USN

18MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020

Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notations prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$. (06 Marks)

b. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$ (06 Marks)

c. Show that the radius of curvature for the catenary of uniform strength $y = a\log\sec\left(\frac{x}{a}\right)$ is $a\sec(x/a)$. (08 Marks)

OR

2 a. Show that the pairs of curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect each other Orthogonally. (06 Marks)

b. Find the pedal equation of the curve $r^n = a^n\cos n\theta$. (06 Marks)

c. Show that the evolute of $y^2 = 4ax$ is $27ay^2 = 4(x+a)^3$. (08 Marks)

Module-2

3 a. Find the Maclaurin's series for tan x upto the term x^4. (06 Marks)

b. Evaluate $\lim_{x\to 0}\left[\frac{a^x+b^x+c^x}{3}\right]^{1/x}$ (07 Marks)

c. If $U = f(x-y, y-z, z-x)$, prove that $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z} = 0$ (07 Marks)

OR

4 a. Expand $\log(\sec x)$ upto the term containing x^4 using Maclaurin's series. (06 Marks)

b. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

c. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2+y^2+z^2$, $v = xy + yz + zx$, $w = x + y + z$. (07 Marks)

Module-3

5 a. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dzdydx$ (06 Marks)

b. Evaluate $\int_{-2}^{2}\int_0^{\sqrt{4-x^2}}(2-x)dydx$ by changing the order of integration. (07 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

6 a. Evaluate $\iint y\,dx\,dy$ over the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. (06 Marks)

b. Find by double integration the area enclosed by the curve $r = a(1+\cos\theta)$ between $\theta = 0$ and $\theta = \pi$. (07 Marks)

c. Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta = \pi$. (07 Marks)

Module-4

7 a. Solve $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x} = 0$ (06 Marks)

b. Solve $r\sin\theta - \cos\theta\frac{dr}{d\theta} = r^2$ (07 Marks)

c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt}+Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t. (07 Marks)

OR

8 a. Solve $(4xy + 3y^2 - x)dx + x(x+2y)dy = 0$. (06 Marks)

b. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$. (07 Marks)

c. Solve $p^2 + 2py\cot x = y^2$. (07 Marks)

Module-5

9 a. Find the rank of $\begin{bmatrix}1 & 2 & 3 & 2\\ 2 & 3 & 5 & 1\\ 1 & 3 & 4 & 5\end{bmatrix}$ by elementary row transformations. (06 Marks)

b. Apply Gauss-Jordan method to solve the system of equations
$2x_1 + x_2 + 3x_3 = 1$,
$4x_1 + 4x_2 + 7x_3 = 1$,
$2x_1 + 5x_2 + 9x_3 = 3$. (07 Marks)

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$ by power method. Using initial vector $(100)^T$. (07 Marks)

OR

10 a. Solve by Gauss elimination method
$x - 2y + 3z = 2$,
$3x - y + 4z = 4$,
$2x + y - 2z = 5$ (06 Marks)

b. Solve the system of equations by Gauss-Seidal method
$20x + y - 2z = 17$,
$3x + 20y - z = -18$,
$2x - 3y + 20z = 25$ (07 Marks)

c. Reduce the matrix $A = \begin{bmatrix}-1 & 3\\ -2 & 4\end{bmatrix}$ to the diagonal form. (07 Marks)

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First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ are intersect orthogonally. (06 Marks)

b. Find the radius of curvature of the curve $y = a \log \sec\left(\frac{x}{a}\right)$ at any point (x, y). (06 Marks)

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. (08 Marks)

OR

2 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. (06 Marks)

b. Find the pedal equation of the curve $r = ae^{\theta\cot\alpha}$ (06 Marks)

c. Find the radius of curvature for the curve $r = a(1+\cos\theta)$. (08 Marks)

Module-2

3 a. Using Maclaurin's expansion. Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$. (06 Marks)

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$. (07 Marks)

c. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.cm. (07 Marks)

OR

4 a. If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

b. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. Find Jacobian $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$. (07 Marks)

c. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $x+y+z = 3a$. (07 Marks)

Module-3

5 a. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$, by changing into polar coordinates. (06 Marks)

b. Find the volume of the tetrahedron bounded by the planes :

$x = 0,\ y = 0,\ z = 0,\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1.$ (07 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

OR

6 a. Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$ by change of order of integration. (06 Marks)

b. Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)dy\,dx\,dz$. (07 Marks)

c. Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\,.d\theta \times \int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}\,.d\theta = \pi$. (07 Marks)

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

c. Solve $xyp^2 - (x^2 + y^2)p + xy = 0$. **(07 Marks)**

OR

8 a. Solve $\frac{dy}{dx} + y\tan x = y^2 \sec x$. **(06 Marks)**

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

c. Find the general solution of the equation $(px - y)(py + x) = 0$ by reducing into Clairaut's from, taking the substitution $X = x^2, Y = y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$ **(07 Marks)**

b. Solve the system of equations :

$$12x + y + z = 31$$
$$2x + 8y - z = 24$$
$$3x + 4y + 10z = 58$$

By Gauss –Siedal method. **(07 Marks)**

c. Diagonalize the matrix :

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}.$$ **(06 Marks)**

OR

10 a. For what values of λ and M the system of equations :

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + \lambda z = M$$

has i) no solution ii) a unique solution iii) infinite number of solution. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by Rayleigh's power method, use $[1\ 1\ 1]^T$ as the initial eigen vector (carry out 6 iterations). **(07 Marks)**

c. Solve the system of equations :

$$x + y + z = 9$$
$$2x + y - z = 0$$
$$2x + 5y + 7z = 52$$

By Gauss elimination method. **(06 Marks)**

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