Adaptive Auctions in Advertising
CS 590 - Final Project

Nitya Raviprakash, Vaishali Jain, Venetia Wong,
Eduardo Schiappa-Pietra, Jia Rong Chua

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Contents

1 Background

1.1 Motivation

Digital advertising and, in particular, search engine advertising, has become one of the most important sources of revenue for tech companies such as Google and Yahoo.

As stated by Pardoe et al. (2010), *"different auction designs can lead to widely differing outcomes"* and hence a key challenge for auctioneers operating in the search engine advertising markets is to identify the 'best design' for revenue maximization.

Moreover, given the availability of large amounts of data in these digital markets and the sequential nature of auctions for keywords, this setting is very suitable for the application of a learning algorithm that allows the auctioneers to iteratively change their estimation of expected revenue by experimenting with different designs as an episode (particular sequence of auctions) progresses.

1.2 How do search-ads auctions work?

1. **Keywords**: An advertiser chooses a set of keywords related to a product / service that she is selling.

2. **Bidding**: She places a bid for each keyword according to her intrinsic valuation, the auction characteristics and her believes about the strategic actions of the other bidders.

3. **Display**: Ads are ranked by their bids. When a user's search query matches one keyword, the set of k ads with the highest bids is displayed. Ads with higher bids have a better placement in the user's screen.

4. **Payment**: The second-price auction is the most widely used auction type in search-advertising auctions. If user clicks on an ad, that advertiser is charged the bid (or a function of it) of the advertiser that is immediately below it in the ranking. VCG is also used by some companies such as Facebook.

1.3 General Setting

The goal is to obtain a sequence of auction designs that maximizes the auctioneer's revenue over the course of an episode (a specific sequence of auctions). Our adaptive approach is based on a data-driven learning process in which the seller adjusts the parameters of the auctions (i.e. The reserve price, the number of slots for the ads, etc) as the episode progresses and she observes the bidders' behavior. We define a design as a unique combination of auction parameters. In addition to varying some parameter values, we also implement the algorithm with two different auction types: GSP and VCG. We then compare the results of the adaptive mechanism with the ones of a traditional fixed mechanism.

1.4 Hypothesis

Overall, we expect the GSP auction to perform at least as well as the VCG auction on average, in line with Edelman et al (2007). Further, we believe that an adaptive auction would choose a design that consistently generates high revenues. This view is consistent with Pardoe et al (2010), who found that the adaptive auction mechanism generates greater revenue for the auctioneer with respect to a fixed auction mechanism.

2 Data

We obtained data on advertiser bids from Yahoo! Search Marketing https://webscope.sandbox.yahoo.com/catalog.php?datatype=a . The data consists of bids on the 1,000 most popular advertising key phrases between June 2002 to June 2003. These bids were collected when they operated on a first price auction.

2.1 Description

Each observation in the dataset corresponds to a bid placed by an advertiser for a specific advertising phrase. For each bid, we have the following information: *Timestamp*, which specifies the time at which the bid was placed, aggregated into 15 minute intervals; *Phrase ID*, which is a unique identifier for the key phrase; *Account ID*, a unique identifier for each bidder account; *Price*, the value of bid placed by the bidder, which is denoted in USD, and *Autobid*, a binary variable indicating whether or not the bid was placed using Yahoo!'s autobbider system, or if the bid was placed manually.

In order to study different auction designs, we define an auction as a *Timestamp, Phrase ID* pair. Intuitively, this means that an auction for a specific key phrase lasts for 15 minutes. While the Yahoo! dataset comprised 6,918,838 auctions distributed across the 1,000 phrases, we are only interested in a small subset of these auctions. To do so, we imposed two criteria on the auctions. First, since we study both the VCG and GSP, we are interested in auctions that have at least 3 bidders, and doing so will generate more variation between the two auction designs. Second, we are only interested in auctions where all the bidders used the autobidding system. After all, the autobidding system guarantees that bidders in the auction were attentive. That is, if a bidder was outbid and the prevailing price was still below his maximum bid, the autobidder system would have updated his bid immediately. Therefore, we know that if a bidder did not win the bid, it is not because the bidder failed to monitor the bid. Overall, this leaves us with 87,309 candidate auctions distributed over 731 phrases.

2.2 Assumptions

Before we proceed with our analysis, we make assumptions on both the bidders' information and the bidders' action. First, we assume that the bidders do not know Yahoo!'s reserve price. In other words, bidders simply place their bids as a function of their true valuation, and if all the bids fall below Yahoo!'s reserve price, Yahoo! will simply not choose to offer the phrase. Second, bidders do not know how bidders will win the auction. Intuitively, we can think of this as there being a number of ads slots on a page, and the winner who placed the highest bid will get the 'best' slot, and the winner with the second highest bid will get the 'next best' slot, and so on. In such a scenario, bidders only bid for a specific slot, and accept the slot they are assigned, if any. Third, bidders do not collude, and bids are independent. Finally, bidders do not change their bid in response to auction designs.

3 Auctions Mechanisms and the Multi-Armed bandit algorithm

3.1 The Adaptive Algorithm

Approach:
The algorithm used to run the adaptive mechanism is taken from Pardoe et al. (2010). It is based on the classic Multi-Armed Bandit algorithm introduced by Robbins in 1952; "The gambler has to decide which of k slot machines to play in a sequence of trials so as to maximize the overall reward." We adapt this problem to our instance of online auctions as follows: The seller has to decide which of k auction designs to pick in a sequence of auctions (episode) so as to maximize the overall revenue. The implementation of this algorithm is based on the fundamental concept of bandit algorithms, exploration and exploitation. In the initial auctions, the seller has less information on the bidders to decide which auction design will generate the highest revenue. Hence, the seller prefers to explore different auction designs. Over time, as multiple auction designs are tried over the auction sequence, the seller's estimation of revenue for some designs increases. Hence, rather than the possibility of losing revenue by picking an "unexplored" design that does not perform well, the seller may prefer to go with a previously explored design that is guaranteed to give them some minimum revenue. In other words, sellers prefer to explore new designs when their revenue estimation is low and prefer to exploit known designs when their revenue estimation is high.

Implementation on Yahoo! Data:
Every phrase id in the dataset has a sequence of auctions and bidders associated with it. The algorithm is run on each phrase id to generate the overall revenue in each case. It is expected that certain phrase are more popular than others. Hence, advertisers may place higher bids for the more popular phrase ids. In real-time, bidders will change their bids according to the auction design. However, as our data is static, we calculate the overall revenue across multiple auction designs given that bidders bid *as given in the data* (which assumes first-price auctions) across all designs.

A set of k auction designs are constructed by selecting different combinations of values for the following three parameters.

- Reserve Price: This is the minimum price of a bid to be eligible. For the purpose of the Yahoo! dataset, the reserve price choices are set dynamically according to the distribution of bid prices in the first 20 auctions of each phrase id i.e. the seller has the option to use the 20th, 30th, or 40th percentile bid value.
- Auction Type: The seller can run a VCG auction or a GSP auction. (Comparative analysis of VCG and GSP mechanisms used can be found in Appendix Section 1 and Section 2)
- Number of Ads: How many ads to display on a page. The first ad has the highest click rate and this rate decreases for subsequent ads. The click-rates set for our implementation are: 20% for the 1st Ad, 10% for the 2nd Ad, and 5% for the 3rd Ad. There is also a fixed cost to the seller based on the number of ads placed for a phrase id (as pages with too many ads may negatively affect viewership).

Following the implementation of Pardoe et al. (2010), the algorithm incorporates a Boltzman distribution to decide the auction design used for a given auction in the sequence. This distribution includes a *temperature* parameter to vary the degree exploration/exploitation linearly over the course of a sequence i.e. the temperature starts at a high value (indicating higher exploration) and gradually decreases as more auctions are run (indicating higher exploitation). Hence at a given timestamp or auction, the probability of a design being chosen depends on its estimated revenue as well as the temperature value. During the exploration phase, the probability of choosing a design that is not frequently selected is higher. During the exploitation phase, the probability of choosing a 'good' design that consistently generates high revenue is higher. More details on the adaptive algorithm and code implementation are available in the Appendix.

4 Results and Final Comments

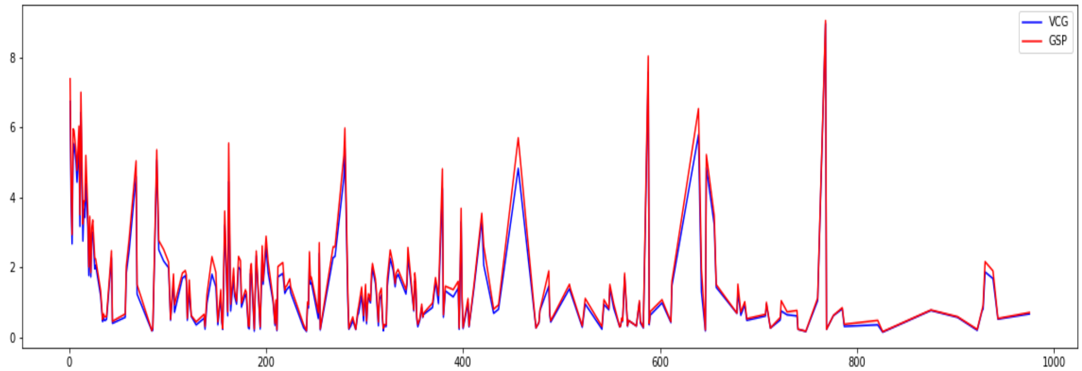
4.1 Results

4.1.1 Comparison between fixed mechanisms: GSP vs VCG

In order to compare the average revenue generated by fixed mechanisms - VCG and GSP for same bid values in the series of auctions, we shortlist around 240 phrases and run the two mechanisms on the dataset. The phrases shortlist is such that each phrase has more than 60 auctions with each auction containing more than 3 unique bidders. For this experiment, we have assumed Reserve Price and Ads Cost as 0 for all auctions and both the mechanisms.

For our data, we get average revenue pretty close to each other for VCG and GSP, with GSP generating slightly better average revenue as shown in figure 1.

Figure 1: Results - VCG vs GSP



4.1.2 Comparison between adaptive mechanism and fixed mechanisms

For each phrase, we decide on 12 fixed mechanism designs and pass it to the adaptive algorithm. The reserve prices and ads cost for these designs are generated dynamically as mentioned earlier. Then we compare the total revenue generated by the adaptive algorithm to the total revenue generated by individual auction design if used for the series of auction for each phrase.

Without any prior info, the Bandit algorithm explores and exploits designs such that it is able to generate total revenue over an episode close to the total revenue generated by the best auction design. On convergence, it has the tendency to assign the maximum probability to the best auction design. Refer the Appendix Section 4 for an example output.

We then run the same experiment for all phrases and try to find out the patterns and statistics across all phrases. The results are reported in the table below.

Description	Value
Frequency of how many times VCG was chosen as final design	77
Frequency of how many times GSP was chosen as final design	166
Average number of slots across all phrases	2.34
Average Reserve Price across all phrases	3.51
Average Ratio of Bandit Revenue to Highest Revenue of all designs	0.89

Based on the statistics, GSP is selected more times than VCG which is in line with our expectations. On average, we are able to achieve around 89% of the highest total revenue through the bandit algorithm.

4.2 Limitations

Our methodology has few limitations as mentioned below:

- Adaptive mechanisms could lead to long-term strategic bidding behavior by the bidders, i.e., a bidder intentionally bids low to make the reserve price be lower in the future. In our set-up and experiments, we are not considering long-term strategic bidding behavior.
- Without real-time data, it's difficult to accurately predict how the algorithm would converge to the best design with dynamic data.
- Bidders will bid differently, depending on whether the VCG and GSP mechanism is used. Our data is based on first price auction, however, we utilize this data for simulation purposes and assume that bidders will bid in the similar way for GSP/VCG to generate our results.

4.3 Closing Remarks

In comparing the two fixed mechanisms - GSP and VCG, we show that GSP performs at least as good as VCG (assuming all advertisers bid the same under both mechanisms) but unlike VCG, truthfulness is not a dominant strategy for GSP (Edelman et al, 2007). In a real-time setting, VCG has the benefit of truthfulness but there may be high costs for transitioning from GSP to VCG and VCG payments are hard to explain to the ad buyers. If we switch from fixed auction mechanism to the adaptive auction mechanism, given multiple fixed designs, the adaptive bandit algorithm is able to select designs such that generated total revenue is close to the maximum total revenue that is possible from the best fixed auction design. On convergence, it has the tendency to stop exploring more designs, but only exploit/use the best design. Our approach also shows how strategically varying other parameters, beside auction type, can change the overall revenue. The overall revenue is highly dependent on the reserve price and ads cost.

We acknowledge that in reality, bidders respond to changes in the auction design. This issue is pertinent in our case, where the adaptive algorithm has to choose over the truthful VCG mechanism and the potentially untruthful GSP mechanism. One way to account for this would be set up the algorithm to begin exploring VCG auctions. Since VCG is a truthful mechanism, we therefore obtain the bidders' true valuation of the phrase. Since we now have information on the true valuations, we can remove the auctions designs to which advertisers place bids are significantly untruthful (relative to a pre-defined threshold).

In reality, sellers care about advertisers' welfare. If the price is too high, bidders will drop out of subsequent auctions. Hence, using an unrestricted reserve price could result in losing advertisers. To handle this situation, our adaptive approach looks at the past history of auctions for each phrase id. The initial choices for reserve price are selected by using prices at specific percentiles of the bid prices distribution for a fixed number of auctions (such as the 20th and 30th percentiles). When these values are specified for auction designs, the adaptive algorithm can also keep track of how many advertisers participate in the auction given a reserve price. If advertisers start dropping drop due to high reserve prices, the algorithm will dynamically lower the reserve price to gain more advertisers. We hope to explore the above considerations in future research.

References

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A Appendix

A.1 General Second Price (GSP)

Following the derivation by Varian et al.(2013), let v_s be the value of a click to an advertiser in slot $s = 1..S$, and let x_s be the click-through rate associated with the slot.

The revealed preferences of an advertiser who purchases slot s would indicate that he/she prefers that slot to the other slots (i.e. slot t), represented by:

$$v_s x_s - p_s x_s \geq v_s x_t - p_t x_t$$

Therefore, advertisers with higher values get more prominent slots

Assuming that the slots have been ordered, starting with the most clicked first, such that $x_1 > x_2 > \ldots > x_s$, the GSP auction will result in a price being charged for each slot. For example, for the three slot case, the GSP auction would yield the following payments:

$$\begin{aligned} p_1 x_1 &= v_2(x_1 - x_2) + p_2 x_2 \\ p_2 x_2 &= v_3(x_2 - x_3) + p_3 x_3 \\ p_3 x_3 &= v_4 x_3 \end{aligned}$$

Implementation: We ordered bidders by their valuation and applied the formula above retrieve the prices charged for each slot

A.2 Vickerey Clarke Groves (VCG)

Following the derivation by Varian et al.(2013), each bidder pays the externality which he/she imposes on the other bidders.

If advertiser 1 participates in the auction, the total payment made by the other advertisers is:

$$v_2 x_2 + v_3 x_3$$

If advertiser 1 does not participate in the auction, the other advertisers will all move up one position and pay:

$$v_2 v_1 + v_3 x_2 + v_4 x_3$$

Therefore the payment for advertiser 1 is the difference:

$$v_2(x_1 - x_2) + v_3(x_2 - x_3) + v_4 x_3$$

Therefore, the VCG payment in the three slot case would be as follows:

$$\begin{aligned} p1x_1 &= v_2(x_1 - x_2) + v_3(x_2 - x_3) + v_4 x_3 \\ p_2 x_2 &= v_3(x_2 - x_3) + v_4 x_3 \\ p_3 x_3 &= v_4 x_3 \end{aligned}$$

Implementation: We ordered bidders based on their bids and calculated the VCG payment based on the formula above.

A.3 Adaptive Algorithm

Code for Adaptive Algorithm:
https://colab.research.google.com/drive/1mT-cuIS-v_UOSZV_qPU1NoC2cvSqvE4j?usp=sharing.

Reason to use the Adaptive Algorithm over a Fixed Design Choice: This algorithm is useful when there are a sequence of online auctions to be run in short periodic intervals and the seller does not have enough information to accurately decide the highest revenue generating mechanism prior to running the auction sequence. It is especially useful if there are a large number of design choices and the seller, or there are certain design parameters that the seller would like to set dynamically, based on bidder behavior. Under this context, giving the seller flexibility to modify the auction design based on which design is observed to generate more revenue (and attracting higher bids) over several auctions would generate higher overall revenue than selecting a constant design, which may not be the highest revenue generating price for all auctions.

Note on Algorithm Convergence: Under the assumptions stated in Part 2, it is expected that bidders will change their bids according to the auction design (but not randomly), and over time there will be some consistency in the way the bid for certain designs, revealing to the seller which designs are better than others. Over an extended period of time, i.e. a number of auctions, the algorithm is expected to converge to the highest revenue generating design. If the bids are constantly changing, regardless of design choice, then the algorithm may not converge. This might happen if many new bidders enter auctions at different points in the sequence or if the assumptions of no collusion and no long-term strategic bidding behavior fail. We discuss this and further limitations of the adaptive approach in Part 4.

A.4 Results

Auction Designs for Phrase 2	Total Revenue
Bandit Algorithm	5222.30
Auction Type: VCG, Number of Slots: 3, Reserved Price: 10, Ads Cost: 0.5	3410.16
Auction Type: VCG, Number of Slots: 2, Reserved Price: 10, Ads Cost: 0.5	4119.30
Auction Type: GSP, Number of Slots: 3, Reserved Price: 10, Ads Cost: 0.5	4496.86
Auction Type: GSP, Number of Slots: 2, Reserved Price: 10, Ads Cost: 0.5	4651.32
Auction Type: VCG, Number of Slots: 3, Reserved Price: 5, Ads Cost: 0.5	4355.95
Auction Type: VCG, Number of Slots: 2, Reserved Price: 5, Ads Cost: 0.5	4951.47
Auction Type: GSP, Number of Slots: 3, Reserved Price: 5, Ads Cost: 0.5	5218.02
Auction Type: GSP, Number of Slots: 2, Reserved Price: 5, Ads Cost: 0.5	5296.75