N^{th} Element of Fibonacci Series using Matrix Exponentiation

Fibonacci series,

$F(n) = F(n-1) + F(n-2)$, for $n > 2$

$F(1) = 1$

$F(2) = 1$

Transition matrix

$$[F1 \quad F2] \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} = [F2 \quad F3] \longrightarrow ①$$

$$[F2 \quad F3] \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} = [F_3 \quad F_4] \longrightarrow ②$$

From ① & ②

$$[F1 \quad F2] \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}^2 = [F_3 \quad F_4]$$

Deriving the transition matrix:

$$[F1 \quad F2] \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} = [F2 \quad F3]$$

$A F1 + C F2 = F2$ $(A=0, C=1)$

$B F1 + D F2 = F3$ $(B=1, D=1) \Rightarrow$ given $F1+F2=F3$

$\therefore$ Transition matrix $= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

Generalising,

$$[F1 \quad F2] \times \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{n-1} = [F_n \quad F_{n+1}]$$

can be done in $O(\log n)$