[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3037

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Unique Paper Code : 2272101102

Name of the Paper : Introductory Mathematical Methods for Economics

Name of the Course : B.A. (Hons.) Economics – DSC-2

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Use of simple calculator is allowed.
4. All parts of a question must be answered together.
5. PwD marked questions are alternatives to be attempted only by PwD students.

1. Answer any **two** of the following : (2×4=8)

   (a) (i) Find all values of x satisfying $(|x| - 2)(x + 5) < 0$.

   (ii) If A and B are two sets containing 4 and 7 elements respectively, find the maximum and minimum number of elements in $A \cup B$.

   (b) Fill in the blank with necessary, sufficient or necessary and sufficient:

   (i) If A is a sufficient condition for B, then ~B is ________ condition for ~A.

   (ii) For a rectangle to be considered a square, having four sides of equal length is ________ condition.

   (iii) $x > 0$ is ________ for $x(x + 4) > 0$.

   (iv) For two sets X and Y, $X \cup Y = X$ is ________ condition for Y to be a subset of X.

   (c) Graph $f(x) = |x^2 - x - 6|$.

PwD(c) Suppose the consumers of a product demand 60 units of a product when the price is ₹ 5 per unit and 40 units when the price per unit has gone up by ₹ 4.

   (i) Find the demand equation for the product, assuming that it is linear.

(ii) Express total revenue as a function of price and find the price for which total revenue is maximum.

2. Answer any **four** of the following : (4×4=16)

(a) The value of a new car depreciates (decreases) after it is purchased, according to an exponential decay model. Suppose that the value of the car is ₹ 12000 at the end of 5 years and that its value has been decreasing at the rate of 9% per year. Find the value of the car when it was new. Find t when the value of the car reduces to half of its value when it was new.

(b) A country exports three goods, wheat W, coal C and palm oil *0*. At time $t = t_0$, the revenue in crores of rupees derived from each of these goods is $W(t_0) = 4$, $C(t_0) = 10$ and $0(t_0) = 7$. W is declining at 3% while *0* and C are growing at 15% and 8% respectively. Find the rate of growth of total export earnings at $t = t_0$.

(c) Examine the inverse demand curve $p = \frac{20}{x+1}$.

Show that the demand increases from 0 to indefinitely large amounts as price falls. Find total revenue and show that it increases to a limiting value.

(d) Consider an infinite series $\sum_{i=1}^{\infty} a_i$. Prove that $\lim_{n\to\infty} a_n = 0$ is necessary for convergence for the series, but not sufficient.

(e) If $f(x) = \frac{x^n}{e^x}$, show that f(x) decreases for $x \geq n > 0$ and find the local maximum value of f(x). Find f(2x) and show that $\frac{2^n x^n}{e^x e^x} \leq \frac{2^n n^n}{e^n e^x}$.

3. Answer any **three** of the following : (4×3=12)

(a) Using Mean Value theorem, prove the inequality, $e^x \geq 1 + x$ for all $x \in R$.

(b) (i) The time in minutes, t, required for a rat to run through a maze depends on the number of trials, n, that the rat has practiced.

$$t(n) = \frac{3n+15}{n+1},\ n \geq 1$$

How does the change in n impact the change in t? Does there appear to be a limiting time in which the rat can complete the maze? How many trials are required so that the rat is able to finish the maze in under 5 minutes?

(ii) National income in two economies X and Y is growing exponentially at $100r_x\%$ and

$100r_y\%$ respectively (compounded continuously), where $r_x > r_y$. In year zero, national income was N_x^0 in economy X and N_y^0 in economy Y. If $N_x^0 < N_y^0$, at what time will the national income become equal in both the economies?

(c) Use Newton's binomial formula to find the approximate value of $\sqrt{217}$, taking the degree of approximation as 2. Also find the upper bound on the absolute error.

(d) An investment project incurs an initial loss of C_0. Thereafter it does not incur any losses and the sum of later profits is greater than the initial loss. Show that the project has a unique positive internal rate of return.

4. Answer any **three** of the following : (4×3=12)

(a) If f is a continuous function on the interval [0,1] with $f(0) > 0$ and $f(1) < 1$, then there is some number $c \in (0,1)$ which satisfies $f(c) = c$.

(b) Suppose that the value of wine W(t) is given as the following function of time: $W(t) = 1000.e^{\sqrt{\frac{t}{4}}}$ in crores of rupees (t = 0 denotes the present). At an interest rate 10% compounded continuously and assuming zero storage costs, what is the optimal time to sell the wine? Interpret the first order condition.

(c) Let $f(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, does the function f have an inverse function g? If yes, find the inverse and $g'\left(\frac{1}{2}\ln 3\right)$.

(d) Given f and g are not differentiable functions, show g(x) = f(ax + b) (where a and b are real numbers) is convex if f is convex.

5. Answer any **three** of the following : (3×5=15)

(a) The graph of the equation $x^2y - 3y^3 = 2x$ passes through the point (x, y) = (−1,1). Find the slope of the graph at this point. Find the points where function is not differentiable. Does the curve have horizontal tangent?

(b) (i) Find the limit: $\lim_{x\to\infty}\left(\frac{2+3x^m}{1-x^n}\right)$ m, n ∈ N. (2)

(ii) Let $f(x)=\frac{\log\left(1+\frac{x}{p}\right)-\log\left(1-\frac{x}{q}\right)}{x}$, where p and q are positive constants. Can you define f(x) at x = 0 so as to make the function continuous at x = 0? (3)

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(c) (i) Consider two cash flows, A and B. For cash flow A, you receive ₹ 10 every year for 5 years with the first payment being a year from now. For cash flow B, you receive ₹ x every year forever with the first payment being today. What is the value of x so that cash flow B has the same present value as cash flow A, given that the rate of interest is 6% per annum (compounded annually)? (3)

(ii) If f(x) and g(x) are differentiable functions of x, express the elasticity of $h(x) = e^{f(x)g(x)}$ w.r.t x in terms of $E_x f$ and $E_x g$ which are the elasticities of f(x) and g(x) w.r.t x respectively. (2)

(d) $Q^d = f(P + t)$ and $Q^s = g(P)$ where f and g are differentiable functions with $f' < 0$ and $g' > 0$. Use the equilibrium condition $Q^d = Q^s$ to find an expression for $\frac{dP}{dt}$. Also comment on its sign. Find the expression for $\frac{d(P+t)}{dt}$ and find its range.

6. Answer any two of the following : (6×2=12)

(a) The monopolist with the cost function $C(x) = \frac{1}{2}x^2$, with quantity x, faces a demand curve $x = 12 - p$, where p is the price.

(i) Find equilibrium price and quantity.

(ii) What would be the equilibrium price and quantity if the monopolist is forced to take the price as given as under perfect competition? Compare the profits under monopoly and perfect competition.

(iii) To ensure that the monopolist acts like a perfectly competitive firm, a specific tax of t per unit is imposed on him. Find the equilibrium output, t and show that it is actually negative. What does it imply?

(b) (i) Let the function $f(x)=\left(6-x^2\right)\sqrt{x^2-4}$ be defined over [-6, -2], Find the extreme points of f.

(ii) Determine the concavity/convexity of the following function $f(x) = (e^{2x} + 4e^{-x})^2$.

(c) Let $f(x) = x - 2\ln(x + 1)$

(i) Determine where f(x) is increasing/decreasing.

(ii) Find possible extreme points and inflexion points. Does the function have global maximum/minimum point(s)?

(iii) Sketch the graph of f(x).

PwD (iv) Determine the intervals of concavity/convexity of the function $g(x) = x^4 - 12x^2$.