### **Load Libraries**

```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
```

## Load dataset & data preprocessing

```
file_path = "MNIST_train.csv"
In [342...
           df_train = pd.read_csv(file_path)
           df_train.head
In [343...
Out[343...
           <bound method NDFrame.head of</pre>
                                                    Unnamed: 0 index labels 0
           5 6 ... 774 775 776 \
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           [60000 rows x 787 columns]>
           file_path = "MNIST_test.csv"
In [344...
           df_test = pd.read_csv(file_path)
In [345...
          df_test.head
```

```
Out[345... <bound method NDFrame.head of
                                       Unnamed: 0 index labels 0 1 2 3 4 5
         6 ... 774 775 776 777 \
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         [10000 rows x 787 columns]>
         df_train.shape
In [346...
Out[346...
         (60000, 787)
In [347...
         df_test.shape
Out[347... (10000, 787)
```

In [348... df\_test.info

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	9997		9997	99	97		4 0	0	0	0	0	0	0		0	0	0	0
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In [349	df_tes	st.des	scribe															
				Frame	.des	cribe	of		U	nna	med	: 0	i	ndex	labels	s 0	1 2	3
In [349 Out[349		d met	hod ND				of 77 \		U	nna	med	: 0	i	ndex	labels	5 0	1 2	3
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	  4 5 0 1 2 3 4  9995 9996 9997 9998	d met	hod ND 77- 0 1 2 3 4 9995 9996 9997	99 99 99	75 7 0 1 2 3 4 995 996		77 \ 7 0 2 0 1 0 0 0 4 0 2 0 3 0 4 0 5 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	  4 5 0 1 2 3 4  9995 9996 9997	d met	hod ND 77 0 1 2 3 4 9995 9996	99 99 99	75 7 0 1 2 3 4 995		77 \ 7 0 2 0 1 0 0 0 4 0 2 0 3 0 4 0	0 0 0 0 0  0	0 0 0 0 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0
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	  4 5 0 1 2 3 4  9995 9996 9997 9998	d met 6 .	hod ND 77 0 1 2 3 4 9995 9996 9997 9998	99 99 99 99	75 7 0 1 2 3 4 995 996 997		77 \\ 7	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9996 9997 9998 9999	d met 6 . 778 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0		75 7 0 1 2 3 4 995 996 997 998 999	782 0	77 \\ 7 \ 0 \\ 2 \ 0 \\ 1 \ 0 \\ 0 \ 0 \\ 4 \ 0 \\ 2 \ 0 \\ 4 \ 0 \\ 5 \ 0 \\ 6 \ 0 \\ 783	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9996 9997 9998 9999 0 1	d met 6 . 778 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0		75 7 0 1 2 3 4 995 996 997 998 999 781 0	782 0	77 \\ 7 \ 0 \\ 2 \ 0 \\ 1 \ 0 \\ 0 \ 0 \\ 4 \ 0 \\ \\ 2 \ 0 \\ 3 \ 0 \\ 4 \ 0 \\ 5 \ 0 \\ 6 \ 0 \\ 783 \\ 0 \\ 0 \\ 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9996 9997 9998 9999 0 1 2	778 0 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0	99 99 99 99 99	75 7 0 1 2 3 4 995 996 997 998 999 781 0 0	782 0 0	77 \\ 7 \ 0 \\ 2 \ 0 \\ 1 \ 0 \\ 0 \ 0 \\ 4 \ 0 \\ \\ 2 \ 0 \\ 3 \ 0 \\ 4 \ 0 \\ 5 \ 0 \\ 6 \ 0 \\ 783 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9996 9997 9998 9999 0 1 2 3 4 	778 0 0 0	hod ND 77. 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0 0 0		75 7 0 1 2 3 4 995 996 997 998 999 781 0 0 0 0	782 0 0 0	77 \\ 7 \ \ \ 7 \ \ \ 2 \ \ \ 0 \ \ 1 \ \ \ \ 0 \ \ \ 4 \ \ \ \ 0 \ \ 4 \ \ \ \ 5 \ \ \ \ 6 \ \ \ \ 783 \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9998 9999 0 1 2 3 4  9995	778 0 0 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0 0 0		75 7 0 1 2 3 4 995 996 997 998 999 781 0 0 0	782 0 0 0 0	77 \\ 7 \ 0 \\ 2 \ 0 \\ 1 \ 0 \\ 0 \ 0 \\ 4 \ 0 \\ 2 \ 0 \\ 4 \ 0 \\ 5 \ 0 \\ 6 \ 0 \\ 783 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9998 9999 0 1 2 3 4  9995 9996 1 2 3 4  9995 9996	778 0 0 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0 0 0		75 7 0 1 2 3 4 995 997 998 999 781 0 0 0 0	782 0 0 0 0	77 \\ 7	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9999 0 1 2 3 4  9995 9996 9997	778 0 0 0  0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0 0 0 0 0		75 7 0 1 2 3 4 995 997 998 999 781 0 0 0 0 0	782 0 0 0 0 0	77 \\ 7	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0 	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0
	   4 5 0 1 2 3 4  9995 9998 9999 0 1 2 3 4  9995 9996 1 2 3 4  9995 9996	778 0 0 0	hod ND 77 0 1 2 3 4 9995 9996 9997 9998 9999 779 0 0 0 0 0		75 7 0 1 2 3 4 995 997 998 999 781 0 0 0 0	782 0 0 0 0	77 \\ 7	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0		0 0 0 0 0 	0 0 0 0 0  0 0	0 0 0 0 0 0 0	0 0 0 0 0 0

[10000 rows x 787 columns]>

 $\label{thm:out} {\tt Out[348...} \quad {\tt cbound method DataFrame.info of} \qquad \qquad {\tt Unnamed: 0 index labels 0 1 2 3 4} \\$ 

In [350...

df\_train

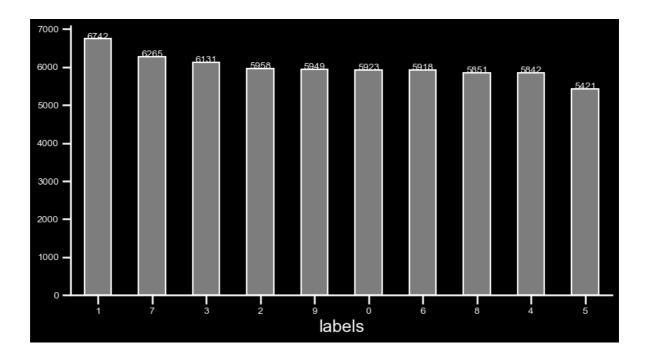
Out[350...

	Unnamed: 0	index	labels	0	1	2	3	4	5	6	•••	774	775	776	777	778
0	0	0	5	0	0	0	0	0	0	0		0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0		0	0	0	0	0
2	2	2	4	0	0	0	0	0	0	0		0	0	0	0	0
3	3	3	1	0	0	0	0	0	0	0		0	0	0	0	0
4	4	4	9	0	0	0	0	0	0	0		0	0	0	0	0
•••						•••	•••									
59995	59995	59995	8	0	0	0	0	0	0	0		0	0	0	0	0
59996	59996	59996	3	0	0	0	0	0	0	0		0	0	0	0	0
59997	59997	59997	5	0	0	0	0	0	0	0		0	0	0	0	0
59998	59998	59998	6	0	0	0	0	0	0	0		0	0	0	0	0
59999	59999	59999	8	0	0	0	0	0	0	0		0	0	0	0	0

60000 rows × 787 columns



## **Checking Target Imbalance**



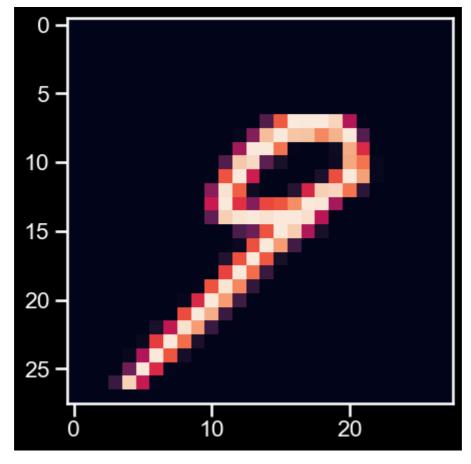
#### Observation

All numbers appear to be balanced. 1 is the most and 5 is the least.

```
df_train = df_train.to_numpy()
In [352...
In [353...
           df_train
                                                            0,
                                                                    0],
Out[353...
           array([[
                        0,
                                0,
                                                     0,
                        1,
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                   4, ...,
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                   [59998, 59998,
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                                        6, ...,
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                   [59999, 59999,
                                        8, ...,
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                                                                    0]],
                                                     0,
                  shape=(60000, 787))
In [354...
           y_train = df_train[:,2]
In [355...
           y_train.shape
Out[355...
           (60000,)
In [356...
           K = set(y_train)
In [357...
Out[357...
           {np.int64(0),
            np.int64(1),
            np.int64(2),
            np.int64(3),
             np.int64(4),
             np.int64(5),
             np.int64(6),
             np.int64(7),
             np.int64(8),
             np.int64(9)}
```

```
X_train = df_train[:,3:]
In [358...
In [359...
           X_train
Out[359...
           array([[0, 0, 0, ..., 0, 0, 0],
                   [0, 0, 0, \ldots, 0, 0, 0],
                   [0, 0, 0, ..., 0, 0, 0]], shape=(60000, 784))
           X_train.shape
In [360...
Out[360...
          (60000, 784)
In [361...
          X_{train1} = X_{train[501]}
           img = X_train1.reshape(28,28)
           #sns.set(style="ticks", context="talk",font_scale = 1)
           #plt.style.use("dark_background")
           fig = plt.figure(figsize = (5,5))
           ax = fig.add_subplot(111)
           ax.imshow(img)
```

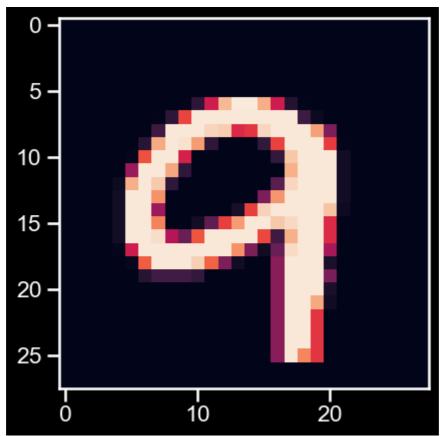
Out[361... <matplotlib.image.AxesImage at 0x21c8d02a240>



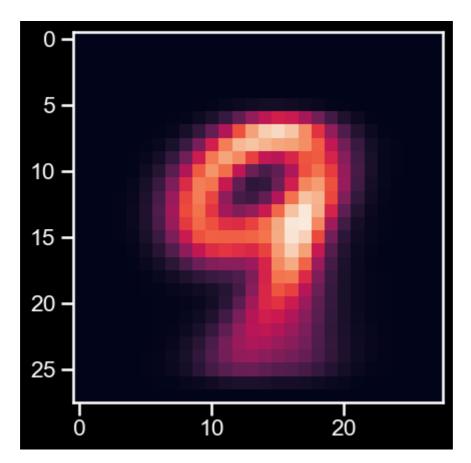
#### Observation

Each number image in MNIST is only a list of 28 \* 28 dimensional numbers.

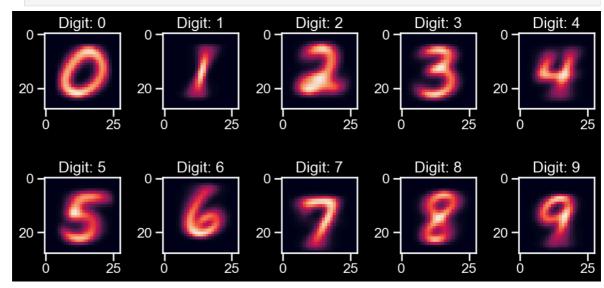
```
df_test = df_test.to_numpy()
In [362...
In [363...
         df_test
Out[363...
                     0,
                           0,
                                 7, ...,
                                                 0,
                                                       0],
          array([[
                                           0,
                     1, 1,
                                 2, ...,
                                         0,
                                               0,
                                                       0],
                 [
                 [ 2,
                         2,
                                 1, ...,
                                          0,
                                               0,
                                                       0],
                                               0,
                 [9997, 9997,
                                 4, ...,
                                           0,
                                                       0],
                 [9998, 9998, 5, ...,
                                          0, 0,
                                                       0],
                 [9999, 9999,
                                                       0]], shape=(10000, 787))
                                 6, ...,
                                           0,
                                                 0,
In [364... y_test = df_test[:,2]
In [365...
         X_{\text{test}} = df_{\text{test}}[:,3:]
In [366...
         df_test
                                                       0],
Out[366...
                           0,
                                                 0,
          array([[
                     0,
                                 7, ...,
                                            0,
                     1,
                          1, 2, ...,
                                         0,
                                               0,
                                                       0],
                          2,
                 [
                    2,
                                 1, ...,
                                         0, 0,
                                                       0],
                 [9997, 9997,
                                 4, ...,
                                                       0],
                                           0,
                                                0,
                 [9998, 9998,
                              5, ...,
                                               0,
                                                       0],
                                           0,
                 [9999, 9999,
                                 6, ...,
                                           0,
                                                 0,
                                                       0]], shape=(10000, 787))
In [367...
         X_{\text{test1}} = X_{\text{test}}[501]
In [368...
         x_test_reshape= X_test1.reshape(28,28)
In [369...
         plt.imshow(x_test_reshape)
Out[369... <matplotlib.image.AxesImage at 0x21c8cfe0f20>
```



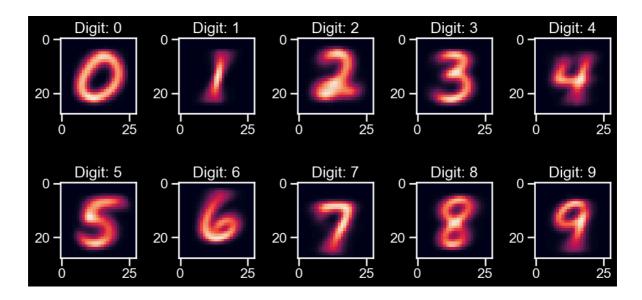
```
In [370... y_test[501]
Out[370... np.int64(9)
In [371... def show_me(X):
    plt.imshow(X.reshape(28,28))
    def show_me_all_mean(X,y,k):
        show_me(sum(X[y==k, :]/len(X[y==k,:])))
In [372... show_me_all_mean(X_train,y_train,9)
```



```
In [373... plt.figure(figsize=(10, 5))
    for digit in range(10):
        plt.subplot(2, 5, digit + 1)
        show_me_all_mean(X_train, y_train, digit)
        plt.title(f'Digit: {digit}')
    plt.tight_layout()
    plt.show()
```



```
In [374... plt.figure(figsize=(10, 5))
for digit in range(10):
    plt.subplot(2, 5, digit + 1)
    show_me_all_mean(X_test, y_test, digit)
    plt.title(f'Digit: {digit}')
plt.tight_layout()
plt.show()
```



```
In [375... sample_image_train = X_train[581]

In [376... plt.figure(figsize=(10, 6))
   plt.hist(sample_image_train, bins=30, color='blue', alpha=0.7, edgecolor='black'
   plt.title(f"Histogram of Pixel Intensities for Digit {y_train[581]}")
   plt.xlabel("Pixel Intensity")
   plt.ylabel("Frequency")
```

plt.grid(axis='y', alpha=0.75)

plt.show()

```
Histogram of Pixel Intensities for Digit 3

600

500

500

100

150

200

250

Pixel Intensity
```

```
In [377...

def CoinFlip(y):
    y_hat=np.zeros(len(y))
    for i in range(len(y)):
        flip = np.random.randn(1)
        if flip>0:
            y_hat[i]=1

    return y_hat
```

# **Naive Buyes Classifier**

```
In [378...
          from scipy.stats import multivariate_normal as mvn
In [379...
          class GauseNB():
               def fit(self, X,y, epsilon = 1e-3):
                   self.likelihoods = dict()
                   self.priors = dict()
                   self.K = set(y.astype(int))
                   for k in self.K:
                       X_k = X[y==k,:]
                       self.likelihoods[k]= {"mean":X_k.mean(axis=0),"cov":X_k.var(axis=0)+
                       self.priors[k]=len(X_k)/len(X)
               def predict(self, X):
                   N, D = X.shape
                   P_hat = np.zeros((N, len(self.K)))
                   for k, l in self.likelihoods.items():
                       P_hat[:, k] = mvn.logpdf(X, 1['mean'], 1['cov']) + np.log(self.prior
                   return P_hat.argmax(axis=1)
In [380...
          def accuracy(y, y_hat):
               return np.mean(y==y_hat)
          Train model
In [381...
          X_train = X_train/255
          X_{\text{test}} = X_{\text{test}}/255
In [382...
          gnb_train = GauseNB()
In [384...
          gnb_train.fit(X_train,y_train)
In [385...
          y_train_hat = gnb_train.predict(X_train)
In [386...
          #plt.figure(figsize = (10,6))
          #plt.scatter(X_train[:,0], X_train[:,1],c=y_train,alpha =0.5,s=10)
In [387...
          #plt.figure(figsize = (10,6))
          #plt.scatter(X_train[:,0], X_train[:,1],c=y_train_hat,alpha =0.5,s=10)
In [388...
          accuracy(y_train,y_train_hat)
```

#### test data

Out[388... np.float64(0.76823333333333333)

```
In [390... y_test_hat = gnb_train.predict(X_test)
```

```
In [391... accuracy(y_test,y_test_hat)
Out[391... np.float64(0.7746)
```

## Non Naive Gausian Bayes

```
In [400...
          class GauseBayes():
               def fit(self, X,y, epsilon = 1e-1):
                   self.likelihoods = dict()
                   self.priors = dict()
                   self.K = set(y.astype(int))
                   for k in self.K:
                      X_k = X[y==k,:]
                       N k_D = X k_shape
                       mu_k = X_k.mean(axis=0)
                       self.likelihoods[k]= {"mean":X_k.mean(axis=0),
                                              "cov":(1/(N_k-1))*np.matmul((X_k-mu_k).T,X_k-m
                       self.priors[k]=len(X_k)/len(X)
               def predict(self, X):
                   N, D = X.shape
                   P_hat = np.zeros((N, len(self.K)))
                   for k, l in self.likelihoods.items():
                       P_{\text{hat}}[:, k] = mvn.logpdf(X, l['mean'], l['cov']) + np.log(self.prior)
                   return P_hat.argmax(axis=1)
               def confusion_matrix_manual(self, y_true, y_pred):
                   classes = np.unique(y_true)
                   matrix = np.zeros((len(classes), len(classes)), dtype=int)
                   class_to_index = {cls: idx for idx, cls in enumerate(classes)}
                   for actual, predicted in zip(y_true, y_pred):
                       matrix[class_to_index[actual], class_to_index[predicted]] += 1
                   return matrix
In [401...
          gbays_naive_train = GauseBayes()
In [402...
          gbays_naive_train.fit(X_train,y_train)
In [403...
         y_hat_train_gbays = gbays_naive_train.predict(X_train)
In [404...
          accuracy(y_train,y_hat_train_gbays)
Out[404...
         np.float64(0.9549333333333333)
In [405...
          con_matrix_gb1 = gbays_naive_train.confusion_matrix_manual(y_train, y_hat_train_
          print(con matrix gb1)
```

```
[[5866
          14
                 1
                       1
                             2
                                   5
                                        15
                                               1
                                                    13
                                                           5]
     0 6683
                33
                       4
                             5
                                   0
                                         1
                                               3
                                                     9
                                                           4]
    27
         114 5588
                      27
                            27
                                   0
                                        18
                                              52
                                                    86
                                                          19]
    14
          69
                45 5775
                             0
                                  29
                                         2
                                              52
                                                    70
                                                          75]
          69
                10
                       0 5523
                                   0
                                        19
                                              16
                                                     6
                                                         193]
     6
    31
          35
                 8
                      56
                             3 5102
                                        66
                                               1
                                                    44
                                                          75]
 Γ
          45
                             3
                                  53 5783
                                                    12
 20
                 1
                       0
                                               1
                                                           0]
    10
          75
                16
                       3
                            12
                                   2
                                         0 5999
                                                    11
                                                         137]
 259
                                              15 5272
 [
    30
                24
                      84
                            11
                                  31
                                        23
                                                         102]
 21
          53
                 9
                      48
                            11
                                   6
                                         1
                                              69
                                                    26 5705]]
```

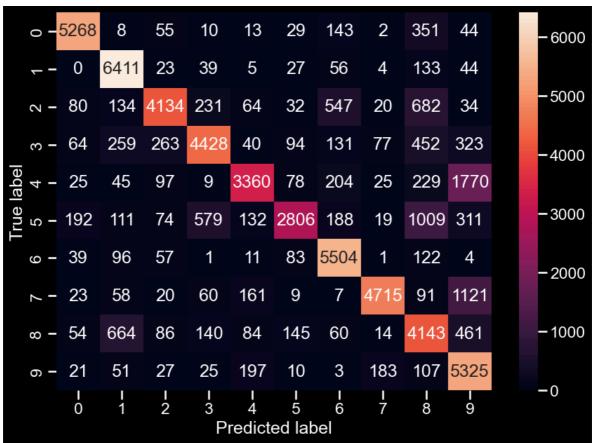
### **Test Data**

```
In [406... y_hat_test_gbays = gbays_naive_train.predict(X_test)
In [407... accuracy(y_test,y_hat_test_gbays)
Out[407... np.float64(0.9542)
```

## **Confusion Matrix**

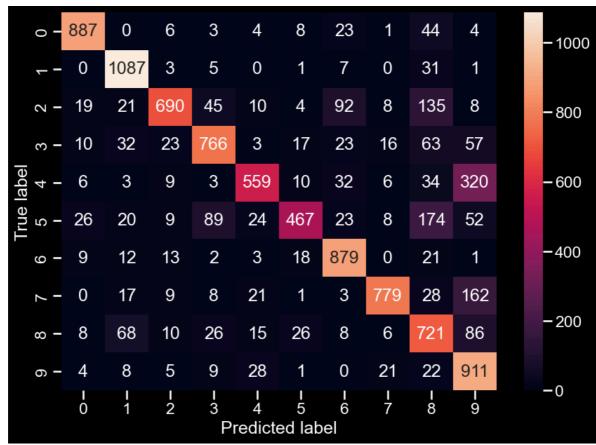
```
In [408... plt.figure(figsize=(10,7))
    y_actu = pd.Series(y_train, name='Actual')
    y_pred = pd.Series(y_train_hat, name='Predicted')
    cm = pd.crosstab(y_actu, y_pred)
    ax = sns.heatmap(cm, annot=True, fmt="d")
    plt.ylabel('True label')
    plt.xlabel('Predicted label')
```

Out[408... Text(0.5, 34.08333333333314, 'Predicted label')



```
In [409... plt.figure(figsize=(10,7))
   ytest_actu = pd.Series(y_test, name='Actual')
   ytest_pred = pd.Series(y_test_hat, name='Predicted')
   cm = pd.crosstab(ytest_actu, ytest_pred)
   ax = sns.heatmap(cm, annot=True, fmt="d")
   plt.ylabel('True label')
   plt.xlabel('Predicted label')
```

Out[409... Text(0.5, 34.0833333333334, 'Predicted label')



# **K Nearest Neighbors**

## **Synthetic Dataset Generation**

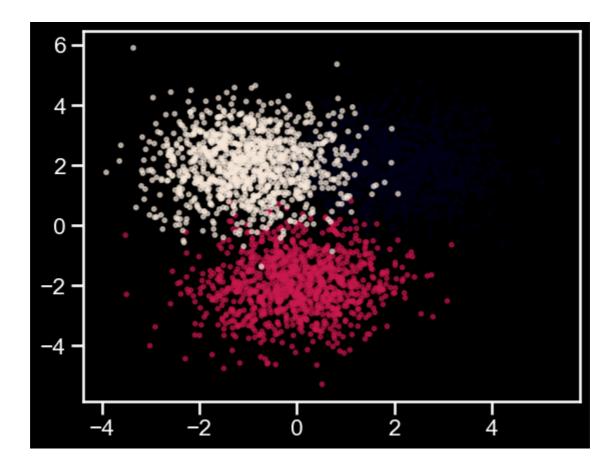
```
In [410... D = 2 #Dimensions
    C = 3 # Classes
    N = int(C*1e3) # Number of points/rows

X0 = np.random.randn((N//C), D) + np.array([2,2])
    X1 = np.random.randn((N//C), D)+ np.array([0,-2])
    X2 = np.random.randn((N//C), D)+ np.array([-1,2])

X = np.vstack((X0, X1, X2))
    y = np.array([0]*(N//C)+ [1]*(N//C)+ [2]*(N//C))

In [411... plt.figure()
    plt.scatter(X[:,0],X[:,1],c=y,alpha=0.6,s=8)
```

Out[411... <matplotlib.collections.PathCollection at 0x21c8dd45fa0>



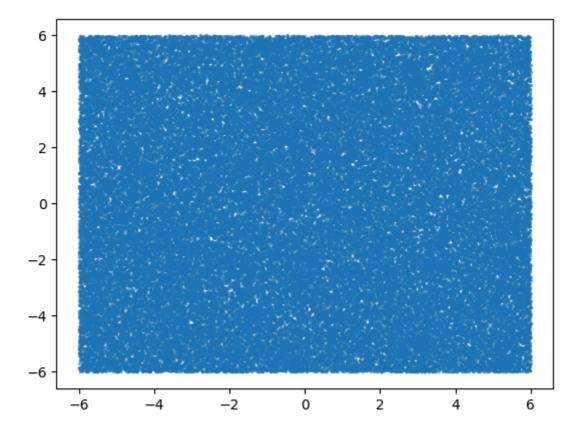
## **KNN Classifier Class**

```
In [412...
          class KNNClassifier():
              def fit(self, X, y):# lazy learner just perform operation
                  self.X = X
                  self.y = y
              def predict(self, X, K , epsilon=1e-3):
                  N = len(X) #number of observations
                  y_hat = np.zeros(N) #
                  for i in range(N):
                      dist2 = np.sum((self.X - X[i])**2,axis =1)
                      idxt = np.argsort(dist2)[:K] # give a list of index from lowest dist
                      gamma_k = 1/(np.sqrt(dist2[idxt] + epsilon))
                      y_hat[i] =np.bincount(self.y[idxt],weights=gamma_k).argmax() # to gi
                  return y_hat # return outside for Loop
In [413...
          knn_instance = KNNClassifier()
         mini_train_data, mini_train_labels = X_train[:10000], y_train[:10000]
In [414...
In [415...
         mini_train_data
```

```
Out[415... array([[0., 0., 0., ..., 0., 0., 0.],
                  [0., 0., 0., ..., 0., 0., 0.]
                  [0., 0., 0., \ldots, 0., 0., 0.]
                  [0., 0., 0., ..., 0., 0., 0.]
                  [0., 0., 0., ..., 0., 0., 0.]
                  [0., 0., 0., ..., 0., 0., 0.]], shape=(10000, 784))
In [442...
          mini_test_data, mini_test_labels = X_test[:10000], y_test[:10000]
In [417...
          knn_instance.fit(mini_train_data,mini_train_labels)
In [423...
          y_hat_knn = knn_instance.predict(y_test,K=200)
In [433...
          mini_train_data[:,0]
Out[433... array([0., 0., 0., ..., 0., 0., 0.], shape=(10000,))
In [435...
          y_hat_knn
           array([0., 0., 0., ..., 0., 0., 0.], shape=(10000,))
Out[435...
In [436...
          accuracy(mini_train_data[:,0],y_hat_knn)
Out[436...
          np.float64(0.902)
In [443...
          accuracy(mini_test_data[:,0],y_hat_knn)
Out[443... np.float64(0.902)
In [104...
          X_{\text{vis}} = \text{np.random.uniform(-6,6,(int(N*30),D))}
In [105...
          plt.scatter(X_vis[:,0],X_vis[:,1],s=2,alpha=0.5)
```

<matplotlib.collections.PathCollection at 0x21c4915b0b0>

Out[105...



In [106... y\_hat\_vis = knn\_instance.predict(X\_vis,K=200)
 plt.figure()
 plt.scatter(X\_vis[:,0],X\_vis[:,1],c=y\_hat\_vis,s=2)

Out[106... <matplotlib.collections.PathCollection at 0x21c490c8d40>

