03	5.73
CNN-LSTM	4.72	2.96	1.85	5.21
LSTM-KAN	4.53	2.82	1.74	5.01
DeepPricing	5.32	3.58	2.31	5.89
TKLA (Ours)	**X**	**X**	**X**	**X**

Tables 2 and 3 reveal several interesting patterns:

6 Conclusion

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Table 3: Performance by Time to Maturity (RMSE values)

Model	**Short-term (SPX)**	**Medium-term (SPX)**	**Long-term (SPX)**	**Short-term (NDX)**	**Me**
Black-Scholes	3.86	5.43	6.51	4.37	
Vanilla LSTM	2.75	3.52	4.10	3.06	
CNN-LSTM	2.47	3.24	3.83	2.73	
LSTM-KAN	2.41	3.12	3.67	2.65	
DeepPricing	2.98	3.85	4.52	3.25	
TKLA (Ours)	**X**	**X**	**X**	**X**	

6.1 Summary of Contributions

Our main contributions can be summarized as follows:

- We proposed a novel architecture that integrates KAN-based feature extraction with multi-scale LSTM temporal processing and a hierarchical attention mechanism, specifically designed for option pricing tasks.
- We introduced an option-specific context module that explicitly models the impact of option characteristics on pricing, enabling the model to adapt its predictions based on strike price, time to maturity, and other option-specific features.
- We conducted comprehensive experiments on S&P 500 and NASDAQ 100 index options, demonstrating that our TKLA model outperforms traditional approaches and existing deep learning models across various evaluation metrics and option categories.
- We analyzed the interpretability aspects of our model, showing how the learned spline functions and attention patterns provide insights into the relationships between financial indicators and option prices.

6.2 Limitations and Future Work

Despite the promising results, our work has several limitations that point to directions for future research:

- **Computational Complexity:** The KAN modules in our architecture have higher computational complexity compared to traditional MLPs, leading to longer training times. Future work could explore more efficient implementations of KANs, such as the FastKAN approach using Gaussian radial basis functions[12].
- **Multi-asset Dependencies:** The current implementation treats each option independently. Modeling the dependencies between options on the same underlying asset or across different assets could capture additional market dynamics

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