17-12-19

Unit - I The Foundations.

Logic and proofs.

I Logic.

→ Proposition logic

A declarative sentence that can said to be either True or false but not both is known as proposition or statement.

Examples

① Delhi is the capital of India → True

② Bombay is the capital of Telangana → False

③ $x + y = 5$ → True / False

④ $x + y = 5$ where $x = 2$ & $y = 4$ → False.

⑤ Are you going → True / False.

1, 2, 4 are propositions and 3, 5 are not propositions.

→ The Truth or falsity of a proposition is called its truth value

- True indicated by T (or) 1
- False indicated by F (or) 0

Propositional Variables: We use letters to denote propositions. That we call as propositional variables.

Ex: p, q, r, s ..... (or) P, Q, R, S ....

Question 1: Identify the propositions in the following declarative sentences.

1. Washington DC capital of U.S → T
2. What time is it? – T/F
3. Thronto is the capital of Canada. – T
4. Read this carefully. – T/F
5. $1+1=2$ – T
6. $2+2=3$ – F
7. $x+1=2$ – T/F
8. $x+y=z$ – T/F

Sol:

1, 3, 5, 6 are propositions and 2, 4, 7, 8 are not proposition.

logical Operators (or) logical connectives

The words are used to make a proposition by combining 2 (or) more propositions are called logical connectives.

- Simple proposition

Without the logic connectives proposition are called Simple proposition.

- Compound proposition

The new proposition obtained by the use of connectives are called the compound proposition.

Types of logical connectives: There are 5 types of logical connectives.

i) Negation ($\neg, \sim, -$)
ii) Conjunction ($\wedge$)
iii) Disjunction ($\vee$)
iv) Implication / conditional ($\rightarrow$)
v) Bi-implication / Bi-conditional ($\leftrightarrow$)

i) Negation ($\sim, \neg, -$) It is a unary operations.

The word "NOT" is inserted at the proper position in the proposition is called "negation of the proposition".

If P is a proposition then the negation of P is denoted as

→ $\sim P$
→ $\neg P$
→ $\overline{P}$

Truth Table:

P	$\sim P$
T	F
F	T

Example:- Find the negation of the Proposition

i) Today is Friday.

Sol i) Today is not Friday.

ii) It is not the case that Today is Friday

iii) It is not Friday ~~Triday~~.

ii) 8 is a Prime number

8 is not a Prime number

ii) Conjunction ($\wedge$)

It is a binary operation

~~The~~ A compound proposition is obtained by combining 2 given proposition by inserting the word "AND" in between them is called Conjunction

· If p and q are two proposition, then conjunction of p and q are denoted by "$p \wedge q$".

p - antecedent (or) hypothesis (or) premises

q - conclusion (or) consequence

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example :- Find the conjunction of following proposition

i) p : Today is Friday

q : It is raining Today

Sol :- $p \wedge q$: Today is friday and It is raining.

ii) p : $\sqrt{2}$ is a irrational number

q : 9 is a prime number

Sol :- $\sqrt{2}$ is a irrational number and 9 is a prime number.

iii) Disjunction :- ($\vee$)

It is binary operator.

A compound proposition obtained by combining the 2 given operations by inserting the word "OR" in between them is called Disjunction of given propositions.

- If P, q are 2 propositions the disjunction of p and q are denoted by "$p \vee q$".

Truth Table:

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:- Find the disjunction of the following proposition.

i) p: Today is Friday
q: It is raining Today.

sol:- Today is Friday or it is raining.

Exclusive OR : ($\oplus$, $\veebar$)

The Exclusive "OR" of p and q denoted by $p \veebar q$ / $p \oplus q$ is the proposition i.e true when excatly one of p and q is true.

Truth Table:

p	q	$p \veebar q$ / $p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

iv) Implication / conditional ($\rightarrow$)

A compound proposition obtained by combining 2 given proposition by using the words If and then at the appropriate place then it is called Conditional/ implication.

The condition of two propositions p, q is denoted by "$p \rightarrow q$" and read as "If p then q" or "p implies q".

Truth Table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example :- Find the conditional of following propositions.

P: Amulya works hards

q: Amulya will pass the exam.

Sol:- If Amulaya works hards then she will pass the exam.

v) Bi-Implication / Bi-conditional ($\leftrightarrow$)

A compound proposition is obtained by combining 2 given propositions by using the word "if and only if" at the proper position is called Bi-conditional / Bi-implication.

If p and q are 2 propositions then the bicondition of p and q is denoted by. $p \leftrightarrow q$.

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Example :- for each of these sentences determine whether an inclusive OR ~~(or)~~ (or) exclusive OR

i) A coffee or tea comes with dinner.

ii) A password must have atleast 8 digits or be atleast 8 characters

sol i) Exclusive "OR"

ii) Inclusive "OR".

Question: Construct the truth Table of the compound proposition.

$$(p \land \neg q) \rightarrow (p \land q).$$

sol:

p	q	$\sim q$	① $p \land \sim q$	② $p \land q$	① $\rightarrow$ ②
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	F	F	T

Question: find the conditional statement of following

i) p: Maria learns Discrete mathematics
q: Maria will find a Good job.

Sol:- If Maria learns DM then she will find a good job.

(or)

Maria will find a good job when she will learn discrete mathematics.

(or)

Maria will find a good job unless she doesnot learn discrete mathematics.

Converse, Inverse and Contrapositive

$p \rightarrow$ hypothesis (or) antecedent

$q \rightarrow$ conclusion (or) consequence

Converse: If $p \rightarrow q$ is a conditional statement then $q \rightarrow p$ is called the converse of $p \rightarrow q$

Inverse: If $p \rightarrow q$ is a conditional statement then $\neg p \rightarrow \neg q$ is called the inverse of $p \rightarrow q$.

Contrapositive: If $p \rightarrow q$ is a conditional statement then $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$.

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Question: Write the Converse, inverse, and contrapositive of the following statement.

i) The home teams wins when ever it's raining'

sol! $p \to q$

If it is raining then the home team wins.

i) p: It is raining

q: The home team wins.

Converse: $q \to p$

If the home team wins then it is raining.

Inverse: $\neg p \to \neg q$

If it is not raining then the home team does not wins.

Contrapositive: $\neg q \to \neg p$

If home team does not wins then it is not raining.

ii) If it is sunny today then we will go to beach.

p:- It is sunny today

q:- we will go to beach.

Converse: $q \rightarrow p$.

· If we go to beach then it is sunny today.

Inverse: $\neg p \rightarrow \neg q$.

If it is not sunny today then we won't go to beach today.

Contrapositive: $\neg q \rightarrow \neg p$.

If we won't go to beach then it is not sunny today.

Question: Construct the truth table for ~~and~~

i) $p \wedge \sim q$

ii) $(\sim p) \vee q$

iii) $(\sim p) \veebar (\sim q)$

iv) $p \rightarrow \sim q$.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$\sim p \veebar \sim q$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	F

$p \to \sim q$
F
T
T
T

Construct the truth Table for the following.

i) $q \wedge [(\sim r) \to p]$

ii) $p \to [q \to (\sim r)]$

iii) $p \to (q \wedge r)$

4) $(\sim p) \Leftrightarrow (q \vee r)$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim r \to p$	$q \wedge (\sim r \to p)$
T	T	T	F	F	F	T	T
T	F	F	F	T	T	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	F
T	T	F	F	F	T	T	T
T	F	T	F	T	F	T	F
F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	F

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$q \to \sim r$	$p \to [q \to (\sim r)]$	$q \wedge r$	$p \to (q \wedge r)$
F	F	T	T
T	T	F	F
T	T	F	T
T	T	F	T
T	T	F	F
T	T	F	F
F	T	T	T
T	T	F	T

$q \vee r$	$\sim p \leftrightarrow (q \vee r)$
T	F
F	T
T	T
T	T
T	F
T	F
T	T
F	F

Precedance of logical operators.

1	$\neg$	negation
2	$\wedge$	AND
3	$\vee$	OR.
4	$\to$	Implication
5	$\leftrightarrow$	Bi-implication

Ex

$(p \vee q) \wedge (\neg r)$

well formed

Question :- Convert the english sentence into symoblic.

i) Identity atomic proposition and represent propositional variables.

ii) Determine logical operators.

Express the following statements in symbolic form:

i) If Ravi does not visit a frd this evening than he studies this evening.

ii) If there is a cricket telecast this evening than ravi does not study and does not vista frd this evening.

iii) If there is no cricket telecast this evening than ravi does not study but visit a frd this evening.

iv) There is no circkect telecast this evening but ravi doesnot ~~visit~~ study and doesnot visit a frd.

p: Ravi does not study this evening.

q: Ravi visit a friend this evening

r: There is a cricket telecast this evening.

Sol:- i) $\neg q \rightarrow \neg p$.

ii) $r \rightarrow (p \wedge \sim q)$

iii) $\sim r \rightarrow (p \wedge q)$

iv) $\sim r \wedge (p \wedge \sim q)$

Write the following statement in symbolic form

i) if either Jerry takes calculus or Ken takes sociology then larry will take english

p: jerry takes calculus
q: Ken takes sociology
r: larry will take english.

$p \rightarrow (q \veebar r)$.

Consider the following propositions p, q, r.

p: A circle is a conic.
q: ~~Yo~~ $\sqrt{5}$ is a irrational number.
r: expontenial series ~~is~~ convergent.

Explain the following propositions in words.

i) $p \wedge (\sim q)$
ii) $(\sim p) \vee q$
iii) $p \veebar \sim q$
iv) $p \rightarrow (q \veebar r)$
v) $q \rightarrow (\sim p)$.

vi) $(\sim p) \leftrightarrow q$

sol:- A circle is a conic and $\sqrt{5}$ is not a irrational number

2:- A circle is not a conic or $\sqrt{5}$ is a Irrational number

3:- either circle is a conic or $\sqrt{5}$ is not a irrational number.

4:- ~~either~~ If circle is a conic then either $\sqrt{5}$ is a irrational number or exponential series is convergent.

5. If $\sqrt{5}$ is a irrational number then circle is not conic.

6. ~~If~~ A circle is not conic if-en-d only -if $\sqrt{5}$ is a irrational number

Let p and q be primitive state. for which the implication $p \to q$ is false. determine the truth values of following

i) $p \wedge q$

ii) $\neg p \vee q$

iii) $q \to p$

iv) $\neg p \to \neg q$

Sol :-

If p, q are primitive stmts.

then $p \to q$ are implication.

$p \to q$ is false when p is true and q is false.

p : T
q : F.

Since $p \to q$ is given to be false, Hence p is to be true and q is to be false. consequently $\sim p$ should be false and $\sim q$ should be true.

i) The truth value $p \wedge q$ is False.

$$p \wedge q \Rightarrow T \wedge F \Rightarrow F.$$

ii) The truth value $\neg p \vee q$ is False.

$$\neg p \vee q = F \vee F \Rightarrow F.$$

iii) $q \to p$.

The truth value of $q \to p$ is True

$$q \to p = F \to T \text{ is True.}$$

iv) $\neg p \to \neg q$

The truth value of $\neg p \to \neg q$ is True.

$$\neg p \to \neg q = F \to T \Rightarrow T.$$

find the possible truth values of p, q and r in the following cases.

i) $p \to (q \vee r)$ is false

ii) $p \wedge (q \to r)$ is true

sol:- $p \to (q \vee r)$ can be false only when p is true and $(q \vee r)$ is false. also $(q \vee r)$ is false only when q is false and r is false.

Hence the truth values of p, q, r are T, F, F respectively.

$p \wedge (q \to r)$ can be True only when p is true and $(q \to r)$ is true. also $(q \to r)$ is true when r is true and q is T or F. If r is false then q is false.

Hence the possible truth values of p, q, r are

p	q	r
T	T	T
T	F	T
T	F	F

Indicate how many rows are needed for truth table of the compound propsition.

$$(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}.$$

find the truth value of the proposition if p and r are true and q, s, T are false.

The given compound proposition contains 5 primitive p, q, r, s & t ∴ the no. of possible combinations of the truth values of these components are $2^5 = 32$ ∴ Hence 32 rows

~~Pu~~ suppose p, ~~r~~ are true and q, s, T are false then

$\sim q$ is T and $\sim r$ is F. Since p is true $\neg q$ is true then $(p \vee \neg q)$ is true.

The $\sim r$ is false and s is false so $(\neg r \wedge s)$ is false.

t is false hence $((\neg r \wedge s) \rightarrow t)$ is True

$(p \vee \neg q)$ is true and $((\neg r \wedge s) \rightarrow t)$ is true then it follows the Truth of the given porposition is True.

$\therefore (p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}$.

Applications of propositional logic

1. Translatting english sentences.
2. System specifications.
3. Boolean searches.
4. Logic puzzles.
5. Logic circuits and Bit operations.

1 Translatting english sentences.

This process helps to clarify what is meant by each sentences after translation we can analyze the argument for validity.

Ex:- How can this english sentences be translated into a logical expression. i) You can access the internet from the campus only if You are a computer science major (or) ~~not~~ you are not a freshman.

p: You can access the internet from the campus only

q: You are a computer science major.

r: You are not a freshman.

$p \rightarrow (q \vee r)$.

ii) You can't ride the roller coaster if u are under 4 feet tall unless you are older than 16 yrs old.

p: You can't ride the roller coaster
q: You are under 4 feet tall
r: You are older than 16 yrs old.

$(q \wedge r) \rightarrow p$

iii) The automated reply cannot be send when the file system is full.

p: The automated reply cannot be send
q: the file system is full.

$q \rightarrow p$.

System Specifications

A list of propositions is consistence if it is possible to assign truth values to the porposition variables so, that each proposition is true

Example: determine whether these system specifications are consistent.

i) The Diagonistic message is stored in the buffer or it is retransmitted.

ii) The diagonistic message is not stored in the buffer.

iii) The diagonstic message is ~~not~~ stored in the buffer than retransimitted.

sol: i) p: the Diagonistic message is stored in the buffer

q: It is retransmitted

i) $p \vee q$

ii) $\neg p$ $\quad\quad P \Rightarrow F$

iii) $p \rightarrow q$ $\quad\quad q \Rightarrow T$

The specifications can be written as $p \vee q$, $\neg p$, $p \rightarrow q$ an assignment of truth value that makes all 3 specifications true must have p false to make $\neg p$ True.

we want $p \vee q$ to be true but p must be false and q must be true. because $p \rightarrow q$ is true when p is false q is true.

we can conclude these specifications are consistent because they all true when p is false and q is true

To the System Specification in the previous ~~expamle~~ example remain consistent if the specification the diagonistic message is not retransmitted is added.

$p \vee q, \neg p \; p \rightarrow q, \neg q$

When p is false and q is true then $\sim q$ is false so these 4 specifications are inconsistent

Boolean searches :-

searches the things like web or large datatbases such as our library catlogs often can done using logical operators.

→ logic puzzles

puzzles can be solved by the rules of logic

→ puzzles that can be solved using logical ~~reatings~~ reasonings are knowns as logic puzzles.

An island that has 2 kinds of inhabit-ance Knight who always tell the truth and their opposite knaves allways ~~tell t~~ lie. You encounter 2 people a & b who are a and b. If a says be b is a knight.

and b says 2 of us are of opposite type

~~Kni~~ Kings

a	b
Knight	Knight ✗.
Knight	naive ✗.
naive	Knight ✗.
naive	naive ✓.

Sol:- Let p and q is the statements that a is a knight and b is knight respectively and ¬p and ¬q are the statement tha a is naive and be is knave.

we 1st consider the posibility that a is night and p is true.

is a is a night then he is telling the truth when ~~be~~ he says that ~~be~~ is a knight then q is true and A and B are same type.

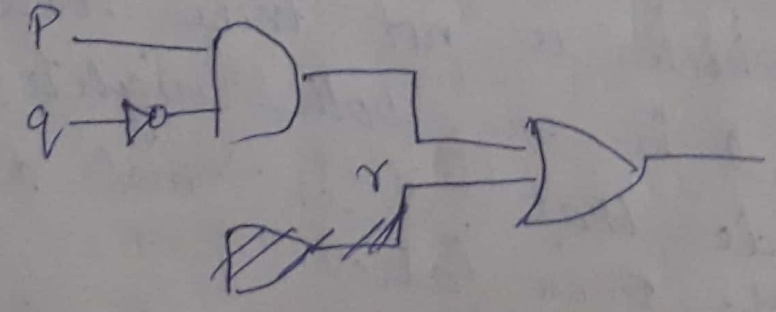
If b is a knight the b statement that a and b are both ~~are~~ opposite type which is not true because a and b are both nights so we conclude that a is not a knight i.e (the p is false).

If a is knave then because everything that knave says is false. A statement that b is a knight that is the q is true is a lie. which means the q is false and b also a nave.

If b is knave then b statement that a & b are opposite types is a lie which is consistent with both a & b being knaves. We can conclude both a and b are naves.

Logic circuits and bitwise operators:
porpositional logic can be applied to design the computer hardware. A logic circuit receives input signal as a bit either 0 or 1 and produce the output signals

$(p \wedge \neg q) \vee r$



Propositional Equivalence:- $\Leftrightarrow$

Two logical expressions are said to be equivalence. If they are have sam truth value is all cases.

Types of Propositions based on the truth values

1) Tautology
2) Contradiction
3) Contigency

i) Tautology:

A compound proposition which is true for all possible values of its components is called tautology (or) logically truth (or) universally valid statement. It is denoted with T_0.

ii) Contradiction:

A compound proposition which is false for all possible values of its components is called a contradiction. It is denoted with F_0

iii) Contigency.

A compound proposition that can be T or F depending upon the truth values of its components is called as contigency. It is neither tautology or contradiction.

NOTE: The compound propositions p and q are called logically equivalent If $p \leftrightarrow q$ is a tautology.

There are 2 methods to determine whether the e statement formulae are equivalent of not.

i) Truth Table method.

ii) Replacement process (i.e using logical equivalence formulae).

Prove the following are ~~equ~~ logically equivalent:

i) $(p \to q) \Leftrightarrow (\sim p \vee q)$

ii) $p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$

iii) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

iv) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

sol:-

P	q	$p \to q$	$\sim p$	$\sim q$	$\sim p \vee q$
1	1	T	0	0	1
1	0	F	0	1	0
0	1	T	1	0	T
0	0	T	1	1	T

$\therefore (p \to q) \Leftrightarrow (\sim p \vee q)$.

p	q	r	$\sim p$	$\sim q$	$\sim r$	$(q \wedge r)$	$p \to (q \wedge r)$
0	0	0	1	1	1	0	1
0	0	1	1	1	0	0	1
0	1	0	1	0	1	0	1
0	1	1	1	0	0	1	1
1	0	0	0	1	1	0	0
1	0	1	0	1	0	0	0
1	1	0	0	0	1	0	0
1	1	1	0	0	0	1	1

$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
1	1	1
1	1	1
1	1	1
1	1	1
0	0	0
0	1	0
1	0	0
1	1	1

$\therefore p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$

iii) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

$\therefore \neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$
0	0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0	0
0	1	0	1	0	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	0	1	1
1	1	1	0	0	0	1	1	1

$p \vee r$	$(p \vee q) \wedge (p \vee r)$
0	0
1	0
0	0
1	1
1	1
1	1
1	1
1	1

$\therefore p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

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Prove the following are Tautologies

i) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

ii) $[\sim(p \vee \sim q)] \to \sim p$

iii) $[p \wedge (p \to q)] \to q$.

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \to q$	$q \to r$
0	0	0	1	1	1	1	1
0	0	1	1	1	0	1	1
0	1	0	1	0	1	1	0
0	1	1	1	0	0	1	1
1	0	0	0	1	1	0	1
1	0	1	0	1	0	0	1
1	1	0	0	0	1	1	0
1	1	1	0	0	0	1	1

$p \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
1	1	T
1	1	T
1	0	T
1	1	T
0	0	T
1	0	T
0	0	T
1	1	T

$[(\sim q) \wedge (p \to q)] \to \sim p$.

p	q	$\sim p$	$\sim q$	$p \to q$	① $\sim q \wedge (p \to q)$	① $\to \sim p$
0	0	1	1	1	1	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	1	0	0	1	0	1

iii) $[p \wedge (p \to q)] \to q$.

p	q	$\sim p$	$\sim q$	$p \to q$	① $p \wedge (p \to q)$	① $\to q$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

Prove the following are controdiction.

i) $p \wedge [(\sim p) \wedge q]$

ii) $[(\sim q) \wedge p] \wedge q$.

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i)

p	q	$\sim p$	$\sim q$	$(\sim p)\wedge q$	$p\wedge[(\sim p\wedge q)]$
0	0	1	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	1	0	0	0	0

ii)

p	q	$\sim p$	$\sim q$	$\sim q\wedge p$	$[(\sim q)\wedge p]\wedge q$
0	0	1	1	0	0
0	1	1	0	0	0
1	0	0	1	1	0
1	1	0	0	0	0

NOTE! The conjunction of 2 tautology is a tautology.

The disjunction of 2 contradiction is a contradiction.

Relpacement process

Equivalence formulas:

① $P \wedge T \cong P$ Identity law
$P \vee F \cong P$

② $P \vee T \cong T$ Domination law
$P \wedge F \cong F$

③ $P \vee P \cong P$ Idempotent law
$P \wedge P \cong P$

④ $\neg(\neg P) \cong P$ double negation law.

⑤ $p \vee q \cong q \vee p$ } → commutative law
$p \wedge q \cong q \wedge p$

⑥ $(p \vee q) \vee r \cong p \vee (q \vee r)$ } Associative law
$(p \wedge q) \wedge r \cong p \wedge (q \wedge r)$

⑦ $p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$ Distributive law.
$p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r)$

⑧ $\neg(p \wedge q) \cong \neg p \vee \neg q$ Demorgan's law
$\neg(p \vee q) \cong \neg p \wedge \neg q$

⑨ $p \vee (p \wedge q) \cong p$ Absorption law
$p \wedge (p \vee q) \cong p$

(10) $p \vee \neg p \cong T$ Negation law
$p \wedge \neg p \cong F$

Logical Equivalences Involving Conditional statements

$p \to q \cong \neg p \vee q$
$p \to q \cong \neg q \to \neg p$
$p \vee q \cong \neg p \to q$
$p \wedge q \cong \neg(p \to \neg q)$
$\neg(p \to q) \cong p \wedge \neg q$
$(p \to q) \wedge (p \to r) \cong p \to (q \wedge r)$
$(p \to q) \wedge (q \to r) \cong (p \vee q) \to r$
$(p \to q) \vee (p \to r) \cong p \to (q \vee r)$
$(p \to r) \vee (q \to r) \cong (p \wedge q) \to r$.

Logical Equivalences Involving Bi-conditional
$p \leftrightarrow q \cong (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \cong \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \cong (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \cong p \leftrightarrow \neg q$.

Use the De-morgan's law to expresse the negation of Miguel has a cell phone and he has a laptop, computer

sol!- p: Miguel has a cell phone

q: Miguel has a laptop, computer.

$p \wedge q$.

$\sim(p \wedge q)$ is $\neg p \vee \neg q$

by using demorgan's law.

Miguel doesnot have a cell phone or he doesnot have a laptop, computer.

Show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

sol!- $\neg(p \to q)$ from conditionale stmts.

$\therefore p \to q \cong \neg p \vee q$

$\neg(\neg p \vee q)$ $\because \neg(p \vee q) \cong \neg p \wedge \neg q$

$\neg(\neg p) \wedge \neg q$ $\because \neg(\neg p) \cong p$.

$p \wedge \neg q$

Show that $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

<u>sol</u>:- $(p \wedge q) \rightarrow (p \vee q)$ $\qquad p \rightarrow q \cong \neg p \vee q$

$\neg(p \wedge q) \vee (p \vee q)$ $\qquad \neg(p \wedge q) \cong \neg p \vee \neg q$

$(\neg p \vee \neg q) \vee (p \vee q)$ $\qquad p \vee q \cong q \vee p$

$\neg p \vee \neg q \vee p \vee q$

$(\neg p \vee p) \vee (\neg q \vee q)$ $\qquad \neg p \vee p \cong T$

$T \vee T$

$= T$

$\therefore (p \wedge q) \rightarrow (p \vee q)$ is tautology.

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q))$ $\qquad \neg(p \vee q) \cong \neg p \wedge \neg q$

$\neg p \wedge \neg(\neg p \wedge q)$ $\qquad \neg(p \wedge q) \cong \neg p \vee \neg q$

$\neg p \wedge (\neg(\neg p) \vee \neg q)$ $\qquad \neg(\neg p) = p$

$\neg p \wedge (p \vee \neg q)$ $\qquad \cancel{p \wedge (q \vee r)}$ $\cancel{= (p \wedge q) \vee (p \wedge r)}$

$\cancel{\neg p \wedge (p \vee \neg q)}$

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$

Scanned with CamScanner

$F \vee (\neg p \wedge \neg q)$ $\quad p \wedge q \cong q \wedge p$

$(\neg p \wedge \neg q) \vee F$ $\quad p \vee F \cong p$

$(\neg p \wedge \neg q)$

Prove the following are logically equivalent

$(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)$ and $p \wedge q$

$\neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$ $\quad p \rightarrow q \cong \neg p \vee q.$

$(\neg(\neg p) \wedge \neg(\neg q)) \vee (p \wedge q \wedge r)$ $\quad \neg(p \vee q) = \neg p \wedge \neg q.$

$\neg(\neg p) = p$

$(p \wedge q) \vee ((p \wedge q) \wedge r)$

$\cancel{((p \wedge q) \vee p) \wedge ((p \wedge q) \vee q)) \wedge (p \wedge q \vee r)}$

$\cancel{p \wedge q}\ (\cancel{r} \vee \cancel{t}$

$\cancel{(p \wedge q)}$

$p \vee (p \wedge q)$

$= p$

absorption law

$= (p \wedge q)$

prove the following contra~~positive~~ diction.

i) $p \wedge (\sim p \wedge q)$

ii) $(\sim q \wedge p) \wedge q$.

i) $p \wedge (\sim p \wedge q)$. Distributive law.

$(p \wedge \sim p) \wedge (p \wedge q)$.

$(p \wedge q) \wedge (p \wedge \sim p$ $\qquad p \wedge q \cong q \wedge p$.

~~$p \wedge (p \wedge q)$~~

$(p \wedge q) \wedge F$ $\qquad p \wedge F \cong F$

$= F$.

ii) $(\sim q \wedge p) \wedge q$

$q \wedge (\sim q \wedge p)$ $\qquad p \wedge q \cong q \wedge p$.

$(q \wedge \sim q) \wedge (q \wedge p)$ $\qquad p \wedge q \cong q \wedge p$.

$(q \wedge p) \wedge (q \wedge \sim q)$ $\qquad p \wedge \neg p \cong F$.

$(q \wedge p) \wedge F$ $\qquad p \wedge F \cong F$.

$= F$.

~~show that~~ when $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent ..

p	q	r	$(p\to q)$	$(p\to q)\to r$	$p\to(q\to r)$ $(q\to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	F	T

$p\to(q\to r)$
T
F
T
T
T
T
T
T

If q is T/F, and p, r are false then the two compound stmts are not logically equivalent.

Predicates and Quantifiers.

Predicate: A part of declarative sentence describing the properties of an object is called a Predicate.

Examples

Statements → $x > 3$, $x = y + 3$, $x + y = z$

These statements are neither true nor false when the values of the variables are not specified.

Ex:- x is greater than 8 has 2 parts. 1-part → is the subject and the 2-part → the predicate of stmt. We can denote the "is greater than 3" statement by P(x) where P is the predicate and x is the variable.

NOTE!- Once a value has been assigned to the variable x. The stmt P(x) become a proposition and has Truth values.

1Q) Let $P(x)$ denote the statement $x > 3$. What are the truth values of $P(4)$ and $P(2)$

Sol:- $P(x)$ is a statement.

statement is $x > 3$.

then $P(4)$ is true

$P(2)$ is false.

2Q) Let $A(x)$ denote the statement "Computer x is under attack by an intruder". Suppose that of the computers on campus only CS2 and MATH1 are currently under attack by intruder. What are truth values of i) A (CS1), A (CS2), A (MATH 1)

Sol:- A (CS1) - False → not mentioned.

A (CS2) - True

A (MATH1) - True } mentioned.

3Q) Let $Q(x, y)$ denote the stmt $x = y + 3$ what are the truth values of proposition $Q(1, 2)$ and $Q(3, 0)$

Sol: $Q(1, 2)$ → false

$Q(3, 0)$ → True.

4Q) Let $A(c, n)$ denote the stmt "computer c is connected to the network n".

where c is a variable representing a computer and n is a variable representing a network.

Suppose that the computer math1 is connected to the network campus 2 but not the network campus 1. What the truth values of A(Math1, CAMPUS 1), A(MATH1, CAMPUS 2).

sol:- A(Math1, campus1) → false.
A(MATH1, CAMPUS2) → True.

5Q) Let $R(x, y, z)$ denote the Statement $x+y=z$ what are truth values of the propostion $R(1,2,3)$, $R(0,0,1)$

sol:- $R(1,2,3)$ → True
$R(0,0,1)$ → false.

NOTE: A stmt involving the n variables $x_1, x_2, x_3 \ldots x_n$ can be denoted by $P(x_1, x_2, x_3 \ldots x_n)$.

A statement of the for $P(x_1, x_2, x_3 \ldots x_n)$ is the values of the propositional function P at the n-tuple $x_1, x_2, x_3 \ldots x_n$. and P is called an n-palace predicate or n-array predicate.

Quantifiers:-

Quantifiers are words that are refer to quantifies such as some or all.

for all
some
there exist
for every.

There are 2 types of Quantifiers:
① Universal Quantifier.
② Existential Quantifier.

① universal Quan $\forall$ for all
② $\exists$ Existential

Universal Quantifier:-

The phrases for all, for every is called the universal Quantifier it is denoted by $\forall$

Ex:- All human beings are mortal gentleman

Let $P(x)$ denote x is a mortal.
then the above sentence written as
$\forall x P(x)$

Existential Quantifier.

→ The pharse that exist is called as existential Quantifier. ~~It~~ is denoted by $\exists$ (There exist)

Ex:- There Exist X such that x^2 equals to 5.

$\exists x \, P(x)$

where $P(x) \Rightarrow x^2 = 5$.

When the variables in the porpositional functions are assigned values then the resulting a proposition with a certain truth value. We can give a Quantification to create a proposition from a propostional function.

Quntifier express the existent to which the predicate is true over a range of elemenents.

Universal which tells us that a predicate is true for every element under consideration and existential Quantification which tell us that there is one or more elements under consideration for which the predicate is true

Stmt	when True?	when false
$\forall x P(x)$	$P(x)$ is true for every x	There is an x for which $P(x)$ is false
$\exists x P(x)$	There is an x for which $P(x)$ is True	$P(x)$ is false for every x

Example: Let $P(x)$ be the statement $x+1 > x$ what is the truth value of the Quantification $\forall x P(x)$ ~~when~~ where the Domain consists of real numbers.

sol:- $P(x)$ is true for all real numbers. So the quantification is true $\forall x P(x)$.

2) Let $Q(x)$ is a statement $x < 2$ what is truth value of the quantification for all x $Q(x)$ (i.e. $\forall x Q(x)$) where the domain consist of all real numbers.

sol: The $Q(x)$ is ~~f~~ not true for every real number coz for instance of $Q(x)$. for instance $Q(3)$ is false. So $\forall x Q(x)$ is false.

$Q(1) \Rightarrow$ True
$Q(2) \Rightarrow$ false
counter example.

note:
→ An element for which $P(x)$ is false is called a counter example of $\forall x P(x)$.

Example 8:- What are the truth values of $\forall x P(x)$ where $P(x)$ is stmt $x^2 < 10$. and the Domain consists of the positive integers not exceeding 4.

sol:- False.

$$\left.\begin{matrix} P(1) \\ P(2) \\ P(3) \end{matrix}\right\} T$$
$P(4) \rightarrow F$.

note:- when all the elements in the domain can be listed $x_1, x_2, \ldots, x_n$. It follows the universal Quantification for all $\forall x P(x)$ is the same as the conjunction of $P(x_1) \wedge P(x_2) \wedge \ldots P(x_n)$.

Example:- Let $P(x)$ denote the statement $x > 3$ what is truth value of the Quantification $\exists x P(x)$ where the domain consist of all real number.

sol:- True

$1 > 3$ ✗
$2 > 3$ ✗
$3 > 3$ ✗
$4 > 3$ ✓

Ex
Let $Q(x)$ denote the statement $x = x+1$ what is the truth value of the quntification $\exists x Q(x)$ where the domain consist of all real numbers,

Existential Quantifier.

→ The pharse that exist is called as existential Quantifier. ~~It~~ is denoted by $\exists$ (there exist)

Ex:- There Exist X such that x^2 equals to 5.

$\exists x\, P(x)$

where $P(x) \Rightarrow x^2 = 5$.

When the variables in the porpositional functions are assigned values then the resulting a proposition with a certain truth value. We can give a Quantification to create a proposition from a propostional function.

Quntifier express the existent to which the predicate is true over a range of elemoments.

Universal which tells us that a predicate is true for every element under considera- tion and existential Quantification which tell us that there is one or more elements under consideration for which the predicate is true

Stmt	When True?	When false
$\forall x\, P(x)$	$P(x)$ is true for every x	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is True	$P(x)$ is false for every x.

Example: Let $P(x)$ be the statement $x+1 > x$ what is the truth value of the Quantification $\forall x\, P(x)$ when where the Domain consists of real numbers.

sol:- $P(x)$ is true for all real numbers. So the quantification is true $\forall x P(x)$.

2) Let $Q(x)$ is a statement $x < 2$ what is truth value of the quantification for all x $Q(x)$ (i.e. $\forall x\, Q(x)$) where the domain consist of all real numbers.

sol:
The $Q(x)$ is ~~≠~~ not true for every real number coz for instance of $Q(x)$. for instance $Q(3)$ is false. So $\forall x Q(x)$ is false.

$Q(1) \Rightarrow$ true
$Q(3) \Rightarrow$ false
counter Example

An element for which $P(x)$ is false is called a counter example of $\forall x P(x)$.

Example :- What are the truth values of $\forall x P(x)$ where $P(x)$ is stmt $x^2 < 10$ and the Domain consists of the positive integers not exceeding 4.

$P(1)$, $P(2)$, $P(3)$ → T
$P(4)$ → F

Sol :- False.

note :- When all the elements in the domain can be listed $x_1, x_2, \ldots x_n$. It follows the universal Quantification for all $\forall x P(x)$ is the same as the conjunction of $P(x_1) \wedge P(x_2) \wedge \ldots P(x_n)$

Example :- Let $P(x)$ denote the statement $x > 3$ what is truth value of the Quantification $\exists x P(x)$ where the domain consist of all real number.

Sol :- True

1 > 3 ✗
2 > 3 ✗
3 > 3 ✗
4 > 3 ✓

Ex
Let $Q(x)$ denote the Statement $x = x + 1$ what is the truth values of the quantification $\exists x Q(x)$ where the domain consist of all real numbers.