sol:- false.

What is the truth value of the $\exists x P(x)$ where $P(x)$ is the statement $x^2 > 10$. and the domain consist of positive integers not exceeding 4.

sol:- True.

Ex:- What does the stmt $\forall x N(x)$ mean if $N(x)$ is "computer x is connected to the network"). and the domain consists of all computers on the campus.

sol:- Every computer is connected to the networke.

True.

X: Let P and Q be the proposition.

P: You drive over 65 miles per hour) X

determine whether each of these conditional statements are T/F?

1) If $1+1=2$ then $2+2=5$ → F
2) If $1+1=3$ then $2+2=4$ → T
3) If monkeys can fly, then $1+1=3$ → T

Q show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

p	q	r
0	0	
0	0	
0	1	
0	1	
1	0	
1	0	
1		
1		

$((p \to q) \wedge (q \to r)) \to (p \to r)$

$p \to q \cong \neg p \vee q$

$\neg((p \to q) \wedge (q \to r)) \vee p \to r$

$\neg(p) = \neg p$

$\neg(p \wedge q) = \neg p \vee \neg q$

$(\neg(p \to q) \vee \neg(q \to r)) \vee p \to r$

$((p \wedge \neg q) \vee (q \wedge \neg r)) \vee p \to r$

$((p \wedge \neg q) \vee (q \wedge \neg r)) \vee p \to r$

$(p \vee q) \vee r \cong p \vee (q \vee r)$

$(p \wedge \neg q) \vee ((q \wedge \neg r) \vee (p \to r))$

$(p \wedge \neg q) \vee ((q \wedge \neg r) \vee (\neg p \vee r))$

$p \wedge \neg q \vee q \wedge \neg r \vee \neg p \vee r$

$T \wedge T \wedge T = T$.

Find the negation of following statement

If p is a square then p is a rectangle

Sol: $p \to q$.

$$\neg(p \to q) = \neg(\neg p \vee q)$$

$$= p \wedge \neg q.$$

If p is a square then p is not a rectangle

Duality

The Dual of a compound proposition that contains logical operation $\vee$, $\wedge$ and $\neg$. is compound proposition obtained by replacing each $\vee$ by $\wedge$, $\wedge$ by $\vee$ and each T by F, F by T.

If s is compound proposition. then dual of s is denoted by s^* or s^d.

Ex:- find the dual of each of the compound propositions.

a) $p \vee \neg q$

b) $p \wedge (q \vee (r \wedge T))$

c) $(p \wedge \neg q) \vee (q \wedge F)$

Sol:- a) $\neg p \wedge q$

b) $\cancel{\neg p \vee (\neg q \wedge (\neg r \vee \neg T))}$

c) $\cancel{(\neg p \vee q)}$

a) $p \wedge \neg q$

b) $p \vee (q \wedge (r \vee F))$

c) $(p \vee \neg q) \wedge (q \vee T)$

NOTE: for any two propositions u and v if u and v logically equivalent then there duals are logically equivalent

↳ $u \Leftrightarrow v \rightarrow u^d \Leftrightarrow v^d$

→ If u is a proposition the dual of dual is equivalent to the $(u^d)^d \Longleftrightarrow u$.

Write the following stmts in symbolic form
a) Something is good
b) Everything is good.

sol:- ~~∃~~ P → is good

x → something and y Everything.

a) $\exists x\, P(x)$

b) $\forall y\, P(y)$

Let $P(x)$ be the statement x spends more then ~~two~~ 5 hours every weekday in class. Where the domain for x consists of all students. Express the each of the quantification in english.

a) $\exists x\, P(x)$
b) $\forall x\, P(x)$

sol:-
a. Some students spends more then 5 hours every week day.

b) All students spends more than 5 hours every week day.

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c) $\exists x \neg P(x)$

some students doesnot spend more than 5 hours every week day.

d) $\forall x \neg P(x)$

All students doesnot spend more than 5 hours every week day.

Q) Let $P(x)$ denote the statement $x \leq 4$ what are truth values of the $P(0)$, $P(4)$, $P(3)$

sol:- T, T, T.

Uniqueness Quantifiers

The phrases for uniqueness Quantification include there is exactly one and there is one and only one. and denoted by $\exists!$

$\exists 1$

<u>Quantifier</u> <u>with</u> <u>restricted</u> <u>domains</u>.

A notation is used to restrict the domain of a quantifier in this notation a conditional variable must satisfy after the quantification.

Let $P(x)$ be ~~a~~ Statement $x = 4$ What is the truth value of the quantification ~~for~~ $\forall x P(x)$ Where the domain consist of all real no.'s but not $x \neq 4$.

Sol:-

False.

Note: The quantifiers ~~for a~~ $\forall$ and $\exists$ have high priority then all logical operator from propositional Calculus.

Binding Variables

When a quantifier is used on a variable x we say that this occurance of the variable is bond and the occurence of the variable that is not bond by a Quantifier or a set equal to a particular value is said to be free.

The part of a logical expresion to which a quantifier is applied is called the scope of this quantifier if a variable is free. If it is outside the scope of all quantifiers

Ex:-
In the statement $\exists x\ (x+y=1)$ the variable x is bond by the existential quantification and variable y is free. It is not bond by quantifier and no value is assigned to this variable.

Logical equivalance involving quantifiers

statements involving predicates and quantifiers are logically equivalent if and only if they have the same truth value. No matter which predicates are subsituted into this stmts and which domain is used for variables in the propositional functions. we use the notation $S \cong T$ To indicate that 2 statements S and T involving predicates and quantifier are logically equivalent.

Negating Quantified Expressions.

$$\boxed{\begin{array}{l} \rightarrow \neg\forall x\, P(x) \cong x \neg P(x) \\ \rightarrow \neg\exists x\, P(x) \cong \forall x\, \neg P(x) \end{array}}$$

Q) What are the negation of the stmts
→ There is an Honest politician.
→ All American's eat cheese burgers

Sol: → $\exists x\, P(x)$

$P(x)$: x is an honest politican

$\neg \exists x\, P(x) \Rightarrow$ ~~All~~ $\forall x\, \neg P(x)$

All politicians are dishonest

→ ~~All~~ $\forall x\, P(x)$ $\qquad$ $P(x)$ x American's eat cheese burger

$\neg \forall x\, P(x) \Rightarrow \exists x \neg P(x)$.

Some American's do not eat chess burgers.

There is one American who do not eat chess burger.

Q) What are the negations of the statements

i) $\forall x (x^2 > x)$

ii) $\exists x (x^2 = 2)$

Sol: i) $\exists x (x^2 \leq x)$

ii) $\forall x (x^2 \neq 2)$.

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Examples:- Express the statements
"Every Student in this class has studied Calculus" using predicates and Quantifiers

Sol:- $P(x) \rightarrow x$ students is in this class has studied

Sol:- $\forall x\, P(x)$.

Q) Express the statements (1) "Some students in the class visted Mexico".

(2) Every Student in this class has visited either Canada or Mexico. using predicates and quantifiers.

Sol:- (1) $P(x) \rightarrow x$ students in the class visted Mexico.

Sol:- $\exists x\, P(x)$.

(2) $P(x) \rightarrow x$ students has visted Canada.

$Q(x) \rightarrow x$ students has visted Mexico

$\forall x(P(x) \vee Q(x))$.

Nested Quantifiers.

Two Quantifiers are Nested if one is within the Scope of other.

Ex:

$\forall x \exists y (x + y = 0)$

Example :- Translate into english statements

$\forall x \forall y \, (x > 0) \wedge (y > 0) \rightarrow xy < 0$.

where the domain for x and y all real no.'s.

Sol. :- for every real no. x and for every real no. y If $x > 0$ and $y > 0$ then $xy < 0$.

Q)

Let $P(x, y)$ be the Statement ~~be~~ $x + y = y + x$

what are the truth values of the quantification $\forall x \forall y \, P(x, y)$. where the domain for variables consists of all real no.'s.

Sol :- True.

Q) Translate the statements "the sum of two positive integers is always positive" into a logical expression where the domain for variables consists of all +ve integers

sol:-

$\forall x \forall y\, (x>0) \wedge (y>0) \rightarrow (x+y>0)$.

Q) Translate the statement every real no except zero has a multiplicative inverse

NOTE :- A multiplicative inverse of a real no. x while is a real no. y such that $xy = 1$.

sol: ~~$\forall x \forall y\, (x \neq 0) \wedge (x>0) \rightarrow (xy=1)$~~.

$\forall x\, (x \neq 0) \rightarrow \exists y\; xy = 1$.

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Rules of Inference.

An argument in propositional logic is a sequence of propositions all but not the final proposition in the arguement are called Premises and final prosposition is called conculsion.

An Arguement is valid if the truth of all its premises Implies that the the conclusion is true.

The Arguement formed with premises $P_1, P_2, P_3 \cdots P_n$ and Conclusion is Q is valid when

$$\boxed{P_1 \land P_2 \land P_3 \cdots \land P_n \rightarrow Q}$$ is a tautology.

premises P_1, P_2, $\vdots$, P_n — argument

$\therefore C$ — conclusion

(A sequence of premises is argument)

NOTE! The Hypothesis are written in a column followed by a horizontal bar followed by a line that begins with the $\therefore$ (therfore symbol) and ends with conclusion.

P_1
P_2
P_3
$\vdots$
P_n

$\therefore q$

~~method~~

Consider the following arguments involing proposition.

① If you have a current password you can log on to the network.

② You have a current password.

Therefore.

③ You can logon to the network

sol:- ① & ② and ③ are arguments

③ is conclusion

① & ② are premesies.

→ Check whether the given argument is valid or not

above Question.

~~P: If~~

1st Premises is $P \rightarrow Q \rightarrow P_1$

2nd premises is $P \rightarrow P_2$ $(P_1 \wedge P_2) \rightarrow C$

3rd conclusion is $Q \rightarrow C$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$ ①	① $\rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Law of Logic:

Rule of Inference	Tautology	Name
① p, $p \to q$, $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
② $\neg q$, $p \to q$, $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
③ $p \to q$, $q \to r$, $\therefore p \to r$	$[(p \to q) \wedge (q \to R)] \to (p \to r)$	Hypothetical Syllogism
④ $p \vee q$, $\neg p$, $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive Syllogism
⑤ p, $\therefore p \vee q$	$p \to (p \vee q)$	Addition
⑥ $p \wedge q$, $\therefore p$	$(p \wedge q) \to p$	Simplification
⑦ p, q, $\therefore p \wedge q$	$(p) \wedge (q) \to (p \wedge q)$	Conjunction

10) $p \vee q$ $\quad (p \vee q) \wedge (\neg p \vee r) \Rightarrow q \vee r$ Resolution.

$$\frac{\begin{array}{c} p \vee q \\ \neg p \vee r \end{array}}{q \vee r}$$

Q State which rule of inteference is Basis of the following argument.

It is below freezing now ∴ it is either below freezig or raining now.

p : It is below freezing now.
q : raining now.

$$\frac{p}{\therefore p \vee q}$$

It is addition rule.

$p \rightarrow (p \vee q)$

This argument uses addition rule

a) State which rule of inference is used in the argument.

If it rains today then we will not have a barbecue today $\quad p \rightarrow q$.

If we do not have a barbecue today then we will have barcbecue tomorrow $\quad q \rightarrow r$.

Therefore

If it rains today, then we will have barbecue tomorrow.

Sol:-

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

This follows hypothetical Syllogism.

2) Show that the hypothesis "If is not sunny this afternoon and it is colder than yesterday" "we will go swiming only if it is sunny" "If we do not go swimming then we will take a canoe trip". and "if we take canoe trip, then we will be home by sunset" lead to the conclusion "we will be home by sunset

Sol:- p: It is sunny this afternoon

q: It is colder than yesterday

r: we will go swimming

s: we will take a canoe trip

t: we will be home by sunset

$\neg p \wedge q$, $r \to p$, $\neg r \to s$, $s \to t$

$\neg p \wedge q$

$r \to p$

$\neg r \to s$

$s \to t$

$\therefore t$

Step	Reason
① $\neg p \wedge q$	Hypothesis (or rule p).
② $\neg p$	Simplification on ①.
③ $r \to p$	Hypothesis
④ $\neg r$	Modus tollen on ② & ③
⑤ $\neg r \to s$	Hypothesis.
⑥ s	Modus ponen on ④ & ⑤
⑦ $s \to t$	Hypothesis.
⑧ t	Modus ponen ⑥ & ⑦.

By using given premeses the conclusion t is derived so the given argument is the valid argument.

Q) Show that the hypothesis "If you send me an E-mail message then I will finish writing the program". If you do not send me an E-mail message, then I will go to sleep early " and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If do not finish writing the program then I will wake up feeling refreshed.

Sol: p: ~~If~~ you send ~~me~~ an E-mail message
q: I will finish writing the program
r: I will go to sleep early.
s: I will wake up feeling refreshed.

$$\begin{array}{l} p \to q \\ \neg p \to r \\ r \to s \\ \hline \therefore \neg q \to s \end{array}$$

or

conditional proof
NOTE :- If conclusion is in the form of implication then antecedent is taken as premises and prove conclusion.

Step	reason
① $p \to q$	Hypothesis.
② $\neg q \to \neg p$	Contra positive on ①
③ $\neg p \to r$	Hypothesis.
④ $\neg q \to r$	Hypothetical syllogism.
⑤ $r \to s$	Hypothesis.
⑥ $\neg q \to s$	Hypothetical syllogism.

~~Not~~
~~If~~

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Q) Test whether the following is a valid argument.

(1) If Sachin hits a century then he gets a free car.

(2) Sachin does not get a free car.

∴ Sachin does not hit a century.

Sol:- $p \to q \to (1)$

$\neg q \to (2)$ by Modus Tollens

$\therefore \neg p$

This arguments are valid.

Q) Show that $p \to s$ can be derived from the premises.

$\neg p \vee q$

$\neg q \vee R$

$R \to S$.

conditional Proof method

$$\begin{array}{l} \neg p \vee q \\ \neg q \vee R \\ R \to S \\ \underline{P} \\ \therefore S \end{array}$$

Sol:-

Step	
$\neg p \vee q$	Hypothesis.
P	Hypothesis.
q	Disjunctive Syllogism
$\neg q \vee R$.	Hypothesis
R	Disjunctive syllogism
$R \to S$	hypothesis
S	

(or)

Step	Reason
① $\neg P \vee Q$	Hypothesis.
② $P \to Q$	equivilance formula on ①.
③ $\neg Q \vee R$	Hypothesis.
④ $Q \to R$	equivilance formula on ③
⑤ $P \to R$	Hypthecical syllogism on ②&④
⑥ $R \to S$	Hypothesis
⑦ $P \to S$	Hypothecial Syllogism.

Q) Domanstrait R is a valid inference from the premesis $P \to Q$, $Q \to R$ and P.

sol: $P \to Q$
$Q \to R$
P
$\therefore R$.

sol:-

Step	Reason
① $P \to Q$	Hypothesis
② $Q \to R$	Hypothesis.
③ $P \to R$	Hypothesis syllogism on ①&②
④ P	Hypothesis.
⑤ R	Modus ponens.

Q) Show that $S \vee R$ is derived from the premesis $P \vee Q$, $P \rightarrow R$, $Q \rightarrow S$.

sol:-
$$\begin{array}{l} P \vee Q \\ P \rightarrow R \\ Q \rightarrow S \\ \hline \therefore S \vee R \end{array}$$

Step	Reson
① $P \vee Q$	Hypothesis
② $P \rightarrow R$	Hypothesis.
③ $\neg P \vee r$	equivalence on ②.
④ $Q \vee R$	① & ③ Resolution.
⑤ $Q \rightarrow S$	Hypothesis.
⑥ $\neg q \vee S$	equivalence on ⑤.
⑦ $R \vee S$	④ & ⑥ Resolution
⑧ $S \vee S$	commutative law on ⑦.

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Rules of inference for Quantified Statements

1) Universal instantiation / specification
2) Universal generalization
3) Existential instantiation / Specification
4) Existential generalization.

Universal Specification

This rule is used to conclude that $P(y)$ is true where y is ~~the~~ a particular member of domain. The give premesis $\forall x P(x)$ the conclusion is $P(y)$.

$$\frac{\forall x P(x)}{\therefore P(y)} \Rightarrow \text{universal Specification}$$

Universal Generalization

It is the rule of inference that states $\forall x P(x)$ is true given the premesis that $P(y)$ is true for all elements y in the domain.

$$\frac{P(y) \text{ for an circularly } y}{\therefore \forall x P(x)} \Rightarrow \text{Universal generalization.}$$

Existential Specification

It is the rule that allows to conclude that there is an element y in the domain for which $P(y)$ is true If ~~there~~ $\exists x P(x)$ is true.

$$\frac{\exists x P(x)}{\therefore P(y) \text{ for some element } y} \Rightarrow \text{existential specification}$$

Existential Generalisation

It is the rule of inference that is used to conclude that $\exists x P(x)$ is true when a particular element y with $P(y)$ is true

$$\frac{P(y) \text{ for some element } y}{\therefore \exists x P(x)} \Rightarrow \text{Existential generalization}$$

Q) Show that the premesis "Everyone in this discrete mathematics class has taken a course in Computer Science" and "Marla is a student in this class" implie the conclusion Marla has taken a course in computer Science.

Sol:- $D(x)$: x is in DM class.

$C(x)$: x is taken a course of Computer Science.

$$\forall x \; D(x) \rightarrow C(x)$$

and $D(\text{Marala})$

$\therefore C(\text{Marla})$

Step	Reason
① $\forall x D(x) \rightarrow C(x)$	Hypothesis
② $D(y) \rightarrow C(y)$	Universal specification on step ①.
③ $D(\text{Marla}$	Hypothesis
④ $D(y)$	Simplify ③.
⑤ $C(y)$	Modus pones ② & ④.
⑥ $C(\text{marla})$.	

Sol:- $D(x)$: x is in DM class.

$C(x)$: x is taken a course of Computer Science.

$\forall x \; D(x) \rightarrow C(x)$

and $D(\text{Manala})$

$\therefore C(\text{Manla})$

Step	Reason
1 $\forall x D(x) \rightarrow C(x)$	Hypothesis
2 $D(y) \rightarrow C(y)$	Universal specification on step 1.
3 $D(\text{Manla})$	Hypothesis
4 $D(y)$	Simplify 3.
5 $C(y)$	Modus pones 2 & 4.
6 $C(\text{manla})$.	

Q) Show that the premises "A student in this class has not read the book" and "Every one in this class passed the first exam" implie the conclusion "Some one who passed the first exam has not read the book."

sol:-

~~$B(x)$: x in this class has not read the book~~

~~$P(x)$: x in this class passed the first exam~~

$C(x)$: x is in this class

$B(x)$: x has read the bok.

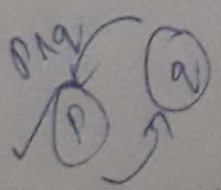
$P(x)$: x passed the first exam.

$\exists x\,(C(x) \wedge \neg B(x))$

$\forall x\,(C(x) \rightarrow P(x))$

$\exists x\,(P(x) \wedge \neg B(x))$.

Step	Reason
① $\exists x(C(x) \land \neg B(x))$	Hypothesis
② $C(a) \land \neg B(a)$ $(p \land q)$	Existential specification.
③ $C(a)$	specification on ②
④ $\forall x\,(C(x) \rightarrow P(x))$	Hypothesis
⑤ $C(a) \rightarrow P(a)$	Universal specification ④
⑥ $P(a)$	Modus ponens on ③ & ⑤
⑦ $\cancel{\exists x(P(x) \land \neg B(x))}$	~~Hypothesis~~
⑧ $\cancel{P(a) \land \neg B(a)}$	~~existential~~
⑦ $\neg(B(a))$	Simplification ②
⑧ $P(a) \land \neg B(a)$	Conjunction ⑥ & ⑦
⑨ $\exists x\, P(x) \land \neg B(x)$	Existential gener ⑧



Introduction to Proof

Theorem :- A theorem is a statement which is true.

Proof :- A Proof is a valid argument that establishes the truth of a theorem,

Two Types of proof methods

1. direct proof
2. indirect proof.

Direct method of proof -

- A direct proof of $p \to q$ is logically valid argument in which we start with the assumption that p is true and then using p as well other axioms show directly that q is true.

Ex:- Give direct proof of the following statement. The product of 2 odd integers is odd.

Sol:- x and y be the 2 odd two integers

$x = 2k+1$
$y = 2k+3$

$xy = (2k+1)(2k+3)$
$= 4k^2+6k+2k+3$
$= 4k^2+8k+3$
$= 2(2k^2+4k)+3$ let $a = 2k^2+4k$.
$= 2(a)+3$

Henceproved.

$\therefore$ The product of 2 odd integers is odd.

Q) Show that square of even no. is a even number. in direct method

Sol:- Let x be an even number.

$x = 2n$.

S.O.B.S.

$x^2 = (2n)^2$
$= 4n^2$
$= 2(2n^2)$
$= 2(a)$ $a = 2n^2$.

Henceproved.

∴ Square of even no. is a even no

Q) Show that sum of 2 odd no.'s is even.

Sol:- Let x and y be 2 odd no.'s

$x = 2n+1$

$y = 2m+1$

$2n+1 + 2m+1$

$2(n+m)+2$

$= 2(n+m+1)$

p: x, y are odd no's

q $x+y$ is even

$x+y = 2m+2n+2$

$= 2(m+n+1)$

$a = m+n+1$

$= 2(a)$

Hence proved.

∴ Sum of 2 odd no.'s is even.

Indirect method of Proof

Proof by contrapositive

It says that $p \to q$ is logically eviquivalent to ~~the~~ contropositive $\neg q \to \neg p$. Now to prove $p \to q$ assume that q is false and show the p is false

Q) Prove that n^2 is odd then ~~prove~~ n is odd.

Sol:- Let $p: n^2$ is odd

$q: n$ is odd

The contrapositive of $p \to q$ is $\neg q \to \neg p$

$\neg q$: n is even.

$\neg p$: n^2 is even

$n = 2x$

$n^2 = 4x^2$

$n^2 = 2(2a^2)$

$= 2a \Rightarrow$ even

$\neg q \Rightarrow \neg p$ is true so $p \to q$ is true

$\therefore$ Hence proved.

Q) P.T If x, y is belongs to Z such that xy is odd then both x & y are odd.

Sol:- $p: xy$ is odd

$q: x, y$ are odd.

$\neg q: x, y$ are even

$\neg p: xy$ is even.

Let $x = 2n$
$y = 2m$

$xy = 2n \times 2m$
$= 4nm$
$= 2(2nm)$ $[a = 2nm]$
$= 2(a)$

$\therefore \neg q \rightarrow \neg p$ is true so $p \rightarrow q$ is true

Proof by Contradiction.

If $p \rightarrow q$ we assume that q is false then by logically argument we arrive at a situation where $\neg q$ is a contradiction. When $\neg q$ is false which implies that q ~~is~~ true.

Q) Show that $\sqrt{2}$ is an irrational no.

Sol:- Let as assume $\sqrt{2}$ is an rational no

The rational no. is expressed as $\frac{p}{q}$ form.

$\therefore \sqrt{2} = \frac{p}{q}$

S.O.B.S

$$2 = \frac{p^2}{q^2}$$

$p^2 = 2q^2 \rightarrow (1)$

$\therefore p^2$ is a even no.

Hence p is also a even.

$p = 2k$ [To prove q^2 is even].

$p^2 = 4k^2 \rightarrow (2)$

$4k^2 = 2q^2 \Rightarrow$ [from (1) & (2)].

$q^2 = 2k^2$.

$\therefore q^2$ is also a even no.

both p and q have common factor 2. Hence it is rational no.

Q) P.T the following stmt $2n+2$ is odd then n is odd using contra positive.

Q) S.T the x is belongs to R. If $x^3+4x=0$ then $x=0$ using contradiction

Q) Ans:- ~~(Let $2n+2$ is a even~~

~~$2n+2 = 2x$.)~~

Let p: $2n+2$ is odd
q: n is odd.

The contrapositive $p \to q$ is $\neg q \to \neg p$.

$\neg q$: n is even.
$\neg p$: $2n+2$ is even.

$n = 2x$.
multiply with 2 on both sides.

$2n = 4x$.
add Two on both sides.

$2n+2 = 4x+2$.
$= 2(2x+1)$.

$\therefore$ $2n+2$ is a even no.

The $\neg q \to \neg p$ ~~the~~ true then $p \to q$ is also true.

Q) Ans:-

Proof methods & Strategies

proof are encap by the names for the approach one takes for several proof types

① Exhaustive proofs :- Some theorems can be proved by examing a relatively small no. of ~~exhausti~~ examples such proofs are called exhaustive proofs.

Q) Prove that $(n+1)^2 \geq 3^n$ if n is a positive integer with $n \leq 2$.

sol :- $n = \{0, 1, 2\}$.

i) $n = 0$

$(0+1)^2 \geq 3^0$

$(1)^2 \geq 1$ True

ii) $n = 1$

$(1+1)^2 \geq 3^1$

$4 \geq 3$ True

iii) $n = 2$

$(2+1)^2 \geq 3^2$

$9 \geq 9$.

True

Q) Prove that only consequetive positive integers not exceeding 100 that are prefect powers are 8 and 9.

sol:- If 'n' integer

$n^2 \to 1, 4, 9, 16, 25, 36, 49, 64, 81, \cancel{100}$

$n^3 \Rightarrow 1, 8, 27, 64$

$n^4 \Rightarrow 1, 16, 81,$

$n^5 = 1, 32$

$n^6 \Rightarrow 1, 64$

$n^8 \Rightarrow 1$

There are no powers of positive integers higher than the 6^{th} power not exceeding 100 other then one. looking the list of powers not exceeding 100. only 3^2 is 9 and 2^3 8 are consecutive number.

$\therefore$ 8 and 9 are two consecutive no.'s are prefect powers not exceeding 100.

Proof by cases

A proof of cases must cover all the possible cases that arises in a theorem. We should check each possible case is covered. ~~So on~~

Q)

proof that n is a integer then $n^2 \geq n$.

Sol:- If $n=0$ then $n^2 \geq n \rightarrow$ true

If $n<0$ (i.e n is -ve integers) then $n^2 \geq n \rightarrow$ true

$(-n)^2 \geq n$

If $n>0$ (i.e n is +ve integers) then $(+n)^2 \geq +n \rightarrow$ true

Hence $n^2 \geq n$ is true in 3 posible cases.

Q) use a proof by cases. S.T $|xy| = |x||y|$ where x & y are real no's.

Sol:- xy i) x +ve y +ve

ii) x +ve y -ve

iii) x -ve y +ve

iv) x -ve y -ve.

i) x any y are +ve.

then

$|xy| = |x||y|$

$xy = xy$.

ii) x is +ve and y is -ve then

$|x-y| = |x||-y|$

$|-xy| = xy$

$xy = xy$.

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iii) x is -ve and y is +ve.

$$|-x\,y| = |-x||y|$$

$$|-xy| = xy$$

$$xy = xy$$

iv) x is -ve and y is -ve.

$$|-x\,-y| = |-x||-y|$$

$$|xy| = xy$$

$$xy = xy.$$

Existence proof:- an existence prove involves simply showing that a particular quanity exist.

Q) Prove that there exist a prime number p such that $p+2$ and $p+6$ are prime numbers.

sol:- Let p is a prime number (i.e 5).

$p = 5$ then $5+2 = 7$

$5+6 = 11$

$\therefore$ 5, 7, 11 are prime numbers hence proved.

Q) Prove that there is a odd intergex that can be written as sum of 2 squares.

sol:- Let a and b be two odd integers

$a = 2, b = 3$

$2^2 + 3^2 = 4 + 9 = 13.$

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Uniqueness Prove:- To prove a statement of this type we need show that an element with this property exist & that no other element has this property.

Q) Show that If a and b real no.'s and $a \neq 0$ then there is a unique real no. r such that $ar+b=0$.

Sol:- $ar+b=0$.

Let s be a real no.

$$as+b=0$$
$$ar+b=as+b$$
$$ar=as$$
$$r=s$$

If $r \neq s$ then $as+b \neq 0$.