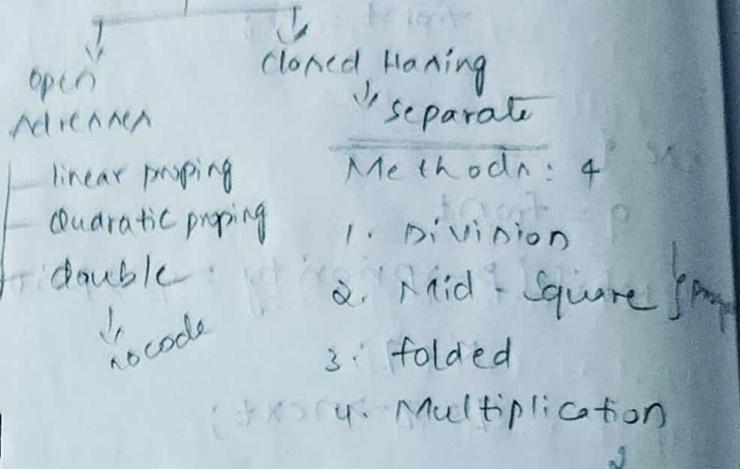


```
else if  
    p = front;  
    front = front->next;  
    free(p);
```

Collision Reduction Technique



1. Hashing
2. Trees
3. Re Hashing
4. Extensible hashing

- Hash function
- Hash methods
- collision resolution techniques
 - linear probing
 - potratic probing
 - seperate change
 - double hashing

Trees :

→ telecom department

Hashing: Hashing is a process of allocating keys into HashTables (memory) using Hash function is called Hashing.

Eg: keys are some integer values
 m stands for size of the table.

$H(k)$ - Hash function.

Methods to implement Hashing:

1. Division method
2. Mid-Square
3. folded method
4. Multiplication method

→ Division Method: → Allocate the following Keys

10, 9, 55, 64, 32, 29 into the Hash table with size i.e $m = 10$ using division Method.

Sol: Step-1: The Hash function for division method is given as $H(k) = k \% m$.

Step-2: Take an array

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(10) = 10 \% 10 \\ = 0$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(9) = 9 \% 10$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	-1	-1	-1	-1	9

$$H(55) = 55 \% 10$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	55	-1	-1	-1	9

$$H(64) = 64 \% 10$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	64	55	-1	-1	-1	9

$$H(32) = 32$$

0	1	2	3	4	5	6	7	8	9
10	-1	32	-1	64	55	-1	-1	-1	9

Q. Mid Square Method:

→ Allocate the following keys into hash table
10, 9, 55, 64, 32, 29
With $m=10$ using Mid square method.

1. The Hash function for mid square method is given as $H(K) = K * K$.

2. Take the array

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(10) = 10 * 10$$

$$= 11\boxed{0}0$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(9) = 9 * 9$$

$$= 0\boxed{8}1$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(55) = 55 * 55$$

$$= 03\boxed{0}25$$

Collision occurred at '0' position

$$H(64) = 64 * 64 = 04\boxed{0}96$$

Collision occurred.

$$H(32) = 32 * 32 = 01\boxed{0}24$$

Collision occurred

$$H(29) = \frac{29 \times 29}{100} \Rightarrow 8\boxed{4}1$$

$$\begin{array}{r} 29 \\ \times 29 \\ \hline 841 \end{array}$$

0	1	2	3	4	5	6	7	8	9
10	-1	-1	-1	29	-1	-1	-1	9	-1

folding method:

→ Allocate the following keys into the Hashtable size 100 using folding method.

8885386142, 9494291146, 9093567812, 6300709101,
9848986142.

Step- 1:

0	1	---	50	---	---	99
-1	-1	-1	-1	-1	-1	-1

H(8885386142)

2
- 88
85 ignore
38
61
42

314

0	1	14	20	20	...	99
-1	-1	888538 6142	-1	-1	-1	-1

H(9494291146)

2
94
94
29
11
46

279

0	1	14	20	20	...	74	...	99
-1	-1	888538 6142	-1	949429 1146	-1	-1	-1	-1

ignore

$H(9093567812)$

ignore
 90 $\boxed{2} \ 9$
 93
 56
 78
 12
 $\frac{329}{}$

0	1	... 10	29	30	... 71	99
-1	-1	-1	888528	1	-1	949999

$H(6300709101)$

63
 00 $\boxed{2} \ 5$
 70
 91
 01
 $\frac{225}{}$

0	1	... 14	25	29	... 21	99
-1	-1	-1	888528	1	-1	949999

$$((1 \times 133 \cdot 01) + 99) \cdot 1001^2$$

$$((41380 \cdot 001) \cdot 1001^2)$$

$$(n \cdot 13) \cdot 1001^2$$

→ Multiplication Method:

→ Allocate the following keys into Hash-table using
 Multiplication method with $m = 100 \rightarrow 10, 9, 55, 64, 32, 29$
 the Hash function for multiplication method is
 given as $H(K) = \text{floor}(m * (K \cdot A \% 1))$

A is constant value

where, A is 0.357840

Initial array :

0	1	... 50	... 99
-1	-1	-1	-1

step-2: $H(10)$

$\text{floor}(100 * (10 * 0.357840 \% 1))$ when we do by $\times 1$
 the remainder part (after point)

$\text{floor}(100 * (3.57840 \% 1))$ we will write

$$\times 100 * (0.57840)$$

$$\Leftrightarrow \text{floor}(57.840)$$

$$\Leftrightarrow 57$$

0	1	... 50	... 52	... 99
-1	-1	-1	10	-1

$$H(9) = \text{floor}(100 * (9 * 0.357840 \% .1))$$

$$= \text{floor}(100 * (3.57840560 \% .1))$$

$$= \text{floor}(100 * (3.57840560))$$

$$= \text{floor}(35.7840560)$$

$$= 35.$$

0	1	22	57	99
-1	-1	9	-1	-1

$$H(55) = \text{floor}(100 * (55 * 0.357840 \% .1))$$

$$= \text{floor}(100 * (19.6812 \% .1))$$

$$= \text{floor}(100 * 0.6812)$$

$$= \text{floor}(68.12)$$

$$= 68$$

0	1	22	57	68	99
-1	-1	9	-1	10	55

$$H(64) = \text{floor}(100 * (64 * 0.357840 \% .1))$$

$$= \text{floor}(100 * (22.90176 \% .1))$$

$$= \text{floor}(100 * 0.90176)$$

$$= \text{floor}(90.176)$$

$$= 90$$

0	1	22	57	68	99
-1	-1	9	-1	10	55

$$H(32) = \text{floor}(100 * (32 * 0.357840 \% .1))$$

$$= \text{floor}(100 * (11.45088 \% .1))$$

$$= \text{floor}(100 * 0.45088)$$

$$= \text{floor}(45.088)$$

$$= 45$$

0	1	22	57	68	99
-1	-1	9	32	10	55

$$H(29) = \text{floor}(100 * (29 * 0.357840 \% .1))$$

$$= \text{floor}(100 * (10.37736 \% .1))$$

$$= \text{floor}(100 * 0.37736)$$

$$= \text{floor}(37.736) = 37$$

0	1	22	37	45	57	68
-1	-1	9	29	32	10	55

→ collision Resolution techniques:

collision: Two keys trying to access the same memory location then we encounter collision.

collision can be resolved in two ways:

(i) open addressing

(ii) closed Hashing

→ open addressing:

(i) linear probing - C

(ii) quadratic probing - C

(iii) double hashing X

→ closed Hashing

(i) separate chaining method - C

→ linear probing: The hash function for the linear probing is given as $H(K, i) = (K + i) \% m$

Allocate the following keys into the hash-table with table size 10 and the keys are 29, 9, 99, 5, 65, 11

st: Initial array is -

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$H(29, 0) = (29 + 0) \% 10 = 9$$

collision occurs

increment the value of i

$$H(9, 1) = (9 + 1) \% 10 = 0$$

0	1	2	3	4	5	6	7	8	9
9	-1	-1	-1	-1	-1	-1	-1	-1	29

$$H(99, 0) = (99 + 0) \% 10 = 9$$

collision occurs

increment the value of i

$$H(99, 1) = (99+1)\%10 = 100\%10 = 0$$

collision occurs.

$$H(99, 2) = (99+2)\%10 = 101\%10 = \frac{1}{2} \quad 8 \quad 9$$

0	1	2	3	4	5	6	7	8	9
9	99	-1	-1	-1	-1	-1	-1	-1	29

$$H(5, 0) = (5+0)\%10 = 5\%10 = 5$$

0	1	2	3	4	5	6	7	8	9
9	99	-1	-1	-1	5	-1	-1	-1	29

$$H(65, 0) = (65+0)\%10 = 65\%10 = 5$$

collision occurs

increment the value of i .

0	1	2	3	4	5	6	7	8	9
9	99	-1	-1	-1	-1	65	-1	-1	29

$$H(65, 1) = (65+1)\%10 = 66\%10 = 6$$

0	1	2	3	4	5	6	7	8	9
9	99	-1	-1	-1	-1	65	-1	-1	29

Program:

```
#include <stdio.h>
int hashfunction(int ele, int size){
    int key = ele % size;
    return key;
}
int hashfunctions(int ele, int size, int i){
    int key = (ele+i) % size;
    return key;
}
int main(){
    int i, j, table[10], m=10, ele, ke, key;
    for(i=0; i<m; i++)
        table[i] = -1;
    // code continues
}
```

```
printf("Enter the no. of key elements:\n");
scanf("%d", &ke);
for(j=0; j<ke; j++) {
    printf("Enter key elements:\n");
    scanf("%d", &ele);
    key = hashfunction(ele, m);
    if (table[key] == -1)
        table[key] = ele;
    else
        i = 1;
    key = hashfunction2(ele, m, i);
    while (table[key] != -1) {
        i++;
        key = hashfunction2(ele, m, i);
    }
    table[key] = ele;
}
printf("Elements in the hash table after linear
probing:\n");
for(i=0; i<m; i++)
    printf("%d", table[i]);
return 0;
}
```

→ Quadratic probing:

The Hash function for quadratic probing is given by

$$H(K, i) = (K + i^2) \% m$$

29, 9, 99, 5, 65, 11. with size 10.

SH: $H(29, 0) = (29 + 0^2) \% 10 = 29 \% 10 = 9$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	29

$$H(9, 0) = (9 + 0^2) \% 10 = 9 \% 10 = 9$$

collision occurs

increment the value of i

$$H(9, 1) = (9 + 1^2) \% 10 = (9 + 1) \% 10 = 10 \% 10 = 0$$

0	1	2	3	4	5	6	7	8	9
9	-1	-1	-1	-1	-1	-1	-1	-1	29

$$H(99, 0) = (99 + 0^2) \% 10 = 99 \% 10 = 9$$

collision occurs

increment the value of i

$$H(99, 1) = (99 + 1^2) \% 10 = 100 \% 10 = 0$$

collision occurs

$$H(99, 2) = (99 + 2^2) \% 10 \Rightarrow (99 + 4) \% 10 = 103 \% 10 = 3$$

0	1	2	3	4	5	6	7	8	9
9	-1	-1	99	-1	-1	-1	-1	-1	29

$$H(5, 0) = (5 + 0^2) \% 10 \Rightarrow 5 \% 10 = 5$$

0	1	2	3	4	5	6	7	8	9
9	-1	-1	99	-1	5	-1	-1	-1	29

$$H(65, 0) = (65 + 0^2) \% 10 \Rightarrow 65 \% 10 = 5$$

collision occurs

increment the value of i

$$H(65, 1) = (65 + 1^2) \% 10 = 66 \% 10 = 6$$

0	1	2	3	4	5	6	7	8	9
9	-1	-1	99	-1	5	65	-1	-1	29

$$H(11, 0) = (11+0^2) \% 10 = 11 \% 10 = 1$$

0	1	2	3	4	5	6	7	8	9
9	11	-11	99	-1	5	165	-11	-11	29

Program :

```
#include <stdio.h>

int hashfunction(int ele, int size)
{
    int key = ele % size;
    return key;
}

int hashfunction2(int ele, int size, int i)
{
    int key = (ele + i * i) % size;
    return key;
}

int main()
{
    int i, j, n, table[10], m = 10, key, ele;
    for(i = 0; i < m; i++)
        table[i] = -1;
    printf("Enter the number of elements: \n");
    scanf("%d", &n);
    for(j = 0; j < n; j++)
    {
        printf("Enter the key elements: \n");
        scanf("%d", &ele);
        key = hashfunction(ele, m);
        if(table[key] == -1)
            table[key] = ele;
        else
        {
            i = 0;
            key = hashfunction2(ele, m, i);
            while(table[key] != -1)
                i++;
            key = hashfunction2(ele, m, i);
            table[key] = ele;
        }
    }
}
```

```

printf("Elements in the hash table after linear
    probing:\n");
for(i=0; i<m; i++) {
    printf("%d", table[i]);
}
return 0;

```

→ Double Hashing:

In double Hashing, we use two hash functions i.e h_1 & h_2 .

Whenever, we encounter a collision then second hash function comes into the picture.

the hash function is given as $h(K, i) = h_1(K)$

$$h(K, i) = (h_1(K) + i * h_2(K)) \% m$$

where,

$$h_1(K) = K \% m$$

$$h_2(K) = \text{PRIME} - (\text{Key \% prime})$$

$$\text{If } m = 10 \Rightarrow 7 - \text{key \% 7}$$

→ construct a hash table for the following keys with table size 10 and the keys are

29, 9, 99, 5, 65, 11

Step-1 :

0	1	2	3	4	5	6	7	8	9
1	7	3	1	5	9	6	2	8	4

Step - 0:

$$H(29, 0) = 29 \times 10 + 0 * H_2(K)$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	29

$$H(9, 0) = 9 \times 10 + 0 * H_2(K)$$

$$= 9 + 0 = 9$$

Collision occurred

$$H(9, 1) = (9 \times 10 + 1 * (7 - 9) \times 10)$$

$$= (9 + 1 * 5) \times 10$$

$$= 14 \times 10 = 14$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	9	-1	-1	-1	-1	29

$$H(99, 0) = 99 \times 10 + 0 * H_2(K)$$

$$= 99 \times 10 = 99$$

Collision occurred

~~$$H(99, 1) = (99 \times 10 + 1 * (7 - 99) \times 10)$$~~

~~$$(9 + 1 * (7 - 2) \times 10) = 9 + 1 * (5) \times 10$$~~
~~$$= 9 + 5 \times 10 = 14 \times 10 = 140$$~~

~~$$H(99, 2) = (99 \times 10 + 2 * (7 - 99) \times 10)$$~~
~~$$= 9 + 2 * ($$~~

$$H(99, 1) = (99 \times 10 + 1 * (7 - 11) \times 10)$$

$$= 9 + 6 \times 10$$

$$= 15 \times 10 = 150$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	9	99	-1	-1	-1	

$$H(5, 0) = 5 \times 10$$

$$= 5$$

Collision occurred

$$\begin{aligned}
 H(5,1) &= (5 \times 10) + 1 * (7 - (5 \times 7) \times 10) \\
 &= (5 + 1 * (7 - 5) \times 10) \\
 &= 5 + 1 * (2) \times 10 \\
 &= 5 + 2 \times 10 \\
 &= 25
 \end{aligned}$$

0	1	2	3	4	5	6	7	8	9
-1	1	-1	-1	9	99	-1	5	-1	-1

$$\begin{aligned}
 H(65,0) &= 65 \times 10 \\
 &= 650
 \end{aligned}$$

Collision occurred

$$\begin{aligned}
 H(65,1) &= (65 \times 10) + 1 * (7 - (65 \times 7) \times 10) \\
 &= 5 + 1 * (7 - 5) \times 10 \\
 &= 5 + 1 * (5) \times 10 \\
 &= 5 + 5 \times 10 \\
 &= 55
 \end{aligned}$$

$$\begin{aligned}
 H(11,0) &= 11 \times 10 = 110 \\
 \hline
 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 -1 & 11 & -1 & -1 & 9 & 99 & -1 & 5 & -1 & -1
 \end{aligned}$$

→ Separate chaining:

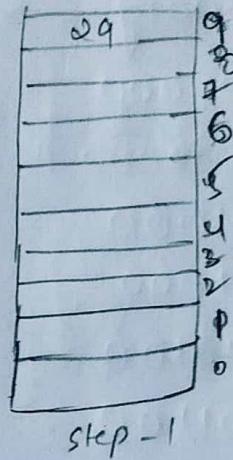
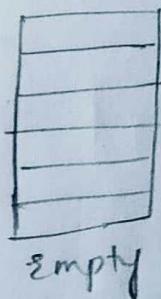
It

(i) In separate chaining we allocate the keys into hash table using linked list.

→ Allocate the following keys in the H.T with table size 10 with $H(K)$ is given as $H(K) = K \% 10$

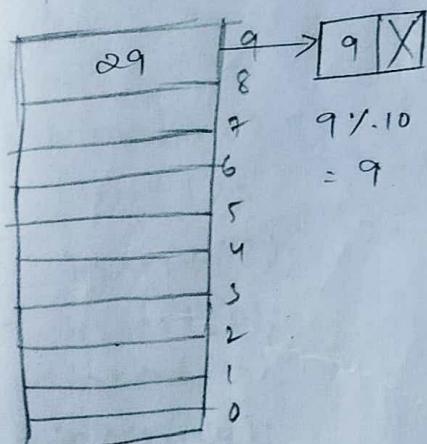
the keys are 29, 9, 99, 12, 72, 54, 34.

so:

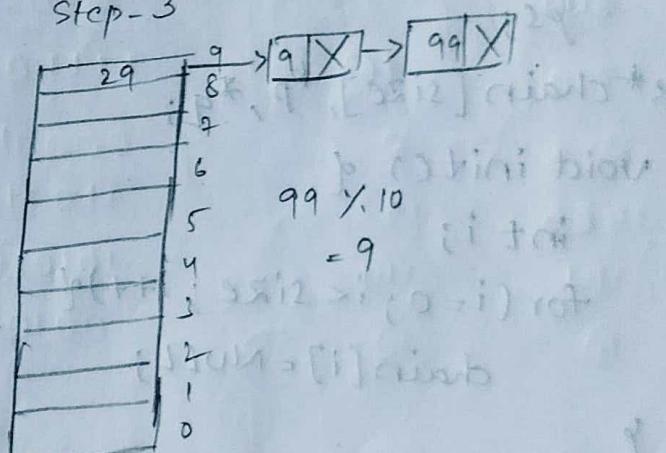


$H(29) = 29 \% 10$

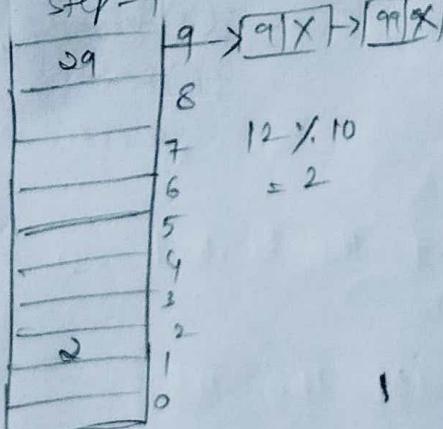
Step - 2



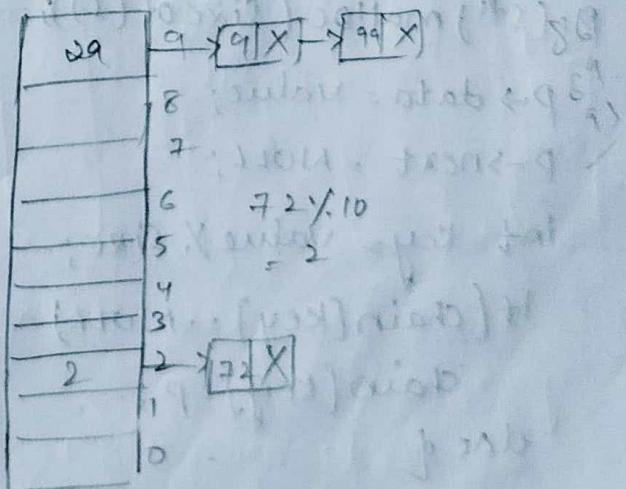
Step - 3



Step - 4

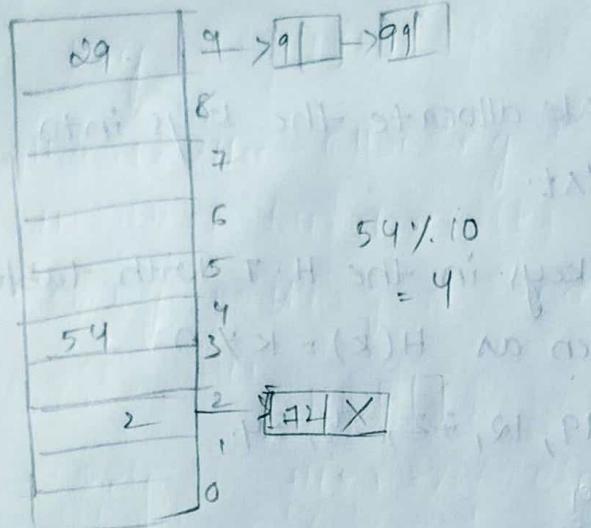


Step - 5

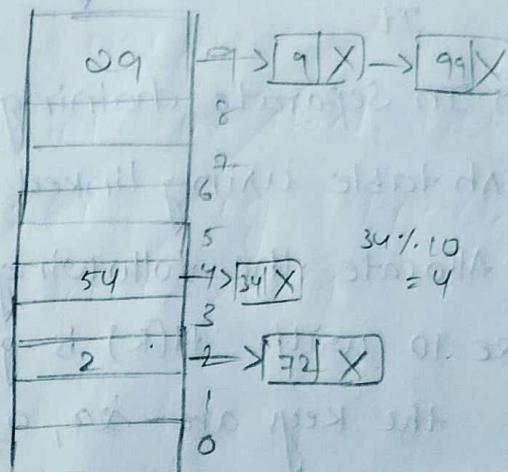


Collision occurred at index 2

Step - 6



Step - 7



```
#include <stdio.h>
#include <stdlib.h>
#define size 7
typedef struct node {
    int data;
    struct node *next;
} s;
s* chain[size], *p, *q;
void init() {
    int i;
    for(i=0; i<size; i++)
        chain[i] = NULL;
}
void insert(int value) {
    p = (s*) malloc(sizeof(s));
    p->data = value;
    p->next = NULL;
    int key = value % size;
    if(chain[key] == NULL)
        chain[key] = p;
    else {
        q = chain[key];
        while(q->next != NULL)
            q = q->next;
        q->next = p;
    }
}
```

```
while (q->next != NULL) q
    q = q->next;
}
q->next = p;
}

void Print() {
    int i;
    for (i=0; i<size; i++) {
        p = chain[i];
        printf("chain [%d].d-->", i);
        for (p=chain[i]; p!=NULL; p=p->next)
            printf("%d.d--> ", p->data);
        printf("NULL\n");
    }
}

int main() {
    init();
    insert(8);
    insert(0);
    insert(16);
    insert(24);
    insert(57);
    insert(4);
    insert(5);
    Print();
    return 0;
}
```

→ Extensible Hashing:

1. Key $H(K)$

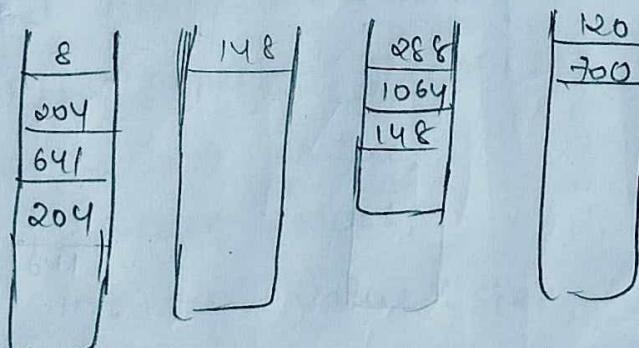
Binary

→ Allocate the following Keys into the Hash table with Hash function $H(K) = K \% 64$ using Extensible Hashing with $g=2$ and Bucket size = 4

1. 288, $H(1)$

Key	$H(K) = K \% 64$	Binary
288	32	1000000
8	8	0010000
1064	40	101000
120	56	111000
148	20	010100
204	12	001100
641	1	000001
700	60	111100
258	2	000010
1586	15	110010
44	44	101010
10	10	001010

Step - 1: 00 01 10 11



Now we have to split the bucket 00, i.e.

$$g = 3.$$

000	001	010	011	100	101	110	111
641 256	8 204 10	148		288 1064 44		100 700	1586

→ Rehashing:

- Whenever the hash table is almost all full and still some keys are left over to hash in such situation we go for rehashing.

i.e. we double the table size.

e.g.: 75, 86, 97, 8, 19, 24

and $m = 5$.

Step 1:

0	1	2	3	4
-1	-1	-1	-1	-1

Step 2: $H(K) = K \% m$

$$H(75) = 75 \% 5$$

0	1	2	3	4
75	-1	-1	-1	-1

$$H(K) = K \% m \Rightarrow H(86) = 86 \% 5$$

0	1	2	3	4	= 1
75	86	-1	-1	-1	

$$H(97) = 97 \% 5 = 2$$

75	86	97	-1	-1
----	----	----	----	----

now, Double the table size with $5 \times 2 = 10$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	75	-1	-1	-1	-1

$$H(86) = 86 \% 10 = 6$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	75	86	-1	-1

$$H(97) = 97 \times 10 \div 7$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	75	86	97	-1	91

$$H(8) = 8 \times 10 \div 8$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	75	86	97	8	-1

$$H(19) = 19 \times 10 \div 9$$

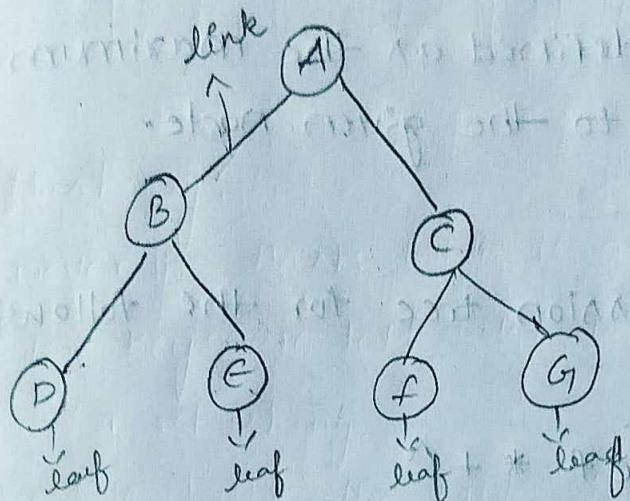
0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	75	86	97	8	19

$$H(24) = 24 \times 10 \div 4$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	24	75	86	97	8	19

Trees:

Tree: A Tree is a non linear (graph) data structure
→ collection of nodes and link is called a tree.



→ Binary tree:

A Binary tree is divided into three subsets

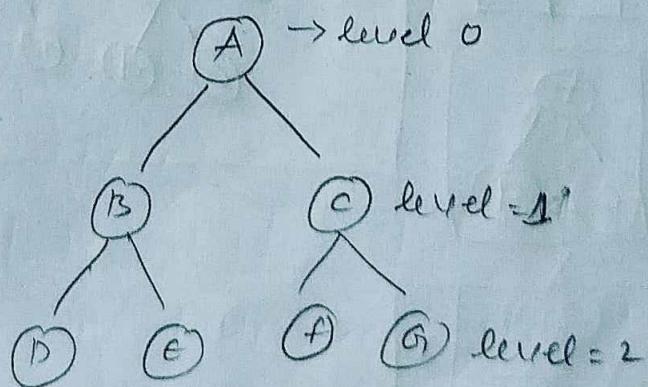
- (i) Root
- (ii) left subtree
- (iii) Right subtree

→ leaf: A node which doesn't have children.

→ siblings: Two nodes having same parent are called siblings.

B, C, D, E, F, G are siblings.

→ level of a node: The level of a node is defined as one + its parent level



→ Height and depth of a tree:

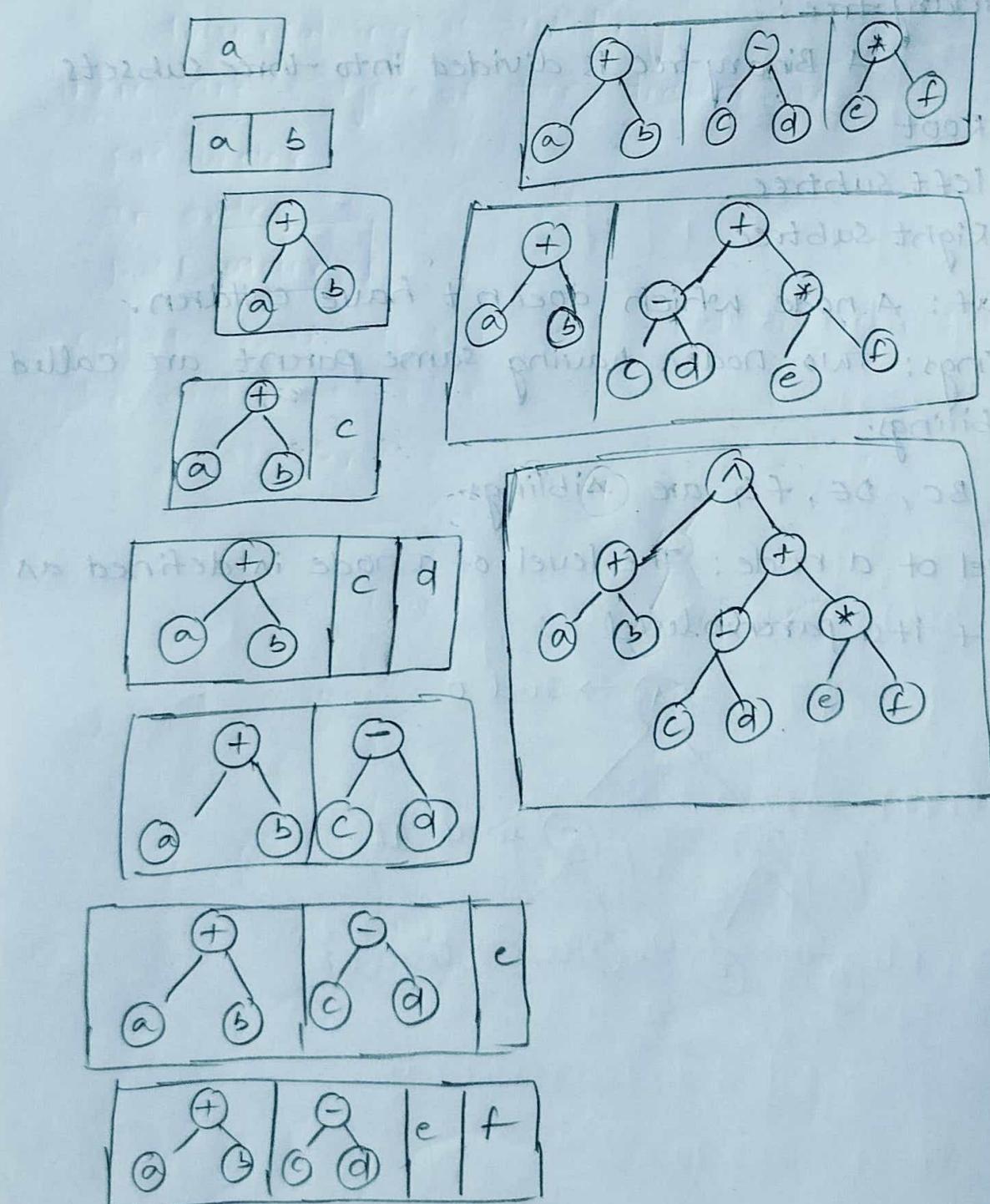
The height is defined as the maximum path from the given node to the root node.

Depth of a tree defined as the maximum path from the root node to the given node.

1. Expression tree :-

construct an expression tree for the following expression.

$$ab + cd - ef * + ^$$

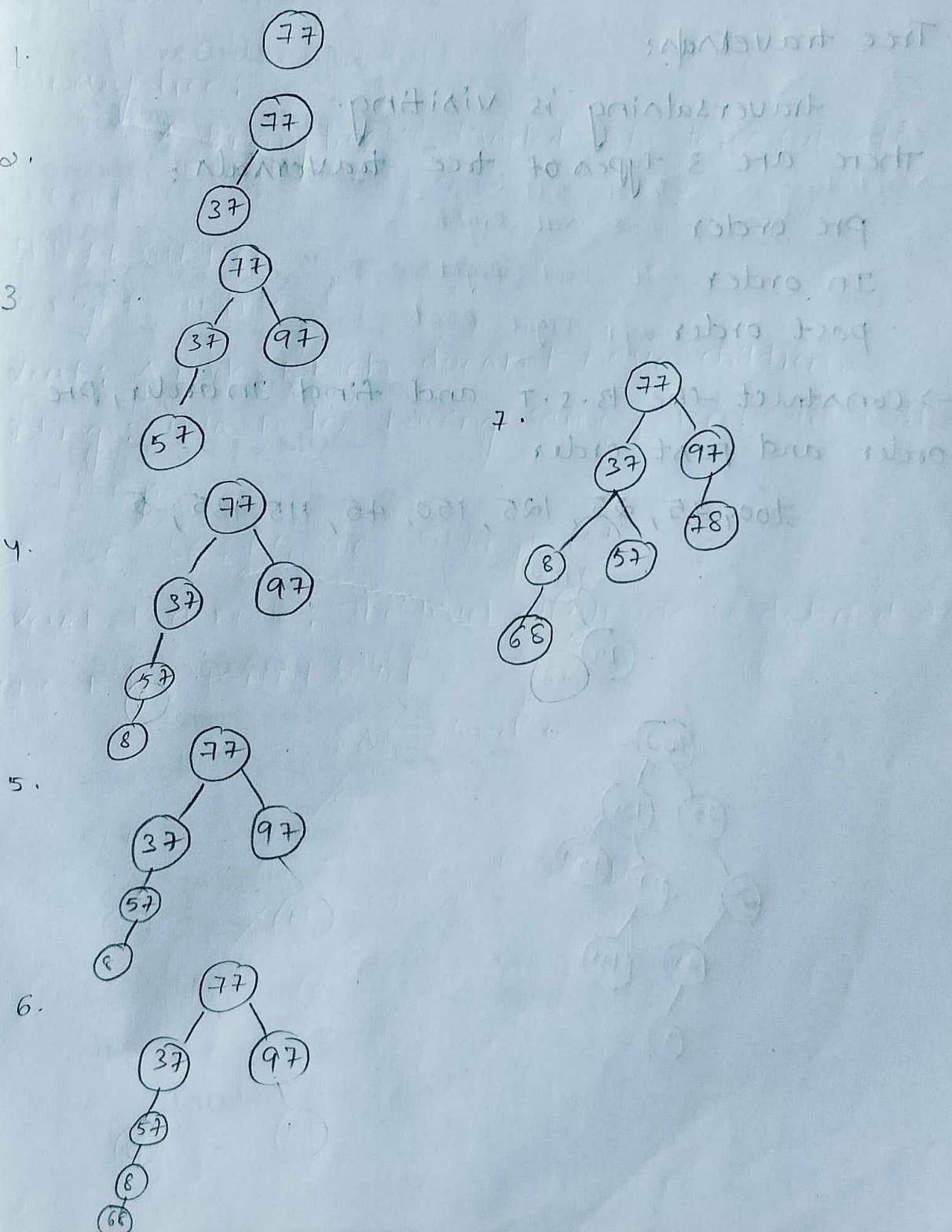


2. BinarySearch tree :

A binary search tree is a binary tree in which the first element is the root node and if the next element is less than the root node then place that node as left child of the root node. If it is greater than the root node then place it as right child of the root node.

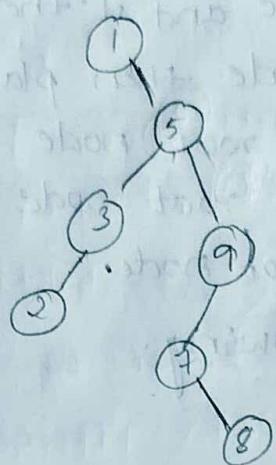
→ construct a B.S.T for the following.

77, 37, 97, 57, 8, 68, 78



→ Construct a BST for the following

Numbers 1, 5, 8, 9, 7, 9, 8



Tree traversal:

traversing is visiting.

There are 3 types of tree traversal:

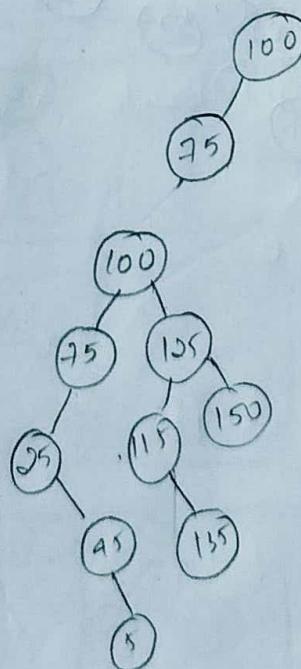
pre order Root left Right

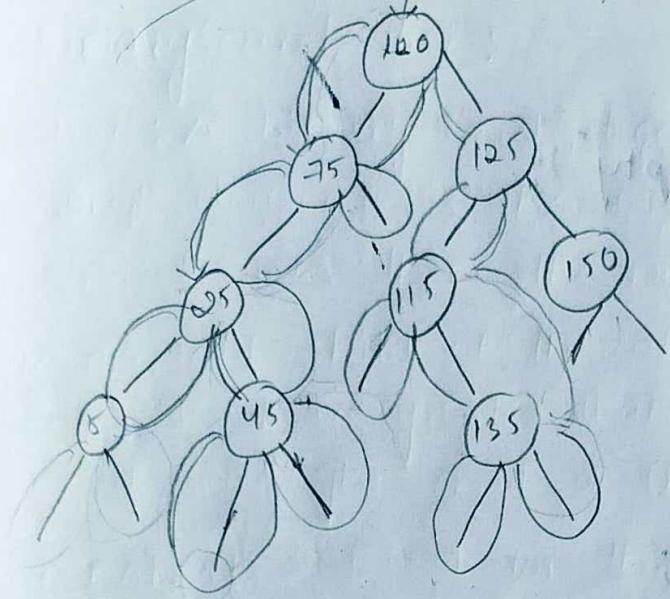
In order Left Root Right

post order left Right Root

→ Construct the B.S.T and find In order, Pre order and post order

100, 75, 25, 125, 150, 45, 115, 135, 5





Pre: 100, 75, 25, 50, 45, 115, 125
 In: 5, 25, 45, 75, 115, 125, 150
 Post: 5, 45, 25, 75, 135, 115, 125, 150, 100

Recursive function for Pre order:

```

void preOrder() {
  if (root == NULL) {
    printf("Tree is empty");
  } else {
    printf("%d-->", T->data);
    if (T->L != NULL)
      preOrder(T->L);
    if (T->R != NULL)
      preOrder(T->R);
  }
}
  
```

```

void Inorder() {
  if (root == NULL) {
    printf("Tree is empty");
  } else {
    if (T->L != NULL)
      Inorder(T->L);
    printf("%d-->", T->data);
    if (T->R != NULL)
      Inorder(T->R);
  }
}
  
```

```
void postOrder(T *t)
{
    if (root == NULL)
        printf("Tree is empty");
    else
        if (t->L != NULL)
            postOrder(t->L);
        if (t->R != NULL)
            postOrder(t->R);
        printf("%d-->%d", t->data);
```

→ Deleting a node in BST:

→ AVL Tree:

AVL stands for Adel'son Velsky Landau.

AVL tree is the balancing tree.

Whenever, we insert a node or delete a node the tree gets imbalanced.

In order to make tree balanced we four types of rotations

We say AVL tree is balanced if the balance factor must be in between $\{-1, 0, 1\}$.

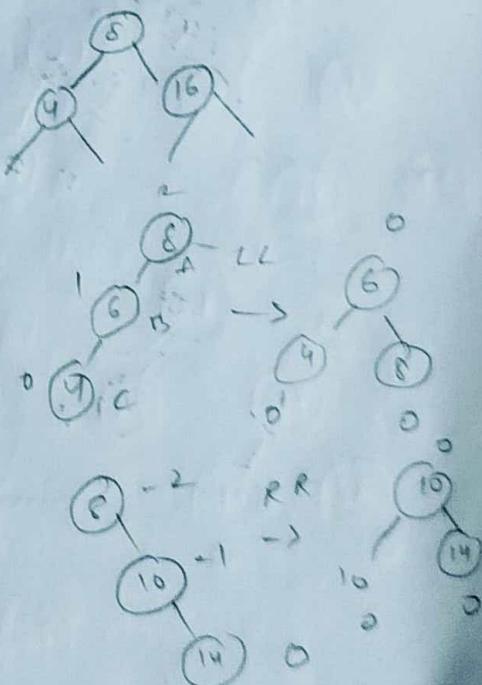
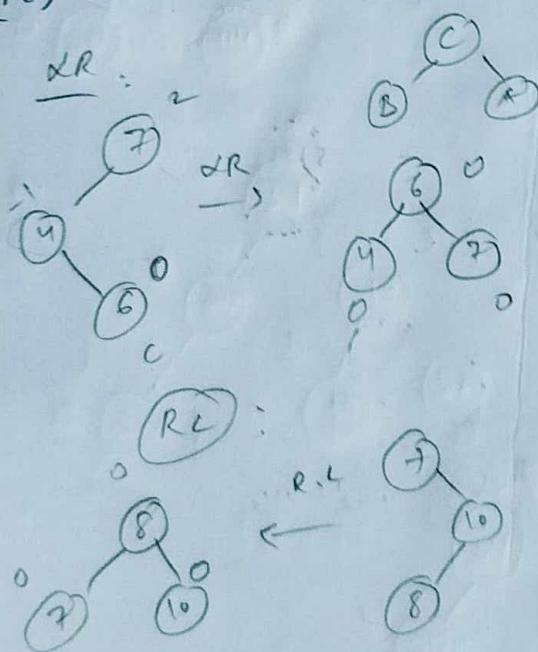
A balance factor of an AVL tree is given as height of the left sub-tree - height of the right sub-tree.

$$B_f = H_L - H_R$$

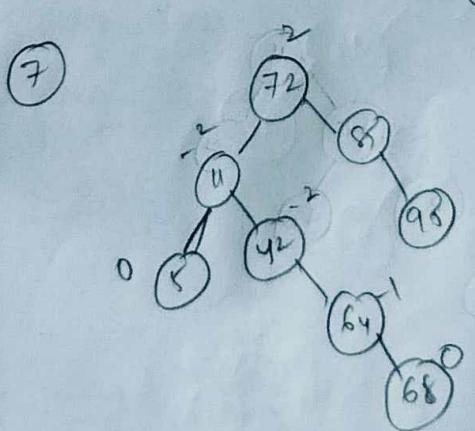
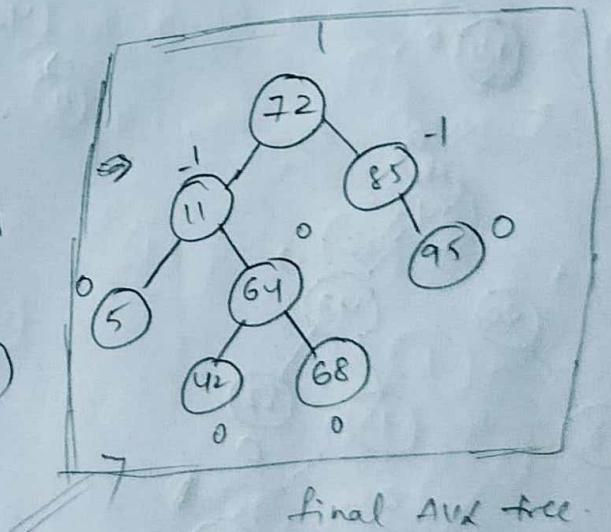
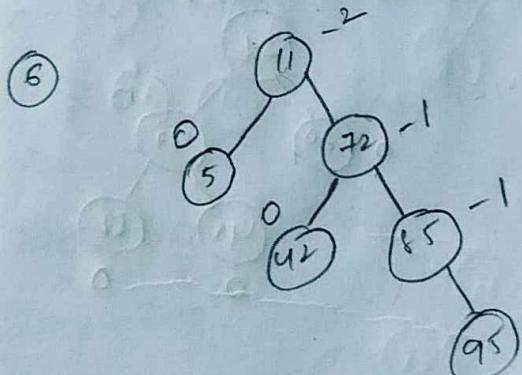
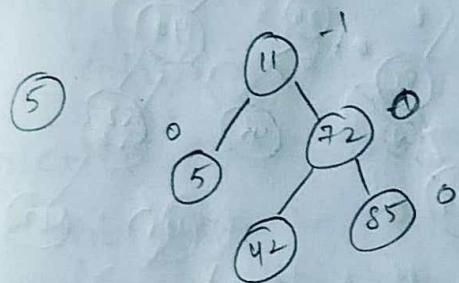
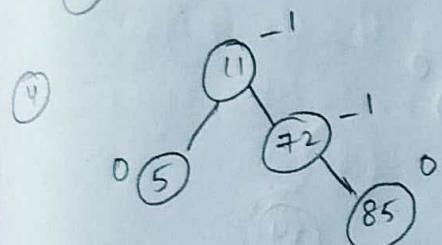
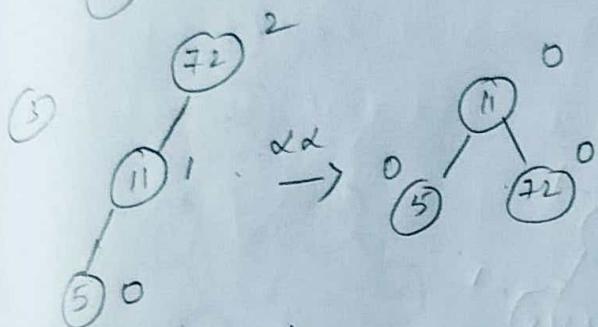
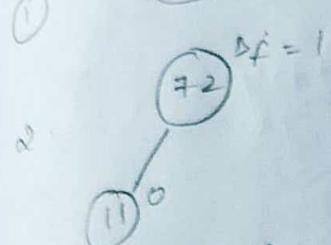
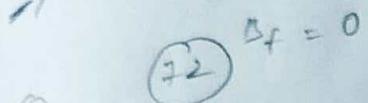
In order to make tree balanced.

The four rotations are:

- (i) LL- rotation
- (ii) RR rotation
- (iii) LR rotation
- (iv) RL rotation

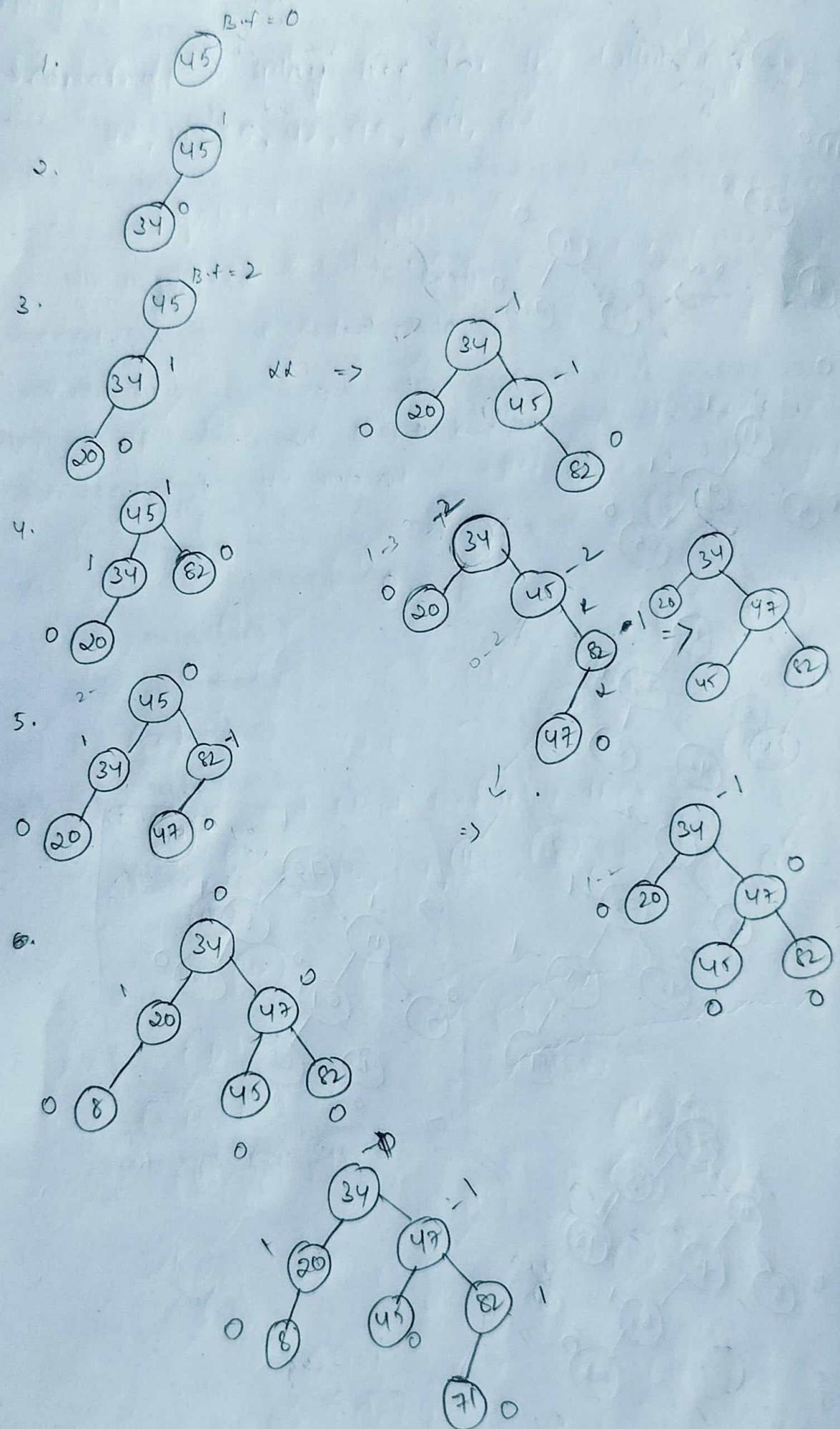


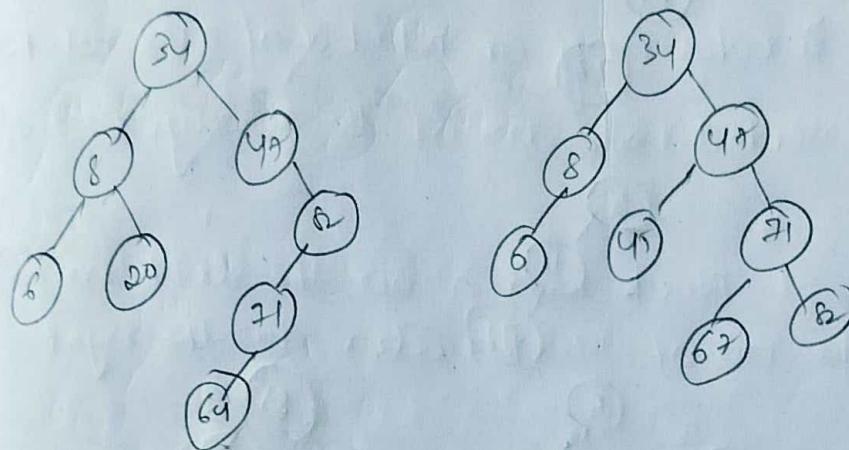
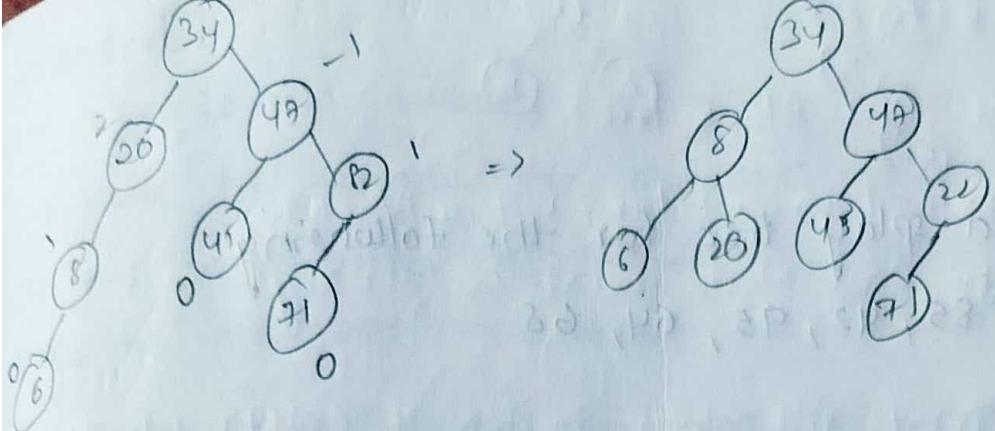
72, 11, 5, 85, 42, 95, 64, 68



→ construct the AVL tree for the following:

45, 34, 20, 82, 47, 8, 71, 6, 67





splay tree:

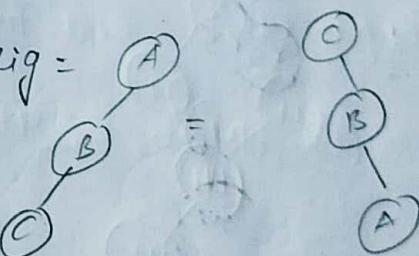
splaying is nothing but taking the last inserted node to the root node.

this can be done by using 6 rotations:

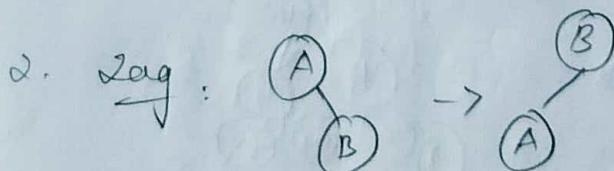
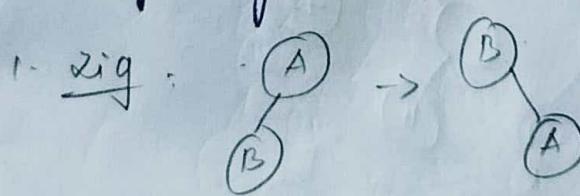
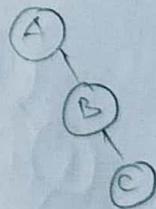
The 6 rotations are:

1. Zig
2. Zag
3. zig-zig
4. zag-zag
5. zig-zag
6. zag-zig

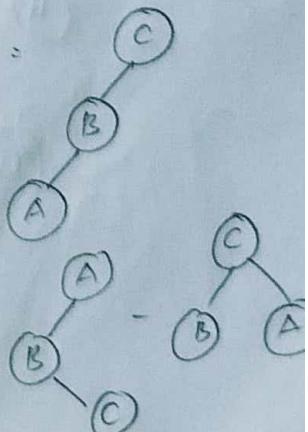
3. zig-zig =



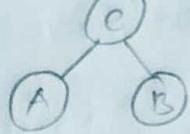
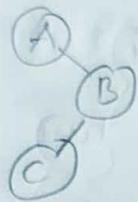
4. zag-zag :



5. zig-zag :

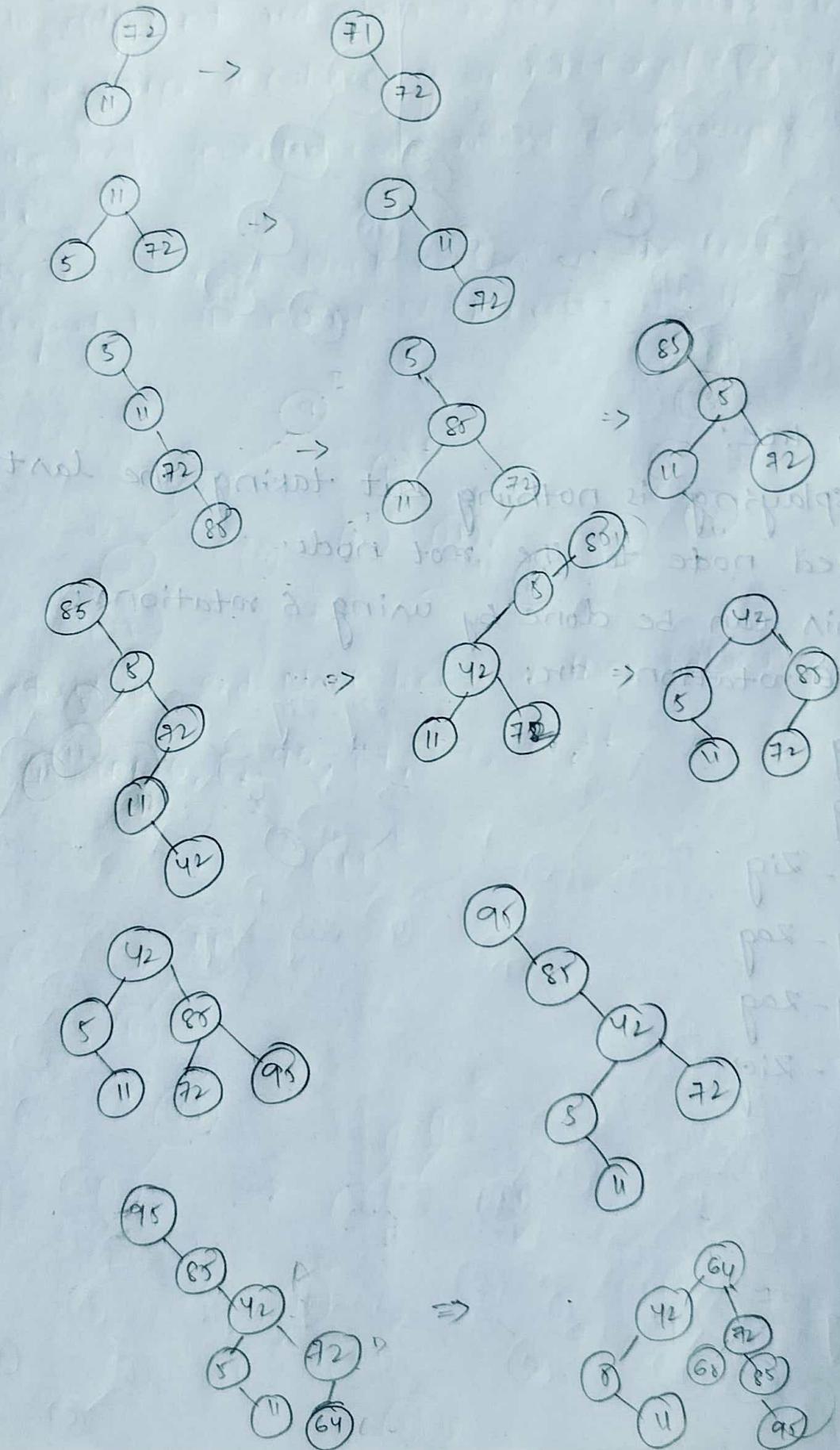


Zag-Zig:



→ construct a splay tree for the following

72, 11, 85, 42, 95, 64, 68



→ B-tree:

It is also known as M-way tree.

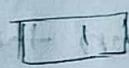
Where M stands for the order of the tree.

If we have order m(3) then we can have m-1 keys in the root node and m children.

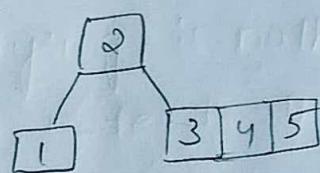
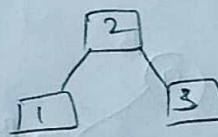
→ construct a B-tree for the following with order 3

1, 2, 3, 4, 5, 6, 7, 8, 9

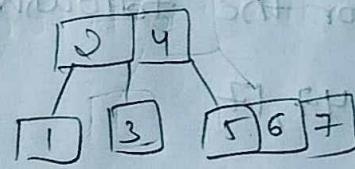
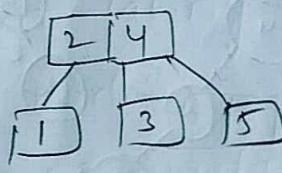
→



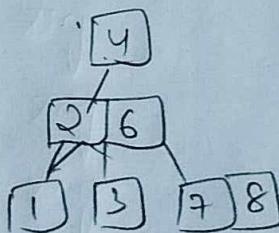
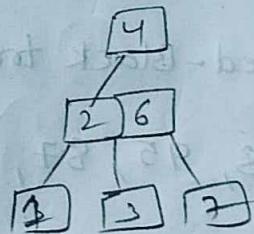
[1 | 2 | 3] →



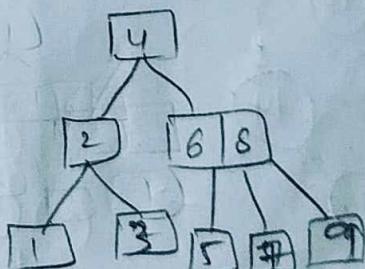
⇒



⇒



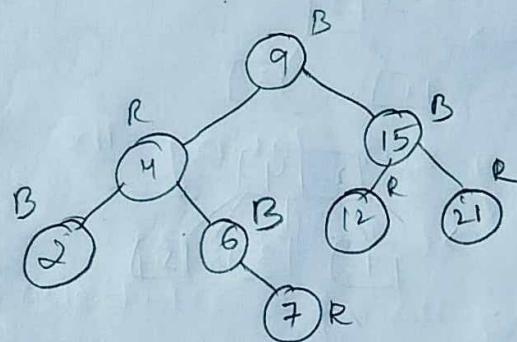
⇒



→ Red-Black tree:

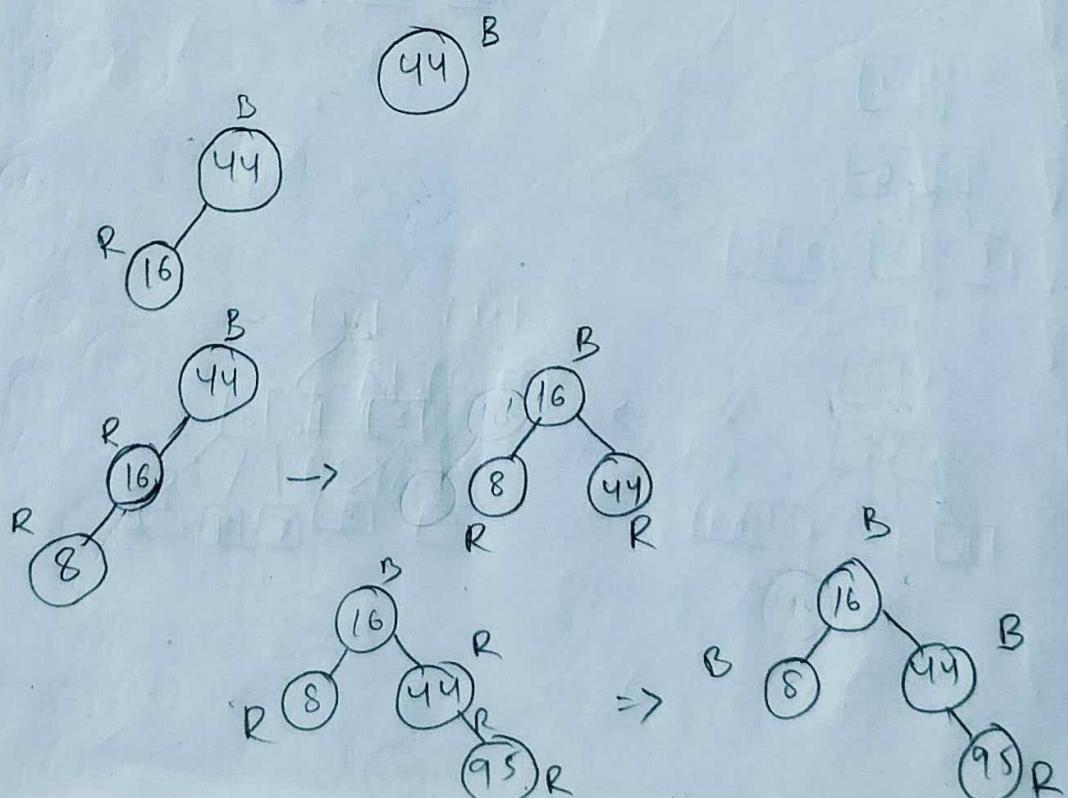
Properties of Red-Black tree:

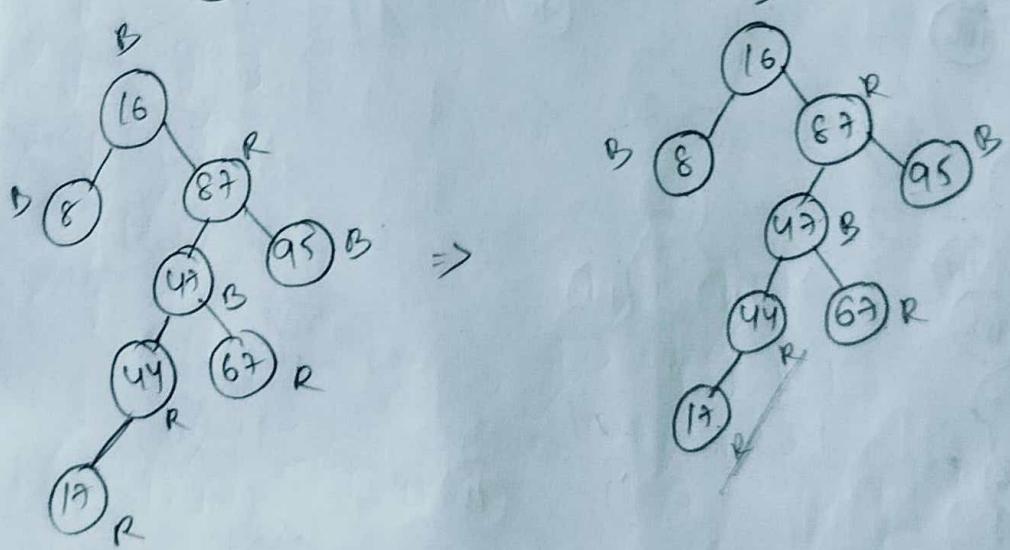
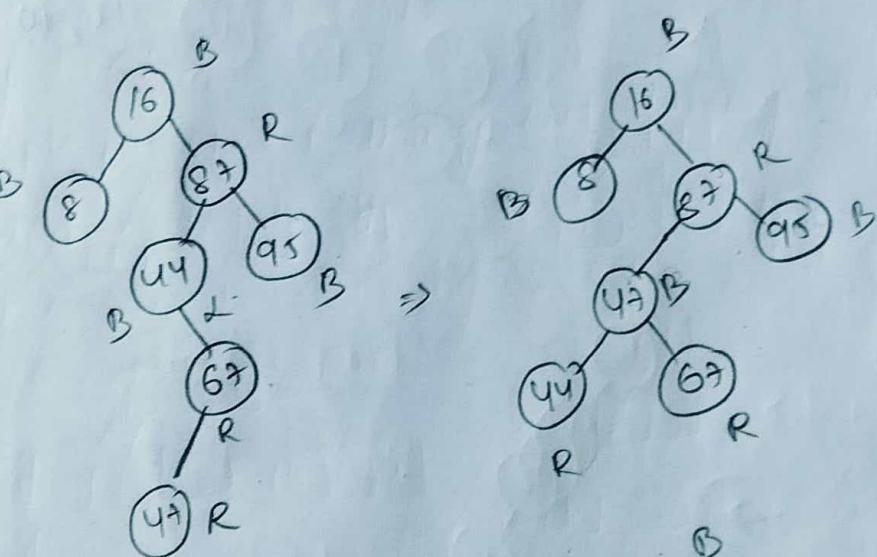
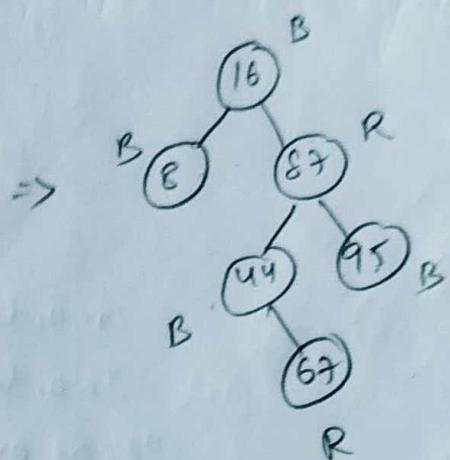
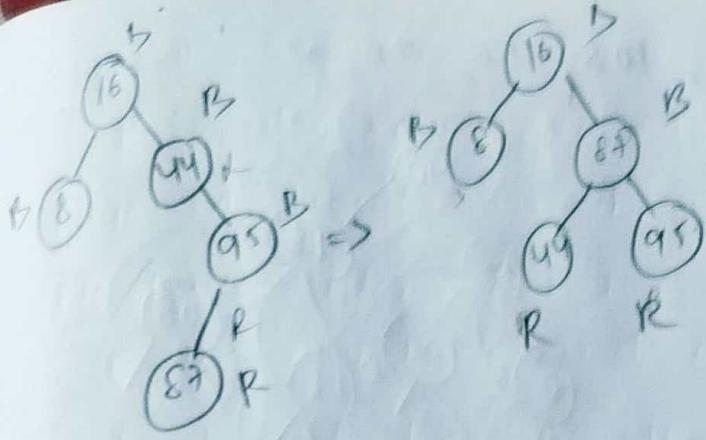
- (i) Every node has a color (Red or Black).
- (ii) (Every) The Root Node must be always Black in colour.
- (iii) No ^{two} adjacent red colour nodes. If exists we have
 - (iv) to perform re-colouring or rotations (AVL rotations)
- (v) The last inserted node must be always Red in colour.
- (vi) The number of Black nodes on the left must be equal to the no. of Black nodes to the right.



→ Construct a Red-Black tree for the following elements

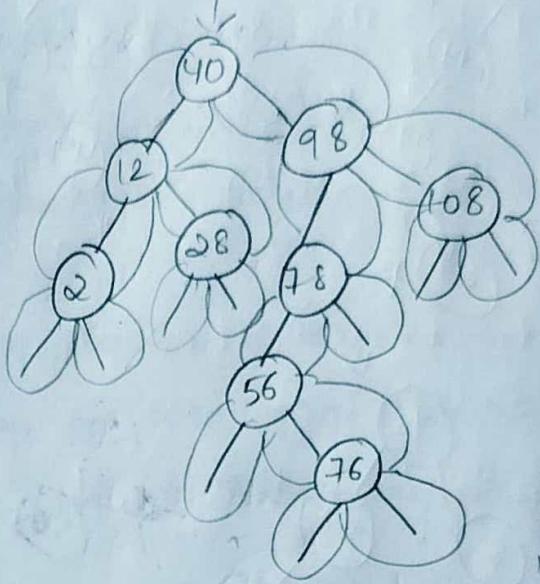
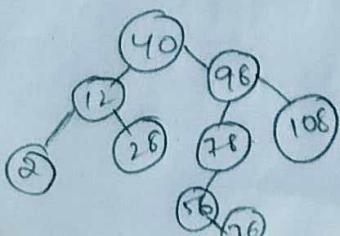
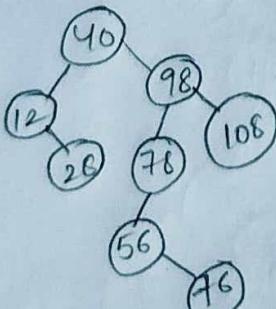
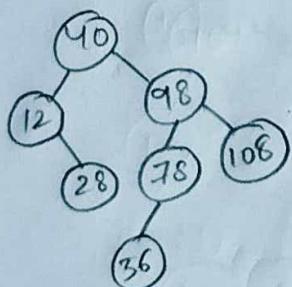
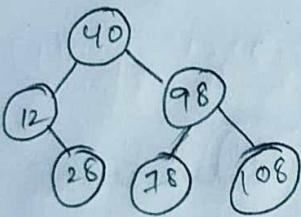
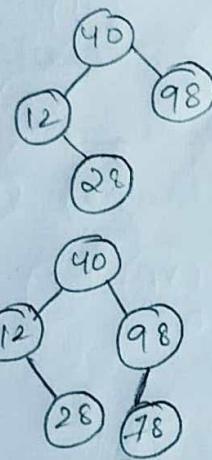
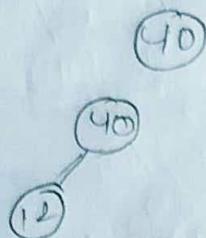
44, 16, 8, 95, 87, 67, 47, 17





$\rightarrow 40, 12, 28, 98, 78, 108, 56, 76, 2$.

Binary Search Tree:

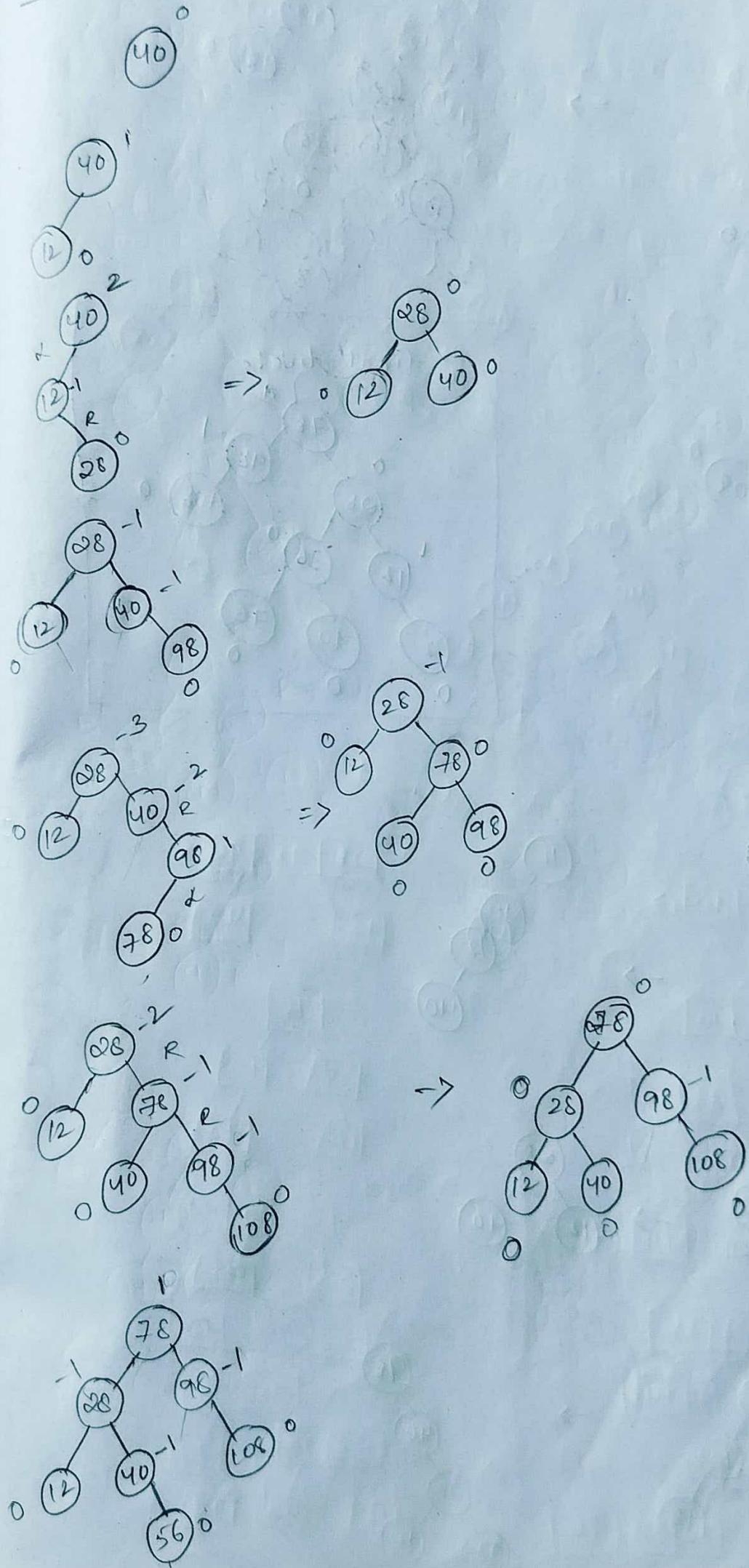


preorder: 40, 12, 2, 28, 98, 78, 56, 76, 108

In order: 2, 12, 28, 40, 56, 76, 78, 98, 108

Post order: 2, 28, 12, 76, 56, 78, 108, 98, 40

AVL Tree :



Graphs

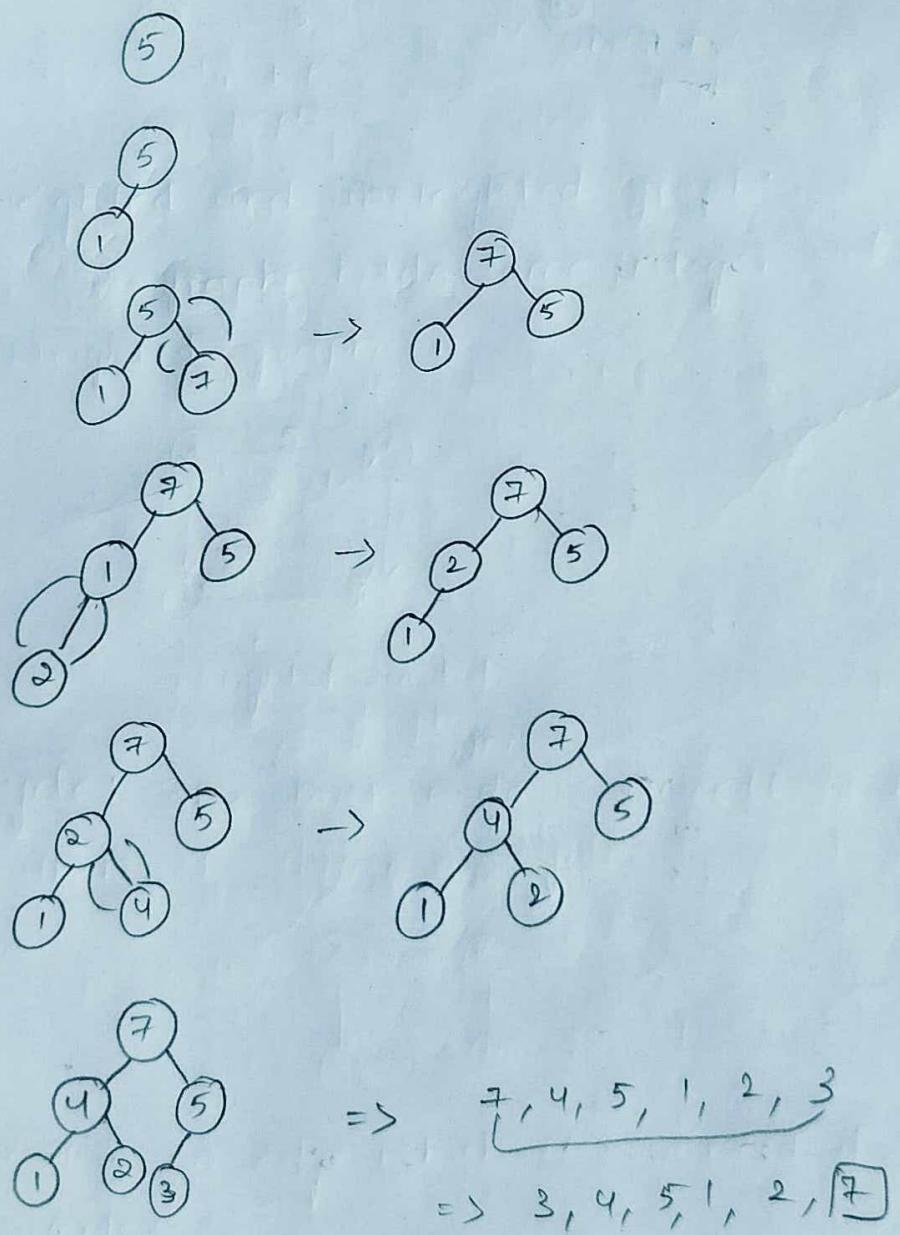
→ Heap Sort : - Code

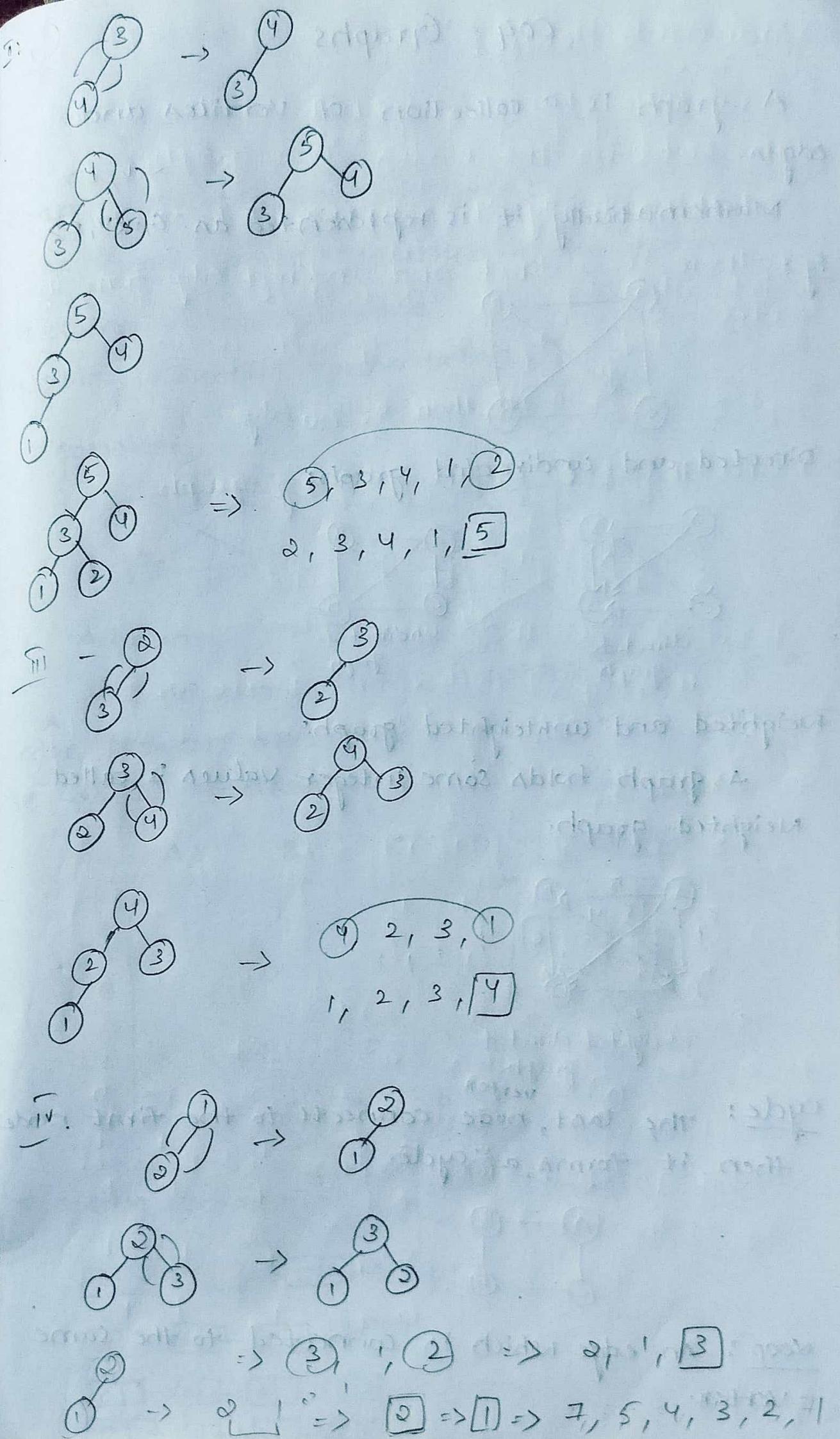
We can construct a heap sort either by using max heap property or min heap property.

Max Heap : In the max heap property the child nodes are always less than its parent nodes.
In Min Heap, the child node are always greater than its parent node.

construct the Max heap for the following and sort the data.

5, 1, 7, 2, 4, 3





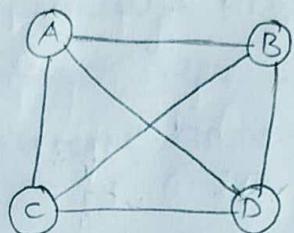
CO4 - Graphs

A graph is a collection of vertices and edges.

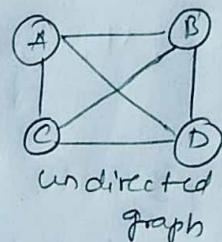
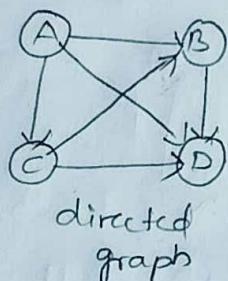
$\langle V, E \rangle$

Mathematically it is represented as $G = \langle V, E \rangle$

Eg :

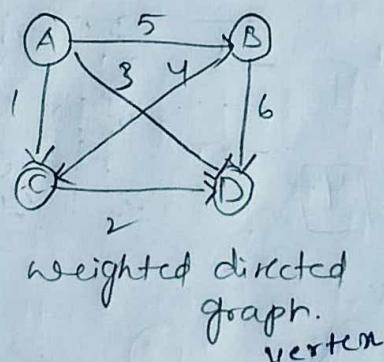


Directed and undirected graph:

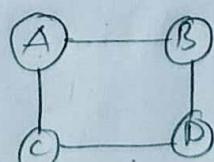


Weighted and unweighted graph:

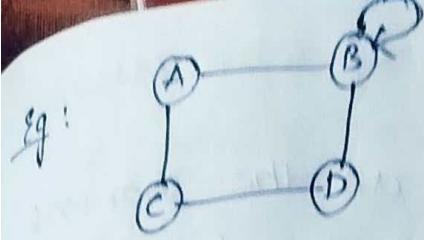
A graph holds some integer values is called weighted graph.



cycle: The last node connects to the first node then it forms a cycle.



loop: An edge which is connected to the same vertex.



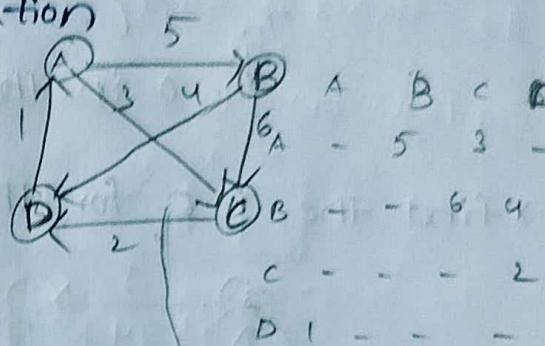
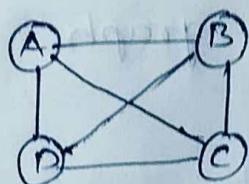
Graph Representation Techniques :

A Graph can be represented in two ways:

i) using Adjacency matrix

ii) using linkedlist representation

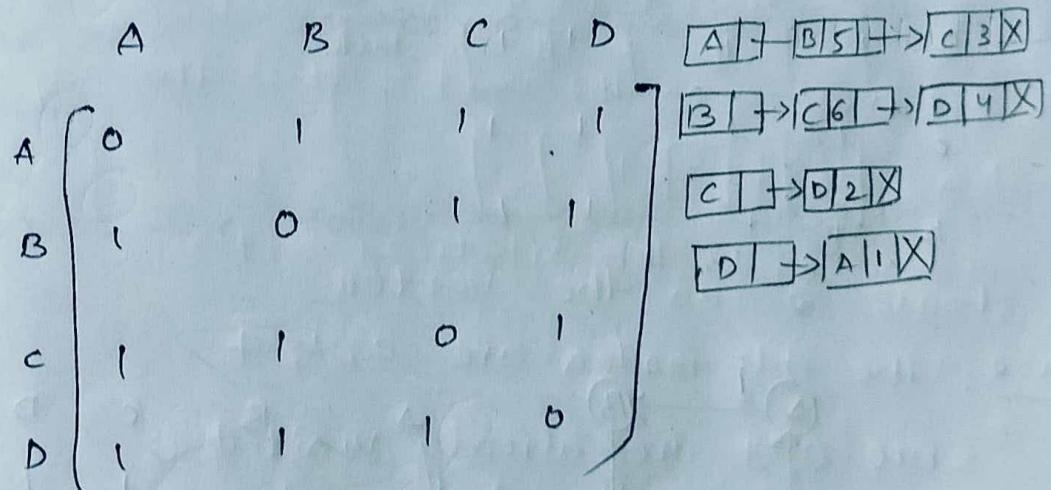
consider



1. Adjacency matrix:

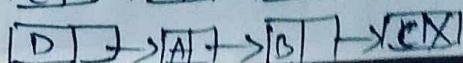
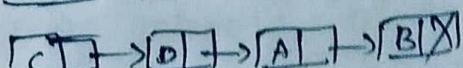
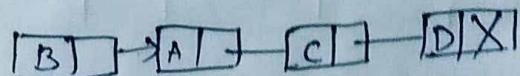
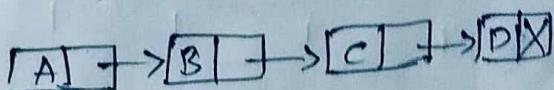
An Adjacency matrix is which if there is a edge between one vertex to another then represent it as 1 otherwise 0.

linked list:



2. using linked list :

AM



→ Transitive closure:

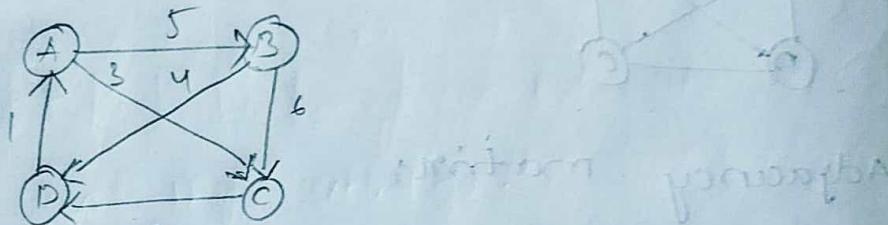
Transitive closure is defined as the shortest path between the nodes in a graph.

Mathematically it is represented as $T(i, j)_{ij}$

If there is a direct and ^(or) indirect edge from that vertex = 0

$T_{ij} = 1$ if there is an edge
 $T_{ij} = 0$ if there is no edge.

→ find the T.C for the following graph.

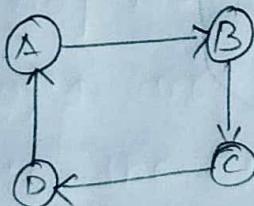


A 101 1001 111-9 SALVADOR L. M.

B 441 1001 01 13 1987 A

D C I W I O)

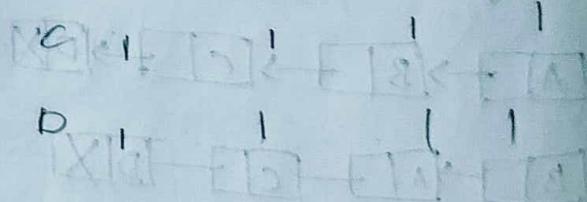
Ex: 2:



A B C D

A | | + +

Biological activity



→ Graph Travels:

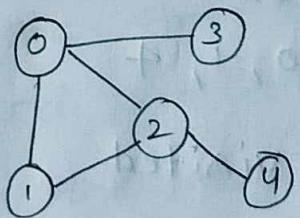
A Graph can be travelled in two ways.

- (i) DFS (Depth first search)
- (ii) BFS (Breadth first search)

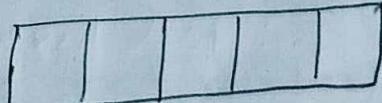
i) DFS :

1. Depth first search uses a strategy called stack.
2. choose any arbitrary vertex from the graph.
(random).
- and put that vertex of the top of stack.
3. Take the top element from the stack and put in the visited list.
4. ~~clear~~ the list of that vertex adjacent nodes.
create
5. Add the ~~onen~~ (vertex) which are not visited on top of the stack.
6. Keep repeating the above two steps until the stack is empty.
7. And all the vertices are visited.
8. Stop.

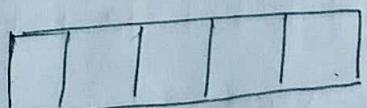
→ Travel the following graph using DFS:



Step - 1:

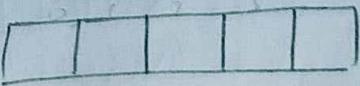


stack

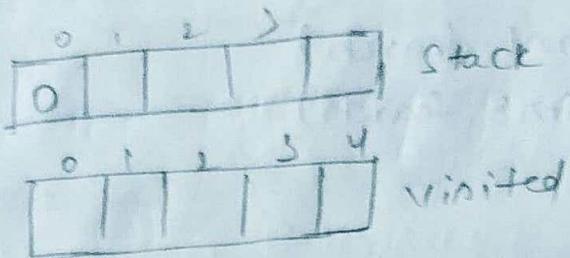


visited

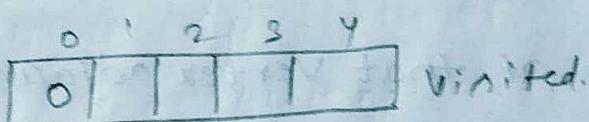
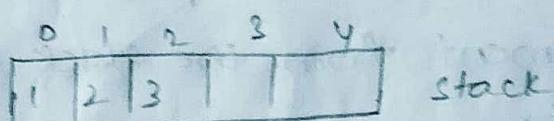
Step - 2: choose '0' as arbitrary vertex, the dfs



algorithm starts by putting the top element in the stack.



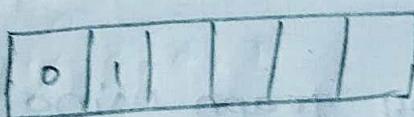
pop the top element in stack and write in the visited, and also adjacent nodes of that vertex on to the stack.



3. choose '1' as the vertex.

the adj of nodes of 1 are 0, 2.

since, 0 is already visited and 2 is already in stack, then,

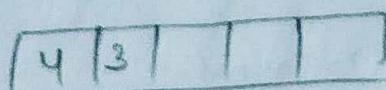


4. choose '2' as the vertex.

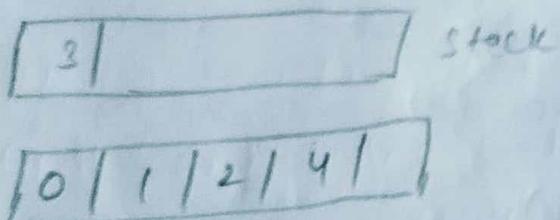
the adj nodes are 0, 1, 4.

since 0, 1 are already visited.

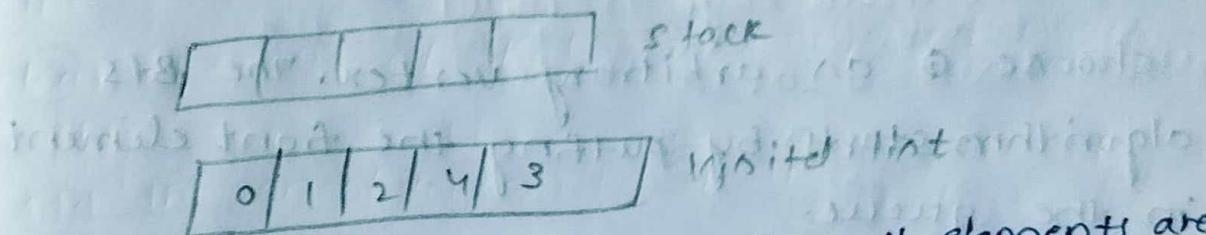
and 4 is the new vertex and it on the top of the stack



5. choose '4' as the vertex
 the adj vertices are 2
 which is already visited then pop the '4' to the
 visited list.



6. choose '3' as the vertex
 the adj vertices are 0
 which is already visited then pop the 3 to the
 visited list.



then the stack is empty. then all elements are visited

$$0 \rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 3$$

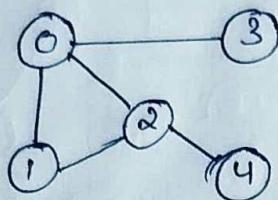
(ii) BFS :

- 1. BFS uses a strategy called queue.
- 2. Start by choosing any arbitrary vertex from the graph and put in the queue.
- 3. Take the first or front element from the queue and put it on the visited list.
- 4. Create the list of that vertex adjacent nodes.
- 5. Add the ones (vertex) which are not visited on the queue.
- 6. Keep repeating the above two steps until the queue is empty.

8. And all the vertices are visited

9. stop.

→ Traverse the following graph using Bfs.



Step - 1 :

0	1	2	3	4

queue

0	1	2	3	4
1	1	1	1	1

visited int
not

choose '0' as arbitrary vertex. The Bfs algorithm starts by putting the front element in the queue.

0	1	2	3	4
1	1	1	1	1

queue

0	1	2	3	4
0	1	1	1	1

visited int

choose 0 as the vertex then the adjacent nodes are 3, 2, 1

3	2	1		
0	1	1	1	1

choose '3' as the arbitrary vertex

2	1	0		
0	1	1	1	1

stack

0	3			
0	1	1	1	1

stack

domain ② → 3 is added (left end) 4 is added (right end)

0	3	2	1		
0	1	1	1	1	1

stack

vertices ① → 3 is added (left end) 4 is added (right end)

0	3	2	1		
0	1	1	1	1	1

stack

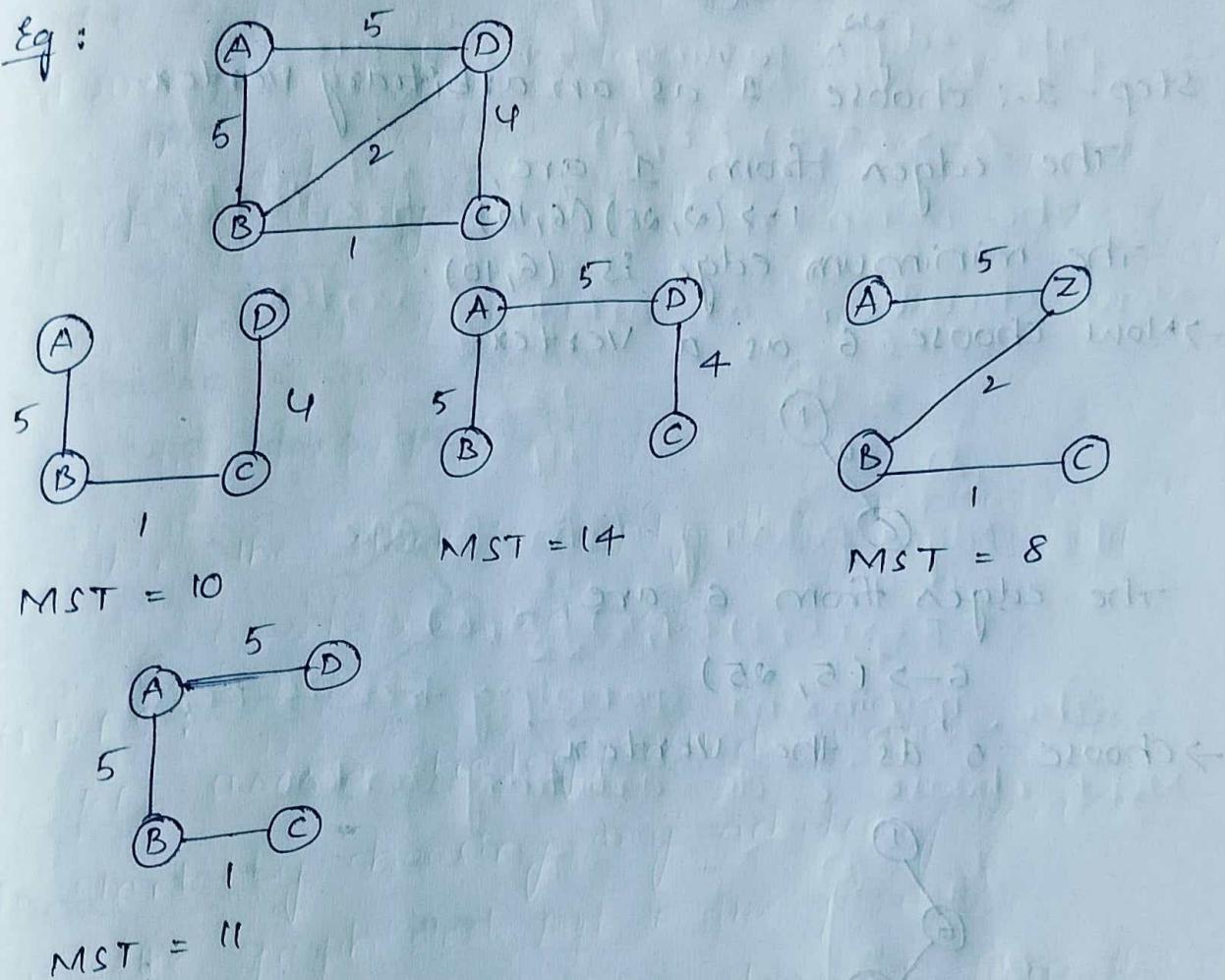
vertices ② → 4 is added (right end)

→ Minimal Spanning Tree (MST):

The minimum spanning tree is a subset of the edges of a connected, weighted graphs, which connects all the vertices together without any cycles and with minimum total edge weights.

for a graph, we can have or construct more than one MST.

Eg:



1. Prims Algorithm:

Prims algorithm comes under (prims) greedy technique.

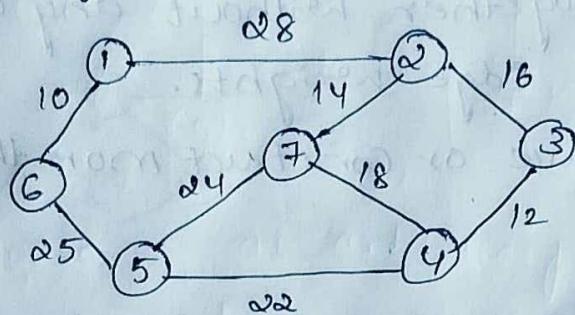
Algorithm:

choose any arbitrary vertex from the given graph.

Now we have to find all the edges that connect the tree to new vertices. from that find the minimum edge and add to the tree.

Repeat step 2 until we get Minimum Spanning Tree.

→ find the MST for the following graph using prim's algorithm.

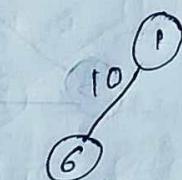


Step - 1: choose '1' as an arbitrary vertex.

The edges from 1 are,
 $1 \rightarrow (2, 28) (6, 10)$

the minimum edge is $(6, 10)$.

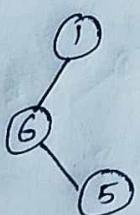
→ Now choose '6' as a vertex.



The edges from 6 are

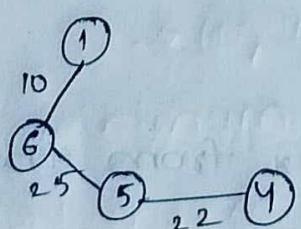
$$6 \rightarrow (5, 25)$$

→ choose '5' as the vertex



The edges from 5 are,

$$5 \rightarrow (4, 22) (7, 24)$$



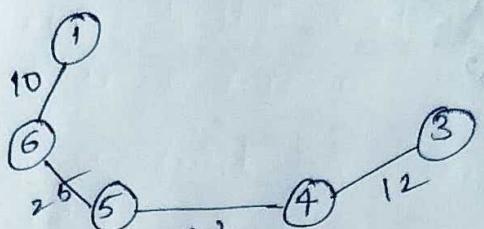
The minimum edge is $(4, 22)$

→ Now choose 4 as the vertex

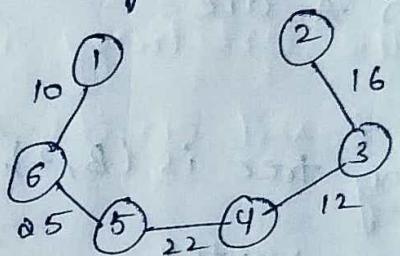
the edges passing from 4 are.

$$4 \rightarrow (7, 18) (3, 12)$$

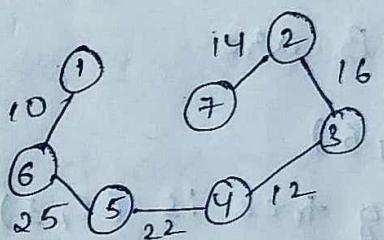
→ choose $(3, 12)$ as a vertex



the edges passing from 3 are $(2, 16)$



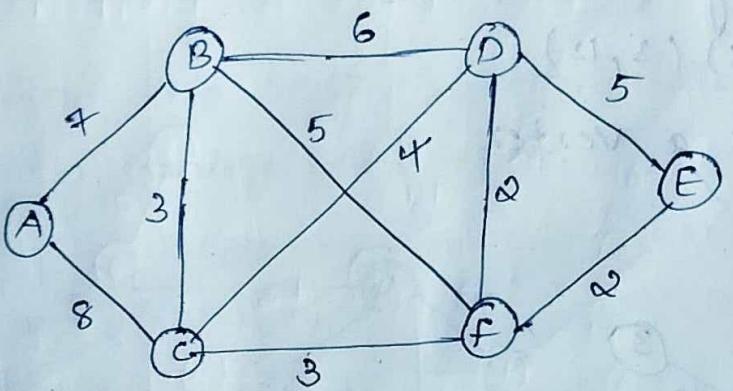
→ choose 2 as a vertex
and edges are $(7, 14)$



since all the vertices have been covered the
final MST = $10 + 25 + 22 + 12 + 16 + 14$
= 99

→ construct a MST for the following graph
using prims algorithm.

(*)



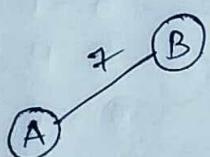
choose 'A' as arbitrary vertex

The edges passing from 'A' are

$$A \rightarrow (B, 7) (C, 8)$$

The minimum spanning tree is $(B, 7)$.

Now choose 'B' as arbitrary vertex

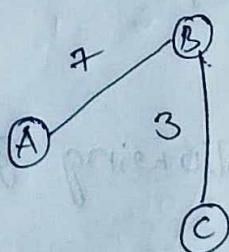


The edges passing from B are

$$B \rightarrow (C, 3) (F, 5) (D, 6)$$

The minimum spanning tree is $(C, 3)$

Now choose 'C' as arbitrary vertex

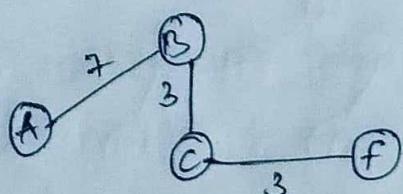


The edges passing from C are

$$C \rightarrow (D, 4), (F, 3)$$

The minimum spanning tree is $(F, 3)$

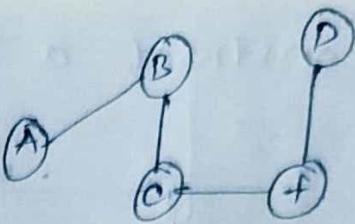
Now choose F as arbitrary vertex



the edges from f are

$$f \rightarrow (D, 2), (E, 2)$$

Now the M.S.T is $(D, 2)$

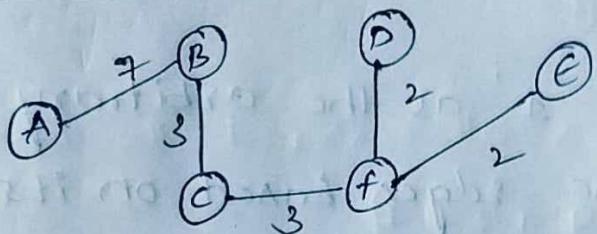


Now choose f as an arbitrary vertex

$$f \rightarrow (D, 5), (E, 2)$$

the minimum M.S.T $(E, 2)$

Now choose E as vertex



since all the vertices are covered the final

$$\text{MST is } 7 + 3 + 3 + 2 + 2 \\ = 17$$

→ Kruskals Algorithm.

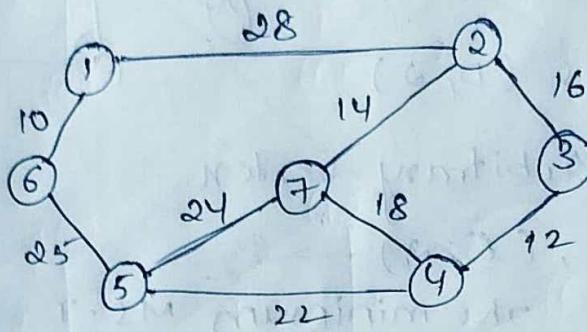
step-1: choose any arbitrary vertex from the given graph.

step-2: Sort all the edges based on its weight

step-3: choose the minimum weight edge and added to the tree until all the edges are covered without cycle.

step-4: stop.

→ Construct a MST for the following graph.



Sol:

- Step-1: choose '1' as the arbitrary vertex
Step-2: Sort the edges based on its weight.

Edge

wt

1-6

10

4-3

12

7-2

14

2-3 (closed)

16

7-4

18

5-4

22

7-5

24

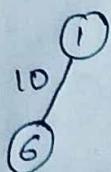
6-5

25

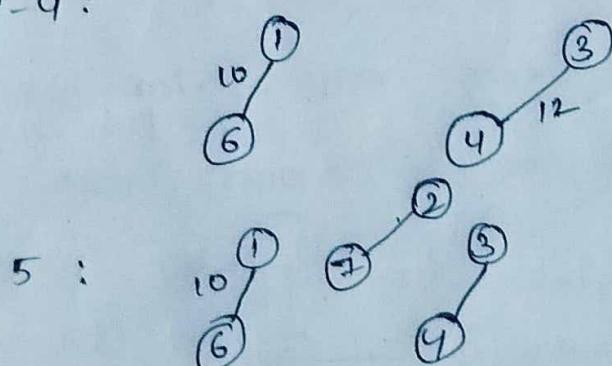
1-2

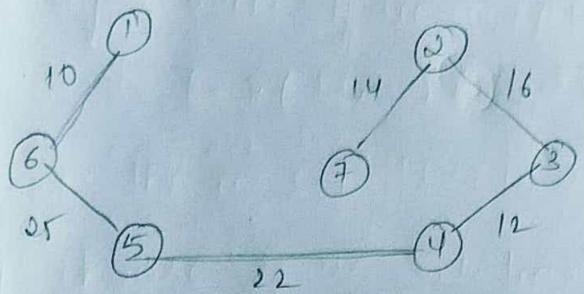
28

Step-3:



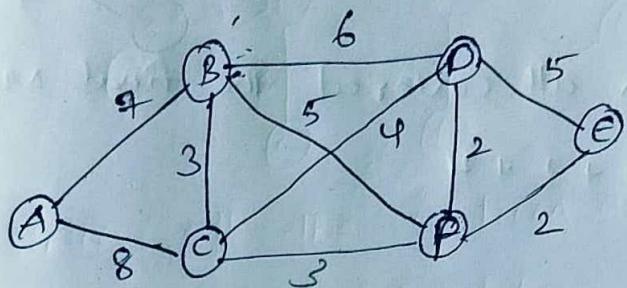
Step-4:





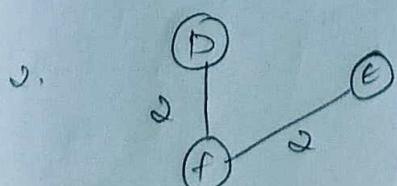
$$10 + 25 + 22 + 12 + 16 + 14 = 99$$

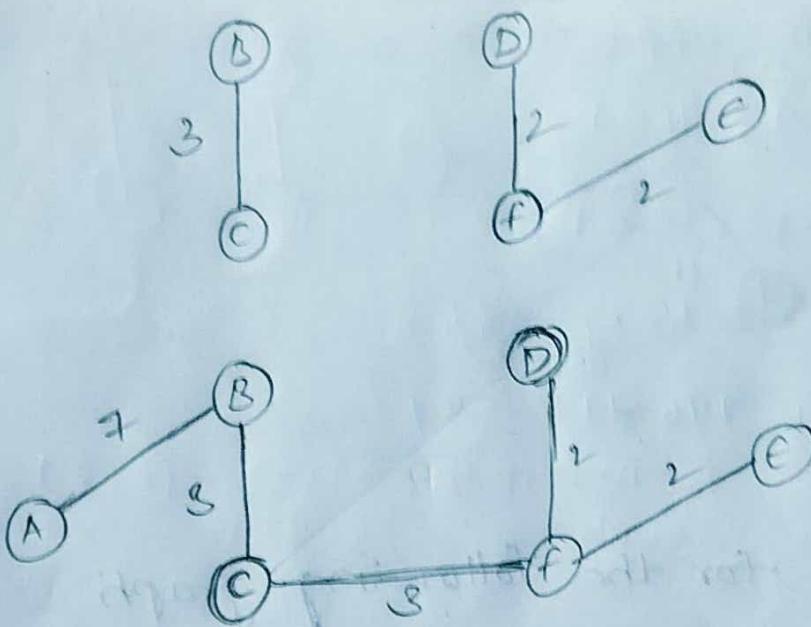
→ construct a MST for the following graph.



1. choose A as arbitrary vertex
2. sort the edges based on its weights.

edge	WT
D - f	2
f - e	2
B - c	3
c - f	4
c - d	5
B - f	5
D - e	6
B - d	7
A - B	8
A - C	





since all edges all covered, the final MST is

$$7 + 3 + 3 + 2 + 2$$

$$= 17$$