[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1963 C

Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : Generic Elective / Other than B.Sc. (H) Mathematics

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve the following differential equations : (6.5)

(i) $\frac{dy}{dx} = y\tan x + x^2\cos x$

(ii) $(x^2 - ay)dx + (y^2 - ax)dy = 0$.

P.T.O.

(b) Solve the Initial Value problem .

$$x\frac{dy}{dx} - 3y = x^5 y^{1/3}, \quad y(0) = 1 .$$

Or

Solve the following differential equations :

$$\sin y\frac{dy}{dx} - 2\cos x\cos y = -\cos x\sin^2 x . \qquad (6.5)$$

(c) Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at angle 60 degrees. (6.5)

2. (a) Consider the differential equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0 .$$

Show that x^2 and $1/x^2$ are linearly independent solutions of this equation on the interval $0 < x < \infty$. Write the general solution of the given equation. Hence find the particular solution satisfying the initial conditions $y(2) = 3,\ \frac{dy}{dx}(2) = -1$. (6)

) Find the suitable integrating factor for the differential equation

$(x^2 + y^2 + x)dx + xydy = 0$ and hence solve it. (6)

c) Given that $y = e^{2x}$ is a solution of the differential equation

$$(2x-1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

(a) Solve the Initial value Problem

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0, \quad y(0) = 1, \quad \frac{dy}{dx}(0) = 5. \qquad (6.5)$$

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 8\left(xe^{-2x}\right) \qquad (6.5)$$

P.T.O.

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}. \qquad (6.5)$$

(a) Given that $e^x \sin 2x$ is a solution of the differential equation

$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 30y = 0$$

find the general solution. (6)

(b) Find the general solution of the differential equation by assuming $x > 0$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4\sin(\log x) \qquad (6)$$

(c) Find the general solution of the given linear system

$$\frac{dx}{dt} = 2x + 5y, \quad \frac{dy}{dt} = 5x + 12.5y \qquad (6)$$

5. (a) Find the solution to the linear partial differential equation

$$x u_x + y u_y = u + 1, \; u(x,y) = x^2 \text{ on } y = x^2.$$ (6.5)

(b) Find the solution of the following partial differential equation by the method of separation of variables

$$u_x + 2u_y = 0, \quad u(0,y) = 3\exp(-2y).$$ (6.5)

(c) Reduce the equation

$$u_x - y u_y - u = 1,$$

into canonical form and obtain the general solution. (6.5)

6. (a) Find the general solution of the differential equation by reducing it into the canonical form

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2 .$$ (6)

(b) Reduce the equation

$$u_{xx} + 4 u_{xy} + 4 u_{yy} = 0$$

into canonical form and hence find its general solution. (6)

(c) (i) Find the partial differential equation by eliminating the arbitrary function f and g from the following equation

$$z = f(x + ay) + g(x - ay).$$

(ii) Find the partial differential equation by eliminating the arbitrary function f from the following equation

$$yz + zx + xy = f\left(\frac{z}{x+y}\right)$$ (6)

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