[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1216 **F**

Unique Paper Code : 2342011202

Name of the Paper : Discrete Mathematical Structures

Name of the Course : **B.Sc. (Hons.) Computer Science (NEP-UGCF-2022)**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1 (Section–A)** is compulsory.
3. Attempt any **four** questions from **Section–B**.
4. Parts of a question should be attempted together.
5. Use of simple calculator is allowed.

P.T.O.

SECTION A

1. (a) **Determine whether the following function is one-to-one and onto from $\mathbb{R}^+$ to $\mathbb{R}^-$**

$$f(x) = -3x^2 + 7$$

Also, check whether it is invertible. If invertible, find its inverse. Justify your answer in each case. (5)

(b) **Show that $\neg(p \vee (\neg p \wedge q)$ and $(\neg p \wedge \neg q)$ are logically equivalent by developing a series of logical equivalences.** (5)

(c) **Evaluate 7^{644} mod 645 using Fast Modular exponentiation algorithm.** (5)

(d) **Prove that if any 14 numbers from 1 to 25 are chosen then one of them will be the multiple of another.** (5)

(e) State whether the K_5 graph is/has a

(i) Tree

(ii) Euler Path

(iii) Euler circuit

Justify your answer. (5)

(f) Let a be a numeric function such that (5)

$$a_r = \begin{cases} 2 & 0 \le r \le 3 \\ 2^{-r} + 5 & r \ge 4 \end{cases}$$

(i) Determine S^2a.

(ii) Determine Va.

SECTION B

2. (a) Prove that the relation "congruence modulo m" over the set of positive integers is an equivalence relation. (7)

(b) If no three diagonals of a convex decagon meet at the same point inside the decagon, into how many line segments are the diagonals divided by their intersections? (8)

3. (a) Prove the following statement using the Direct Proof method :

If m and n both are perfect squares, then m * n is also a perfect square. (7)

(b) Using the principle of mathematical induction, prove that

$1.2.3 + 2.3.4 + \cdots + n.(n+1).(n+2) = n(n+1)(n+2)/3$ (8)

4. (a) Using the Euclidean algorithm, find the GCD of 1529 and 14039. (7)

(b) The interest for money deposited in a saving bank account is paid at a rate of 0.5% per month, with interest compounded monthly. \$50 is deposited in the saving account each month for a period of 3 years, followed by \$20 each month for next 2 years. What is the total amount in the account

(i) 4 years after the first deposit?

(ii) 20 years after the first deposit?

Formulate the numeric functions for each. (8)

5. (a) Prove that a tree with n vertices has n – 1 edges. (7)

P.T.O.

(b) For the following numeric functions : (8)

$$a_r = 2^r \text{ for all } r$$

$$b_r = \begin{cases} 0 & 0 \le r \le 2 \\ 2^r & r \ge 3 \end{cases}$$

Determine a * b in either sketch or closed form expression.

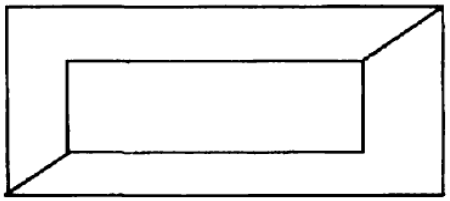
6. (a) In how many ways can a cricket team of eleven be chosen out of a batch of 14 players? How many of them will:

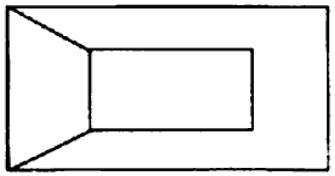
(i) include a particular player?

(ii) exclude a particular player? (7)

(b) Define graph isomorphism. Check whether the

following pair of graphs are isomorphic. Give justification in support of your answer. (8)





7. (a) Is Q_3 a planar graph? If planar, draw it in such a form. Verily your result using Euler formula also. (7)

(b) Draw Hasse Diagram for the relation R on A = {1, 2, 3, 4, 5}, whose relation matrix is given below

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Is it a totally ordered set? Justify your answer. (8)