CBCS SCHEME

USN

18MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ are intersect orthogonally. **(06 Marks)**

b. Find the radius of curvature of the curve $y = a \log \sec\left(\frac{x}{a}\right)$ at any point (x, y). **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

OR

2 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the pedal equation of the curve $r = ae^{\theta\cot\alpha}$. **(06 Marks)**

c. Find the radius of curvature for the curve $r = a(1+\cos\theta)$. **(08 Marks)**

Module-2

3 a. Using Maclaurin's expansion. Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$. **(07 Marks)**

c. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.cm. **(07 Marks)**

OR

4 a. If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(06 Marks)**

b. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. Find Jacobian $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

c. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $x+y+z = 3a$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$, by changing into polar coordinates. **(06 Marks)**

b. Find the volume of the tetrahedron bounded by the planes :

$x = 0$, $y = 0$, $z = 0$, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(07 Marks)**

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

OR

6 a. Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$ by change of order of integration. **(06 Marks)**

b. Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)dy\,dx\,dz$. **(07 Marks)**

c. Prove that $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$. **(07 Marks)**

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

c. Solve $xyp^2 - (x^2+y^2)p + xy = 0$. **(07 Marks)**

OR

8 a. Solve $\frac{dy}{dx} + y\tan x = y^2\sec x$. **(06 Marks)**

b. Show that the family of parabolas $y^2 = 4a(x+a)$ is self orthogonal. **(07 Marks)**

c. Find the general solution of the equation $(px-y)(py+x) = 0$ by reducing into Clairaut's from, taking the substitution $X = x^2$, $Y = y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$
(07 Marks)

b. Solve the system of equations :

$$12x + y + z = 31$$
$$2x + 8y - z = 24$$
$$3x + 4y + 10z = 58$$

By Gauss –Siedal method. **(07 Marks)**

c. Diagonalize the matrix :

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}.$$
(06 Marks)

OR

10 a. For what values of λ and M the system of equations :

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + \lambda z = M$$

has i) no solution ii) a unique solution iii) infinite number of solution. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by Rayleigh's power method, use $[1\ 1\ 1]^T$ as the initial eigen vector (carry out 6 iterations). **(07 Marks)**

c. Solve the system of equations :

$$x + y + z = 9$$
$$2x + y - z = 0$$
$$2x + 5y + 7z = 52$$

By Gauss elimination method. **(06 Marks)**

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