UNIT 2

BASIC STRUCTURES, SETS, FUNCTIONS, SEQUENCES, SUMS, MATRICES AND RELATIONS

Marketed by

SIA GROUP

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. Use set builder notation to give a description of each of these sets.

(a) {0, 3, 6, 9, 12}

(b) {–3, –2, –1, 0, 1, 2, 3}

Answer :

(a) {0, 3, 6, 9, 12}

Given set is,

$\{0, 3, 6, 9, 12\}$

The above set is multiples of '3'.

∴ The set builder notation is given as,

$\{3n | n \in N, 0 \leq n \leq 4\}$

Where,

N is set of natural numbers.

(b) {– 3, – 2, – 1, 0, 1, 2, 3}

Given set is,

$\{-3, -2, -1, 0, 1, 2, 3\}$

The above set consists of integers starting from –3 and ending with 3.

∴ The set builder notation is given as,

$\{n | n \in Z, -3 \leq n \leq 3\}$

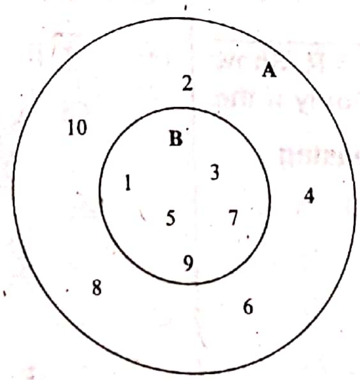
Where,

Z is set of integers.

Q2. Use a Venn diagram to illustrate the subset of odd integers in the set of all positive integers not exceeding 10.

Answer :

Let 'A' be the set that contains all positive integers which do not exceed 10 i.e., $A = \{1, 2, ... 10\}$ and B be the set which is the subset of 'A' that contains odd positive integers. i.e., $B = \{1, 3, 5, 7, 9\}$. The venn diagram to illustrate these sets is shown in figure.



Firure: Venn Diagram

Q3. Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find

(a) C × B × A

(b) B × B × B

Answer :

Given sets are,

$A = \{a, b, c\}$

$B = \{x, y\}$

$C = \{0, 1\}$

(a) C × B × A

$C \times B \times A = \{0, 1\} \times \{x, y\} \times \{a, b, c\}$

$\Rightarrow C \times B \times A = \{(0, x, a), (0, x, b)\ (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

(b) B × B × B

$B \times B \times B = \{x, y\} \times \{x, y\} \times \{x, y\}$

$\Rightarrow B \times B \times B = \{(x, x, x), (x, x, y)\ (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$

Q4. Prove the idempotent laws.

(i) A ∪ A = A

(ii) A ∩ A = A

Answer : Model Paper-1, Q1(c)

(i) A ∪ A = A

Consider,

$A \cup A = \{x | x \in A \cup A\}$

$= \{x | x \in A \vee x \in A\}$ $[\because A \cup B = \{x | x \in A \vee x \in B\}]$

$= \{x | x \in A\}$

$= A$

$\therefore A \cup A = A$

(ii) A ∩ A = A

Consider,

$A \cap A = \{x | x \in A \cap A\}$

$= \{x | x \in A \wedge x \in A\}$ $[\because A \cap B = \{x | x \in A \wedge x \in B\}]$

$= \{x | x \in A\}$

$= A$

$\therefore A \cap A = A.$

Q5. Let f : R → R and let f(x) > 0 for all x ∈ R. Show that f(x) is strictly increasing if and only if the function $g(x) = \dfrac{1}{f(x)}$ is strictly decreasing.

Answer :

Given that,

$f : R \to R, f(x) > 0$ for all $x \in R$.

$$g(x) = \frac{1}{f(x)} \quad \text{... (1)}$$

Case (i)

Assuming that the function $f(x)$ is strictly increasing i.e., if $x < y$ then $f(x) < f(y)$.

$\Rightarrow \dfrac{1}{f(x)} > \dfrac{1}{f(y)}$

$\Rightarrow g(x) > g(y)$ [∵ From equation (1)]

i.e., $g(x)$ is strictly decreasing.

Case (ii)

Assuming that the function $g(x)$ is strictly decreasing i.e., if $x < y$ then $g(x) > g(y)$.

$\Rightarrow \dfrac{1}{g(x)} < \dfrac{1}{g(y)}$

$\Rightarrow f(x) < f(y)$ [∵ From equation (1)]

i.e., $f(x)$ is strictly increasing.

$\therefore$ $f(x)$ is strictly increasing if and only if the function $g(x) = \dfrac{1}{f(x)}$ is strictly decreasing.

Q6. Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that the inverse of the composition fog is given by $(fog)^{-1} = g^{-1}of^{-1}$.

Answer :

Given that,

$f : Y \to Z$ and $g : X \to Y$ are invertible functions.

Let $z \in Z$ and x be the corresponding image of the function $(fog)^{-1}$ i.e., the composition of $(fog)^{-1}$ be z.

$\Rightarrow (fog)^{-1}(z) = x$

$\Rightarrow (fog)(x) = z$ [∵ By inverse property]

$\Rightarrow f(g(x)) = z$

$\Rightarrow g(x) = f^{-1}(z)$ [∵ f is invertible]

$\Rightarrow x = g^{-1}(f^{-1}(z))$ [∵ g is invertible]

$\Rightarrow x = (g^{-1} o f^{-1})(z)$

For all $z \in Z, (fog)^{-1}(z) = x = (g^{-1}of^{-1})(z)$

$\Rightarrow (fog)^{-1}(z) = (g^{-1}of^{-1})(z)$

$\Rightarrow (fog)^{-1} = (g^{-1}of^{-1})$

$\therefore (fog)^{-1} = (g^{-1}of^{-1}).$

Q7. Find

(a) $\sum_{j=0}^{4} j!$ (b) $\prod_{j=0}^{4} j!$

Answer :

(a) $\sum_{j=0}^{4} j!$

Given summation is,

$$\sum_{j=0}^{4} j!$$

$\Rightarrow \sum_{j=0}^{4} j! = 0! + 1! + 2! + 3! + 4!$

$= 1 + 1 + 2 + 6 + 24 = 34$

$\therefore \sum_{j=0}^{4} j! = 34$

(b) $\prod_{j=0}^{4} j!$

Given product is,

$$\prod_{j=0}^{4} j!$$

$$\Rightarrow \prod_{j=0}^{4} j! = (0!)\,(1!)\,(2!)\,(3!)\,(4!)$$

$$= (1)\,(1)\,(2)\,(6)\,(24)$$

$$= 288$$

$$\therefore \prod_{j=0}^{4} j! = 288$$

Q8. Show that the set $Z^+ \times Z^+$ is countable.

Answer : Model Paper-2, Q1(c)

Given set is,

$$Z^+ \times Z^+$$

Let the function is defined as,

$$f: Z^+ \times Z^+ \to Z^+, f(m, n) = 2^m . 3^n$$

Let, $f(a, b) = f(m, n)$

$\Rightarrow$ $2^a.3^b = 2^m.3^n$

$\Rightarrow$ $2^a = 2^m,\ 3^b = 3^n$ [$\because$ 2, 3 are prime numbers]

$\Rightarrow$ $a = m,\ b = n$

[$\because$ Bases are equal, powers also equal]

$\therefore$ f is one-to-one

From the definition of f,

$$|Z^+ \times Z^+| \leq |Z^+|$$

The set 'A' is said to be countable if and only if $|A| \leq |Z^+|$

Since, $|Z^+ \times Z^+| \leq |Z^+|$, the set $Z^+ \times Z^+$ is countable.

Q9. Show that the set of all finite bit strings is countable.

Answer : Model Paper-4, Q1(c)

Let, S be the set of all finite bit strings. The function is defined as,

$$f: S \to Z^+ \ ;\ f(s) = \text{decimal representation of } S.$$

i.e., each string is translated to its decimal representation by f.

Let $f(s) = f(t)$

$\Rightarrow s = t$

i.e., s and t represent the same positive integer such that their decimal and binary representations are same.

$\therefore$ f is one-to-one

From the definition of f,

$$|S| \leq |Z^+|$$

i.e., S is countable

$\therefore$ The set of all finite bit strings is countable.

Q10. Find the boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Answer : Model Paper-1, Q1(d)

Given matrices are,

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A \odot B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 1) \vee (0 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 \vee 0 \\ 0 \vee 0 \vee 0 \vee 1 & 0 \vee 1 \vee 0 \vee 0 \\ 1 \vee 0 \vee 1 \vee 1 & 0 \vee 1 \vee 1 \vee 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\therefore \ A \odot B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Q11. Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)} and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S o R.

Answer : Model Paper-2, Q1(d)

Given relations are,

$$R = \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$$

$$S = \{(2, 1), (3, 1), (3, 2), (4, 2)\}.$$

The relation S o R is obtained such that the second element of ordered pair in R must agree with the first element of ordered pair in S.

Then, $(1, 2) \in R$ and $(2, 1) \in S \Rightarrow (1, 1) \in S \text{ o } R$

$(1, 3) \in R$ and $(3, 1) \in S \Rightarrow (1, 1) \in S \text{ o } R$

$(1, 3) \in R$ and $(3, 2) \in S \Rightarrow (1, 2) \in S \text{ o } R$

$(2, 3) \in R$ and $(3, 1) \in S \Rightarrow (2, 1) \in S \text{ o } R$

$(2, 3) \in R$ and $(3, 2) \in S \Rightarrow (2, 2) \in S \text{ o } R$

$(2, 4) \in R$ and $(4, 2) \in S \Rightarrow (2, 2) \in S \text{ o } R$

$\therefore$ $S \text{ o } R = \{(1, 1), (1, 2), (2,1), (2, 2)\}$

Q12. Define n-ary relation with example.

Answer :

If $A_1, A_2, A_3, \ldots\ldots, A_n$ be the sets (i.e., the domains of the relation) with degree 'n', then the n-ary relation on $A_1, A_2, A_3, \ldots\ldots, A_n$ is a subset of $A_1 \times A_2 \times A_3 \times \ldots\ldots \times A_n$.

Example

If R is a relation on $N \times N \times N$ consisting of triples (a, b, c) with $a < b < c$, where a, b, c are positive integers then the domains are $(1, 2, 3) \in R$, $(2, 3, 4) \in R$, $(3, 4, 5) \in R$ and so on.

Hence, the domains are the set of natural numbers.

Q13. The 3 tuples in a 3-ary relation represent the following attributes of a student database: student ID number, name, phone number.

(a) Is student ID number likely to be a primary key.

(b) Is name likely to be a primary key.

(c) Is phone number likely to be a primary key.

Answer :

Given that,

The attributes of database are student *ID* number, name, phone number.

(a) Every student has unique *ID* number. Therefore, student *ID* number is likely to be a primary key.

(b) As more than one student may have the same name, name is not likely to be a primary key.

(c) The students belonging to the same family can have the same phone number. Hence, phone number is not likely to be a primary key.

Q14. The 5-tuples in a 5-ary relation represent these attributes of all people in the United States: name, social security number, street address, city, state.

(a) Determine a primary key for this relation

(b) Under what conditions would (name, street address) be a composite key

(c) Under what conditions would (name, street address, city) be a composite key.

Answer : Model Paper-3, Q1(c)

Given that,

The attributes of the people in United States are name, social security number, street address, city, state.

(a) Different people may have the same name. Therefore, name is not a primary key. As each person is assigned with a unique social security number, it is likely to be a primary key. Street address, city and state are not likely to be the primary keys because different people can live at same street address, city and state.

(b) If no two people with same name live at the same street address then (name, street address) will be a composite key.

(c) If no two people with same name live at the same street address in same city then (name, street address, city) will be a composite key.

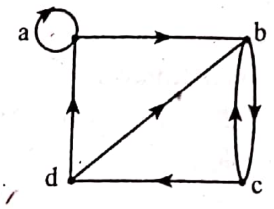
Q15. Draw the directed graph that represents the relation {(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)}

Answer :

Given relation is,

$$R = \{(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)\}$$

The directed graph representing the given relation is as shown in figure.



Figure

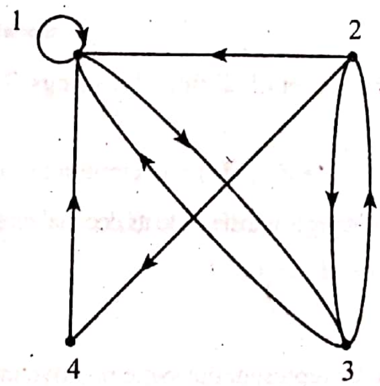
Q16. Draw the directed graph of the relation R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)} on the set {1, 2, 3, 4}.

Answer : Model Paper-3, Q1(d)

Given relation is,

$$R = \{(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1)\ (3, 2), (4, 1)\}$$

The directed graph of the relation R is as shown in the figure.



Figure

Q17. Let R_1 and R_2 be relations on a set A represented by the matrices $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Find the matrix that represent $R_1 \oplus R_2$

Answer :

Given matrices are,

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and}$$

$$M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_1 \oplus R_2} = M_{R_1} \oplus M_{R_2}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \oplus 0 & 1 \oplus 1 & 0 \oplus 0 \\ 1 \oplus 0 & 1 \oplus 1 & 1 \oplus 1 \\ 1 \oplus 1 & 0 \oplus 1 & 0 \oplus 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore M_{R_1 \oplus R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Q18. Let R be the relation (a, b) | (a ≠ b) on the set of integers. What is the reflexive closure of R?

Model Paper-5, Q1(c)

Answer :

Given relation is,

$$R = \{(a, b) \mid a \neq b\}$$

If R is a relation defined on a set A then the reflexive closure of R is given by,

$$R \cup \Delta$$

Where,

$$\Delta = \{(a, a) \mid a \in A\}$$

$\therefore$ Reflexive closure of $R = R \cup \Delta$

$$= \{(a, b) \mid a \neq b\} \cup \{(a, a) \mid a \in A\}$$

$$= \{(a, b) \mid a \neq b\} \cup \{(a, a) \mid a \in Z\} \quad [\because A = Z]$$

$$= \{(a, b) \mid a, b \in Z\} = Z \times Z = Z^2$$

$\therefore$ The reflexive closure of R is the set of all ordered pairs (a, b) where a and b are integers.

Q19. Let R be the relation {(a, b) | a divides b} on the set of integers. What is the symmetric closure of R?

Answer :

Given relation is,

$$R = \{(a, b) \mid a \text{ divides } b\}$$

The inverse of above relation is given by,

$$R^{-1} = \{(a, b) \mid b \text{ divides } a\}$$

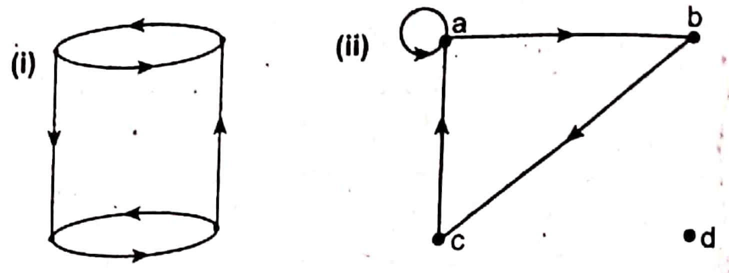
Symmetric closure of $R = R \cup R^{-1}$

$$= \{(a, b) \mid a \text{ divides } b\} \cup \{(a, b) \mid b \text{ divides } a\}$$

$$= \{(a, b) \mid a \text{ divides } b \text{ or } b \text{ divides } a\}$$

$\therefore$ The symmetric closure of R is the set of all ordered pairs (a, b) of integers which are composite to each other.

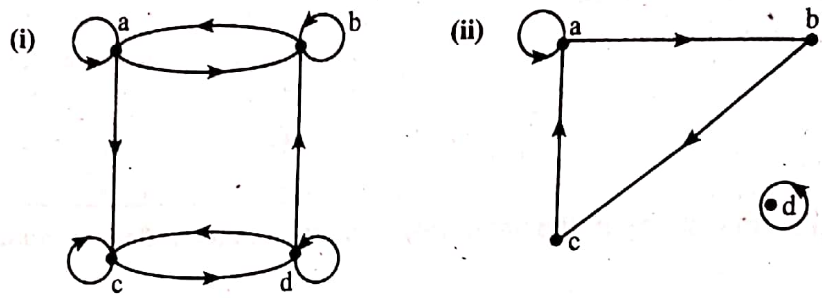
Q20. Draw the directed graph of the reflexive closure of the relations with the directed graphs shown.



Figure

Answer :

The directed graph of the reflexive closure of the relation is obtained by adding loops as shown in figures.



Figure

Q21. Determine the relation {(0, 0), (1, 1), (2, 2), (3, 3)} on {0, 1, 2, 3} is an equivalence relation?

Answer : Model Paper-4, Q1(d)

Given relation is,

$$R = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$$

Reflexive

R is reflexive because every element in A belongs to R i.e., $(a, a) \in R$.

Symmetric

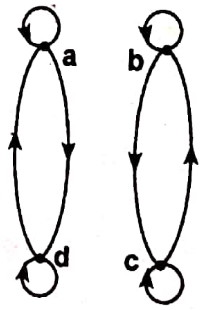
R is symmetric because $(b, a) \in R$ then $(a, b) \in R \ \forall a, b \in A$.

Transitive

R is transitive because $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R \ \forall a, b, c \in A$.

Hence, the given relation is an equivalence relation.

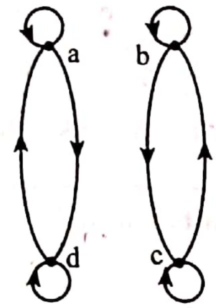
Q22. Determine whether the relation with the direct graph is an equivalence relation.



Figure

Answer :

Given directed graph is,



Figure

The given directed graph is said to be reflexive, because it contains loops at each vertex.

The given directed graph is said to be symmetric because all edges are in pairs and there are no single arrows between any pair of points.

The given directed graph is said to be transitive, if there is no relation between any elements of $\{a, d\}$ and $\{b, c\}$.

Thus, the relation with the directed graph is an equivalence relation.

Q23. Which of these are posets?

(a) (Z, =)

(b) (Z, $\nmid$)

Answer :

(a) $(Z, =)$

Given set is,

$(Z, =)$

The above set is a poset since it contains equality relation. The equality relation on any set is reflexive, antisymmetric and transitive.

(b) $(Z, \nmid)$

Given set is,

$(Z, \nmid)$

The above set is not a poset since the relation $\nmid$ is not reflexive, not antisymmetric and not transitive.

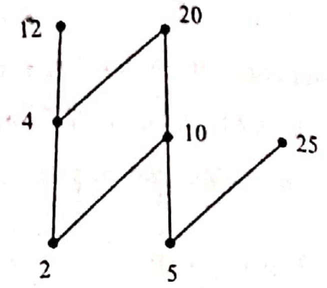
Q24. Which elements of the poset ({2,4,5,10,12,20,25},|) are maximal and which are minimal?

Answer :

Given poset is,

$(\{2, 4, 5, 10, 12, 20, 25\}, |)$

Hasse diagram for the above poset can be drawn as follows.



Figure

From the diagram, it is clear that the minimal elements are 2, 5 and the maximal elements are 12, 20 and 25.

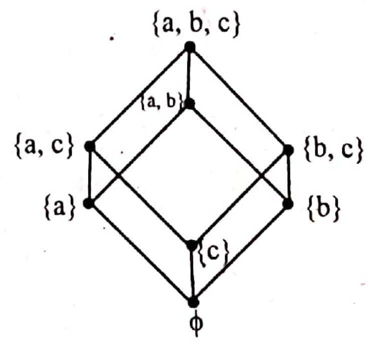
Q25. Draw the Hasse diagram for the partial ordering {(A, B)|A ⊆ B} on the power set P(S), where S = {a, b, c}.

Answer : **Model Paper-5, Q1(d)**

Given set is,

$S = \{a, b, c\}$

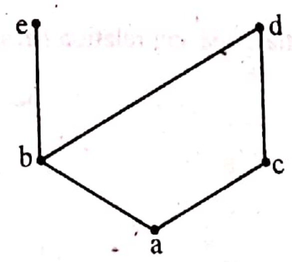
Hasse diagram of $(P(S), \subseteq)$ for $S = \{a, b, c\}$ is given as,



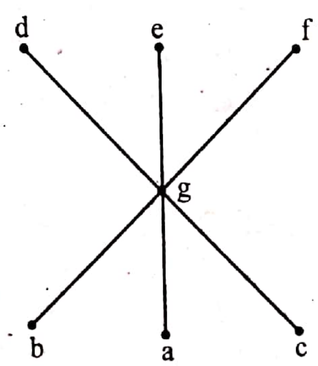
Figure

Q26. List all ordered pairs in the partial ordering with the accompanying Hasse diagram.

(a)



(b)



Answer :

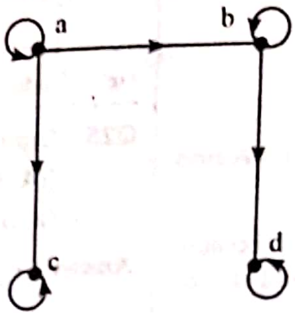
(a) For the given figure 1 the ordered pairs can be given by,

$R = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, d), (b, e), (c, c), (c, d), (d, d), (e, e)\}$

(b) For the given figure, the ordered pairs can be given by,

$R = \{(a, a), (a, g), (a, d), (a, e), (a, f), (b, b), (b, g), (b, d), (b, e), (b, f), (c, c), (c, g) (c, d), (c, e), (c, f), (g, d), (g, e), (g, f$
$(g, g), (d, d), (e, e), (f, f)\}$

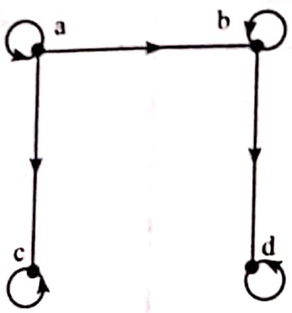
Q27. Determine whether the relation with the directed graph shown as a partial order.



Figure

Answer :

Given directed graph is,



Figure

R is reflexive because the directed graph consists of loops at every vertex.

R is antisymmetric because it does not contain any pair of arrows between any pair of vertices.

R is not transitive, because there is an arrow from a to b and from b to d but there is no arrow from a to d.

$\therefore$ R is not a partial ordering relation because R is not transitive.

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PART-B

ESSAY QUESTIONS WITH SOLUTIONS

2.1 SETS

Q28. Define set. How is it denoted and what are the two different ways of representing a set along with an example?

Answer :

Sets

The collection of finite or infinite number of objects with some common property is called set. The objects belonging to the set are called members or elements of the set.

Examples

(i) A group of people

(ii) A collection of stamps

(iii) A pair of glasses.

Notations

A set is usually denoted by capital letters with or without subscripts. Lower case letters are used to signify the elements of the set.

Representation of Set

There are two ways for representing a set. They are,

(a) Roaster method

(b) Set-builder form.

(a) Roaster Method

In this method, the elements are enclosed within the "{'}" brackets.

Example

Set of natural numbers

$N = \{1, 2, 3, 4, ...\}$

(b) Set-builder Form

In this method, the elements are described with respect to their common property.

Example

$\{x/x$ is a natural number$\}$

Q29. Explain with examples different operations that can be applied on the sets.

Answer :

The different operations performed on binary sets are,

(i) Union of sets ($\cup$)

(ii) Intersection of sets ($\cap$)

(iii) Relative complement of sets ($-$)

(iv) Symmetric difference of sets ($\oplus$)

(v) Absolute complement.

(i) Union of Sets

If A and B are two sets, then union of set creates a new set that consists of all the elements present either in A or in B or in both. Union of sets is denoted by $A \cup B$.

Symbolically, it is represented as,

$$A \cup B = \{x | x \in A \vee x \in B\}$$

Properties of Union of Sets

(a) Commutative: $A \cup B = B \cup A$

(b) Associative: $A \cup (B \cup C) = (A \cup B) \cup C$

(c) $A \cup A = A$ (Idempotent)

(d) $A \cup \phi = A$.

(ii) Intersection of Sets

If A and B are two sets, then intersection of sets creates a new set that consists of the common elements that are in A as well as in B. Intersection of sets is denoted by $A \cap B$.

Symbolically, it is represented as,

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

Properties of Intersection of Sets

(a) Commutative $A \cap B = B \cap A$

(b) Associative $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cap A = A$ (Idempotent)

(d) $A \cap \phi = \phi$

(iii) Relative Complement (Difference)

If A and B are two sets, then relative complement of B with respect to A creates a new set that consists of elements belonging to A, but not B. Relative complement is denoted by $A - B$.

$$A - B = \{x | (x \in A \wedge x \notin B)\}$$

Properties of Relative Complement

(a) Commutative: $A - B \neq B - A$

(b) Associative: $A - (B - C) \neq (A - B) - C$

(iv) Symmetric Difference (Boolean Sum)

If A and B are two sets, then their symmetric difference creates a new set that consists of elements present either in A or in B but not both.

It is denoted as $A \oplus B = (A - B) \cup (B - A)$.

Symbolically, it is represented as,

$$A \oplus B = \{x | (x \in A \vee x \in B) \wedge x \notin (A \cap B)\}$$

Properties of Symmetric Difference

(a) Commutative: $A \oplus B = B \oplus A$

(b) Associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

(v) Absolute Complement

$\overline{A}$ (or) $U - A$ (i.e., relative complement of A with respect to U) is called as absolute complement or complement of A with respect to U. Symbolically it is represented as,

$$\overline{A} = \{x | x \notin A\}$$

Q30. Let A = {1, 2, 3, 4, 5) and B = {0, 3, 6}. Find

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

Answer :

Given sets are,

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 3, 6\}$$

(a) $A \cup B$

$$A \cup B = \{1, 2, 3, 4, 5\} \cup \{0, 3, 6\}$$
$$\therefore A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

(b) $A \cap B$

$$A \cap B = \{1, 2, 3, 4, 5\} \cap \{0, 3, 6\}$$
$$\therefore A \cap B = \{3\}$$

(c) $A - B$

$$A - B = \{1, 2, 3, 4, 5\} - \{0, 3, 6\}$$
$$\therefore A - B = \{1, 2, 4, 5\}$$

(d) $B - A$

$$B - A = \{0, 3, 6\} - \{1, 2, 3, 4, 5\}$$
$$\therefore B - A = \{0, 6\}$$

Q31. Find the sets A and B if A – B = {1, 5, 7, 8}, B – A = {2, 10}, and A ∩ B = {3, 6, 9}.

Answer :

Given that,

$$A - B = \{1, 5, 7, 8\}$$
$$B - A = \{2, 10\}$$
$$A \cap B = \{3, 6, 9\}$$

The above values can be represented in the form of Venn diagram as,

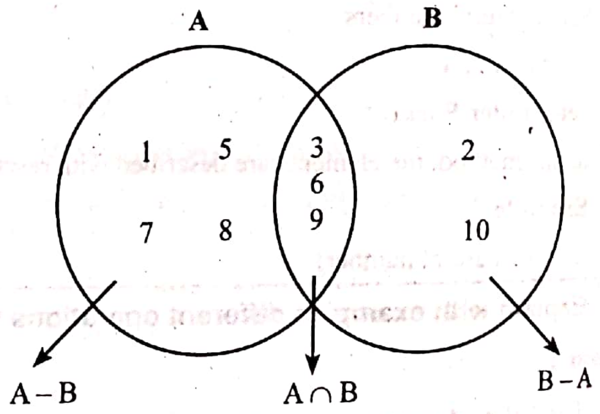


Figure: Venn Diagram

Set A

Set A contains the elements of $A \cap B$ and also the elements of set A but not the elements of set B.

i.e., $A = (A \cap B) \cup (A - B)$

$= \{3, 6, 9\} \cup \{1, 5, 7, 8\}$

$= \{1, 3, 5, 6, 7, 8, 9\}$

$\therefore A = \{1, 3, 5, 6, 7, 8, 9\}$

Set B

Set B contains the elements of $A \cap B$ and also the elements of set B but not the elements of set A.

i.e., $B = (A \cap B) \cup (B - A)$

$= \{3, 6, 9\} \cup \{2, 10\}$

$= \{2, 3, 6, 9, 10\}$

$\therefore \quad B = \{2, 3, 6, 9, 10\}$

Q32. Let A, B and C be sets. Show that (A – B) – C = (A – C) – (B – C).

Model Paper-1, Q4(a)

Answer :

Consider,

$(A - C) - (B - C) = \{x | x \in (A - C) - (B - C)\}$

$= \{x | x \in (A - C) \wedge x \notin (B - C)\}$

$= \{x | x \in (A - C) \wedge x \in \overline{(B - C)}\}$

$= \{x | x \in (A \cap \overline{C}) \wedge x \in \overline{(B \cap \overline{C})}\}$ $\quad [\because A - B = A \cap \overline{B}]$

$= \{x | x \in (A \cap \overline{C}) \wedge x \in (\overline{B} \cup \overline{\overline{C}})\}$ $\quad [\because \overline{A \cap B} = \overline{A} \cup \overline{B}]$

$= \{x | x \in (A \cap \overline{C}) \wedge x \in (\overline{B} \cup C)\}$

$= \{x | x \in (A \cap \overline{C}) \cap (\overline{B} \cup C)\}$ $\quad [\because$ Intersection of two sets$]$ ... (1)

From associative law,

$(A \cap \overline{C}) \cap (\overline{B} \cup C) = ((A \cap \overline{C}) \cap \overline{B}) \cup ((A \cap \overline{C}) \cap C)$ ... (2)

Substituting the equation (2) in equation (1),

$(A - C) - (B - C) = \{x | x \in ((A \cap \overline{C}) \cap \overline{B}) \cup ((A \cap \overline{C}) \cap C)\}$

$= \{x | x \in ((A \cap \overline{C}) \cap \overline{B}) \vee x \in ((A \cap \overline{C}) \cap C)\}$ $\quad [\because$ Union of sets$]$

$= \{x | x \in (A \cap (\overline{C} \cap \overline{B})) \vee x \in (A \cap (\overline{C} \cap C)\}$ $\quad [\because$ Associative law$]$

$= \{x | x \in (A \cap (\overline{B} \cap \overline{C})) \vee x \in (A \cap \phi)\}$ $\quad [\because C \cap \overline{C} = \phi]$

$= \{x | x \in (A \cap (\overline{B} \cap \overline{C})) \vee x \in \phi\}$ $\quad [\because A \cap \phi = \phi]$

$= \{x | x \in (A \cap \overline{B}) \cap \overline{C}\}$

$= \{x | x \in (A - B) \cap \overline{C}\}$ $\quad [\because A \cap B = A - \overline{B}]$

$= \{x | x \in (A - B) - C\}$

$= (A - B) - C$

$\therefore \quad (A - C) - (B - C) = (A - B) - C$

Q33. Prove the complement laws,

(i) $A \cup \overline{A} = U$

(ii) $A \cap \overline{A} = \phi$

Answer :

(i) $A \cup \overline{A} = U$

Consider,

$A \cup \overline{A} = \{x | x \in A \vee x \in \overline{A}\}$

$= \{x | x \in A \vee x \notin A\}$ $\quad [\because$ Complement law$]$

$= \{x | x \in A \vee \neg(x \in A)\}$

$= \{x | T\}$ $\quad [\because A \vee \neg A \equiv T]$

$= \{x | x \in U\}$

$= U$

$\therefore \quad A \cup \overline{A} = U$

(ii) $A \cap \bar{A} = \phi$

Consider,

$$A \cap \bar{A} = \{x | x \in A \wedge x \in \bar{A}\}$$
$$= \{x | x \in A \wedge x \notin A)\}$$
$$= \{x | x \in A \wedge \neg (x \in A)\}$$
$$= \{x | F\} \qquad [\because A \cap \bar{A} \equiv F]$$
$$= \{x | x \in \phi\}$$
$$= \phi$$
$$\therefore A \cap \bar{A} = \phi$$

Q34. If A and B are sets, then prove $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

(a) By showing each side is a subset of the other side.

(b) Using a membership table.

Answer :

Given,

A and B are sets.

(a) **Case (i)**

Consider,

$$\overline{A \cup B}$$

Let, $x \in \overline{A \cup B}$

$\Rightarrow \quad x \notin (A \cup B)$

[$\because$ From the definition of complement]

$\Rightarrow \quad x \notin A \wedge x \notin B$

$\Rightarrow \quad x \in \bar{A} \wedge x \in \bar{B}$

$\Rightarrow \quad x \in \bar{A} \cap \bar{B}$

$\therefore \quad \overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$

Case (ii)

Consider,

$$\bar{A} \cap \bar{B}$$

Let, $x \in (\bar{A} \wedge \bar{B})$

$\Rightarrow \quad x \in \bar{A} \wedge x \in \bar{B}$

$\Rightarrow \quad x \notin A \wedge x \notin B$

$\Rightarrow \quad \neg (x \in A) \wedge \neg (x \in B)$

$\Rightarrow \quad \neg (x \in A \vee x \in B)$

$\Rightarrow \quad \neg (x \in (A \vee B))$

$\Rightarrow \quad x \notin A \cup B$

$\Rightarrow \quad x \in \overline{A \cup B}$

[$\because$ From the definition of complement]

$\Rightarrow \quad \bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$

From cases (i) and (ii), two sets are said to be equal.

$\therefore \quad \overline{A \cup B} = \bar{A} \cap \bar{B}$

(b) Membership table can be obtained as,

A	B	$A \cup B$	$\bar{A}$	$\bar{B}$	$\overline{A \cup B}$	$\bar{A} \cap \bar{B}$
0	0	0	1	1	1	1
0	1	1	1	0	0	0
1	0	1	0	1	0	0
1	1	1	0	0	0	0

Table

From the above table, last two columns are equal.

$\therefore \quad \overline{A \cup B} = \bar{A} \cap \bar{B}$.

Q35. Let A and B be sets. Show that,

(i) $A - B \subseteq A$

(ii) $A \cap (B - A) = \phi$

Answer :

Given,

A and B are sets.

(i) $A - B \subseteq A$

Consider,

$$A - B = \{x | x \in (A - B)\}$$
$$= \{x | x \in A \text{ and } x \notin B\}$$
$$= \{x | x \in A \wedge x \notin B\}$$
$$= \{x | x \in A \wedge x \in \bar{B}\}$$
$$= \{x | x \in A\}$$
$$= A$$
$$\therefore A - B \subseteq A$$

(ii) $A \cap (B - A) = \phi$

Consider,

$$B - A = \{x | x \in B \text{ and } x \notin A\}$$
$$= \{x | x \in B \wedge x \notin A\}$$
$$= \{x | x \in B \wedge x \in \bar{A}\}$$
$$= \{x | x \in (B \cap \bar{A})\}$$
$$\Rightarrow B - A = B \cap \bar{A}$$

Then,

$$A \cap (B - A) = A \cap (B \cap \bar{A})$$
$$= A \cap (\bar{A} \cap B) \quad [\because \text{Commutative law}]$$
$$= (A \cap \bar{A}) \cap B \quad [\because \text{Associative law}]$$
$$= \phi \cap B$$
$$= \phi$$
$$\therefore A \cap (B - A) = \phi$$

2.2 FUNCTIONS

Q36. Define the following

(i) One-to-one function

(ii) Onto function

(iii) Bijective function

(iv) Inverse function

Answer :

Let 'f' is a function from set A to set B, then f is defined as $f : A \rightarrow B$.

(i) **One-to-One Function**

A function 'f' is said to be one-to-one function if and only if, $f(a) = f(b) \Rightarrow a = b \ \forall \ a$ and b in the domain of f. It is also called as injective function.

(ii) **Onto Function**

'f' is said to be onto function if and only if, for every element $b \in B$, there exists an element $a \in A$ with $f(a) = b$. It is also called as surjective function.

(iii) **Bijective Function**

If 'f' is both one-to-one and onto functions then f is said to be a bijective function. It is also called as one-to-one correspondence function.

(iv) **Inverse Function**

Inverse function is the function which assigns to the element $b \in B$, a unique element $a \in A$ such that

$f^{-1}(b) = a$ when $f(a) = b$.

f^{-1} is the inverse function of f.

Q37. Determine whether each of the functions are one-to-one and onto.

(i) f(a) = b, f(b) = a, f(c) = c, f(d) = d

(ii) f(a) = d, f(b) = b, f(c) = c, f(d) = d.

Answer :

(i) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$

The above values are represented as,

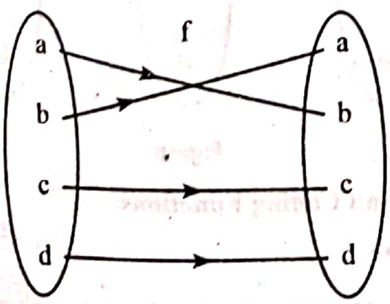


Figure (1)

The above function is both one-to-one and onto as f is assigned to different values for all the elements in codomain.

(ii) $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

The above values are represented as,

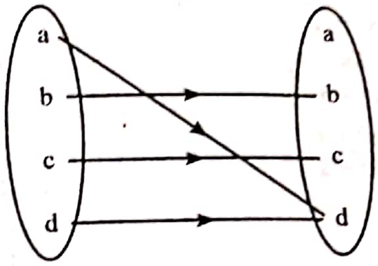


Figure (2)

The above function is neither one-to-one nor onto as f is assigned to same values and 'a' in codomain is not the image of any element.

Q38. Suppose that g is a function from A to B and f is a function from B to C. Show that,

(i) If both f and g are one-to-one functions, then fog is also one-to-one.

(ii) If both f and g are onto functions, then fog is also onto.

Answer : **Model Paper-4, Q4(a)**

Given functions are,

$$g : A \rightarrow B, \ f : B \rightarrow C$$

(i) **If both f and g are one-to-one functions, then fog is also one-to-one.**

If f is one-to-one function then $f(x) = f(y) \Rightarrow x = y$

If g is one-to-one function then $g(x) = g(y) \Rightarrow x = y$

Consider,

$$(fog)(a) = (fog)(b)$$

$\Rightarrow \quad f(g(a)) = f(g(b))$

$\Rightarrow \quad g(a) = g(b) \qquad [\because f \text{ is one-to-one}]$

$\Rightarrow \quad a = b \qquad [\because g \text{ is one-to-one}]$

$\therefore \quad (fog)$ is a one-to-one function.

(ii) **If both f and g are onto functions, then fog is also onto**

If f is onto function, $\forall c \in C, \exists \ b \in B$ such that $f(b) = c$

If g is onto function, $\forall b \in B, \exists \ a \in A$ such that $g(a) = b$

Let, $x \in C$, then there exists an element $y \in B$ such that $f(y) = x$, where f is onto function.

As g is onto function, there exists an element $z \in A$ such that $g(z) = y$.

Let the composition of f and g be z.

Consider,

$$(fog)(z) = f(g(z))$$
$$= f(y)$$
$$= x$$

$\Rightarrow \quad (fog)(z) = x$

Thus, for every element c in C, there exists an element a in A such that $(fog)(a) = c$.

$\therefore \quad (fog)$ is onto function.

Q39. Determine whether each of these functions is a bijection from R to R.

(a) $x^2 + 1$

(b) $-3x + 4$

(c) $x^5 + 1$

(d) $\dfrac{x^2+1}{x^2+2}$

Answer :

(a) $x^2 + 1$

Given function is,

$$f(x) = x^2 + 1$$

Let, $f(x) = f(y)$

$\Rightarrow \quad x^2 + 1 = y^2 + 1$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad x = \pm y$

f is not one-to-one function,

Hence, f is not a bijection.

(b) $-3x + 4$

Given function is,

$$f(x) = -3x + 4$$

Let, $f(x) = f(y)$

$\Rightarrow \quad -3x + 4 = -3y + 4$

$\Rightarrow \quad -3x = -3y$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-to-one function

Let, $y = f(x)$

$\Rightarrow \quad y = -3x + 4$

$\Rightarrow \quad x = \dfrac{y-4}{-3}$

$\Rightarrow \quad x = \dfrac{4-y}{3}$

$$f\left(\frac{4-y}{3}\right) = -3\left(\frac{4-y}{3}\right) + 4$$

$\Rightarrow \quad f\left(\dfrac{4-y}{3}\right) = y - 4 + 4$

$\Rightarrow \quad f\left(\dfrac{4-y}{3}\right) = y$

$\therefore \quad f$ is onto function.

As f is both one-to-one and onto function, f is a bijection.

(c) $x^5 + 1$

Given function is,

$$f(x) = x^5 + 1$$

Let, $f(x) = f(y)$

$\Rightarrow \quad x^5 + 1 = y^5 + 1$

$\Rightarrow \quad x^5 = y^5$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-to-one function

Let, $y = f(x)$

$\Rightarrow \quad y = x^5 + 1$

$\Rightarrow \quad x^5 = y - 1$

$\Rightarrow \quad x = (y-1)^{1/5}$

$$f((y-1)^{1/5}) = ((y-1)^{1/5})^5 + 1$$

$\Rightarrow \quad f((y-1)^{1/5}) = y - 1 + 1$

$\Rightarrow \quad f((y-1)^{1/5}) = y$

$\therefore \quad f$ is onto function.

As f is both one-to-one and onto function, f is a bijection.

(d) $\dfrac{x^2+1}{x^2+2}$

Given function is,

$$f(x) = \frac{x^2+1}{x^2+2}$$

Let, $f(x) = f(y)$

$\Rightarrow \quad \dfrac{x^2+1}{x^2+2} = \dfrac{y^2+1}{y^2+2}$

$\Rightarrow \quad (x^2+1)(y^2+2) = (y^2+1)(x^2+2)$

$\Rightarrow \quad x^2y^2 + 2x^2 + y^2 + 2 = x^2y^2 + 2y^2 + x^2 + 2$

$\Rightarrow \quad 2x^2 + y^2 = 2y^2 + x^2$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad x = \pm y$

f is not one-to-one function, hence f is not a bijection.

Q40. Define the following terms,

(a) Composite function

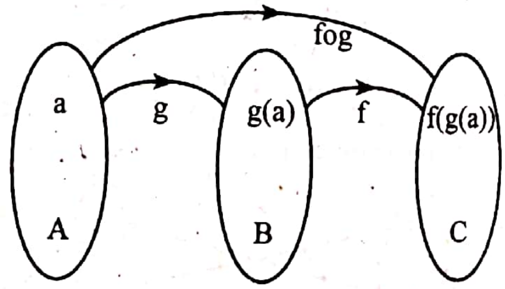
(b) Floor and ceiling functions.

Answer :

(a) Composite Function

If $g : A \to B$ and $f : B \to C$ are the two functions then the composition of the functions f and g is defined as

$$fog: A \to C,\ (fog)(a) = f(g(a)).$$



Figure

(b) Floor and Ceiling Functions

Floor Function

The floor function assigns the real number 'x' to its next smallest integer. It is denoted as $\lfloor x \rfloor$.

Example: $\lfloor 0.5 \rfloor = 0$

Ceiling Function

The ceiling function assigns the real number 'x' to its next largest integer. It is denoted as $\lceil x \rceil$.

Example: $\lceil 0.5 \rceil = 1$

Q41. Find the following for the functions f(x) = x² + 1 and g(x) = x + 2.

(i) f + g

(ii) fg

(iii) fog

(iv) gof

Answer :

Given functions are,

$$f(x) = x^2 + 1 \text{ and } g(x) = x + 2$$

(i) f + g

$$(f+g)(x) = f(x) + g(x)$$
$$= x^2 + 1 + x + 2$$
$$= x^2 + x + 3$$
$$\therefore (f+g)(x) = x^2 + x + 3$$

(ii) fg

$$(fg)(x) = f(x) \cdot g(x)$$
$$= (x^2 + 1)(x + 2)$$
$$= x^3 + 2x^2 + x + 2$$
$$\therefore (fg)(x) = x^3 + 2x^2 + x + 2$$

(iii) fog

$$(fog)(x) = f(g(x))$$
$$= f(x + 2)$$
$$= (x + 2)^2 + 1$$
$$= x^2 + 4 + 4x + 1$$
$$= x^2 + 4x + 5$$
$$\therefore (fog)(x) = x^2 + 4x + 5$$

(iv) gof

$$(gof)(x) = g(f(x))$$
$$= g(x^2 + 1)$$
$$= x^2 + 1 + 2$$
$$= x^2 + 3$$
$$\therefore (gof)(x) = x^2 + 3$$

Q42. Let f(x) = ax + b and g(x) = cx + d, where a, b, c and d are constants. Determine for which constants a, b, c and d it is true that fog = gof

Answer :

Given functions are,

$$f(x) = ax + b \quad \text{... (1)}$$
$$g(x) = cx + d \quad \text{... (2)}$$

And $fog = gof$ ... (3)

Consider,

$$(fog) = f(g(x))$$
$$\Rightarrow (fog)(x) = f(cx + d) \quad [\because \text{From equation (2)}]$$
$$\Rightarrow (fog)(x) = a(cx + d) + b$$
$$\Rightarrow (fog)(x) = acx + ad + b$$

Consider,

$$(gof) = g(f(x))$$
$$\Rightarrow (gof)(x) = g(ax + b) \quad [\because \text{From equation (1)}]$$
$$\Rightarrow (gof)(x) = c(ax + b) + d$$
$$\Rightarrow (gof)(x) = acx + bc + d$$

Substituting the corresponding values in equation (3),

$$acx + ad + b = acx + bc + d$$
$$\Rightarrow ad + b = bc + d$$

$\therefore$ The constants a, b, c and d can be determined from the equation $ad + b = bc + d$.

Q43. Let g be the function from the set {a, b, c} to itself such that g(a) = b, g(b) = c and g(c) = a. Let f be the function from the set {a, b, c} to the set {1, 2, 3} such that f(a) = 3, f(b) = 2 and f(c) = 1. What is the composition of

(i) f and g

(ii) g and f.

Answer : **Model Paper-1, Q4(b)**

Given that,

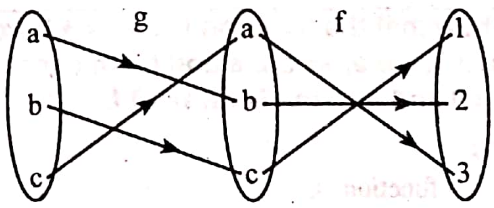
g is the function from the set $\{a, b, c\}$ to itself.

$$g(a) = b$$
$$g(b) = c$$
$$g(c) = a$$

f is the function from the set $\{a, b, c\}$ to $\{1, 2, 3\}$

$$f(a) = 3$$
$$f(b) = 2$$
$$f(c) = 1$$

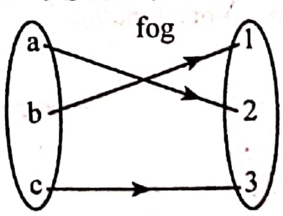
The functions f and g are defined as,



Figure(1)

(i) **Composition of f and g**

$$(fog)(a) = f(g(a)) = f(b) = 2$$
$$(fog)(b) = f(g(b)) = f(c) = 1$$
$$(fog)(c) = f(g(c)) = f(a) = 3$$



Figure(2)

(ii) Composition of g and f

$(gof)(a) = g(f(a)) = g(3) \rightarrow$ Not defined

$(gof)(b) = g(f(b)) = g(2) \rightarrow$ Not defined

$(gof)(c) = g(f(c)) = g(1) \rightarrow$ Not defined

Hence gof is not defined, as the range of f is not a subset of the domain of g.

Q44. Let f and g be the functions from the set of integers to the set of integers defined by f(x) = 2x + 3 and g(x) = 3x + 2. What is the composition of,

(i) f and g

(ii) g and f

Answer :

Given functions are,

$f(x) = 2x + 3$ ... (1)

$g(x) = 3x + 2$ ... (2)

(i) Composition of f and g

$(fog)(x) = f(g(x))$

$= f(3x + 2)$ [∵ From equation (2)]

$= 2(3x + 2) + 3$

$= 6x + 4 + 3$

$= 6x + 7$

$\therefore (fog)(x) = 6x + 7$

(ii) Composition of g and f

$(gof)(x) = g(f(x))$

$= g(2x + 3)$ [∵ From equation (1)]

$= 3(2x + 3) + 2$

$= 6x + 9 + 2$

$= 6x + 11$

$\therefore (gof)(x) = 6x + 11$

Q45. Show that the function f(x) = ax + b from R to R is invertible, where a and b are constants, with a ≠ 0 and find the inverse of f.

Answer :

Given function is,

$f(x) = ax + b$ ... (1)

Let, $f(x) = f(y)$

$\Rightarrow ax + b = ay + b$

$\Rightarrow ax = ay$

$\Rightarrow x = y$

$\therefore$ f is one-to-one function

Let, $y = f(x)$

$\Rightarrow y = ax + b$ [∵ From equation (1)] ... (2)

$\Rightarrow x = \dfrac{y-b}{a}$

$\Rightarrow f^{-1}(y) = \dfrac{y-b}{a}$ ... (3)

Consider,

$(fof^{-1})(y) = f(f^{-1}(y))$

$\Rightarrow (fof^{-1})(y) = f\left(\dfrac{y-b}{a}\right)$ [∵ From equation (3)]

$\Rightarrow (fof^{-1})(y) = a\left(\dfrac{y-b}{a}\right) + b$

$\Rightarrow (fof^{-1})(y) = y$

Consider,

$(f^{-1}of)(x) = f^{-1}(f(x))$

$\Rightarrow (f^{-1}of)(x) = f^{-1}(ax + b)$ [∵ From equation (1)]

$\Rightarrow (f^{-1}of)(x) = \dfrac{ax+b-b}{a}$

$\Rightarrow (f^{-1}of)(x) = x$

As $(fof^{-1})(y) = y$ and $(f^{-1}of)(x) = x$, f is onto function

∵ f is one-to-one and onto function, f is invertible.

The inverse of a function is obtained by interchanging x and y in equation (2).

i.e., $x = ay + b$

$\Rightarrow y = \dfrac{x-b}{a}$

$\Rightarrow f^{-1}(x) = \dfrac{x-b}{a}$

$\therefore$ Inverse of the function is $f^{-1}(x) = \dfrac{x-b}{a}$.

Q46. Find the values of the following functions,

(i) $\lfloor 1.1 \rfloor$

(ii) $\lfloor -0.1 \rfloor$

(iii) $\lceil 2.99 \rceil$

(iv) $\left\lceil \dfrac{-3}{4} \right\rceil$

(v) $\lceil 3 \rceil$

(vi) $\lfloor -1 \rfloor$

(vii) $\left\lfloor \dfrac{1}{2} + \left\lceil \dfrac{3}{2} \right\rceil \right\rfloor$

(viii) $\left\lceil \left\lfloor \dfrac{1}{2} \right\rfloor + \left\lceil \dfrac{1}{2} \right\rceil + \dfrac{1}{2} \right\rceil$

(ix) $\left\lfloor \dfrac{1}{2} \cdot \left\lfloor \dfrac{5}{2} \right\rfloor \right\rfloor$

Answer :

(i) $\lfloor 1.1 \rfloor$

Given function is,

$\lfloor 1.1 \rfloor$

The next lowest integer of 1.1 is 1

$\therefore$ Value of $\lfloor 1.1 \rfloor = 1$

(ii) $\lfloor -0.1 \rfloor$

Given function is,

$\lfloor -0.1 \rfloor$

The next lowest integer of -0.1 is -1.

$\therefore$ Value of $\lfloor -0.1 \rfloor = -1$.

(iii) $\lceil 2.99 \rceil$

Given function is,

$$\lceil 2.99 \rceil$$

The next highest integer of 2.99 is 3.

$\therefore$ Value of $\lceil 2.99 \rceil = 3$

(iv) $\left\lceil \frac{-3}{4} \right\rceil$

Given function is,

$$\left\lceil \frac{-3}{4} \right\rceil = \lceil -0.75 \rceil$$

The next highest integer of -0.75 is 0

$\therefore$ Value of $\left\lceil \frac{-3}{4} \right\rceil = 0$

(v) $\lceil 3 \rceil$

Given function is,

$$\lceil 3 \rceil$$

The number '3' itself is an integer.

$\therefore$ Value of $\lceil 3 \rceil = 3$

(vi) $\lfloor -1 \rfloor$

Given function is,

$$\lfloor -1 \rfloor$$

The number '-1' itself is an integer

$\therefore$ Value of $\lfloor -1 \rfloor = -1$

(vii) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$

Given function is,

$$\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor = \lfloor 0.5 + \lceil 1.5 \rceil \rfloor$$

The next highest integer of 1.5 is 2

$$\lfloor 0.5 + \lceil 1.5 \rceil \rfloor = \lfloor 0.5 + 2 \rfloor$$
$$= \lfloor 2.5 \rfloor$$

The next lowest integer of 2.5 is 2.

$\therefore$ Value of $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor = 2$

(viii) $\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{1}{2} \right\rceil$

Given function is,

$$\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{1}{2} \right\rceil = \lceil \lfloor 0.5 \rfloor + \lceil 0.5 \rceil + 0.5 \rceil$$

The next highest integer of 0.5 is 1 and the next lowest integer of 0.5 is 0.

$$\lceil \lfloor 0.5 \rfloor + \lceil 0.5 \rceil + 0.5 \rceil = \lceil 0 + 1 + 0.5 \rceil$$
$$= \lceil 1.5 \rceil$$

The next highest integer of 1.5 is 2

$\therefore$ Value of $\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{1}{2} \right\rceil = 2.$

(ix) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$

Given function is,

$$\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor = \lfloor 0.5 \lfloor 2.5 \rfloor \rfloor$$

The next lowest integer of 2.5 is 2

$$\lfloor 0.5 \lfloor 2.5 \rfloor \rfloor = \lfloor 0.5 \times 2 \rfloor$$
$$= \lfloor 1.0 \rfloor$$
$$= \lfloor 1 \rfloor$$

As the number '1' itself is an integer,

$$\lfloor 1 \rfloor = 1$$

$\therefore$ Value of $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor = 1$

Q47. Let s = {–1, 0, 2, 4, 7}. Find f(s) if

(a) f(x) = 1

(b) f(x) = 2x + 1

(c) f(x) = $\lceil x/5 \rceil$

(d) f(x) = $\left\lfloor \frac{x^2+1}{3} \right\rfloor$

Answer :

Given set is,

$$s = \{-1, 0, 2, 4, 7\}$$

(a) f(x) = 1

Given function is,

$$f(x) = 1$$
$$f(-1) = 1$$
$$f(0) = 1$$
$$f(2) = 1$$
$$f(4) = 1$$
$$f(7) = 1$$
$$\therefore f(s) = \{1\}$$

(b) f(x) = 2x + 1

Given function is,

$$f(x) = 2x + 1$$
$$f(-1) = 2(-1) + 1$$
$$\Rightarrow f(-1) = -1$$
$$f(0) = 2(0) + 1$$
$$\Rightarrow f(0) = 1$$
$$f(2) = 2(2) + 1$$
$$\Rightarrow f(2) = 5$$
$$f(4) = 2(4) + 1$$
$$\Rightarrow f(4) = 9$$
$$f(7) = 2(7) + 1$$
$$\Rightarrow f(7) = 15$$
$$\therefore f(s) = \{-1, 1, 5, 9, 15\}$$

(c) $f(x) = \lceil x/5 \rceil$

Given function is,

$$f(x) = \lceil x/5 \rceil$$

$$f(-1) = \lceil -1/5 \rceil = \lceil -0.2 \rceil$$

The next highest integer of – 0.2 is 0.

$\Rightarrow$ $f(-1) = 0$

$$f(0) = \lceil 0/5 \rceil = \lceil 0 \rceil$$

The number '0' itself is an integer.

$\Rightarrow$ $f(0) = 0$

$$f(2) = \lceil 2/5 \rceil = \lceil 0.4 \rceil$$

The next higher integer of 0.4 is 1.

$\Rightarrow$ $f(2) = 1$

$$f(4) = \lceil 4/5 \rceil = \lceil 0.8 \rceil$$

The next highest integer of 0.8 is 1

$\Rightarrow$ $f(4) = 1$

$$f(7) = \lceil 7/5 \rceil = \lceil 1.4 \rceil$$

The next highest integer of 1.4 is 2

$\Rightarrow$ $f(7) = 2$

$\therefore$ $f(s) = \{0, 1, 2\}$

(d) $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$

Given function is,

$$f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$$

$$f(-1) = \left\lfloor \frac{(-1)^2+1}{3} \right\rfloor = \lfloor 0.66 \rfloor$$

The next lowest integer of 0.66 is 0.

$\Rightarrow$ $f(-1) = 0$

$$f(0) = \left\lfloor \frac{(0)^2+1}{3} \right\rfloor = \lfloor 0.33 \rfloor$$

The next lowest integer of 0.33 is 0.

$\Rightarrow$ $f(0) = 0$

$$f(2) = \left\lfloor \frac{(2)^2+1}{3} \right\rfloor = \lfloor 1.66 \rfloor$$

The next lowest integer of 1.66 is 1.

$\Rightarrow$ $f(2) = 1$

$$f(4) = \left\lfloor \frac{(4)^2+1}{3} \right\rfloor = \lfloor 5.66 \rfloor$$

The next lowest integer of 5.66 is 5.

$\Rightarrow$ $f(4) = 5$

$$f(7) = \left\lfloor \frac{(7)^2+1}{3} \right\rfloor = \lfloor 16.66 \rfloor$$

The next highest integer of 16.66 is 16.

$\Rightarrow$ $f(7) = 16$

$\therefore$ $f(s) = \{0, 1, 5, 16\}$

Q48. Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(s)$ if,

(a) $s = \{-2, -1, 0, 1, 2, 3\}$

(b) $s = \{1, 5, 7, 11\}$

Model Paper-3, Q4(b)

Answer :

Given function is,

$$f(x) = \lfloor x^2/3 \rfloor$$

(a) $s = \{-2, -1, 0, 1, 2, 3\}$

$$f(-2) = \left\lfloor \frac{(-2)^2}{3} \right\rfloor = \lfloor 1.33 \rfloor$$

The next lowest integer of 1.33 is 1

$\Rightarrow$ $f(-2) = 1$

$$f(-1) = \left\lfloor \frac{(-1)^2}{3} \right\rfloor = \lfloor 0.33 \rfloor$$

The next lowest integer of 0.33 is 0

$\Rightarrow$ $f(-1) = 0$

$$f(0) = \left\lfloor \frac{0^2}{3} \right\rfloor = \lfloor 0 \rfloor$$

The number '0' itself is an integer.

$\Rightarrow$ $f(0) = 0$

$$f(1) = \left\lfloor \frac{1^2}{3} \right\rfloor = \lfloor 0.33 \rfloor$$

The next lowest integer of 0.33 is 0

$\Rightarrow$ $f(1) = 0$

$$f(2) = \left\lfloor \frac{2^2}{3} \right\rfloor = \lfloor 1.33 \rfloor$$

The next lowest integer of 1.33 is 1

$\Rightarrow$ $f(2) = 1$

$$f(3) = \left\lfloor \frac{3^2}{3} \right\rfloor = \lfloor 3 \rfloor$$

The number '3' itself is an integer.

$\Rightarrow$ $f(3) = 3$

$\therefore$ $f(s) = \{0, 1, 3\}$

(b) $s = \{1, 5, 7, 11\}$

$$f(1) = \left\lfloor \frac{(1)^2}{3} \right\rfloor = \lfloor 0.33 \rfloor$$

The next lowest integer of 0.33 is 0

$\Rightarrow$ $f(1) = 0$

$$f(5) = \left\lfloor \frac{(5)^2}{3} \right\rfloor = \lfloor 8.33 \rfloor$$

The next lowest integer of 8.33 is 8

$\Rightarrow$ $f(5) = 8$

$$f(7) = \left\lfloor \frac{(7)^2}{3} \right\rfloor = \lfloor 16.33 \rfloor$$

The next lowest integer of 16.33 is 16

$\Rightarrow$ $f(7) = 16$

$$f(11) = \left\lfloor \frac{(11)^2}{3} \right\rfloor = \lfloor 40.33 \rfloor$$

The next lowest integer of 40.33 is 40

$\Rightarrow$ $f(11) = 40$

$\therefore$ $f(s) = \{0, 8, 16, 40\}$.

2.3 SEQUENCES AND SUMMATIONS

Q49. What is the term a_8 of the sequence $\{a_n\}$ if a_n equals

(i) 2^{n-1}

(ii) 7

(iii) $1+(-1)^n$

Answer :

(i) 2^{n-1}

Given sequence is,

$a_n = 2^{n-1}$

$a_8 = 2^{8-1}$

$\Rightarrow \quad a_8 = 2^7 = 128$

$\therefore \quad a_8 = 128$

(ii) 7

Given sequence is,

$a_n = 7$

$\Rightarrow \quad a_8 = 7$

(iii) $1+(-1)^n$

Given sequence is,

$a_n = 1+(-1)^n$

$a_8 = 1+(-1)^8$

$\Rightarrow \quad a_8 = 1+1 = 2$

$\therefore \quad a_8 = 2$

Q50. What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$, where a_n equals

(i) $(n+1)^{n+1}$

(ii) $\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil$ **Model Paper-2, Q4(a)**

(iii) $2^n + (-2)^n$

Answer :

(i) $(n+1)^{n+1}$

Given sequence is,

$a_n = (n+1)^{n+1}$

$a_0 = (0+1)^{0+1} = (1)^1 = 1$

$a_1 = (1+1)^{1+1} = 2^2 = 4$

$a_2 = (2+1)^{2+1} = 3^3 = 27$

$a_3 = (3+1)^{3+1} = 4^4 = 256$

$\therefore \quad a_0 = 1, a_1 = 4, \ a_2 = 27, a_3 = 256$

(ii) $\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil$

Given sequence is,

$a_n = \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil$

$a_0 = \left\lfloor \frac{0}{2} \right\rfloor + \left\lceil \frac{0}{2} \right\rceil = \lfloor 0 \rfloor + \lceil 0 \rceil = 0$

$a_1 = \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil = \lfloor 0.5 \rfloor + \lceil 0.5 \rceil = 0 + 1 = 1$

$a_2 = \left\lfloor \frac{2}{2} \right\rfloor + \left\lceil \frac{2}{2} \right\rceil = \lfloor 1 \rfloor + \lceil 1 \rceil = 1 + 1 = 2$

$a_3 = \left\lfloor \frac{3}{2} \right\rfloor + \left\lceil \frac{3}{2} \right\rceil = \lfloor 1.5 \rfloor + \lceil 1.5 \rceil = 1 + 2 = 3$

$\therefore \quad a_0 = 0, \ a_1 = 1, \ a_2 = 2, \ a_3 = 3$

(iii) $2^n + (-2)^n$

Given sequence is,

$a_n = 2^n + (-2)^n$

$a_0 = 2^0 + (-2)^0 = 1 + 1 = 2$

$a_1 = 2^1 + (-2)^1 = 2 - 2 = 0$

$a_2 = 2^2 + (-2)^2 = 4 + 4 = 8$

$a_3 = 2^3 + (-2)^3 = 8 - 8 = 0$

$\therefore \quad a_0 = 2, a_1 = 0, a_2 = 8, a_3 = 0.$

Q51. List the first 10 terms of each of these sequences

(i) The sequence that begins with 2 and in which each successive term is 3 more than the preceding term.

(ii) The sequence whose n^{th} term is $n! - 2^n$

(iii) The sequence whose first two terms are 1 and each succeeding term is the sum of two preceding terms.

(iv) The sequence whose n^{th} term is the number of letters in the English word for the index n.

(v) The sequence obtained by starting with 10 and obtaining each term by substracting 3 from the previous terms.

(vi) The sequence whose n^{th} term is the sum of the first n positive integers.

(vii) The sequence whose terms are constructed sequentially as follows: start with 1, then add 1, then multiply by 1, then add 2, then multiply by 2, and so on.

Answer :

(i) **The sequence that begins with 2 and in which each successive term is 3 more than the preceding term.**

The sequence begins with 2 i.e., first term $a_1 = 2$.

Each successive term is increased by 3, then

$a_2 = a_1 + 3 = 2 + 3 = 5$

$a_3 = a_2 + 3 = 5 + 3 = 8$

$a_4 = a_3 + 3 = 8 + 3 = 11$

$a_5 = a_4 + 3 = 11 + 3 = 14$

$a_6 = a_5 + 3 = 14 + 3 = 17$

$a_7 = a_6 + 3 = 17 + 3 = 20$

$a_8 = a_7 + 3 = 20 + 3 = 23$

$a_9 = a_8 + 3 = 23 + 3 = 26$

$a_{10} = a_9 + 3 = 26 + 3 = 29$

$\therefore$ The first 10 terms of the sequence are 2, 5, 8, 11, 14, 17, 20, 23, 26, 29.

(ii) The sequence whose n^{th} term is $n! - 2^n$

Given sequence is,

$$a_n = n! - 2^n$$
$$a_1 = 1! - 2^1 = 1 - 2 = -1$$
$$a_2 = 2! - 2^2 = 2 - 4 = -2$$
$$a_3 = 3! - 2^3 = 6 - 8 = -2$$
$$a_4 = 4! - 2^4 = 24 - 16 = 8$$
$$a_5 = 5! - 2^5 = 120 - 32 = 88$$
$$a_6 = 6! - 2^6 = 720 - 64 = 656$$
$$a_7 = 7! - 2^7 = 5040 - 128 = 4912$$
$$a_8 = 8! - 2^8 = 40320 - 256 = 40064$$
$$a_9 = 9! - 2^9 = 362880 - 512 = 362368$$
$$a_{10} = 10! - 2^{10} = 3628800 - 1024 = 3627776$$

$\therefore$ The first 10 terms of the sequence are $-1, -2, -2, 8, 88, 656, 4912, 40064, 362368, 3627776$.

(iii) The sequence whose first two terms are 1 and each succeeding term is the sum of two preceding terms

The first two terms are 1 i.e., $a_1 = 1, a_2 = 1$.

Each succeeding term is the sum of two preceding terms.

$$a_3 = a_1 + a_2 = 1 + 1 = 2$$
$$a_4 = a_3 + a_2 = 2 + 1 = 3$$
$$a_5 = a_4 + a_3 = 3 + 2 = 5$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8$$
$$a_7 = a_6 + a_5 = 8 + 5 = 13$$
$$a_8 = a_7 + a_6 = 13 + 8 = 21$$
$$a_9 = a_8 + a_7 = 21 + 13 = 34$$
$$a_{10} = a_9 + a_8 = 34 + 21 = 55$$

$\therefore$ The first 10 terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55.

(iv) The sequence whose n^{th} term is the number of letters in the English word for the index n

The n^{th} term is the number of letters in English word for the index 'n'.

One = 3 $\rightarrow a_1$

Two = 3 $\rightarrow a_2$

Three = 5 $\rightarrow a_3$

Four = 4 $\rightarrow a_4$

Five = 4 $\rightarrow a_5$

Six = 3 $\rightarrow a_6$

Seven = 5 $\rightarrow a_7$

Eight = 5 $\rightarrow a_8$

Nine = 4 $\rightarrow a_9$

Ten = 3 $\rightarrow a_{10}$

$\therefore$ The first 10 terms of the sequence are 3, 3, 5, 4, 4, 3, 5, 5, 4, 3.

(v) The sequence obtained by starting with 10 and obtaining each term by substracting 3 from the previous term

The sequence starts with 10 i.e., first term $a_1 = 10$.

Each term is obtained by substracting 3 from previous term, then

$$a_2 = a_1 - 3 = 10 - 3 = 7$$
$$a_3 = a_2 - 3 = 7 - 3 = 4$$
$$a_4 = a_3 - 3 = 4 - 3 = 1$$
$$a_5 = a_4 - 3 = 1 - 3 = -2$$
$$a_6 = a_5 - 3 = -2 - 3 = -5$$
$$a_7 = a_6 - 3 = -5 - 3 = -8$$
$$a_8 = a_7 - 3 = -8 - 3 = -11$$
$$a_9 = a_8 - 3 = -11 - 3 = -14$$
$$a_{10} = a_9 - 3 = -14 - 3 = -17$$

$\therefore$ The first 10 terms of the sequence are 10, 7, 4, 1, –2, –5, –8, –11, –14, –17.

(vi) The sequence whose n^{th} term is the sum of the first n positive integers

The n^{th} terms is the sum of the first n positive integers.

$$a_1 = 1$$
$$a_2 = 1 + 2 = 3$$
$$a_3 = 1 + 2 + 3 = 6$$
$$a_4 = 1 + 2 + 3 + 4 = 10$$
$$a_5 = 1 + 2 + 3 + 4 + 5 = 15$$
$$a_6 = 1 + 2 + 3 + 4 + 5 + 6 = 21$$
$$a_7 = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$
$$a_8 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$$
$$a_9 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$
$$a_{10} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$$

$\therefore$ The first 10 terms of the sequence are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55.

(vii) The sequence whose terms are constructed sequentially as follows: starts with 1, then add 1, then multiply by 1, then add 2, then multiply by 2 and so on

The sequence starts with 1 i.e., $a_1 = 1$.

$$a_2 = a_1 + 1 = 1 + 1 = 2 \quad \text{(Adding by 1)}$$
$$a_3 = 1.a_2 = 1(2) = 2 \quad \text{(Multiplying by 1)}$$
$$a_4 = a_3 + 2 = 2 + 2 = 4 \quad \text{(Adding by 2)}$$
$$a_5 = 2.a_4 = 2(4) = 8 \quad \text{(Multiplying by 2)}$$
$$a_6 = a_5 + 3 = 8 + 3 = 11 \quad \text{(Adding by 3)}$$
$$a_7 = 3.a_6 = 3(11) = 33 \quad \text{(Multiplying by 3)}$$
$$a_8 = a_7 + 4 = 33 + 4 = 37 \quad \text{(Adding by 4)}$$
$$a_9 = 4.a_8 = 4(37) = 148 \quad \text{(Multiplying by 4)}$$
$$a_{10} = a_9 + 5 = 148 + 5 = 153 \quad \text{(Adding by 5)}$$

$\therefore$ The first 10 terms of the sequence are 1, 2, 2, 4, 8, 11, 33, 37, 148, 153.

Q52. Find atleast three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

Answer :

Given sequence is,

3, 5, 7. . . . . .

(i) As the terms 3, 5, 7 are consecutive odd numbers, the n^{th} term of sequence can be written as $a_n = 2n + 1$.

$\therefore$ a_n is the sequence of consecutive odd numbers starting from 3.

(ii) $a_1 = 3, a_2 = 5, a_3 = 7$.

Second term (a_2) is increased by 2 with first term (a_1).

Third term (a_3) is increased by 2 with second term (a_2).

i.e., $a_n = a_{n-1} + 2$

$\therefore$ a_n is the sequence begins with 3 and in which each successive term is 2 more than the preceding term.

(iii) The terms 3, 5, 7 are prime numbers

$\therefore$ a_n is the sequence of prime numbers starting from 3.

(iv) Third term (a_3) is the sum of two preceding terms decreased by 1.

i.e., $a_n = a_{n-1} + a_{n-2} - 1$

$\therefore$ a_n is the sequence with first two terms 3, 5 and in which each successive term is the sum of two preceding terms decreased by 1.

Q53. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

(i) 3, 6, 12, 24, 48, 96, 192, ...

(ii) 2, 3, 7, 25, 121, 721, 5041, 40321, ...

(iii) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, ...

(iv) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...

(v) 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, ...

Answer :

(i) 3, 6, 12, 24, 48, 96, 192, ...

Given sequence is,

3, 6, 12, 24, 48, 96, 192, ...

$a_1 = 3, a_2 = 6, a_3 = 12, a_4 = 24, a_5 = 48, a_6 = 96, a_7 = 192$

Each successive term is twice the preceding term, starting with 3.

$a_2 = 6 = 2(3) = 2(a_1)$

$a_3 = 12 = 2(6) = 2(a_2)$

$a_4 = 24 = 2(12) = 2(a_3)$

$a_5 = 48 = 2(24) = 2(a_4)$

$a_6 = 96 = 2(48) = 2(a_5)$

$a_7 = 192 = 2(96) = 2(a_6)$

Then, n^{th} term is $a_n = 2a_{n-1}$

$a_8 = 2(a_7) = 2(192) = 384$

$a_9 = 2(a_8) = 2(384) = 768$

$a_{10} = 2(a_9) = 2(768) = 1536$

$\therefore$ The next three terms of the sequence a_8, a_9, a_{10} are 384, 768, 1536 respectively.

(ii) 2, 3, 7, 25, 121, 721, 5041, 40321, ...

Given sequence is,

2, 3, 7, 25, 121, 721, 5041, 40321, ...

$a_1 = 2, a_2 = 3, a_3 = 7, a_4 = 25, a_5 = 121, a_6 = 721, a_7 = 5041, a_8 = 40321$

Each successive term is increased by 1 with the sequence of $n!$

$a_1 = 2 = 1! + 1$

$a_2 = 3 = 2! + 1$

$a_3 = 7 = 3! + 1$

$a_4 = 25 = 4! + 1$

$a_5 = 121 = 5! + 1$

$a_6 = 721 = 6! + 1$

$a_7 = 5041 = 7! + 1$

$a_8 = 40321 = 8! + 1$

Then, n^{th} term is $a_n = n! + 1$

$a_9 = 9! + 1 = 362880 + 1 = 362881$

$a_{10} = 10! + 1 = 3628800 + 1 = 3628801$

$a_{11} = 11! + 1 = 39916800 + 1 = 39916801$

$\therefore$ The next three terms of the sequence a_9, a_{10}, a_{11} are 362881, 3628801, 39916801 respectively.

(iii) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, ...

Given sequence is,

1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, ...

$a_1 = 1 = 0001$

$a_2 = 10 = 0010$

$a_3 = 11 = 0011$

$a_4 = 100 = 0100$

$a_5 = 101 = 0101$

$a_6 = 110 = 0110$

$a_7 = 111 = 0111$

$a_8 = 1000$

$a_9 = 1001$

$a_{10} = 1010$

$a_{11} = 1011$

The above sequence is the increasing order of positive integers in binary form.

$$a_{12} = 1100$$
$$a_{13} = 1101$$
$$a_{14} = 1110$$

$\therefore$ The next three terms of the sequence a_{12}, a_{13}, a_{14} are 1100, 1101, 1110 respectively.

(iv) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...

Given sequence is,

0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...

$a_1 = 0, a_2 = 2, a_3 = 8, a_4 = 26, a_5 = 80, a_6 = 242, a_7 = 728, a_8 = 2186, a_9 = 6560, a_{10} = 19682.$

Each successive term is decreased by 1 with the sequence of 3^{n-1}.

$$a_1 = 0 = 3^{1-1} - 1$$
$$a_2 = 2 = 3^{2-1} - 1$$
$$a_3 = 8 = 3^{3-1} - 1$$
$$a_4 = 26 = 3^{4-1} - 1$$
$$a_5 = 80 = 3^{5-1} - 1$$
$$a_6 = 242 = 3^{6-1} - 1$$
$$a_7 = 728 = 3^{7-1} - 1$$
$$a_8 = 2186 = 3^{8-1} - 1$$
$$a_9 = 6560 = 3^{9-1} - 1$$
$$a_{10} = 19682 = 3^{10-1} - 1$$

Then, n^{th} term is $a_n = 3^{n-1} - 1$

$$a_{11} = 3^{11-1} - 1 = 3^{10} - 1 = 59049 - 1 = 59048$$
$$a_{12} = 3^{12-1} - 1 = 3^{11} - 1 = 177147 - 1 = 177146$$
$$a_{13} = 3^{13-1} - 1 = 3^{12} - 1 = 531441 - 1 = 531440$$

$\therefore$ The next three terms of the sequence a_{11}, a_{12}, a_{13} are 59048, 177146, 531440 respectively.

(v) 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1 ....

Given sequence is,

1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1 ....

$a_1 = 1, a_2 = 0, a_3 = 0, a_4 = 1, a_5 = 1, a_6 = 1, a_7 = 0, a_8 = 0, a_9 = 0, a_{10} = 0, a_{11} = 1, a_{12} = 1, a_{13} = 1, a_{14} = 1, a_{15} = 1.$

The sequence is alternates of 1 and 0, and the number of times for the repetition of each term is increased i.e., 1 one, 2 zeroes, 3 ones, 4 zeroes ....

$$a_{16} = 0$$
$$a_{17} = 0$$
$$a_{18} = 0$$

$\therefore$ The next three terms of the sequence a_{16}, a_{17}, a_{18} are 0, 0, 0 respectively.

Q54. What are the values of these sums.

(a) $\sum_{k=1}^{5}(k+1)$

(b) $\sum_{i=1}^{10} 3$

(c) $\sum_{j=0}^{8}(2^{j+1} - 2^j)$

Answer : Model Paper-4, Q4(b)

(a) $\sum_{k=1}^{5}(k+1)$

Given summation is,

$$\sum_{k=1}^{5}(k+1)$$

$$\sum_{k=1}^{5}(k+1) = (1+1) + (2+1) + (3+1) + (4+1) + (5+1)$$
$$= 2 + 3 + 4 + 5 + 6$$
$$= 20$$

$$\therefore \sum_{k=1}^{5}(k+1) = 20$$

(b) $\sum_{i=1}^{10} 3$

Given summation is,

$$\sum_{i=1}^{10} 3$$

$$\sum_{i=1}^{10} 3 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$$
$$= 30$$

$$\therefore \sum_{i=1}^{10} 3 = 30$$

(c) $\sum_{j=0}^{8}(2^{j+1} - 2^j)$

Given summation is,

$$\sum_{j=0}^{8}(2^{j+1} - 2^j)$$

$$\sum_{j=0}^{8}(2^{j+1} - 2^j) = (2^{0+1} - 2^0) + (2^{1+1} - 2^1) + (2^{2+1} - 2^2) + (2^{3+1} - 2^3) + (2^{4+1} - 2^4) + (2^{5+1} - 2^5) + (2^{6+1} - 2^6) + (2^{7+1} - 2^7) + (2^{8+1} - 2^8)$$
$$= 2^1 - 2^0 + 2^2 - 2^1 + 2^3 - 2^2 + 2^4 - 2^3 + 2^5 - 2^4 + 2^6 - 2^5 + 2^7 - 2^6 + 2^8 - 2^7 + 2^9 - 2^8$$
$$= -2^0 + 2^9$$
$$= -1 + 512 = 511$$

$$\therefore \sum_{j=0}^{8}(2^{j+1} - 2^j) = 511.$$

Q55. What are the values of these sums, where $S = \{1, 3, 5, 7\}$

(a) $\sum_{j \in S} j$ (b) $\sum_{j \in S} j^2$

(c) $\sum_{j \in S} (1/j)$ (d) $\sum_{j \in S} 1$

Answer :

Given sequence is,

$S = \{1, 3, 5, 7\}$

(a) $\sum_{j \in S} j$

Given summation is,

$$\sum_{j \in S} j$$

$$\sum_{j \in \{1,3,5,7\}} j = 1 + 3 + 5 + 7$$

$$= 16$$

$$\therefore \sum_{j \in S} j = 16$$

(b) $\sum_{j \in S} j^2$

Given summation is,

$$\sum_{j \in S} j^2$$

$$\sum_{j \in \{1,3,5,7\}} j^2 = 1^2 + 3^2 + 5^2 + 7^2$$

$$= 1 + 9 + 25 + 49$$

$$= 84$$

$$\therefore \sum_{j \in S} j^2 = 84$$

(c) $\sum_{j \in S} (1/j)$

Given summation is,

$$\sum_{j \in S} (1/j)$$

$$\sum_{j \in \{1,3,5,7\}} \frac{1}{j} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}$$

$$= \frac{105 + 35 + 21 + 15}{105}$$

$$= \frac{176}{105}$$

$$\therefore \sum_{j \in S} (1/j) = \frac{176}{105}$$

(d) $\sum_{j \in S} 1$

Given summation is,

$$\sum_{j \in S} 1$$

$$\sum_{j \in \{1,3,5,7\}} 1 = 1 + 1 + 1 + 1$$

$$= 4$$

$$\therefore \sum_{j \in S} 1 = 4$$

Q56. Compute each of these double sums,

(a) $\sum_{i=1}^{3}\sum_{j=1}^{2} (i - j)$

(b) $\sum_{i=0}^{3}\sum_{j=0}^{2} (3i + 2j)$

(c) $\sum_{i=1}^{3}\sum_{j=0}^{2} i$

(d) $\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$

Answer : Model Paper-1, Q5(a)

(a) $\sum_{i=1}^{3}\sum_{j=1}^{2} (i - j)$

Given summation is,

$$\sum_{i=1}^{3}\sum_{j=1}^{2} (i - j)$$

$$\sum_{i=1}^{3}\sum_{j=1}^{2} (i - j) = \sum_{i=1}^{3} (i - 1 + i - 2)$$

$$= \sum_{i=1}^{3} (2i - 3)$$

$$= 2(1) - 3 + 2(2) - 3 + 2(3) - 3$$

$$= 2 - 3 + 4 - 3 + 6 - 3$$

$$= 3$$

$$\therefore \sum_{i=1}^{3}\sum_{j=1}^{2} (i - j) = 3$$

(b) $\sum_{i=0}^{3}\sum_{j=0}^{2} (3i + 2j)$

Given summation is,

$$\sum_{i=0}^{3}\sum_{j=0}^{2} (3i + 2j)$$

$$\sum_{i=0}^{3}\sum_{j=0}^{2} (3i + 2j) = \sum_{i=0}^{3} (3i + 2(0) + 3i + 2(1) + 3i + 2(2))$$

$$= \sum_{i=0}^{3} (3i + 0 + 3i + 2 + 3i + 4)$$

$$= \sum_{i=0}^{3} (9i + 6)$$

$$= 9(0) + 6 + 9(1) + 6 + 9(2) + 6 + 9(3) + 6$$

$$= 0 + 6 + 9 + 6 + 18 + 6 + 27 + 6$$

$$= 78$$

$$\therefore \sum_{i=0}^{3}\sum_{j=0}^{2} (3i + 2j) = 78$$

(c) $\sum_{i=1}^{3}\sum_{j=0}^{2} i$

Given summation is,

$$\sum_{i=1}^{3}\sum_{j=0}^{2} i$$

$$\sum_{i=1}^{3}\sum_{j=0}^{2} i = \sum_{i=1}^{3}(0+1+2)$$

$$= \sum_{i=1}^{3} 3$$

$$= 3+3+3 = 9$$

$$\therefore \quad \sum_{i=1}^{3}\sum_{j=0}^{2} i = 9$$

(d) $\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$

Given summation is,

$$\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$$

$$\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3 = \sum_{i=0}^{2}(i^2(0)^3 + i^2(1)^3 + i^2(2)^3 + i^2(3)^3)$$

$$= \sum_{i=0}^{2}(0 + i^2 + 8i^2 + 27i^2)$$

$$= \sum_{i=0}^{2} 36\, i^2$$

$$= 36(0)^2 + 36(1)^2 + 36(2)^2$$

$$= 0 + 36 + 144$$

$$= 180$$

$$\therefore \quad \sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3 = 180$$

Q57. Sum both sides of the identity $k^2-(k-1)^2 = 2k-1$ from $k = 1$ to $k = n$ and find a formula for

(i) $\sum_{k=1}^{n}(2k-1)$ (ii) $\sum_{k=1}^{n} k$

Answer : **Model Paper-5, Q4(a)**

Given identity is,

$$\sum(k^2-(k-1)^2) = \sum 2k-1; \text{ from } k = 1 \text{ to } k = n$$

$$\Rightarrow \sum_{k=1}^{n}(k^2-(k-1)^2) = \sum_{k=1}^{n}(2k-1)$$

(i) $\sum_{k=1}^{n} 2k-1$

Let $a_k = k^2$ and $a_{k-1} = (k-1)^2$

Then,

$$\sum_{k=1}^{n}(a_k - a_{k-1}) = \sum_{k=1}^{n}(2k-1)$$

$$\Rightarrow a_n - a_0 = \sum_{k=1}^{n}(2k-1)$$

$$\Rightarrow n^2 - 0^2 = \sum_{k=1}^{n}(2k-1)$$

$$\Rightarrow n^2 = \sum_{k=1}^{n}(2k-1)$$

$$\therefore \quad \sum_{k=1}^{n}(2k-1) = n^2$$

(ii) $\sum_{k=1}^{n} k$

Consider,

$$\sum_{k=1}^{n}(2k-1) = n^2$$

$$\Rightarrow \sum_{k=1}^{n} 2k - \sum_{k=1}^{n} 1 = n^2$$

$$\Rightarrow 2\sum_{k=1}^{n} k - (1+1+1+\ldots\ldots+n) = n^2$$

$$\Rightarrow 2\sum_{k=1}^{n} k - n = n^2$$

$$\Rightarrow 2\sum_{k=1}^{n} k = n^2 + n$$

$$\Rightarrow \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\therefore \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Q58. Derive the formula for $\sum_{k=1}^{n} k^2$ (Hint: Take $a_k = k^3$).

Answer :

Consider the identity

$$\sum_{k=1}^{n}(k^3-(k-1)^3) = \sum_{k=1}^{n}(3k^2-3k+1)$$

Let $a_k = k^3$ and $a_{k-1} = (k-1)^3$

Then,

$$\sum_{k=1}^{n}(a_k - a_{k-1}) = \sum_{k=1}^{n} 3k^2 - \sum_{k=1}^{n} 3k + \sum_{k=1}^{n} 1$$

$$\Rightarrow a_n - a_0 = 3\sum_{k=1}^{n} k^2 - 3\sum_{k=1}^{n} k + (1+1+1+\ldots\ldots+n)$$

$$\Rightarrow n^3 - 0^3 = 3\sum_{k=1}^{n} k^2 - 3\left(\frac{n(n+1)}{2}\right) + n$$

$$\Rightarrow n^3 = \sum_{k=1}^{n} k^2\left(\frac{3n(n+1)}{2}\right) + n$$

$$\Rightarrow 3\sum_{k=1}^{n} k^2 = n^3 + \frac{3n(n+1)}{2} - n$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{1}{3}\left(\frac{2n^3+3n^2+3n-2n}{2}\right)$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n}{6}(2n^2+3n+3-2)$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n}{6}(2n^2+3n+1)$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n}{6}(2n^2+2n+n+1)$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n}{6}(2n(n+1)+1(n+1))$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n}{6}(n+1)(2n+1)$$

$$\therefore \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Q59. Find the values of

(i) $\sum_{k=100}^{200} k$ (ii) $\sum_{k=99}^{200} k^3$

Answer :

(i) $\sum_{k=100}^{200} k$

Given summation is,

$$\sum_{k=100}^{200} k$$

$$\sum_{k=100}^{200} k = \sum_{k=1}^{200} k - \sum_{k=1}^{99} k$$

$$= \frac{200(200+1)}{2} - 99\frac{(99+1)}{2} \quad \left[\because \sum_{k=1}^{n} k = \frac{n(n+1)}{2}\right]$$

$$= 20100 - 4950$$

$$= 15150$$

$$\therefore \sum_{k=100}^{200} k = 15150$$

(ii) $\sum_{k=99}^{200} k^3$

Given summation is,

$$\sum_{k=99}^{200} k^3$$

$$\sum_{k=99}^{200} k^3 = \sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3$$

$$= \frac{(200)^2(200+1)^2}{4} - \frac{(98)^2(98+1)^2}{4} \quad \left[\because \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}\right]$$

$$= 404010000 - 23532201$$

$$= 380477799$$

$$\therefore \sum_{k=99}^{200} k^3 = 380477799.$$

Q60. What are the values of the following products.

(a) $\prod_{i=0}^{10} i$ (b) $\prod_{i=1}^{100} (-1)^i$ (c) $\prod_{i=1}^{10} 2$

Answer :

(a) $\prod_{i=0}^{10} i$

Given product is,

$$\prod_{i=0}^{10} i$$

$$\prod_{i=0}^{10} i = (0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(10)$$

$$= 0$$

$$\therefore \prod_{i=0}^{10} i = 0$$

(b) $\prod_{i=1}^{100} (-1)^i$

Given product is,

$$\prod_{i=1}^{100} (-1)^i$$

$$\prod_{i=1}^{100} (-1)^i = (-1)^1(-1)^2(-1)^3(-1)^4(-1)^5(-1)^6 \ldots (-1)^{99}(-1)^{100}$$

$$= (-1)(1)(-1)(1)(-1)(1) \ldots (-1)(1)$$

$$= (-1)^{50}(1)^{50}$$

$$= (1)(1) = 1$$

$$\therefore \prod_{i=1}^{100} (-1)^i = 1$$

(c) $\prod_{i=1}^{10} 2$

Given product is,

$$\prod_{i=1}^{10} 2$$

$$\prod_{i=1}^{10} 2 = 2(2)(2)(2)(2)(2)(2)(2)(2)(2)$$

$$= 2^{10} = 1024$$

$$\therefore \prod_{i=1}^{10} 2 = 1024$$

2.4 CARDINALITY OF SETS AND MATRICES

Q61. Determine whether each of these sets is countable or uncountable. For those that are countable, exhibhit a one-to-one correspondence between the set of natural numbers and that set.

(a) The negative integers

(b) The even integers

(c) The real numbers between 0 and 1/2

(d) Integers that are multiples of 7

(e) The integers greater than 10.

(f) All bit strings not containing the bit 0

(g) The real numbers containing only a finite number of 1's in their decimal representation.

Answer :

(a) **The negative integers**

The set of negative integers is countably infinite i.e., $-1, -2, -3, -4, -5, \ldots\ldots$

One-to-one correspondence between the positive integers (i.e., set of natural numbers) and negative integers is given as,

$$f : Z^+ \rightarrow Z^- ; f(n) = -n$$

Let, $f(a) = f(b)$

$\Rightarrow \quad -a = -b$

$\Rightarrow \quad a = b$

$\therefore \quad f$ is one-to-one

Let $b \in Z^-$, then $-b \in Z^+$ with $f(-b)$ i.e., $f(-b) = -(-b) = b$

$\therefore \quad f$ is onto

(b) The even integers

The set of even integers is countably infinite i.e., ... 4, – 2, 0, 2, 4, ...

One-to-one correspondence between the positive integers (i.e., set of natural numbers) and even integers (A) is given as,

$$f : Z^+ \rightarrow A ; f(n) = \begin{cases} 0 \text{ if } n = 1 \\ n \text{ if } n \text{ is even} \\ -(n-1) \text{ if } n \text{ is odd}, n \neq 1 \end{cases}$$

Let, $f(a) = f(b)$

- $a = b = 1$ (if $f(a) = f(b) = 0$)
- $a = b$ (if $f(a) = f(b)$ = positive)
- $-(a-1) = -(b-1) \Rightarrow a = b$ (if $f(a) = f(b)$ = negative)

$\therefore \quad f$ is one-to-one

Let $b \in A$ then,

- $b \in Z^+$ with $f(b) = b$ if b is even
- $1 \in Z^+$ with $f(1) = b$ if b is zero
- $(-b+1) \in Z^+$ with $f(-b+1) = -(-b+1-1) = b$ if b is odd

$\therefore$ f is onto

(c) The Real Numbers between 0 and 1/2

The list of all real numbers between 0 and 1/2 are not possible to count. Hence, the set is uncountable.

(d) Integers that are Multiples of 7

The set of integers that are multiples of 7 are countably infinite i.e., ... –14, – 7, 0, 7, 14,............

One-to-one correspondence between the positive integers (i.e., set of natural numbers) and the integers that are multiples of 7, (A) is given as,

$$f : Z^+ \rightarrow A, f(n) = \begin{cases} 0 \text{ if } n = 1 \\ \dfrac{7n}{2} \text{ if } n \text{ is even} \\ \dfrac{-7(n-1)}{2} \text{ if } n \text{ is odd and } n \neq 1 \end{cases}$$

Let, $f(a) = f(b)$,

- $a = b = 1$, if $f(a) = f(b) = 0$
- $\dfrac{7a}{2} = \dfrac{7b}{2} \Rightarrow a = b$ if $f(a) = f(b)$ = positive
- $\dfrac{-7(a-1)}{2} = \dfrac{-7(b-1)}{2} \Rightarrow a = b$ if $f(a) = f(b)$ = negative

$\therefore$ f is one-to-one

Let $b \in A$, then

- $\frac{2b}{7} \in Z^+$ with $f\left(\frac{2b}{7}\right) = b$ if b is even
- $1 \in Z^+$ with $f(1) = b$ if $b = 0$
- $\frac{-2b}{7+1} \in Z^+$ with $f\left(\frac{-2b}{7+1}\right) = -7\left(\frac{-2b}{7+1-1}\right) = b$ if b is odd

$\therefore$ f is onto

(e) **The integers greater than 10**

The set of integers greater than 10 are countably infinite.

One-to-one correspondence between the positive integers and the integers greater than 10, (A) is given as,

$f : Z^+ \rightarrow A, f(n) = n + 10$

Let, $f(a) = f(b)$

$\Rightarrow$ $a + 10 = b + 10$

$\Rightarrow$ $a = b$

$\therefore$ f is one-to-one

Let $b \in A$, then $b - 10 \in Z^+$ with $f(b - 10) = b - 10 + 10 = b$

$\therefore$ f is onto

(f) **All Bit strings not containing the bit 0**

The set of all bit strings not containing the bit 0 is countably infinite.

One-to-one correspondence between the positive integers (i.e., set of natural numbers) and all bit strings not containing the bit 0, (A) is given as,

$f : Z^+ \rightarrow A, f(n) =$ String containing n 1's

Let, $f(a) = f(b)$

$\Rightarrow$ $a = b$, as a and b contain the same number of 1's.

$\therefore$ f is one-to-one

Let, $b \in A$, then $z \in Z^+$ with $f(n) = b$, as b contains n 1's.

$\therefore$ f is onto

(g) **The real numbers containing only a finite number of 1's in their decimal representation**

The listing of all real numbers from smallest to largest is not possible. Hence, the set is uncountable.

Q62. If A is an uncountable set and B is a countable set, must A – B be uncountable.

Answer : Model Paper-2, Q4(b)

Given that,

A is an uncountable set and B is a countable set.

Assuming that $A - B$ is countable.

The set 'A' can be written as,

$$A = (A - B) \cup (A \cap B)$$

$$\Rightarrow \quad A = (A - B) \cup B$$

As B is countable, $A \cap B$ is countably infinite.

Since $A - B$ and $A \cap B$ are countably infinite then, there exists a listing for each $A - B$ and $A \cap B$. Hence, the elements of A can be listed in a sequence by alternating the elements of $A - B$ and $A \cap B$. i.e., A is countable.

But, given that 'A' is uncountable. Thus, the above assumption is wrong and $A - B$ must be uncountable.

Q63. Show that if S is a set, then there does not exist an onto function f from S to p(S), the power set of S.

Answer :

Given that,

$P(S)$ = Power set of S

Assuming that there exists an onto function f from S to $P(S)$

Consider a set 'T', as subset of S,

i.e., $T = \{s \in S \mid s \notin f(s)\}$

From the definition of power set,

$T \in P(S)$

$\Rightarrow$ $f(s) \neq T$ for all $s \in S$

From the above assumption, if $f(s) = T$ for some $s \in S$, then $s \notin f(s)$

$\Rightarrow$ $s \notin T$ i.e., $f(s) = T$ cannot be true when $s \in S$ such that f cannot be onto.

Hence, the above assumption is incorrect and then there doesn't exist an onto function f from S to $P(S)$.

Q64. Define the following matrices.

(i) Identity matrix

(ii) Transpose of matrix

(iii) Symmetric matrix

(iv) Zero-one matrix.

Answer :

(i) Identity Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is an identity matrix if,

$$a_{ij} = \begin{cases} 1 \text{ for } i = j \\ 0 \text{ for } i \neq j \end{cases}$$

$$\text{i.e.,} \quad I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

(ii) Transpose of a Matrix

For a matrix $A = [a_{ij}]_{m \times n}$, the transpose is obtained as $[b_{ij}]_{n \times m}$ by interchanging its rows and columns.

i.e., $A^T = [b_{ij}]_{n \times m}$

(iii) Symmetric Matrix

A square matrix $A = [a_{ij}]_{m \times m}$ is symmetric if $a_{ij} = a_{ji}$ $\forall$ i and j with $1 \leq i \leq n$ and $i \leq j \leq n$.

$$\text{i.e., } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

(iv) Zero-one Matrix

A zero-one matrix is a matrix which contains the elements either 0 or 1.

Example

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q65. Find A + B, where

(i) $A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$

(ii) $A = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$

Answer :

(i) Given matrices are,

$$A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0+3 & 4+5 \\ -1+2 & 2+2 & 2-3 \\ 0+2 & -2-3 & -3+0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$$

$$\therefore \quad A + B = \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$$

(ii) Given matrices are,

$$A = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 0+9 & 5-3 & 6+4 \\ -4+0 & -3-2 & 5-1 & -2+2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$$

$$\therefore \quad A + B = \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}.$$

Q66. Find the product AB, where

(i) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 4 & -3 \\ 3 & -1 \\ 0 & -2 \\ -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 2 & -2 \\ 0 & -1 & 4 & -3 \end{bmatrix}$

Answer :

(i) Given matrices are,

$$A=\begin{bmatrix}2&1\\3&2\end{bmatrix} \text{ and}$$

$$B=\begin{bmatrix}0&4\\1&3\end{bmatrix}$$

$$AB=\begin{bmatrix}2&1\\3&2\end{bmatrix}\begin{bmatrix}0&4\\1&3\end{bmatrix}$$

$$=\begin{bmatrix}2\times0+1\times1 & 2\times4+1\times3\\3\times0+2\times1 & 3\times4+2\times3\end{bmatrix}$$

$$=\begin{bmatrix}0+1 & 8+3\\0+2 & 12+6\end{bmatrix}=\begin{bmatrix}1&11\\2&18\end{bmatrix}$$

$$\therefore\ AB=\begin{bmatrix}1&11\\2&18\end{bmatrix}$$

(ii) Given matrices are,

$$A=\begin{bmatrix}1&0&1\\0&-1&-1\\-1&1&0\end{bmatrix} \text{ and}$$

$$B=\begin{bmatrix}0&1&-1\\1&-1&0\\-1&0&1\end{bmatrix}$$

$$AB=\begin{bmatrix}1&0&1\\0&-1&-1\\-1&1&0\end{bmatrix}\begin{bmatrix}0&1&-1\\1&-1&0\\-1&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times0+0\times1+1\times-1 & 1\times1+0\times-1+1\times0 & 1\times-1+0\times0+1\times1\\0\times0-1\times1-1\times-1 & 0\times1-1\times-1-1\times0 & 0\times-1-1\times0-1\times1\\-1\times0+1\times1+0\times-1 & -1\times1+1\times-1+0\times0 & -1\times-1+1\times0+0\times1\end{bmatrix}$$

$$=\begin{bmatrix}0+0-1 & 1-0+0 & -1+0+1\\0-1+1 & 0+1-0 & 0-0-1\\0+1-0 & -1-1+0 & 1+0+0\end{bmatrix}$$

$$=\begin{bmatrix}-1&1&0\\0&1&-1\\1&-2&1\end{bmatrix}$$

$$\therefore\ AB=\begin{bmatrix}-1&1&0\\0&1&-1\\1&-2&1\end{bmatrix}$$

(iii) Given matrices are

$$A=\begin{bmatrix}4&-3\\3&-1\\0&-2\\-1&5\end{bmatrix} \text{ and}$$

$$B=\begin{bmatrix}-1&3&2&-2\\0&-1&4&-3\end{bmatrix}$$

$$AB=\begin{bmatrix}4&-3\\3&-1\\0&-2\\-1&5\end{bmatrix}\begin{bmatrix}-1&3&2&-2\\0&-1&4&-3\end{bmatrix}$$

$$=\begin{bmatrix}4\times-1-3\times0 & 4\times3-3\times-1 & 4\times2-3\times4 & 4\times-2-3\times-3\\3\times-1-1\times0 & 3\times3-1\times-1 & 3\times2-1\times4 & 3\times-2-1\times-3\\0\times-1-2\times0 & 0\times3-2\times-1 & 0\times2-2\times4 & 0\times-2-2\times-3\\-1\times-1+5\times0 & -1\times3+5\times-1 & -1\times2+5\times4 & -1\times-2+5\times-3\end{bmatrix}$$

$$=\begin{bmatrix}-4-0 & 12+3 & 8-12 & -8+9\\-3-0 & 9+1 & 6-4 & -6+3\\-0-0 & 0+2 & 0-8 & 0+6\\1+0 & -3-5 & -2+20 & 2-15\end{bmatrix}$$

$$=\begin{bmatrix}-4&15&-4&1\\-3&10&2&-3\\0&2&-8&6\\1&-8&18&-13\end{bmatrix}$$

$$\therefore\ AB=\begin{bmatrix}-4&15&-4&1\\-3&10&2&-3\\0&2&-8&6\\1&-8&18&-13\end{bmatrix}$$

Q67. Find the matrix A such that,

(i) $\begin{bmatrix}2&3\\1&4\end{bmatrix}A=\begin{bmatrix}3&0\\1&2\end{bmatrix}$

(ii) $\begin{bmatrix}1&3&2\\2&1&1\\4&0&3\end{bmatrix}A=\begin{bmatrix}7&1&3\\1&0&3\\-1&-3&7\end{bmatrix}$

Answer :

(i) Given that,

$$\begin{bmatrix}2&3\\1&4\end{bmatrix}A=\begin{bmatrix}3&0\\1&2\end{bmatrix}$$

Let, $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$

Then,

$$\begin{bmatrix}2&3\\1&4\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}3&0\\1&2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\times a+3\times c & 2\times b+3\times d\\1\times a+4\times c & 1\times b+4\times d\end{bmatrix}=\begin{bmatrix}3&0\\1&2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2a+3c & 2b+3d\\a+4c & b+4d\end{bmatrix}=\begin{bmatrix}3&0\\1&2\end{bmatrix}$$

Equating the corresponding elements on both sides,

$$2a+3c=3 \qquad \text{... (1)}$$

$$2b+3d=0 \qquad \text{... (2)}$$

$$a+4c=1 \qquad \text{... (3)}$$

$$b+4d=2 \qquad \text{... (4)}$$

Solving equations (1) & (3),

$c=-1/5,\ a=9/5$

Solving equations (2) & (4),

$b=-6/5,\ d=4/5$

Substituting the corresponding values in matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 9/5 & -6/5 \\ -1/5 & 4/5 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} 9/5 & -6/5 \\ -1/5 & 4/5 \end{bmatrix}$$

(ii) Given that,

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} A = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$$

Let, $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Then,

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1\times a+3\times d+2\times g & 1\times b+3\times e+2\times h & 1\times c+3\times f+2\times i \\ 2\times a+1\times d+1\times g & 2\times b+1\times e+1\times h & 2\times c+1\times f+1\times i \\ 4\times a+0\times d+3\times g & 4\times b+0\times e+3\times h & 4\times c+0\times f+3\times i \end{bmatrix} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+3d+2g & b+3e+2h & c+3f+2i \\ 2a+d+g & 2b+e+h & 2c+f+i \\ 4a+0+3g & 4b+0+3h & 4c+0+3i \end{bmatrix} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+3d+2g & b+3e+2h & c+3f+2i \\ 2a+d+g & 2b+e+h & 2c+f+i \\ 4a+3g & 4b+3h & 4c+3i \end{bmatrix} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$$

Equating the corresponding elements on both sides,

$a + 3d + 2g = 7$ ... (1)

$2a + d + g = 1$ ... (2)

$4a + 3g = -1$ ... (3)

$b + 3e + 2h = 1$ ... (4)

$2b + e + h = 0$ ... (5)

$4b + 3h = -3$ ... (6)

$c + 3f + 2i = 3$ ... (7)

$2c + f + i = 3$ ... (8)

$4c + 3i = 7$ ... (9)

Solving equations (1), (2) & (3),

$a = -1, d = 2, g = 1$

Solving equations (4), (5) & (6),

$b = 0, e = 1, h = -1$

Solving equations (7), (8) & (9),

$c = 1, f = 0, i = 1$

Substituting the corresponding values in matrix A,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Q68. Find the formula for A^n if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, whenever n is a positive integer.

Answer :

Given matrix is,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\times1+1\times0 & 1\times1+1\times1 \\ 0\times1+1\times0 & 0\times1+1\times1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1\times1+2\times0 & 1\times1+2\times1 \\ 0\times1+1\times0 & 0\times1+1\times1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = A^3 \times A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 1\times1+3\times0 & 1\times1+3\times1 \\ 0\times1+1\times0 & 0\times1+1\times1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 1+0 & 1+3 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

Similarly, $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$\therefore$ Formula for $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Q69. Let A be an m × n matrix and let O be the m × n matrix that has all entries equal to zero show that A = O + A = A + O.

Answer :

Given that,

A and O are $m \times n$ matrices

Let the matrix 'A' be written as $[a_{ij}]$

Consider $O + A$,

The addition of O and A matrices add their corresponding elements i.e., zero is added to every element (real number) in $[a_{ij}]$ results in real number.

$$O + A = 0 + [a_{ij}]$$
$$= [0 + a_{ij}]$$
$$= [a_{ij}]$$
$$= A$$

$$\Rightarrow O + A = A$$

Consider $A + O$,

The addition of A and O matrices add their corresponding elements i.e., every element (real number) is added to zero results in real number.

$$A + O = [a_{ij}] + 0$$
$$= [a_{ij} + 0]$$
$$= [a_{ij}]$$
$$= A$$

$$\Rightarrow A + O = A$$

$$\therefore A = O + A = A + O$$

Q70. Let A be the 2 × 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that if $ad - bc \neq 0$, then

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Answer :

Given matrices are,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and}$$

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\text{Adjoint of } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Determinant of $A = ad - bc$.

$$\Rightarrow |A| = ad - bc \neq 0$$

$$A^{-1} = \frac{1}{|A|}(\text{Adjoint of } A)$$

$$= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Q71. Let $A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

(a) Find A^{-1}

(b) Find A^3

(c) Find $(A^{-1})^3$

(d) Show that $(A^{-1})^3$ is the inverse of A^3.

Answer : Model Paper-1, Q5(b)

Given matrix is,

$$A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$$

(a) A^{-1}

Adjoint of $A = \begin{bmatrix} 3 & -2 \\ -1 & -1 \end{bmatrix}$

Determinant of $A = 3(-1) - 2(1)$

$\Rightarrow \quad |A| = -3 - 2$

$\Rightarrow \quad |A| = -5$

$$A^{-1} = \frac{1}{|A|}(\text{adjoint of } A)$$

$$= \frac{1}{-5}\begin{bmatrix} 3 & -2 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3/5 & -2/-5 \\ -1/-5 & -1/-5 \end{bmatrix}$$

$$= \begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

(b) A^3

$$A^2 = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times-1+2\times1 & -1\times2+2\times3 \\ 1\times-1+3\times1 & 1\times2+3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & -2+6 \\ -1+3 & 2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times-1+4\times1 & 3\times2+4\times3 \\ 2\times-1+11\times1 & 2\times2+11\times3 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4 & 6+12 \\ -2+11 & 4+33 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$$

$$\therefore \quad A^3 = \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$$

(c) $(A^{-1})^3$

$$(A^{-1})^2 = \begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix}\begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{5}\times\frac{-3}{5}+\frac{2}{5}\times\frac{1}{5} & \frac{-3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5} \\ \frac{1}{5}\times\frac{-3}{5}+\frac{1}{5}\times\frac{1}{5} & \frac{1}{5}\times\frac{2}{5}+\frac{1}{5}\times\frac{1}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{25}+\frac{2}{25} & \frac{-6}{25}+\frac{2}{25} \\ \frac{-3}{25}+\frac{1}{25} & \frac{2}{25}+\frac{1}{25} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{25} & \frac{-4}{25} \\ \frac{-2}{25} & \frac{3}{25} \end{bmatrix}$$

$$(A^{-1})^3 = (A^{-1})^2 \times A^{-1}$$

$$= \begin{bmatrix} \frac{11}{25} & \frac{-4}{25} \\ \frac{-2}{25} & \frac{3}{25} \end{bmatrix}\begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{25}\times\frac{-3}{5}-\frac{4}{25}\times\frac{1}{5} & \frac{11}{25}\times\frac{2}{5}-\frac{4}{25}\times\frac{1}{5} \\ \frac{-2}{25}\times\frac{-3}{5}+\frac{3}{25}\times\frac{1}{5} & \frac{-2}{25}\times\frac{2}{5}+\frac{3}{25}\times\frac{1}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-33}{125}-\frac{4}{125} & \frac{22}{125}-\frac{4}{125} \\ \frac{6}{125}+\frac{3}{125} & \frac{-4}{125}+\frac{3}{125} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-37}{125} & \frac{18}{125} \\ \frac{9}{125} & \frac{-1}{125} \end{bmatrix}$$

$$\therefore \quad (A^{-1})^3 = \begin{bmatrix} \frac{-37}{125} & \frac{18}{125} \\ \frac{9}{125} & \frac{-1}{125} \end{bmatrix}$$

(d) Determinant of $A^3 = 37(1) - 18(9)$

$\Rightarrow \quad |A^3| = 37 - 162$

$\Rightarrow \quad |A^3| = -125$

Adjoint of $A^3 = \begin{bmatrix} 37 & -18 \\ -9 & 1 \end{bmatrix}$

$$(A^3)^{-1} = \frac{1}{|A^3|}(\text{adjoint of } A^3)$$

$$= \frac{1}{-125}\begin{bmatrix} 37 & -18 \\ -9 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-37}{125} & \frac{18}{125} \\ \frac{9}{125} & \frac{-1}{125} \end{bmatrix}$$

$$= (A^{-1})^3$$

$\therefore \quad (A^{-1})^3$ is the inverse of A^3.

Q72. Show that $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$.

Answer : **Model Paper-3, Q4(b)**

Given matrices are,

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$$

Let the matrices be,

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times7+3\times-4-1\times1 & 2\times-8+3\times5-1\times-1 & 2\times5+3\times-3-1\times1 \\ 1\times7+2\times-4+1\times1 & 1\times-8+2\times5+1\times-1 & 1\times5+2\times-3+1\times1 \\ -1\times7-1\times-4+3\times1 & -1\times-8-1\times5+3\times-1 & -1\times5-1\times-3+3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 14-12-1 & -16+15+1 & 10-9-1 \\ 7-8+1 & -8+10-1 & 5-6+1 \\ -7+4+3 & 8-5-3 & -5+3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Since the product of $AB = I$, A and B are inverse to each other.

Hence, $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$

Q73. Solve the system $7x_1 - 8x_2 + 5x_3 = 5$, $-4x_1 + 5x_2 - 3x_3 = -3$, $x_1 - x_2 + x_3 = 0$.

Answer : **Model Paper-5, Q4(b)**

Given system of equations are,

$$7x_1 - 8x_2 + 5x_3 = 5,$$

$$-4x_1 + 5x_2 - 3x_3 = -3,$$

$$x_1 - x_2 + x_3 = 0$$

The above equations can be written in matrix form as,

$$\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix} \text{ and}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

From Unit-II, Q72,

The inverse of A is given as,

$$A^{-1} = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$$

The solution of system of equations can be determined as,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2\times5+3\times-3-1\times0 \\ 1\times5+2\times-3+1\times0 \\ -1\times5-1\times-3+3\times0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10-9-0 \\ 5-6+0 \\ -5+3+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$\Rightarrow x_1 = 1, x_2 = -1, x_3 = -2.$$

$\therefore$ The solution set is $x_1 = 1$, $x_2 = -1$, $x_3 = -2$.

Q74. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find

(a) $A \vee B$

(b) $A \wedge B$

(c) $A \odot B$

Answer : **Model Paper-2, Q5(a)**

Given matrices are,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) $\mathbf{A \vee B}$

$$A \vee B = \begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \vee \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1\vee 0 & 0\vee 1 & 1\vee 1\\ 1\vee 1 & 1\vee 0 & 0\vee 1\\ 0\vee 1 & 0\vee 0 & 1\vee 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$$

$$\therefore \quad A \vee B = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$$

(b) $\mathbf{A \wedge B}$

$$A \wedge B = \begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \wedge \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1\wedge 0 & 0\wedge 1 & 1\wedge 1\\ 1\wedge 1 & 1\wedge 0 & 0\wedge 1\\ 0\wedge 1 & 0\wedge 0 & 1\wedge 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore \quad A \wedge B = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

(c) $\mathbf{A \odot B}$

$$A \odot B = \begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \odot \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}(1\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1) & (1\wedge 1)\vee(0\wedge 0)\vee(1\wedge 0) & (1\wedge 1)\vee(0\wedge 1)\vee(1\wedge 1)\\ (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 0) & (1\wedge 1)\vee(1\wedge 1)\vee(0\wedge 1)\\ (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1) & (0\wedge 1)\vee(0\wedge 0)\vee(1\wedge 0) & (0\wedge 1)\vee(0\wedge 1)\vee(1\wedge 1)\end{bmatrix}$$

$$= \begin{bmatrix}0\vee 0\vee 1 & 1\vee 0\vee 0 & 1\vee 0\vee 1\\ 0\vee 1\vee 0 & 1\vee 0\vee 0 & 1\vee 1\vee 0\\ 0\vee 0\vee 1 & 0\vee 0\vee 0 & 0\vee 0\vee 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$$

$$\therefore \quad A \odot B = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$$

Q75. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Find,

(a) $A^{[2]}$

(b) $A^{[3]}$

(c) $A \vee A^{[2]} \vee A^{[3]}$

Model Paper-4, Q5(a)

Answer :

Given matrix is,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) $A^{[2]}$

$$A^{[2]} = A \odot A$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 1) \vee (0 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 0) \\ (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 0) & (0 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 0 \vee 0 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \\ 1 \vee 0 \vee 0 & 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 \\ 0 \vee 1 \vee 0 & 0 \vee 0 \vee 0 & 0 \vee 1 \vee 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad A^{[2]} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) $A^{[3]}$

$$A^{[3]} = A^2 \odot A$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 1) \vee (0 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 0) \\ (1 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 0) \\ (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 0 \vee 0 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \\ 1 \vee 1 \vee 0 & 0 \vee 0 \vee 0 & 0 \vee 1 \vee 0 \\ 1 \vee 0 \vee 0 & 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad A^{[3]} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(c) $A \vee A^{[2]} \vee A^{[3]}$

$$A \vee A^{[2]} \vee A^{[3]} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 1 \vee 1 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \\ 1 \vee 1 \vee 1 & 0 \vee 1 \vee 0 & 1 \vee 0 \vee 1 \\ 0 \vee 1 \vee 1 & 1 \vee 0 \vee 1 & 0 \vee 1 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad A \vee A^{[2]} \vee A^{[3]} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2.5 RELATIONS: RELATIONS AND THEIR PROPERTIES

Q76. For the relation R = {(a, b) | a divides b} on the set {1, 2, 3, 4, 5, 6},

(a) List all the ordered pairs in R

(b) Display the relation graphically

(c) Display the relation in tabular form.

Answer :

Given set is,

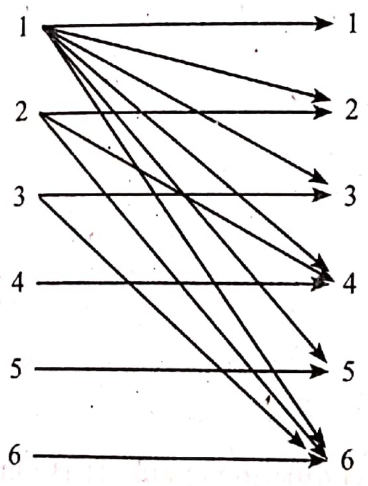
$R = \{(a, b) \mid a$ divides $b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$

(a) R contains the points of a and b, not exceeding 6 such that a divides b.

$\Rightarrow$ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2,2), (2, 4), (2, 6), (3,3), (3, 6), (4, 4), (5, 5), (6, 6)

$\therefore$ $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$

(b) The graphical representation of relation is shown in figure below.



Figure

(c) The tabular form representation of relation is shown in table below.

R	1	2	3	4	5	6
1	×	×	×	×	×	×
2		×		×		×
3			×			×
4				×		
5					×	
6						×

Table

Q77. List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b) ∈ R if and only if

(i) a = b

(ii) a + b = 4

(iii) a > b

(iv) a | b

Answer :

Given sets are,

$A = \{0, 1, 2, 3, 4\}$ and

$B = \{0, 1, 2, 3\}$

(i) **a = b**

$R = \{(a, b) \mid a = b\}$ i.e., R contains the points with $a = b$ (element of A equals to element of B)

$\Rightarrow$ (0, 0), (1, 1), (2, 2), (3, 3)

$\therefore$ $R = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$

(ii) **a + b = 4**

$R = \{(a, b) \mid a + b = 4\}$ i.e., R contains the points with sum of elements in A and B must be equal to 4.

$\Rightarrow$ (1, 3), (2, 2), (3, 1), (4, 0)

$\therefore$ $R = \{(1, 3), (2, 2), (3, 1), (4, 0)\}$

(iii) **a > b**

$R = \{(a, b) \mid a > b\}$ i.e., R contains the points with $a > b$ (element of A should be greater than the element of B).

$\Rightarrow$ (1, 0), (2, 0), (3, 0), (4, 0), (2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)

$\therefore$ $R = \{(1, 0), (2, 0), (3, 0), (4, 0), (2, 1), (3,1), (4, 1), (3, 2), (4, 2), (4, 3)\}$.

(iv) **a|b**

$R = \{(a, b) \mid a \mid b)$ i.e., R contains the points with $a \mid b$(element of A should be divisor of element of B)

$\Rightarrow$ (1, 0), (1, 1), (1, 2), (1, 3); (2, 0), (2, 2), (3, 0), (3, 3), (4, 0)

$\therefore$ $R = \{(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3,0), (3, 3), (4, 0)\}$

Q78. Define the following properties of binary relations.

(i) Reflexive

(ii) Irreflexive

(iii) Symmetric

(iv) Anti symmetric

(v) Asymmetric

(vi) Transitive

Answer :

Let 'R' be the relation defined on set 'A'.

(i) **Reflexive**

R is said to be reflexive if, $(a, a) \in R$ for every element $a \in A$.

(ii) **Irreflexive**

R is said to be irreflexive if, $(a, a) \notin R$ for every element $a \in A$.

(iii) **Symmetric**

R is said to be symmetric if, $(b, a) \in R$ then $(a, b) \in R\ \forall\ a, b \in A$.

(iv) **Antisymmetric**

R is said to be antisymmetric if, $(a, b) \in R$ and $(b, a) \in R$ then $a = b\ \forall\, a, b \in A$

(v) **Asymmetric**

R is said to be asymmetric if, $(a, b) \in R$ then $(b, a) \notin R$

(vi) **Transitive**

R is said to be transitive if, $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R \ \forall\ a, b, c \in A$.

Q79. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric and whether it is transitive.

(a) (1, 2), (2, 3), (3, 4)

(b) (1, 1), (1, 2), (2,1), (2, 2), (3, 3), (4, 4)

(c) (1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)

Answer : Model Paper-2, Q5(b)

Given set is,

$\{1, 2, 3, 4\}$

(a) **(1, 2), (2, 3), (3, 4)**

Given relation is,

$R = \{(1, 2), (2, 3), (3, 4)\}$

Reflexive

R does not contain (1, 1), (2, 2), (3, 3) and (4, 4)

$\therefore$ R is not reflexive

Symmetric

R does not contain (2, 1) i.e., $(2, 1) \notin R$ whereas $(1, 2) \in R$

$\therefore$ R is not symmetric

Antisymmetric

As $(1, 2) \in R$ and $(2, 1) \notin R$, R is antisymmetric.

Transitive

$(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$

$\therefore$ R is not transitive

(b) **(1, 1), (1, 2), (2,1), (2, 2), (3, 3), (4, 4)**

Given relation is,

$R = \{(1, 1), (1, 2), (2,1), (2, 2), (3, 3), (4, 4)\}$

Reflexive

R contains (1, 1), (2, 2), (3, 3) and (4, 4)

$\therefore$ R is Reflexive

Symmetric

As $(1, 2) \in R$ and $(2,1) \in R$, R is symmetric.

Antisymmetric

As $(1, 2) \in R$ and $(2, 1) \in R$ while $2 \neq 1$, R is not antisymmetric.

Transitive

$(2, 1) \in R$ and $(1, 2) \in R \Rightarrow (2, 2) \in R$

$\therefore$ R is transitive

(c) **(1,3), (1,4), (2, 3), (2, 4), (3, 1), (3, 4)**

Given relation is,

$R = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

Reflexive

R does not contain (1, 1), (2, 2), (3, 3) and (4, 4)

$\therefore$ R is not reflexive

Symmetric

R does not contain (4, 1) i.e., $(1, 4) \in R$ but $(4, 1) \notin R$

$\therefore$ R is not symmetric

Antisymmetric

$(1, 3) \in R$ and $(3, 1) \in R$ while $1 \neq 3$

$\therefore$ R is not antisymmetric

Transitive

$(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ R is not transitive.

Q80. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric transitive, and/or asymmetric, where (x, y) ∈ R if and only if

(a) $x + y = 0$

(b) $x = \pm y$

(c) $x - y$ is a rational number

(d) $x = 2y$

(e) $xy \geq 0$

(f) $x = 1$ or $y = 1$

Answer :

Give that,

R is a relation on set of all real numbers

(a) $x + y = 0$

$R = \{(x, y) \mid x + y = 0\}$

Reflexive

$x + x = 0$ is true when $x = 0$ and false for all other real numbers.

$\therefore$ R is not reflexive

Symmetric

By commutative property of addition, if $x + y = 0$ then $y + x = 0$

$\therefore$ R is symmetric

Antisymmetric

$1 + (-1) = 0$ and $(-1) + 1 = 0$ while $1 \neq -1$

$\therefore$ R is not antisymmetric

Transitive

if $1 + (-1) = 0$ and $(-1) + 1 = 0$ then $1 + 1 \neq 0$

$\therefore$ R is not transitive

Asymmetric

For $x + y = 0$, $(x, y) \in R$ and for $y + x = 0$, $(y, x) \in R$

$\therefore$ R is not asymmetric

(b) $x = \pm y$

$R = \{(x, y) \mid x = \pm y\}$

Reflexive

$x = x$ is true for all real numbers

$\therefore$ R is reflexive

Symmetric

If $x = \pm y$ then $y = \pm x$

$\therefore$ R is symmetric

Antisymmetric

$(-1) = -1$ and $1 = -1(-1)$ while $1 \neq -1$

$\therefore$ R is not antisymmetric

Transitive

if $x = \pm y$ and $y = \pm z$ then $x = \pm(\pm z) \Rightarrow x = \pm z$

$\therefore$ R is transitive

(b) **Asymmetric**

For $x = \pm y$, $(x, y) \in R$ and for $y = \pm x$, $(y, x) \in R$

$\therefore$ R is not asymmetric

(c) **x – y is a rational number**

$R = \{(x, y) \mid x - y$ is a rational number$\}$

Reflexive

For $x - x = 0$, 0 is always a rational number

$\therefore$ R is reflexive

Symmetric

If $x - y$ is a rational number, then $y - x = -(x - y)$ is also a rational number.

$\therefore$ R is symmetric

Antisymmetric

$(1 - 2)$ is a rational number and $(2 - 1)$ is a rational number while $1 \neq 2$

$\therefore$ R is not antisymmetric

Transitive

If $(x - y)$ and $(y - z)$ are rational numbers then $x - z = (x - y) - (y - z)$ is also a rational number.

$\therefore$ R is transitive

Asymmetric

If $x - y$ is a rational number then $(x, y) \in R$ and $(y - x)$ is a rational number then $(y, x) \in R$

$\therefore$ R is not asymmetric

(d) **x = 2y**

$R = \{(x, y) \mid x = 2y\}$

Reflexive

$x = 2x$ is only true when $x = 0$ and false for all other real numbers.

$\therefore$ R is not reflexive

Symmetric

Let $x = 2$ and $y = 1$

$2 = 2(1)$ but $1 \neq 2(2)$

$\therefore$ R is not symmetric

Antisymmetric

For $x = y = 0$, if $x = 2y$ then $y = 2x$

$\therefore$ R is antisymmetric

Transitive

if $x = 2y$ and $y = 2z$ then $x = 2(2z) = 4z$ i.e., $x \neq 2z$

$\therefore$ R is not transitive

Asymmetric

Let $x = 0, y = 0$

For $x = 2y$, $(0, 0) \in R$

$\therefore$ R is not asymmetric

(e) **xy ≥ 0**

$R = \{(x, y) \mid xy \geq 0\}$

Reflexive

$x \cdot x = x^2 \geq 0$, for all real numbers

$\therefore$ R is reflexive

Symmetric

By commutative property of multiplication, if $xy \geq 0$ then $yx \geq 0$

$\therefore$ R is symmetric

Antisymmetric

$(-2)(-1) \geq 0$ and $(-1)(-2) \geq 0$ while $-2 \neq -1$

$\therefore$ R is not antisymmetric

Transitive

Let $x = -1, y = 0, z = 1$

$xy \geq 0$, $yz \geq 0$ but $xz < 0$

$\therefore$ R is not transitive

Asymmetric

For $xy \geq 0$, $(x, y) \in R$ and $(y, x) \in R$

$\therefore$ R is not asymmetric

(f) **x = 1 or y = 1**

$R = \{(x, y) \mid x = 1$ or $y = 1\}$

Reflexive

R does not contain $(0, 0)$ since $0 \neq 1$

$\therefore$ R is not reflexive

Symmetric

If $(x, y) \in R$ then $(y, x) \in R$

$\therefore$ R is symmetric

Antisymmetric

Let $x = 1$ and $y = 0$

$(x, y) \in R$ and $(y, x) \in R$ while $x \neq y$

$\therefore$ R is not antisymmetric

Transitive

Let $x = 0, y = 1, z = 2$

$(x, y) \in R$ and $(y, z) \in R$ while $(x, y) \notin R$

$\therefore$ R is not transitive

Asymmetric

Let $x = 1, y = 1$

Then $(1, 1) \in R$

$\therefore$ R is not asymmetric

Q81. (a) **List the 16 different relations on the set {0, 1}**

(b) **How many of the 16 different relations on {0, 1} contain the pair (0, 1)**

(c) **Which of the 16 relations on {0, 1} are**

(i) **Reflexive**

(ii) **Irreflexive**

(iii) **Symmetric**

(iv) **Antisymmetric**

(v) **Asymmetric**

(vi) **Transitive**

Answer :

(a) Given set is,

$\{0, 1\}$

The ordered pairs of $\{0, 1\}$ are $(0, 0), (0, 1), (1, 0)$ and $(1, 1)$.

The 16 different relations on the set $\{0, 1\}$ are given as,

$R_1 = \{\ \}$

$R_2 = \{(0, 0)\}$

$R_3 = \{(0, 1)\}$

$R_4 = \{(1, 0)\}$

$R_5 = \{(1, 1)\}$

$R_6 = \{(0, 0), (0, 1)\}$

$R_7 = \{(0, 0), (1, 0)\}$

$R_8 = \{(0, 0), (1, 1)\}$

$R_9 = \{(0, 1), (1, 0)\}$

$R_{10} = \{(0, 1), (1, 1)\}$

$R_{11} = \{(1, 0), (1, 1)\}$

$R_{12} = \{(0, 0), (0, 1), (1, 0)\}$

$R_{13} = \{(0, 0), (0, 1), (1, 1)\}$

$R_{14} = \{(0, 0), (1, 0), (1, 1)\}$

$R_{15} = \{(0, 1), (1, 0), (1, 1)\}$

$R_{16} = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

(b) The relations containing the pair $(0, 1)$ are,

$R_3 = \{(0, 1)\}$

$R_6 = \{(0, 0), (0, 1)\}$

$R_9 = \{(0, 1), (1, 0)\}$

$R_{10} = \{(0, 1), (1, 1)\}$

$R_{12} = \{(0, 0), (0, 1), (1, 0)\}$

$R_{13} = \{(0, 0), (0, 1), (1, 1)\}$

$R_{15} = \{(0, 1), (1, 0), (1, 1)\}$

$R_{16} = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

$\therefore$ Eight relations on set $\{0, 1\}$ contains the pair $(0, 1)$.

(c) **(i)** ....

R is reflexive if it contains $(0, 0)$ and $(1, 1)$

$R_8 = \{(0, 0), (1, 1)\}$

$R_{13} = \{(0, 0), (0,1), (1, 1)\}$

$R_{14} = \{(0, 0), (1, 0), (1, 1)\}$

$R_{16} = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

$\therefore$ R_8, R_{13}, R_{14} and R_{16} are reflexive.

(ii) **Irreflexive**

R is irreflexive if it does not contain $(0, 0)$ and $(1, 1)$

$R_1 = \{\ \}$

$R_3 = \{(0, 1)\}$

$R_4 = \{(1, 0)\}$

$R_9 = \{(0, 1), (1, 0)\}$

$\therefore$ R_1, R_3, R_4 and R_9 are irreflexive.

(iii) **Symmetric**

R is symmetric if $(0, 1) \in R$ then $(1, 0) \in R$ and vice versa.

$R_1 = \{\ \}$

$R_2 = \{(0, 0)\}$

$R_5 = \{(1, 1)\}$

$R_8 = \{(0, 0), (1, 1)\}$

$R_9 = \{(0, 1), (1, 0)\}$

$R_{12} = \{(0, 0), (0, 1), (1, 0)\}$

$R_{15} = \{(0, 1), (1, 0), (1, 1)\}$

$R_{16} = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

$\therefore$ $R_1, R_2, R_5, R_8, R_9, R_{12}, R_{15}$ and R_{16} are symmetric

(iv) **Antisymmetric**

R is antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ while $a = b$. In otherwords, if $(0, 1) \in R$ then $(1, 0) \notin R$ and vice versa

$R_1 = \{\ \}$

$R_2 = \{(0, 0)\}$

$R_3 = \{(0, 1)\}$

$R_4 = \{(1, 0)\}$

$R_5 = \{(1, 1)\}$

$R_6 = \{(0, 0), (0, 1)\}$

$R_7 = \{(0, 0), (1, 0)\}$

$R_8 = \{(0, 0), (1, 1)\}$

$R_{10} = \{(0, 1), (1, 1)\}$

$R_{11} = \{(1, 0), (1, 1)\}$

$R_{13} = \{(0, 0), (0, 1), (1, 1)\}$

$R_{14} = \{(0, 0), (1, 0), (1, 1)\}$

$\therefore$ $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_{10}, R_{11}, R_{13}$ and R are antisymmetric.

(v) **Asymmetric**

R is asymmetric if $(a, b) \in R$ then $(b, a) \notin R$

$R_1 = \{ \}$

$R_3 = \{(0, 1)\}$

$R_4 = \{(1, 0)\}$

$\therefore$ R_1, R_3 and R_4 are asymmetric.

(vi) **Transitive**

R is transitive if $(0, 1) \in R$ and $(1, 0) \in R$, then $(0, 0) \in R$ and $(1, 1) \in R$

$R_1 = \{ \}$

$R_2 = \{(0, 0)\}$

$R_3 = \{(0, 1)\}$

$R_4 = \{(1, 0)\}$

$R_5 = \{(1, 1)\}$

$R_6 = \{(0, 0), (0, 1)\}$

$R_7 = \{(0, 0), (1, 0)\}$

$R_8 = \{(0, 0), (1, 1)\}$

$R_{10} = \{(0, 1), (1, 1)\}$

$R_{11} = \{(1, 0), (1, 1)\}$

$R_{13} = \{(0, 0), (0, 1), (1, 1)\}$

$R_{14} = \{(0, 0), (1, 0), (1, 1)\}$

$R_{16} = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

$\therefore$ $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_{10}, R_{11}, R_{13}, R_{14}$ and R_{16} are transitive.

Q82. Let R be the relation R = {(a, b) | a < b} on the set of integers. Find (i) R⁻¹ (ii) $\overline{R}$.

Answer :

Given relation is,

$R = \{(a, b) \mid a < b\}$

(i) R^{-1}

From the definition of inverse relation,

$R^{-1} = \{(b, a) \mid (a, b) \in R\}$

$\Rightarrow$ $R^{-1} = \{(b, a) \mid a < b\}$

Interchanging a and b,

$R^{-1} = \{(a, b) \mid b < a\}$

$\Rightarrow$ $R^{-1} = \{(a, b) \mid a > b\}$

$\therefore$ Inverse relation, $R^{-1} = \{(a, b) \mid a > b\}$

(ii) $\overline{R}$

From the definition of complementary relation,

$\overline{R} = \{(a, b) \mid (a, b) \notin R\}$

$\Rightarrow$ $\overline{R} = \{(a, b) \mid a \not< b\}$

$\Rightarrow$ $\overline{R} = \{(a, b) \mid a \geq b\}$

$\therefore$ Complementary relation, $\overline{R} = \{(a, b) \mid a \geq b\}$

Q83. Show that the relation R on a set A is reflexive if and only if the complementary relation $\overline{R}$ is irreflexive.

Model Paper-4, Q5(b)

Answer :

Given that,

R is a relation on set A.

Case (i)

Assuming that the relation R is reflexive

Let $a \in A$ such that $(a, a) \in R$

From the definition of complementary relation,

$(a, a) \notin \overline{R}$

From the definition of reflexive, $\overline{R}$ is irreflexive (since $(a, a) \notin \overline{R}$)

$\therefore$ If R is reflexive then $\overline{R}$ is irreflexive

Case (ii)

Assuming that the relation $\overline{R}$ is irreflexive

Let $a \in A$ such that $(a, a) \notin \overline{R}$

From the definition of complementary relation,

$(a, a) \in R$

From the definition of reflexive, R is reflexive (since $(a, a) \in R$)

$\therefore$ If $\overline{R}$ is irreflexive then R is reflexive.

Thus, R is reflexive if and only if $\overline{R}$ is irreflexive.

Q84. Show that the relation R on a set A is reflexive if and only if the inverse relation R⁻¹ is reflexive.

Answer :

Given that,

R is a relation on set A

Case (i)

Assuming that the relation R is reflexive

Let $a \in A$, such that $(a, a) \in R$

From the definition of inverse relation,

$(a, a) \in R^{-1}$

From the definition of reflexive, R^{-1} is reflexive (since $(a, a) \in R^{-1}$).

$\therefore$ If R is reflexive then R^{-1} is reflexive.

Case (ii)

Assuming that the relation R^{-1} is reflexive

Let $a \in A$ such that $(a, a) \in R^{-1}$

From the definition of inverse relation,

$(a, a) \in R$

From the definition of relfexive, R is reflexive (since $(a, a) \in R$)

$\therefore$ If R^{-1} is reflexive then R is reflexive.

Thus, R is reflexive if and only if R^{-1} is reflexive.

Q85. Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find

(a) $R_1 \cup R_2$
(b) $R_1 \cap R_2$
(c) $R_1 - R_2$
(d) $R_2 - R_1$

Answer :

Given relations are,

$R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and

$R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$

(a) $R_1 \cup R_2$

$R_1 \cup R_2 = \{(1, 2), (2, 3), (3, 4)\} \cup \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$

$\Rightarrow$ $R_1 \cup R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$

(b) $R_1 \cap R_2$

$R_1 \cap R_2 = \{(1, 2), (2, 3), (3, 4)\} \cap \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$

$\Rightarrow$ $R_1 \cap R_2 = \{(1, 2), (2, 3), (3, 4)\}$

(c) $R_1 - R_2$

$R_1 - R_2 = \{(1, 2), (2, 3), (3, 4)\} - \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$

$\Rightarrow$ $R_1 - R_2 = \{\ \} = \phi$

(d) $R_2 - R_1$

$R_2 - R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\} - \{(1, 2), (2, 3), (3, 4)\}$

$\Rightarrow$ $R_2 - R_1 = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}$

Q86. The relations on the set of real numbers,

$R_1 = \{(a, b) \in R^2 \mid a > b\}$, the greater than relation

$R_2 = \{(a, b) \in R^2 \mid a \geq b\}$, the "greater than or equal to" relation.

$R_3 = \{(a, b) \in R^2 \mid a < b\}$, the "less than" relation

$R_4 = \{(a, b) \in R^2 \mid a \leq b\}$, the "less than or equal to" relation.

$R_5 = \{(a, b) \in R^2 \mid a = b\}$, the "equal to" relation

$R_6 = \{(a, b) \in R^2 \mid a \neq b\}$, the "unequal to" relation.

Find

(i) $R_1 \cup R_3$ (ii) $R_2 \cap R_4$ (iii) $R_3 - R_6$

(iv) $R_3 \oplus R_5$ (v) $R_1 \circ R_4$ (vi) $R_3 \circ R_6$

Answer : **Model Paper**

Given relations are,

$R_1 = \{(a, b) \in R^2 \mid a > b\}$

$R_2 = \{(a, b) \in R^2 \mid a \geq b\}$

$R_3 = \{(a, b) \in R^2 \mid a < b\}$

$R_4 = \{(a, b) \in R^2 \mid a \leq b\}$

$R_5 = \{(a, b) \in R^2 \mid a = b\}$ and

$R_6 = \{(a, b) \in R^2 \mid a \neq b\}$

(i) $R_1 \cup R_3$

$R_1 \cup R_3 = \{(a, b) \in R^2 \mid a > b\} \cup \{(a, b) \in R^2 \mid a < b\}$

$= \{(a, b) \in R^2 \mid a > b \text{ or } a < b\}$

$= \{(a, b) \in R^2 \mid a \neq b\}$

$= R_6$

$\therefore$ $R_1 \cup R_3 = R_6$

(ii) $\mathbf{R_2 \cap R_4}$

$$R_2 \cap R_4 = \{(a, b) \in R^2 \mid a \geq b\} \cap \{(a, b) \in R^2 \mid a \leq b\}$$
$$= \{(a, b) \in R^2 \mid a \geq b \text{ and } a \leq b\}$$
$$= \{(a, b) \in R^2 \mid a = b\}$$
$$= R_5$$

$\therefore \quad R_2 \cap R_4 = R_5$

(iii) $\mathbf{R_3 - R_6}$

$$R_3 - R_6 = \{(a, b) \in R^2 \mid a < b\} - \{(a, b) \in R^2 \mid a \neq b\}$$
$$= \{(a, b) \in R^2 \mid a < b \text{ and not } a \neq b\}$$
$$= \{(a, b) \in R^2 \mid a < b \text{ and } a = b\}$$
$$= \phi$$

$\therefore \quad R_3 - R_6 = \phi$

(iv) $\mathbf{R_3 \oplus R_5}$

$$R_3 \oplus R_5 = \{(a, b) \in R^2 \mid a < b\} \oplus \{(a, b) \in R^2 \mid a = b\}$$
$$= \{(a, b) \in R^2 \mid (a < b \text{ or } a = b) \text{ and not } (a < b \text{ and } a = b)\}$$
$$= \{(a, b) \in R^2 \mid a \leq b \text{ and not } F\}$$
$$= \{(a, b) \in R^2 \mid a \leq b \text{ and } \sim F\}$$
$$= \{(a, b) \in R^2 \mid a \leq b \text{ and } T\}$$
$$= \{(a, b) \in R^2 \mid a \leq b\} \qquad [\because P \wedge T \equiv P]$$
$$= R_4$$

$\therefore \quad R_3 \oplus R_5 = R_4$

(v) $\mathbf{R_1 \circ R_4}$

$$R_1 \circ R_4 = \{(a, b) \in R^2 \mid a > b\} \circ \{(a, b) \in R^2 \mid a \leq b\}$$
$$= \{(a, c) \in R^2 \mid \exists\ b \in R : (a, b) \in R_4 \text{ and } (b, c\} \in R_1\}$$
$$= \{(a, c) \in R^2 \mid \exists\ b \in R : a \leq b \text{ and } b > c\}$$
$$= \{(a, c) \in R^2 \mid T\}$$
$$= \{(a, c) \in R^2\}$$
$$= R^2$$

$\therefore \quad R_1 \circ R_4 = R^2$

(vi) $\mathbf{R_3 \circ R_6}$

$$R_3 \circ R_6 = \{(a, b) \in R^2 \mid a < b\} \circ \{(a, b) \in R^2 \mid a \neq b\}$$
$$= \{(a, c) \in R^2 \mid \exists\ b \in R : (a, b) \in R_6 \text{ and } (b, c\} \in R_3\}$$
$$= \{(a, c) \in R^2 \mid \exists\ b \in R : a \neq b \text{ and } b < c\}$$
$$= \{(a, c) \in R^2 \mid T\}$$
$$= \{(a, c) \in R^2\}$$
$$= R^2$$

$\therefore \quad R_3 \circ R_6 = R^2$

Q87. Let R_1 and R_2 be the "divides" and "is a multiple of" relations on the set of all positive integers respectively. That is, R_1 = {(a, b) | a divides b} and R_2 = {(a , b) | a is a multiple of b}. Find,

Find

(i) $R_1 \cup R_2$ (ii) $R_1 \cap R_2$ (iii) $R_1 - R_2$ (iv) $R_2 - R_1$ (v) $R_1 \oplus R_2$

Answer :

Given relations are,

$R_1 = \{(a, b) \mid a \text{ divides } b\}$ and

$R_2 = \{(a, b) \mid a \text{ is a multiple of } b\}$

Where, R_1 and R_2 are relations on the set of all positive integers.

Thus,

$R_1 = \{(a, b) \in R^2 \mid a \text{ divides } b\}$

$R_2 = \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\}$

(i) $R_1 \cup R_2$

$R_1 \cup R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b\} \cup \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ or } a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ or } b \text{ divides } a\}$

$\therefore \quad R_1 \cup R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b \text{ or } b \text{ divides } a\}$

(ii) $R_1 \cap R_2$

$R_1 \cap R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b\} \cap \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } b \text{ divides } a\}$

$= \{(a, b) \in R^2 \mid a = \pm b \text{ and } a \neq 0\}$

$\therefore \quad R_1 \cap R_2 = \{(a, b) \in R^2 \mid a = \pm b \text{ and } a \neq 0\}$

(iii) $R_1 - R_2$

$R_1 - R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b\} - \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } a \text{ is not a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } b \text{ does not divide } a\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } a \neq \pm b\}$

$\therefore \quad R_1 - R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b \text{ and } a \neq \pm b\}$

(iv) $R_2 - R_1$

$R_2 - R_1 = \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\} - \{(a, b) \in R^2 \mid a \text{ divides } b\}$

$= \{(a, b) \in R^2 \mid a \text{ is a multiple of } b \text{ and } a \text{ does not divide } b\}$

$= \{(a, b) \in R^2 \mid b \text{ divides } a \text{ and } a \text{ does not divide } b\}$

$= \{(a, b) \in R^2 \mid b \text{ divides } a \text{ and } a \neq \pm b\}$

$\therefore \quad R_2 - R_1 = \{(a, b) \in R^2 \mid b \text{ divides } a \text{ and } a \neq \pm b\}$

(v) $R_1 \oplus R_2$

$R_1 \oplus R_2 = \{(a, b) \in R^2 \mid a \text{ divides } b\} \oplus \{(a, b) \in R^2 \mid a \text{ is a multiple of } b\}$

$= \{(a, b) \in R^2 \mid a \text{ divides } b \text{ or } b \text{ is a multiple of } a \text{ but not both}\}$

$= \{(a, b) \in R^2 \mid (a \text{ divides } b \text{ or } b \text{ divides } a) \text{ and } a \neq \pm b\}$

$\therefore \quad R_1 \oplus R_2 = \{(a, b) \in R^2 \mid (a \text{ divides } b \text{ or } b \text{ divides } a) \text{ and } a \neq \pm b\}$

2.6 N-ARY RELATIONS AND THEIR APPLICATIONS

Q88. List the triples in the relation {(a, b, c) | a, b and c are integers with 0 < a < b < c < 5}.

Answer :

Given relation is,

$R = \{(a, b, c) \mid a, b$ and c are integers with $0 < a < b < c < 5\}$.

From the condition $0 < a < b < c < 5$, a can have the value either 1 or 2; b, c must be larger than a and smaller than 5.

Case(i) when a = 1

For $a = 1$, the possible values of b are either 2 or 3; c should be larger than b and smaller than 5.

* **When a = 1, b = 2**

For $a = 1$ and $b = 2$, c can have either 3 or 4.

i.e., $(1, 2, 3) \in R$

$(1, 2, 4) \in R$

* **When a = 1, b = 3**

For $a = 1$ and $b = 3$, c has the value of 4.

i.e., $(1, 3, 4) \in R$

Case(ii) when a = 2

For $a = 2$, the only values of b and c are 3 and 4 respectively.

i.e., $(2, 3, 4) \in R$

$\therefore$ The set of triples are $\{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)\}$.

Q89. Which 4-tuples are in the relation {(a, b, c, d)|a, b, c and d are positive integers with abcd = 6}.

Model Paper-3, Q5(b)

Answer :

Given relation is,

$R = \{(a, b, c, d) \mid a, b, c$ and d are positive integers with $abcd = 6\}$.

The product of 6 can be obtained by multiplying with positive integers either 1 and 6 or 2 and 3.

Case (i) : Multiplying with 1 and 6

For 4-tuples of product 6, one of the four integers must be 6 and remaining three integers be 1.

i.e., $(6, 1, 1, 1) \in R$

$(1, 6, 1, 1) \in R$

$(1, 1, 6, 1) \in R$

$(1, 1, 1, 6) \in R$

Case (ii) : Multiplying with 2 and 3

For 4-tuples of product 6, one of the four integers must be 3, other integer should be 2 and remaining all integers be 1.

i.e., $(3, 2, 1, 1) \in R$

$(3, 1, 2, 1) \in R$

$(3, 1, 1, 2) \in R$

$(2, 3, 1, 1) \in R$

$(1, 3, 2, 1) \in R$

$(1, 3, 1, 2) \in R$

$(2, 1, 3, 1) \in R$

$(1, 2, 3, 1) \in R$

$(1, 1, 3, 2) \in R$

$(2, 1, 1, 3) \in R$

$(1, 2, 1, 3) \in R$

$(1, 1, 2, 3) \in R$

$\therefore$ The set of 4-tuples are $\{(6, 1, 1, 1), (1, 6, 1, 1), (1, 1, 6, 1), (1, 1, 1, 6), (3, 2, 1, 1), (3, 1, 2, 1), (3, 1, 1, 2), (2, 3, 1, 1), (1, 3, 2, 1), (1, 3, 1, 2), (2, 1, 3, 1), (1, 2, 3, 1), (1, 1, 3, 2), (2, 1, 1, 3), (1, 2, 1, 3), (1, 1, 2, 3)\}$.

Q90. For the following table,

(i) List the 5-tuples in the relation

(ii) Find the primary key for the relation displayed in table

(iii) Find a composite key with two fields containing the Airline field for the database.

(iv) Apply the selection operator s_C, where C is the condition Destination = Detroit, to the databas

(v) Apply the selection operator S_C, where C_c is the condition (Airline = Nadir) $\vee$ (Destination = Denv to the database.

(vi) Display the table produced by applying the projection $P_{1,2,4}$.

Airline	Flight-number	Gate	Destination	Departure-time
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08:47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

Table

Answer :

Given table is,

Airline	Flight-number	Gate	Destination	Departure-time
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08:47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

Table (1)

(i) (Nadir, 122, 34, Detroit, 08:10)

(Acme, 221, 22, Denver, 08:17)

(Acme, 122, 33, Anchorage, 08:22)

(Acme, 323, 34, Honolulu, 08:30)

(Nadir, 199, 13, Detroit, 08:47)

(Acme, 222, 22, Denver, 09:10)

(Nadir, 322, 34, Detroit, 09:44)

(ii) **Primary Key**

As Nadir and Acme are repeated, Airline is not a primary key.

As the number 122 is specified twice, flight-number is not a primary key.

As 34 and 22 are mentioned more than once, gate is not a primary key.

As more than one 5-tuples contain the same destination, it is also not a primary key.

The entire column of departure-time is different. Hence the primary key is departure-time.

(iii) **Composite Key**

As no two 5-tuples have same airline and flight-number, airline-flight-number forms a composite key.

Gate number and destination cannot be the composite keys as Nadir Airline has multiple flights departing from gate 34 to the destination Detroit.

The departure-time for every airline is different. So, airline-departure-time can be a composite key.

(iv) Given selection operator is,

S_C, where C is the condition

Destination = Detroit.

From the table, Detroit is assigned as the destination for three 5-tuples. They are,

1. (Nadir, 122, 34, Detroit, 08:10)
2. (Nadir, 199, 13, Detroit, 08:47)
3. (Nadir, 322, 34, Detroit, 09:44)

(v) Given selection operator is,

S_C, where C is the condition (Airline = Nadir) $\vee$ (Destination = Denver)

From the table, the 5-tuples having either the airline Nadir or the destination Denver are given as,

1. (Nadir, 122, 34, Detroit, 08:10)
2. (Nadir, 199, 13, Detroit, 08:47)
3. (Nadir, 322, 34, Detroit, 09:44)
4. (Acme, 221, 22, Denver, 08:17)
5. (Acme, 222, 22, Denver, 09:10)

(vi) The table obtained by applying the projection $P_{1,2,4}$ is shown below.

Airline	Flight-number	Destination
Nadir	122	Detroit
Acme	221	Denver
Acme	122	Anchorage
Acme	323	Honolulu
Nadir	199	Detroit
Acme	222	Denver
Nadir	322	Detroit

Table (2)

Q91. **Table 1: part needs :**

Supplier	Part-number	Project
23	1092	1
23	1101	3
23	9048	4
31	4975	3
31	3477	2
32	6984	4
32	9191	2
33	1001	1

Table 2: parts - inventory

Part-number	Project	Quantity	Color Code
1001	1	14	8
1092	1	2	2
1101	3	1	1
3477	2	25	2
4975	3	6	2
6984	4	10	1
9048	4	12	2
9191	2	80	4

(a) Construct the table obtained by applying the join operator J_2 to the relations in tables 1 and 2.

(b) What are the operations that correspond to the query expressed using the SQL statement

SELECT Supplier, Project

FROM Part needs, Parts inventory

WHERE Quantity $\leq$ 10

(c) What is the output of the query.

Answer :

Given tables are,

Supplier	Part-number	Project
23	1092	1
23	1101	3
23	9048	4
31	4975	3
31	3477	2
32	6984	4
32	9191	2
33	1001	1

Table (1)

Part-number	Project	Quantity	Color Code
1001	1	14	8
1092	1	2	2
1101	3	1	1
3477	2	25	2
4975	3	6	2
6984	4	10	1
9048	4	12	2
9191	2	80	4

Table (2)

(a) The second, third columns in table (1) and first, second columns in table (2) represent the same fields i.e., part-number and project.

By applying the join operator J_2, the two fields (i.e., part-number and project) are combined such that a new table is formed with the columns as supplier, part-number, project, quantity and color code as shown below

Supplier	Part-number	Project	Quantity	Color Code
23	1092	1	2	2
23	1101	3	1	1
23	9048	4	12	2
31	4975	3	6	2
31	3477	2	25	2
32	6984	4	10	1
32	9191	2	80	4
33	1001	1	14	8

Table (3)

(b) Given SQL statement is,

SELECT Supplier, Project

FROM Part needs, Part inventory

WHERE Quantity ≤ 10

As the two columns part-number, project are common in both the tables, join operator J_2 is used to combined them as shown in table (3).

The information of supplier and project is needed from the 5-tuples satisfying the condition C = {Quantity ≤ 10} i.e., $S_C(J_2(R))$. Projection $P_{1,3}$ is used to the above operation, as the columns 1 and 3 represent supplier and project i.e., $P_{1,3}(S_C(J_2(R)))$.

Hence, the operations that correspond to the given SQL statement are $P_{1,3}(S_C(J_2(R)))$.

(c) The output of the query is the information of supplier and project satisfying the condition C = Quantity ≤ 10.

The data sets of supplier and project with quantity ≤ 10, are represented in table below, which is obtained from table (3).

Supplier	Part-number	Project	Quantity	Color Code
23	1092	1	2	2
23	1101	3	1	1
31	4975	3	6	2
32	6984	4	10	1

Table (4)

Hence, the output is (23, 1), (23, 3), (31, 3), (32, 4).

2.7 REPRESENTING RELATIONS, CLOSURES OF RELATIONS

Q92. Explain the two alternative methods for representing relations with examples.

Answer :

The two alternative methods for representing relations are,

(i) Matrix method

(ii) Digraph (or) directed graph method.

(i) **Matrix Method**

Any relation R from sets A to B (i.e., $R : A \to B$) is defined using matrices such that for any $A = \{a_1, a_2, ..., a_n\}$ and $B = \{b_1, b_2, ..., b_n\}$, the elements of rows are headed by $A = \{a_1, a_2, ..., a_n\}$ and that of columns by $B = \{b_1, b_2, ..., b_n\}$. The elements of matrix are chosen such that

$$M_R = [m_{ij}]$$

Where,

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

Example

If $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$ and R is relation from A to B such that $R = \{(a_1, b_1)\ (a_2, b_1)\ (a_3, b_2)\ (a_2, b_2)\}$. Then matrix representation is,

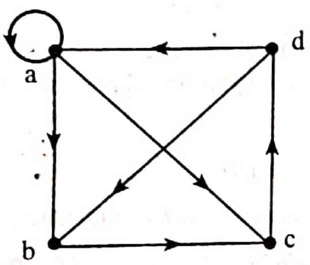
$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(ii) Digraph (or) Directed Graph Method

Any relation $R : A \to B$ is represented using graph where the elements of the set A are represented by points usually known as nodes (vertices) and ordered pairs arcs indicating the directions with an arrow.

Example

Let $R = \{(a, a), (a, b), (b, c), (c, d), (d, a), (a, c), (d, b)\}$. The digraph for '$R$' is,



Figure

Q93. Represent each of these relations on {1, 2, 3} with a matrix.

(a) {(1, 1), (1, 2), (1, 3)}

(b) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

Answer :

Given set is,

$$A = \{1, 2, 3\}$$

(a) Given relation is,

$$R = \{(1, 1), (1, 2), (1, 3)\}$$

The given relation can be expressed in matrix form as,

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Given relation is,

$$R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$$

The given relation can be expressed in matrix form as,

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Q94. List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Answer :

Given set is,

$$A = \{1, 2, 3\}$$

(a) Given matrix is,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The ordered pairs of the given matrix M_R is,

$$R = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$$

(b) Given matrix is,

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The ordered pairs of the given matrix M_R is,

$$R = \{(1, 2), (2, 2), (3, 2)\}$$

(c) Given matrix is,

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The ordered pairs of the given matrix M_R is,

$$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Q95. Suppose that the relations R_1 and R_2 on a set A are represented by the matrices,

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find $M_{R_1 \cup R_2}$ and $M_{R_1 \cap R_2}$

Answer :

Given matrices are,

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and}$$

$$M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 1 & 0 \vee 0 & 1 \vee 1 \\ 1 \vee 0 & 0 \vee 1 & 0 \vee 1 \\ 0 \vee 1 & 1 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \wedge 1 & 0 \wedge 0 & 1 \wedge 1 \\ 1 \wedge 0 & 0 \wedge 1 & 0 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 0 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q96. Find the matrix representing the relations S o R, where the matrices representing R and S are,

$$\mathbf{M_R} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{M_S} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Model Paper-5, Q5(a)

Answer :

Given matrices are,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The matrix for $S \circ R$ is given as,

$$M_{S \circ R} = M_R \odot M_S$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 0) \vee (0 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) \\ (1 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 0) & (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) \\ (0 \wedge 0) \vee (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 0) \vee (0 \wedge 0) & (0 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \vee 0 \vee 1 & 1 \vee 0 \vee 0 & 0 \vee 0 \vee 1 \\ 0 \vee 0 \vee 0 & 1 \vee 0 \vee 0 & 0 \vee 1 \vee 0 \\ 0 \vee 0 \vee 0 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad M_{S \circ R} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Q97. Let R be the relation on the set (1, 2, 3, 4, 5) containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2) and (5, 4). Find R^3

Answer :

Given relation is,

$R = \{(1, 3), (2, 4), (3, 1), (3, 5) (4, 3), (5, 1), (5, 2), (5, 4)\}$

The matrix representing the relation R is given as,

$$M_r = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The matrix for R^2 is obtained by the boolean product of M_r.

i.e., $M_r^{[2]} = M_r \odot M_r$

$$M_r^{[2]} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\vee0\vee1\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee1\vee0\vee0 \\ 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee1\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 \\ 1\vee1\vee1\vee1\vee0 & 0\vee0\vee0\vee0\vee1 & 1\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee1 & 0\vee0\vee0\vee0\vee0 \\ 0\vee0\vee1\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 0\vee0\vee1\vee0\vee0 \\ 0\vee0\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 & 1\vee0\vee0\vee0\vee0 & 0\vee1\vee0\vee0\vee0 & 0\vee0\vee0\vee0\vee0 \end{bmatrix}$$

$$\Rightarrow M_r^{[2]} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$R^2 = \{(1, 1), (1, 5), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 5), (5, 3), (5, 4)\}$

Q98. How many nonzero entries does the matrix representing the relation R on A = {1, 2, 3, ..., 100} consisting of the first 100 positive integers have if R is {(a, b) | a > b}?

Answer :

Given set is,

$A = \{1, 2, 3, \ldots\ldots, 100\}$

The relation is,

$R = \{(a, b) \mid a > b\}$

Let,

The possible values for $a = 100$

The possible values for $b = 100$

Total number of entries representing the relation R is,

$a.b = 100.100 = 10,000$

The given relation $a > b$ is satisfied for the elements below the main diagonal.

$\therefore$ The matrix will contain 1's only below the main diagnoal as,

$$R = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 1 & 1 & 1 & 1 & \cdots & 0 \end{bmatrix}$$

Number of elements in the main diagonal = 10,000 – 100 = 9,900

Number of elements below the main diagonal is exactly half of the elements in the main diagonal.

i.e., $\frac{9900}{2} = 4950$

$\therefore$ The matrix contains 4950 nonzero entries.

Q99. How many nonzero entries does the matrix representing the relation R on A = {1, 2, 3, ..., 1000} consisting of the first 1000 positive integers have if R is {(a, b) | a + b = 1000}.

Answer :

Given set is,

$A = \{1, 2, 3, \ldots\ldots, 1000\}$

The relation is,

$R = \{(a, b) \mid a + b = 1000\}$

Let,

The possible values for $a = 1000$

The possible values for $b = 1000$

Total number of entries representing the relation R is,

$a.b = 1000.1000 = 10,00,000$

The given relation $a + b = 1000$ is satisfied for the elements above the non-main diagonal.

$\therefore$ The non-main diagonal contains only 1's as,

$$R = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}$$

From the above matrix, the main diagonal contains 1000 elements and the non-main diagonal also contains 1000 elements.

$\therefore$ The matrix contains 999 entries of nonzero elements directly above the non-main diagonal.

Q100. Let R be the relation represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Find the matrix representing

(a) R^{-1}

(b) $\overline{R}$

(c) R^2

Answer :

Given matrix is,

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) R^{-1}

The matrix corresponding to R^{-1} is obtained by taking transpose of the given matrix M_R.

$$M_{R^{-1}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^T$$

$$\Rightarrow M_{R^{-1}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) $\bar{R}$

The matrix corresponding to $\bar{R}$ is obtained by changing every 0 to 1 and every 1 to 0.

$$\Rightarrow \quad M_{\bar{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) R^2

The matrix corresponding to R^2 is obtained by taking the boolean product of two matrices.

$$M_R^{[2]} = M_R \odot M_R$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (0\wedge 0)\vee(1\wedge 1)\vee(1\wedge 1) & (0\wedge 1)\vee(1\wedge 1)\vee(1\wedge 0) & (0\wedge 1)\vee(1\wedge 0)\vee(1\wedge 1) \\ (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) \\ (1\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1) & (1\wedge 1)\vee(0\wedge 1)\vee(1\wedge 0) & (1\wedge 1)\vee(0\wedge 0)\vee(1\wedge 1) \end{bmatrix}$$

$$= \begin{bmatrix} 0\vee 1\vee 1 & 0\vee 1\vee 0 & 0\vee 0\vee 1 \\ 0\vee 1\vee 0 & 1\vee 1\vee 0 & 1\vee 0\vee 0 \\ 0\vee 0\vee 1 & 1\vee 0\vee 0 & 1\vee 0\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad M_R^{[2]} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q101. (a) Define transitive closure

(b) Find the zero-one matrix of the transitive closure of the relation R where

$$\mathbf{M_R} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Answer :

(a) Transitive Closure

Transitive closure of a binary relation 'R', denoted by R^*, is the smallest transitive relation that consists of R. It can be defined as a set of unions of $R^1, R^2, R^3, \ldots, R^n$ i.e.,

$$R^* = \bigcup_{n=1}^{\infty} R^n$$

(b) Given matrix is, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

The zero-one matrix of the transitive closure R^* is given by,

$$M_R^* = M_R \vee M_R^{[2]} \vee M_R^{[3]}$$

$$M_R^{[2]} = M_R \odot M_R$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\wedge 1)\vee(0\wedge 0)\vee(1\wedge 1) & (1\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1) & (1\wedge 1)\vee(0\wedge 0)\vee(1\wedge 0) \\ (0\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (0\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (0\wedge 1)\vee(1\wedge 0)\vee(0\wedge 0) \\ (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1\vee 0\vee 1 & 0\vee 0\vee 1 & 1\vee 0\vee 0 \\ 0\vee 0\vee 0 & 0\vee 1\vee 0 & 0\vee 0\vee 0 \\ 1\vee 0\vee 0 & 0\vee 1\vee 0 & 1\vee 0\vee 0 \end{bmatrix}$$

$$\therefore \quad M_R^{[2]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[3]} = M_R^{[2]} \odot M_R$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\wedge 1)\vee(1\wedge 0)\vee(1\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(1\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(1\wedge 0) \\ (0\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (0\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (0\wedge 1)\vee(1\wedge 0)\vee(0\wedge 0) \\ (1\wedge 1)\vee(1\wedge 0)\vee(1\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(1\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(1\wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1\vee 0\vee 1 & 0\vee 1\vee 1 & 1\vee 0\vee 0 \\ 0\vee 0\vee 0 & 0\vee 1\vee 0 & 0\vee 0\vee 0 \\ 1\vee 0\vee 1 & 0\vee 1\vee 1 & 1\vee 0\vee 0 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

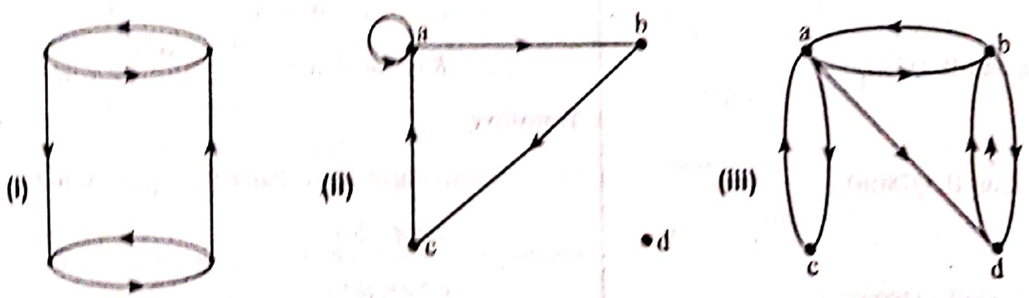
$$M_R^{*} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\vee 1\vee 1 & 0\vee 1\vee 1 & 1\vee 1\vee 1 \\ 0\vee 0\vee 0 & 1\vee 1\vee 1 & 0\vee 0\vee 0 \\ 1\vee 1\vee 1 & 1\vee 1\vee 1 & 0\vee 1\vee 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Transitive closure of R is, $M_R^{*} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

Q102. Find the directed graphs of the symmetric closures of the relation with directed graphs shown in figures.



Answer :

The directed graph of the symmetric closure is obtained by appending an edge in the opposite direction to the given edge as shown in figures.

(i)

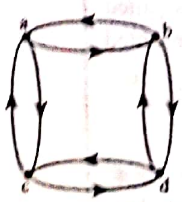


Figure (1)

(ii)

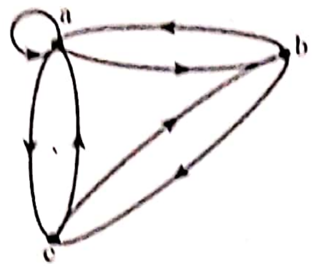


Figure (2)

(iii)

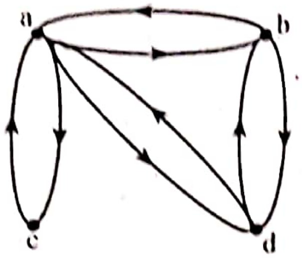


Figure (3)

2.8 EQUIVALENCE RELATIONS, PARTIAL ORDERINGS

Q103. Define,

(i) Equivalence relation

(ii) Equivalence class

(iii) Partition.

Answer :

(i) Equivalence Relation

A relation 'R' defined on any set 'A' is said to be an equivalent relation if it satisfies the following properties.

Reflexive Property

For answer refer Unit-II, Q78(i).

Symmetric Property

For answer refer Unit-II, Q78(iii).

Transitive Property

For answer refer Unit-II, Q78(vi).

(ii) Equivalence Class

If R is a relation defined on set A such that the set of all the elements that are related to an element $a \in A$ is called equivalence class of a and is denoted by $[a]_R$. It is represented as $[a]_R = \{s|(a, s) \in R\}$

(iii) Partition

Any set 'P' is defined such that, if $A_1, A_2, ..., A_n$ equivalence classes of an equivalence relation A are such that,

(a) $A = A_1 \cup A_2 \cup ... A_n$ and

(b) $\exists\, A_i, A_j \in A_i \cap A_j = \phi$ (or) $A_i = A_j$ then set $P = \{A_1, A_2, ..., A_n\}$ is called partition of a set.

Any equivalence relation R on P induces a partition of P and any partition of P will generate an equivalence relation R on P such that the set of equivalence relations and the set of partitions have one-to-one correspondence between each other.

Q104. Let R be the relation on the set of ordered pairs of positive integers such that ((a, b), (c, d)) ∈ R if and only if a + d = b + c. Show that R is an equivalence relation.

Answer :

Given relation is,

$$R = \{((a, b), (c, d)) \mid a + d = b + c\}$$

Reflexive

Consider, $(a, b) \in A$

If $a + b = b + a$ then $((a, b), (a, b)) \in R$

$\therefore$ R is reflexive.

Symmetric

Consider,

$((a, b), (c, d)) \in R$

$\Rightarrow$ $a + d = b + c$

$\Rightarrow$ $d + a = c + b$

[By commutative property of addition]

$\Rightarrow$ $c + b = d + a$

$\Rightarrow$ $((c, d), (a, b)) \in R$

$\therefore$ R is symmetric.

Transitive

Let $((a, b), (c, d)) \in R$ and $((c, d), (e, f)) \in R$.

$$a + d = b + c \quad ...(1)$$

$$c + f = d + e \quad ...(2)$$

Adding equations (1) and (2),

$$a + d + c + f = b + c + d + e$$

$\Rightarrow$ $a + f = b + e$

$\Rightarrow$ $((a, b), (e, f)) \in R$

$\therefore$ R is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.

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Q105. Let m be a positive integer with m > 1. Show that the relation R = {(a, b) | a ≡ b(mod m)} is an equivalence relation on the set of integers.

Answer : **Model Paper-5, Q5(b)**

Given relation is,

$R = \{(a, b) \mid a \equiv b (\bmod m)\}$

Consider $a \equiv b \pmod m$

Let $a - b$ is divisible by m

$\Rightarrow \quad a - a = 0$

Reflexive

$a - a$ is divisible by m

$\Rightarrow \quad a \equiv a \pmod m \qquad [\because 0 = 0.m]$

$\therefore$ Congruence modulo m is reflexive.

Symmetric

$a - b = \bmod m \qquad [\because a - b$ is divisible by $m]$

$\Rightarrow \quad a - b = mk \qquad [\because \bmod m = mk]$

$\Rightarrow \quad -(a - b) = -mk$

$\Rightarrow \quad b - a = m(-k)$

$\therefore \quad b \equiv a \bmod m$

Transitive

Consider,

$a \equiv b \pmod m,\ b \equiv c \pmod m$

Let, $a - b,\ b - c$ are divisible by m.

Then $a - b = km,\ b - c = lm$

Adding the above equations,

$a - b + b - c = km + lm$

$\Rightarrow \quad a - c = (k + l)m$

$\Rightarrow \quad a \equiv c \pmod m$

$\therefore$ Congruence modulo m is transitive.

Hence the given relation is an equivalence relation as m is reflexive, symmetric and transitive.

Q106. (a) Show that the relation R on the set of all differentiable functions from R to R consisting of all pairs (f, g) such that f'(x) = g'(x) for all real numbers x is an equivalence relation.

(b) Which functions are in the same equivalence class as the function f(x) = x²?

Answer :

Let, A be the set of all differentiable functions from R to R.

$R = \{(f, g) \mid f'(x) = g'(x)\}$ for all $x \in R$.

(a) Reflexive

Let, $f \in A$

Since $f'(x) = f'(x)$ for all $x \in R$

$\Rightarrow \quad (f, f) \in R$

$\therefore \quad R$ is reflexive.

Symmetric

Let $(f, g) \in R$.

Since $f'(x) = g'(x)$ for all $x \in R$

$\Rightarrow \quad g'(x) = f'(x)$

$\Rightarrow \quad (g, f) \in R$

$\therefore \quad R$ is symmetric.

Transitive

Let $(f, g) \in R$ and $(g, h) \in R$

Here,

$f'(x) = g'(x)$, for all $x \in R$

$g'(x) = h'(x)$, for all $x \in R$

$\Rightarrow \quad f'(x) = g'(x) = h'(x)$, for all $x \in R$

$\Rightarrow \quad (f, h) \in R$

$\therefore \quad R$ is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation on A.

(b) Given function is,

$f(x) = x^2$

Derivative is given by,

$f'(x) = 2x$

The equivalence relation of x contains all functions in the set A that also contains $f'(x)$.

Then,

$[f]_R = \{g \in A \mid g'(x) = 2x\}$

If $g'(x) = 2x$ then $g(x) = x^2 + C$ where C is any real number.

$[f]_R = \{g \in A \mid g(x) = x^2 + C \text{ and } C \in R\}$

Q107. Determine whether the relation represented by the zero-one matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is an equivalence relation or not.

Answer :

Given matrix is,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Reflexive

The matrix A is said to be reflexive, because it contains only 1's as the diagonal elements.

Symmetric

The given matrix A is not a symmetric matrix.

Transitive

The matrix A is not a transitive matrix, because $A^{[2]} \neq A$.

i.e., $A^{[2]} = A \odot A$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) \\ (0 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) \\ (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 0 \vee 1 & 1 \vee 1 \vee 1 & 1 \vee 1 \vee 1 \\ 0 \vee 0 \vee 1 & 0 \vee 1 \vee 1 & 0 \vee 1 \vee 1 \\ 1 \vee 0 \vee 1 & 1 \vee 1 \vee 1 & 1 \vee 1 \vee 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\Rightarrow \quad A^{[2]} \neq A$

$\therefore$ The given matrix is not an equivalence relative since it is not symmetric and transitive.

Q108. Define partial ordering and total ordering of relations.

Answer :

Partial Ordering Relation

If S is any set on which the binary relation R is defined such that R is reflexive, antisymmetric and transitive then R is called partial ordering relation (or) partial order. Partial ordering relation is denoted as (S, R) where R is partial ordering on S, called as Partially Ordered Set (or) Poset.

Examples

1. On the set of integers the relations 'greater than or equal to' and 'less than or equal to' are partial ordering relations.
2. Inclusion relation is a partial ordering on any power set P.

Total Ordering Relation

If $(S, \leq)$ represents a partially ordered set, then S is said to be totally ordered (or) linearly ordered on S when $a, b \in S$ are comparable.

Q109. Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.

(a) {(0, 0), (2, 2), (3, 3)}

(b) {(0,0), (1, 1), (2, 0), (2, 2), (2, 3) (3, 3)}

Answer :

Given set is,

$A = \{0, 1, 2, 3\}$

(a) $R = \{(0, 0), (2, 2), (3, 3)\}$

Reflexive

R is not reflexive because $(1, 1) \notin R$ but $1 \in A$.

Antisymmetric

R is antisymmetric because $(a, b) \in R$ and $(b, a) \in R$ while $a = b$ for every element in A.

Transitive

R is transitive because $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for every element in A.

$\therefore$ R is not partially ordering as it is not reflexive.

(b) $R = \{(0, 0), (1, 1), (2, 0), (2,2), (2, 3), (3, 3)\}$

Reflexive

R is reflexive because $(a, a) \in R$ for every element $a \in A$.

Antisymmetric

R is antisymmetric because $(a, b) \in R$ and $(b, a) \in R$ while $a = b$ for every element in A.

Transitive

R is transitive because $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for every element in A.

$\therefore$ R is a partial ordering relation as it is reflexive, antisymmetric and transitive.

Q110. Write short notes on Hasse diagram and explain the procedure for drawing Hasse diagram.

Answer :

Hasse Diagram

If $(A, \leq)$ is a partial order set, then any $b \in A$ is said to cover $a \in A$ if $a < b$ and there is no other element c is such that $a < c$, i.e., b covers $a \Leftrightarrow (a < b) \wedge (a \leq c \leq b \Rightarrow a = c \text{ or } b = c)$.

Procedure for Drawing a Hasse Diagram

- The elements of partial order set are represented using dot or circles.
- For any two element a, b if $a < b$, then a is represented below b and a line is used for joining a and b if b covers a.
- If $a < b$ but b does not cover a then a and b are not connected directly using a line but are connected using other elements of partial order set.

Q111. Draw the Hasse diagram for divisibility on the set {1, 2, 3, 4, 5, 6}.

Answer :

Given set is,

$$A = \{1, 2, 3, 4, 5, 6\}$$

Let, R be the divisibility on the set A.

Step 1

The directed graph R needs to contain loops at every vertex, because an element is always divisible by itself.

If a is divisible by b, then draw an arrow from integer a to integer b.

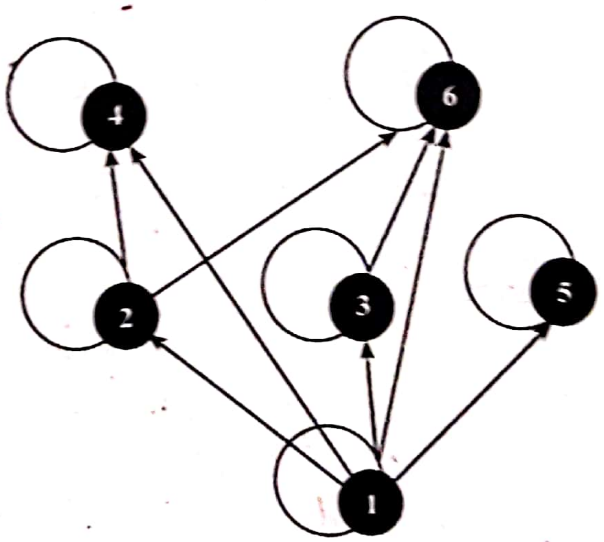


Figure (1)

Step 2

Remove all loops since loops are always present in partial order. Here *R* is a partial order.

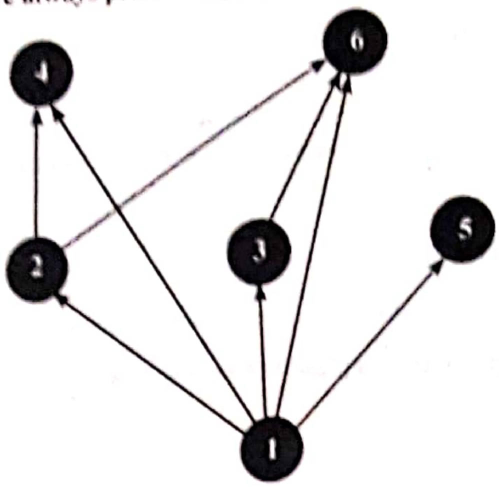


Figure (2)

Step 3

Then, remove all arrows on the directed edges since edges are derived from the transitivity.

Hence, the corresponding Hasse diagram can be given as,

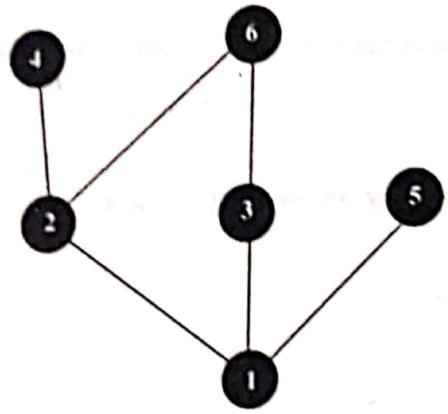


Figure (3)

Q112. Let A be a given finite set and P(A) its power set. Let be the inclusion relation on the elements of P(A). Draw Hasse diagram of (P(A), ⊆) for,

(i) A = {a}

(ii) A = {a, b}

(iii) A = {a, b, c}

(iv) A = {a, b, c, d}.

Answer :

(i) A = {a}

The Hasse diagram of $(P(A), \subseteq)$ is shown below.

Figure (1)

(ii) $A = \{a, b\}$

The Hasse diagram of $(P(A), \subseteq)$ is shown below.

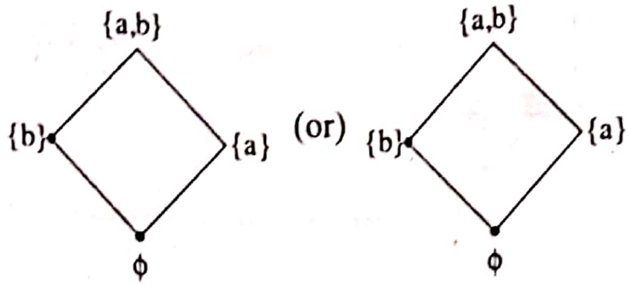


Figure (2)

(iii) $A = \{a, b, c\}$

The Hasse diagram of $(P(A), \subseteq)$ is shown below.

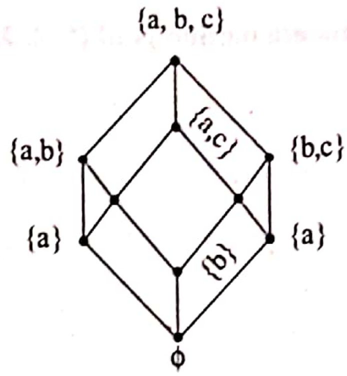


Figure (3)

(iv) $A = \{a, b, c, d\}$

The Hasse diagram of $(P(A), \subseteq)$ is shown below.

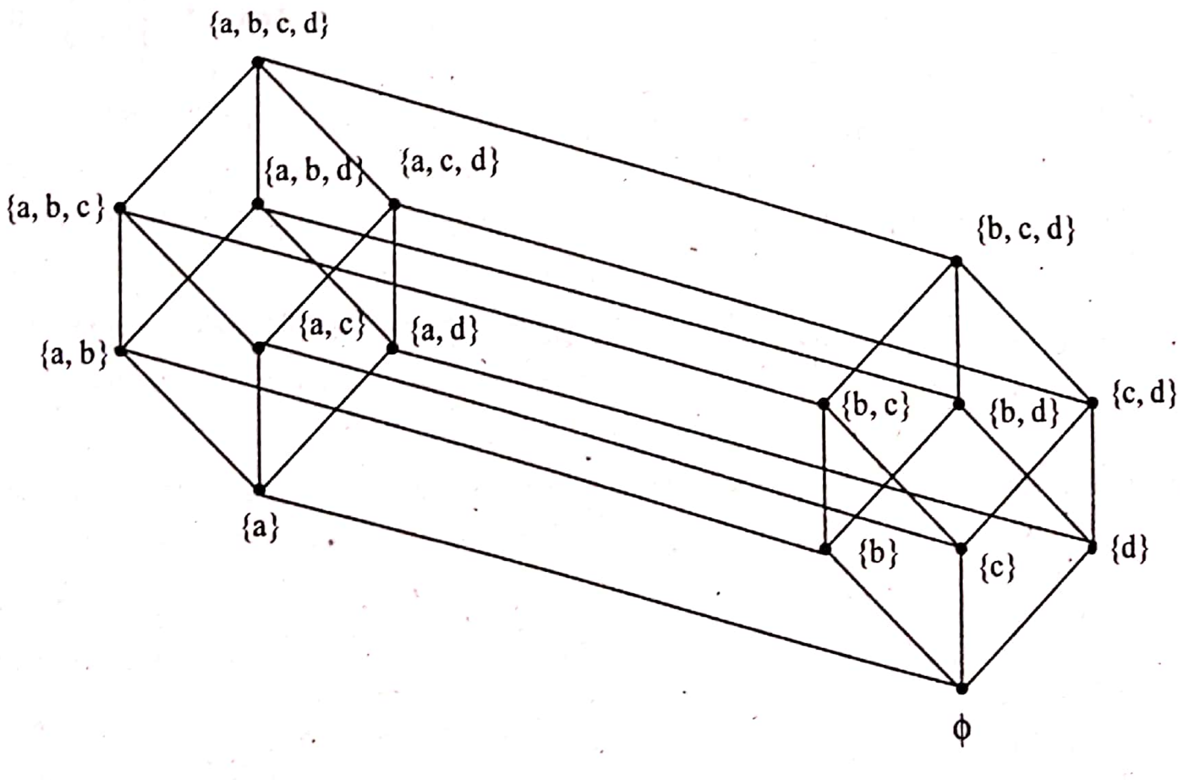


Figure (4)

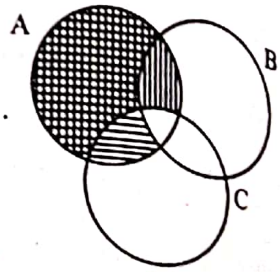
EXERCISE QUESTIONS

1. Let A = {a, b, c d} and B = (y, z}. Find A × B.

Ans : {(a, y), (b, y), (c, y), (d, y), (a, z), (b, z), (c, z), (d, z)}

2. Draw the Venn diagram for $(A \cap \bar{B})$ $(A \cap \bar{C})$.

Ans : .



3. Find the value of the sum $\sum_{j=0}^{4}(-2j)$.

4. Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}?

(a) {1, 2}, {2, 3, 4}, {4, 5, 6}

(b) {1}, {2, 3, 6}, {4}, {5}

(c) {2, 4, 6}, {1, 3, 5}

(d) {1, 4, 5}, {2, 6},

Ans : (a) No. (b) Yes (c) Yes (d) No

5. Find the join and meet of the zero-one matrices

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Ans : $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$