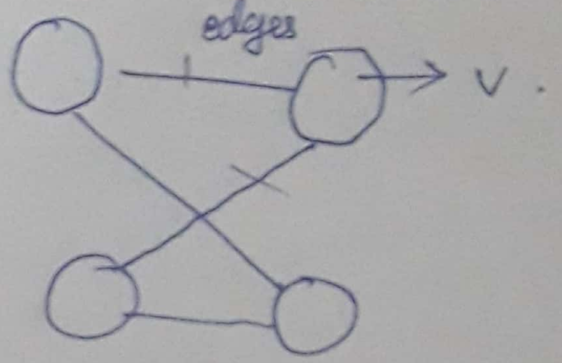
Unit - V **Graphs**

Nodes.

$G = (V, E)$



Applications.

→ Google maps.

→ Networks (computer networks).

→ If 2 vertices are connected to single edge then such a graph is called simple graph

→ If the vertices are connected with out direction then graph is called undirected graph and vertices are called unordered pair of vertices $((u, v), (v, u))$.

→ Graph that contain more than one edge between same pair of vertices then graph is called multigraph

→ If we have m edges b/w same unordered pair of vertices then the cardinality of pair of vertices is m — (or) multiplicity

→ The graphs which has self loops that graph is called pseudo graph

→ Graphs that contains loop and possibly multiple edges connecting the same pair of vertices are called pusedo graph.

→ Allowing the data to flow only in one direction then it is called single duplex line.

The directed Graph is called as Dia-graph

directed Graph:- The directed Graph consists of set of vertices and directed set of edges.

The pair of vertices is called order pair of vertices.

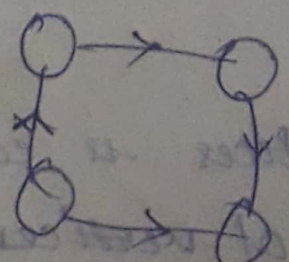
→ If the order pair of vertices are connected to single directed edge then such a graph is called simple directed Graph:

→ If more than one edge are connetected to ordered pair of vertices then the graph is called directed multigraphs.

→ Combination of directed and undirected edges ~~that~~ graph is called mixed graph

Graph models.

1. Niche overlap Graph in ecology.
2. A acquaintanceship Graphs
   ↳ friend
3. Influence graph (no multiple edges and no selfloop) (It is directed graph)
4. Round Robin Tournament (no loops and no multiple edges)

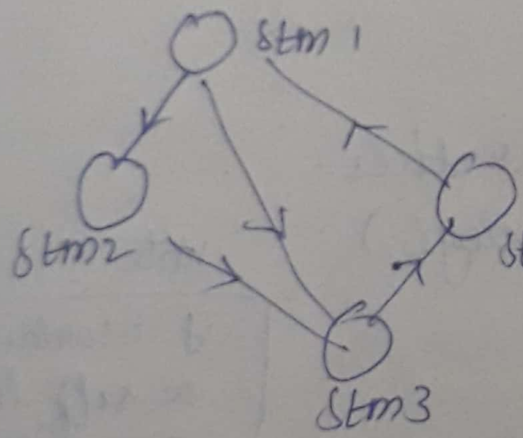


5. Hollywood graph.
6. call Graphs (directed graph, multigrap

7. Web graph (links b/w 2 web pages) (directed graph) (changes with time).

8. Precedence Graph and concurrent Processing.

[computer programs are executed concurrently. If there are 5 stmts. 3rd stmt is executed if 2nd stmt is executed then stmt 3 is dependent on 2nd stmt].



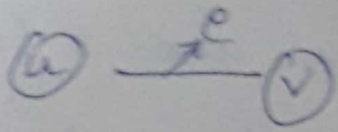
[precedence graph].

9. Road maps [vertices are intersection of 2 roads and roads are edges (we have directed, undirected and loop edges).

Graph Terminology and Special Types of Graph

Adjacent :- When 2 vertices are connected to a common edge.

Incident :-



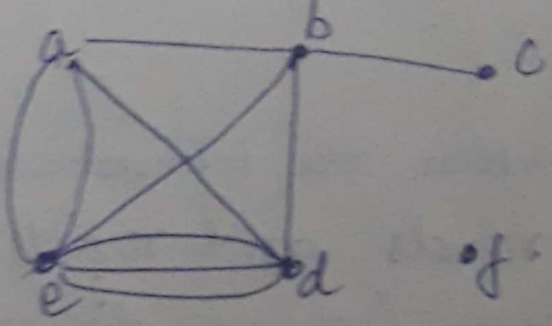
e is incident with u, v.

u, v are end points of the e.

degree

no. of edges of a vertices.
It is denoted by deg( ).

note :-
If it contain a self loo count it tw



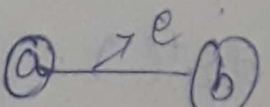
deg (a) = 4
(b) = 6
(c) = 1
(d) = 5
(e) = 6

isolated :- The vertex with degree zero (0) then is called isolated. Ex :- "f"

pendent : The vertex with degree one

Ex :- "c".

NOTE :-
Each edge contributes 2 to the sum of degrees of vertices because an edge is incident with exactly 2 vertices. This means the sum of the degrees of the vertices is twice ~~2~~ the no. of edges.



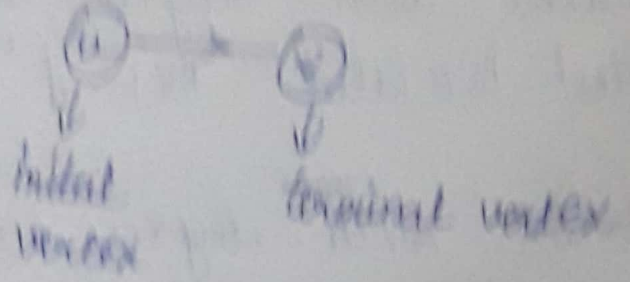
This theorm is called Handshaking theorem.

$$2e = \sum_{\in V} \deg(\ ).$$

twice the no. of edges = sum of the degrees.

Ex :-
How many edges are there in the graph with 10 vertices each of degree ~~vertex with~~ 6.

$$2e = 10(6)$$
$$= 30$$



u is adjacent to v and v is adjacent from v

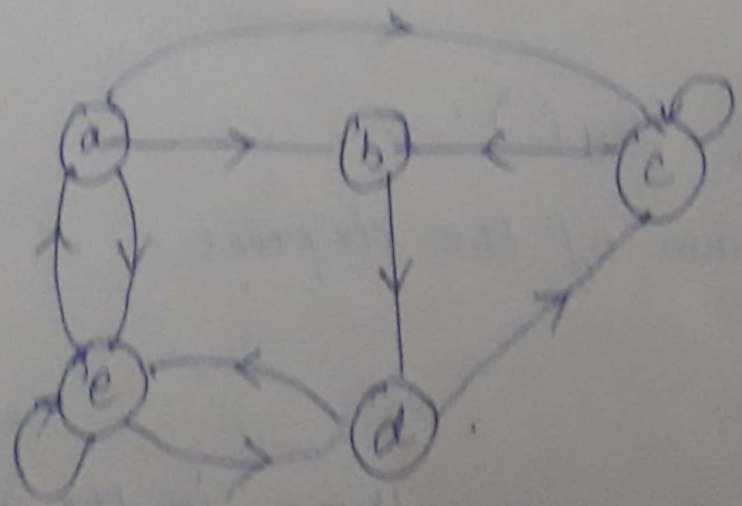
→ In degree

It is denoted by $deg^-( )$

→ Out degree

It is denoted by $deg^+( )$

loop is taken only once in Indegree and out degree



In degree

$deg^-(a) = 2$

$(b) = 2$

$(c) = 3$

$(d) = 2$

$(e) = 3$

Out degree

$deg^+(a) = 4$

$(b) = 1$

$(c) = 2$

$(d) = 2$

$(e) = 3$

Some Special Simple graph

1) Complete graph.

It is denoted by K_n

where n = no. of vertices.

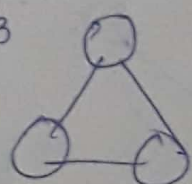
Complete grah is a simple graph.

K_n $[n = 1, 2, 3, 4, 5]$.

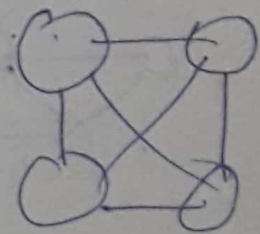
K_1

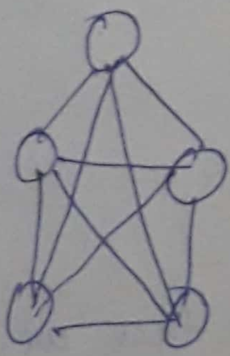
K_2

K_3



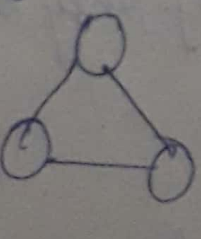
K_4



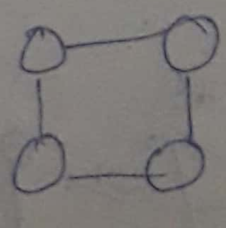


2) Cycles (C_n) where $n \geq 3$.

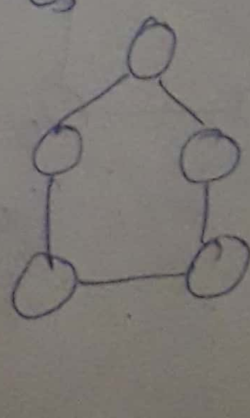
C_3



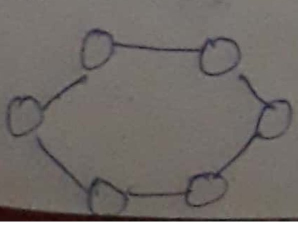
C_4



C_5

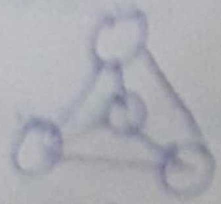


C_6

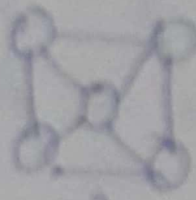


Wheel (W_n)

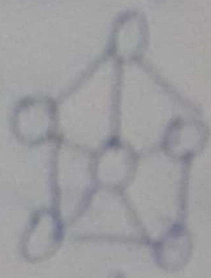
adding a new vertex to the existing vert[ices] of a cycle and joining the edges



W_4



W_5



W_6

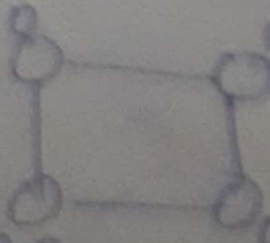
W_7

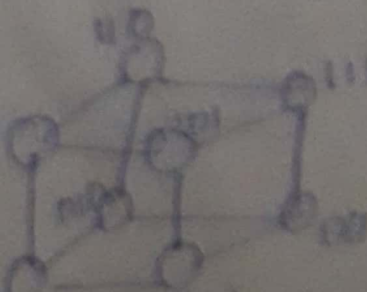
n - cube (Q_n) ↳ (dimensions)

Q_1

In this we say that 2 vertices are adjacent if they differ by only one bi[t]

Q_2





Q_3

Scanned with CamScanner

Bi - partited Graph

A bipartite graph also called a bigraph, It is a set of graph vertices decomposed into 2 disjoint sets such that no two graph vertices with in the same set are adjacent

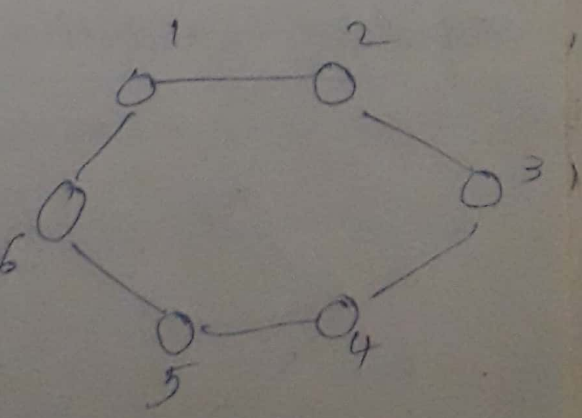
→ It is special case of a K-partite graph with $K = 2$.

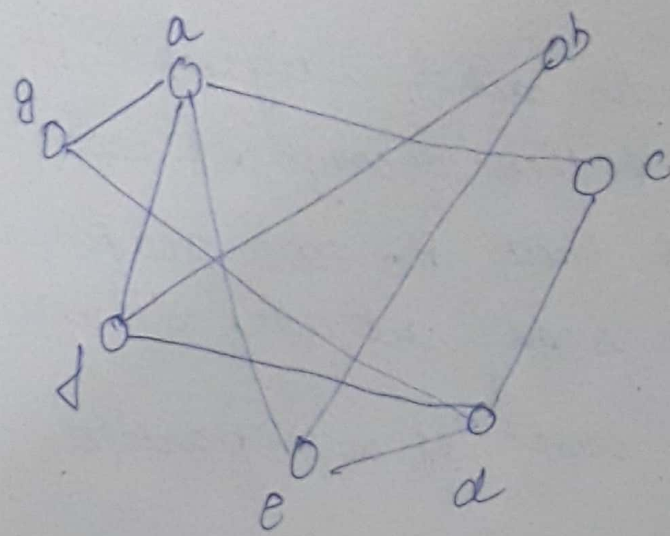
→ Bipartite graphs are equivalent to two colorable graphs.

A simple graph is called Bi-partited if its vertex set V can be partitioned into 2 disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in a V_1 and vertex in V_2. (So that no edge in G connects either 2 vertices in V_1 or 2 vertices in V_2)

V_1 (1, 3, 5)

V_2 (2, 4, 6)





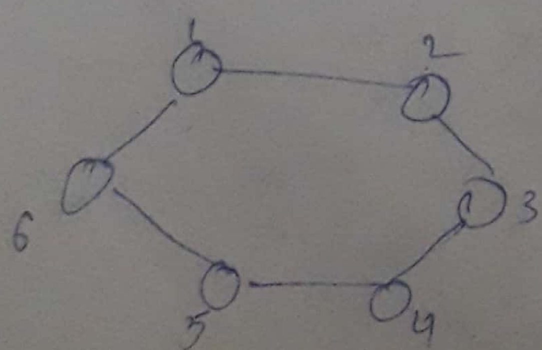
(a, b, d)

(g, f, c, e)

Theorem:-

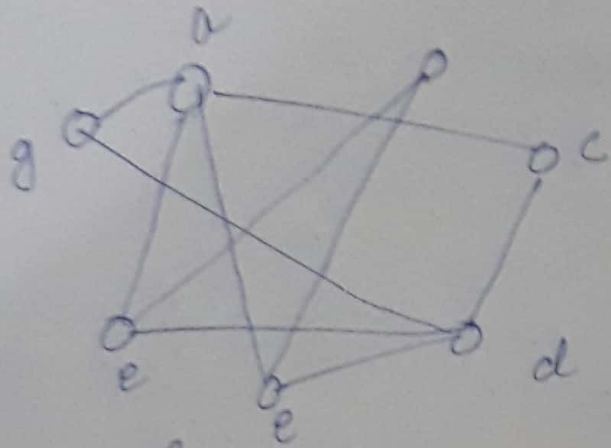
A simple graph is bi-partite iff and only if it is possible to assign one of 2 different colours to each vertex of the graph so that no two adjacent vertices are assigned the same colour.

<u>Complete Bi-Partite Graph</u> ($K_{m,n}$)



V_1 (1, 3, 5)
V_2 (2, 4, 6).
$K_{3,3}$

The complete Bi-partite graph $K_{m,n}$ is the graph that has its vertex set partitioned into 2 subsets of m and n respectively.



(a, b, d)

(e, c, d, g)

$K_{3,4}$.

Applications of Special graphs.

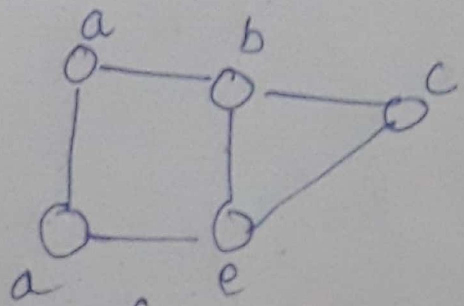
1) If there are m employees and j jobs in a company where $m < j$ so one employee can have more than one job. The vertices are disjoint, (one set of employees and other set of job so it is called Bi-partite graph).

ii) local area network.

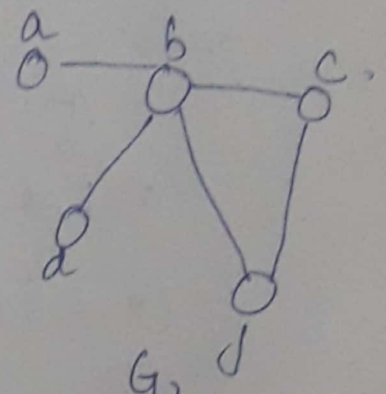
Union of 2 Graphs :-

Combination of two graphs and resulting in a single graph with all vertices and edges of 2 Graphs.

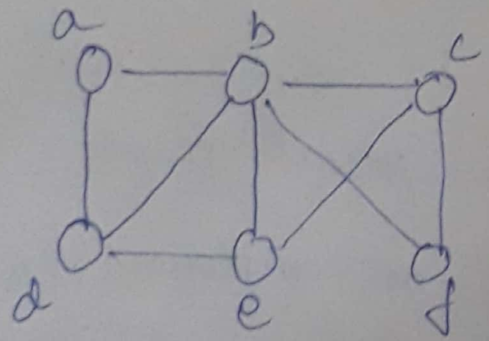
Ex:-



G_1



G_2

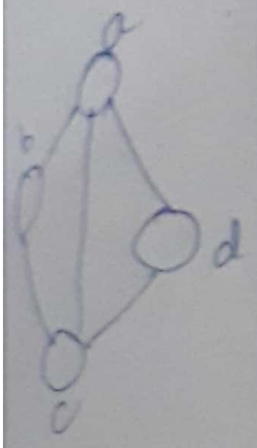


$G_1 \cup G_2$.

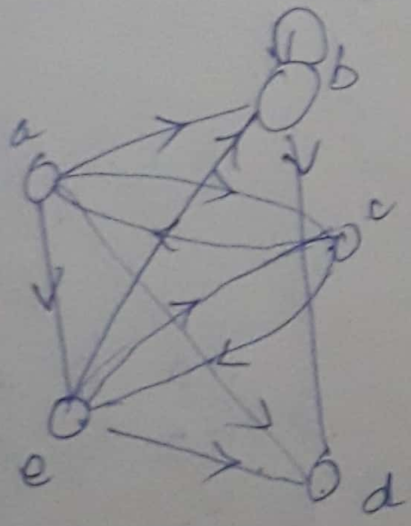
Representing Graphs.

1. To list all the edges $(e_1, e_2, e_3 \cdots e_n)$

2. Adjacency list.



vertices	adjacency list
a	b, c, d
b	a, c
c	a, b, d
d	a, c



vertices	adjacency list
a	b, c, d, e
b	b, d,
c	a, e
d	—
e	b, c, d

3. Adjacency matrix

1) Adjacency of vertices → dimension is according to vertices (n) then matrix is $n \times n$

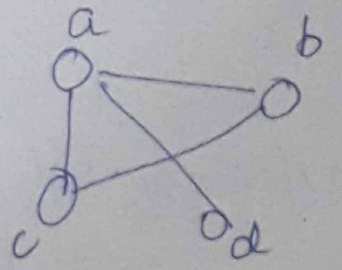
2) Incidence of edges and vertices.

Here we have to takes edges also (m) and vertices (n).

→ $n \times m$.

$a_{ij} - \begin{cases} 1 & \text{if } (i,j) \text{ is an edge of G} \\ 0 & \text{otherwise.} \end{cases}$

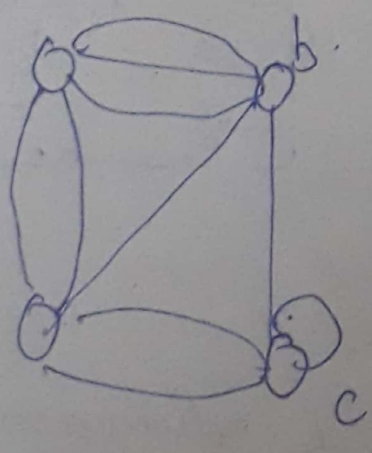
Ex:-



simple graph.

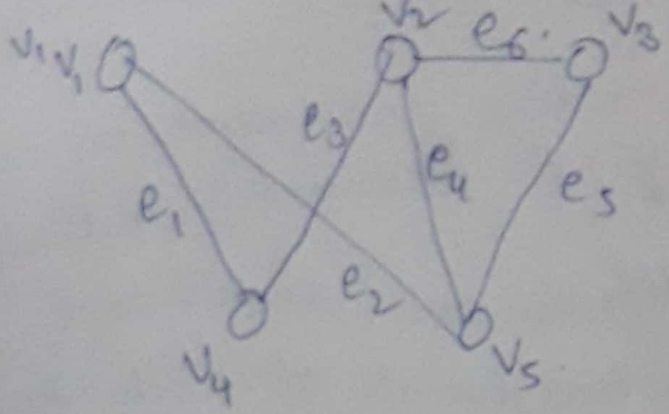
$$\begin{array}{c|cccc|} & a & b & c & d \\ a & 0 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 1 & 0 & 0 \\ d & 1 & 0 & 0 & 0 \end{array}$$

<u>pseudo graph</u>

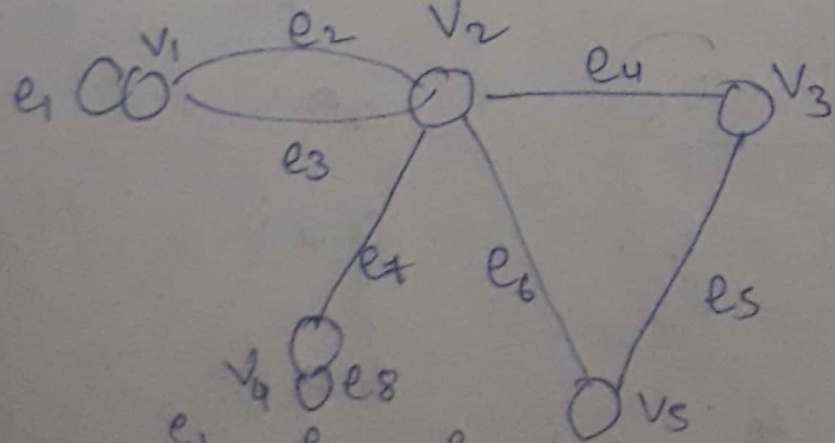


$$\begin{array}{c|cccc|} & a & b & c & d \\ a & 0 & 3 & 0 & 2 \\ b & 3 & 0 & 1 & 1 \\ c & 0 & 1 & 1 & 2 \\ d & 2 & 1 & 2 & 0 \end{array}$$

1) Incidence of Edges and vertices.



$$\begin{array}{c|c} & \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



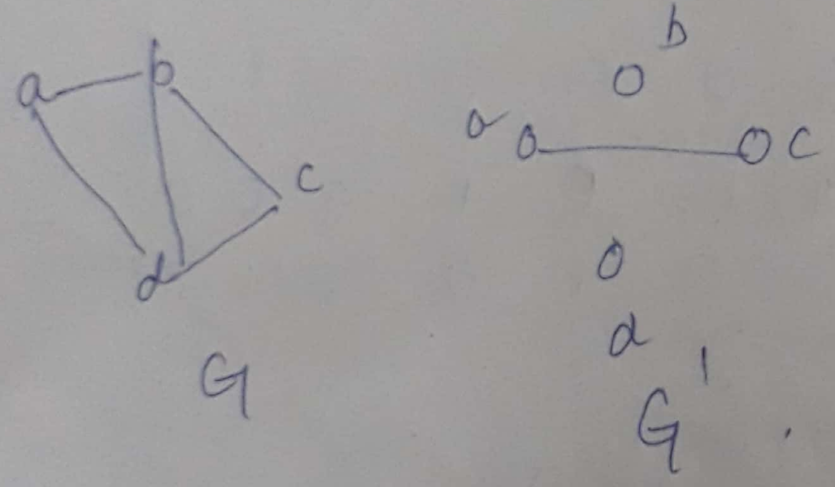
$$\begin{array}{c|c} & \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

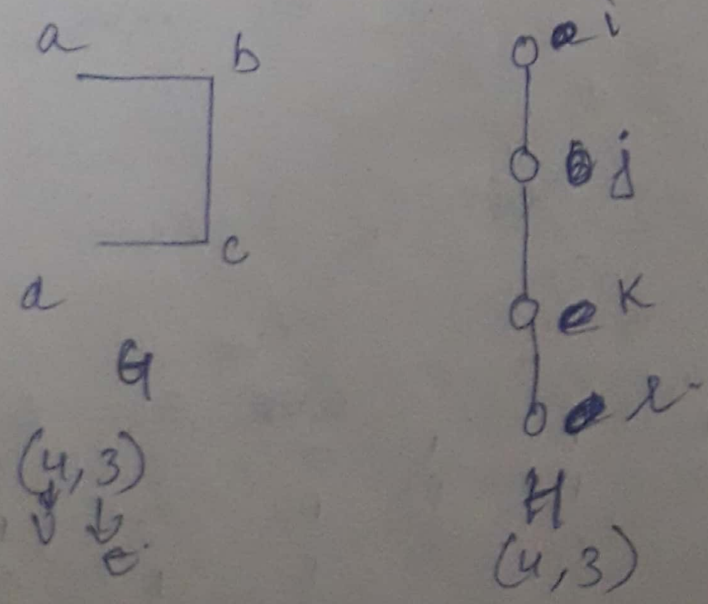
Isomorphism of graph

(same formulae but different structure.

same no. of vertices and same no. of edges and degree but different structure & and adjacency also should satisfies and length and compliment [do compliment of both then if their structure is same then it is isomorphic

compliment of graph.





degree of $G(a) = 1$ deg of $H(i) = 1$

$G(b) = 2$ $H(j) = 2$

$G(c) = 2$ $H(k) = 2$

$G(d) = 1$ $H(l) = 1$

now mapping.

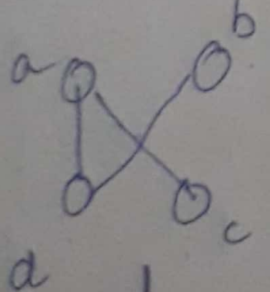
$f(a) = i$ $f(a) = l$

$f(b) = j$ (or) $f(b) = k$

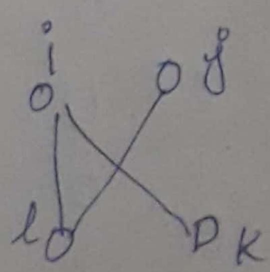
$f(c) = k$ $f(c) = j$

$f(d = l$. $f(d) = i$

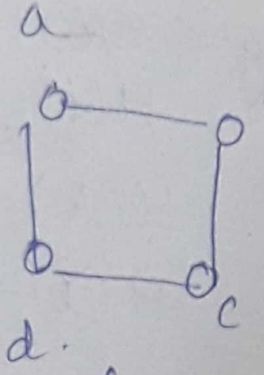
compliment



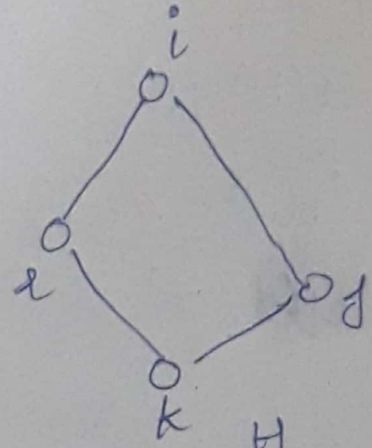
G'



H'



G



H

(4, 4)

(4, 4).

$f(a) = i$
$f(b) = j$
$f(c) = k$
$f(d) = l$

(or)

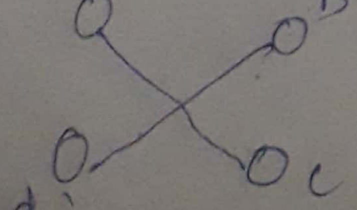
$f(a) = l$
$f(b) = k$
$f(c) = j$
$f(d) = i$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

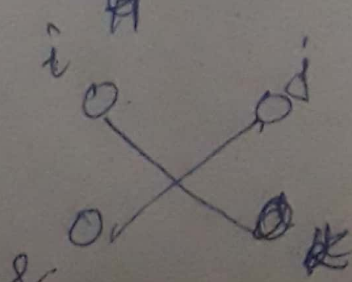
G

$$\begin{array}{c|cccc} & l & k & j & i \\ \hline l & 0 & 1 & 0 & 1 \\ k & 1 & 0 & 1 & 0 \\ j & 0 & 1 & 0 & 1 \\ i & 1 & 0 & 1 & 0 \end{array}$$

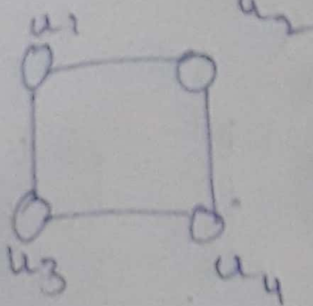
G'



H'

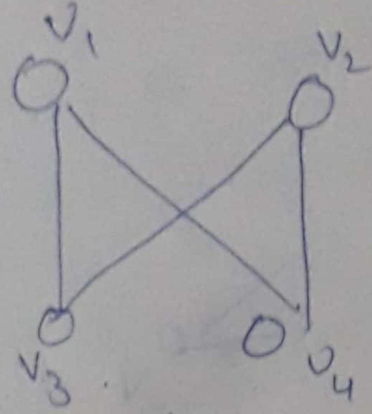


1. The two graphs are isomorphic



G

$(4,4)$



H

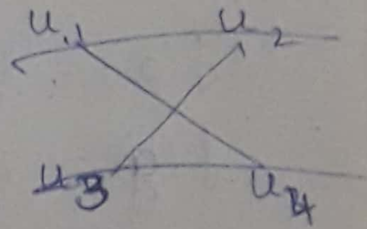
$(4,4)$.

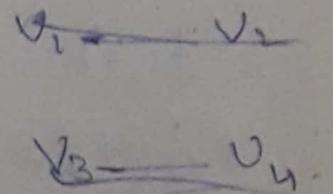
$f(u_1) = v_1$

$f(u_2) = v_3$.

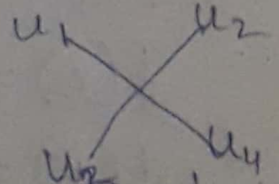
$f(u_4) = v_2$

$f(u_3) = v_4$



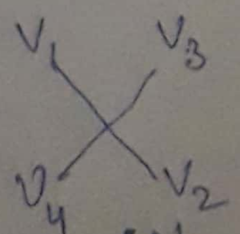


$$\begin{array}{c|cccc} & u_1 & u_2 & u_4 & u_3 \\ \hline u_1 & 0 & 1 & 0 & 1 \\ u_2 & 1 & 0 & 1 & 0 \\ u_4 & 0 & 1 & 0 & 1 \\ u_3 & 1 & 0 & 1 & 0 \end{array}$$

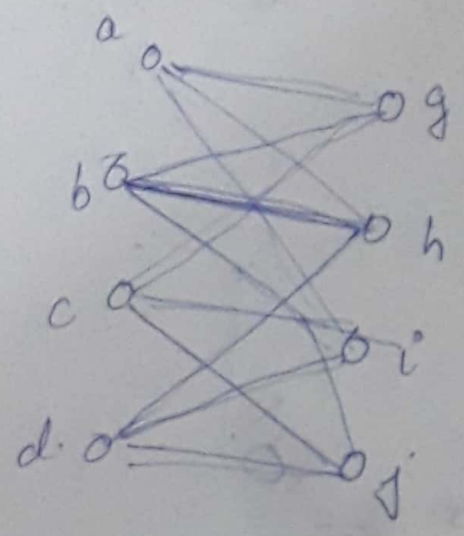


G'

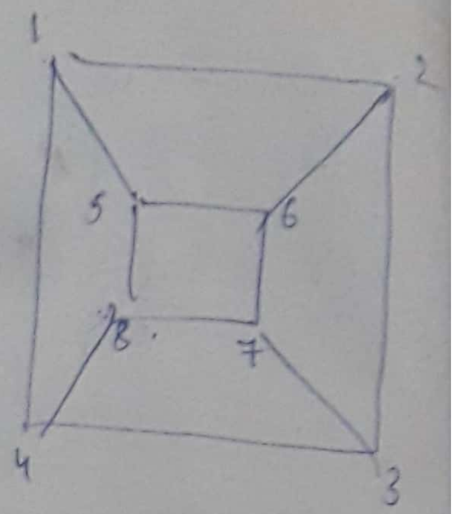
$$\begin{array}{c|cccc} & v_1 & v_3 & v_2 & v_4 \\ \hline v_1 & 0 & 1 & 0 & 1 \\ v_3 & 1 & 0 & 1 & 0 \\ v_2 & 0 & 1 & 0 & 1 \\ v_4 & 1 & 0 & 1 & 0 \end{array}$$



H'



(8, 12).



(8, 12).

d(a) = 1 1
d(g) = 2 3
d(b) = 3 6
d(h) = 4 8
d(d) = 8 2
d(j) = 5 4
d(c) = 6 5
d(i) = 7 7

$$
\begin{array}{c|cccccccc}
 & a & g & b & h & d & j & c & i \\
\hline
a & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
g & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
b & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
h & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
d & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
j & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
c & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
i & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0
\end{array}
$$

path:-

let 'n' be a non-negative Integer and 'G' on undirected graph

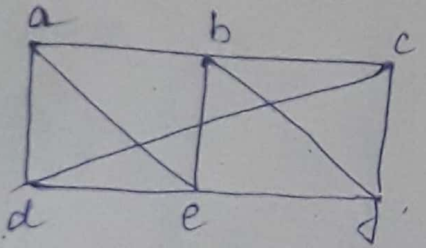
circuit: path is a circuit when starting point to ending point will be the same

$\boxed{u = v}$

→ path or circuit is said to be simple if the one of ~~the~~ edge is not repeated more than once.

★ only edge b/w two circlices

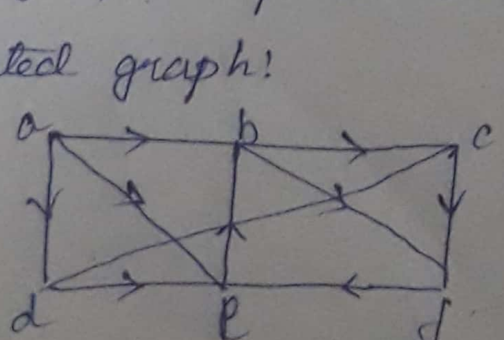
Eg:



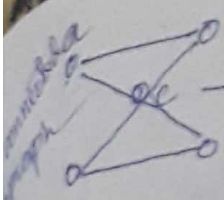
a, d, c, f, e → length = 4
a, e → 1
} simple path

a, d, e, c - not a path
a, b, e, d, a - circuit
* b, c, f, e, b - circuit → length = 4.
becoz begins at 'b' and ends at 'b'.
* a, b, e, d, a, b - not a circuit.
coz a, b - repeated twice.

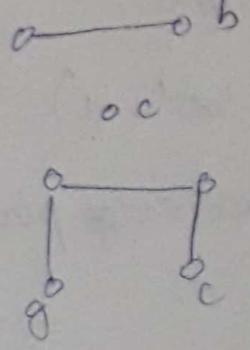
directed graph:



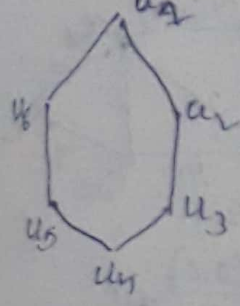
If there is a path b/w all the distinct vertices then an undirected graph is said to be connected graph.



→ arthimetic point.



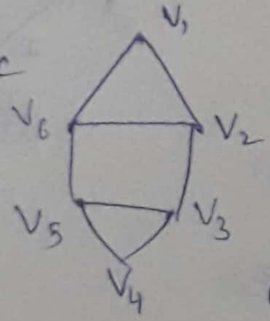
Isomorphic using path:



Degree (2, 3, 3, 2, 3, 3)
circuit (u_1, u_2, u_3, u_6, u_1)
length - 4

V - 6, E - 8

not isomorphic

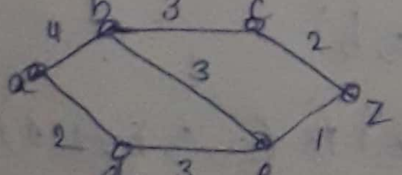


(∵ ckt length are not mapping)
V - 6, E - 8
Degree - (2, 3, 3, 2, 3, 3)
circuit (V_6, V_2, V_3, V_5, V_6)
length 4.
circuit (V_1, V_2, V_6, V_1)
length = 5.

shortest path problem:

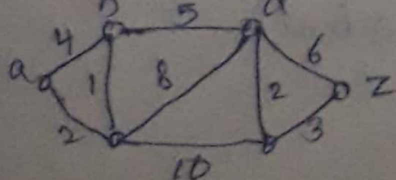
choosing shortest path:

→ Dijkstras Algorithm:-



$a \to z$: a - d - e - z
$a \to z$: 2 + 3 + 1 = 6
distance

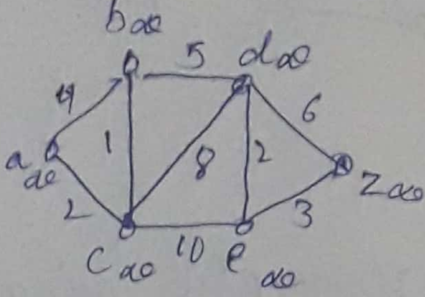
Q)



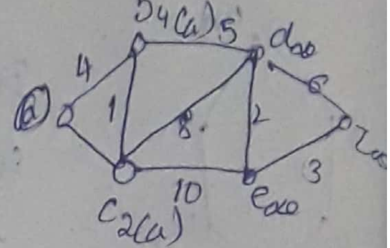
use dijkstras algorithm to find the length of the shortest path b/w the vertices a and z in the weighted graph -

(system '∞' - - 1)

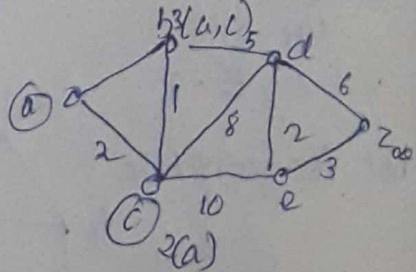
→ step I each eadge place '∞'



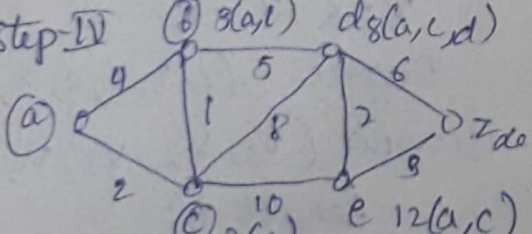
→ step II starting vertix make it a circle and mark length



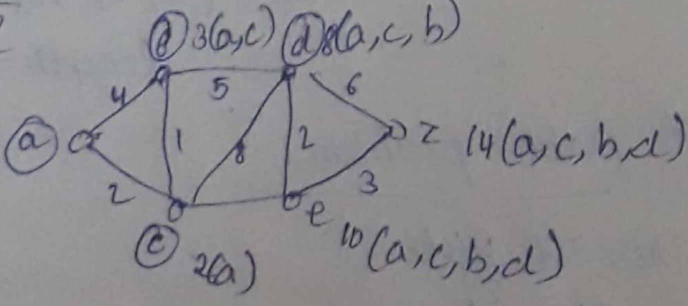
→ step III select shortest path i.e minimum length and circle it.



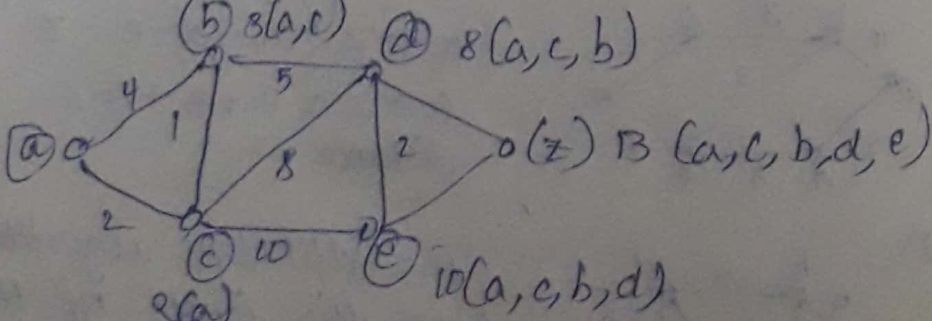
→ step-IV



→ step V



→ step-VI



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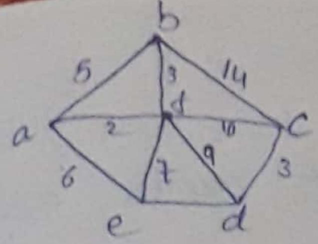
Q)



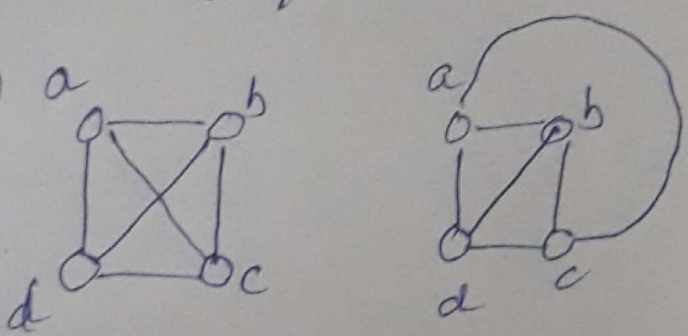
Planar Graphs

When 2 edges does not cross each other is called planar graph.

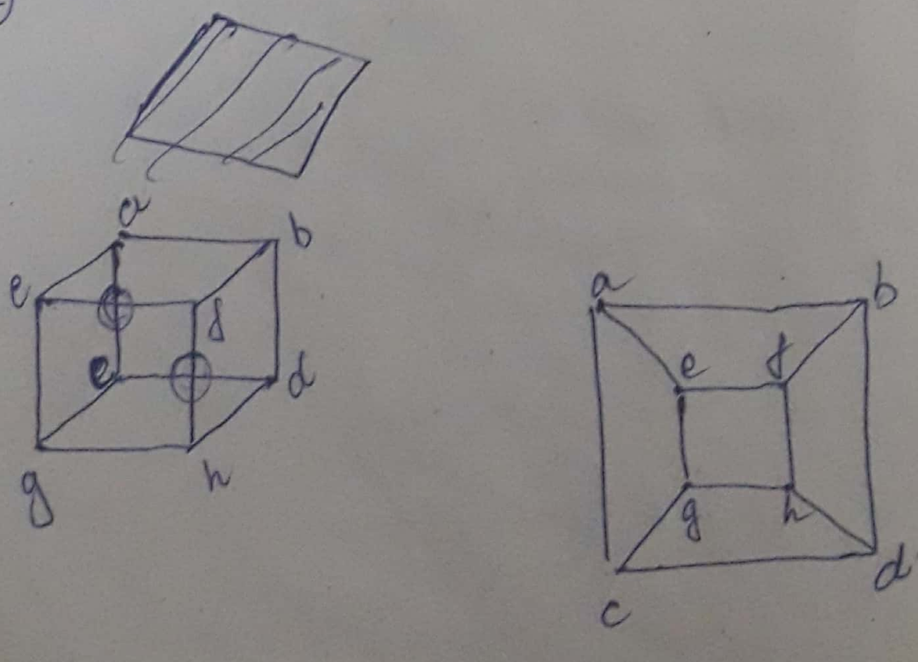
(or)

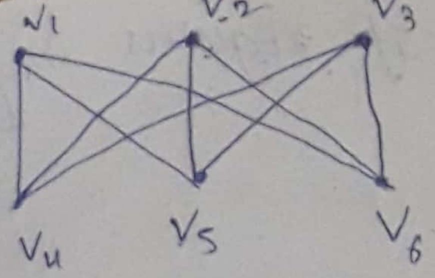
If the edges cross each other, then if we can re-draw the graph such that no 2 edges cross each other then it is called planar graph.

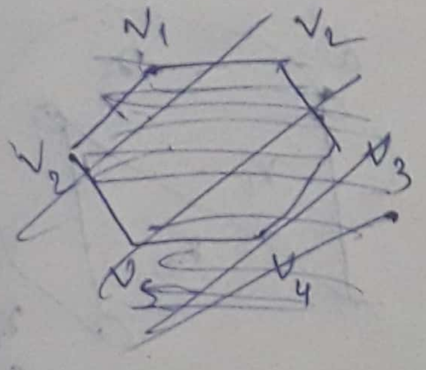
Ex:- ①

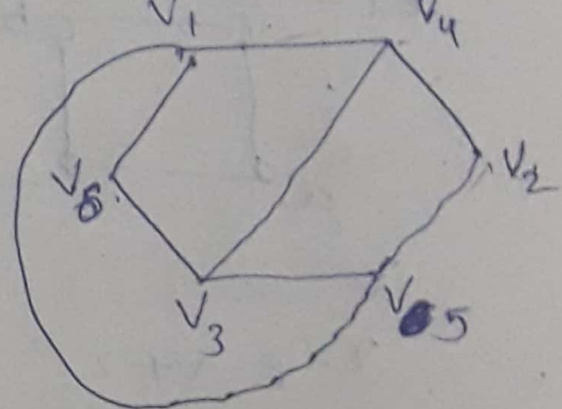


②









It is not plannar graph.

Euler's formula

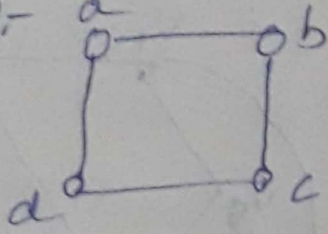
A plannar representation of a graph splits the plane into regions including an undounded region

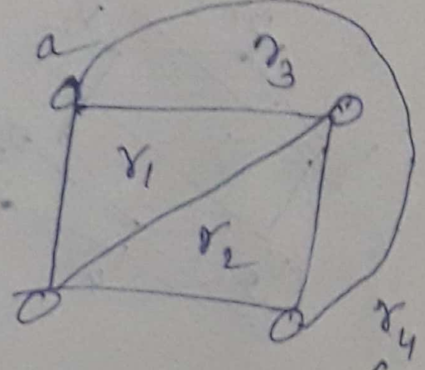
Euler showed that all planar repr-esentation of a graph split the plane into sum of regions. He accomplished this by finding a relation, ship among no. of regions, no. of vertice and the no. of edges.

Let G be a connected planar simple graph with E edges and V vertices let r be the no. of regions in plane representation of G

Then $\boxed{r = e - v + 2}$

Ex:-





$r = e - v + 2$

$= 6 - 4 + 2$

$r = 4$

unbounded region.

(It complusory it include unbounded region in any graph)

Suppose that a connected plannar simple graph as 20 vertices each of degree 3 into how many regions does our representation of these planar graph split the plane.

Sol: sum of the degrees of graph $= \begin{array}{r} 20 \\ \times 3 \\ \hline 60 \end{array} = 60$

$\therefore 2e = $ s.o.t.d.o.g $= 60$

$2e = 60$

$\boxed{e = 30}$

$r = 30 - 20 + 2$

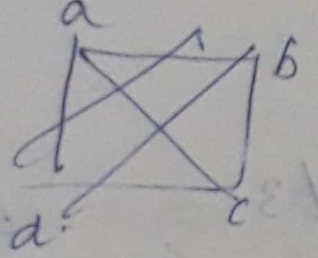
$= 10 + 2$

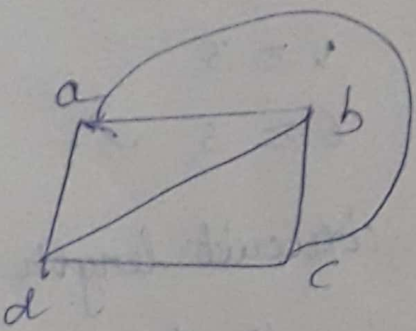
$= 12$

Theorem 1:-

If 'G' is a connected planar simple graph of 'e' edges and 'v' vertices where $\boxed{v \geq 3}$ then $\boxed{e \leq 3v-6}$

Ex:-





$v \geq 3$

$4 \geq 3$ then $e \leq 3v-6$

$6 \leq 3(4)-6$

$6 \leq 6$.

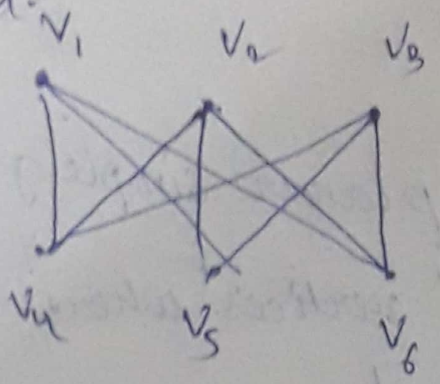
Theorem 2:-

If G is a connected plannar simple graph then G has a vertex of degree not exceeding 5.

If a connected planar simple graph has 'E' edges and 'V' vertices with $v \geq 3$ and no circuits of length 3 then

$e \leq 2v-4$

① Ex:-



$v \geq 3$

$6 \geq 3$ ✓

cirucuit length $\neq 3$.

$4 \neq 3$ ✓

circuit ($V_1 - V_6 - V_3 - V_4 - V_1$)

$e \leq 2v - 4$.

$9 \leq 2(6) - 4$

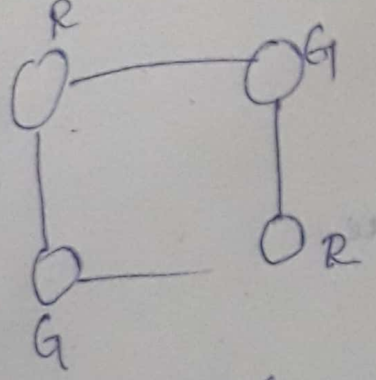
$9 \leq 12 - 4$

$9 \leq 8$ ✗.

$\therefore$ It is not plannar

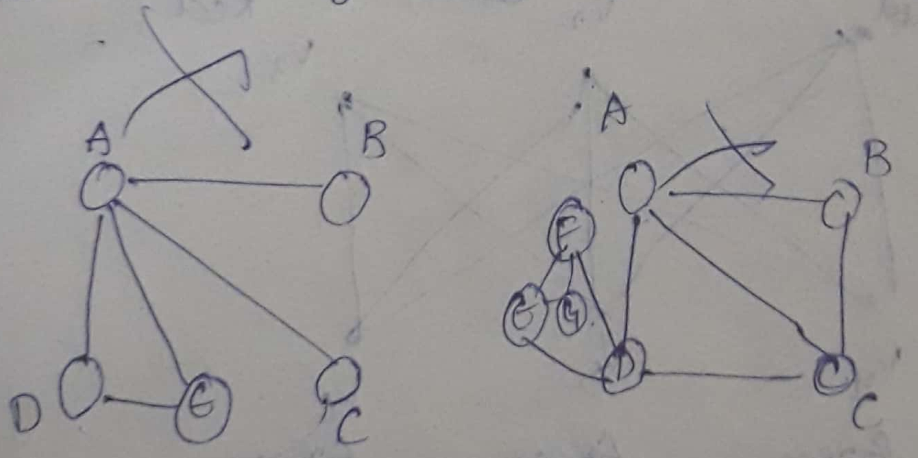
Graph coloring (chromatic number)

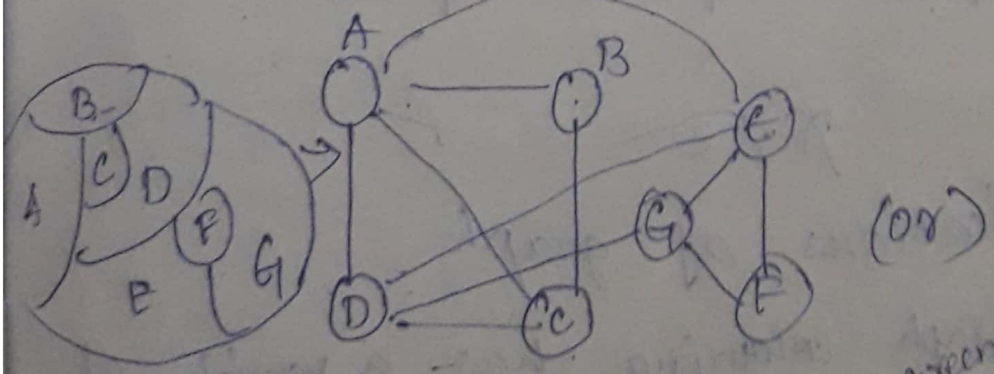
minimum no. of colours required to color the graph such that no 2 adjacent vertices have same color

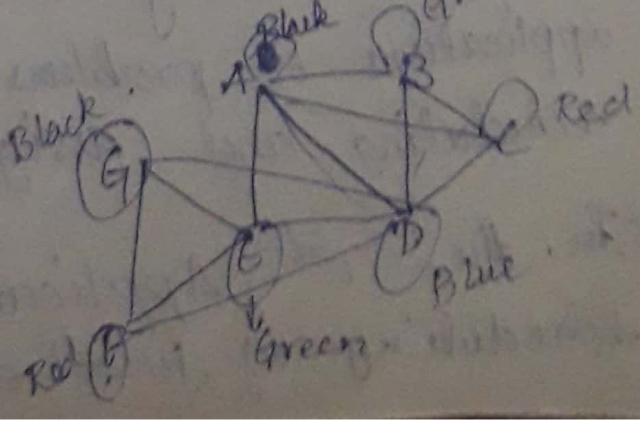


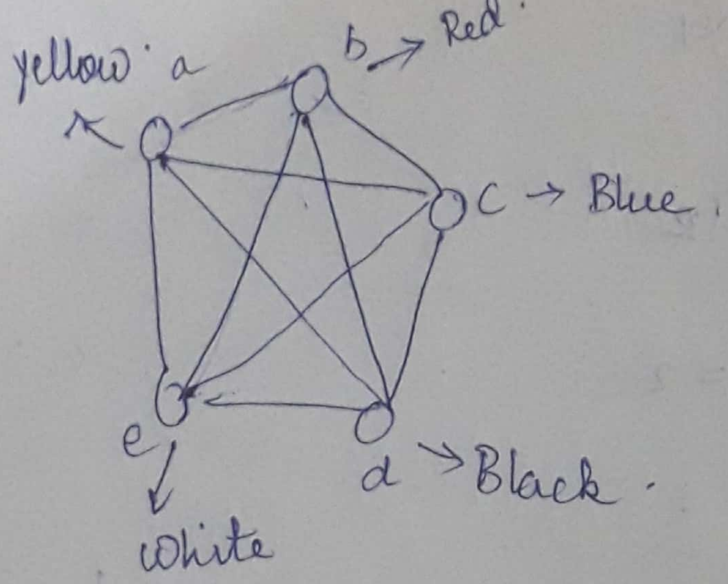
$\chi = 2$
↓
chi

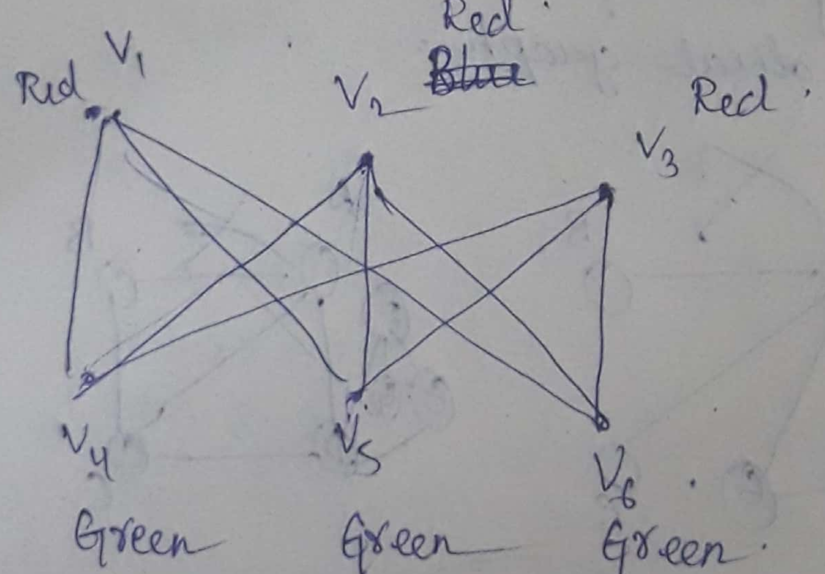
→ converting a map into a graph is called dual graph











$\chi = 2$

Applications of graph

i) → Graph colouring has a variety of applications to problems involving scheduling and assignments.

→ In the 1st Application deals with scheduling of final exams.

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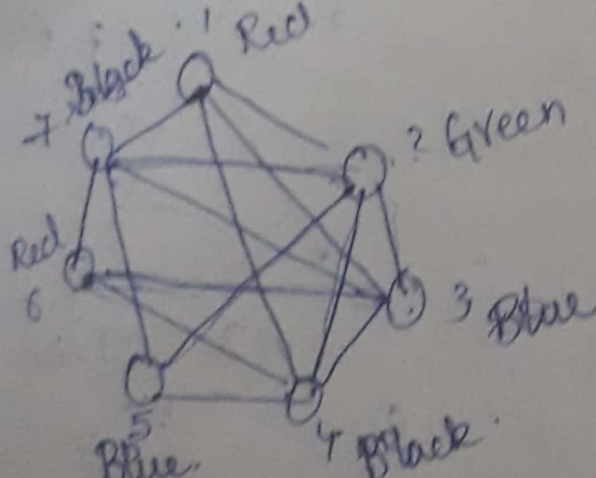
How can the final exam at a university be scheduled so that no student has 2 exams at the same time

Suppose that the following pairs of courses have common students.

1 and 2
1 and 3
1 and 4
1 and 7
2 and 3
2 and 4
2 and 5
2 and 7
3 and 4
3 and 6
3 and 7
4 and 5
4 and 6
5 and 7
6 and 7.



$\chi = 4$.

we have to create 4 slots

I - 1, 6
II - 2
III - 4, 7
IV - 3, 5.