# Practical -No -1 :-



**Topic** :- Limits & Continuity.

1) $\lim_{x \to a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to 0} \left[ \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[ \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4) $\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2-1}}$

5) Examine the continuity of the following function at given points.

i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos x}}$ for $0 < x \le \pi/2$
$\phantom{f(x)} = \frac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$
at $x = \pi/2$

ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$\phantom{f(x)} = x + 3$ $\quad 3 \le x < 6$
$\phantom{f(x)} = \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$
at $x = 3$ & $x = 6$

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Q) Find the value of $k$, so that the $f^n$ $f(x)$ is at the indicated point.

i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$
$= k$ $x = 0$ $\}$ at $x = 0$.

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= k$ $x = 0$ $\}$ at $x = 0$.

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$
$= k$ $x = \pi/3$ $\}$ at $x = \pi/3$

Q) Continuity of the following of function have a removable discontinuity.

i) $f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$
$= 9$ $x = 0$ $\}$ at $x = 0$

ii) $f(x) = \frac{(e^{3x} - 1)\sin x^\circ}{x^2}$ $x \neq 0$
$= \frac{\pi}{6}$ $x = 0$ $\}$ at $x = 0$.

iii) If $f(x) = (e^{3x} - 1)\sin x^\circ$, for $x \neq 0$ is continuous at $x = 0$ find $f(0)$.

iv) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continuous at $x = \pi/2$ find $f(\pi/2)$



Solⁿ:-

i) $\lim_{x \to a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{4^2\sqrt{a}}{2\sqrt{3}\sqrt{a}}$

$= \frac{2}{3\sqrt{3}}$ //

Q2) $\lim_{y \to 0} \left[ \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left( \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$

$\lim_{y \to 0} \dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \dfrac{\cancel{y}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$\dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})}$

$\dfrac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})}$

$\dfrac{1}{\sqrt{a}\,(2\sqrt{a})}$

$= \dfrac{1}{2a}$

Q3) $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

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by substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$ where $h \to 0$

$\lim_{h \to 0} \dfrac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3}\sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h + \frac{\pi}{6}\right)}$

using $\cos(A+B) = \cos A\cos B - \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\lim_{h \to 0} \dfrac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cosh \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(6h + \frac{\pi}{6}\right)}$

$\cos\frac{\pi}{6} = \cos 30^\circ = \sqrt{3}/2$

$\sin\frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}$

$\lim_{h \to 0} \dfrac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \cdot \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$

$\lim_{h \to 0} \dfrac{\cos\frac{\sqrt{3}}{2}h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}h}{-6h}$

$\lim_{h \to 0} \dfrac{-\sin\frac{4h}{2}}{-6h}$

$\lim_{h \to 0} \dfrac{\sin 4h}{\sin 12h}$

$\frac{1}{3} \lim_{h \to 0} \dfrac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$

Q4) $\lim \left[ \frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \right]$

=

by rationalising numr & denr both,

$$\lim_{x\to\infty} \left[ \frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \right]$$

$$\lim_{x\to\infty} \left[ \frac{(\cancel{x^2}+5-\cancel{x^2}+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\cancel{x^2}+3-\cancel{x^2}-1)(\sqrt{x^2+5}+\sqrt{x^2-3})} \right]$$

$$\lim_{x\to\infty} \frac{\cancel{8}^4}{\cancel{2}_1} \frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})}+\sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-\frac{3}{x^2})}}$$

After applying limit, we get,

$$4\lim_{x\to\infty} . 0 .$$

$= 4$

Q7) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$, for $\frac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \quad \therefore f(\pi/2) = 0$$

f at $x = \pi/2$ define.

ii) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$.

by substituting method,

$x - \frac{\pi}{2} = h$.

$x = h + \frac{\pi}{2}$

where $h \to 0$.

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - \cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)}$$

$$\lim_{h\to 0} \cos\left(h+\frac{\pi}{2}\right)$$

$$\lim_{h\to 0} \frac{\cos h . \cos\frac{\pi}{2} - \sinh . \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h . 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$= \frac{1}{2}$ //

iii) $f(3) = \frac{x^2-9}{x-3} = 0$

$f$ at $x=3$ is defined.

iv) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

$f$ is defined at $x=3$.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

$f$ is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$.

$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL.

$\therefore f^n$ is not continuous.

(6) i) Sol$^n$:-

$f$ is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$.

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$.

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$k = 8$.

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$

using $\sec^2\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1+\tan^2 x$ & $\cot^2 x = \frac{1}{\tan^2 x}$.

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$.

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that,

$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$= e$

$\therefore k = e$.

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iii) $f(3) = \frac{x^2-9}{x-3} = 0$.

f at $x=3$ is defined.

iv) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

f is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}3 = 3$.

⑥ $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL.

$f^n$ is not continuous.

(6) i) sol$^n$:-

f is continuous at $x=0$.

$\lim_{x\to 0} f(x) = f(0)$.

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$.

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$.

$2(2)^2 = k$

$k = 8$.

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$

using $\sec^2 \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$ & $\cot^2 x = \frac{1}{\tan^2 x}$.

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$.

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that,

$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$= e$.

$\therefore k = e$.

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$

Let $x - \frac{\pi}{3} = h$.

$x = h + \frac{\pi}{3}$

where $h \to 0$.

$f\left(\frac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$.

$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$.

using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$.

$\lim_{h \to 0} \frac{\sqrt{3} - \frac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$.

using $\tan\frac{\pi}{3} = \tan 60 = \sqrt{3}$

$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$.

$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - 3 - \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$

$\lim_{h \to 0} \frac{\frac{(\cancel{\sqrt{3}} - 3\tan h) - \cancel{\sqrt{3}} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$

$\lim_{h \to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$.



$\lim_{h \to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$

$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \left(\frac{1}{1 - \sqrt{3}\tan h}\right)$

$\frac{\tan h}{h} = 1$.

$= \frac{4}{3} \frac{1}{(1 - \sqrt{3}(0))}$

$= \frac{4}{3} \left(\frac{1}{1}\right) = \frac{4}{3}$.

17) Soln:-

$f(x) = \frac{1 - \cos 3x}{x \tan x}$

$\lim_{x \to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$.

$\lim_{x \to 0} = \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$

$= 2\lim_{x \to 0} \left(\frac{3}{2}\right)^2 = \cancel{2} \times \frac{9}{\cancel{4}2} = \frac{9}{2}$

$\lim_{x \to 0} \cancel{f(x) = \frac{9}{2}}$ $\quad 9 = f(0)$

$\therefore f$ is not continuous at $x = 0$.

Redefine $f^n$

$f(x) = \begin{cases} \frac{1 - \cos 3x}{x \tan x} & x \neq 0. \\ \frac{9}{2} & x = 0 \end{cases}$.

Now $\lim_{x\to0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$

$= \frac{\pi}{6}$ $x=0$.

Sol^n:-

$\lim_{x\to0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x\to0} \frac{e^{3x}-1}{x} \lim_{x\to0} \sin\left(\frac{\pi x}{180}\right)$

$\lim_{x\to0} 3\cdot\frac{e^{3x}-1}{3x} \lim_{x\to0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$.

$3\lim_{x\to0} \frac{e^{3x}-1}{3x} \lim_{x\to0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3\log_e \frac{\pi}{180_{60}} = \frac{\pi}{60} = f(0)$.

f is continuous at $x=0$.

iii) $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is continuous at $x=\pi/2$

Sol^n:- $\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$\lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$ //.

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# Practical-02

## Derivatives:-

Q1) Show that the following $f^n$ defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$.

$f(x) = \cot x$.

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}.$$

put $x - a = h$

$x = a + h$.

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h - a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula:- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a - a - h)(1 + \tan a \tan(a+h))}{h \times \tan(a+h) \tan a}.$$

$\lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$

$= \frac{-\sec^2 a}{\tan^2 a}$

$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\operatorname{cosec}^2 a$.

$\therefore f$ is differentiable $\forall, a \in R$.

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)(\sin a)(\sin x)}$

Put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

Formula:- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$.

$= \lim_{h\to 0} -\frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$

iii) $\sec x$

$f(x) = \sec x$.

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$.

$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$

$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$

Put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$.

$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

Formula:- $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

$= \lim_{h\to 0} \dfrac{2\sin\left(\frac{2a+h}{2}\right)\sin\frac{h}{2}}{\cos a \cos(a+h) \times \frac{h}{2}} \times \frac{1}{2}$

$= \frac{1}{2} \times \dfrac{2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$

$= \frac{1}{2} \times 2 \dfrac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$.

Q2) If $f(x) = 4x+1,\ x \le 2$
$= x^2+5\quad x > 0$, at $x = 2$, then, find $f(x)$ is differentiable or not.

Soln:-

LHD:-

$Df(2^-) = \lim_{x\to 2^-} \dfrac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x+1-(4\times 2+1)}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x+1-9}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x-8}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$

RHD:-

$Df(2^+) = \lim_{x\to 2^+} \dfrac{x^2+5-9}{x-2}$

$= \lim_{x\to 2^+} \dfrac{x^2-4}{x-2}$

$= \lim_{x\to 2^+} \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 2+2 = 4.$

$Df(2^+) = 4$

RHD = LHD.

$f$ is differentiable at $x = 2$.

Q3) If $f(x) = 4x+7,\ x < 3$
$= x^2+3x+1,\ x \ge 3$ at $x = 3$, then find $f$ is differentiable or not!

Soln:-

RHD:-

$Df(3^+) = \lim_{x\to 3^+} \dfrac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x+1-19}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x-18}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x-3}$

$= \lim_{x \to 3^-} \frac{4x - 12}{x-3}$

$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$

$Df(3^-) = 4$

RHD $\neq$ LHD

$\therefore f$ is not differentiable at $x = 3$.

Q4) If $f(x) = 8x - 5$, $x \le 2$

$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$, then, find $f$ is differentiable or not.

Soln:- $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$.

RHD:-

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{(x-2)}$

$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:-

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$= 8$

$Df(2^-) = 8$

LHD = RHD.

$\therefore f$ is differentiable at $x = 2$.

Practical 3 :-

Topic :- Application of Derivatives :-

1) Find the intervals in which $f^n$ is increasing or decreasing.

a) $f(x) = x^3 - 5x - 11$

Sol$^n$ :- $f$ is increasing if $f$ only if

$f'(x) > 0$

$\therefore f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

$-\infty \quad -\sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}} \quad \infty$

$\therefore x \in [-\infty, \sqrt{\frac{5}{3}}) \cup (\sqrt{\frac{5}{3}}, \infty)$

Now $f$ is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 5 < 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

$-\infty \quad \sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}} \quad \infty$

$\therefore x \in \left(-\frac{\sqrt{5}}{\sqrt{3}}, \sqrt{\frac{5}{3}}\right)$.



b) $f(x) = x^2 - 4x$.

Sol$^n$ :- $f$ is increasing iff $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$

$-\infty \quad 2 \quad \infty$

$\therefore x \in (2, \infty)$

Now $f$ is decreasing iff.

$f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$.

$-\infty \quad 2 \quad \infty$

$\therefore x \in (-\infty, 2)$.

c) $f(x) = 2x^3 + x^2 - 20x + 4$

Sol$^n$ - $f$ is increasing iff

$f'(x) > 0$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$.

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$\therefore x = 2, \frac{5}{3}$.

$-\infty \quad -2 \quad \frac{5}{3} \quad \infty$

$\therefore x \in (-\infty, -2) \cup (\frac{5}{3}, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$\therefore x = -2, 5/3$

$-\infty \quad -2 \quad \frac{5}{3} \quad \infty$

$x \in (-2, \frac{5}{3})$

eg $f(x) = x^3 - 27x + 5$

Sol$^n$:- f is increasing iff

$f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$

$-\infty \quad -3 \quad 3 \quad \infty$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$

$-\infty \quad -3 \quad 3 \quad \infty$

$\therefore x \in (-3, 3)$

eg $f(x) = 69 - 24x - 9x^2 + 2x^3$

Soln:- f is increasing iff $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24 - 18x + 6x^2 > 0$

$\therefore 6(-4 - 3x + x^2) > 0$

$\therefore x^2 - 3x - 4 = 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x = 4, -1$

$-\infty \quad -1 \quad 4 \quad \infty$

$x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$\therefore (x-4)(x+1) < 0$

$\therefore x = 4, -1$

$-\infty \quad -1 \quad 4 \quad \infty$

$x \in (-1, 4)$

Scanned with CamScanner

Q.7 Find the intervals in which $f^n$ is concave upwards & concave downwards.

a) $y = 3x^2 - 2x^3$

Sol^n - $y = f(x)$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

$\therefore f$ is concave upward iff

$f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1 - 2x) > 0$

$(1 - 2x) > 0$

$\therefore -12x - 1 > 0$

$-\infty \quad + \quad \frac{1}{2} \quad - \quad \infty$

$x \in (-\infty, \frac{1}{2})$.

$\therefore f$ is ~~$x \in (-\infty$~~ concave downwards iff

$f''(x) < 0$

$\therefore 6(1 - 2x) < 0$

$\therefore 6 - (2x - 1) < 0$

$-\infty \quad + \quad \frac{1}{2} \quad - \quad \infty$

$x \in (\frac{1}{2}, \infty)$.



b) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

Sol^n - $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$.

$\therefore f$ is concave upwards iff

$f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x - 2)(x - 1) > 0$

$\therefore x = 2, 1$.

$-\infty \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

$x \in (-\infty, 1) \cup (2, \infty)$

$f$ is concave downward iff

$f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x - 2)(x - 1) < 0$

$\therefore x = 2, 1$

$-\infty \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

$x \in (2, 2)$.

(7) $y = x^3 - 27x + 5$

Soln :- $y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore f''(x) = 6x$

$\therefore f$ is concave upward iff

$f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$-\infty \quad 0 \quad \infty$

$\therefore x \in (0, \infty)$

$\therefore f$ is concave downward iff

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0.$

$-\infty \quad 0 \quad \infty$

$\therefore x \in (-\infty, 0)$



(8) $y = 69 - 24x - 9x^2 + 2x^3$

Soln :- $y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x.$

$\therefore f$ is concave upwards iff

$f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x - 3) > 0$

$\therefore 2x - 3 > 0$

$\therefore x = 3/2.$

$-\infty \quad - \quad 3/2 \quad + \quad \infty$

$\therefore x \in \left(\frac{3}{2}, \infty\right).$

$f$ is concave downwards iff.

$f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x - 3) < 0$

$\therefore 2x - 3 < 0$

$\therefore x = 3/2$

$-\infty \quad 3/2 \quad \infty$

$\therefore x \in \left(-\infty, \frac{3}{2}\right)$

e) $y = 2x^3 + x^2 - 20x + 4$.

Sol$^n$:- $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

$\therefore f$ is concave upwards iff

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x + 1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$.

$-\infty \quad \frac{-1}{6} \quad \infty$

$\therefore x \in (\frac{-1}{6}, \infty)$

$\therefore f$ is concave downwards iff

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 1) < 0$

$\therefore x = -1/6$

$-\infty \quad \frac{-1}{6} \quad \infty$

$\therefore x \in (-\infty, \frac{-1}{6})$

05/01/2020

Practical No. 4 :-

053

Topic:- Application of Derivative & Newton's Method.

I) Find maximum and minimum values of foll. func$^n$s.

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$ minima = 1 maxima = 5

iii) $f(x) = x^3 - 3x^2 + 1$ in $[\frac{-1}{2}, -1]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

II) Find the root of following equation by Newton's Method (Take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$.

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$.

1 Sol$^n$ -

$f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

For maxima/minima

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3} \Rightarrow 2x \cdot x^3 = 32 \Rightarrow 2x^4 = 32 \Rightarrow x^4 = \frac{32}{2}$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$\therefore f''(2) = f''(-2) = 2 + \frac{96}{(\pm 2)^4} = 2 + \frac{96}{16} = 8 > 0$

$\therefore f(x)$ is minimum at $x = \pm 2$

$\therefore f(2) = 8$ is the minimum value.



2) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^4 - 15x^2$

For Maxima/Minima

$f'(x) = 15x^4 - 15x^2 = 0$

$\therefore x^4 - x^2 = 0$

$\therefore x^2(x^2 - 1) = 0$

$\therefore x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 10 - 30 = 30 > 0$

$\therefore f(x)$ is maximum at $-1$ and minimum at $1$.

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

Q3) $f(x) = x^3 - 3x^3 + 1$

$f'(x) = 3x^2 - 6$

for maxima/minima

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$\therefore f(x)$ is maximum at $x = 0$ & minimum at $x =$

$\therefore f(0) = 1$

$\therefore f(2) = -3$

$f(x) = 2x^3 - 3x^2 - 12x + 1$.

$f'(x) = 6x^2 - 6x - 12$

for maxima/minima

$\therefore f'(x) = 0$.

$\therefore 6x^2 - 6x - 12 = 0$.

$\therefore x^2 - x - 2 = 0$.

$\therefore x^2 - 2x + x - 2 = 0$

$\therefore x(x-2) + 1(x-2) = 0$.

$\therefore (x+1)(x-2) = 0$.

$\therefore x = -1, 2$.

$f''(x) = 12x - 6$.

$f''(-1) = -12 - 6 = -18 < 0$.

$f''(2) = 24 - 6 = 18 > 0$.

$\therefore f(x)$ is maximum at $x = -1$ & minimum at $x = 2$

$\therefore f(-1) = 8$.

$\therefore f(2) = -19$.



$f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_0 = 0$.

$f(x_0) = 9.5$

$f'(x_0) = -55$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$.

$= 0 - \dfrac{9.5}{-55}$

$\therefore x_1 = 0.1727$.

$f(x_1) = -0.0828$

$f'(x_1) = -55.9467$.

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \dfrac{0.0828}{-55.9467}$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$f'(x_2) = -55.9393$.

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 0.1712 - \dfrac{0.0011}{-55.9393}$

$\therefore x_3 = 0.1712$

$\therefore x = 0.1712$ is the root of the eqn.

(2) $f(x) = x^3 - 4x - 9$.

$f'(x) = 3x^2 - 4$.

$f(2) = -9$.

$f(3) = 6$.

$\therefore x_0 = 3$ odd

$f(x_0) = 6$

$f'(x_0) = 23$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5742$

$f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7391 - \frac{0.5742}{18.508}$

$x_2 = 2.707$

$\therefore f(x_2) = 0.0085$

$\therefore f'(x_2) = 17.9835$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.707 - \frac{0.0085}{17.9835}$

$x_3 = 2.7065\ldots$

$f(x) = 0.0005$

$f'(x_3) = 17.9757$

$\therefore x_4 = \frac{x_3 - f(x_3)}{f'(x_3)}$

$= 2.7065 - \frac{0.0005}{17.9757}$

$x_4 = 2.7065$



(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

$\therefore -2.2$ is C

[1, 2]

$\therefore x_0 = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= 2 - \frac{-2.2}{-5.2}$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5767) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.6592$

$\therefore f(x_2) = 0.0204$

$\therefore f'(x_2) = 2.7143$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= -1.6572 - \frac{0.0204}{2.7143}$

$x_3 = -1.6618$

$\therefore f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$\therefore f(x_3) = 0$

$\therefore$ Ans $= 1.6618$

Practical - 5



## Topic - Integration

i) Solve the following integration :-

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$ iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2t^4)\,dt$ vi) $\int \sqrt{x}(x^2-1)\,dx$

vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$ viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x}\sin 2x\,dx$ x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

① $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$I = \int \frac{1}{\sqrt{x^2-2x+1-4}} \, dx$

$I = \int \frac{1}{\sqrt{(x+1)^2-2^2}} \, dx$

$I = \ln|x+1 \sqrt{x^2+2x-3}| + C$

② $I = \int (4e^{3x}+1) \, dx$

$I = 4\int e^{3x} dx + \int 1 \, dx$

$I = \frac{4e^{3x}}{3} + x + c$

③ $I = \int (2x^2 - 3\sin x + 5\sqrt{x}) \, dx$

$I = 2\int x^2 dx - 3\int \sin x \, dx + 5\int \sqrt{x} \, dx$

$I = \frac{2x^3}{3} + 3\cos x + 5 \frac{\cancel{x}2}{3} x^{3/2} + c$

$I = \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x\sqrt{x} + C$

4) $I = \int \frac{x^3+3x+4}{\sqrt{x}} \, dx$

Put $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx} = \frac{dx}{\sqrt{x}} = 2dt$

$I = 2\int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}} \, dx$

$I = 2\int t^6 + 3t^2 + 4 \, dt$

$I = 2\int t^6 + 3t^2 + 4 \, dt$

$I = 2\left[\frac{t^7}{7} + \frac{3t^3}{3} + 4t\right] + c$

$I = 2\left[\frac{x^{7/2}}{7} + x^{3/2} + 4\sqrt{x}\right] + c$

5) $I = \int t^7 \sin(2t^4) \, dt$

$I = \int t^4 \sin(2t^4) \times t^3 \, dt$

Put $t^4 = x$

$\therefore 4t^3 = \frac{dx}{dt}$

$\therefore t^3 \, dt = \frac{1}{4} dx$

~~$\therefore I = \frac{1}{4}\int x \cdot \sin(2x) \, dx$~~

$\therefore I = \frac{1}{4}\left[x \int \sin 2x \, dx - \int\left(\frac{dx}{dx} \int \sin 2x \, dx\right) dx\right]$

$= \frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \, dx\right]$

$= \frac{1}{16} \sin 2x - \frac{x \cdot \cos 2x}{8} + C$

$\therefore I = \frac{1}{16} \sin 2t^4 - \frac{t^4 \cdot \cos 2t^4}{8} + c$

054

$\Rightarrow I = \int \sqrt{x}\,(x^2 - 1)\,dx$

$\Rightarrow I = \int x^2 \sqrt{x}\,dx - \int \sqrt{x}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \dfrac{x^{7/2}}{7/2} - \dfrac{x^{3/2}}{3/2} + c$

$= \dfrac{2}{7} x^3\sqrt{x} - \dfrac{2}{3} x\sqrt{x} + c$

---

$\Rightarrow \int \dfrac{1}{x^3} \sin\left(\dfrac{1}{x^2}\right) dx$

$\dfrac{1}{x^2} = t$

$\therefore -\dfrac{2}{x^3} = \dfrac{dt}{dx}$

$\therefore \dfrac{dx}{x^3} = \dfrac{-1}{2}\,dt$

$I = \dfrac{-1}{2} \int \sin t\,dt$

$I = \dfrac{-1}{2} \times -\cos t + c$

$I = \dfrac{\cos t}{2} + c$

$I = \dfrac{\cos(1/x^2)}{2} + c$

---

$I = \int \dfrac{\cos x}{\sqrt{\sin x}}\,dx$

Put $\sin x = t$

$\therefore \cos x\,dx = dt$

$\therefore I = \int \dfrac{1}{t^{1/2}}\,dt$

$I = \int t^{-1/2}\,dt$

$= \dfrac{t^{-1/2+1}}{-1/2+1} + c$

$I = 2t^{1/2} + c$

$I = 2\sqrt{\sin x} + c$

---

$I = \int e^{\cos^2 x} \cdot \sin 2x\,dx$

Put $\cos^2 x = t$

$\therefore -2\cos x \cdot \sin x = \dfrac{dt}{dx}$

$\therefore \sin 2x\,dx = -dt$

$\therefore I = -\int e^t\,dt$

$\therefore I = -e^t + c$

$\therefore I = -e^{\cos^2 x} + c$

Q7) $I = \int \left( \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

$x^3 - 3x^2 + 1 = t$

$\therefore 3x^2 - 6x = \frac{dt}{dx}$

$\therefore (x^2 - 2x)\,dx = \frac{dt}{3}$

$\therefore I = \frac{1}{3}\int \frac{1}{t}\,dt$

$= \frac{1}{3}\log|t| + C$

$\therefore I = \frac{1}{3}\log|x^3 - 3x^2 + 1| + C$

Practical-6

060

Topic :- Application of Integration & Numerical Integration

Q1) Find the length of the following curve.

① $x = t\sin t,\ y = 1 - \cos t,\ t \in [0, 2\pi]$

② $y = \sqrt{4 - x^2},\ x \in [-2, 2]$.

③ $y = x^{3/2}$ in $[0, 4]$

④ $x = 3\sin t,\ y = 3\cos t\ \ t \in [0, 2\pi]$

⑤ $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$.

Q2) Using Simpson's Rule, solve the following:-

① $\int_0^2 e^{x^2}\,dx$ with $n = 4$

② $\int_0^4 x^2\,dx$ with $n = 4$.

③ $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n = 6$.

Q17 $x = 3t - \sin t,\ y = 1 - \cos t \quad [0, 2\pi]$.

Ans:- arc length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad \left(\because \sin^2 \frac{t}{2} = \frac{1-\cos t}{2}\right)$

$= \int_0^{2\pi} 2 \sin \frac{t}{2}\, dt \quad \left(\because \sin \frac{t}{2} \geq 0, \text{ when } 0 \leq t \leq 2\pi\right)$

$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi}$

$= (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

Q) $y = \sqrt{4 - x^2}$.

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$.

$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

$= \int_{-2}^{2} \sqrt{\frac{1 + x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$

$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$

$L = 2\pi$

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3) $y = x^{3/2}$ $\quad x \in [0, 4]$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$\therefore L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$

$= \frac{1}{2} \int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2} \left[ \frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9} \right]_0^4$

$= \frac{1}{27} \left[ (4+9x)^{3/2} \right]_0^4$

$= \frac{1}{27} \left[ (4+0)^{3/2} - (4+36)^{3/2} \right]$

$\frac{40\sqrt{40}}{27}$ ~~$40\sqrt{40}$~~

4) $x = 3\sin t$ $\quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\sqrt{1}\, dt$

$= 3 \int_0^{2\pi} dt$

$= 3 [t]_0^{2\pi}$

$= 3(2\pi - 0)$

$L = 6\pi$ units

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$= \frac{1}{6} y^3 + \frac{1}{2y}$.

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{1+\frac{(y^4-1)^2}{4y^2}}\, dy$.

$= \int_1^2 \sqrt{\frac{(y^4-1)^2+4 \times y^4 \times 1}{4y}}\, dy$ $\quad (a-b)^2+4ab = (a+b)^2$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$.

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$.

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$.

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$L = \frac{17}{12}$ units

$\int_0^2 e^{x^2} dx$ where $n = 4$.

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
| $y$ | 1 | 1.284 | 2.7183 | 9.4877 | 54.5982 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |

$\int_0^2 e^{x^2} dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$.

$= \frac{0.5}{3}\left[(1+54.5982)+4(1.284+9.4877)+2(2.7183)\right]$.

$= \frac{0.5}{3}\left[55.5982+43.0868+5.4366\right]$.

$\int_0^2 e^{x^2} dx = 17.3535$

ii) $\int_0^4 x^2 dx$ $n = 4$.

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$.

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 0 | 1 | 4 | 9 | 16 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |

$\int_0^4 x^2 dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$

$= \frac{1}{3}\left[(0+16) + 4(1+9) + 2\times4\right]$.

$= \frac{1}{3}[16 + 40 + 8]$.

$= \frac{64}{3}$.

$\int_0^4 x^2 dx = 21.333$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ $n = 6$.

$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 0 | $\pi/18$ | $2\pi/18$ | $3\pi/18$ | $4\pi/18$ | $5\pi/18$ | $6\pi/18 = \pi/3$ |
| y | 0 | 0.4167 | 0.584 | 0.707 | 0.801 | 0.875 | 0.93 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[(y_0) + 4(y_1+y_3+y_5) + 2(y_2+y_4) + y_6\right]$.

$= \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 2(0.584 + 0.801) + 0.584\right)$.

$\int_0^{\pi/3} \sqrt{\sin x} \approx 0.681$

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Practical- 7.

Solve the following differential equation :-

$x\frac{dy}{dx} + y = e^x$.

$x\frac{dy}{dx} + y = e^x$.

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$.

$P(x) = 1/x \quad Q(x) = \frac{e^x}{x}$.

$IF = e^{\int P(x)dx}$.

$= e^{\int 1/x \, dx}$.

$= e^{\ln x}$.

$IF = x$.

$y(IF) = \int Q(x)(IF)dx + c$.

$\therefore yx = \int \frac{e^x}{x} \cdot x \cdot dx + c$.

$= \int e^x dx + c$.

$\therefore xy = e^x + c$

---

$e^x\frac{dy}{dx} + 2e^x y = 1$.

$\frac{dy}{dx} + \frac{2e^x y}{e^x} = \frac{1}{e^x}$.

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$.

$\frac{dy}{dx} + 2y = e^{-x}$.

$P(x) = 2 \quad Q(x) = e^{-x}$.

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$.

$= e^{2x}$.

$y(IF) = \int Q(x)(IF)dx + c$.

$y.e^{2x} = \int e^{-x} e^{2x} dx + c$.

$= \int e^{-x+2x} dx + c$

$= \int e^x dx + c$

$y.e^{2x} = e^x + c$

$dx + \frac{dy}{dx} = c$

$\frac{dy}{dx} + P(x)y =$

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$e^{\ln x^2}$

$x^2$

iii) Soln:- $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2/x$. $Q(x) = \frac{\cos x}{x^2}$.

$IF = e^{\int P(x)dx}$

$= e^{\int 2/x \, dx}$

$= e^{2\ln x}$

$= e^{\ln x^2}$

$IF = x^2$

$y(IF) = \int Q(x)(IF)dx + c$

$y \cdot x^2 = \int \frac{\cos x}{x^2} \cdot x^2 \, dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Soln:-

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$.

$P(x) = 3/x$ $Q(x) = \sin x / x^3$.

$IF = e^{\int P(x)dx}$

$= e^{\int 3/x \, dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$IF = x^3$

$y(IF) = \int Q(x)(IF)\,dx + c$.

$x^3 y = \int \frac{\sin x}{x^3} x^3 \, dx + c$

$= \int \sin x \, dx + c$.

$x^3 y = -\cos x + c$.

7) Q. $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$.

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$.

$P(x) = 2$ . $Q(x) = 2x/e^{2x} = 2xe^{-2x}$.

$IF = e^{\int P(x)dx}$.

$= e^{\int 2dx}$.

$= e^{2x}$.

$y(IF) = \int Q(x)(IF)dx + c$.

$= \int 2xe^{-2x}e^{2x}dx + c$.

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$.



Q. $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \tan x\,dy$.

$\frac{\sec^2 x\,dx}{\tan x} = -\frac{\sec^2 y}{\tan y}dy$.

$\therefore \int \frac{\sec^2 x\,dx}{\tan x} = \int \frac{-\sec^2 y}{\tan y}dy$.

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\therefore \tan x \cdot \tan y = e^c$.

$\frac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

~~Differentiating~~ both sides.

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \frac{dy}{dx}$.

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$.

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\therefore \frac{dv}{\cos^2 v} = dx$

$\therefore \int \sec^2 v \, dv = \int dx$

$\therefore \tan v = x + c$

$\therefore \tan(x + y - 1) = x + c$

87 $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Put $(2x + 3y) = v$

$\therefore 2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\therefore \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\therefore \frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\therefore \frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v+2}$

$= \frac{3v + 3}{v + 2}$

$= 3\left(\frac{v+1}{v+2}\right)$

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$\therefore \int \left(\frac{v+2}{v+1}\right) dv = 3\int dx$

$\therefore \int \frac{v+2}{v+1} dv + \int \frac{1}{v+1} dv = 3x + c$

$= v + \log|v+1| = 3x + c$

$\therefore 2x + 3y + \log|2x + 3y + 1| = 3x + c$

$3y = x - \log|2x + 3y + 1| + c$

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Practical-8 — Euler's Method

Using Euler's method; find the following:-

1) $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$. find $y(2)$.

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$. Find $y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$. Find $y(1)$

4) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$ for $h = 0.5$ & $h = 0.25$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$. find $y(1.2)$ with $h = 0.2$

Soln:-



$\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

$f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 2.5 |
| 1 | 0.5 | 2.5 | 2.1487 | 3.5743 |
| 2 | 1 | 3.5743 | 4.2925 | 5.7205 |
| 3 | 1.5 | 5.7205 | 8.2021 | 9.8215 |
| 4 | 2 | 9.8215 | | |

$\therefore y(2) = 9.8215$

$\frac{dy}{dx} = 1 + y2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y_0 = 0$, $h = 0.2$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0.2 |
| 1 | 0.2 | 0.2 | 1.04 | 0.408 |
| 2 | 0.4 | 0.408 | 1.1664 | 0.6412 |
| 3 | 0.6 | 0.6412 | 1.4111 | 0.9234 |
| 4 | 0.8 | 0.9234 | 1.8526 | 1.2939 |
| 5 | 1 | 01.2939 | | |

$y(1) = 01.2939$

$y_{n+1} = 0.408 + 0.2 \times (1 + (0.4)^2)^2$

$=$

$y_{n+1} = y_n + h f(x_n, y_n)$

$f(0+0)$

$y(1.2)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$, $h=0.2$ find $y(1)=?$

$x_0 = 0$ $y(0)=1$ $h=0.2$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0.2 | 1 | 0.4472 | 1.0894 |
| 2 | 0.4 | 1.0894 | 0.6057 | 1.2105 |
| 3 | 0.6 | 1.2105 | 0.7040 | 1.3513 |
| 4 | 0.8 | 1.3513 | 0.7696 | 1.5051 |
| 5 | 1 | 1.5051 | | |

$\therefore y(1) = 1.5051$

4) $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$

$h=0.25$

$y_0 = 2$ $x_0 = 1$ $h=0.5$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 4 |
| 1 | 1.5 | 4 | 7.75 | 7.875 |
| 2 | 2 | 7.875 | | |

$y(2) = 7.875$

$y_0 = 2$ $x_0 = 1$ $h=0.25$

| $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|
| 1 | 2 | 4 | |
| 1.25 | 3 | 5.6875 | 3 |
| 1.5 | 4.4218 | 59.6569 | 4.4218 |
| 1.75 | 19.3360 | 1122.6426 | 19.3360 |
| 2 | 299.9960 | | 299.9960 |

$y(2) = 299.9960$

$\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1)=1$ $h=0.2$

$x_0 = 1$ $y_0 = 1$ $h=0.2$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 3.6 |
| 1 | 1.2 | 3.6 | | |

$y(1.2) = 3.6$

# Practical- 9

Topics:- Limits & Partial order derivatives.

Q) Evaluate the following limits.

i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3x+y^2-1}{xy+5}$

$= \lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3x+y^2-1}{xy+5}$

Apply limit,

$= \dfrac{(-4)^3-3(-4)+(-1)^2-1}{-(4)(-1)+5} = \dfrac{-64+12+1-1}{4+5} = \dfrac{-52}{9}$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$= \lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2-y^2-4x)}{x+3y}$ # Apply limit

$= \dfrac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)} = \dfrac{1(4+0-8)}{2} = \dfrac{4-8}{2} = \dfrac{-4}{2} = -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3x^2yz}$



Apply limit,

$= \dfrac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$ ∴ limit does not exist

Find $fx, fy$ for each of the foll. of

i) $f(x,y) = xy\,e^{x^2+y^2}$

$fx = y(1e^{x^2+y^2}) + xy(e^{x^2+y^2}.2x)$

$= y\,e^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$fy = x(1.e^{x^2+y^2}) + xy(e^{x^2+y^2}.2y)$

$= x.e^{x^2+y^2} + 2xy^2e^{x^2+y^2}$

$fx = ye^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$\therefore fy = xe^{x^2+y^2} + 2xy^2e^{x^2+y^2}$.

ii) $f(x,y) = e^x\cos y$

$fx = \cos y\,e^x$

$fy = e^x - \sin y$

$\therefore fy = -\sin y\,e^x$.

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$fx = y^2 3x^2 - 3y2x + 0 + 0$.

$= 3x^2y^2 - 6xy$

$fy = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$.

Q5) Using defⁿ find values of $f_x$, $f_y$ at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$.

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

According to given $(a,b) = (0,0)$.

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h-0}{h} = 2$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0.$$

$\therefore f_x = 2,\ f_y = 0.$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h-0}{h} = 2$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0.$$

$\therefore f_x = 2 \quad f_y = 0$

Q) Find all second order partial derivatives of $f(x,y)$. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f_{xx} = \frac{d^2f}{dx^2} \qquad f_{yy} = \frac{d^2f}{dy^2}$

Applying $\frac{u}{v}$ rule,

$$f(x) = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$$

$$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$$

$$\therefore f_x = \frac{x^2y - 2xy^2}{x^4}$$

$$\therefore f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2y - 2xy^2)(4x^3)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$$

$$= \frac{-2x^5y + 6x^4y^2}{x^8}$$

$$f_{xx} = \frac{6y^2 - 2xy}{x^4}$$

$$f_y = \frac{1}{x^2}(2y - x) \quad \therefore f_y = \frac{(2y-x)}{x^2}$$

$$\therefore f_{yy} = \frac{1}{x^2} 2 = \frac{2}{x^2}$$

$$\therefore f_{xy} = \frac{2y-x}{x^2}$$

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$$= \frac{x^2(-1) - (2y-x)(2x)}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$\therefore f_{xy} = \frac{x-4y}{x^3}$$

Q2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f_x = \frac{d}{dx}\, x^3 + 3x^2y^2 - \log(x^2+1)$$

$$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{d}{dy}\, x^3 + 3x^2y^2 - \log(x^2+1)$$

$$= 0 + 6x^2y - 0$$

$$f_y = 6x^2y$$

$$f_{xx} = \frac{d}{dx} f_x$$

$$= \frac{d}{dx}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$f_{xx} = 6x + 6y^2 - \frac{4x - 2x^2 + 2}{(x^2+1)^2}$$

$$f_{yy} = \frac{d}{dy} f_y$$

$$= \frac{d}{dy}\, 6x^2y.$$

$$f_{yy} = 6x^2$$

$$f_{xy} = \frac{d}{dy} f_x$$

$$= \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$f_{xy} = 12xy$$

$$f_{yx} = \frac{d}{dx} f_y$$

$$= \frac{d}{dx}\, 6x^2y.$$

$$f_{xy} = 12xy$$

$f_{xy} = f_{yx}$.

$$\cancel{f_{xy} = \frac{d}{dy} f_x}$$

$$\cancel{= \frac{d}{dy}\, y \cos(xy) + e^{x} \cdot e^{x}}$$

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Q7 Soln: -

$f(x,y) = \sin(xy) + e^{x+y}$.

$f(x,y) = \sin(xy) + e^x \cdot e^y$.

$f_x = \frac{d}{dx}(\sin(xy) + e^x e^y)$

$f_x = y\cos(xy) + e^x \cdot e^y$.

$f_y = \frac{d}{dy}(\sin(xy) + e^x \cdot e^y)$.

$= (x\cos(xy) + e^x \cdot e^y)$.

$f_{xx} = \frac{d}{dx} f_x$

$= \frac{d}{dx} y\cos(xy) + e^x \cdot e^y$.

$f_{xx} = -y^2 \sin(xy) + e^{x+y}$.

$f_{yy} = \frac{d}{dy} f_y$

$= \frac{d}{dy}(x\cos(xy) + e^x \cdot e^y)$

$f_{yy} = -x^2 \sin^2(xy) + e^x \cdot e^y$

Q5]

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$.

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$.

iii) $f(x,y) = \log x + \log y$ at $(1,1)$.

i) Soln:-

$f(x,y) = \sqrt{x^2+y^2}$

$(a,b) = (1,1)$.

$f(1,1) = \sqrt{1+1} = \sqrt{2}$

$\therefore f_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$

$\therefore f_x = \frac{x}{\sqrt{x^2+y^2}}$

$\therefore f_x(1,1) = \frac{1}{\sqrt{2}}$

$f_y = \frac{2y}{2\sqrt{x^2+y^2}}$

$\therefore f_y = \frac{y}{\sqrt{x^2+y^2}}$

$\therefore f_y(1,1) = \frac{1}{\sqrt{2}}$.

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{2 + x + y - 2}{\sqrt{2}}$

$L(x,y) = \frac{x+y}{\sqrt{2}}$

Q2) Soln :-

$(a,b) = (\pi/2, 0)$

$f(x,y) = 1 - x + y\sin x$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 + \sin \pi/2$

$f(\pi/2, 0) = \frac{2-\pi}{2}$

$f_x = -1 + y\cos x \qquad f_y = \sin x$

$f_x(\pi/2, 0) = -1 + 0 \times \cos\pi/2 \qquad f_y(\pi/2, 0) = \sin \pi/2$

$f_x(\pi/2, 0) = -1 \qquad f_y(\pi/2, 0) = 1.$

$L(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2) + f_y(\pi/2, 0)(y-0).$

$\therefore L(x,y) = \frac{2-\pi}{2} + (-1)(x - \pi/2) + 1(y)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y.$

$L(x,y) = 1 - x + y.$

Q3) Soln :-

$f(x,y) = \log x + \log y. \qquad (a,b) = (1,1)$

$f(1,1) = \log 1 + \log 1$

$f(1,1) = 0$

$f_x = \frac{1}{x} \qquad f_y = \frac{1}{y}$

$f_x(1,1) = 1 \qquad f_y(1,1) = 1$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + 1(y-1)$

$L(x,y) = \underline{x + y - 2}.$

AV 22/01/2

Practical-10 :



Aim :- Directional derivatives, gradient vector & maxima, minima & Tangent & normal vectors.

Find the directional derivative of the following function at given points & in the direction of given vector.

$f(x,y) = x + 2y - 3 \qquad a = (1,-1) \quad u = 3i - j$

Here, $u = (3i - j)$

$|u| = \sqrt{9+1} = \sqrt{10}$

$\therefore$ unit vector along $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right) - 3$

$= f(1,-1) + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) - 3$

$= f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3h}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h} \qquad f(a) = f(1,-1)$

$= 1 - 2 - 3$

$= 1 - 5 = -4.$

$= \lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$

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iii) $f(x,y) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

then, $v = (1,5)$

$|v| = \sqrt{1+25} = \sqrt{26}$

$= \frac{v}{|v|} = \frac{1}{\sqrt{26}}(1,5)$

$f(a) = (4)^2 - 4(3) + 1$

$= 16 - 12 + 1$

$= 4 + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

iv) $f(x,y) = 2x + 3y \quad a = (1,2) \quad u = 3i + 4j$

$v = (3,4)$

$|u| = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \frac{3}{5}, \frac{4}{5}$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$

$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = 8 + \frac{18h}{5}$

$f(a) = 2(1) + 3(2) = 2 + 6 = 8$

$\frac{f(a+hu) - f(a)}{h} = \frac{8 + \frac{18h}{5} - 8}{h} = 18/5$

Q) Find gradient vector for the following function a given point.

i) $f(x,y) = x^y + y^x, \quad a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (y \cdot x^{y-1} + y^x \log y,\ x^y(\log x + x y^{x-1}))$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$.

$f(x,y) = (\tan^{-1} x) \cdot y^2$ & $a = (1,-1)$

$f_x = y^2 \left(\frac{1}{1+x^2}\right)$

$f_y = \tan^{-1} x \cdot 2y$

$\nabla f(x,y) = \left(\frac{y^2}{1+x^2}, \tan^{-1} x \, 2y\right)$

$\nabla f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4} \cdot (-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

$f(x,y) = xyz - e^{x+y+z}$ at $(1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = \left(yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}\right)$

$\nabla f(1,-1,0) = \left(1(0) - e^{1-1+0}, (1)(0) - e^{1-1+0}, (1)(-1) - e^{1-1+0}\right)$

$= (0-1, 0-1, (-1)-1)$

$= (-1, -1, -2)$

Find the equation of tangent & normal to each of the following curve at given point.

$x \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y + e^{xy} y$

$f_y = x(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1, y_0 = 0$.

Eqn of tangent

$f_x (x - x_0) + f_y (y - y_0) = 0$

$f_x (x_0, y_0) = \cos 0 + e^0 (0)$

$= 1 + 0$

$= 1$

$f_y (x_0, y_0) = (1)(-\sin 0) + e^0 (1)$

$= 0 + 1 \cdot 1$

$= 1$.

$1(x-1) + 1(y-0) = 0$

$x - 1 + y = 0$

$x + y - 1 = 0$ $\therefore$ It is the required eqn of tangent.

Eqn of normal.

$= ax + by + c = 0$

$= bx - ay + d = 0$

$1(x) - 1(y) + d = 0$

$= x - y + d = 0$ at $(1,0)$

$= 1 - (0) + d = 0$

$d + 1 = 0$ $d = -1$

.in

$x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f(x) = 2x+0-2-2+0+0$

$= 2x-2$

$fy = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$f(x)(x_0, y_0) = 2(2)-2 = 2$

$f(y)(x_0, y_0) = 2(-2)+3 = -1$

eqn of tangent

$f(x)(x-x_0) + fy(y-y_0) = 0$

$2(x-2)(-1(y+2)) = 0$

$2x-2-y-2=0$

$2x-y-4=0$ It is required eqn.

eqn of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$= (-1)(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$.

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore \underline{d=6}$

Find the eqn of tangent at & normal line to each of the following surface.

078

$x^2-2yz+3y+xz=7$ at $(2,1,0)$.

$fx = 2x-0+0+z$

$fx = 2x+z$

$fy = 0-2z+3+0$

$= 2z+3$

$fz = 0-2y+0+x$

$= -2y+x$

$(x_0, y_0, z_0) = (2,1,0)$.

$\therefore x_0 = 2, y_0 = 1, z_0 = 0$.

$fx(x_0, y_0, z_0) = 2(2)+0 = 4$

$fy(x_0, y_0, z_0) = 2(0)+3 = 3$

$fz(x_0, y_0, z_0) = -2(1)+2 = 0$.

eqn of tangent

$fx(x_0-x_0) + fy(y_0-y_0) + fz(z_0-z_0) = 0$

$= 4(x-2)+3(y-1)+0(z-0) = 0$

$= 4x-8+3y-3=0$

$= 4x+3y-11=0$ → reqd. eqn of tangent.

eqn of normal at $(4,3,-11)$.

$\frac{x-x_0}{fx} = \frac{y-y_0}{fy} = \frac{z-z_0}{fz}$

$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$.

iii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x + y + z + 4 = 0$ at $(1,-1,2)$.

$f_x : 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y : 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$.

$(x_0, y_0, z_0) = (1, -1, 2)$

$\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) + 1 = 7$
$f_y(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$
$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Eq$^n$ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$
$-7x + 7 + 5y + 5 - 2z + 4 = 0$
$-7x + 5y - 2z + 16 = 0$
$\hookrightarrow$ eq$^n$ of tangent required.

Eq$^n$ of normal

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$



Find the local maxima & minima for the follow

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$.
$f_x = 6x + 0 - 3y + 6 + 0$
$= 6x - 3y + 6$
$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$.

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ — (1)

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ (2)

Multiply eqn (1) with 2
$4x - 2y = -4$
$2y - 3x = 4$

$\boxed{x = 0}$.

Substitute value of $x$ in eq$^n$ (1)
$2(0) - y = -2$
$-y = -2$
$\therefore y = 2$

$\therefore$ critical points are $(0,2)$
$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$
Here $r > 0$.
$\therefore rt - s^2$

$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$
$\therefore f$ has maximum at $(0,2)$
$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$\therefore 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$= 0 + 4 - 0 + 0 - 8 = \underline{-4}$

ii) $f(x,y) = 2x^4 + 3x^2 y - y^2$
$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$
$f_x = 0$
$\therefore 8x^3 + 6xy = 0$
$\therefore 2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0$ — (1)
$f_y = 0$
$3x^2 - 2y = 0$ — (2).
Multiply $eq^n$ (1) with 3 & $eq^n$ (2) with 4.
$12x^2 + 9y = 0$
$- 12x^2 - 8y = 0$.
$\boxed{y = 0}$.

Substitute value of $y$ in $eq^n$ (1)
$4x^2 + 3(0) = 0$
$4x^2 = 0$
$\boxed{x = 0}$

critical point is $(0,0)$.
$r = f_{xx} = 24x^2 + 6y$
$t = f_{yy} = 0 - 2 = -2$



$s = f_{xy} = 6x - 0 = 6x = (6)(0) = 0$.
$r$ at $(0,0)$
$= 24(0) + 6(0) = 0$
$\therefore r = 0$.
$rt - s^2 = 0(-2) - (0)^2$
$0 - 0 = 0$
$r = 0$ & $rt - s^2 = 0$.
$f(x,y)$ at $(0,0)$ (nothing to say).
$2(0)^4 + 3(0)^2(0) - 0$
$0 + 0 - 0 = 0$.

$f(x,y) = x^2 - y^2 + 2x + 8y - 70 = 0$.
$f_x = 2x + 2$
$f_y = -2y + 8$
$f_x = 0$
$\therefore 2x + 2 = 0$
$x = \frac{-2}{2} = -1$.

$f_y = 0$ $\quad -2y + 8 = 0$.
$y = \frac{-8}{-2} = +4$

critical point is $(-1, 4)$.

$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$.
$r > 0$
$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0$.

$f_{xy}$ at $(-1, 4)$
$(-1)^2 - (4)^2 + 2(-1) + 8(-4) - 70$
$1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 37 - 70$
$= \underline{-33}$