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Your Roll No.....

Sr. No. of Question Paper : 3924

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Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : **Generic Elective / Other than B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

- Write your Roll No. on the top immediately on receipt of this question paper.
- All the **six** questions are compulsory.
- Attempt any **two** parts from each question.

- (a) Solve the following differential equations :

$$(4x + 3y + 1)dx + (x + y + 1)dy = 0, \quad y(3) = -4. \quad (6.5)$$

- (b) Solve the Initial Value problem :

$$x^2 \frac{dy}{dx} + xy = \frac{y^3}{x}, \quad y(1) = 1 \quad (6.5)$$

P.T.O.

- (c) Find the orthogonal trajectories of the family of ellipse having centre at the origin, a focus at the point (c,0) and semimajor axis of length 2c.

(6.5)

- (a) (i) Find the Wronskian of the set $\{1 - x, 1 + x, 1 - 3x\}$ and hence find their linear independence or dependence on $(-\infty, \infty)$. (6)

- (ii) Solve the differential equation :

$$\frac{dy}{dx} = \frac{2 + ye^{xy}}{2y - xe^{xy}}.$$

- (b) Given that $y = x$ is a solution of the differential equation

$$(x^2 - x + 1) \frac{d^2y}{dx^2} - (x^2 + x) \frac{dy}{dx} + (x + 1)y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

- (c) Solve the following differential equations :

$$(i) \quad x \frac{dy}{dx} - 2y = 2x^4, \quad y(2) = 8$$

$$(ii) \quad (2xy^2 - y)dx + (2y^3 - x)dy = 0. \quad (6)$$

- (a) Solve the Initial value Problem

$$\frac{d^3y}{dx^3} + 3 \frac{d^2y}{dx^2} - 10 \frac{dy}{dx} = 0, \quad y(0) = 7, \quad \frac{dy}{dx}(0) = 0,$$

$$\frac{d^2y}{dx^2}(0) = 70. \quad (6.5)$$

- (b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8\sin(3x). \quad (6.5)$$

- (c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec}(x). \quad (6.5)$$

4. (a) Given that $\sin x$ is a solution of the differential equation

$$\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$

find the general solution. (6)

- (b) Find the general solution of the differential equation by assuming $x > 0$,

$$x^3 \frac{d^3y}{dx^3} - 4x^2 \frac{d^2y}{dx^2} + 8x \frac{dy}{dx} - 8y = 0 \quad (6)$$

- (c) Find the general solution of the given linear system

$$2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}, \quad \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^t. \quad (6)$$

P.T.O.

5. (a) Find the solution of the linear partial differential equation

$$(y-u)u_x + (u-x)u_y = x-y, \text{ with Cauchy data } u=0 \text{ on } xy=1. \quad (6)$$

- (b) Find the solution of the following partial differential equation by the method of separation of variables

$$u_x + u = u_y, \quad u(x, 0) = 4\exp(-3y). \quad (6)$$

- (c) Reduce the equation to canonical form and obtain the general solution

$$u_x + 2xy u_y = x. \quad (6)$$

6. (a) Find the general solution of the linear partial differential equation

$$yz u_x + xz u_y + xy(x^2 + y^2) u_z = 0. \quad (6.5)$$

- (b) Reduce the equation

$$u_{xx} + \frac{1}{c^2} u_{yy} = 0, \quad c \neq 0 \text{ where } c \text{ is a constant,}$$

into canonical form and hence find the general solution. (6.5)

- (c) Reduce the following partial differential equation with constant coefficients,

$$3 u_{xx} + 10 u_{xy} + 3 u_{yy} = 0$$

into canonical form and hence find the general solution. (6.5)

(1000)