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3 (i) Given, matrix $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

characteristic polynomial =

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

now, det $\begin{vmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = 0$

$$-\lambda(3-\lambda) - (-1)(2) = 0$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$
$$\lambda(\lambda-2) - 1(\lambda-2) = 0$$
$$\boxed{\lambda = 1, 2}$$

when $\lambda = 1$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$-x - y = 0$ $\qquad$ $-x = +y$

$2x + 2y = 0$ $\qquad$ $x = -y$

$\therefore$ the eigen vectors are the vectors with x and y component with equal magnitude but eigen vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ for $\lambda = 1$

when $\lambda = 2$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-2x - y = 0$$
$$2x + y = 0$$

$$\Rightarrow 4x + 2y = 0$$
$$\Rightarrow y = -2x.$$

$\therefore$, the corresponding eigen vector are the vector with ~~y~~ -component being -2 times the x component

$$= \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$ for $x = 1$

(2) file: eigen_value_vector_800990636.py

obtained eigen value $= \boxed{1, 2}$ $\Leftarrow$ same

obtained eigen vector $= \begin{bmatrix} -0.70710678 \\ 0.7070678 \end{bmatrix}$ and $\begin{bmatrix} 0.447213 6 \\ -0.89442719 \end{bmatrix}$

The vectors are different as we can have countably infinite vectors such that for $\lambda = 1$, $x = -y$ holds and for $\lambda = 2$, $y = -2x$ holds.