5
14
30
55

3 - 14

Basic Strutures

The discrete structures are sets, functions, sequences, summations, matrices.

Sets

set is an unoder collection of objects. Each object in a set is called member/ or element of the set.
The elements are written in flower brackets and separated by comma.

Membership of a set:

The symbol $\in$ indicates the membership in a set if $a \in A$ denote a is an element of set 'A'. and $a \notin A$ denote that a is not a element of set 'A'.

Example → The set ~~V~~ of all vowels in the english ~~apt~~ alphabet can be written as:

$$V = \{a, e, i, o, u\}.$$

→ The set O of odd positive integers less than 10 can be expressed by

$O = \{1, 3, 5, 7, 9\}$.

Cardinality of Sets :

The no. of elements in a set is called cardinality of set. It is denoted with $|A|$.

where A is a set.

Representation of sets

There are 2 methods to represent a set.

1) Tabular or Roster Representation.

A set is defined ~~by~~ by listing of its elements

Ex:- $\{1, 2, 3, 4, 5\}$.

2) Set builder representation

Specify the properties for the elements of the set.

$A = \{x \mid x \in N \text{ and } x < 6\}$

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Types of sets

i) Subset: A and B are 2 sets. A is a subset of B if and only if every element of A is also a element of B. It is denoted with $A \subset B$

Note :- A set "A" is always a subset of "A". "A" is not a subset of "B" can be represented by "$A \not\subset B$".

ii) Equal sets: If A and B are 2 sets $A \subseteq B$ equals to $B \subseteq A$ then A and B are equal sets. It is denoted by $A = B$.

iii) Proper subset: A set "A" is called proper subset of the set "B". If "A" $\subset$ "B" and $B \not\subset A$. Then "A" is ~~ca~~ said to be Proper subset of B.

Ex:- $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$.

iv) Super Set :- If "A" is subset of B then B is called a super set of A.

v) Null Set :- The set with no elements is called a empty set or Null set.
The Null set is denoted by ϕ.

vi) Universal Set :- A set that contains all sets in a given context is called a Universal set. It is denoted with U/E/μ.

NOTE :- The Null set is subset for every set and universal set is superset for every set.

vii) Infinite Set :- A set is said to be infinite if it is not finite.
Ex:- The set of all positive integers.

viii) Power Set :- If "A" is set then set of all the subsets of "A" is called the power set of "A". It is denoted by P(A). If A have n elements then P(A) has 2^n elements

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Q) What is the power set of a set $\{0,1,2\}$

Sol:- $2^n = 2^3 = 8$

$\{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$

Q) What is the power set of empty set.

Sol:- $\{\phi\}$ $\quad 2^0 = 1$

Disjoint Sets.

Two sets are said to be disjoint. If they have no ~~com~~ element in common.

Single ~~ton~~ ton set:-

A set contains a single element is called Single ton.

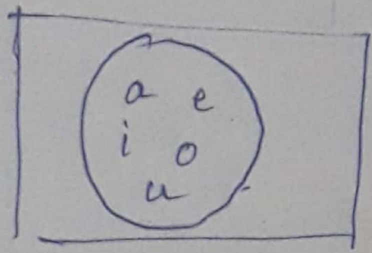
NOTE :- The cardinality of null set is zero.

The cardinality of Single ton is one.

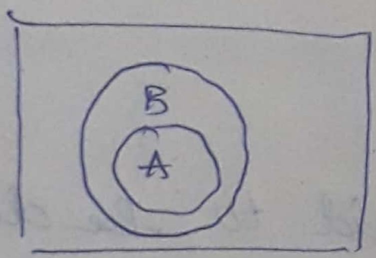
Q) 1.
Draw a Ven diagram for V the set of vowels in English alphabets.

2.
Q) "A" is a subset of B ($A \subset B$) draw the Ven diagram.

1 Sol :-



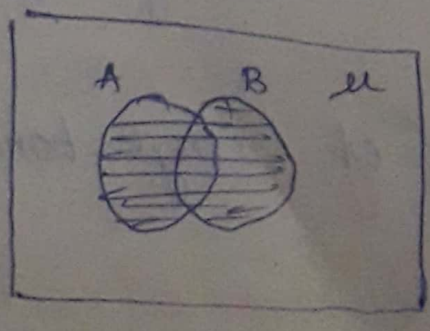
2 Sol :-



Set operations

1. Union of two sets :- the union of two sets A & B is the set the elements are all the elements in A or B. The union of sets A & B denoted by $A \cup B$

$A \cup B = \{x \mid x \in A \lor x \in B\}$



let A = {1, 2, 3} B = {4, 5, 6} the A ∪ B is the {1, 2, 3, 4, 5, 6}.

2. Intersection :- Intersection of 2 sets A and B is the set whose elements are all of the elements common to both A and B. The Intersection of A and B is

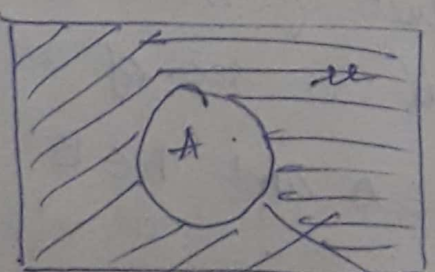
~~3. The is~~ A ∩ B.

$$A \cap B = \{x \mid x \in A \wedge x \in B\}.$$

let A = {1, 2, 3} and B = {2, 3, 4} ~~is the~~ then A ∩ B is the {2, 3}.

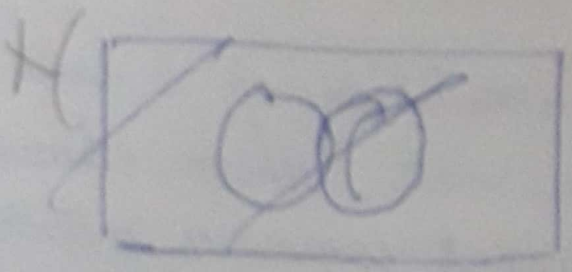
3. Compliment :- let A is a set the elements which are not belongs to A is called the complement of A. It is denoted by $\bar{A}$/A'/A^c.

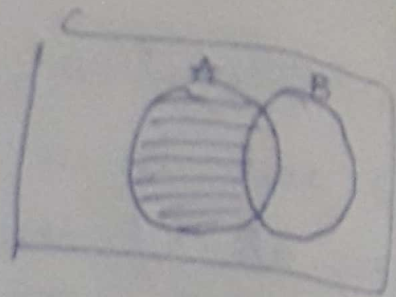
$$\bar{A} = \{x / x \notin A\}.$$



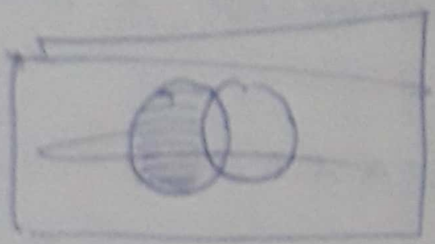
4. Difference :- let A and B be sets. the difference of A and B denoted by A-B is the set contains elements that are in A not be. The difference or A-B is called the ~~B~~ compliment of B ($\bar{B}$) w.r.t A.

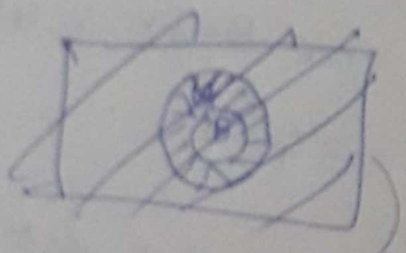
$A - B = \{x \mid x \in A \wedge x \notin B\}$





correct





X wrong

Let $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$ then $A - B$ is $\{5\}$.

5. Carstian product :- Let A and B be sets the carstian product of A and B is denoted by ~~set~~ $A \times B$ is a set of all order pairs (a, b) where $a \in A$ and $b \in B$.

What is carstian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$. $A \times B$?

Sol :- $\{\{1, a\}, \{1, b\}, \{1, c\}, \{2, a\}, \{2, b\}, \{2, c\}\}$

$B \times A$

$\{\{a, 1\}, \{a, 2\}, \{b, 1\}, \{b, 2\}, \{c, 1\}, \{c, 2\}\}$

NOTE :- The cartisan $A \times B$ and $B \times A$ are not equal.

2. What is the certian product of $A \times B \times C$ where A is $= \{0, 1\}$ $B = \{1, 2\}$ $C = \{0, 1, 2\}$.

$\{\{0,1,0\}, \{0,1,1\}, \{0,1,2\}, \{0,2,0\}$
$\{0,2,1\}$ $\{0,2,2\}$ $\{1,1,0\}$ $\{1,1,1\}$
$\{1,1,2\}$ $\{1,2,0\}$ $\{1,2,1\}$ $\{1,2,2\}\}$

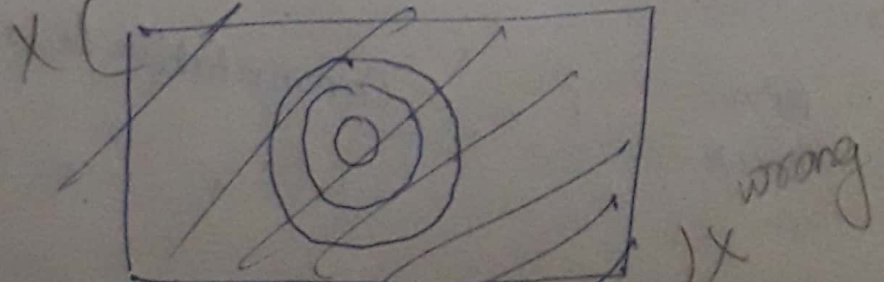
NOTE :- A set A hers m elements and B has n elemente then the cardinality of $A \times B$ is mn.

3. Let $A = \{a, e, i, o, u\}$ ~~where A~~ ~~and the~~ and ~~is~~ the universal set ~~of~~ is the all english alphabets.

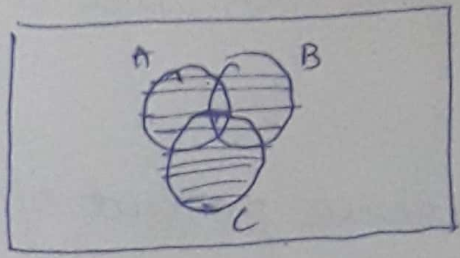
sol :- $\bar{A} = \{b, c, d, f, g, h, j, k, l, m, n$
$p, q, r, s, t, u, v, w, x, y, z\}$

4. Draw the ven diagrams of $A \cap B \cup C$ and $A \cap B \cap C$.

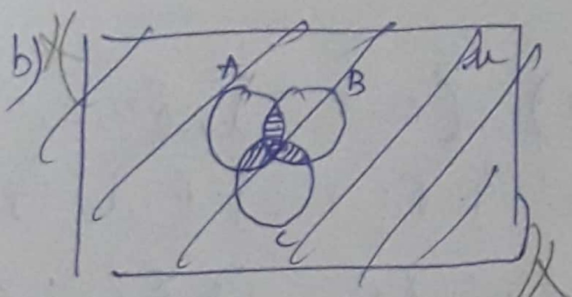
sol :-



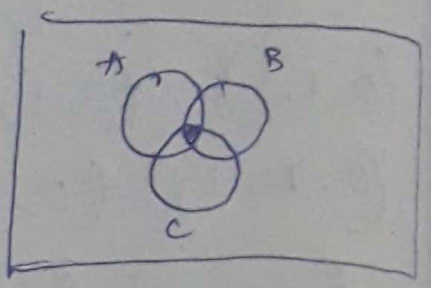
a)



b)



wrong



NOTE :- Two sets are called ~~are~~ disjoint if the intersection is empty set.

Set Identities

Identity	Name
$A \cup \phi = A$ $A \cap U = A$	Identity law
$A \cup U = U$ $A \cap \phi = \phi$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent law
$\overline{\overline{A}} = A$	complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative law

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$ } → Associative law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive law

---

$\overline{A \cup B} = \bar{A} \cap \bar{B}$
$\overline{A \cap B} = \bar{A} \cup \bar{B}$ Demorgon's law

$A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$ Absorption law

$A \cup \bar{A} = U$
$A \cap \bar{A} = \phi$ complement law

Q) ~~St~~ Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

T.P ⇒ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

L.H.R $\overline{A \cap B}$

⇒ $\{x \notin A \cap B\}$ By the definition complement.

⇒ $\{\neg(x \in A \cap B)\}$

⇒ $\{\neg(x \in A \wedge x \in B)\}$ By the definition of intersection.

⇒ $\{\neg(x \in A) \vee \neg(x \in B)\}$ by the $\neg(p \wedge q) = \neg p \vee \neg q$

⇒ $\{x \notin A \vee x \notin B\}$ by the definition of compliment.

⇒ $\{\bar{A} \cup \bar{B}\}$ by the union definition.

Q)

Prove the distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for all sets

A, B and C.

Sol:- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$

$= \{x \in A \wedge x \in (B \cup C)\}$ By the intersection def.

$= \{x \in A \wedge (x \in B \vee x \in C)\}$ By the union defin

$= \{[(x \in A \wedge x \in B)] \vee [(x \in A) \wedge (x \in C)]\}$ $p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r)$

$\Rightarrow \{x \in (A \cap B) \vee x \in (A \cap C)\}$ By the def intersection

$\Rightarrow (A \cap B) \cup (A \cap C)$ By the defination of union.

Computer representation of sets.

Assume that the universal set U is finite specifying the order of element of U for ~~an~~ instance $a_1, a_2, \ldots a_n$. Represent a subset a of U with the bit string of length where i^{th} bit in the string is one. If ~~a(i)~~ $a_i \in A$ and is zero $a_i \notin A$.

Example:- Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the ordering of elements U has the elements in increasing order. What are bit string to represent all odd integers in U.

→ The subset of all even integers in U and the subset of integers not exceeding 5.

Sol:- i) $\{1, 0, 1, 0, 1, 0, 1, 0, 1, 0\}$.

ii) $\{0, 1, 0, 1, 0, 1, 0, 1, 0, 1\}$.

iii) $\{1, 1, 1, 1, 1, 0, 0, 0, 0, 0\}$.

Q) What is bit string for the compliment of the set. the set is = {1, 3, 5, 7, 9} with universal set = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Sol:-

$A = \{1, 3, 5, 7, 9\}$

$\bar{A} = \{2, 4, 6, 8, 10\}$.

$\bar{A} = 0101010101$

Note:-

The bit string for union is bit-wise OR of the bit string for the 2 sets.

The bit string for intersection is bit-wise AND of the bit strings for the 2 sets

Q) The bit strings for the sets {1,2,3,4,5} and {1,3,5,7,9} are 1111100000 and 1010101010. use ~~the~~ bit strings to find union and intersection of sets.

Sol:- Union (OR operation).

$1111100000 \vee 1010101010$.

$= 1111101010$

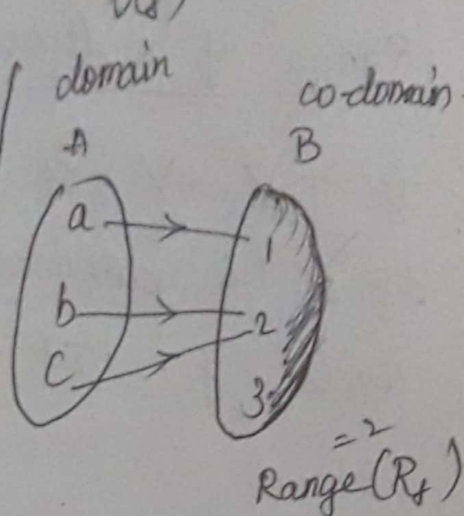
i.e $= \{1, 2, 3, 4, 5, 7, 9\}$.

ii) Intersection (And operation).
1111100000 $\wedge$ 1010101010.

1010100000.

$\Rightarrow \{1, 3, 5\}$.

DQ)



1 is image of a
a is ~~pre-image~~ of 1

__functions:-__

Let X and Y be 2 not empty sets. A relation $f: X \to Y$ is called a funtion if ~~every~~ $x \in X$ there is a unique element $y \in Y$ such that $(x, y) \in f$.

A function requires a relation must statisfy 2 conditions.

i) $\forall x \in X$ must be relate to some $y \in Y$.

ii) uniqueness
$(x, y) \in f$ and $(x, z) \in f$ then $y = z$

A function $f: X \to Y$ the set X is called domain of a function ~~to~~ d and set Y is called co-domain of a function

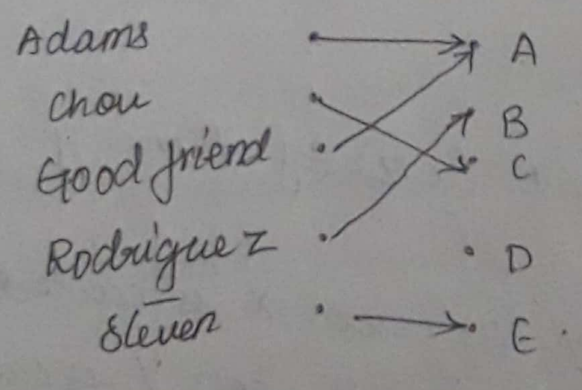
The domain of function f is denoted by $D(f)$

In a function $f: X \to Y$ then $f(x) = y$ where $x \in X$ & $y \in Y$ then y is called the image of x and x is called the pre-image of y.

Range of a function

If $f: X \to Y$ is a function then the range of x is defined as the set of all images under f. It is denoted by 'R_f'.

Q) Write are the domain, co-domain and range of function that assign grades of the student in the following :-



Sol:-

Domain :- Adams, chou, Good friend, Rodriguez, Steven

Codomain : A, B, C, D, E

Range : A, B, C, E (4)

Let f_1 and f_2 be the functions from $A \to R$ such that $f_1 + f_2$ and $f_1 f_2$ are also the functions from $A \to R$ defined by

$$\boxed{\begin{aligned} f_1+f_2(x) &= f_1(x)+f_2(x) \\ f_1 f_2(x) &= f_1(x)\cdot f_2(x). \end{aligned}}$$

Q) Let f_1 and f_2 be the $f: R \to R$ such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are function $f_1 + f_2$ and $f_1 f_2$.

Sol:-

$f_1(x) = x^2$

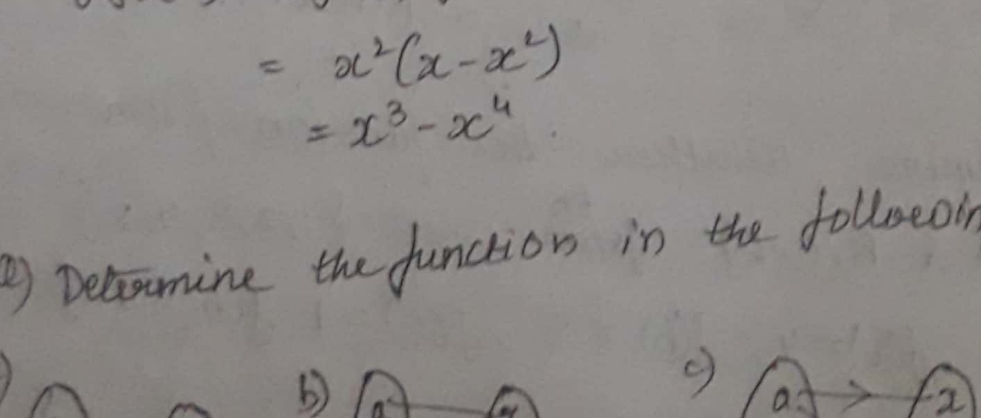
$f_2(x) = x - x^2$.

$$f_1+f_2(x) = f_1(x) + f_2(x)$$
$$= x^2 + (x - x^2)$$
$$= x.$$

$$f_1 f_2(x) = f_1(x)\cdot f_2(x).$$
$$= x^2(x - x^2)$$
$$= x^3 - x^4.$$

Q) Determine the function in the following.

a) b) c)



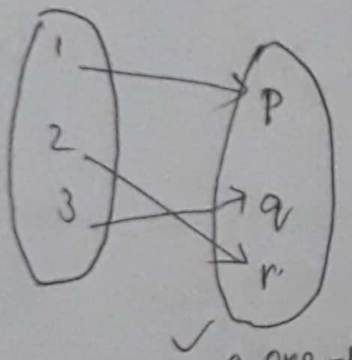
X not a fun ✓ It is fun X not a fun

Types of functions

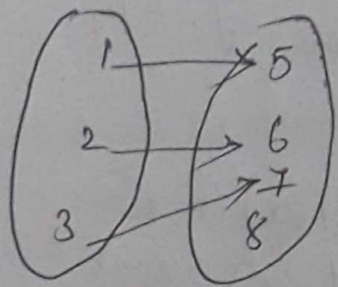
① one-to-one / Injective
② onto / surjective
③ Bi-jective / one to-one correspondance

① One-to-one / Injective :- A function $f: A \to B$ is called one-to-one if every element of A is exactly mapped to one-element of B.

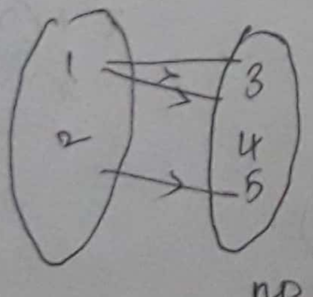
Ex:-



✓ a one-to-one func



✓ a one-to-one func.



✗. not a function.

Q) determine whether the function f from set {A, B, C, D} ~~from~~ to set {1, 2, 3, 4, 5} with $f(a) = 4$ $f(b) = 5$, $f(c) = 1$ $f(b) = 5$, $f(c) = 1$ $f(d) = 3$.

A function f is one-to-one because

f takes on different values of the four elements of its domain.

Q) determine whether the fun $f(x)=x^2$ from the set of Integers to the set of integers is one-to-one.

Sol:- The function $f(x)=x^2$ is not one-to-one because $f(1)=1$ and $f(-1)=1$.

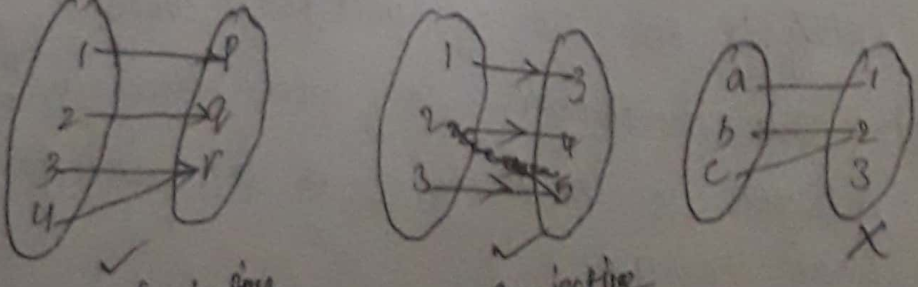
Q) determine whether the function $f(x)=x+1$ from the set of real no.'s to itself is one to one.

Sol:- The function $f(x)=x+1$ is one-to-one function because $x+1 \neq y+1$ when $x \neq y$.

Onto/ surjective

A function $f: A \rightarrow B$ is called onto if every element of B is mapped to some element of A.

Exs:-

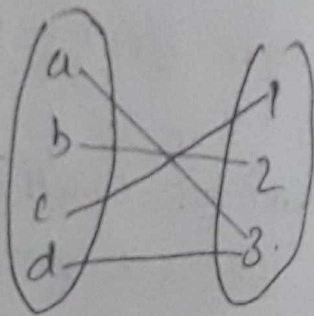


✓ Surjective Surjective ✗

NOTE:- In this mapping the codomain and range is same.

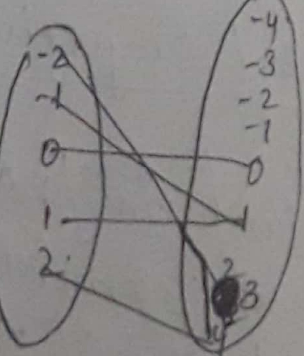
Q) let f be a function from $\{a, b, c, d\}$ to set $\{1, 2, 3\}$ difined by $f(a)=3$, $f(b)=2$, $f(c)=1$, $f(d)=3$. is f is an onto fun?

sol:- The function f is onto because all the 3 elements of the co-domain are images of elements in domain.



Q) is the function $f(x) = x^2$ from the set of integers to set of integers onto..

sol:- The fun $f(x)=x^2$ is not onto because there is no integer x with $x^2 = -1$



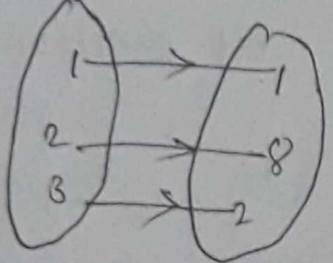
Q) Is the function $f(x) = x+1$ from the set of integers to set of integers onto?

Sol:- This function is onto because for every integer y there is an integer x such that $x = y$.

Bi-jective

A function which satisfies injective mapping and surjective mapping then that func is said to be Bi-jective or one-to-one correspondance.

Ex:-



Bi-jective.

Q) Let f be the func from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a)=4$, $f(b)=2$, $f(c)=1$, $f(d)=3$ is a bijective.

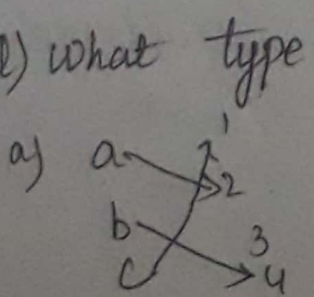
Sol

The function ~~f(x)~~ f is one-to-one and onto.

It is one-to-one because no-two values in the ~~it~~ ~~onto~~ domain are assigned the same func value. It is onto because all the 4 elements of to co-domain are images of the elements in the domaine. Hence it is a bijective

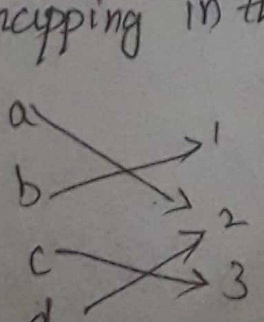
Q) what type of mapping in the following.

a)



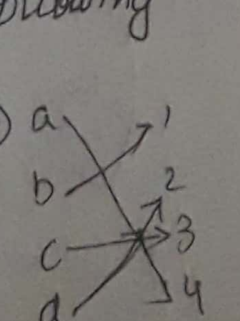
One-one

b)



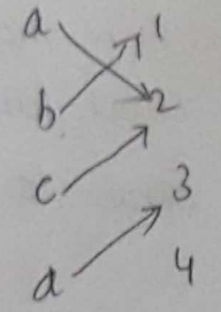
bi-jective.

c)



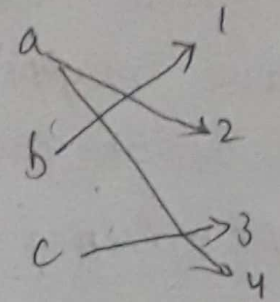
bijective.

d)



one - to - one

e)

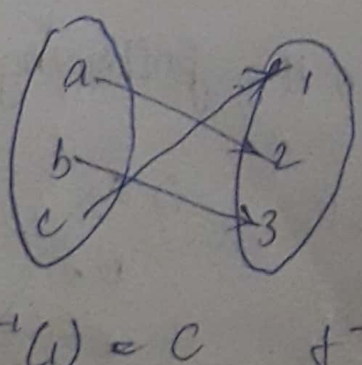


not function.

Inverse functions :- let f be a one to one correspondance from the set A to the set B. The inverse function of f is the fun that assign to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse fun of f is denoted by f^{-1} hence $f^{-1}(b) = a$ when $f(a) = b$.

Ex:- let f be the fun from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, $f(c) = 1$. Is f invertiable and if it is what is its invertiable.

sol :-



It is one-one and onto so it is invertiable

$f^{-1}(1) = c$ $\quad f^{-1}(3) = b$

$f^{-1}(2) = a$

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Composition function: let g be a fun from set A to set B and let f be a fun from set B to set C. The composition of the functions f and g denoted by $f o g$ is defined by

$(fog)(a) = f(g(a))$. In other words fog is a fun that assigns to the function a to the element $a(A)$, the element assign by a to $g(a)$. i.e to find $fog(a)$ we apply the fun $g \to a$ to obtain $g(a)$ and then we apply the fun $g \to a$ to obtain $g(a)$ and then we apply f to the result $g(a)$ to obtain

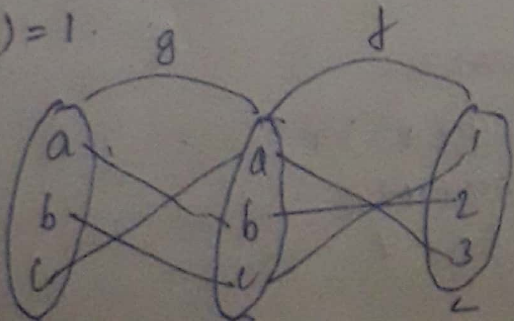
$f(g(a)) = fog(a)$

NOTE :-

The composition fog cannot be define unless the range of g is a subset of domain f.

Eg: let g be the function from set $\{a, b, c\}$ to itself. Such that $g(a) = b$, $g(b) = c$, $g(c) = a$

let f be the function from set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, $f(c) = 1$.



What is the composition of $f \circ g$ and $g \circ f$

Sol:- $f \circ g(a) = f(g(a)) = f(b) = 2$.

$f \circ g(b) = f(g(b)) = f(c) = 1$

$f \circ g(c) = f(g(c)) = f(a) = 3$.

$$\left.\begin{aligned} g \circ f(a) &= g(f(a)) = g(3) \\ g \circ f(b) &= g(f(b)) = g(2) \\ g \circ f(c) &= g(f(c)) = g(1) \end{aligned}\right\} \text{not possible.}$$

Note that $g \circ f$ is not define because the range of f is not a subset of domain of g

Q) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$. What is the composition of $f \circ g$ and $g \circ f$

$x = 2x+3$

Sol:- $g \circ f = g(f(x)) = g(2x+3) = 3(2x+3)+2$

$= 6x+9+2$

$= 6x+11$

$f \circ g = f(g(x)) = 2x+3$

$= 2(3x+2)+3$

$= 6x+4+3$

$= 6x+7$.

Important functions

There are 2 imp functions known as floor and ceiling.

Let x be a real no. The floor function rounds x down to the closest integer less than or equal to x and it is denoted by $\lfloor x \rfloor$

ceiling fun rounds x up to the closest integer greater than or equal to x and it is denoted by $\lceil x \rceil$. These func are used when objects are wanted.

Ex:- $\lfloor \frac{1}{2} \rfloor = 0$

$\lceil \frac{1}{2} \rceil = 1$

$\lfloor \frac{-1}{2} \rfloor = -1$

$\lceil \frac{-1}{2} \rceil = 0$

$\lfloor 3.1 \rfloor = 3$

$\lceil 3.1 \rceil = 4$

The floor & ceiling function are useful in a wide variety of appilcations including those involving data storage and data transmition.

Eg:- data stored on a computer disk or transmited over a data network are usually reported as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data

Sol:- To determine the no. of bites needs we de

$\lceil 100 \div 8 \rceil = \lceil 12.5 \rceil = 13$ bytes.

Sequences and Summations

A sequence a discrete structure used to represent an ordered list.

Eg:- 1, 2, 3, 4, 5 is a sequence with 5 terms.

We use the notation a_n to represent sequence.

Ex:- Consider the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$

Sol:- The list of the terms of this sequence begining with $a_1, a_2, a_3, a_4 \ldots a_n$ starts with

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n}$

Geometric progression :-

* G.P is the sequence of the form a, ar, ar^2, ar^n where the initial term 'a' and the common ratio 'r' are real numbers.

Ex:- The sequence $\{b_n\}$ with $b_n = (-1)^n$, $\{c_n\}$ with $c_n = 2 \times 5^n$ and $\{d_n\}$ with $d_n = 6 \times (\frac{1}{3})^n$ are geometric progression with initial term and common ~~term~~ ratio equal to $\underline{a=1}$ and $r = -1$

$\underline{2}$ and $\underline{5}$

$\underline{6}$ and $\underline{1/3}$

and begins (starts) with $n=0$

$b_n = (-1)^n \Rightarrow (-1)^0 = +1$ $\quad 2 \times 5^n = 2 \times 5^0 = 2$ $\quad 6 \times (\frac{1}{3})^0 = 6$

$(-1)^1 = -1$ $\quad 2 \times 5^1 = 10$ $\quad 6 \times (\frac{1}{3})^1 = 2$

$(-1)^2 = 1$ $\quad 2 \times 5^2 = 50$ $\quad 6 \times (\frac{1}{3})^2 = \frac{2}{3}$

$(-1)^4 = 1$ $\quad 2 \times 5^3 = 250$ $\quad 6 \times (\frac{1}{3})^3 = \frac{2}{9}$

$(-1)^5 = -1$ $\quad 2 \times 5^4 = 1250$ $\quad 6 \times (\frac{1}{4})^4 = \frac{2}{27}$

$2 \times 5^5 = 6250$

Arithmetic progression :-

An arithmetic progression is a sequence of a form $a, a+d, a+2d, \cdots a+nd$. where the initial term 'a' and the common difference 'd' are real numbers.

Ex:- The sequence $\{s_n\}$ with $s_n = -1+4n$

$\{t_n\}$ with $t_n = 7-3n$ are an arithmetic progression with initial term and common differences equals to $\underline{-1}$ and $\underline{4}$, $\underline{7}$ and $\underline{-3}$.

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Starts with $n = 0$

$-1+4n = -1+4(0)$
$= -1$
$-1+4(1) \Rightarrow 3$
$-1+4(2) = 7$
$-1+4(3) = 11$
$-1+4(4) = 15$
$-1+4(5) = 19$

$7-3n \Rightarrow 7-3(0) = 7$
$7-3(1) = 4$
$7-3(2) = 1$
$7-3(3) = -2$
$7-3(4) = -5$
$7-3(5) = -7$.

Special integer sequences :-

Find formulae for the sequences with the following first 5 terms.

a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \rightarrow a_n = 1/2^n \Rightarrow$ where $n = 0, 1, 2, \ldots$

b. $1, 3, 5, 7, 9 \rightarrow a_n = 1+2n$ where $n = 0, 1, 2, \ldots$

c. $1, -1, 1, -1, 1, -1 \rightarrow a_n = (-1)^n$ where $n = 0, 1, 2 \ldots$

Q) construct a simple formulae for a_n if the 1st 5 terms of the sequence 'a_n' are $1, 7, 25, 79, 2$

Sol:- $a_n = 3^n - 2$ where $n = 1, 2, 3, 4, 5$.

Summations :-

The notation used to express the sum of the terms $a_m, a_{m+1}, \ldots a_n$ from the sequence $\{a_n\}$ we use the notation $\sum_{j=m}^{n} a_j \,/\, \sum_{j=m}^{n} a_j$ to represent $a_m + a_{m+1} + \ldots \ldots + a_n$.

Here the variable j is called index of summation and the choose of letter j as the variable is arbitary (change).

Here the index of summation runs through all integers starting with its lower limit m

and ending with its upper limit n.
a large uppercase greek letter sigma is used to denote summation.

Ex: 1) Express the sum of first 100 terms of the sequence.

$\{a_n\}$ where $a_n = \frac{1}{n}$ for $n = 1, 2, 3 \ldots$

sol:- $\Rightarrow \sum_{i=1}^{100} \frac{1}{i}$ where $i = 1, 2, 3, \ldots 100$

2) What is the value of $\sum_{j=1}^{5} j^2$

$j^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25$

$\Rightarrow 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$.

3) What is the value of $\sum_{k=4}^{8} (-1)^k$

$(-1)^4 = 1,\ (-1)^5 = -1,\ (-1)^6 = 1,\ (-1)^7 = -1,\ (-1)^8 = 1$

$\cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1 = 1$

4) H.W find $\sum_{k=50}^{100} k^2$ $\left[\text{formulae } \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \right]$

sol $\sum_{k=1}^{100} k^2 - \sum_{k=1}^{49} k^2 = \sum_{k=50}^{100} k^2$

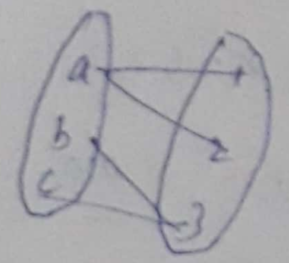
$\Rightarrow 338350 - 40425$

$= 297925$.

Relations :-

Let A and B be sets. a binary relation from A to B is a sub set of $A \times B$

Ex :-



(a,1)
(a,2)
(b,3), (c,3) → A, B are relations and subsets of $A \times B$.

$A \times B = \{(a,1), (a,2), (a,3), (b,1) (b,2), (b,3), (c,1) (c,2), (c,3)\}$

Q) let A be the set {1,2,3,4} which ordered pairs are in the relation R = {(a,b)/a divides b}

Sol :- A {1, 2, 3, 4}

⇒ {(1,2) (1,3) (1,4) (1,1) (2,2) (2,4) (3,3) (4,4)}

Properties of Relations :-

1) Reflexive - A relation R on a set A is called reflexive if (a,a) belongs to R for every element $a \in A$.

Q) consider the following relations on {1, 2, 3, 4}

$R_1 = \{(1,1) (1,2) (2,1) (2,2) (3,4) (4,1) (4,4)\}$

$R_2 = \{(1,1) (1,2) (2,1)\}$

$R_3 = \{(1,1) (1,2) (1,4) (2,1) (2,2) (3,3) (4,1) (4,4)$

Sol:- R_3 is reflexive coz it contains ordered pairs. i.e $(1,1)$ $(2,2)$ $(3,3)$ $(4,4)$.

2) irReflexive - A relation R on a set A is said to be reflexive if $(a \in A)$, and $(a,a) \notin R$. It is also called as anti Reflexive relation.

Q) consider $a \in A$, $(a,a) \notin R$

$A = \{1, 2, 3, 4\}$

R_1 $\{(1,2)(2,1)(3,3)(4,4)\}$

R_2 $\{(1,2)(1,3)(2,4)\}$

Sol:- R_2 is irreflexive.

3) Symmetric relation - let R be a relation in set A. then R is said to be symmetric relation if $(a,b) \in R$ & $(b,a) \in R$

Ex:- A $\{1, 2, 3, 4\}$

R_3 $\{(1,1)(2,2)(3,3)(4,4)\}$

R_1 $\{(1,2)(2,1)(3,4)(4,3)\}$

R_2 $\{(1,3)(3,1)(3,4)(4,4)\}$

$\therefore$ R_1, R_3 are Symmetric.

4) Anti Symmetric - A relation R on a set A said to be anti symmentric if $(a,b) \in R$ and $(b,a) \in R$ then $a=b$.

→ their should be ordered pair (reflexive)

→ $(a,b) \in R$ and $(b,a) \notin R$ then it is a anti symmetric

Ex:- A {1,2,3}

R_1 {(1,1)(2,2)} ✓ AS

R_2 {(1,1) (2,2) (3,1)} AS

R_3 {(1,1), (2,2) (3,1) (1,3)} x.

5) Transitive Relation - a relation R on a set A is called transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $(a,b,c) \in A$.

A {1, 2, 3, 4}

R_2 {(1,1)(1, 2) (1, 3)(1,4)(2,2)(2,3) (2,4) (3,3)(3,4)(4,4)}

R_1 {(2,1) (3,1)(3,2)(4,1)(4,2)(4,3)}

∴ R_2 is Transitive

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