Yashas.H.D

As per Revised Syllabus of

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

Basic Electrical Engineering

Semester - I/II (Common to All Branches)

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PREFACE

The importance of **Basic Electrical Engineering** is well known in various engineering fields. Overwhelming response to our books on various subjects inspired us to write this book. The book is structured to cover the key aspects of the subject **Basic Electrical Engineering.**

The book uses plain, lucid language to explain fundamentals of this subject. The book provides logical method of explaining various complicated concepts and stepwise methods to explain the important topics. Each chapter is well supported with necessary illustrations, practical examples and solved problems. All the chapters in the book are arranged in a proper sequence that permits each topic to build upon earlier studies. All care has been taken to make students comfortable in understanding the basic concepts of the subject.

Representative questions have been added at the biginning of each section to help the students in picking important points from that section.

The book not only covers the entire scope of the subject but explains the philosophy of the subject. This makes the understanding of this subject more clear and makes it more interesting. The book will be very useful not only to the students but also to the subject teachers. The students have to omit nothing and possibly have to cover nothing more.

We wish to express our profound thanks to all those who helped in making this book a reality. Much needed moral support and encouragement is provided on numerous occasions by our whole family. We wish to thank the **Publisher** and the entire team of **Technical Publications** who have taken immense pain to get this book in time with quality printing.

Any suggestion for the improvement of the book will be acknowledged and well appreciated.

Authors
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Dedicated to Laxmi and Narayana

SYLLABUS

Basic Electrical Engineering [18ELE13/18ELE23]

Module-1

D.C. Circuits : Ohm's Law and Kirchhoff's Laws, analysis of series, parallel and series - parallel circuits excited by independent voltage sources. Power and Energy.

A.C. Fundamentals : Generation of sinusoidal voltage, frequency of generated voltage, definition and numerical values of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities. **(Chapters-1, 2)**

Module-2

Single Phase Circuits : Analysis, with phasor diagram, of circuits with R, L, C, R-L, RC, R-L-C for series and parallel configurations. Real power, reactive power, apparent power and power factor.

Three Phase circuits : Advantages of 3-phase power, Generation of 3-phase power, Three-phase balanced circuits, voltage and current relations in star and delta connections. Measurement of three phase power using two wattmeter method. **(Chapters-3, 4)**

Module-3

Single Phase Transformers : Necessity of transformer, Principle of operation, Types and construction of transformers. emf equation, losses, variation of losses with respect to load, efficiency, Condition for maximum efficiency.

Domestic Wiring : Service mains, meter board and distribution board. Brief discussion on concealed conduit wiring. Two-way and three-way control Elementary discussion on circuit protective devices : Fuse and Miniature Circuit Breaker (MCB 's),electric shock, precautions against shock. Earthing : Pipe and Plate earthing. **(Chapters-5, 6)**

Module-4

DC Generators : Principle of operation, Construction of D.C. Generators. Expression for induced emf,Types of D.C. Generators,Relation between induced emf and terminal voltage.

DC motors : Principle of operation,Back emf,Torque equation, Types of dc motors, Characteristics of dc motors (shunt and series motors only) and Applications. **(Chapters-7, 8)**

Module-5

Three Phase Synchronous Generators : Principle of operation, Constructional details, Synchronous speed, Frequency of generated voltage, emf equation, Concept of winding factor (excluding the derivation and calculation of distribution and pitch factors).

Three Phase Induction Motors : Principle of operation, Generation of rotating magnetic field, Construction and working of three-phase induction motor, Slip and its significance. Necessity of starter, star-delta starter.

(Chapters-9, 10)

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MODULE - 1

1

D.C. Circuits

Syllabus

Ohm's Law and Kirchhoff's Laws, analysis of series, parallel and series - parallel circuits excited by independent voltage sources. Power and Energy. Illustrative Examples.

Contents

1.1 Concept of Electric Charge

➤ ***Explain the concept of an electric charge. State the unit of charge.***

- The matter on the earth which occupies the space may be solid, liquid or gaseous and is made up of many atoms which are of similar nature.
- According to modern electron theory, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron**, the **proton** and the **electron**.
- The proton is positively charged while the electron is negatively charged. The neutron is electrically neutral i.e. possessing no charge.
- Atom as a whole is electrically neutral as the number of protons is always equal to the number of electrons.
- If by some means electron is removed from an atom, it will loose negative charge and will become positively charged. Such positively charged atom is called **cation**. As against this, if excess electron is added to an atom, it will become negatively charged. Such negatively charged atom is called **anion.**

> • This total deficiency or addition of excess electrons in an atom is called as its **charge** and the atom is said to be **charged.** The unit of charge is **coulomb.**

- The change on one electron and one proton is equal in magnitude and given by 1.602×10^{-19}C. Hence the coulomb of charge means the total charge possessed by $\frac{1}{1.602 \times 10^{-19}}$ electrons ie. 6.24×10^{18} number of electrons.

Thus, $\boxed{1 \text{ coulomb} = \text{Charge on } 6.24 \times 10^{18} \text{ electrons}}$

1.2 Electromotive Force and Current

➤ ***Explain the concept of electromotive force and current.***

- A conductor is one which has abundant free electrons. The fr ee electrons in such a conductor are always moving in random directions as shown in the Fig. 1.2.1.

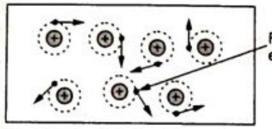


Fig. 1.2.1 Inside the piece of a conductor

- The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction.
- This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected

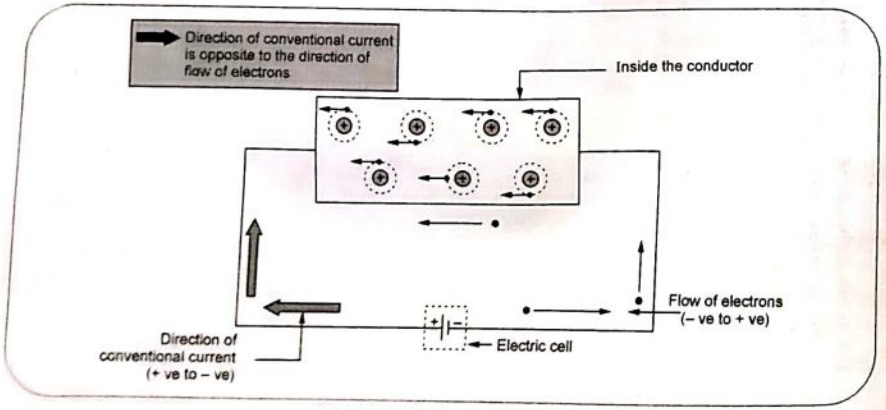


Fig. 1.2.2 The flow of current

across the two ends of a conductor. Such physical phenomenon is represented in the Fig. 1.2.2.

> • An electrical effort required to drift the free electrons in one particular direction, in a conductor is called **Electromotive Force** (e.m.f.). **It is measured in volts.**

- When free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor.
- So there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an **Electric Current.**
- The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called **direction of conventional current.**

1.3 Relation between Charge and Current

➤ ***State the relation between charge and current. Define unit of a current.***

> • The current can be defined as **rate of flow of charge in an electric circuit** or in any medium in which charges are subjected to an external electric field.

- The unit for the current is **Amperes** which is nothing but coulombs/sec.
- Mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$I = \frac{Q}{t} \quad \text{Amperes}$$

Where I = Average current flowing while

Q = Total charge transferred

t = Time required for transfer of charge.

> **Definition of 1 Ampere :** *A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.*

1.4 Electric Potential and Potential Difference

➤ ***Define electric potential. State its unit.***

- When two similarly charged particles are brought near, they try to repel each other while dissimilar charges attract each other. This means, every charged particle has a tendency to do work.

> • This ability of a charged particle to do the work is called its **electric potential**. The unit of electric potential is **volt.**

- The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge i.e. positive charge of one coulomb from infinity to that point.
- Mathematically it is expressed as,

$$\boxed{\text{Electrical Potential} = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}}$$

➤ ***Define potential difference and mention its unit.***

- It is well known that, flow of water is always from higher level to lower level, flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on, also exists in electric circuits. In electric circuits, flow of current is always from higher electric potential to lower electric potential.

> • The difference between the electric potentials at any two given points in a circuit is known as **Potential Difference (p.d.).** This is also called voltage between the two points and measured in **volts.** The symbol for voltage is V.

- To maintain the flow of electrons i.e. flow of electric current, there must exist a potential difference between the two points.

> • No current can flow if the potential difference between the two points is zero.

1.5 Resistance

➤ ***Define resistance. State and define its unit.***

> • The property of an electric circuit opposing the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.

• Higher the availability of the free electrons, lesser will be the opposition to the flow of current. A conductor having high number of free electrons offer less resistance to the flow of current.

• The resistance is denoted by the symbol 'R' and is measured in **ohm** symbolically represented as Ω . We can define unit ohm as below :

> **Definition : 1 Ohm :** The resistance of a circuit, in which a current of 1 Ampere generates the heat at the rate of one joules per second is said to be 1 ohm.

4.186 joules	= 1 calorie
and 1 joule	= 0.24 calorie

• Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

➤ ***State and explain the factors affecting the resistance.***

1. Length of the material : The resistance of a material is directly proportional to the length. Length is denoted by 'l' .

2. Cross-sectional area : The resistance of a material is inversely proportional to the cross-sectional area of the material. The cross sectional area is denoted by 'a'.

3. The type and nature of the material : If the material is conductor, its resistance is less while if it is insulator, its resistance is very high.

4. Temperature : As temperature changes, the value of the resistance of the material changes.

• So for a certain material at a certain constant temperature we can write a mathematical expression as,

$$\boxed{R \propto \frac{l}{a}}$$

• The effect of nature of material is considered through the constant of proportionality denoted by ρ (rho) called **resistivity** or **specific resistance** of the material.

So finally,

$$\boxed{R = \frac{\rho l}{a}}$$

Where l = Length in metres,
a = Cross-sectional area in square metres
ρ = Resistivity in ohms-metres,
R = Resistance in ohms

1.6 Energy Sources

• There are two types of energy sources : Voltage source and current source.

• These are further classified as ideal source and practical source.

➤ ***Explain the ideal and practical voltage source.*** VTU : Mar.-02, Marks 4

• Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals.

• The symbol for ideal voltage source is shown in the Fig. 1.6.1 (a). This is connected to the load as shown in Fig. 1.6.1 (b). The Fig. 1.6.1 (c) shows the V- I characteristics of ideal voltage source.

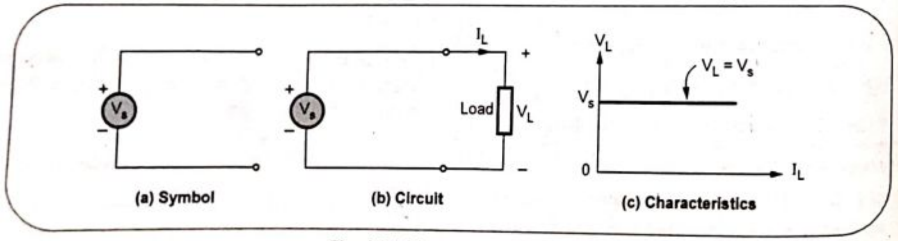


Fig. 1.6.1 Ideal voltage source

• But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.6.2.

• Because of the R_{se}, voltage across terminals decreases with increase in the load current and it is given by expression,

$$\boxed{V_L = -(R_{se})\, I_L + V_s = V_s - I_L\, R_{se}}$$

> • *For ideal voltage source,* $R_{se} = 0$ and for practical voltage source it is as small as possible.

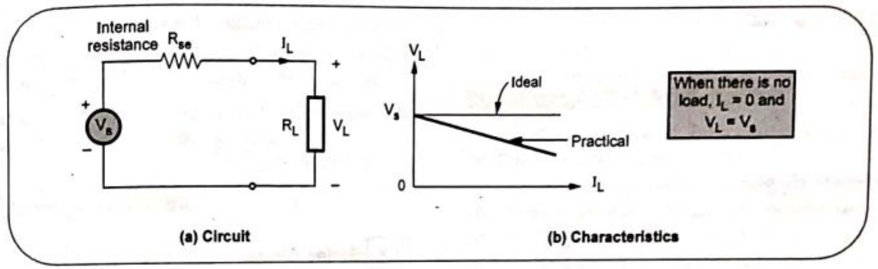


Fig. 1.6.2 Practical voltage source

➤ ***Explain the ideal and practical current source.*** VTU : Aug. 01, 03, Marks 5

• Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals.

• The symbol for ideal current source is shown in the Fig. 1.6.3 (a). This is connected to the load as shown in the Fig. 1.6.3 (b). The Fig. 1.6.3 (c) shows the V-I characteristics of ideal current source.

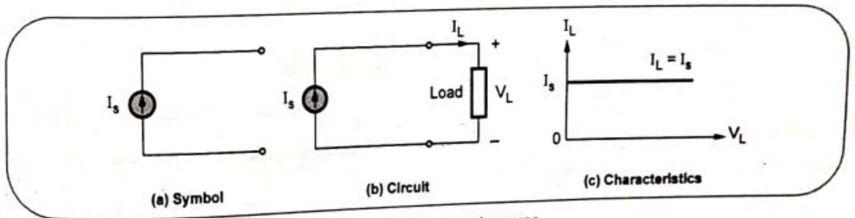


Fig. 1.6.3 Ideal current source

• But practically, every current source has high internal resistance, which is in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.6.4.

• Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

> • For ideal current source, $R_{sh} = \infty$ while for practical current source it is as high as possible.

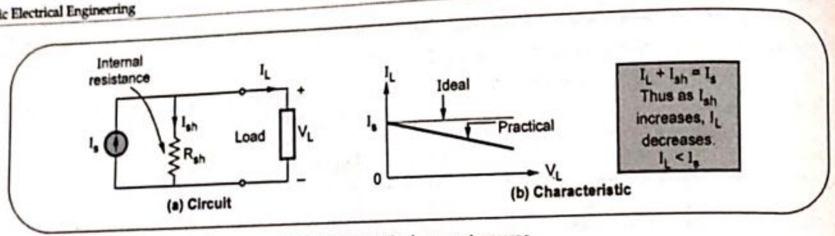


Fig. 1.6.4 Practical current source

1.7 Ohm's Law

➤ ***State Ohm's law and its limitations.***

VTU : Jan.-08, 16, 18, July-08, 16, 17, Marks 6

- The Ohm's law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit.
- It states that, the current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.
- Mathematically, $I \propto \frac{V}{R}$ Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 1.7.1.

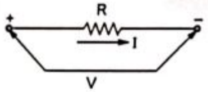


Fig. 1.7.1 Ohm's law

Then $I = \frac{V}{R}$

- The unit of potential difference is defined in such a way that the constant of proportionality is unity.

> - Ohm's Law is, $I = \frac{V}{R}$ amperes or $V = IR$ volts
> or $\frac{V}{I}$ = Constant = R ohms

- The Ohm's law can be defined as, the ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

- The **limitations of the Ohm's law** are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = k\, I^m \quad \text{where k, m are constants.}$$

1.8 Series Circuit

➤ ***Derive an expression for an equivalent resistance of n resistances connected in series.***

- A series circuit is one in which several resistances are connected one after the other. Such connection is also called end to end connection or **cascade** connection. There is only one path for the flow of current.
- Consider the resistances shown in the Fig. 1.8.1.

Current same voltage division

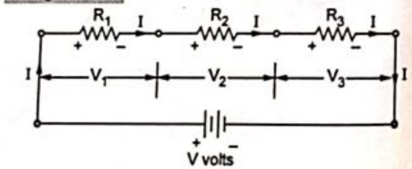


Fig. 1.8.1 A series circuit

- The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts.
- Naturally the current flowing through all of them is same indicated as I amperes.

- Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively then, $V = V_1 + V_2 + V_3$.
- Now according to Ohm's law, $V_1 = I R_1$, $V_2 = I R_2$, $V_3 = I R_3$
- Current through all of them is same i.e. I.

$$\therefore \quad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

- Applying Ohm's law to overall circuit,

$$V = I\, R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit.

- By comparison of two equations, $R_{eq} = R_1 + R_2 + R_3$
- Thus the total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

> For n resistances in series,
> $$R_{eq} = R_1 + R_2 + R_3 + \ldots\ldots + R_n$$

➤ ***List the characteristics of series circuit.***

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots. + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $R > R_1$, $R > R_2$, ... $R > R_n$

1.9 Parallel Circuit

➤ ***Derive an expression for an equivalent resistance of 'n' resistances connected in parallel.***

- The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.
- Consider a parallel circuit shown in the Fig. 1.9.1.
- In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel

Voltage same current division

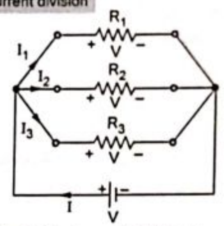


Fig. 1.9.1 A parallel circuit

and combination is connected across a source of voltage 'V'.

- In parallel circuit current passing through each resistance is different. Let total current drawn is say 'I' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. These individual currents are shown as I_1, I_2 and I_3.
- The voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V. Hence apply Ohm's law to each resistance,

$V = I_1 R_1$, $V = I_2 R_2$, $V = I_3 R_3$

i.e. $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, $I_3 = \frac{V}{R_3}$

But $$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \ldots (1.9.1)$$

- For overall circuit the Ohm's law can be applied as,

$$V = I\, R_{eq} \quad \text{and} \quad I = \frac{V}{R_{eq}} \quad \ldots (1.9.2)$$

where R_{eq} = Total or equivalent resistance of the circuit

- Comparing the two equations, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

> In general if 'n' resistances are connected in parallel,
> $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

➤ ***List the characteristics of parallel circuit.***

VTU : June-12, Marks 4

1) The voltage or potential difference across all the resistances in parallel is always same.

2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

4) The equivalent resistance is the smallest of all the resistances.

$$R < R_1, \quad R < R_2, \ldots\ldots, R < R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

> **Key Point** The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.

Ex. 1.9.1 *Find the resistance of the circuit shown (R_{AD}).* **VTU : June-13, Marks 5**

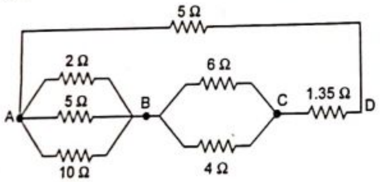


Fig. 1.9.2

Sol. : The resistances 2 Ω, 5 Ω, 10 Ω are in parallel giving,

$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} \text{ i.e. } R' = 1.25\ \Omega$$

The resistances of 6 Ω and 4 Ω are in parallel giving,

$$R'' = \frac{6 \times 4}{6+4} = 2.4\ \Omega$$

Hence the circuit becomes as shown in the Fig. 1.9.2 (a).

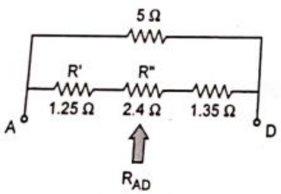


Fig. 1.9.2 (a)

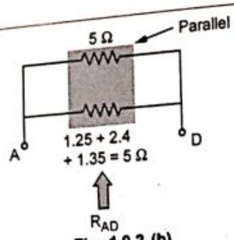


Fig. 1.9.2 (b)

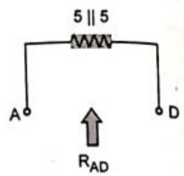


Fig. 1.9.2 (c)

$$\therefore \quad R_{AD} = \frac{5 \times 5}{5+5} = 2.5\ \Omega$$

Ex. 1.9.2 *A resistance of 10 Ω is connected in series with the two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken will be 1.5 A from 20 V supply applied ?* **VTU : Aug. 95, Marks 6**

Sol. : The arrangement is shown in the Fig. 1.9.3. The resistance R is required to be shunted across parallel combination.

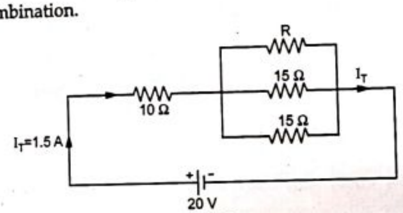


Fig. 1.9.3

The parallel combination of R, 15 Ω and 15 Ω is say R_p.

$$\therefore \quad \frac{1}{R_p} = \frac{1}{R} + \frac{1}{15} + \frac{1}{15} = \frac{1}{R} + \frac{2}{15}$$

$$\text{i.e.} \quad R_p = \frac{15R}{15+2R}$$

Hence circuit reduces as,

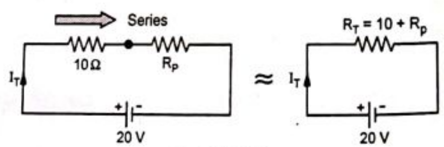


Fig. 1.9.3 (a)

$$\text{Now } R_T = 10 + R_p = 10 + \frac{15R}{15+2R} = \frac{150+20R+15R}{15+2R}$$

$$\therefore \quad R_T = \frac{150+35R}{15+2R} \quad \ldots(1)$$

$$\text{Now} \quad I_T = 1.5 \text{ A required} \quad \ldots\text{(Given)}$$

$$\therefore \quad R_T = \frac{V}{I_T} = \frac{20}{1.5} = 13.33\ \Omega \quad \ldots(2)$$

Equating equation (1) and equation (2),

$$\frac{150+35R}{15+2R} = 13.33$$

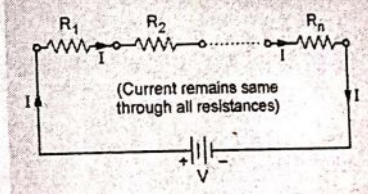
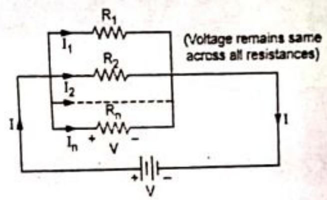
$$\therefore \quad 150 + 35R = 199.95 + 26.66R$$

$$\therefore \quad 8.34R = 49.95$$

$$\therefore \quad R = 6\ \Omega \quad \ldots \text{Resistance to be shunted}$$

1.10 Comparison of Series and Parallel Circuits

➤ ***Compare series and parallel circuits of resistances.***

Sr. No.	Series Circuit	Parallel Circuit
1.	The connection is as shown, 	The connection is as shown, 
2.	The same current flows through each resistance.	The same voltage exists across all the resistances in parallel.
3.	The voltage across each resistance is different.	The current through each resistance is different.
4.	The sum of the voltages across all the resistances is the supply voltage.	The sum of the currents through all the resistances is the supply current.
5.	The equivalent resistance is, $R_{eq} = R_1 + R_2 + \ldots + R_n$	The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
6.	The equivalent resistance is the largest than each of the resistances in series.	The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

1.11 Kirchhoff's Laws

VTU : July-07, 11, 16, Marks 6; Jan.-09, 11, 13, 16, 18, Marks 5
VTU : Jan-17, July 17, Marks 5

➤ ***State and explain Kirchhoff's laws.***

➤ ***Define KCL and KVL with example.***

- There are two Kirchhoff's laws.

1. Kirchhoff's Current Law (KCL)

- The law can be stated as,

  The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

- Another way to state the law is,

  The algebraic sum of all the current meeting at a junction point is always zero.

- The word algebraic means considering the signs of various currents.

$$\boxed{\sum I \text{ at junction point } = 0}$$

> *Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.*

- Consider a junction point in a complex network as shown in the Fig. 1.11.1. The currents I_1 and I_2 are positive as entering the junction while I_3 and I_4 are negative as leaving the junction.

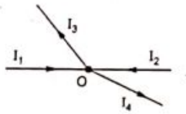


Fig. 1.11.1 Junction point

- Applying KCL, $\sum I$ at junction O = 0

$$I_1 + I_2 - I_3 - I_4 = 0 \quad \text{i.e.} \quad I_1 + I_2 = I_3 + I_4$$

- The law is very helpful in network simplification.

2. Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f. s in the path"

➤ ***In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."***

$$\boxed{\text{Around a closed path } \sum V = 0}$$

- The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.
- Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

➤ ***Explain the sign convention used for the application of KVL.***

- Sign conventions for KVL can be stated as :
- When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.
- In the Fig. 1.11.2 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.
- In the Fig. 1.11.2 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

(a) Fig. 1.11.2 (b)

- Now while tracing a closed path for KVL, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise**. For example, if the branch AB is traced from A to B in the Fig. 1.11.2 (a) then the drop across it must be considered as rise and must be taken as + IR.
- While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop**. For example, in the Fig. 1.11.2 (a) only, if the branch is traced from B to A then it should be taken as negative, as – IR while in the Fig. 1.11.2 (b) if the branch is traced from A to B then it should be taken as negative, as – IR.

➤ ***Explain the steps to apply Kirchhoff's laws for solving the circuit.***

Step 1 : Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2 : Mark all the branch currents with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

- A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3 : Mark all the polarities of voltage drops and rises as per the directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4 : Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equations.

> - *KVL must be applied to sufficient number of loops such that each element of the network is included at least once in any of the equations.*

Step 5 : Solve the simultaneous equations for the unknown currents. From these currents, unknown voltages and power consumption in different resistances can be calculated.

Ex.1.11.1 *For the Fig. 1.11.3 calculate the current in 2 Ω resistor.* **VTU : Feb.-2000, Jan.-16, Marks 6**

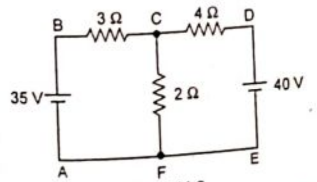


Fig. 1.11.3

Sol. : Step 1 : The currents are shown in the Fig. 1.11.3 (a).

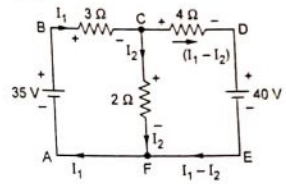


Fig. 1.11.3 (a)

Step 2 : Mark the polarities of voltage drops.

Step 3 : Apply KVL to the loops ABCFA and CDEFC,

$$35 - 3I_1 - 2I_2 = 0 \quad \text{i.e.} \quad 3I_1 + 2I_2 = 35 \qquad \text{... (1)}$$

$$-4(I_1 - I_2) - 40 + 2I_2 = 0 \quad \text{i.e.} \quad -4I_1 + 6I_2 = 40 \qquad \text{... (2)}$$

Step 4 : Solving, I_1 = 5 A, I_2 = 10 A

Step 5 : Current in 2 Ω = I_2 = 10 A

Ex. 1.11.2 *For the circuit shown in Fig. 1.11.4. Obtain voltage between points X and Y.* **VTU : Aug.-99, July-09,15,17, Marks 6**

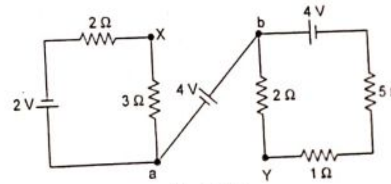


Fig. 1.11.4

Sol. : The branch currents are shown in the Fig. 1.11.4 (a). Apply KVL to the loop cXaac,

$$+2I_1 + 3I_1 - 2 = 0 \quad \text{i.e.} \quad I_1 = 0.4 \text{ A}$$

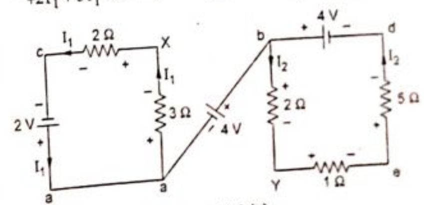


Fig. 1.11.4 (a)

Apply KVL to the loop bdeYb,

$-4+5I_2+I_2+2I_2 = 0$

i.e. $I_2 = 0.5$ A.

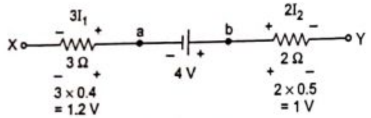


Fig. 1.11.4 (b)

To find V_{XY}, consider path XabY and write the voltage drops as shown in the Fig. 1.11.4 (b) across various elements. Adding algebraically the various voltages,

$V_{XY} = 4+1.2-1$

= 4.2 V with X negative w.r.t. Y.

Ex. 1.11.3 *Find the currents in the various branches of the given network shown in Fig. 1.11.5*

VTU : July-16, Jan.-15, Marks 6

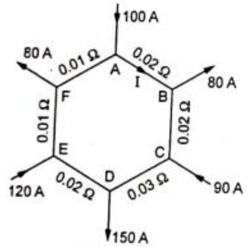


Fig. 1.11.5

Sol. : Let the current distribution is as shown in the Fig. 1.11.5 (a).

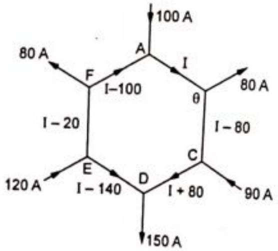


Fig. 1.11.5 (a)

Using KCL at various nodes. Applying KVL to the loop ABCDEFA,

$I(0.02)+(I-80)\,0.02+(I+10)\,0.03$
$+(I-140)\,0.02+(I-20)\,0.01+(I-100)\,0.01 = 0$

Solving, $0.11\,I = 5.3$ i.e. $I = 48.1818$ A

Hence currents in the various branches are,

AB = **48.1818 A**, BC = **– 31.8182 A**,

CD = **58.1818 A**, DE = **– 91.8182 A**,

EF = **28.1818 A**, FA = **– 51.8182 A**

Negative sign indicates that the direction is opposite to that assumed in the Fig. 1.11.5 (a).

Ex. 1.11.4 *Find current in the battery, the current in each branch and p.d. across AB in the network shown in Fig. 1.11.6.*

VTU : June-12, Marks 6

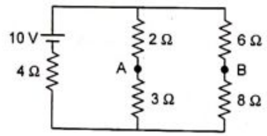


Fig. 1.11.6

Sol. : The currents are shown in the Fig. 1.11.6 (a).

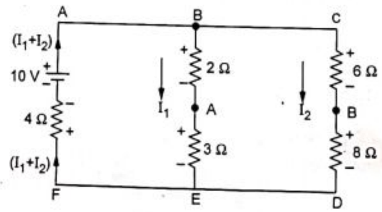


Fig. 1.11.6 (a)

Applying KVL to the two loops,

$$-2I_1-3I_1-4(I_1+I_2)+10 = 0$$

i.e.

$$9I_1+4I_2 = 10 \qquad \text{...(1)}$$

$$-6I_2-8I_2+3I_1+2I_1 = 0$$

i.e.

$$5I_1-14I_2 = 0 \qquad \text{...(2)}$$

Solving, I_1 = **0.9589 A,**

I_2 = **0.34246 A**

∴ Battery current = I_1+I_2 = **1.3013 A**

Tracing path AEDB as shown in the Fig. 1.11.6 (b).

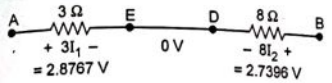


Fig. 1.11.6 (b)

∴ V_{AB} = 2.8767 – 2.7396

= **0.13702 V with A +ve**

Ex. 1.11.5 *In the parallel arrangement of resistors shown the current flowing in the 8 Ω resistor is 2.5 amperes. Find i) Current in other resistors ii) Resistor X iii) The equivalent resistance. Refer Fig. 1.11.7.*

VTU : June-13, Marks 6

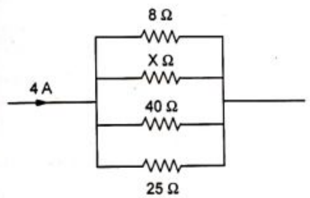


Fig. 1.11.7

Sol. :

The drop across 8 Ω = $8\times I_{8\,\Omega}$ = 8×2.5 = 20 V.

This voltage remains same across all the parallel resistances.

i) $I_{40\,\Omega} = \dfrac{20}{40}$ = **0.5 A,**

$I_{25\,\Omega} = \dfrac{20}{25}$ = **0.8 A,** $I_X = \dfrac{20}{X}$ ... (1)

Now $I_X = I_T - I_{8\,\Omega} - I_{40\,\Omega} - I_{25\,\Omega}$ (KCL)

... I_T = Total current = 4 A

∴ I_X = 4 – 2.5 – 0.5 – 0.8 = **0.2 A**

ii) Using equation (1),

$0.2 = \dfrac{20}{X}$ = i.e. X = 100 Ω

iii) $\dfrac{1}{R_{eq}} = \dfrac{1}{8}+\dfrac{1}{100}+\dfrac{1}{40}+\dfrac{1}{25} = 0.2$

∴ R_{eq} = 5 Ω ... Equivalent resistance

Ex. 1.11.6 *Find the value of resistance 'R' as shown in Fig. 1.11.8, so that current drawn from the source is 250 mA. All the resistor values are in ohm.*

VTU : Jan.-14, Marks 6

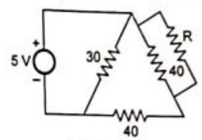


Fig. 1.11.8

Sol. : The resistance R and 40 Ω are in parallel.

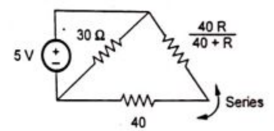


Fig. 1.11.8 (a)

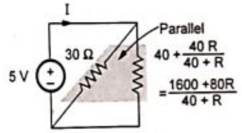


Fig. 1.11.8 (b)

$$\therefore \quad I = \frac{5}{(30)\,||\left(\frac{1600+80\,R}{40+R}\right)}$$

$$= \frac{5}{\dfrac{30\times\left[\frac{1600+80\,R}{40+R}\right]}{30+\frac{1600+80\,R}{40+R}}}$$

$$\therefore \quad I = \frac{5\,[1200+30\,R+1600+80\,R]}{30\,(1600+80\,R)}$$

but I = 250 mA given

∴ $250\times10^{-3}\times30\times[1600+80\,R] = 5\,[2800+110\,R]$

Solving, $R = \dfrac{2000}{50} = 40\ \Omega$

Ex. 1.11.7 *In the network shown in Fig. 1.11.9, find the currents flowing in each branch using Kirchhoff's laws.*

VTU : July-16, Marks 6

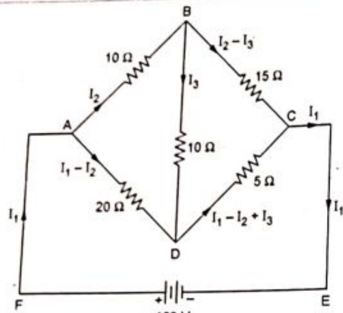


Fig. 1.11.9

Ans. : The various polarities are marked as shown in the Fig. 1.11.9 (a).

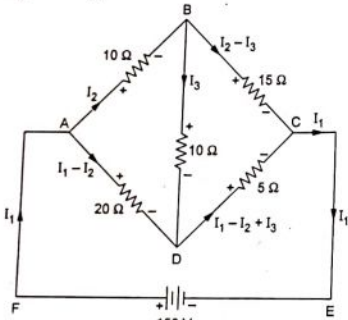


Fig. 1.11.9 (a)

Applying KVL to the loops ABDA, BCDB, ADCEFA,

$$-10I_2 - 10I_3 + 20(I_1 - I_2) = 0$$

i.e. $$20I_1 - 30I_2 - 10I_3 = 0 \quad ...(1)$$

$$-15(I_2 - I_3) + 5(I_1 - I_2 + I_3) + 10I_3 = 0$$

i.e. $$5I_1 - 20I_2 + 30I_3 = 0 \quad ...(2)$$

$$-20(I_1 - I_2) - 5(I_1 - I_2 + I_3) + 150 = 0$$

i.e. $$25I_1 - 25I_2 + 5I_3 = 150 \quad ...(3)$$

Solving, $I_1 = 13.2$ A, $I_2 = 7.8$ A, $I_3 = 3$ A

Current flowing in various branches is,

AB = 7.8 A, BC = 4.8 A, CD = 8.4 A,

AD = 5.4 A, BD = 3 A

Ex. 1.11.8 *In the network shown in Fig. 1.11.10, determine current flow in the ammeter 'A' having resistance of 10 Ω.*

VTU : Jan-16, Marks 5

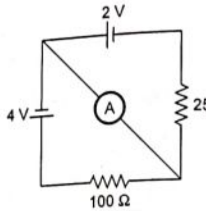


Fig. 1.11.10

Sol. : The various branch currents are shown in the Fig. 1.11.10 (a)

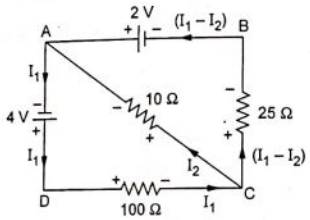


Fig. 1.11.10 (a)

Applying KVL to the two loops ADCA and ABCA,

$$+4 - 100\ I_1 - 10\ I_2 = 0$$

i.e. $$100\ I_1 + 10\ I_2 = 4 \quad ...(1)$$

$$-2 + 25(I_1 - I_2) - 10\ I_2 = 0$$

i.e. $$25\ I_1 - 35\ I_2 = 2 \quad ...(2)$$

$$I_1 = 0.0426 \text{ A}, \quad I_2 = -0.0266 \text{ A}$$

$\therefore$ Ammeter current $= I_2 = -0.0266$ A

Negative sign indicates that direction of I_2 is from A to C and magnitude **0.0266 A**

1.12 Electrical Work

> ***Define electrical work done and its unit.***

In an electric circuit, movement of electrons i.e. transfer of charge is an electric current. The electrical work is done when there is a transfer of charge. The unit of such work is Joule.

- *One joule of electrical work done is that work done in moving a charge of 1 coulomb through a potential difference of 1 volt.*
- So if V is potential difference in volts and Q is charge in coulombs then we can write,

$$\text{Electrical work } W = V \times Q \text{ J} \quad \text{But } I = \frac{Q}{t}$$

$$\therefore \quad W = V I t \text{ J} \qquad \text{Where } t = \text{Time in seconds}$$

1.13 Electrical Power

> ***Define electrical power and obtain its expression. State its unit.***

- The rate at which electrical work is done in an electric circuit is called an electrical power.

$$\text{Electrical power} \quad P = \frac{\text{Electrical work}}{\text{Time}} = \frac{W}{t} = \frac{V I t}{t}$$

$$\therefore \quad P = V I \text{ J/sec i.e. watts}$$

- Thus power consumed in an electric circuit is 1 watt if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.

Remember, 1 watt = 1 joule/sec

- As unit of power watt is small, many a times power is expressed as kW (1000 watts) or MW (1×10^6 watts).
- According to Ohm's law, $V = I R$ or $I = V/R$
- Using this, power can be expressed as,

$$P = V I = I^2 R = \frac{V^2}{R} \qquad \text{Where R = Resistance in } \Omega$$

Ex. 1.13.1 *The total power consumed by the network shown in Fig. 1.13.1 if 16 W. Find the value of R and the total current.*

VTU : April-97, May-10, Jan.-18, Marks 6

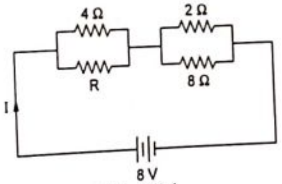


Fig. 1.13.1

Sol. : The resistances 4 and R in parallel while 2 and 8 are in parallel. So combining them circuit reduces as shown in the Fig. 1.13.1 (a).

$$I_T = \frac{8}{\frac{4R}{4+R} + 1.6} = \frac{8(4+R)}{5.6R + 6.4} \quad ...(1)$$

The total power dissipated in the network is 16 W.

$$\therefore \quad V \times I_T = 16$$

i.e. $$\frac{8 \times 8(4+R)}{5.6R + 6.4} = 16 \qquad \text{... Using (1)}$$

$$\therefore \quad 256 + 64\ R = 89.6\ R + 102.4$$

i.e. $$25.6\ R = 153.6$$

$$\therefore \quad R = 6\ \Omega$$

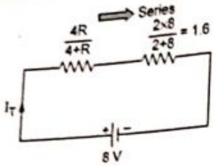


Fig. 1.13.1 (a)

Substituting in the equation (1),

$$I_T = \frac{8(4+6)}{5.6\times6+6.4} = 2\text{ A}$$

Ex. 1.13.2 *A 8 ohm resistor is in series with a parallel combination of two resistors 12 ohm and 6 ohm. If the current in the 6 ohm resistor is 5 A, determine the total power dissipated in the circuit.* **VTU : Feb. 09, Marks 6**

Sol. : The arrangement is shown in the Fig. 1.13.2.

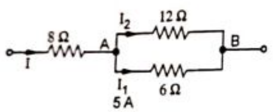


Fig. 1.13.2

$V_{AB} = I_1 \times 6 = 5\times 6 = 30$ V

V_{AB} = 30 V remains **same** across 12 Ω and 6 Ω as they are in **parallel**.

$$I_2 = \frac{V_{AB}}{12} = \frac{30}{12} = 2.5\text{ A}$$

$\therefore \quad I = I_1 + I_2 = 7.5$ A

$\therefore \quad P_{12} = I_2^2 \times 12 = (2.5)^2\times 12 = 75$ W,

$P_6 = I_1^2 \times 6 = (5)^2 \times 6 = 150$ W

$\therefore \quad P_8 = I^2\times 8 = (7.5)^2 \times 8 = 450$ W,

$P_T = P_{12} + P_6 + P_8 =$ **675 W.**

Ex. 1.13.3 *For the circuit shown in Fig. 1.13.3, find the current supplied by each battery and power dissipated in 1 Ω resistor.* **VTU : Dec.-11, Marks 8, Jan.-12, Marks 8**

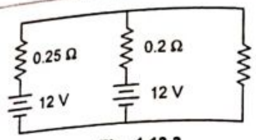


Fig. 1.13.3

Sol. :

Step 1 : Mark the branch currents.

Step 2 : Show the polarities of voltage drops due to branch currents as shown in the Fig. 1.13.3 (a).

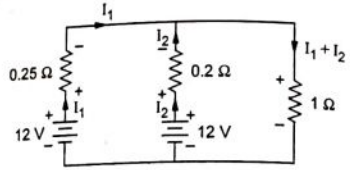


Fig. 1.13.3 (a)

Step 3 : Apply KVL to two loops.

$$+0.2\,I_2 - 12 + 12 - 0.25\,I_1 = 0$$

i.e. $\quad -0.25\,I_1 + 0.2\,I_2 = 0 \quad$ ... (1)

$$-0.2\,I_2 - (I_1 + I_2)\times 1 + 12 = 0$$

i.e. $\quad -I_1 - 1.2\,I_2 = -12 \quad$ ... (2)

Solving, $\quad I_1 =$ **4.8 A**, $\quad I_2 =$ **6 A**

$\therefore \quad I_{1\,\Omega} = I_1 + I_2 = 10.8$ A

$\therefore \quad P_{1\,\Omega} = (I_{1\,\Omega})^2 \times 1 = (10.8)^2\times 1 =$ **116.64 W**

Ex. 1.13.4 *A circuit consists of two parallel resistors having resistances of 20 Ω and 30 Ω respectively, connected in series with a 15 Ω resistor. If the current through 30 Ω resistor is 1.2 A, find*

i) Currents in 20 Ω and 15 Ω resistors.

ii) The voltage across the whole circuit.

iii) Voltage across 15 Ω resistor and 20 Ω resistor.

iv) Total power consumed in the circuit.

VTU : Jan.-15, Marks 8

Sol. : The circuit is shown in the Fig. 1.13.4.

i) The drop across parallel resistors is same

$\therefore \quad 1.2\times 30 = 20\times I_2$

$\therefore \quad I_2 =$ **1.8 A**

$\therefore \quad I_{15\,\Omega} = I = I_1 + I_2 =$ **3 A**

ii) $\quad V$ = Drop across 30 Ω + Drop across 15 Ω

$= 1.2\times 30 + 3\times 15 =$ **81 V**

iii) Voltage across 15 Ω $= 15\times 3 =$ **45 V**

Voltage across 20 Ω $= 20\times 1.8 =$ **36 V**

iv) Power consumed in the circuit $= I_1^2\times 30 + I_2^2\times 20 + I^2\times 15 \quad$ or $\quad = V\times I$

$= 43.2 + 64.8 + 135 = 81\times 3 =$ **243 W**

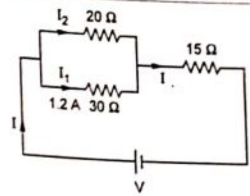


Fig. 1.13.4

Ex. 1.13.5 *For the circuit shown in Fig. 1.13.5, the total power dissipated is 488 W. Calculate the current flowing in each resistance and pd between A and B.* **VTU : Jan. 16, Marks 5**

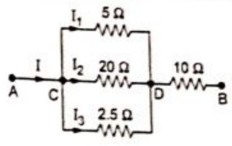


Fig. 1.13.5

Sol. : The parallel combination of 5 Ω, 20 Ω and 2.5 Ω be R'.

$$\therefore \quad \frac{1}{R'} = \frac{1}{5}+\frac{1}{20}+\frac{1}{2.5}$$

i.e. $\quad R' = 1.5384\ \Omega$

The circuit reduces as shown in the Fig. 1.13.5 (a).

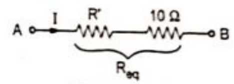


Fig. 1.13.5 (a)

$R_{eq} = R' + 10 = 11.5384\ \Omega$

P = 488 W given

$\therefore \quad P = I^2 R_{eq} \quad$ i.e. $\quad I^2 = \frac{488}{11.5384} \quad$ i.e. $\quad I =$ **6.5033 A**

$\therefore \quad V_{AB} = I\times R_{eq} = 6.5033\times 11.5384 =$ **75.038 V**

$\therefore$ Drop across R' $= V_{AB} - 10I = 75.038 - 65.033 = 10$ V

Drop across all parallel resistances is same hence,

$I_1 = \frac{10}{5} =$ **2 A**, $\quad I_2 = \frac{10}{20} =$ **0.5 A**, $\quad I_3 = \frac{10}{2.5} =$ **4 A**

1.14 Electrical Energy

➤ ***Define electrical energy and explain its commercial unit.***

- An electrical energy is the total amount of electrical work done in an electric circuit.

$$\therefore \quad \boxed{\text{Electrical energy } E = \text{Power}\times\text{Time} = V\,I\,t \quad \text{joules}}$$

- The **unit of energy is joules or watt-sec.**
- The energy consumed by an electric circuit is said to be 1 joule or watt-sec when it utilises power of 1 watt for 1 second.

- As watt-sec unit is very small, the electrical energy is measured in bigger units as watt-hour (Wh) and kilo watt-hour (kWh).

$$1 \text{ Wh} = 1 \text{ watt} \times 1 \text{ hour} = 1 \text{ watt} \times 3600 \text{ sec} = 3600 \text{ watt-sec i.e. J}$$

and $$1 \text{ kWh} = 1000 \text{ Wh} = 1 \times 10^3 \times 3600 \text{ J} = 3.6 \times 10^6 \text{ J}$$

- When a power of 1 kW is utilised for 1 hour the energy consumed is said to be 1 kWh. This unit is called **Board of Trade Unit**.

- The electricity bills we are getting are charged based on this commercial unit of energy i.e. **kWh or unit.**

1.15 Current Division in Parallel Circuit of Resistors

➤ ***Explain the current division in parallel circuit of resistors.***

- Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.
- Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$\therefore$ $$I_T = I_1 + I_2$$

But $$I_1 = \frac{V}{R_1},\quad I_2 = \frac{V}{R_2} \quad \text{i.e.}$$

$$V = I_1 R_1 = I_2 R_2$$

$\therefore$ $$I_1 = I_2\left(\frac{R_2}{R_1}\right)$$

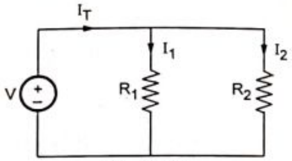


Fig. 1.15.1

- Substituting value of I_1 in I_T,

$\therefore$ $$I_T = I_2\left(\frac{R_2}{R_1}\right) + I_2 = I_2\left[\frac{R_2}{R_1}+1\right] = I_2\left[\frac{R_1+R_2}{R_1}\right]$$

$\therefore$ $$\boxed{I_2 = \left[\frac{R_1}{R_1+R_2}\right] I_T}$$

Now $$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T = \left[\frac{R_1+R_2-R_1}{R_1+R_2}\right] I_T$$

$\therefore$ $$\boxed{I_1 = \left[\frac{R_2}{R_1+R_2}\right] I_T}$$

- In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.

Ex. 1.15.1 *A circuit of two parallel resistors having resistance of 20 Ω and 30 Ω respectively, connected in series with 15 Ω. If the current through 15 Ω resistor is 3 A, find i) current in 20 Ω and 30 Ω resistors, ii) voltage across the whole circuit, iii) the total power and power consumed in all resistors.*

VTU : Aug.-07, Jan.-13, 16, Marks 6

Sol. : The circuit is shown in the Fig. 1.15.2.

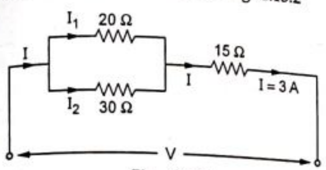


Fig. 1.15.2

i) Using current division rule,

$$I_1 = I \times \frac{30}{30+20} = \frac{3 \times 30}{50} = \mathbf{1.8\ A}$$

$$I_2 = I \times \frac{20}{20+30} = \frac{3 \times 20}{50} = \mathbf{1.2\ A}$$

ii) Drop across parallel combination :

$$I_1 \times 20 = I_2 \times 30 = 1.8 \times 20 = 36 \text{ V}$$

Drop across 15 Ω = I × 15 = 3 × 15 = 45 V

$\therefore$ $$V = 36 + 45 = \mathbf{81\ V} \quad \text{...Supply voltage.}$$

iii) Total power = V I = 81 × 3 = **243 W**

$$P_{20} = I_1^2 \times 20 = \mathbf{64.8\ W}$$

$$P_{30} = I_2^2 \times 30 = \mathbf{43.2\ W}$$

$$P_{15} = I^2 \times 15 = \mathbf{135\ W}$$

Ex. 1.15.2 *A current of 20 A flows through two ammeters A and B in series. The potential difference across A is 0.2 V and across B is 0.3 V. Find how the same current will divide between A and B when they are in parallel.*

VTU : Feb.-08, Jan.-15, Marks 8

Sol. :

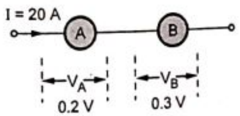


Fig. 1.15.3 (a) Ammeters in series

$$R_A = \text{Resistance of ammeter A}$$

$$= \frac{V_A}{I} = \frac{0.2}{20} = 0.01\ \Omega$$

$\therefore$ $$R_B = \text{Resistance of ammeter B}$$

$$= \frac{V_B}{I} = \frac{0.3}{20} = 0.015\ \Omega$$

Now the ammeters are connected in parallel as shown in the Fig. 1.15.3 (b).

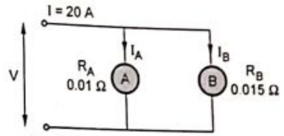


Fig. 1.15.3 (b) Ammeters in parallel

$$R_{eq} = R_A \| R_B = \frac{0.01 \times 0.015}{0.01 + 0.015} = 0.006\ \Omega$$

$\therefore$ $$V = \text{Voltage across both the ammeters}$$

$\therefore$ $$V = I_A \times R_A = I_B \times R_B$$

While $$V = I \times R_{eq} = 20 \times 0.006 = 0.12 \text{ V}$$

$\therefore$ $$I_A = \frac{V}{R_A} = \frac{0.12}{0.01} = \mathbf{12\ A}$$

and $$I_B = \frac{V}{R_B} = \frac{0.12}{0.015} = \mathbf{8\ A}$$

Note : This can be verified using current division rule as,

$$I_A = I \times \frac{R_B}{R_A + R_B} = \mathbf{12\ A}$$

and $$I_B = I \times \frac{R_A}{R_A + R_B} = \mathbf{8\ A}$$

Ex. 1.15.3 *If the total power dissipated in the circuit shown in 18 W. Find the value of 'R' and its current.*

VTU : July-15 Marks 8

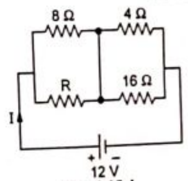


Fig. 1.15.4

Sol. : It can be seen that 8 Ω and R Ω are in parallel while 4 Ω and 16 Ω in parallel. Hence circuit reduces as shown in the Fig. 1.15.4 (a).

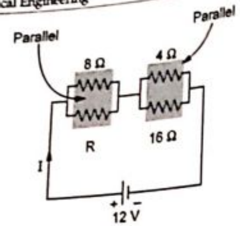


Fig. 1.15.4 (a)

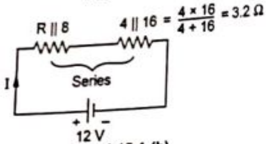


Fig. 1.15.4 (b)

$$\therefore \quad I = \frac{V}{(R \parallel 8) + 3.2} = \frac{V}{\frac{8R}{R+8} + 3.2} = 1.5 \text{ A}$$

$$\therefore \quad I = \frac{12\ (R+8)}{8R + 3.2\,R + 25.6} = \frac{12\ (R+8)}{11.2\,R + 25.6}$$

$$P = VI = \frac{12 \times 12\ (R+8)}{11.2\,R + 25.6} = 18 \text{ (given)} \qquad \text{...Power}$$

$$\therefore \quad 8\ (R+8) = 11.2R + 25.6 \text{ i.e. } \mathbf{R = 12\ \Omega}$$

Let current through R be I_R.

As R and 8 Ω in parallel, using current division rule,

$$I_R = I \times \frac{8}{8+R} \text{ and } I = \frac{V}{\frac{8R}{R+8} + 3.2} = 1.5 \text{ A}$$

$$\therefore \quad I_R = \frac{1.5 \times 8}{(8+12)} = \mathbf{0.6\ A}$$

Ex. 1.15.4 *From the given below circuit, find the current through 6 Ω resistor.* **VTU : Jan.-17, Marks 5**

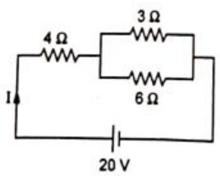


Fig. 1.15.5

Sol. : The resistances 3Ω and 6Ω are in parallel

$$\therefore \quad 3 \| 6 = \frac{3 \times 6}{3+6} = 2\ \Omega$$

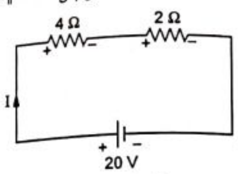


Fig. 1.15.5 (a)

From Fig. 1.15.5 (a)

$$I = \frac{20}{4+2} = 3.333 \text{ A}$$

Using current division in parallel resistances for the circuit shown in the Fig. 1.15.5 (b).

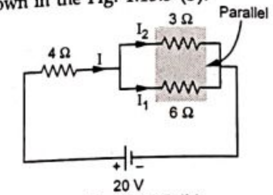


Fig. 1.15.5 (b)

$$I_1 = I \times \frac{3}{3+6} = \frac{3.333 \times 3}{9}$$

$$\therefore \quad I_1 = \mathbf{1.111\ A} = I_{6\Omega}$$

Ex. 1.15.5 *An 8 ohms resistor in series with a parallel combination of two resistors 12 ohms and 6 ohms. If the current in the 6 ohm resistor is 5 A, determine the total power dissipated in the circuit and also power consumed in all individual resistors.* **VTU : Jan.-18, Marks 8**

Sol. : The circuit is shown in the Fig. 1.15.6

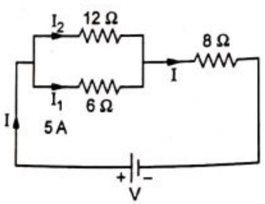


Fig. 1.15.6

Applying current distribution rule to parallel resistances,

$$I_1 = I \times \frac{12}{12+6}$$

$$\therefore \quad 5 = I \times \frac{12}{18} \text{ i.e. } I = 7.5 \text{ A}$$

$$\therefore \quad I_2 = I \times \frac{6}{6+12} = \frac{7.5 \times 6}{18} = 2.5 \text{ A}$$

$\therefore$ Drop across parallel resistance $= I_1 \times 6 = I_2 \times 12 = 30$ V

Drop across 8 Ω resistance $= I \times 8 = 7.5 \times 8 = 60$ V

$$\therefore \quad V = 30 + 60 = 90 \text{ V}$$

$\therefore$ Total power consumed $= VI = 90 \times 7.5 = \mathbf{675\ W}$

$$P_{12\Omega} = I_1^2 \times 12 = 2.5^2 \times 12 = \mathbf{75\ W}$$

$$P_{6\Omega} = I_1^2 \times 6 = 5^2 \times 6 = \mathbf{150\ W}$$

$$P_{8\Omega} = I_1^2 \times 8 = 7.5^2 \times 8 = \mathbf{450\ W}$$

Ex. 1.15.6 *A 10 Ω resistance is connected in series with a parallel combination of 15 Ω and 20 Ω resistors. The circuit is applied with V volts. The power taken by the circuit is 150 watts. Find the total current through the circuit and power consumed in all the resistors.* **VTU : Jan.-17, Marks 8**

Sol. : The circuit is shown in the Fig. 1.15.7

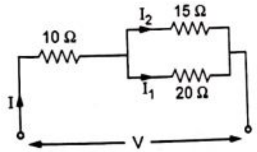


Fig. 1.15.7

$$15 \parallel 20 = \frac{15 \times 20}{15 + 20} = \mathbf{8.5714\ \Omega}$$

$$\therefore \quad R_T = 10 + (15 \parallel 20) = \mathbf{18.5714\ \Omega}$$

$$\therefore \quad I = \frac{V}{R_T} = \frac{V}{18.5714} \text{ A}$$

$$P = VI = V \times \frac{V}{18.5714}$$

and $\quad P = 150$ W given

$$\therefore \quad V^2 = 150 \times 18.5714 \text{ i.e. } \mathbf{V = 52.8\ V}$$

$$\therefore \quad I = \frac{V}{R_T} = \mathbf{2.842\ A}$$

$$\therefore \quad I_1 = I \times \frac{15}{15+20} = 1.218 \text{ A},$$

$$I_2 = I \times \frac{20}{15+20} = 1.624 \text{ A}$$

$$\therefore \quad P_{10\Omega} = I^2 \times 10 = \mathbf{80.769\ W},$$

$$P_{15\Omega} = I_2^2 \times 15 = \mathbf{39.5606\ W}$$

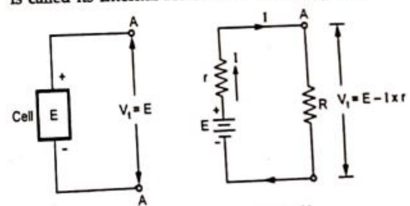
$$P_{20\Omega} = I_1^2 \times 20 = \mathbf{29.6704\ W},$$

$$P = P_{10\Omega} + P_{15\Omega} + P_{20\Omega}$$

1.16 Concept of the Terminal Voltage of a Cell

➤ ***Explain the concept of the terminal voltage of a cell.***

- Consider a cell having e.m.f. of E volts.
- This e.m.f. and the potential difference between its terminals are same as long as it is not driving any current through the circuit. This is shown in Fig. 1.16.1 (a).
- Now consider the resistance R connected across the terminals A-A of the cell, as shown in the Fig. 1.16.1 (b).
- The cell itself opposes to drive the current through the resistance R. Such opposition from the cell itself is called its **internal resistance** denoted by r Ω .



(a) $I = 0, V_t = E$ (b) $E \neq V_t$

Fig. 1.16.1

- Due to this internal resistance, according to Ohm's law there is potential drop across it, which is (I×r) volts.
- After supplying this drop, remaining voltage is available across the terminals denoted by V_t. This is called **terminal voltage.** In such case it is less than the electromotive force E.

So electromotive force when drives a current I, has two parts,
1) Drop across its internal resistance r ohms. 2) Terminal voltage available V_t volts.

$\therefore$ Mathematically we can write as, $E = I\,r + V_t$

$\therefore$ $\boxed{\text{Terminal voltage } V_t = E - I\,r}$

1.17 University Questions with Answers

June - 2012

Q.1 *List the characteristics of parallel circuit.* **[Refer section 1.9]** **[4]**

Jan. - 2013

Q.2 *State and explain Kirchhoff's laws.* **[Refer section 1.11]** **[5]**

June - 2013

Q.3 *State and explain Kirchhoff's laws.* **[Refer section 1.11]** **[5]**

CBCS Scheme

Jan. - 2016

Q.4 *State and explain Kirchhoff's laws.* **[Refer section 1.11]** **[4]**

Q.5 *State Ohm's law and its limitations.* **[Refer section 1.7]** **[5]**

Q.6 *State and explain Kirchhoff's current law and Kirchhoff's voltage law.* **[Refer section 1.11]** **[6]**

July - 2016

Q.7 *State and explain Kirchhoff's laws.* **[Refer section 1.11]** **[5]**

Q.8 *State Ohm's law and its limitations.* **[Refer section 1.7]** **[5]**

Jan. - 2017

Q.9 *Define KCL and KVL with an example.* **[Refer section 1.11]** **[5]**

July - 2017

Q.10 *State and explain Ohm's Law with an illustration. Also list its's limitations.* **[Refer section 1.7]** **[5]**

Jan. - 2018

Q.11 *Explain Ohm's law and its limitations.* **[Refer section 1.7]** **[4]**

Q.12 *State and explain Kirchhoff's laws.* **[Refer section 1.11]** **[6]**

MODULE - 1

2

A. C. Fundamentals

Syllabus

Generation of sinusoidal voltage, frequency of generated voltage, definition and numerical values of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities.

Contents

2.1 Introduction to A.C. Fundamentals

➤ ***What is a.c. ? How it differs from d.c. ?***

- Electrical supply used for commercial and domestic purposes is alternating.

The d.c. supply has constant magnitude with respect to time. The Fig. 2.1.1 (a) shows a graph of such current with respect to time.

> • *An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.*

- In alternating waveform there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly. The Fig. 2.1.1 (b) shows a graph of alternating current against time.
- In practice some waveforms are available in which magnitude changes but its direction remains same as positive or negative. This is shown in the Fig. 2.1.1 (c). Such waveform is called **pulsating d.c.** The waveform obtained as the output of full wave rectifier is an example of pulsating d.c.

2.1.1 Advantages of A.C.

➤ ***State the advantages of a.c.***

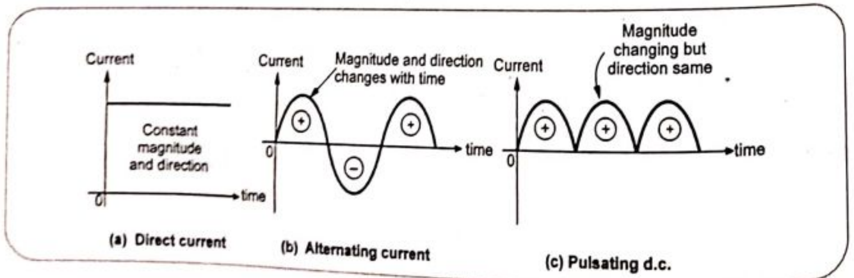
1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high a.c. voltage and hence high speed a.c. generators of large capacities. The construction and cost of such generators are very low. High a.c. voltages of about 11 kV can be generated and can be raised up to 220 kV for transmission purpose.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, a.c. supply can be easily converted to obtain d.c. supply.

- Due to these advantages, a.c. is used extensively in practice and hence, it is necessary to study a.c. principles.

2.1.2 Advantages of Purely Sinusoidal Waveform

➤ ***State the advantages of sinusoidal alternating waveform.***

- In practice, a quantity which undergoes variations in its instantaneous values, in magnitude as well as direction with respect to some zero reference is called an **alternating quantity.**
- There can be rectangular a.c., triangular a.c. and so on. Out of all these types of alternating waveforms, **purely sinusoidal waveform** is preferred for a.c. system.



(a) Direct current (b) Alternating current (c) Pulsating d.c.

Fig. 2.1.1

- The advantages of using sinusoidal waveform are,

1) Mathematically, it is very easy to write the equations for purely sinusoidal waveform.
2) Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original wave form. Hence, it is always better to have sinusoidal waveform as the standard waveform.
3) The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through a linear circuit.
4) The integration and derivative of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical networks with sinusoidal inputs, very easy.

2.2 Generation of A.C. Voltage

➤ ***With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field.***

VTU : Mar.-01, July-03, Aug.-05, Marks 6

- The basic principle of an a.c. generation is the principle of electromagnetic induction. The sine wave is generated according to **Faraday's law of electromagnetic induction.**
- Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called **single turn** or **single loop alternator.**
- It consists of a permanent magnet having two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet.
- The coil is made up of two conductors namely a-b and c-d. Such two conductors are connected at one end to form a coil.
- The coil is so placed that it can be rotated about its own axis.
- The remaining two ends C_1 and C_2 of the coil are connected to the rings mounted on the shaft called **slip rings.** Slip rings are also rotating members of the alternator.
- The two brushes P and Q are resting on the slip rings. The **brushes are stationary** and just making contact with the slip rings. The overall construction is shown in the Fig. 2.2.1.
- **Working :** The coil is rotated in anticlockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, an e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected

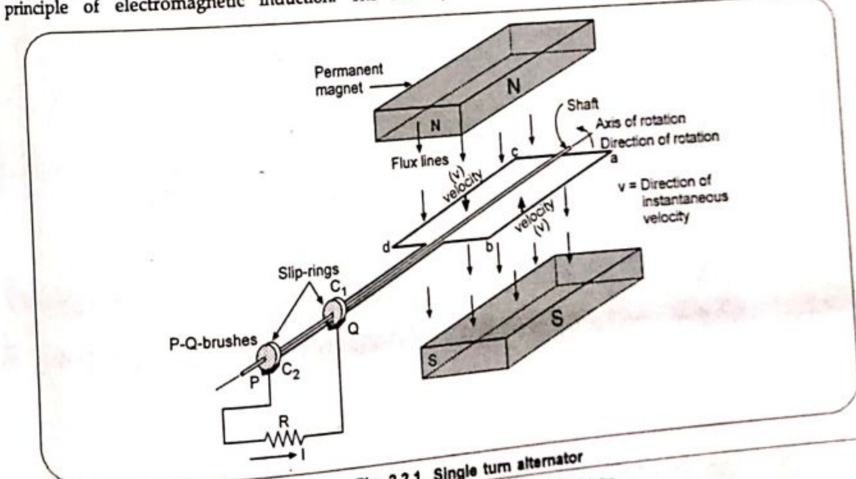


Fig. 2.2.1 Single turn alternator

across the brushes P and Q. The magnitude of the **induced e.m.f. depends on the position of the coil in the magnetic field.**

Consider different instants and the different positions of the coil :

- **Instant 1 :** Let the initial position of the coil be as shown in the Fig. 2.2.1. The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductors ab and cd, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence, no e.m.f. will be generated in the conductors ab and cd. This shown in the Fig. 2.2.2 (a).
- **The angle θ is measured from plane of the magnetic flux.**
- **Instant 2 :** When the coil is rotated in anticlockwise direction through some angle θ, then the velocity will have two components vsinθ perpendicular to flux lines and vcosθ parallel to the flux lines. Due to vsinθ component, there will be cutting of the flux and proportionally, there will be induced e.m.f. in the conductors ab and cd. This is shown in the Fig. 2.2.2 (b).
- **Instant 3 :** As angle 'θ' increases, the component of velocity acting perpendicular to flux lines increases, hence induced e.m.f. also increases. At θ = 90°, the plane of the coil is parallel to the plane of the

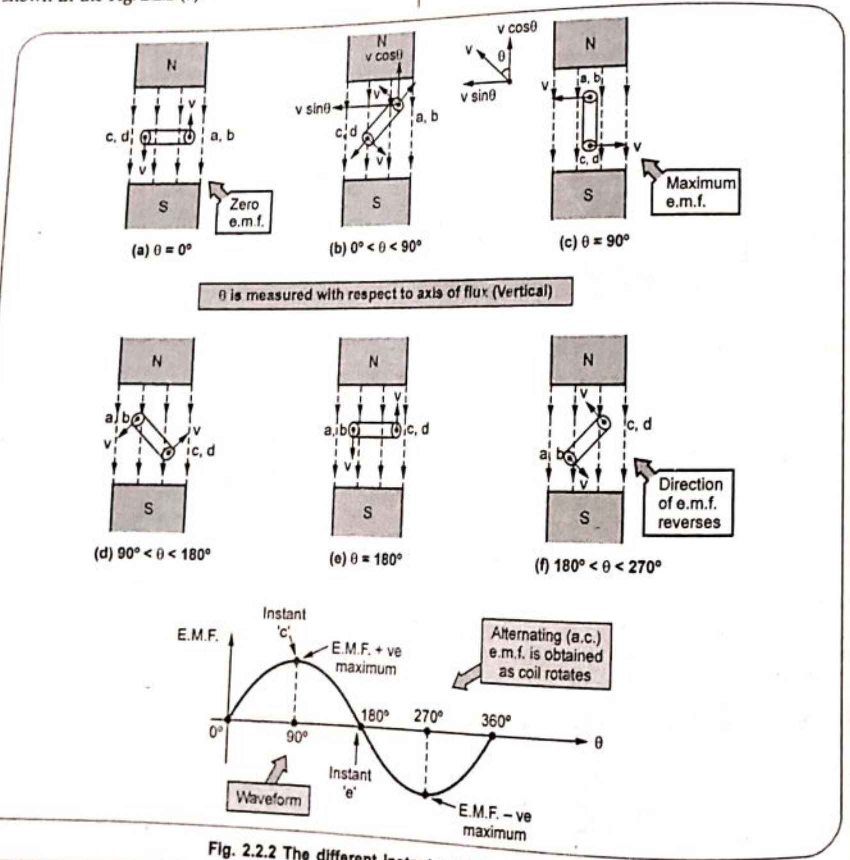


Fig. 2.2.2 The different instants of induced e.m.f.

magnetic field while the component of velocity cutting the lines of flux is at its maximum. So, **induced e.m.f. in this position, is at its maximum value.** This is shown in the Fig. 2.2.2 (c).

- So, as θ increases from 0° to 90°, e.m.f. induced in the conductors increases gradually from 0 to maximum value.
- **Instant 4 :** As the coil continues to rotate further from θ = 90° to 180°, the component of velocity, perpendicular to magnetic field starts decreasing, hence, gradually decreasing the magnitude of the induced e.m.f. This is shown in the Fig. 2.2.2 (d).
- **Instant 5 :** In this position, the velocity component is fully parallel to the lines of flux similar to the instant 1. Hence, there is no cutting of flux and hence, no induced e.m.f. in both the conductors.
- **Instant 6 :** As the coil rotates beyond θ = 180°, the conductor ab uptil now cutting flux lines in one particular direction reverses the direction of cutting the flux lines. Similar is the behaviour of conductor cd. This change in direction of induced e.m.f. occurs because the direction of rotation of conductors ab and cd reverses with respect to the field as θ varies from 180° to 360°. This process continues as coil rotates further.
- **So, as θ varies from 0° to 360°, the e.m.f. in a conductor ab or cd varies in an alternating manner i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductors rotate in a circular motion with a certain speed.**
- This variation of e.m.f. in a conductor can be graphically represented as shown in the Fig. 2.2.2. From the waveform, it is clear that the waveform generated by the instantaneous values of the induced e.m.f. in any conductor (ab or cd) is **purely sinusoidal** in nature.
- The angle θ in radians and the angular velocity ω in radians/seconds of the coil are related to each other through time t by the equation,

$$\boxed{\theta = \omega t \quad \text{radians}}$$

- Hence the waveform of alternating quantity can be shown with respect to time t rather than θ as ω is constant.
- There are 2 poles and for one complete rotation of conductor, one electrical cycle of an induced e.m.f. is generated.

> • Thus in general for P number of poles, for one complete rotation of conductor P/2 electrical cycles of an induced e.m.f. will be generated.

2.3 Standard Definitions Related to Alternating Quantity

➤ ***Sketch the sinusoidal alternating current waveform and define the following terms :***
i) Instantaneous value ii) Waveform
iii) Time period iv) Cycle v) Frequency
vi) Amplitude or peak value or maximum value. **VTU : Jan-03, 08, 16, Marks 5**

- The Fig. 2.3.1 shows the graphical representation of an alternating quantity.

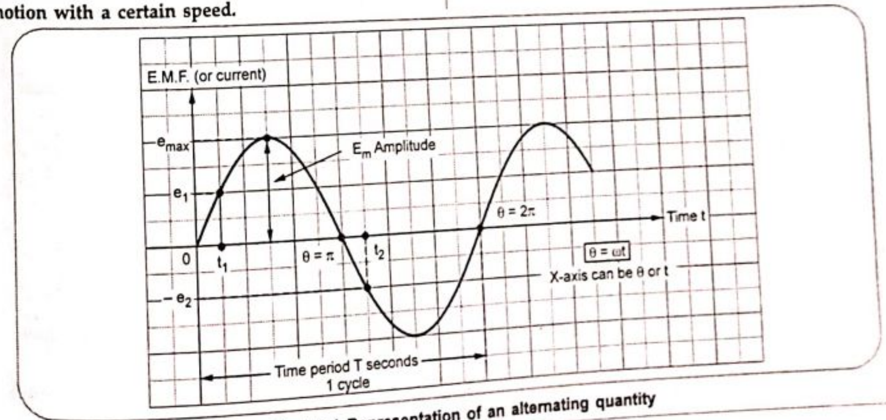


Fig. 2.3.1 Representation of an alternating quantity

1. Instantaneous value : The value of an alternating quantity at a particular instant is known as its **instantaneous value.** e.g. e_1 and $-e_2$ are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 2.3.1.

2. Waveform : The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

3. Cycle : Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

- Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**
- A **cycle** can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops).
- One such cycle of the alternating quantity is shown in the Fig. 2.3.1.

One cycle corresponds to 2π radians or 360°.

4. Time period (T) : The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds.

- After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 2.3.1.

5. Frequency (f) : The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by **f** and it is measured in **cycles / second** which is known as **Hertz**, denoted as **Hz.**

- As time period T is time for one cycle i.e. seconds/cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$\boxed{f = \frac{1}{T} \text{ Hz}}$$

- As time period increases, frequency decreases while as time period decreases, frequency increases.

- **In our nation, standard frequency of alternating voltages and currents is 50 Hz.**

6. Amplitude : The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

- Thus E_m is called peak value of the voltage while I_m is called peak value of the current.
- The amplitude is also called peak value or maximum value of an alternating quantity.

7. Angular frequency (ω) : It is the frequency expressed in electrical radians per second.

- As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as $(2\pi \times \text{cycles/sec.})$. It is denoted by 'ω' and its unit is radians/second. The relation between frequency 'f' and angular frequency 'ω' is,

$$\boxed{\begin{array}{ll} & \omega = 2\pi f \quad \text{radians/sec.} \\ \text{or} & \omega = \frac{2\pi}{T} \quad \text{radians/sec.} \end{array}}$$

- The angle θ and the angular frequency ω are related to each other through time as,

$$\boxed{\theta = \omega t \quad \text{radians} \quad \text{or} \quad \theta = 2\pi f t \quad \text{radians}}$$

- Thus the waveform of an alternating quantity can be shown with respect to time 't' or angle 'θ' as 'ω' is constant.

8. Peak to Peak value : The value of an alternating quantity from its positive peak to negative peak is called its peak to peak value. It is denoted as I_{p-p} or V_{p-p}.

$$\boxed{\text{Amplitude} = \frac{\text{Peak to Peak Value}}{2}}$$

2.4 Equation of an Alternating Quantity

➤ ***State the equation of an alternating quantity. State its various forms.***

- As the standard waveform of an alternating quantity is purely sinusoidal, the equation of an alternating voltage can be expressed as,

$$\boxed{e = E_m \sin\theta \text{ volts}}$$

where E_m = Amplitude or maximum or peak value of the voltage

e = Instantaneous value of an alternating voltage

- Similarly equation of an alternating current can be expressed as,

$$\boxed{i = I_m \sin\theta}$$

where I_m = Amplitude or maximum or peak value of the current.

i = Instantaneous value of an alternating current

The equation can be expressed in various forms as :

Now, $\theta = \omega t$ radians

$$\therefore \quad \boxed{e = E_m \sin(\omega t)} \quad \ldots(2.4.1)$$

But, $\omega = 2\pi f$ rad / sec.

$$\therefore \quad \boxed{e = E_m \sin(2\pi f t)} \quad \ldots(2.4.2)$$

But, $f = \frac{1}{T}$ seconds

$$\therefore \quad \boxed{e = E_m \sin\left(\frac{2\pi}{T}t\right)} \quad \ldots(2.4.3)$$

Important Note : *In all the above equations, the angle θ is expressed in* ***radians.*** *Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.*

Ex. 2.4.1 *An alternating current of frequency 60 Hz has a maximum value of 12 A : i) Write down the equation for instantaneous values. ii) Find the value of the current after 1/360 second. iii) Time taken to reach 9.6 A for the first time. In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction.* **VTU : June 12, Marks 4**

Sol. : $f = 60$ Hz, $I_m = 12$ A,

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec

i) Equation of instantaneous value is,

$$i = I_m \sin\omega t = 12 \sin 377 t$$

ii) $t = \frac{1}{360}$ sec i.e. $i = 12 \sin 377 \frac{1}{360} = 12 \sin 1.0472$

$$= 10.3924 \text{ A}$$

Note sin of 1.0472 must be calculated in **radian** mode.

iii) $i = 9.6$ A i.e. $9.6 = 12 \sin 377 t$

i.e. $\sin 377 t = 0.8$

$\therefore \quad 377 t = 0.9272$ i.e. $t = 2.459 \times 10^{-3}$ **sec**

... $\sin^{-1}$ in **radian** mode

Ex. 5.4.2 *A sinusoidally varying alternating current has maximum value of $2\sqrt{2}$ A and time period of 20 msec. If the waveform enters into its positive half cycle at t = 0, find the instantaneous value of the current of t = 6 msec.*

Sol. : $I_m = 2\sqrt{2}$ A, T = 20 msec, $f = \frac{1}{T} = 50$ Hz

∴ Equation of current is, $i(t) = I_m \sin(2\pi ft)$

$\therefore \quad i(t) = 2.8284 \sin(314.1592t)$ A

At t = 6 msec, $i(t) = 2.8284 \sin(314.1592 \times 6 \times 10^{-3})$ = **2.6899 A**

Note : that sin must be calcualted in radian mode.

2.5 Effective Value or R.M.S. Value

➤ ***Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.***

VTU : July-03, 04, 06, 16; Jan.-04, 07, 13, 14, 18; Feb.-05; July-11, 13, 14, 16, Marks 4

- An alternating current varies from instant to instant, while the direct current is constant, with respect to time.
- For the comparison of the two, a common effect to both the type of currents can be considered. Such an effect is heat produced by the two currents flowing through the resistance. The heating effect can be used to compare the alternating and direct current. From this, r.m.s. value of an alternating current can be defined as,

- *The effective or r.m.s. value of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

2.5.1 Analytical Method of Obtaining R.M.S. Value

Steps to find r.m.s. value of an a.c. quantity :

1. Write the equation of an a.c. quantity bserve its behaviour during various time intervals.
2. Find square of the a.c. quantity from its equation.
3. Find average value of square of an alternating quantity as,

$$\text{Average} = \frac{\text{Area of curve over one cycle of squared waveform}}{\text{Length of the cycle}}$$

4. Find square root of average value which gives r.m.s. value of an alternating quantity.

- Consider sinusoidally varying alternating current and square of this current as shown in the Fig. 2.5.1.

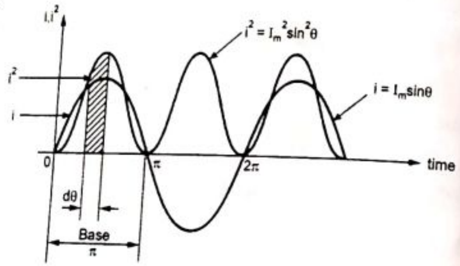


Fig. 2.5.1 Waveform of current and square of the current

Step 1 : The current $i = I_m \sin\theta$

Step 2 : Square of current $i^2 = I_m^2 \sin^2\theta$

- The area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle $= \int_0^{\pi} i^2 \, d\theta$ and length of the base is π.

Step 3 : ∴ Average value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

Step 4 : Root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

∴ $$\boxed{I_{r.m.s.} = 0.707\, I_m}$$

- **The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.**

- *The **instantaneous** values are denoted by small letters like i, e etc. while **r.m.s.** values are represented by **capital** letters like I, E etc.*

- The above result is also applicable to **sinusoidal** alternating voltages.

∴ $$\boxed{V_{r.m.s.} = 0.707\, V_m}$$

2.5.2 Importance of R.M.S. Value

➤ ***State the practical significance of r.m.s. value.***

1. In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quantities unless and otherwise specified to be other than r.m.s.

- In practice, everywhere, r.m.s. values are used to analyze alternating quantities.

2. The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3. The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

2.6 Average Value

➤ ***Define average value of an alternating quantity. Obtain the relation between average value and the maximum value of an alternating quantity.***

VTU : July-03, 04, 06, 16; Jan.-04, 07, 18; Feb.-05; July-11, 16, Marks 4

- The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

- **For a symmetrical a.c., the average value over a complete cycle is zero** *as both positive and negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.*

- **Average value** can also be expressed by that steady current which transfers across any circuit, the same amount of charge as is transferred by that alternating current during the same time.

2.6.1 Analytical Method of Obtaining Average Value

- Consider sinusoidally varying current, $I = I_m \sin\theta$
- Consider the elementary interval of instant 'dθ' as shown in the Fig. 2.6.1. The average instantaneous value of current in this interval is say, 'i' as shown.

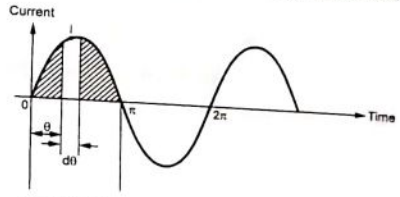


Fig. 2.6.1 Average value of an alternating current

- The average value can be obtained by taking ratio of area under curve over **half cycle** to length of the base for half cycle.

∴ $$\boxed{I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}}$$

$$I_{av} = \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi + \cos 0] = \frac{I_m}{\pi}[2]$$

$$= \frac{2 I_m}{\pi} = 0.637\, I_m$$

- For a purely sinusoidal waveform, the average value is expressed in terms of its maximum value as,

∴ $$\boxed{I_{av} = 0.637\, I_m \quad \text{and} \quad V_{av} = 0.637\, V_m}$$

2.6.2 Importance of Average Value

➤ ***State the importance of average value.***

1. The average value is used for applications like battery charging.
2. The charge transferred in capacitor circuits is measured using average values.

3. The average values of voltages and currents play an important role in analysis of the rectifier circuits.
4. The average value is indicated by d.c. ammeters and voltmeters.
5. The average value of purely sinusoidal waveform is always zero.

2.7 Form Factor (K_f)

➤ ***Define form factor. Derive its value for sinusoidal quantity.*** VTU : July-09, 15, Jan. 16, Marks 2

- The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value,

Form factor, $\boxed{K_f = \dfrac{\text{r.m.s. value}}{\text{Average value}}}$

- The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\ I_m}{0.637\ I_m}$$

$$= 1.11 \quad \text{for sinusoidally varying quantity}$$

2.8 Crest or Peak Factor (K_p)

➤ ***Define form factor.*** VTU : Mar.-01; Aug.-03, Jan. 16, Marks 2

- The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

Peak factor $\boxed{K_p = \dfrac{\text{maximum value}}{\text{r.m.s. value}}}$

- The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{I_m}{0.707\ I_m} = 1.414 \quad \text{for sinusoidal waveform}$$

Ex. 2.8.1 *The equation of an alternating current is given by i = 42.42 sin 628t. Calculate its i) Maximum value ii) Frequency iii) RMS value iv) Average value v) Form factor* VTU : Feb. 06, Marks 5

Sol. : Compare given equation with $i = I_m \sin(\omega t)$

i) $I_m = 42.42$ A

ii) $f = \dfrac{\omega}{2\pi} = \dfrac{628}{2\pi} = 100$ Hz

iii) $I_{r.m.s.} = \dfrac{I_m}{\sqrt{2}} = 30$ A

iv) $I_{av} = 0.637\ I_m = 27.0215$ A

v) $K_f = \dfrac{\text{r.m.s.}}{\text{Average}} = \dfrac{30}{27.0215} = 1.11$

Ex. 2.8.2 *For the current wave shown in Fig. 2.8.1. Find i) Peak current ii) Average value iii) Frequency iv) Periodic time v) Instantaneous value at t = 3 ms.* VTU : Jan. 07, Marks 6

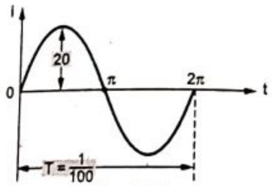


Fig. 2.8.1

Sol. : i) Amplitude or peak value of current waveform = 20 A.

i.e. $I_m = 20$ A

ii) Average value

$$(I_{av}) = \frac{2I_m}{\pi} = \frac{2\times 20}{\pi} = 12.732 \text{ A}$$

iii) Frequency, $f = \dfrac{1}{\text{Time period}} = \dfrac{1}{T}$

$$f = \frac{1}{1/100} = 100 \text{ Hz}$$

iv) Periodic time,

$$T = \frac{1}{100} = 0.01 \text{ sec.}$$

v) Instantaneous value at

$$t = 3 \text{ msec} = 3\times 10^{-3} \text{ sec}$$

$$i = I_m \sin \omega t$$

$$= 20\sin(2\pi f t) = 20 \sin(628.3185\ t)$$

$$= 20\sin(628.3185\times 3\times 10^{-3}) \quad \text{... Use radian mode}$$

$$i = 19.0211 \text{ A.}$$

Ex. 2.8.3 *Obtain the form factor of full rectified sine wave.* VTU : July-09, Marks 5

Sol. : The full rectified sine wave is shown in the Fig. 2.8.2.

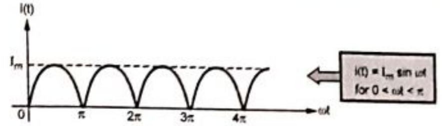


Fig. 2.8.2

i) R.M.S. value : Consider the first cycle,

$$I_{r.m.s.} = \sqrt{\frac{\int_0^{\pi} i^2(t)\,d(\omega t)}{\pi}}$$

$$= \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2(\omega t)\,d(\omega t)}{\pi}} = \frac{I_m}{\sqrt{\pi}} \sqrt{\int_0^{\pi} \sin^2(\omega t)\,d(\omega t)}$$

$$= \frac{I_m}{\sqrt{\pi}} \sqrt{\int_0^{\pi}\left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)} = \frac{I_m}{\sqrt{2}\sqrt{\pi}} \sqrt{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \frac{I_m}{\sqrt{2}\sqrt{\pi}}\sqrt{\pi - 0} = \frac{I_m}{\sqrt{2}}$$

ii) Average value :

$$I_{av} = \frac{\text{Area of one cycle}}{\text{Base}} = \frac{\int_0^{\pi} i(t)\,d(\omega t)}{\pi} = \frac{\int_0^{\pi} I_m \sin(\omega t)\,d(\omega t)}{\pi}$$

$$= \frac{I_m}{\pi}\left[-\cos(\omega t)\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos \pi - (-\cos 0)\right] = \frac{2I_m}{\pi}$$

$$\therefore \quad K_f = \frac{I_{r.m.s.}}{I_{av}} = \frac{I_m/\sqrt{2}}{\frac{2I_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.1107$$

Ex. 2.8.4 *Obtain the form factor of a half rectified sinewave.* VTU : July-08, Marks 5

Sol. : The following waveform represented as shown in Fig. 2.8.3.

i) R.M.S. value : Consider the first cycle,

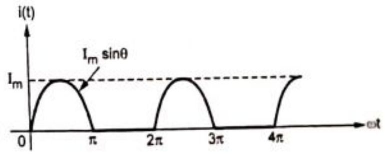


Fig. 2.8.3

$$I_{r.m.s.} = \sqrt{\frac{\int_0^{2\pi} i^2(t)\,d\theta}{2\pi}} = \sqrt{\frac{\int_0^{\pi} i^2(t)\,d\theta + \int_{\pi}^{2\pi} i^2(t)\,d\theta}{2\pi}}$$

$$= \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta + 0}{2\pi}}$$

$$= \frac{I_m}{\sqrt{2\pi}}\sqrt{\int_0^{\pi} \sin^2 \theta d\theta}$$

$$= \frac{I_m}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}} = \frac{I_m}{2} \qquad \because \int_0^{\pi} \sin^2 \theta d\theta = \frac{\pi}{2}$$

ii) Average value

$$I_{av} = \frac{\text{Area of 1 cycle}}{\text{base}} = \frac{\int_0^{2\pi} i(t)d\theta}{2\pi}$$

$$= \frac{\int_0^{\pi} i(t)d\theta + \int_{\pi}^{2\pi} i(t)d\theta}{2\pi}$$

$$= \frac{\int_0^{\pi} I_m \sin\theta \, d\theta}{2\pi}$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} \sin\theta \, d\theta = \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi}[-\cos\pi + \cos 0^\circ] = \frac{2 I_m}{2\pi} = \frac{I_m}{\pi}$$

iii) $K_f = \frac{\text{r.m.s.}}{\text{Average}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$

iv) $K_p = \frac{\text{Maximum}}{\text{r.m.s.}} = \frac{I_m}{I_m/2} = 2.$

2.9 R.M.S. Value of Combined Waveform

- Consider a wire carrying simultaneously more than one alternating current of different magnitudes and frequencies alongwith certain d.c. current. It is required to calculate resultant **r.m.s. value** i.e. **effective value** of the current.
- Let the wire carries three different currents as shown in the Fig. 2.9.1.
- It is required to obtain resultant I_{rms} through the wire.
- Method is based on heating effect of various currents.
- Let R = Resistance of wire
- I_{rms} = Resultant r.m.s. value of current and t = Time for which current is flowing

$\therefore$ H = Heat produced by resultant

$$= I_{rms}^2 \times R \times t \qquad \ldots(5.9.1)$$

- This heat produced is sum of the heats produced by the individual current components flowing for the same time t.

H_1 = Heat produced by d.c. component

$$= I_{dc}^2 \times R \times t$$

H_2 = Heat produced by first a.c. component

$$= I_{rms2}^2 \times R \times t = \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t$$

H_3 = Heat produced by second a.c. component

$$= I_{rms3}^2 \times R \times t = \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

Note that for alternating currents $I_{rms} = I_m/\sqrt{2}$.

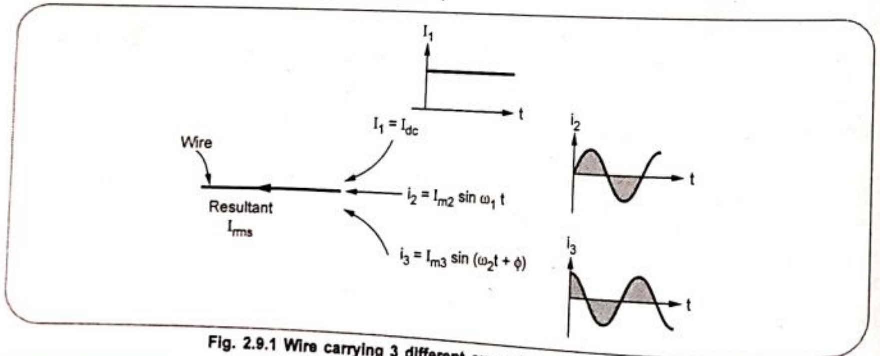


Fig. 2.9.1 Wire carrying 3 different currents simultaneously

- Thus equating the total heat produced to sum of the individual heats produced,

$$H = H_1 + H_2 + H_3$$

i.e. $$I_{rms}^2 Rt = I_{dc}^2 Rt + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 Rt + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 Rt$$

$$\therefore \quad \boxed{I_{rms} = \sqrt{I_{dc}^2 + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2}}$$

- The result can be extended to n number of current components flowing through the wire.

Ex. 2.9.1 *An a.c. current is given by i = 10 sin ωt + 3 sin 3 ωt + 2 sin 5 ωt. Find the r.m.s. value of the current.* **VTU : July-08, Marks 5**

Sol. : When a coil carries d.c. current and more than one alternating signals then the total heat produced is the sum of the heats produced by d.c. component and all the alternating components.

Let R = Resistance of wire

t = Time for which signals are flowing

$$\therefore \quad H_{total} = H_{dc} + H_1 + H_2 + \ldots$$

$$H_{total} = I_{rms}^2 \times R \times t$$

where I_{rms} = Total r.m.s. value

$$H_{dc} = I_{dc}^2 \times R \times t$$

$$H_1 = I_{rms1}^2 \times R \times t, \; H_2 = I_{rms2}^2 \times R \times t \; \ldots$$

$$\therefore \; I_{rms}^2 \times R \times t = I_{dc}^2 Rt + I_{rms1}^2 Rt + I_{rms2}^2 Rt + \ldots$$

$$\therefore \quad I_{rms} = \sqrt{I_{dc}^2 + I_{rms1}^2 + I_{rms2}^2 + \ldots}$$

For given example, $I_{dc} = 0$

$$I_{rms1} = \frac{I_{m1}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07106 \text{ A}, \; I_{rms2} = \frac{3}{\sqrt{2}} = 2.1213 \text{ A},$$

$$I_{rms3} = \frac{2}{\sqrt{2}} = 1.4142 \text{ A}$$

$$I_{rms} = \sqrt{0 + (7.07106)^2 + (2.1213)^2 + (1.4142)^2} = 7.5166 \text{ A}$$

2.10 Phasor Representation of an Alternating Quantity

➤ ***What is phasor ? How a rotating phasor represents an alternating quantity ?***

- The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in the phasor representation method.
- The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

- The phasors are assumed to be rotated in anticlockwise direction with a constant speed ω rad/sec.

- One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.
- Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 2.10.1.
- If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as θ = ω t), we get a sine waveform.

Consider the various positions shown in the Fig. 2.10.1. (See Fig. 2.10.1 on next page)

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [*l* (ob) sin θ)]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is i = I_m sin θ, represented in the waveform.
3. At point 'c', the Y-axis projection 'oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes I_m sin θ which is the instantaneous value of the current at that instant.
5. At point 'e', the Y-axis projection is zero and instantaneous value of the current is zero at this instant.

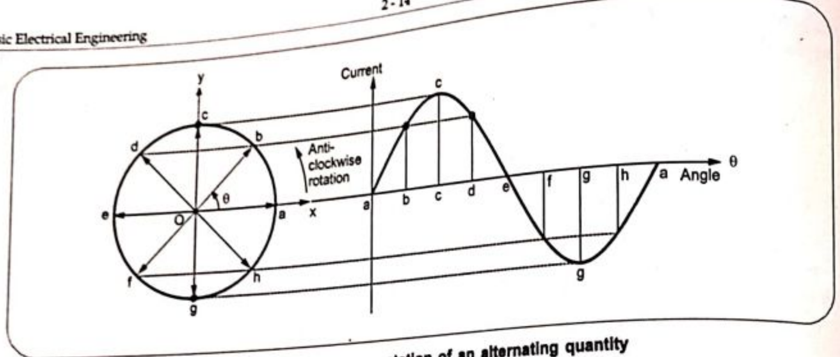


Fig. 2.10.1 Phasor representation of an alternating quantity

6. Similarly, at points f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants and when plotted, give us negative half cycle of the alternating quantity.

- Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y-axis gives the instantaneous value of the alternating quantity at that particular instant.
- The angular velocity 'ω' in an anticlockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle i.e. $\theta = \omega t$, where $\omega = 2\pi f$ rad/sec.

Points to Remember :

- In practice, the alternating quantities are represented by their r.m.s. values. Hence, the length of the phasor represents r.m.s. value of the alternating quantity. In such case, projection on Y-axis does not give directly the instantaneous value but as $I_m = \sqrt{2} I_{r.m.s}$, the projection on Y-axis must be multiplied by $\sqrt{2}$ to get an instantaneous value of that alternating quantity.
- Phasors are always assumed to be rotated in **anticlockwise** direction.
- Two alternating quantities of same frequencies can be represented on same phasor diagram.

2.11 Concept of Phase of an Alternating Quantity

➤ ***Define phase of an alternating quantity.***

In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant.

- It is represented in terms of angle θ in radians or degrees, measured from certain reference.
- Thus, phase can be defined as,

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

- Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 2.11.1 at the instant A is $\phi = 0°$.
- While the phase of the current at the instant B is the angle ϕ through which the phasor has travelled, measured from the reference axis i.e. X-axis.
- In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.
- In terms of phase, the equation of an alternating quantity can be modified as,

$$e = E_m \sin(\omega t \pm \phi)$$

where ϕ = Phase of the alternating quantity

- Let us consider three cases;

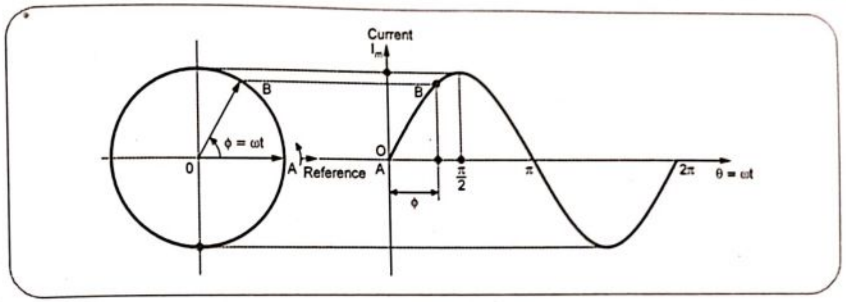


Fig. 2.11.1 Concept of phase

Case 1 : ϕ = 0° :

- When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 2.11.2 (a).

Case 2 : Positive phase ϕ :

- When phase of an alternating quantity is positive it means that quantity has **some positive instantaneous value at t = 0.** This is shown in the Fig. 2.11.2 (b).

Case 3 : Negative phase ϕ :

When phase of an alternating quantity is negative it means that quantity has **some negative instantaneous value at t = 0.** This is shown in the Fig. 2.11.2 (c).

1. The **phase** is measured with respect to **reference direction** i.e. positive X-axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the **phase** measured in **clockwise** direction is **negative.**

2.11.1 Phase Difference

➤ ***Explain the concept of phase difference.***

1. Zero Phase Difference : Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t) \quad \text{and} \quad i = I_m \sin(\omega t)$$

where $E_m > I_m$

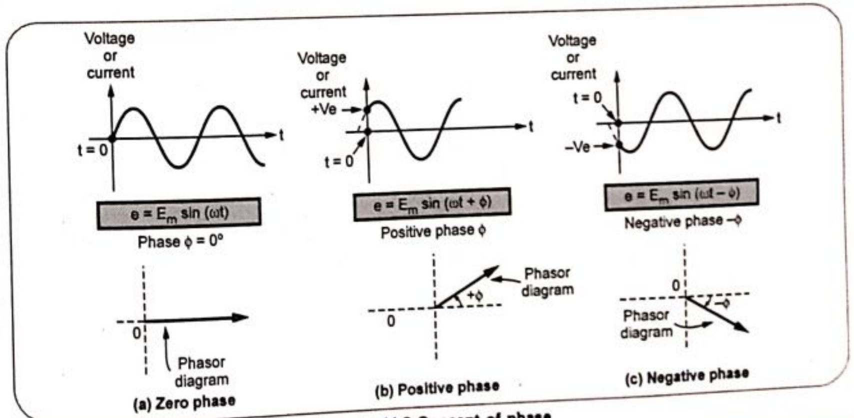


Fig. 2.11.2 Concept of phase

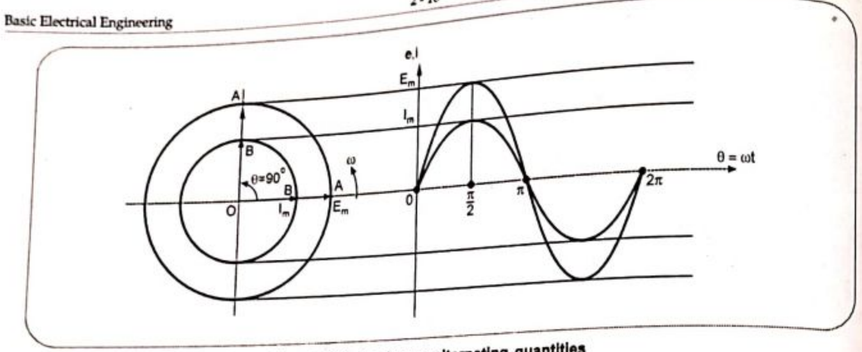


Fig. 2.11.3 In phasse alternating quantities

- The phasor representation and waveforms of both the quantities are shown in the Fig. 2.11.3.

The phasors $OA = E_m$ and $OB = I_m$

- After $\theta = \frac{\pi}{2}$ radians, the OA phasor achieves its maximum E_m while at the same instant, the OB phasor achieves its maximum I_m.
- So, at any instant, we can say that the phase of voltage e will be same as phase of i. The difference between the phases of the two quantities is zero at any instant.
- **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

> • *When such phase difference between the two alternating quantities is* ***zero****, the two quantities are said to be* ***in phase****.*

- In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value, at the same instant, the other is having some negative value or positive value. Such two quantities are said to have **phase difference** between them.

2. Lagging Phase Difference : Consider an e.m.f. having maximum value E_m and current having maximum value I_m.

- Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig. 2.11.4.
- Thus, there exists a phase difference ϕ between the two phasors.
- Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the instants by angle ϕ. This is called **lagging phase difference.** The current i is said to lag the voltage e by angle ϕ.

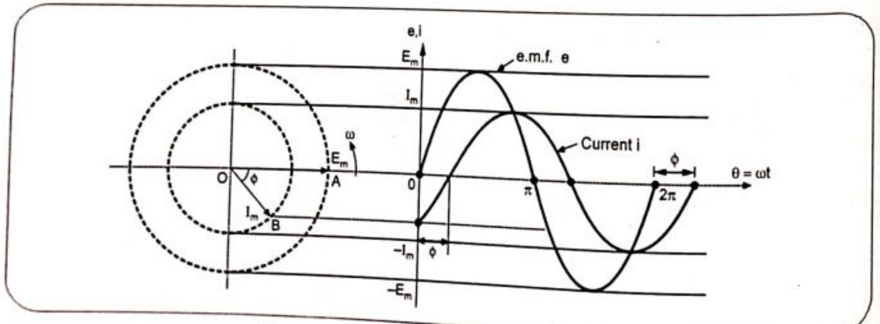


Fig. 2.11.4 Concept of phase difference (Lag)

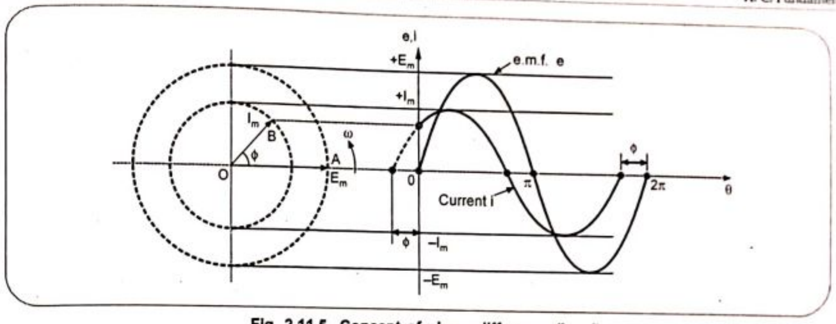


Fig. 2.11.5 Concept of phase difference (Lead)

- The current i achieves its maximum and zero values, ϕ angle later than the corresponding maximum and zero values of voltage.
- The equations of the two quantities are written as,

> $e = E_m \sin \omega t$ and $i = I_m \sin (\omega t - \phi)$ and 'i' is said to lag 'e' by angle ϕ

3. Leading Phase Difference :

It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig. 2.11.5

- It can be seen that there exists a phase difference of ϕ angle between the two. But in this case, current 'i' is ahead of voltage 'e', as both are rotating in anticlockwise direction with same speed.
- Thus, current is said to be leading with respect to voltage and the phase difference is called **leading phase difference.**
- At all instants, current i is going to remain ahead of voltage 'e' by angle 'ϕ'.
- The equations of such two quantities are written as

> $e = E_m \sin \omega t$ and $i = I_m \sin (\omega t + \phi)$
>
> and 'i' is said to lead 'e' by angle ϕ

2.11.2 Phasor Diagram

➤ ***What is phasor diagram ?***

- The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**
- All phasors have a particular fixed position with respect to each other.

> Phasor diagram can be considered as a still picture of the phasors at a particular instant.

Important Points Regarding Phasor Diagram :

1) As phasor diagram can be drawn at any instant. X and Y axis are not included in it. But, generally, the reference phasor chosen is shown along the positive X axis direction and at that instant other phasors are shown. This is just from convenience point of view. **The individual phase of an alternating quantity is always referred with respect to the positive x-axis direction.**
2) There may be more than two quantities represented in phasor diagram. Some of them may be current and some may be voltages or any other alternating quantities like flux, etc. **The frequency of all of them must be the same.**
3) Generally, length of phasor is drawn equal to r.m.s. value of an alternating quantity, rather than maximum value.
4) The phasors which are ahead, in anticlockwise direction, with respect to reference phasor are said to be leading with respect to reference and phasors behind are said to be lagging.
5) Different arrow heads may be used to differentiate phasors drawn for different alternating quantities like current, voltage, flux, etc.

Ex. 2.11.1 *Two sinusoidal currents are given by, $i_1 = 10 \sin(\omega t + \pi/3)$ and $i_2 = 15 \sin(\omega t - \pi/4)$. Calculate the phase difference between them in degrees.*

Sol. : The phase of current i_1 is $\pi/3$ radians i.e. $60°$ while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 2.11.6.

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = 105°$$

And i_2 lags i_1.

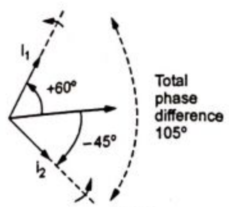


Fig. 2.11.6

Ex. 2.11.2 *In a circuit supplied from 50 Hz the voltage and current have maximum values of 500 V and 10 A respectively. At t = 0, their respective values are 400 V and 4 A both increasing positively. i) Write expressions for their instantaneous values. ii) Find the angle between V and I and iii) I at t = 0.015 sec.* VTU : Feb. 06, Marks 8

Sol. : f = 50 Hz, V_m = 500 V, I_m = 10 A

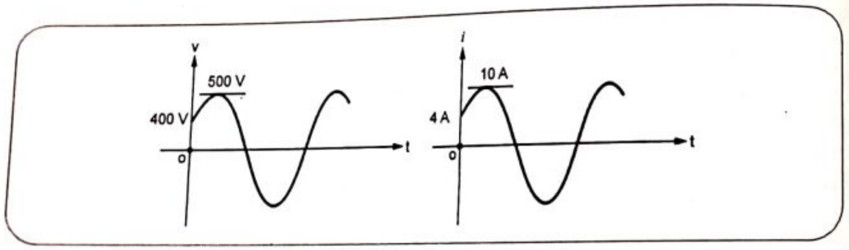


Fig. 2.11.7

$v = V_m \sin(2\pi f t + \phi_1)$ ...As voltage is +ve at t = 0

$\therefore$ $400 = 500 \sin(0 + \phi_1)$ ... t = 0

$\therefore$ $\phi_1 = 53.13° = 0.9272$ rad.

$\therefore$ $\mathbf{v = 500 \sin(100\pi t + 0.9272)\ V}$

$i = I_m \sin(2\pi f t + \phi_2)$ i.e. $4 = 10 \sin(0 + \phi_2)$ ... t = 0

$\therefore$ $\phi_2 = 23.57° = 0.4115$ rad. $\therefore$ i.e. $\mathbf{i = 10 \sin(100\pi t + 0.4115)\ A}$

ii) $\phi_1 = 53.13°$ for voltage, $\phi_2 = 23.57°$ for current

$\therefore$ ϕ = angle between V and I = $53.13° - 23.57° = \mathbf{29.56°}$

iii) At t = 0.015 sec,

$i = 10 \sin(100\pi \times 0.015 + 0.4115) = \mathbf{-9.1652\ A}$ **...Use radian mode**

2.12 Mathematical Representation of Phasor

- The algebraic operations such as addition, subtraction etc. with waveforms are much complicated and time consuming. Hence it is necessary to represent the phasors mathematically.
- Any phasor can be represented mathematically in two ways,

1) Polar co-ordinate system and

2) Rectangular co-ordinate system.

2.12.1 Polar Co-ordinate System

- Consider an alternating current given by, $i = I_m \sin(\omega t + \phi)$
- Thus its maximum value is I_m and phase is $+\phi$. The phase ϕ is always measured with respect to positive X-axis direction.
- While representing this phasor by polar system, it is represented as $r \angle \phi$.

$r = I_m$ and ϕ is angle with respect to + ve X-axis.

- So draw a line at an angle ϕ measured with respect to + ve X-axis from the origin. And measure a distance equal to $r = I_m$ on it.
- The line OA represents polar representation of phasor. This is shown in the Fig. 2.12.1.
- The angle ϕ can be positive or negative.
- The **positive** ϕ is measured in **anticlockwise** direction while the **negative** ϕ is measured in **clockwise** direction.
- While r is always positive and called **magnitude** of the phasor. The angle ϕ is called **phase** of that phasor.
- Thus mathematically the polar representation of a phasor is,

Polar representation = $r \angle \pm \phi$

- **Practically instead of $r = I_m$, r.m.s. value is used as the magnitude r.**
- The polar form of an alternating quantity can be easily obtained from its instantaneous equation directly.

If $e = E_m \sin(\omega t \pm \phi)$ then polar form is,

$E = E \angle \pm \phi$ where E is r.m.s. value = $\frac{E_m}{\sqrt{2}}$.

2.12.2 Rectangular Co-ordinate System

- Mathematically an alternating quantity can be divided into two components, X-component and Y-component.
- If an alternating current is $i = I_m \sin(\omega t + \phi)$ then,

X-component = $I_m \cos\phi$ and Y-component = $I_m \sin\phi$

- Thus to represent the phasor, travel ($I_m \cos\phi$) in +X direction then travel ($I_m \sin\phi$) in +Y direction. Joining final point to origin gives the required phasor OA as shown in the Fig. 2.12.2.
- The X and Y components can be positive or negative. To indicate that X and Y components are perpendicular to each other, the operator 'j' is used in mathematical representation of phasor in rectangular co-ordinate system.
- Thus mathematically the rectangular representation of a phasor is,

Rectangular representation = $\pm X \pm j Y$

- **Practically instead of maximum value, r.m.s. value is used to find X and Y components.**

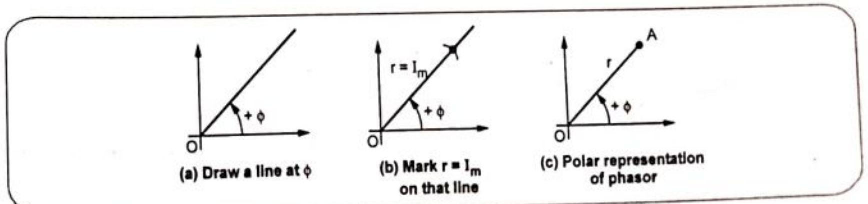


Fig. 2.12.1

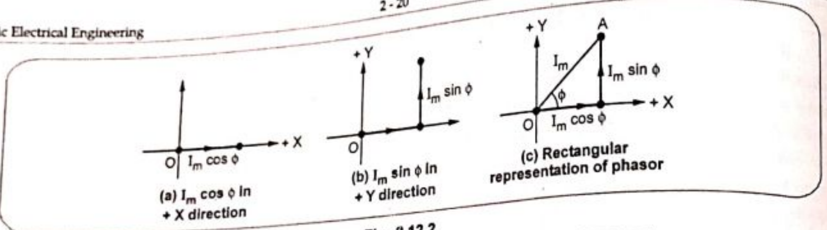


Fig. 2.12.2

- The mathematical value of operator j is $\sqrt{-1}$ but in the phasor representation, multiplication by 'j' indicates the rotation of a phasor through 90° in anticlockwise direction.

2.12.3 Polar to Rectangular Conversion

- Let a phasor is represented in polar form as shown in the Fig. 2.12.3.

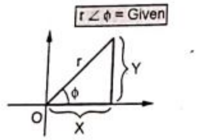


Fig. 2.12.3

- It is necessary to find X and Y components in terms of r and ϕ.

From the Fig. 2.12.3,

$$\text{X component} = r \cos \phi$$

and

$$\text{Y component} = r \sin \phi$$

$\therefore$ $\boxed{\text{Rectangular representation} = r \cos \phi + j\, r \sin \phi}$

2.12.4 Rectangular to Polar Conversion

- Let a phasor is represented in rectangular form X + j Y, as shown in the Fig. 2.12.4.

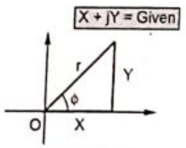


Fig. 2.12.4

- It is necessary to find r and ϕ in terms of X and Y.
- From the Fig. 2.12.4,

$$r = \sqrt{X^2 + Y^2} \quad \text{and} \quad \phi = \tan^{-1} \frac{Y}{X}$$

$\therefore$ $\boxed{\text{Polar representation} = r \angle \phi = \sqrt{X^2 + Y^2} \angle \tan^{-1} \frac{Y}{X}}$

- The polar form always gives r.m.s. value of an alternating quantity.

Important Note : To obtain polar form from the instantaneous equation, express the given equation in sine form instead of cosine form.

If, $e = E_m \cos (\omega t \pm \phi)$ then express it as,

$$e = E_m \sin (\omega t + 90° \pm \phi)$$

$\therefore$ Phase of alternating quantity = 90° ± ϕ.

- *Instead of using above relations, use the polar to rectangular (P → R) and rectangular to polar (R → P) functions available on calculator for the required conversions.*

Ex. 2.12.1 *Write the polar form of the voltage given by, V = 100 sin (100 π t + π/6) V*

Obtain its rectangular form.

Sol. : V_m = 100 V and $\phi = +\frac{\pi}{6}$ rad = + 30°,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 70.7106 \text{ V}$$

$\therefore$ In polar form = **70.7106 ∠ + 30° V**

$\therefore$ Rectangular form = **61.2371 + j 35.3553 V**

Key Point : The r.m.s. vlaue of an alternating quantity exists in its polar form and not in rectangular form. **Thus to find r.m.s. value of an alternating quantity express it in polar form.**

Ex. 2.12.2 *Find r.m.s. value and phase of the current I = 25 + j 40 A.*

Sol. : The r.m.s. value is not 25 or 40 as it exists in polar form.

Converting it to polar form,

$$I = 47.1699 \angle 57.99° \text{ A} = I_{rms} \angle \phi \text{ A}$$

$\therefore$ r.m.s. value of current = **47.1699 A**

and Phase = **57.99°**

Key Point : To obtain phase, express the equation in **sine** form if given in **cosine** as,

If $e = E_m \cos (\omega t)$

then $e = E_m \sin (\omega t + 90°)$

as $\sin(90° + \theta) = \cos \theta$

Thus the phase is 90° and not zero.

Ex. 2.12.3 *A voltage is defined as $- E_m \cos \omega t$. Express it in polar form.*

Sol. : To express a voltage in polar form express it in the form, $e = E_m \sin \omega t$

Now $e = - E_m \cos \omega t$

$$= - E_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$= E_m \sin\left(\omega t + \frac{3\pi}{2}\right) \quad \text{as } \sin(\pi + \theta) = -\sin \theta$$

Now it can be expressed in polar form as,

$$e = E_m \angle + \frac{3\pi}{2} \text{ rad} = E_m \angle + 270° \text{ V}$$

But + 270° phase is nothing but − 90°

$$\therefore \quad e = E_m \angle - 90° \text{ V}$$

2.13 Addition and Subtraction of Alternating Quantities

- The addition and subtraction of phasors is called **vector addition** or **vector subtraction** of the alternating quantities. It is possible by analytical method or graphical method.
- As the graphical method is time consuming which includes plotting the phasors to the scale, the analytical method is used. Also, graphical method may give certain error which will vary from person to person depending upon the skills of plotting the phasors. The answer by analytical method is always accurate.
- In the analytical method, represent all the phasors in the rectangular form. For this, **represent each phasor to be added as sine function if any of the phasors is represented as cosine function in its equation form.**
- Let $P = X_1 + j Y_1$ and $Q = X_2 + j Y_2$
- Then analytically while adding P and Q, their X components get added and corresponding Y components get added. Hence the resultant R is,

$$\boxed{R = P + Q = (X_1 + X_2) + j (Y_1 + Y_2)}$$

- While subtracting, their X components get subtracted and corresponding Y components get subtracted. Hence the resultant is,

$$\boxed{R = P - Q = (X_1 - X_2) + j (Y_1 - Y_2)}$$

Important : While performing addition and subtraction use rectangular form of representation of phasors.

- The result of the addition and subtraction, finally can be expressed in the polar form, but their individual polar forms cannot be used to perform addition and subtraction.

Ex. 2.13.1 *Three voltages are connected as shown in the Fig. 2.13.1.*

If V_a = 17.32 + j 10 V, V_b = 30 ∠80° V, V_c = 15 ∠−100° V. Find.

i) V_{12} ii) V_{23} iii) V_{34}.

VTU : Aug.- 05, Marks 8

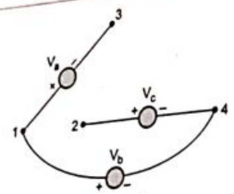


Fig. 2.13.1

Sol. : Looking at the polarities of the voltages V_a, V_b and V_c,

i) $$V_{12} = \bar{V}_b - \bar{V}_c = (30\angle 80^\circ) - (15\angle -100^\circ) = [5.2094 + j\,29.5442] - [-2.6047 - j\,13.7721]$$
$$= 7.8141 + j\,43.3162 \text{ V} = 45\angle 80^\circ \text{ V}$$

ii) $$V_{23} = \bar{V}_a + \bar{V}_c - \bar{V}_b = [17.32 + j\,10] + [-2.6047 - j\,14.7721] - [5.2094 + j\,29.5442]$$
$$= 9.5059 - j\,34.3163 \text{ V} = 35.6085\angle -74.51^\circ \text{ V}$$

iii) $$V_{34} = \bar{V}_b - \bar{V}_a = [5.2094 + j\,29.5442] - [17.32 + j\,10]$$
$$= -12.1106 + j\,19.5442 \text{ V} = 23\angle 121.78^\circ \text{ V}$$

2.14 Multiplication and Division of Phasors

- The multiplication and division of phasors is performed using **polar form** of representation.
- Let P and Q be the two phasors such that,

$$P = X_1 + jY_1 \quad \text{and} \quad Q = X_2 + jY_2$$

- To obtain the multiplication $P \times Q$, both must be expressed in polar form

$\therefore$ $$P = r_1\angle\phi_1 \quad \text{and} \quad Q = r_2\angle\phi_2$$

Then
$$\boxed{\begin{aligned} P \times Q &= [r_1\angle\phi_1] \times [r_2\angle\phi_2] \\ &= [r_1 \times r_2]\angle\phi_1 + \phi_2 \end{aligned}}$$

- *Thus in multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.*

- The result then can be expressed back to rectangular form, if required.

Now consider the division of the phasors P and Q.

$$\boxed{\frac{P}{Q} = \frac{r_1\angle\phi_1}{r_2\angle\phi_2} = \left|\frac{r_1}{r_2}\right| \angle\phi_1 - \phi_2}$$

- *Thus in division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.*

Important : While performing multiplication and division use polar form of representation of phasors.

- The final result then can be expressed in rectangular form if required.

Remember :

- While **addition and subtraction**, use **rectangular** form.
- While **multiplication and division**, use **polar** form.

Ex. 2.14.2 *If $A = 4 + j\,7$; $B = 8 + j\,9$ and $C = 5 - j\,6$ then calculate*

i) $\frac{A+B}{C}$ ii) $\frac{A \times B}{C}$ iii) $\frac{A+B}{B+C}$ iv) $\frac{B-C}{A}$

Sol. : $A = 4 + j\,7 = 8.062\angle 60.255^\circ$,

$B = 8 + j\,9 = 12.041\angle 48.366^\circ$

$C = 5 - j\,6 = 7.8102\angle -50.194^\circ$

i) $$\frac{A+B}{C} = \frac{4 + j\,7 + 8 + j\,9}{7.8102\angle -50.194^\circ} = \frac{12 + j\,16}{7.8102\angle -50.194^\circ}$$
$$= \frac{20\angle 53.13^\circ}{7.8102\angle -50.194^\circ}$$
$$= \mathbf{2.5607\angle 103.324^\circ = -0.5901 + j\,2.4917}$$

ii) $$\frac{A \times B}{C} = \frac{8.062\angle 60.255^\circ \times 12.041\angle 48.366^\circ}{7.8102\angle -50.194^\circ}$$
$$= \frac{97.0745\angle 108.621^\circ}{7.8102\angle -50.194^\circ}$$
$$= \mathbf{12.4291\angle 158.815^\circ}$$
$$= \mathbf{-11.5891 + j\,4.4916}$$

iii) $$\frac{A+B}{B+C} = \frac{20\angle 53.13^\circ}{8 + j\,9 + 5 - j\,6} = \frac{20\angle 53.13^\circ}{13 + j\,3}$$

...Using A+B calculated in (i)

$$= \frac{20\angle 53.13^\circ}{13.341\angle 12.99^\circ} = \mathbf{1.499\angle 40.14^\circ}$$
$$= \mathbf{1.1459 + j\,0.966}$$

iv) $$\frac{B-C}{A} = \frac{(8 + j\,9) - (5 - j\,6)}{8.062\angle 60.255^\circ} = \frac{3 + j\,15}{8.062\angle 60.255^\circ}$$
$$= \frac{15.297\angle 78.69^\circ}{8.062\angle 60.255^\circ}$$
$$= \mathbf{1.8974\angle 18.435^\circ = 1.8 + j\,0.6}$$

2.15 University Questions with Answers

July - 2011

Q.1 *Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.* **[Refer section 2.5]** **[4]**

Q.2 *Define average value of an alternating quantity. Obtain the relation between average value and the maximum value of an alternating quantity.* **[Refer section 2.6]** **[4]**

Jan. - 2013

Q.3 *Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.* **[Refer section 2.5]** **[4]**

July - 2013

Q.4 *Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.* **[Refer section 2.5]** **[4]**

Jan. - 2014

Q.5 *Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.* **[Refer section 2.5]** **[4]**

July - 2014

Q.6 *Define R.M.S. value of an alternating quantity. Obtain the relation between r.m.s. value and the maximum value of an alternating quantity.* **[Refer section 2.5]** **[4]**

July - 2015

Q.7 *Define form factor. Derive its value for sinusoidal quantity.* **[Refer section 2.7]** **[2]**

CBCS Scheme

Jan. - 2016

Q.8 *Define the following with reference to AC quantities : i) Instantaneous value ii) Frequency iii) Time period iv) Form factor v) Peak factor.*
[Refer sections 2.3, 2.7 and 2.8] **[5]**

July - 2016

Q.9 *Define RMS value of sinusoidally verying current and find its relation with its maximum value.* **[Refer section 2.5]**

Q.10 *Derive an expression for average value of an alternating quantity.* **[Refer section 2.6]**

Jan. - 2018

Q.11 *Derive equations for the rms value and average value of a sinusoidally verying current.*
[Refer sections 2.5, 2.6]

❑❑❑

MODULE - 2

3

Single Phase A.C. Circuits

Syllabus

Analysis with phasor diagram of circuits with R, L, C, R - L, RC, R - L - C for series and parallel configurations. Real power, reactive power, apparent power and power factor, Illustrative Examples,

Contents

3.1 A.C. through Pure Resistance

➢ ***With the help of phasor diagram and waveform comment on the phase realtion between voltage and current in purely resistive circuit.***

➢ ***Derive an expression for the instantaneous power in a pure resistor energised by sinusoidal voltage.*** **VTU : Aug.-02; Mar.-04 Marks 6**

- Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$.
- The circuit is shown in the Fig. 3.1.1.
- According to Ohm's law, we can find the equation for the current i as,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \quad \text{i.e. } i = \left(\frac{V_m}{R}\right) \sin (\omega t)$$

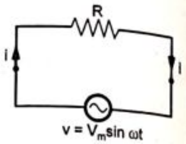


Fig. 3.1.1 Pure resistive circuit

- This is the equation giving instantaneous value of the current.
- Comparing this with standard equation, $i = I_m \sin (\omega t + \phi)$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0°}$$

- So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while as $\phi = 0$, it indicates that it is in phase with the voltage applied.

> - *In purely resistive circuit, the current and the voltage applied are in phase with each other.*

- The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 3.1.2 (a) and (b).

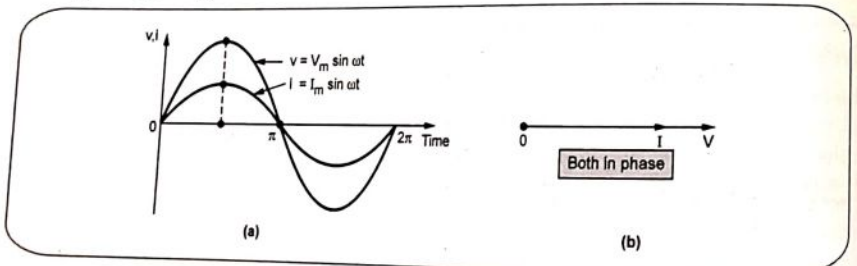


Fig. 3.1.2 A.C. through purely resistive circuit

- In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the **r.m.s. values** of alternating quantities.

3.1.1 Power

- The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t = V_m I_m \sin^2(\omega t) = \frac{V_m I_m}{2} (1 - \cos 2 \omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos (2 \omega t)}$$

- From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2} \cos(2 \omega t)\right]$ having frequency, double the frequency of the applied voltage.

- **The average value of the fluctuating cosine component of double frequency is zero, over one complete cycle.**
- So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$ which is half of the peak power $V_m I_m$.

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \quad \text{i.e.} \quad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

- Generally, r.m.s. values are indicated by capital letters.
- The Fig. 3.1.3 shows the waveforms of voltage, current and power.

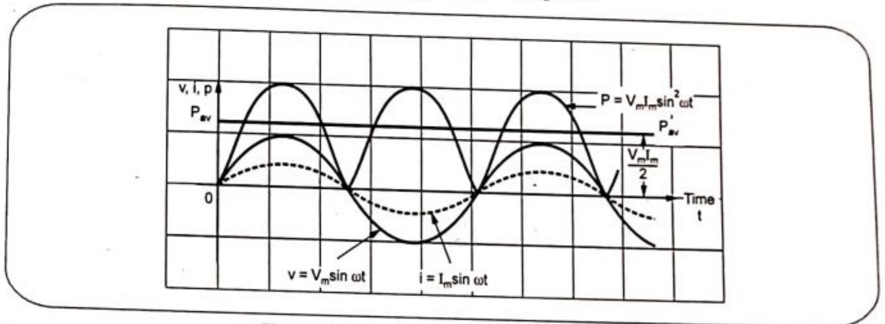


Fig. 3.1.3 v, i and p for purely resistive circuit

3.2 A.C. through Pure Inductance

➢ ***Obtain expression for the current through the pure inductor, if the voltage across it is $V = V_m \sin \omega t$.*** **VTU : Jan.-15, Marks 6**

$$V = V_m \sin wt$$

➢ ***Prove that in a purely inductive circuit the current lags voltage by 90°.*** **VTU : Jan.-10, Marks 6**

- Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$.
- The circuit is shown in the Fig. 3.2.1.
- Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

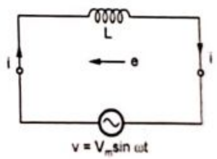


Fig. 3.2.1 Purely inductive circuit

- When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance.
- This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.
- The self induced e.m.f. in the coil is given by, $e = -L\frac{di}{dt}$.
- At all instants, applied voltage, V is equal and opposite to the self induced e.m.f., e

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad v = L\frac{di}{dt} \quad \text{i.e.} \quad V_m \sin \omega t = L\frac{di}{dt} \quad \text{i.e.} \quad di = \frac{V_m}{L}\sin \omega t \, dt$$

$$\therefore \quad i = \int di = \int \frac{V_m}{L}\sin \omega t \, dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

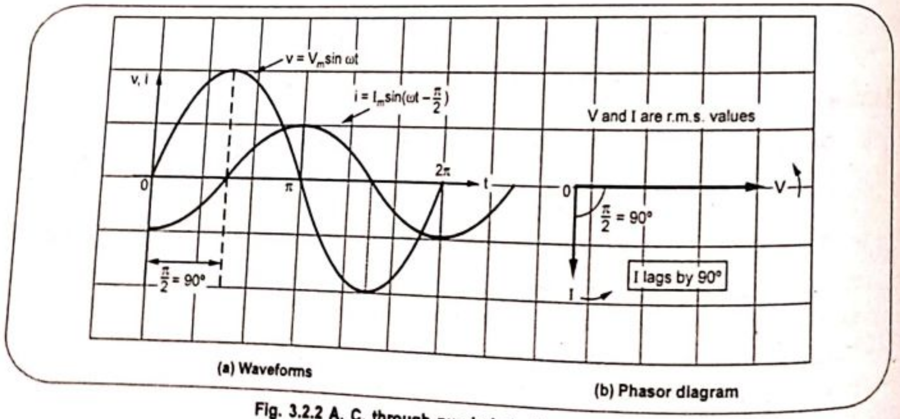
$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \quad \text{as } \cos \omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\therefore \quad i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{as } \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\therefore \quad \boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \text{ where } I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L} \text{ and } X_L = \omega L = 2\pi f L \;\; \Omega}$$

- The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. $-90°$. This means that **the current lags voltage applied by 90°.**
- The negative sign indicates lagging nature of the current.
- The Fig. 3.2.2 shows the waveforms and the corresponding phasor diagram.

> - *In purely inductive circuit, current lags voltage by 90°.*



(a) Waveforms (b) Phasor diagram

Fig. 3.2.2 A. C. through purely inductive circuit

3.2.1 Concept of Inductive Reactance

➤ ***Explain the concept of inductive reactance. How it depends on the frequency ?***

- It is shown that, $\boxed{X_L = \omega L = 2\pi f L \; \Omega}$
- The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**
- The **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.
- It is measured in ohms and it depends on the frequency of the applied voltage.

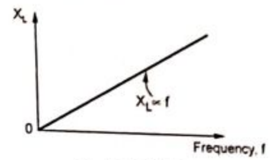


Fig. 3.2.3 X_L Vs f

- The inductive reactance is directly proportional to the frequency for constant L.

$X_L \propto f$, for constant L

- So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 3.2.3.

> - *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

3.2.2 Power

➤ ***Show that the average power consumed by pure inductor is zero.*** **VTU : Jan.-02, 06 Marks 5**

- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t)\cos(\omega t) \quad \text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \quad \boxed{P = -\frac{V_m I_m}{2}\sin(2\omega t)} \quad \text{as } 2\sin \omega t \cos \omega t = \sin 2\omega t$$

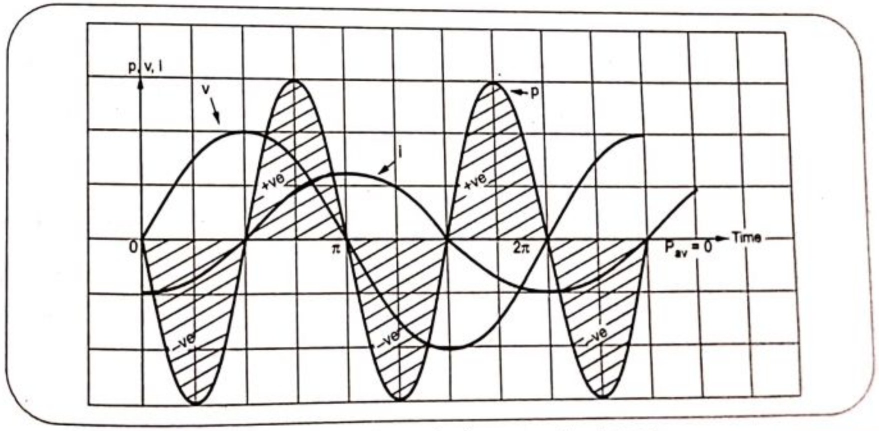


Fig. 3.2.4 Waveforms of voltage, current and power

- This power curve is a sine curve of frequency double than that of applied voltage.
- The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$

- The Fig. 3.2.4 shows voltage, current and power waveforms.
- It can be observed from it that when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply. The areas of positive loop and negative loop are exactly same and hence, **average power consumption is zero.**

- Pure inductance never consumes power.

- The average energy stored in an inductor is given by $E = \frac{1}{2} L I^2$ joules.

3.3 A.C. through Pure Capacitance

> ***Show that the current through purely capacitive circuit leads the applied voltage by 90°.*** VTU : Mar.-01, Aug.-03 Marks 6

- Consider a simple circuit consisting of a pure capacitor of C- farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 3.3.1.
- The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C\,v = C\,V_m \sin \omega t$$

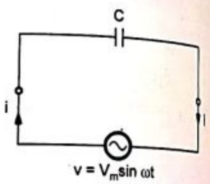


Fig. 3.3.1 Purely capacitive circuit

- Current is rate of flow of charge.

$$i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t) = C\,V_m \frac{d}{dt}(\sin \omega t) = C\,V_m\, \omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $\boxed{I_m = \frac{V_m}{X_C} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega}$

- The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.
- This means **current leads voltage applied by 90°.** The positive sign indicates leading nature of the current.
- The Fig. 3.3.2 shows waveforms of voltage and current and the corresponding phasor diagram.
- The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

- *In purely capacitive circuit, current leads voltage by 90°.*

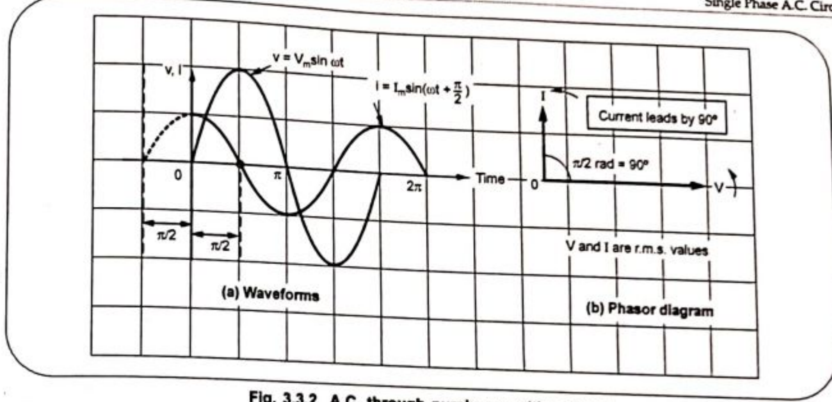


Fig. 3.3.2 A.C. through purely capacitive circuit

3.3.1 Concept of Capacitive Reactance

> ***Explain the concept of inductive reactance. How it depends on the frequency ?***

- It is shown that, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$
- The term X_C is called **Capacitive Reactance** and is measured in ohms.
- The **capacitive reactance** is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.
- X_C is measured in ohms and it depends on the frequency of the applied voltage.
- The capacitive reactance is inversely proportional to the frequency for constant capacitor C.

$$X_C \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_C Vs f is a rectangular hyperbola as shown in Fig. 3.3.3.

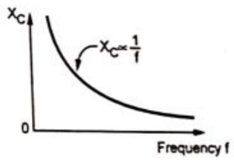


Fig. 3.3.3 X_C Vs f

- *If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers open circuit to the d.c. or it blocks d.c.*

3.3.2 Power

> ***Show that the average power consumed by pure capacitor is zero.*** VTU : Feb.-05, July-15, Jan-16, Marks 6

> ***Derive the expression for the instantaneous power in a pure capacitor energised by sinusoidal voltage. Draw the wave shapes of current, voltage and power.*** VTU : Jan.-01, 03, 17; May-10, July-17, Marks 8

- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t) \cos(\omega t) \quad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} \sin (2\omega t)} \quad \text{as } 2 \sin \omega t \cos \omega t = \sin 2 \omega t$$

- Thus, power curve is a sine wave of frequency double that of applied voltage.
- **The average value of sine curve over a complete cycle is always zero.**

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin (2\omega t)\, d(\omega t) = 0$$

- The Fig. 3.3.4 shows waveforms of current, voltage and power.

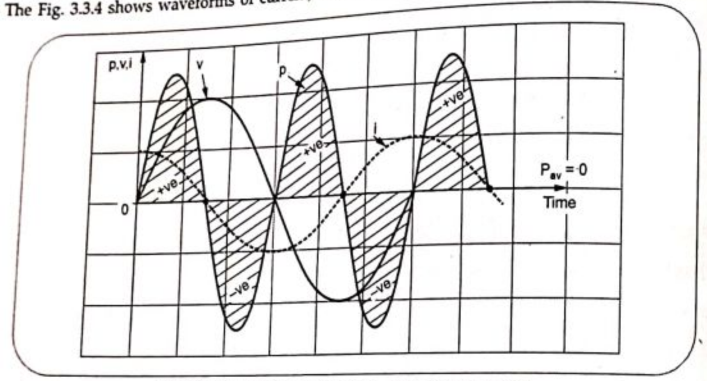


Fig. 3.3.4 Waveforms of voltage, current and power

- It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging.
- The areas of positive and negative loops are exactly the same and hence, **average power consumption is zero.**

• Pure capacitance never consumes power.

- The average energy stored in a capacitor is given by, $E = \frac{1}{2} C V^2$ joules.

Ex. 3.3.1 *The current drawn by a pure capacitor of 20 μF is 1.382 A from 220 V a.c. supply. What is the supply frequency ?* **VTU : Jan.-03, Marks 6**

Sol. : $C = 20\ \mu F, I = 1.382\ A, V = 220\ V$

$$X_C = \frac{V}{I} = \frac{220}{1.382} = 159.1895\ \Omega$$

But $\quad X_C = \frac{1}{2\pi f C} \quad$ i.e. $\quad f = \frac{1}{2\pi C X_C}$

$$\therefore \quad f = \frac{1}{2\pi \times 20 \times 10^{-6} \times 159.1895} = 49.99 \approx \mathbf{50\ Hz} \quad \text{...Supply frequency}$$

3.4 Impedance

➤ ***Define impedance.***

- The opposition offered by an electric circuit to the flow of an alternating current is called an impedance. It is denoted by Z. It is the ratio of an alternating voltage to an alternating current through the circuit.
- Impedance is complex and is expressed in polar or rectangular form.
- For pure resistance voltage and current are in phase hence impedance does not introduce any phase angle. So impedance of a pure resistance can be expressed in polar and rectangular form as,

$$\boxed{Z = R + j0 = R\angle 0^\circ \text{ ohms.}}$$

- For a pure inductance, the current lags voltage by 90° hence the inductive reactance X_L produces a phase lag of 90°.
- For a pure inductance, if voltage is $V\angle 0^\circ$ then current is $I\angle -90^\circ$ hence its impedance in polar and reactangular form is given by,

$$\boxed{Z = \frac{V\angle 0^\circ}{I\angle -90^\circ} = \frac{V}{I}\angle 90^\circ = X_L\angle 90^\circ = 0 + j\,X_L \text{ ohms}}$$

- For a pure capacitance, the current leads voltage by 90° hence the capacitive reactance X_C produces a phase lead of 90°.
- For a pure capacitance, if voltage is $V\angle 0^\circ$ then current is $I\angle +90^\circ$ hence its impedance is given by,

$$\boxed{Z = \frac{V\angle 0^\circ}{I\angle +90^\circ} = \frac{V}{I}\angle -90^\circ = X_C\angle -90^\circ = 0 - j\,X_C \text{ ohms}}$$

3.5 A.C. through Series R-L Circuit

➤ ***Explain the behaviour of series R-L circuit. Draw its phasor diagram.***

➤ ***Show that current lags behind the voltage in series R-L circuit.*** **VTU : Jan.-04, Marks 6**

➤ ***With the help of circuit diagram and phasor diagram, find the phase angle, impedance and power in case of R-L series circuit.***

- Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L henries as shown in the Fig. 3.5.1 (a).
- The series combination is connected across a.c. supply given by $v = V_m \sin \omega t$.

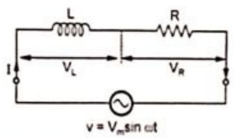


Fig. 3.5.1 (a) Series R-L circuit

- Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $\quad V_R = I \times R$

b) Drop across pure inductance, $\quad V_L = I \times X_L \quad$ where $X_L = 2\pi f L$

I = R.M.S. value of current drawn

V_R, V_L = R.M.S. values of the voltage drops.

- The Kirchhoff's voltage law can be applied to the a.c. circuit but important point to remember is that the **addition of voltages is a phasor (vector) addition and no longer algebraic as in case of d.c.** (Phasor addition)

$\therefore \quad \overline{V} = \overline{V}_R + \overline{V}_L = \overline{IR} + \overline{IX}_L$

- *For series a.c. circuits, generally, current is taken as the reference phasor as it is common to both the elements.*

- Following are the **steps to draw the phasor diagram :**

1) Take current as a reference phasor.
2) In case of resistance, voltage and current are in phase, so V_R will be along current phasor.
3) In case of inductance, current lags voltage by 90°. But, as current is reference, V_L must be shown leading with respect to current by 90°.
4) The supply voltage being vector sum of these two vectors V_L and V_R obtained by law of parallelogram.

- The phasor diagram and the voltage triangle is shown in the Fig. 3.5.1 (b) and (c).

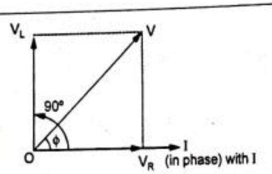


Fig. 3.5.1 (b) Phasor diagram

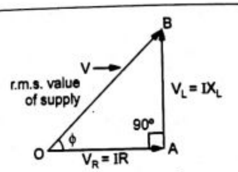


Fig. 3.5.1 (c) Voltage triangle

- From the voltage triangle, we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (I \times X_L)^2} = I\sqrt{(R)^2 + (X_L)^2}$$

$$\therefore \quad V = IZ \quad \text{i.e.} \quad Z = \frac{V}{I}$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_L)^2} \text{ (magnitude)}}$ ... **impedance** of the circuit

- The impedance Z is measured in ohms.

3.5.1 Impedance

> ***Derive the impedance of a series R-L circuit.***

- Impedance is defined as the opposition of circuit to flow of alternating current. It is denoted by Z and its unit is ohms.
- For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle ϕ. From the voltage triangle,

we can write,

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

- If all the sides of the voltage triangle are divided by current, we get a triangle called **impedance triangle** as shown in the Fig. 3.5.2.
- Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.
- From this impedance triangle, we can see that,

X component of $Z = R = Z\cos\phi$

Y component of $Z = X_L = Z\sin\phi$

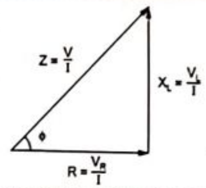


Fig. 3.5.2 Impedance triangle

- In rectangular form the impedance is denoted as,

$$\boxed{Z = R + jX_L\ \Omega}$$

- While in polar form, it is denoted as,

$$Z = |Z| \angle\phi \ \Omega$$
$$\text{where } |Z| = \sqrt{R^2 + X_L^2}, \qquad \phi = \tan^{-1}\left[\frac{X_L}{R}\right]$$

- *Thus ϕ is positive for inductive impedance.*

3.5.2 Power and Power Triangle

> *Derive the expression for the instantaneous power in a series R-L circuit.* VTU : Dec.-11, Marks 6

> *Draw the power triangle and define active power, reactive power and apparent power. State their units.* VTU : July-03, 06, Marks 6

> *Show that power consumed in AC circuit is P = VI cos ϕ, where V is RMS value of the applied voltage, I is the RMS value of current and ϕ is the angle between voltage V and current I.* VTU : Jan-16, Marks 5

- The expression for the current in the series R-L circuit is,

$i = I_m \sin(\omega t - \phi)$ as current lags voltage.

- The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t - \phi) = V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

- The second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$\therefore \quad \boxed{P = VI\cos\phi \text{ watts}}$ where V and I are r.m.s. values

- If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R}\,\overline{I} + \overline{V_L}\,\overline{I} \quad \text{i.e. } \overline{VI} = \overline{V\cos\phi I} + \overline{V\sin\phi I}$$

- From this equation, power triangle can be obtained as shown in the Fig. 3.5.3.

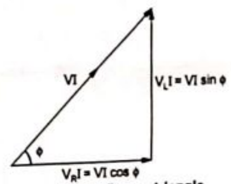


Fig. 3.5.3 Power triangle

- So, three sides of this triangle are,
  1) VI 2) VI cos ϕ 3) VI sin ϕ

1. Real or True or Active Power (P) :

- It is defined as the product of the applied voltage and the active component of the current.
- It is measured in unit watts (W) or kilowatts (kW).
- It is real component of the apparent power.

$$\boxed{P = V I \cos\phi \quad \text{watts}}$$

2. Apparent Power (S) :

- It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\therefore \quad \boxed{S = VI \text{ VA}}$$

- It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

3. Reactive Power (Q) :

- It is defined as product of the applied voltage and the reactive component of the current.
- It is also defined as imaginary component of the apparent power.
- It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$\boxed{Q = V I \sin\phi \quad \text{VAR}}$$

3.5.3 Power Factor (cos ϕ)

➤ ***Define power factor and explain its significance in a.c. circuit.***

VTU : Jan.-02, 04, 17; July-04 Marks 5; July-09, 15, Marks

- It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.
- It is the ratio of true power to apparent power.

$$\boxed{\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{V I \cos\phi}{V I} = \cos\phi}$$

- It is the factor which decides the true power consumption in the circuit.
- **The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor.**
- It cannot be greater than 1.
- It is also defined as the ratio of resistance to the impedance.

$$\boxed{\cos\phi = \frac{R}{Z}}$$

- The nature of power factor is always determined by position of current with respect to the voltage.

- If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

- So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. $\phi = 0$. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\boxed{\text{Power factor} = \cos\phi \text{ where } \phi \text{ is the angle between supply voltage and current.}}$$

- Nature of power factor always tells position of current with respect to voltage.

Ex. 3.5.1 *Given $v = 200 \sin 377t$ volts and $i = 8 \sin (377t - 30°)$ amps for an a.c. circuit, determine : i) Power factor ii) True power iii) Apparent power iv) Reactive power Indicate the unit of power calculated.*

VTU : Feb.-07, Marks 8

Sol. : Given : $v = 200 \sin 377t$ and $i = 8 \sin (377t - 30°)$

We know that $v = V_m \sin(\omega t - \phi)$ and $I = I_m \sin(\omega t - \phi)$

$\therefore$ From above equation $V_m = 200$ V, $I_m = 8$ A and $\omega = 377 = 2\pi f$

$$\therefore \quad f = \frac{377}{2\pi} = \frac{377}{2 \times 3.14} = 60 \text{ Hz}$$

i) Phase of voltge is 0° while that of current is 30° lagging as shown in the Fig. 3.5.4.

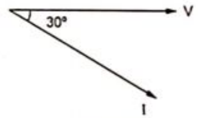


Fig. 3.5.4

$\therefore \quad \phi$ = Angle between V and I = 30°

$\therefore$ p.f. = $\cos\phi = \cos 30° =$ **0.866 lagging**

ii) True power (active power) = $VI\cos\phi$

where $V = \frac{V_m}{\sqrt{2}}$, $I = \frac{I_m}{\sqrt{2}}$ (r.m.s)

$$\text{or} \quad = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos\phi = \frac{200}{\sqrt{2}} \times \frac{8}{\sqrt{2}} \times 0.866 = \textbf{692.8 watts}$$

iii) $$\text{Apparent power} = V \times I = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} \times \frac{8}{\sqrt{2}} = \textbf{800 VA} \text{ (volt-ampere)}$$

iv) $$\text{Reactive power} = VI \sin\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \sin\phi = \frac{200}{\sqrt{2}} \times \frac{8}{\sqrt{2}} \sin 30°$$

$$= \textbf{400 VAR} \text{ (volt-ampere reactive)}$$

Ex. 3.5.2 *An e.m.f. whose instantaneous value is 100 sin (314 t − π /4) volts is applied to a circuit and the current flowing through it is 20 sin (314 t − 1.5708) Amperes. Find the frequency and the values of circuit elements, assuming a series combination of circuit elements.*

VTU : Jan.-03, Marks 6

Sol. : Given voltage and current are,

$$e = 100 \sin\left(314t - \frac{\pi}{4}\right) \text{V} \quad \text{and} \quad i = 20 \sin(314\,t - 1.5708) \text{ A}$$

Compare with, $e = E_m \sin(\omega t - \phi_1)$ and $i = I_m \sin(\omega t - \phi_2)$

$\therefore \quad \omega = 314 \text{ rad/sec} = 2\pi f \quad \text{i.e.} \quad f = \frac{314}{2\pi} = 49.97 \approx 50 \text{ Hz}$ ... frequency

The r.m.s. values of voltage and current are,

$$E = \frac{E_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7106 \text{ V}, \quad I = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.1421 \text{ A}$$

The polar forms of voltage and current are,

$$E = 70.7106 \angle -\frac{\pi}{4} \text{ rad} = 70.7106 \angle -45° \text{ V}$$

$$I = 14.1421 \angle -1.5708 \text{ rad} = 14.1421 \angle -90° \text{ A}$$

$$\therefore \quad Z = \frac{E}{I} = \frac{70.7106 \angle -45°}{14.1421 \angle -90°} = 5 \angle +45° \ \Omega = 3.5355 + j\ 3.5355 \ \Omega$$

Compare with, $Z = R + j X_L$, the circuit elements are,

$$R = 3.5355 \ \Omega, \quad X_L = 3.5355 \ \Omega$$

But $\quad X_L = 2\pi f L \quad \text{i.e.} \quad L = \frac{X_L}{2\pi f} = \frac{3.5355}{2\pi \times 50} = \mathbf{11.2538 \text{ mH}}$

Ex. 3.5.3 *A non-inductive resistance is connected in series with a coil across a 230 volts 50 Hz supply. The current is 1.8 A and the potential difference across the resistance and coil are 80 volts and 170 volts respectively. Calculate the resistance and inductance of the coil and the phase difference between the current and the supply voltage and the power dissipated in the coil. Draw the phasor diagram.*

VTU : Jan.-96, 99, 18, Marks 8

Sol. : The circuit is shown in the Fig. 3.5.5.

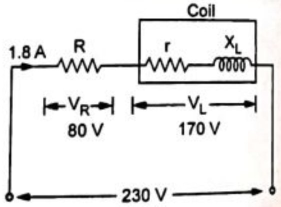


Fig. 3.5.5

$$V_R = I R$$

$$\therefore \quad 80 = 1.8 R$$

$$\therefore \quad R = 44.44 \ \Omega$$

The impedence of only coil is,

$$Z_L = r + j X_L$$

$$V_L = |Z_L| \times I = \sqrt{r^2 + X_L^2} \times 1.8 \quad \dots V_L = 170 \text{ V}$$

$$\therefore \quad \sqrt{r^2 + X_L^2} = \frac{170}{1.8} \quad \text{i.e.} \quad r^2 + X_L^2 = 8919.75 \quad \dots (1)$$

Total current $I = \frac{|V|}{|Z_T|} = 1.8 \text{ A}$

$$Z_T = (R + r) + j x_L \quad \text{i.e.} \quad |Z_T| = \sqrt{(R + r)^2 + (X_L)^2}$$

$$1.8 = \frac{230}{\sqrt{(44.44 + r)^2 + X_L^2}}$$

$$(44.44 + r)^2 + X_L^2 = 16327.16$$

$$\therefore (44.44)^2 + 2 \times 44.44 \times r + r^2 + X_L^2 = 16327.16$$

Using (1), $88.88 r = 5432.496 \quad \text{i.e.} \quad r = 61.12 \Omega$

From (1), $X_L = 72 \ \Omega = 2\pi f L \quad \text{i.e.} \quad L = 0.229 \text{ H}$

$$\therefore \quad Z_T = (44.44 + 61.12) + j\ 72$$

$$Z_T = 127.777 \angle 34.29° \Omega$$

Phase difference between Current and V = 34.29°

$$P_{coil} = |I|^2 \times r = 1.8^2 \times 61.12$$

$$= \mathbf{198.029 \text{ W}}$$

The phasor diagram is shown in the Fig. 3.5.5 (a).

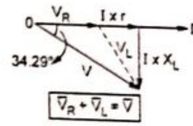


Fig. 3.5.5 (a)

Ex. 3.5.4 *An inductor coil is connected to supply of 250 V at 50 Hz and takes a current of 5 A. The coil dissipates 750 W. Calculate power factor, resistance and inductance of the coil.*

VTU : Jan.-14, Marks 6

Sol. : $V = 250 \text{ V}, I = 5 \text{ A}, P = 750 \text{ W}$

$$|I| = \frac{V}{|Z|} \quad \text{i.e.} \quad |Z| = \frac{V}{|I|} = \frac{250}{5} = 50 \ \Omega$$

$$P = I^2 \times R \quad \text{i.e.} \quad 750 = 5^2 \times R \quad \text{i.e.} \quad R = 30 \ \Omega$$

$$|Z| = \sqrt{R^2 + X_L^2} \quad \text{i.e.} \quad 50 = \sqrt{30^2 + X_L^2}$$

$$\therefore \quad X_L = 40 \ \Omega = 2\pi f L \ \text{i.e.} \quad L = \frac{40}{2\pi \times 50} = \mathbf{127.32 \text{ mH}}$$

$$\cos \phi = \frac{R}{|Z|} = \frac{30}{50} = \mathbf{0.6 \text{ lagging}}$$

3.6 A.C. through Series R-C Circuit

➤ **For a.c. circuit consisting of R and C, draw the phasor diagram and show that the current leads the voltage.**

VTU : Mar.-99; Aug.-02, Jan.-13, Marks 4

- Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. 3.6.1.
- The series combination is connected across a.c. supply, $v = V_m \sin \omega t$
- Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance $\quad V_R = I \times R$

b) Drop across pure capacitance $\quad V_C = I \times X_C$

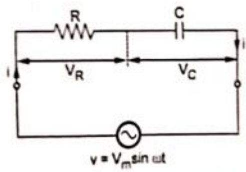


Fig. 3.6.1 Series R-C circuit

where $X_C = \frac{1}{2\pi f C}$ and I, V_R, V_C are the r.m.s. values

- The Kirchhoff's voltage law can be applied to get,

$$\bar{V} = \bar{V}_R + \bar{V}_C = \bar{I}R + \bar{I}X_C \quad \text{... (Phasor addition)}$$

- Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.
- Following are the **steps to draw the phasor diagram :**

1) Take current as reference phasor.
2) In case of resistance, voltage and current are in phase. So, V_R will be along current phasor.
3) In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4) The supply voltage being vector sum of these two voltages V_C and V_R obtained by completing parallelogram.

- The phasor diagram and voltage triangle are shown in the Fig. 3.6.2.
- It can be seen from the phasor diagram that the current leads the voltage by angle ϕ which is decided by the circuit components R and C.

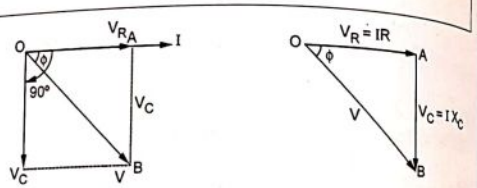


Fig. 3.6.2

3.6.1 Impedance

➤ ***Define impedance of a series R-C circuit.***

- From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = IZ$$

where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2} \ \text{(magnitude)}}$ is the impedance of the circuit.

- Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current.
- It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_C)^2}$ where $X_C = \frac{1}{2\pi f C}\,\Omega$ called **capacitive reactance.**
- In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 3.6.3.
- From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

- If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.** It is shown in the Fig. 3.6.3.
- Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.

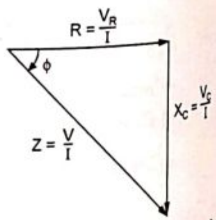


Fig. 3.6.3 Impedance triangle

X component of Z = R = Z cos ϕ

Y component of Z = X_C = Z sin ϕ

- But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - jX_C \ \Omega}$$

➤ ***While in polar form, it is denoted as,***

$$Z = R - jX_C = |Z| \angle -\phi$$

$$\text{where } |Z| = \sqrt{R^2 + X_C^2}, \ \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

- Thus ϕ is negative for capacitive impedance.

3.6.2 Power and Power Triangle

➤ ***Obtain the expression of power for series R-C circuit.***

- The current leads voltage by angle ϕ hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

- The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t)\cdot\sin(\omega t + \phi)] = V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2}\cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

- The second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$\therefore$ $\boxed{P = VI\cos\phi \ \text{watts}}$ where V and I are r.m.s. values

- If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

- Hence, the power triangle can be shown as in the Fig. 3.6.4.
- Thus, the various powers are,

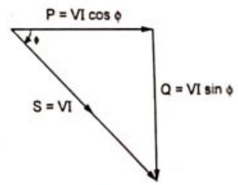


Fig. 3.6.4 Power triangle

Apparent power,	S = V I VA
True or average power,	P = V I cos ϕ W
Reactive power,	Q = V I sin ϕ VAR

For any single phase a.c. circuit, the average power is given by,

$P = V I \cos\phi$ **watts** where **V, I are r.m.s. values**

$\cos\phi$ = Power factor of circuit

$\cos\phi$ is **lagging** for **inductive** circuit and $\cos\phi$ is **leading** for **capacitive** circuit.

Ex. 3.6.1 *An alternating voltage (80 + j 60) V is applied to a circuit and the current flowing is (– 4 + j 10) A. Find : i) The impedance of the circuit, ii) The phase angle, iii) Power consumed.*

VTU : Jan. 16, July 17, Marks 5

Sol. : $V = 80 + j\,60 = 100\angle 36.87^\circ$ V, $I = -4 + j\,10 = 10.77\angle 111.8^\circ$ A

i) $Z = \dfrac{V}{I} = \dfrac{100\angle 36.87^\circ}{10.77\angle 111.8^\circ} = 9.285\angle -74.931^\circ\ \Omega$

ii) Phase angle = **74.931° leading**

iii) $P = V I \cos\phi = 100 \times 10.77 \times \cos(74.931^\circ) = \mathbf{280\ W}$

Ex. 3.6.2 *A voltage v = 100 sin 314 t is applied to a circuit consisting of a 25 ohm resistor and an 80 μF capacitor in series. Determine i) Peak value of current ii) Power factor iii) Total power consumed by the circuit.*

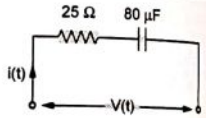


Fig. 3.6.5

Sol. : Comparing given voltage with $V_m \sin\omega t$,

$$V_m = 100\text{ V},\ \omega = 314\text{ rad/s}$$

$$\therefore\quad X_C = \frac{1}{\omega C} = \frac{1}{314\times 80\times 10^{-6}} = 39.8089\ \Omega$$

$$\therefore\quad Z = R - jX_C = 25 - j\,39.8089 = 47\angle -57.87^\circ\ \Omega$$

i) $I_m = \dfrac{V_m}{|Z|} = \dfrac{100}{47} = \mathbf{2.1276\ A}$ ... Peak value

ii) $\cos\phi = \dfrac{R}{Z} = \dfrac{25}{47} = \mathbf{0.5319\ leading}$ ... Power factor

iii) $P = V I \cos\phi = \dfrac{V_m}{\sqrt{2}} \times \dfrac{I_m}{\sqrt{2}} \cos\phi = \dfrac{100\times 2.1276}{2}\times 0.5319 = \mathbf{56.5851\ W}$

Ex. 3.6.3 *125 volts at 60 Hz is applied across a capacitance connected in series with a non inductive resistor. The combination carries a current of 2.2 A and causes a power loss of 96.8 W in the resistor. Power loss in the capacitor is negligible. Calculate the resistance and capacitance.*

VTU : June 13, Marks 5

Sol. : The circuit is shown in the Fig. 3.6.6.

$$P_R = I^2 R = 96.8\text{ W (given)}$$

$$R = \frac{96.8}{(2.2)^2} = 20\ \Omega$$

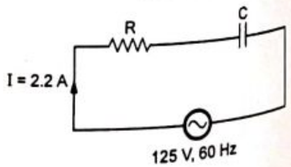


Fig. 3.6.6

$$\therefore\quad |I| = \frac{|V|}{|Z|}$$

where $|Z| = \sqrt{R^2 + X_C^2}$

$$\therefore\quad 2.2 = \frac{125}{\sqrt{20^2 + X_C^2}} \quad \text{i.e. } X_C = 45.8257\ \Omega = \frac{1}{2\pi f C}$$

$$\therefore\quad C = \frac{1}{2\pi\times 60\times 45.8257} = 57.884\ \mu F$$

3.7 A.C. through Series R-L-C Circuit

➤ ***Derive an expression for impedacne, phase angle and power for series R-L-C circuit energised by sinusoidal voltage.***

VTU : July 06, Jan. 09, 18, June 13, Marks 8

➤ ***For a R-L-C series circuit discuss the nature of the power factor for i) $X_L > X_C$ ii) $X_L < X_C$ iii) $X_L = X_C$***

VTU : Jan. 07, Marks 6

- Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit is shown in the Fig. 3.7.1.

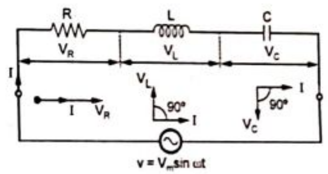


Fig. 3.7.1 R-L-C series circuit

- The a.c. supply is given by,
  $v = V_m \sin\omega t$.
- The circuit draws a current I.
- Due to current I, there are different voltage drops across R, L and C which are given by,
  - a) Drop across resistance R is $V_R = I R$
  - b) Drop across inductance L is $V_L = I X_L$
  - c) Drop across capacitance C is $V_C = I X_C$
- The values of I, V_R, V_L and V_C are r.m.s. values
- The characteristics of three drops are,
  a) V_R in phase with I b) V_L leads I by 90° c) V_C lags I by 90°
- According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

- Current I is taken as reference as it is common to all the elements.
- **Following are the steps to draw the phasor diagram :**

1) Take current as reference.
2) V_R is in phase with I.
3) V_L leads current I by 90°.
4) V_C lags current I by 90°.
5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition.
6) Add that with V_R by law of parallelogram to get the supply voltage.

- The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C.
- Let us consider the different cases.

3.7.1 $X_L > X_C$

- When $X_L > X_C$, obviously, IX_L i.e. V_L is greater than IX_C i.e. V_C.
- So, resultant of V_L and V_C will be directed towards V_L. Current I will lag the resultant of the voltages V_L and V_C i.e. $(V_L - V_C)$.
- The circuit is said to be **inductive** in nature.
- The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig. 3.7.2.

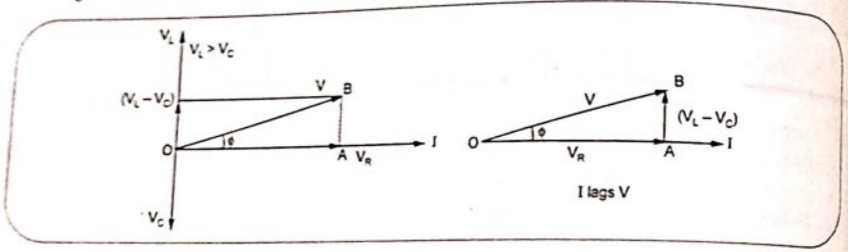


Fig. 3.7.2 Phasor diagram and voltage triangle for $X_L > X_C$

- From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(I R)^2 + (I X_L - I X_C)^2}$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2} = I Z$$

where $$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

- So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \phi)$ as current **lags** voltage by angle ϕ.

3.7.2 $X_L < X_C$

- When $X_L < X_C$, obviously, IX_L i.e. V_L is less than IX_C i.e. V_C. So, the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.
- The circuit is said to be **capacitive** in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the Fig. 3.7.3

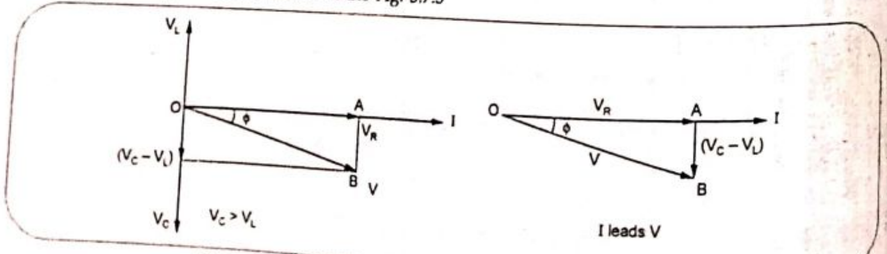


Fig. 3.7.3 Phasor diagram and voltage triangle for $X_L < X_C$

3.7.3 $X_L = X_C$

- When $X_L = X_C$, obviously, $V_L = V_C$. So, V_L and V_C will cancel each other and their resultant is zero.
- So, $V_R = V$ in such case and overall circuit is purely resistive in nature. The phasor diagram is shown in the Fig. 3.7.4.
- From phasor diagram,

$$V = V_R = I R = I Z$$

Where $Z = R$

- The circuit is purely resistive with **unity power factor.**

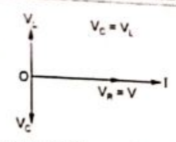


Fig. 3.7.4 Phasor diagram for $X_L = X_C$

3.7.4 Impedance

- In general, for RLC series circuit impedance is given by,

$$\boxed{Z = R + jX \quad \text{where } X = X_L - X_C = \text{Total reactance of circuit}}$$

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan\phi = \left[\frac{X_L - X_C}{R}\right], \quad \cos\phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

3.7.5 Impedance Triangle

- The impedance is expressed as,

$$Z = R + jX \quad \text{where} \quad X = X_L - X_C$$

- For $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in the Fig. 3.7.5 (a).
- For $X_L < X_C$, $X_L - X_C$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig. 3.7.5 (b).
- In both the cases, $R = Z\cos\phi$ and $X = Z\sin\phi$

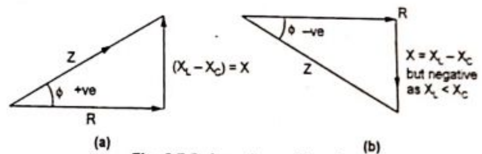


Fig. 3.7.5 Impedance triangles

3.7.6 Power and Power Triangle

- The average power consumed by the circuit is,

P_{av} = Average power consumed by R + Average power consumed by L + Average power consumed by C

- But, pure L and C never consume any power.

$\therefore \quad P_{av}$ = Power taken by R $= I^2 R = I(I R) = I V_R$

But, $\quad V_R = V\cos\phi$ in both the cases

$\therefore \quad P = V I \cos\phi \quad W$

- Thus, for any condition, $X_L > X_C$ or $X_L < X_C$, in general power can be expressed as,

$$\boxed{P = V\, I \cos\phi \quad \text{watts}}$$

where magnitude of current is given by,

$$I = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad \text{(only magnitude)}$$

- While ϕ is decided by the total impedance of the circuit.

No.	Circuit	Impedance (Z)		ϕ	p.f. $\cos\phi$	Remark
		Polar	Rectangular			
1.	Pure R	$R\angle 0°\,\Omega$	$R + j\,0\,\Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90°\,\Omega$	$0 + j\,X_L\,\Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90°\,\Omega$	$0 - j\,X_C\,\Omega$	− 90°	0	Zero leading
4.	Series RL	$\|Z\| \angle +\phi°\,\Omega$	$R + j\,X_L\,\Omega$	$0° \angle \phi \angle 90°$	$\cos\phi$	Lagging
5.	Series RC	$\|Z\| \angle -\phi°\,\Omega$	$R - j\,X_C\,\Omega$	$-90° \angle \phi \angle 0°$	$\cos\phi$	Leading
6.	Series RLC	$\|Z\| \angle \pm\phi°\,\Omega$	$R + j\,X\,\Omega$, $X = X_L - X_C$	ϕ	$\cos\phi$	$X_L > X_C$ Lagging; $X_L < X_C$ Leading; $X_L = X_C$ Unity

Table 3.7.1 Summary of R, L and C circuits

Ex. 3.7.1 *A series circuit with a resistor of 100 Ω, capacitor of 25 μF and inductance of 0.15 H is connected across 220 V, 50 Hz supply. Calculate impedance, current, power and p.f. of circuit.*

VTU : Jan. 17, July 17, Marks

Sol. : The circuit is shown in the Fig. 3.7.6.

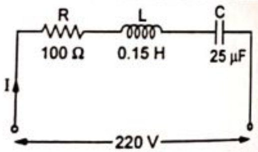


Fig. 3.7.6

$$X_L = 2\pi fL = 47.124\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = 127.324\ \Omega$$

$$\therefore \quad Z = R + j\,X_L - j\,X_C = 100 + j\,47.124 - j\,127.324$$

$$= 100 - j\,80.2\ \Omega = 128.187\angle -38.73°\,\Omega \qquad \text{... Impedance}$$

$$I = \frac{V}{Z} = \frac{220\angle 0°}{128.187\angle -38.73°} = 1.716\angle +38.73°\,\text{A} \qquad \text{... Current}$$

$$P = I^2R = (1.716)^2 \times 100 = 294.466\ \text{W}$$

$$\cos\phi = \cos(-38.73°) = \mathbf{0.78\ leading} \qquad \text{... } X_C > X_L$$

$$P = VI\cos\phi = 220 \times 1.716 \times 0.78 = \mathbf{294.466\ W}$$

As pure L and C, do not consume power, the power can be obtained as $P = I^2R$ or $P = VI\cos\phi$

Ex. 3.7.1 *A coil of p.f. 0.6 is in series with a 100 μF capacitor. When connected to a 50 Hz supply the p.d. across the coil is the p.d. across the capacitor. Find the resistance and inductance of the coil for the circuit shown in Fig. 3.7.7.*

VTU : Jan. 16, Marks 6

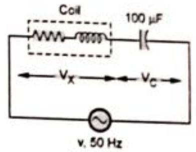


Fig. 3.7.7

Sol. : Coil p.f. = 0.6, C = 100 μF, f = 50 Hz, $|V_X| = |V_C|$

$$Z_X = \text{Coil impedance} = R + j\,X_L,\ \text{coil p.f.} = \frac{R}{Z_X}$$

$$\therefore \quad 0.6 = \frac{R}{Z_X} \quad \text{i.e.} \quad 0.6 = \frac{R}{\sqrt{R^2 + X_L^2}} \quad \text{i.e.} \quad \sqrt{R^2 + X_L^2} = 1.667\ R$$

$$\therefore \quad R^2 + X_L^2 = 2.777\ R^2 \quad \text{i.e.} \quad 1.777\ R^2 = X_L^2$$

$$\therefore \quad R = 0.75\ X_L \qquad \text{...(1)}$$

$$|V_X| = |V_C| \quad \text{i.e.} \quad I\,|Z_X| = I\,|X_C| \quad \text{i.e.} \quad |Z_X| = |X_C|$$

$$\text{But} \quad X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.831\ \Omega$$

$$\therefore \quad |Z_X| = 31.831 \quad \text{i.e.} \quad \sqrt{R^2 + X_L^2} = 31.831$$

$$\therefore \quad R^2 + X_L^2 = 1013.2118 \qquad \text{...(2)}$$

Solving (1) and (2),

$$(0.75\,X_L)^2 + X_L^2 = 1013.2118$$

$$\therefore \quad X_L = 25.4648\ \Omega = 2\pi fL \quad \text{i.e.} \quad \mathbf{L = 0.081\ H}$$

$$\therefore \quad R = 0.75 \times 25.4648 = \mathbf{19.098\ \Omega}$$

Ex. 3.7.3 *A voltage of 200 V is applied to a series circuit consisting of a resistor, an inductor and a capacitor. The respective voltages across these components are 170 V, 150 V and 100 V and the current is 4 A. Find i) The power factor ii) Resistance iii) Impedance iv) Inductive reactance and capacitive reactance.*

VTU : Jan. 15, Marks [illegible]

Sol. : The circuit is shown in the Fig. 3.7.8.

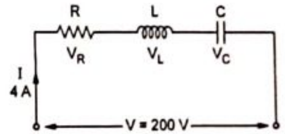


Fig. 3.7.8

$$V_R = 170\ \text{V},\ V_L = 150\ \text{V},\ V_C = 100\ \text{V}$$

$$V_R = I\,R \quad \text{i.e.} \quad R = \frac{170}{4} = 42.5\ \Omega$$

$$|Z| = \frac{|V|}{|I|} = \frac{200}{4} = 50\ \Omega$$

Let R_x be the resistance of an inductor

$$\therefore \quad Z_L = R_x + j\,X_L$$

$$= |Z_L|\angle\,\phi_L \ \text{and}\ |Z_L| = \frac{|V_L|}{|I|} = 37.5\ \Omega$$

$\therefore \quad \sqrt{R_x^2 + X_L^2} = 37.5 \quad$ i.e. $R_x^2 + X_L^2 = 1406.25 \quad \ldots (1)$

$$Z = R + R_x + jX_L - jX_C = (R+R_x) + j(X_L - X_C)$$

And $\quad |X_C| = \dfrac{|V_C|}{|I|} = \dfrac{100}{4} = 25\ \Omega$

$$|Z| = \sqrt{(R+R_x)^2 + (X_L - X_C)^2} \text{ and } |Z| = 50$$

$\therefore \quad 50^2 = (42.5+R_x)^2 + (X_L - 25)^2 \quad \ldots (2)$

$\therefore \quad 2500 = 1806.25 + 85 R_x^2 + R_x^2 + X_L^2 - 50X_L + 625$ and use equation (1)

$\therefore \quad 85 R_x - 50 X_L = -1337.5 \quad$ i.e. $\quad R_x = \dfrac{-1337.5 + 50X_L}{85} \quad \ldots (3)$

Use equation (3) in equation (1), $\left[\dfrac{(-1337.5 + 50X_L)}{85}\right]^2 + X_L^2 = 1406.25$

$\therefore \quad 247.5965 - 18.5045 X_L + 0.34574 X_L^2 + X_L^2 = 1406.25$

... Neglecting negative value

Solving, $\quad X_L = 37.01\ \Omega$

$\therefore \quad R_x = 6.035\ \Omega$

$\therefore \quad Z = (R+R_x) + jX_L - jX_C = 48.535 + j\ 12.01\ \Omega = \mathbf{50 \angle 13.898^\circ\ \Omega}$

$\therefore \quad \cos\phi = \cos(13.898^\circ) = \mathbf{0.9702\ lagging.}$

Ex. 3.7.4 *For the circuit shown in Fig. 3.7.9, find the values of R and C so that $V_b = 3V_a$ and V_b and V_a are in quadrature.* **VTU : Dec.-11, Marks 6**

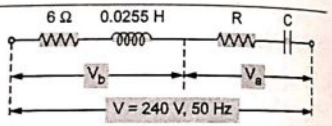


Fig. 3.7.9

Sol. : $Z_1 = 6 + j\ (2\pi \times 50 \times 0.0255) = 6 + j\ 8.011\ \Omega = 10 \angle 53.16^\circ\ \Omega$

Let the current through the circuit is I.

$$Z_2 = R - jX_C \quad \text{and} \quad |V_b| = I\,|Z_1| \quad \text{and} \quad |V_a| = I\,|Z_2|$$

$$V_b = 3\,V_a \quad \text{i.e.} \quad I\,|Z_1| = 3\,I\,|Z_2|$$

$\therefore \quad 10 = 3 \times \sqrt{R^2 + X_C^2} \quad$ i.e. $\quad R^2 + X_C^2 = 11.1111 \quad \ldots (1)$

$$Z_2 = |Z_2| \angle \phi_2{}^\circ\ \Omega$$

where $\quad |Z_2| = \sqrt{R^2 + X_C^2} \quad$ and $\quad \phi_2 = \tan^{-1}\left(\dfrac{-X_C}{R}\right)$

The phasor diagram is shown in the Fig. 3.7.9 (a) with current as reference.

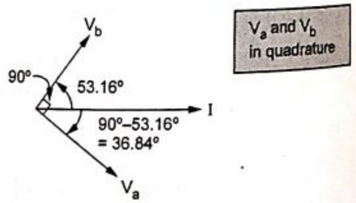


Fig. 3.7.9 (a)

$$V_b = I\,Z_1 = I \angle 0^\circ \times 10 \angle 53.16^\circ$$

$\therefore \quad V_b = 10\,I \angle 53.16^\circ\ V$

$$V_a = I\,Z_2 = |I|\ |Z_1| \angle \phi_2$$

where $\quad \phi_2 = -36.84^\circ$ from Fig. 3.7.9 (a)

But $\quad \phi_2 = \tan^{-1}\dfrac{-X_C}{R} \quad$ i.e. $\quad \tan\phi_2 = \dfrac{-X_C}{R}$

$\therefore \quad \tan(-36.84^\circ) = \dfrac{-X_C}{R} \quad$ i.e. $\quad X_C = +\,0.7492\,R \quad \ldots (2)$

Solving (1) and (2), $R^2 + (0.7492\,R)^2 = 11.111$

$\therefore \quad R = \mathbf{2.667\ \Omega}, \quad X_C = 2\ \Omega = \dfrac{1}{2\pi fC} \quad$ i.e. $\quad C = \mathbf{1.5915 \times 10^{-3}\ F}$

Ex. 3.7.5 *A series circuit with resistance of 10 Ω, inductance of 0.2 H and capacitance of 40 μF is supplied with a 100 V supply at 50 Hz. Find the current, power and power factor of the circuit.* **VTU : July-15, Marks 8**

Sol. : R = 10 Ω, L = 0.2 H, C = 40 μF, f = 50 Hz

$\therefore \quad X_L = 2\pi fL = 62.8318\ \Omega, \; X_C = \dfrac{1}{2\pi fC} = 79.5774\ \Omega$

$\therefore \quad Z = R + jX_L - jX_C = 10 - j\ 16.7456\ \Omega = 19.504 \angle -59.155^\circ\ \Omega$

$I = \dfrac{V}{Z} = \dfrac{100 \angle 0^\circ}{19.504 \angle -59.155^\circ} = \mathbf{5.127 \angle +59.155^\circ\ A} \quad$ ...Current

$$P = VI\cos\phi \text{ or } P = I^2R = 5.127^2 \times 10 = \mathbf{262.8612\ W}$$

$$\cos\phi = \cos[-59.155^\circ] = \mathbf{0.5127\ leading}$$

Ex. 3.7.6 *A circuit consists of a resistance of 10 Ω, an inductance of 16 mH and a capacitance of 150 μF connected in series. A supply of 100 V at 50 Hz is given to the circuit. Find the current, pf and power consumed by the circuit. Draw the vector diagram.*

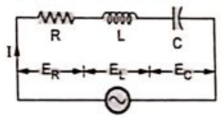


Fig. 3.7.10

Sol. : f = 50 Hz, $X_L = 2\pi fL = 2\pi \times 50 \times 16 \times 10^{-3} = 5.026\Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.22\Omega$$

$$Z = R + jX_L - jX_C = 10 + j\ 5.026 - j\ 21.22$$

$$= 10 - j\ 16.194\ \Omega = 19.032 \angle -58.3^\circ\ \Omega$$

Let V be reference i.e. $V = 100 \angle 0^\circ V$

$$I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{19.032 \angle -58.3^\circ} = \mathbf{5.254 \angle +58.3^\circ A}$$

$\therefore \quad E_R = IR = 5.254 \angle 58.3^\circ \times 10 \angle 0^\circ = 52.54 \angle 58.3^\circ V$

$\therefore \quad E_L = I(j\,X_L) = 5.254 \angle 58.3^\circ \times 5.026 \angle 90^\circ = 26.41 \angle 148.3^\circ V$

$\therefore \quad E_C = I(-j\,X_C) = 5.254 \angle 58.3^\circ \times 21.22 \angle -90^\circ = 111.49 \angle -31.7^\circ V$

$\phi = 58.3°$ ...Power factor

$\cos \phi = 0.525$ leading

The vector diagram is shown in the Fig. 3.7.10 (a).

$P = V I \cos \phi = 100 \times 5.254 \times 0.525$ ...Power

$= 275.851$ W

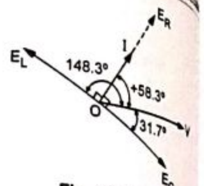


Fig. 3.7.10 (a)

3.8 A.C. Parallel Circuit

VTU : July 02, 01

➤ ***Define admittance, susceptance and conductance.***

- A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage.
- Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.
- The Fig. 3.8.1 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

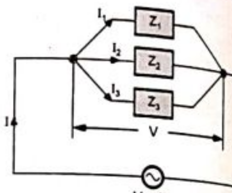


Fig. 3.8.1 A.C. parallel circuit

- The voltage across all the impedances is same as supply voltage of V volts.

- The current taken by each impedance is different.
- Applying Kirchhoff's law, $\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$... (Phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$ where Z is called **equivalent impedance.**

- This result is applicable for 'n' such impedances connected in parallel.
- Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I_1} = \frac{\bar{V}}{\bar{Z_1}}, \quad \bar{I_2} = \frac{\bar{V}}{\bar{Z_2}}, ..., \quad \bar{I_n} = \frac{\bar{V}}{\bar{Z_n}}$$

While the individual phase angles can be calculated by the relation,

$$\tan \phi_1 = \frac{X_1}{R_1}, \quad \tan \phi_2 = \frac{X_2}{R_2}, \quad ..., \quad \tan \phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.

3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in **rectangular form.** This is the resultant current drawn from the supply.

4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit.

3.8.1 Concept of Admittance

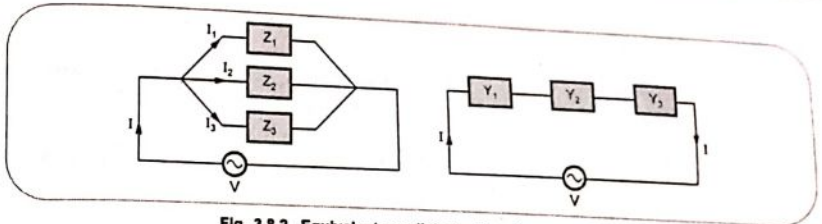


Fig. 3.8.2 Equivalent parallel circuit using admittances

- Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit **siemens or mho.**
- Now, current equation for the circuit shown in the Fig. 3.8.2 is,

$$\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$$

$$\bar{I} = \bar{V} \times \left(\frac{1}{\bar{Z_1}}\right) + \bar{V} \times \left(\frac{1}{\bar{Z_2}}\right) + \bar{V} \times \left(\frac{1}{\bar{Z_3}}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y_1} + \bar{Y_2} + \bar{Y_3}$$ where **Y is the total admittance** of circuit.

- **The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 3.8.2.

3.8.2 Components of Admittance

➤ ***Define susceptance and conductance. State their units.***

- Consider an impedance given as, $Z = R \pm j X$
- Positive sign for inductive and negative sign for capacitive circuit.
- Admittance $Y = \frac{1}{Z} = \frac{1}{R \pm j X}$
- Rationalising the above expression,

$$Y = \frac{R \mp j X}{(R \pm j X)(R \mp j X)} = \frac{R \mp j X}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2 + X^2}\right) \mp j\left(\frac{X}{R^2 + X^2}\right) = \frac{R}{Z^2} \mp j \frac{X}{Z^2}$$

$$\therefore \quad \boxed{Y = G \mp j B, \quad |Y| = \sqrt{G^2 + B^2}, \quad \phi = \tan^{-1} \frac{B}{G}}$$

In the above expression, $\boxed{G = \text{Conductance} = \frac{R}{Z^2}, \quad B = \text{Susceptance} = \frac{X}{Z^2}}$

Conductance (G) :

- It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit **siemens.**

Susceptance (B) :

- It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit **siemens.**

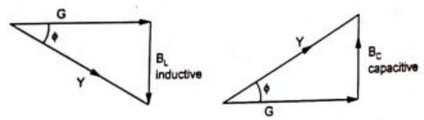
> • **The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.**

> **Note** The sign convention for the reactance and the susceptance are opposite to each other.

3.8.3 Admittance Triangles

➢ ***What is admittance triangle ?***

- The sides of the triangle representing the conductance, susceptance and admittance of the circuit, it is known as **admittance triangle.** The Fig. 3.8.3 shows such admittance triangles.



(a) Inductive, B_L negative (b) Capacitive, B_C positive

Fig. 3.8.3 Admittance triangles

3.8.4 Admittance Method to Solve Parallel Circuit

➢ *Explain the admittance method to solve parallel a.c. circuit.*

- The various steps to solve the parallel circuit by admittance method are,

Step 1 : Calculate the admittance of each branch from the respective impedance.

$$Y_1 = \frac{1}{Z_1},\ Y_2 = \frac{1}{Z_2},\ Y_3 = \frac{1}{Z_3}\ ......$$

Step 2 : Convert all the admittances to the respective rectangular form.

Step 3 : Calculate the equivalent admittance of the circuit by adding the individual admittances of the branches.

$$Y_{eq} = Y_1 + Y_2 + Y_3 ..... = G_{eq} + B_{eq}$$

Step 4 : The total current drawn from the supply is then given by,

$$I_T = V \times Y_{eq}$$

Step 5 : The individual branch currents can be obtained as,

$$I_1 = V \times Y_1,\quad I_2 = V \times Y_2,\quad I_3 = V \times Y_3\ .....$$

- It can be crosschecked that the vector addition of all the above currents gives the total current calculated in step 4.

Step 6 : The angle between V and I_T is the power factor angle ϕ. The cosine of this angle is the power factor of the circuit. The power factor of the circuit can also be obtained as,

$$\cos\phi = \frac{G_{eq}}{Y_{eq}}$$

- The nature of the power factor is to be decided from the sign of B_{eq}. If it is negative power factor is lagging while if it is positive the power factor is leading.

Step 7 : Voltage must be taken as reference phasor as it is common to all branches to draw the phasor diagram.

3.8.5 Two Impedances in Parallel

- If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\boxed{\bar{I}_1 = \bar{I}_T \times \frac{\overline{Z_2}}{\overline{Z_1}+\overline{Z_2}} \quad \text{and} \quad \bar{I}_2 = \bar{I}_T \times \frac{\overline{Z_1}}{\overline{Z_1}+\overline{Z_2}}}$$

Ex. 3.8.1 *Two admittances, Y_1 = (0.167 – j0.167) siemen, and Y_2 = (0.1 + j 0.05) siemen are connected in parallel across a 100 V, 50-Hz single-phase supply. Find the current in each branch and the total current. Also find the power-factor of the combination. Sketch a neat phasor diagram.*

Sol. : The circuit is shown in the Fig. 3.8.4.

Let $V = 100 \angle 0°$ V

$Y_1 = 0.167 - j\,0.167 = 0.23617 \angle -45°$ S

$Y_2 = 0.1 + j\,0.05 = 0.1118 \angle 26.56°$ S

$I_1 = VY_1$

$= 100 \angle 0° \times 0.23617 \angle -45°$

$= 23.617 \angle -45°$ A

$= \mathbf{16.699 - j\,16.699}$ **A**

$I_2 = VY_2 = 100\angle 0° \times 0.1118 \angle +26.56°$

$= 11.18 \angle +26.56°$ A $= \mathbf{10 + j\,5}$ **A**

$\therefore \quad I_T = \bar{I}_1 + \bar{I}_2 = 16.699 - j\,16.699 + 10 + j\,5$

$= 26.699 - j\,11.699$ A

$= \mathbf{29.15 \angle -23.66°}$ **A**

p.f. $= \cos(-23.66°) = $ **0.9159 lagging.**

The phasor diagram is shown in the Fig. 3.8.4 (a).

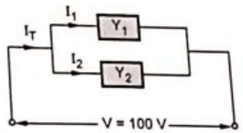


Fig. 3.8.4

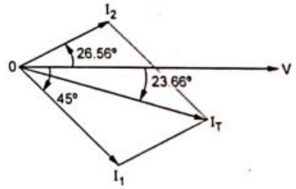


Fig. 3.8.4 (a)

Ex. 3.8.2 *How is a current of 10 A shared by three impedances $Z_1 = 2 - j\,5\ \Omega$, $Z_2 = 6.708 \angle 26.56°\ \Omega$ and $Z_3 = 3 + j\,4\ \Omega$, all connected in parallel ?* **VTU : Aug.-05, Marks 8**

Sol. :

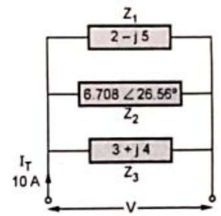


Fig. 3.8.5

$$Y_1 = \frac{1}{Z_1} = \frac{1}{2-j5} = \frac{1}{5.3851\angle -68.198°}$$

$= 0.1856 \angle +68.198°$ ℧

$= 0.069 + j\,0.1723$ ℧

$$Y_2 = \frac{1}{Z_2} = \frac{1}{2-j5} = \frac{1}{6.708\angle 26.56°}$$

$= 0.149 \angle -26.56°$ ℧

$= 0.1332 - j\,0.066$ ℧

$$Y_3 = \frac{1}{Z_3} = \frac{1}{3+j4} = \frac{1}{5\angle 53.13°}$$

$= 0.2 \angle -53.13°$ ℧ $= 0.12 - j\,0.16$ ℧

$\therefore \quad Y_T = Y_1 + Y_2 + Y_3 = 0.3222 - j\,0.537 = 0.3266 \angle -9.4623°$ ℧

$$\therefore \quad V = \frac{I_T}{Y_T} = \frac{10\angle 0°}{0.3266\angle -9.4623°} = 30.6185 \angle +9.4623° \text{ V}$$

$\therefore \quad I_1 = VY_1 = 30.6185 \angle 9.4623° \times 0.1856 \angle 68.198° = \mathbf{5.6827 \angle 77.66°}$ **A**

$\therefore \quad I_2 = VY_2 = 30.6185 \angle 9.4623° \times 0.149 \angle -25.56° = \mathbf{4.5621 \angle -17.09°}$ **A**

$\therefore \quad I_3 = VY_3 = 30.6185 \angle 9.4623° \times 0.2 \angle -53.13° = \mathbf{6.1237 \angle -43.66°}$ **A**

Cross-check : $\bar{I}_T = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 10 \angle 0°$ A

Ex. 3.8.3 *A resistance of 10 Ω and inductive reactance of 8 Ω and capacitive reactance of 15 Ω are connected in parallel across 120 V, 50 Hz mains. Determine :*
i) The total current ii) Power factor of the circuit and iii) The power.

VTU : July-06, Marks 8

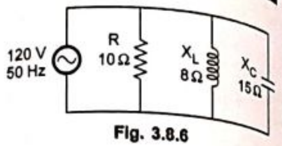


Fig. 3.8.6

Sol. :

$$Z_1 = 10 + j0\,\Omega$$
$$Z_2 = 0 + j8\,\Omega$$
$$Z_3 = 0 - j15\,\Omega$$

Using admittance method,

$$Y_1 = \frac{1}{Z_1},\ Y_2 = \frac{1}{Z_2},\ Y_3 = \frac{1}{Z_3}$$

$$Y = Y_1 + Y_2 + Y_3 \qquad \text{... Parallel connection}$$

$$Y_1 = \frac{1}{10\angle 0^\circ} = 0.1\angle 0^\circ\ \mho = 0.1 + j0\ \mho$$

$$Y_2 = \frac{1}{0+j8} = \frac{1}{8\angle 90^\circ} = 0.125\angle -90^\circ\ \mho = 0 - j0.125\ \mho$$

$$Y_3 = \frac{1}{0-j15} = \frac{1}{15\angle -90^\circ} = 0.067\angle +90^\circ\ \mho = 0 + j0.067\ \mho$$

$$Y_T = 0.1 + j0 + 0 - j0.125 + 0 + j0.067 = 0.1 - j0.058\ \mho = 0.1156\angle -30.11^\circ\ \mho$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.1156\angle -30.11^\circ} = 8.65\angle +30.11^\circ\,\Omega = 7.482 + j4.339\ \Omega$$

Total current $\quad I_T = \dfrac{V\angle 0^\circ}{Z_T\angle\phi_T} = \dfrac{120\angle 0^\circ}{8.65\angle 30.11^\circ} = \mathbf{13.8728\angle -30.11^\circ\ A}$

Power factor of the circuit is $\cos\phi_T$, where $\phi_T = -30.11^\circ$

$\cos\phi_T = \cos(-30.11) =$ **0.865 lagging as ϕ_T is negative**

$P = VI_T\cos\phi_T = 120\times 13.8728\times 0.865 =$ **1440 W.**

Ex. 3.8.4 *The circuit shown in Fig. 3.8.7 is operating at ω = 50 rad/sec. Construct two phasor diagrams one for 3 voltages and other for 3 currents.*

VTU : July-07, Marks 6

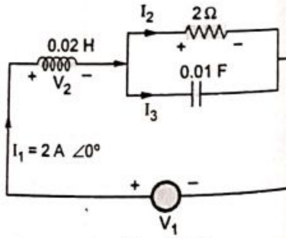


Fig. 3.8.7

Sol. : $\omega = 50$ rad/sec, $I_1 = 2\angle 0^\circ$ A

$\therefore \quad X_L = \omega L = 50\times 0.02 = 1\ \Omega$

$\therefore \quad X_C = \dfrac{1}{\omega C} = \dfrac{1}{50\times 0.01} = 2\ \Omega$

$\therefore \quad V_2 = I_1(0 + jX_L) = 2\angle 0^\circ(0 + j1) = 2\angle 0^\circ \times 1\angle 90^\circ = 2\angle 90^\circ\ V = 0 + j2\ V$

$$I_2 = I_1 \times \frac{(0 - jX_C)}{(R + j0 + 0 - jX_C)} = \frac{2\angle 0^\circ \times [0 - j2]}{(2 + j0 + 0 - j2)} = \frac{2\angle 0^\circ \times 2\angle -90^\circ}{2.8284\angle -45^\circ} = 1.4142\angle -45^\circ\ A$$

$$I_3 = I_1 \times \frac{(R + j0)}{[R + j0 + 0 - jX_C]} = \frac{2\angle 0^\circ \times 2\angle 0^\circ}{2.8284\angle -45^\circ} = 1.4142\angle +45^\circ\ A$$

V_3 = Voltage across 2 Ω or voltage across 0.01 F

$= I_2 \times (2 + j0) = 1.4142\angle -45^\circ \times 2\angle 0^\circ = 2.8284\angle -45^\circ\ V = 2 - j2\ V$

$\therefore \quad V_1 = \bar{V}_2 + \bar{V}_3 = 0 + j2 + 2 - j2 = 2 + j0\ V = 2\angle 0^\circ\ V$

The two phasor diagrams are shown in the Fig. 3.8.7 (a) and (b).

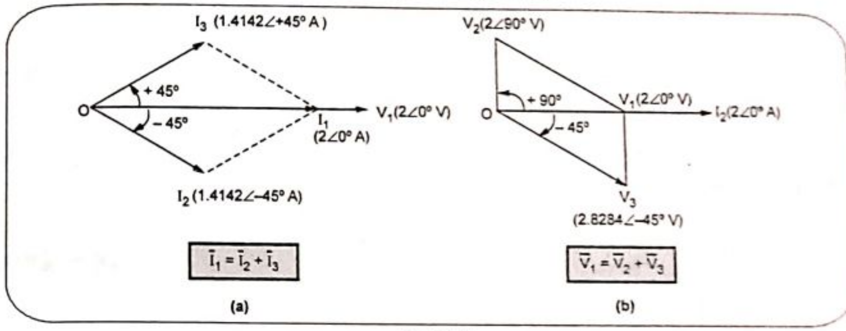


Fig. 3.8.7

Ex. 3.8.5 *An impedance coil in parallel with a 100 μF capacitor is connected across a 200 V, 50 Hz supply. The coil takes a current of 4 A and the power loss in the coil is 600 W. Calculate : i) The resistance of the coil ii) The inductance of the coil iii) The power factor of the entire circuit.*

VTU : Jan.-08, Jan.-13, Marks 8

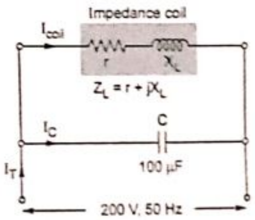


Fig. 3.8.8

Sol. : $I_{coil} = 4$ A, $P_{coil} = 600$ W, f = 50 Hz

$\therefore \quad X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi\times 50\times 100\times 10^{-6}}$

$= 31.8309\ \Omega$

The power loss in a coil is only because of its resistance r as inductive part never consumes power.

i) $\quad P_{coil} = I_{coil}^2 \times r$

$\therefore \quad 600 = (4)^2 \times r$

$\therefore \quad r = \dfrac{600}{16} = 37.5\ \Omega \qquad$ ...Resistance of the coil

ii) $\quad |Z_L| = \dfrac{V}{I_{coil}} = \dfrac{200}{4} = 50\ \Omega$

But $\quad |Z_L| = \sqrt{r^2 + X_L^2}$

$\therefore \quad 50 = \sqrt{(37.5)^2 + X_L^2}$ i.e. $2500 = 1406.25 + X_L^2$

$\therefore \quad X_L^2 = 1093.75$ i.e. $X_L = 33.071\ \Omega$

But $\quad X_L = 2\pi f L$ i.e. $L = \dfrac{33.071}{2\pi \times 50} = \mathbf{0.1052\ H}$ ...Inductance of the coil

iii) $Z_L = r + j X_L = 37.5 + j\,33.071\ \Omega = 50\ \angle 41.4088°\ \Omega$

$Z_C = 0 - j X_C = 0 - j\,31.8309\ \Omega = 31.8309\ \angle -90°\ \Omega$

$$\therefore \quad Z_{eq} = (Z_L \| Z_C) = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{(50\angle 41.4088°)\ (31.8309\angle -90°)}{37.5 + j33.071 + 0 - j31.8309}$$

$$= \frac{1591.545\angle -48.5912°}{37.5 + j1.2401} = \frac{1591.545\angle -48.5912°}{32.5205\angle 1.894°}$$

$= 42.418\ \angle -50.4852°\ \Omega$ ...Total impedance

$\therefore \quad \phi$ = Total p.f. angle = $-50.4852°$

$\therefore \quad \cos\phi = \cos(-50.4852°) = \mathbf{0.6362\ leading}$ ...Power factor of circuit

The p.f. is leading as ϕ is negative.

Ex. 3.8.6 *Two impedances $Z_1 = (10 + j\,15)\ \Omega$ and $Z_2 = (6 - j\,8)\ \Omega$ are connected in parallel. If the total current supplied is 15 A, what is power taken by each branch ?* **VTU : Jan.-16, Marks**

Sol. : The circuit is shown in the Fig. 3.8.9.

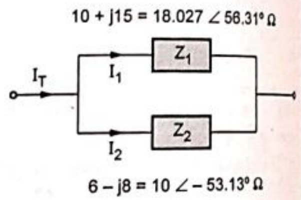


Fig. 3.8.9

$I_T = 15$ A

Let I_T is reference i.e.

$I_T = 15\ \angle 0°$ A

Using current distribution rule,

$$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{15\angle 0° \times 10\angle -53.13°}{(10 + j\,15 + 6 - j\,8)} = \frac{150\ \angle -53.13°}{16 + j\,7}$$

$$= \frac{150\ \angle -53.13°}{17.464\ \angle 23.63°} = 8.589\ \angle 76.76°\ \text{A}$$

$$I_2 = \frac{I_T \times Z_1}{Z_1 + Z_2} = \frac{15\ \angle 0° \times 18.027\ \angle 56.31°}{17.464\ \angle 23.63°} = 15.483\ \angle 32.68°\ \text{A}$$

Voltage across both Z_1 and $Z_2 = I_1 Z_1$ or $I_2 Z_2$

$= 8.589\ \angle -76.76° \times 18.027\ \angle 56.31° = \mathbf{154.833\ \angle -20.45°\ V}$

Power in $Z_1 = V I_1 \cos(\phi_1) = 154.833 \times 8.589 \times \cos(56.31°) = \mathbf{737.67\ W}$

Power in $Z_2 = V I_2 \cos\phi_2 = 154.833 \times 15.483 \times \cos(-53.13°) = \mathbf{1438.37\ W}$

Ex. 3.8.7 *When 220 V A.C. supply is applied across AB terminals in the circuit shown in Fig. 3.8.10, the total power input is 3.25 kW and the current is 20 amps. Find the current through Z_3.* **VTU : July-09, Jan-16, Marks 7**

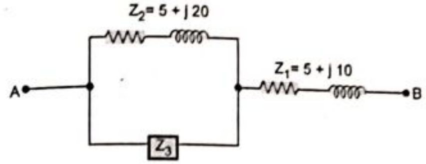


Fig. 3.8.10

Sol. : $|I| = 20$ A

$P_{in} = 3.25$ kW

$Z_1 = 11.18\ \angle 63.435°\ \Omega$

$Z_2 = 20.615\ \angle 75.963°\ \Omega$

$P_{in} = V I \cos\phi$ i.e. 3.25×10^3

$= 220 \times 20 \times \cos\phi$

$\therefore \quad \cos\phi = 0.7386$ i.e. $\phi = 42.3846°$ lagging

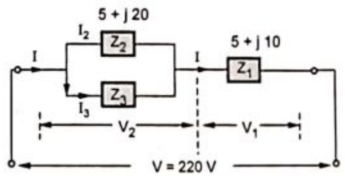


Fig. 3.8.10 (a)

So I lags V by 42.3846°.

Let voltage V be reference, $V = 220\ \angle 0°$ V

$\therefore \quad I = 20\ \angle -42.3846°\ \text{A} = 14.7727 - j\,13.482$ A

$\therefore \quad V_1 = I \times Z_1 = 20\ \angle -42.3846° \times 11.18\ \angle 63.435°$

$= 223.6\ \angle 21.0504°\ \text{V} = 208.678 + j\,80.3146$ V

Now $\bar{V} = \bar{V}_1 + \bar{V}_2$ i.e. $\bar{V}_2 = \bar{V} - \bar{V}_1$

$\therefore \quad \bar{V}_2 = [220 + j\,0] - [208.678 + j\,80.3146]$

$= 11.322 - j\,80.3146\ \text{V} = 81.1087\ \angle -81.9758°$ V

As Z_2 and Z_3 are in parallel, voltage across them is same.

$$\therefore \quad \bar{I}_2 = \frac{V_2}{Z_2} = \frac{81.1087\angle -81.9758°}{20.615\angle 75.963°} = 3.9344\ \angle -157.9208°\ \text{A}$$

$= -3.6458 - j\,1.4788$ A

But, $\quad \bar{I} = \bar{I}_2 + \bar{I}_3$ i.e. $\bar{I}_3 = \bar{I} - \bar{I}_2$

$\therefore \quad \bar{I}_3 = [14.7727 - j\,13.482] - [-3.6458 - j\,1.4788]$

$= 18.4185 - j\,12\ \text{A} = \mathbf{21.9827\ \angle -33.085°\ A}$

So magnitude of current through Z_3 is **21.9827 A** and it lags voltage by **33.085°**.

Ex. 3.8.8 *Two circuits A and B are connected in parallel across 200 V, 50 Hz supply. Circuit A consists of 10 Ω resistance and 0.12 H inductance in series while circuit B consists of 20 Ω resistance in series with 40 μF capacitance. Calculate : i) Current in each branch ii) Supply current iii) Total power. Draw the phasor diagram.* **VTU : Jan.-10, July-17, Marks 8**

Sol. : The circuit is shown in the Fig. 3.8.11.

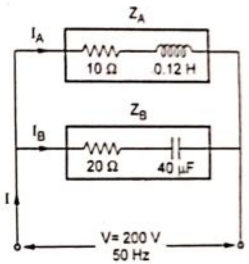


Fig. 3.8.11

$Z_A = 10 + j\,(2\pi f L) = 10 + j\,(2\pi \times 50 \times 0.12)$

$= 10 + j\,37.7\Omega = 39\angle 75.144°\Omega$

$\therefore \quad Y_A = \dfrac{1}{Z_A} = 0.0256\angle -75.144°$ ℧

$$Z_B = 20 - j\left(\frac{1}{2\pi f C}\right) = 20 - j\left(\frac{1}{2\pi \times 50 \times 40 \times 10^{-6}}\right)$$

$= 20 - j\,79.5774\Omega = 82.052\angle -75.892°\Omega$

$\therefore \quad Y_B = \dfrac{1}{Z_B} = 0.01218\angle 75.892°$ ℧

i) $I_A = V Y_A = 200\angle 0° \times 0.0256\angle -75.144° = 5.12\angle -75.144°A = 1.3127 - j\,4.9488$ A

$I_B = V Y_B = 200\angle 0° \times 0.01218\angle 75.892° = 2.437\angle 75.892°A = 0.5937 + j\,2.3625$ A

ii) $I = I_A + I_B = 1.3127 - j\,4.9488 + 0.5937 + j\,2.3625$

$= 1.9064 - j\,2.5863$ A

$= 3.2129\angle -53.605°$A

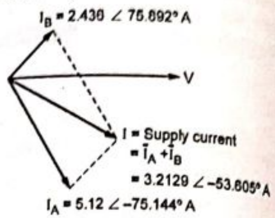


Fig. 3.8.11 (a)

iii) Total power factor is,

$\cos\phi_T = \cos(-53.605°) = 0.5933$ **lagging**

iv) The phasor diagram is as shown in Fig. 3.8.11 (a).

Ex. 3.8.9 *In a series parallel circuit, the two parallel branches A and B are in series with C. The impedances are $Z_A = (10 - j8)\,\Omega$, $Z_B = (9 - j6)\,\Omega$ and $Z_C = (3 + j2)\,\Omega$. The voltage across branch C is 100V. Find the current I_A and I_B and the phase difference between them.* **VTU : July 11**

Sol. : The circuit is shown in the Fig. 3.8.12.

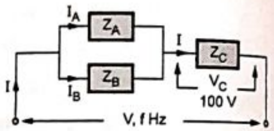


Fig. 3.8.12

$Z_A = 10 - j\,8\,\Omega = 12.806\angle -38.65°\,\Omega$

$Z_B = 9 - j\,6\,\Omega = 10.817\angle -33.69°\,\Omega$

$Z_C = 3 + j\,2\,\Omega = 3.6055\angle -33.69°\,\Omega$

Let voltage across coil C is reference i.e. $V_C = 100\angle 0°$V

$\therefore \quad I = \dfrac{V_C}{Z_C} = \dfrac{100\angle 0°}{3.6055\angle 33.69°} = 27.7354\angle -33.69°$ A

Using current division rule,

$I_A = I \times \dfrac{Z_B}{Z_A + Z_B} = \dfrac{27.7354\angle -33.69° \times 10.817\angle -33.69°}{(10 - j8 + 9 - j6)}$

$= \dfrac{300.013\angle -67.38°}{23.6\angle -36.384°}$

$\therefore \quad I_A = 12.7124\angle -30.996°$ **A**

$I_B = I \times \dfrac{Z_A}{Z_A + Z_B} = \dfrac{27.7354\angle -33.69° \times 12.806\angle -38.65°}{23.6\angle -36.384°}$

$= 15.05\angle -35.956°$ **A**

The phase difference between I_A and I_B is $[-30.996° - (-35.956°)] = 4.96°$ such that I_A **Leads** I_B.

Ex. 3.8.10 *Find the total current, power and power factor of the circuit given in Fig. 3.8.13.*

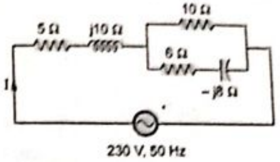


Fig. 3.8.13

Sol. : The circuit is redrawn as shown in the Fig. 3.8.13 (a).

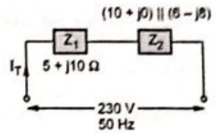


Fig. 3.8.13 (a)

$Z_2 = (10 + j0)\,||\,(6 - j8)$

$= \dfrac{10\angle 0° \times 10\angle -53.13°}{10 + j0 + 6 - j8} \quad \ldots \dfrac{Z_a Z_b}{Z_a + Z_b}$

$= \dfrac{100\angle -53.13°}{17.888\angle -26.56°}$

$= 5.5908\angle -26.57°\,\Omega = 5 - j2.5\,\Omega$

$\therefore \quad Z_T = Z_1 + Z_2 = 5 + j\,10 + 5 - j\,2.5$

$= 10 - j2.5\,\Omega = 10.3077\angle -14.03°\,\Omega$

$\therefore \quad I_T = \dfrac{V}{Z_T} = \dfrac{230\angle 0°}{10.3077\angle -14.03°} = 22.3134\angle +14.03°$ **A**

Power factor $= \cos\phi = \cos(14.03°) = 0.9701$ **leading**

$P = VI_T\cos\phi = 230 \times 22.3134 \times 0.9701 = 4978.986$ **W**

Ex. 3.8.11 *A 60 Ω resistor is connected in parallel with an inductive reactance of 80 Ω to a 240 V, 50 Hz supply. Calculate : i) The current through the resistor and inductance, ii) The supply current, iii) The circuit phase angle. Draw phasor diagram.* **VTU : June-12, Marks 4**

Sol. : The circuit is shown in the Fig. 3.8.14 (a).

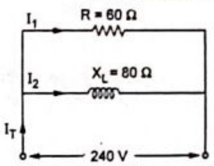


Fig. 3.8.14 (a)

i) Let voltage be reference, $240\angle 0°$ **V**

$\therefore \quad I_1 = \dfrac{240\angle 0°}{R + j0} = \dfrac{240\angle 0°}{60\angle 0°} = 4\angle 0°$ **A**

$I_2 = \dfrac{240\angle 0°}{0 + jX_L} = \dfrac{240\angle 0°}{80\angle 90°} = 3\angle -90°$ **A**

ii) $\bar{I}_T = \bar{I}_1 + \bar{I}_2 = (4 + j0) + (0 - j3)$

$= 4 - j3$ A $= 5\angle -36.86°$ **A**

iii) ϕ = Angle between V and I_T

$= 36.86°$ **with I_T lagging**

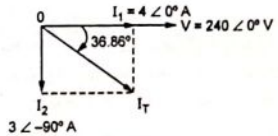


Fig. 3.8.14 (b)

The phasor diagram is shown in the Fig. 3.8.14 (b).

Ex. 3.8.12 *A capacitor of 50 μF shunted across a non inductive resistance of 100 Ω is connected in series with a resistor of 50 Ω to a 200 V, 50 Hz supply. Find circuit current and power factor.* **VTU : Jan.-14, Marks 6**

Sol. : $X_C = \dfrac{1}{2\pi fC} = 63.66\,\Omega$ i.e. $Z_C = -j\,63.66\,\Omega$

$R = 100\,\Omega$ and Z_C are in parallel

$$\therefore \quad Z' = \frac{R\times(-jX_C)}{R-jX_C} = \frac{100\times(-j63.66)}{100-j63.66}$$

$$= \frac{6366\angle-90^\circ}{118.543\angle-32.48^\circ}$$

$$= 53.702\angle-57.52^\circ = 28.838 - j\,45.302\ \Omega$$

$$\therefore \quad Z_T = Z' + 50 = 78.838 - j\,45.302\ \Omega = 90.927\angle-29.882^\circ\ \Omega$$

$$\therefore \quad I = \frac{V}{Z_T} = \frac{200\angle0^\circ}{90.927\angle-29.882^\circ} = \mathbf{2.2\angle+29.882^\circ\ A}$$

$\therefore$ Power factor = cos (29.882°) = **0.867 leading**

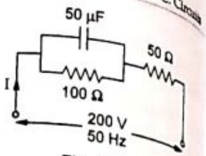


Fig. 3.8.15

Ex. 3.8.13 *Two impedances Z_1 = (6 – j8) ohms and Z_2 = (16 + j 12) ohms are connected in parallel. If the total current of the combination is (20 + j 10) amperes, find*
i) Voltage across the combination. ii) Currents in the two branches iii) Power factor **VTU : Jan.-06, 15, July-06, Dec.-11, Marks**

Sol. : The circuit is shown in the Fig. 3.8.16.

$$I_T = 20 + j\,10\ A = 22.36\angle26.565^\circ\ A$$

$$Z_1 \parallel Z_2 = \frac{Z_1\ Z_2}{Z_1 + Z_2} = Z_T$$

$$\therefore \quad Z_T = \frac{(10\angle-53.13^\circ)\,(20\angle36.86^\circ)}{6-j\,8+16+j\,12}$$

$$= \frac{200\angle-16.27^\circ}{22.36\angle10.304^\circ}$$

$$= 8.9445\angle-26.574^\circ\ \Omega$$

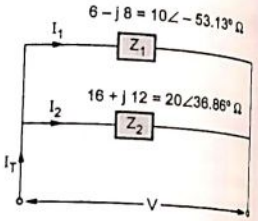


Fig. 3.8.16 (a)

i) $V = I_T \times Z_T$

$$= (22.36\angle26.56^\circ)\times(8.945\angle-26.57^\circ) = \mathbf{200\angle0^\circ\ V}$$

ii) $$I_1 = \frac{V}{Z_1} = \frac{200\angle0^\circ}{10\angle-53.13^\circ} = \mathbf{10\angle53.13^\circ\ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{200\angle0^\circ}{20\angle36.86^\circ} = \mathbf{10\angle-36.86^\circ\ A}$$

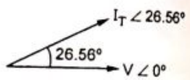


Fig. 3.8.16 (b)

iii)) cos ϕ = cos (26.565°) = **0.894 leading**

Ex. 3.8.14 *Two impedances (150 – 157j)Ω and (100 + 110j)Ω are connected in parallel across 200 V, 50 Hz supply. Find branch currents, total current and total power consumed in the circuit. Draw the phasor diagram.* **VTU : Jan.-17, Marks**

Sol. : The circuit is shown in the Fig. 3.8.17.

$$Z_1 = 150 - j\,157\ \Omega = 217.14\angle-46.31^\circ\ \Omega$$

$$Z_2 = 100 - j\,110\ \Omega = 148.66\angle47.73^\circ\ \Omega$$

$$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{200\angle0^\circ}{217.14\angle-46.31^\circ}$$

$$= \mathbf{0.921\angle+46.31^\circ\ A = 0.636 + j\,0.666\ A}$$

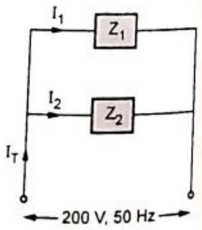


Fig. 3.8.17

$$\therefore \quad I_2 = \frac{V}{Z_2} = \frac{200\angle0^\circ}{148.66\angle47.73^\circ}$$

$$= \mathbf{1.345\angle-47.73^\circ\ A}$$

$$= \mathbf{0.905 + j\,0.995\ A}$$

$$\therefore \quad I_T = \bar{I}_1 + \bar{I}_2$$

$$= (0.636 + 0.905) + j\,(0.666 - 0.995)$$

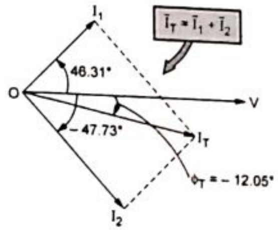
$$= 1.541 - j\,0.329\ A$$

$$= 1.576\angle-12.05^\circ\ A$$

$$P_T = VI_T\cos\phi_T$$

$$= 200\times1.576\times\cos(-12.05)^\circ$$

$$= \mathbf{308.255\ W}$$



The phasor diagram is shown in Fig. 3.8.17 (a).

Fig. 3.8.17 (a)

Ex. 3.8.15 *A circuit having a series combination of 20 Ω and inductance of 0.07 H is connected parallel with a series combination of 50 Ω resistance and 60 μF capacitance. Calculate the total current, when the parallel combination is connected across 230 volts, 50 Hz supply.* **VTU : Jan.-18, Marks**

Sol. : The circuit is shown in the Fig. 3.8.18.

$$Z_1 = 10 + j\,2\pi fL \qquad \ldots f = 50\ Hz$$

$$= 20 + j\,22 = 29.73\angle47.73^\circ\Omega$$

$$Z_2 = 50 - j\left(\frac{1}{2\pi fC}\right)$$

$$= 50 - j\,53.05 = 72.9\angle-46.7^\circ\Omega$$

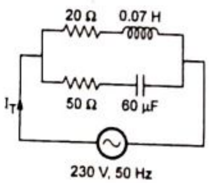


Fig. 3.8.18

$$\therefore \quad Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{29.73\angle47.73^\circ\times72.9\angle-46.7^\circ}{20 + j\,22 + 50 - j\,53.05} = \frac{2167.317\angle1.03^\circ}{76.577\angle-23.92^\circ}$$

$$= 28.302\angle24.95^\circ\ \Omega$$

$$\therefore \quad I_T = \frac{V}{Z_T} = \frac{230\angle0^\circ}{28.302\angle24.95^\circ} = \mathbf{8.127\angle-24.95^\circ A} \qquad \ldots\text{Current}$$

3.9 University Questions with Answers

Jan. - 2010

Q.1 *Prove that in a purely inductive circuit the current lags voltage by 90°.* **[Refer section 3.2]** **[8]**

May - 2010

Q.2 *Derive the expression for the instantaneous power in a pure capacitor energised by sinusoidal voltage. Draw the wave shapes of current, voltage and power.* **[Refer section 3.3.2]** **[8]**

Dec. - 2011

Q.3 *Derive the expression for the instantaneous power in a series R-L circuit.* **[Refer section 3.5.2]** **[6]**

Jan. - 2013

Q.4 *For a.c. circuit consisting of R and C, draw the phasor diagram and show that the current leads the voltage.* **[Refer section 3.6]** **[4]**

June - 2013

Q.5 *Derive an expression for impedacne, phase angle and power for series R-L-C circuit energised by sinusoidal voltage.* **[Refer section 3.7]** **[8]**

Jan. - 2015

Q.6 *Obtain expression for the current through the pure inductor, if the voltage across it is* $V = V_m \sin \omega t$. **[Refer section 3.2]** **[6]**

July - 2015

Q.7 *Show that th1e average power consumed by pure capacitor is zero.* **[Refer section 3.3.2]** **[6]**

Q.8 *Define power factor and explain its significance in a.c. circuit.* **[Refer section 3.5.3]** **[2]**

CBCS Scheme

Jan. - 2016

Q.9 *Prove that a pure capacitor do not consume any power.* **[Refer section 3.3.2]** **[6]**

Q.10 *Show that power consumed in an AC circuit is* $P = VI \cos \phi$ *where V is RMS value of the applied voltage, I is the RMS value of current and* ϕ *is the angle between voltage V and current I.* **[Refer section 3.5.2]** **[5]**

Jan. - 2017

Q.11 *Derive an expression for power in pure capacitor circuit and draw voltage, current and power waveforms.* **[Refer section 3.3.2]** **[5]**

Q.12 *Define power factor and mention its practical importance.* **[Refer section 3.5.3]** **[6]**

July - 2017

Q.13 *Show that the power consumed by a pure capacitor is zero. Draw the voltage, current and power waveforms.* **[Refer section 3.3.2]** **[5]**

Jan. - 2018

Q.14 *Obtain an expression for power in a series RLC circuit when applied by an ac.* **[Refer section 3.7]** **[6]**

□□□

MODULE - 2

4

Three Phase Circuits and Power Measurement

Syllabus

Three - phase balanced circuits, voltage and current relations in star and delta connections. Measurement of three phase power using two wattmeter method.

Contents

4.1 Introduction

➤ ***What is three phase system ?***

- There are certain loads which require polyphase supply. **Phase** means **branch, circuit** or **winding** while **poly** means **many**. So such applications need a supply having many a.c. voltages present in it simultaneously. Such a system is called **polyphase system.**
- To develop polyphase system, the armature winding in a generator is divided into number of phases required.
- In each section, a separate a.c. voltage gets induced. So there are many independent a.c. voltages present equal to number of phases of winding.
- The various phases of winding are arranged in such a manner that the magnitudes and frequencies of all these voltages is same but they have definite phase difference with respect to each other.
- The phase difference depends on number of phases in which winding is divided. For example, if winding is divided into 'n' phases then 'n' separate a.c. voltages will be available having same magnitude and frequency but they will have a phase difference of (360°/n) with respect to each other.
- Thus in a three phase supply system, there are three voltages with a same magnitude and freuqency but having a phase difference of 360°/3 = 120° between them. Such a supply system is called **three phase system.**

- In practice a three phase system is found to be more economical and it has certain advantages over other polyphase systems. Hence three phase system is very popularly used everywhere in practice.

4.2 Advantages of Three Phase System

➤ ***List the advantages of three phase system over single phase system.***

VTU : July-03, 04, 17; Jan.-04, 08, 11, 14, 15, 16, 17, Marks 4

- A three phase system has following advantages over single phase system :

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.
4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5) Three phase system give steady output.
6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.
7) Power factor of single phase motor is poor than three phase motors of same rating.
8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

- But it is found that optimum number of phases required to get all above said advantages is three. Hence three phase system is accepted as standard system throughout the world.

4.3 Generation of Three Phase Voltage System

➤ ***Explain the generation of three phase voltages in an alternator.***

March-01; Jan.-07, July-15, Marks 4

- It is already discussed that alternator consisting of one group of coils on armature produces one alternating voltage. But if armature coils are divided

into three groups such that they are **displaced by the angle 120°** from each other, three separate alternating voltages get developed.

- Consider armature of alternator divided into three groups as shown in the Fig. 4.3.1. The coils are named as R_1 - R_2, Y_1 - Y_2 and B_1 - B_2 and mounted on same shaft.

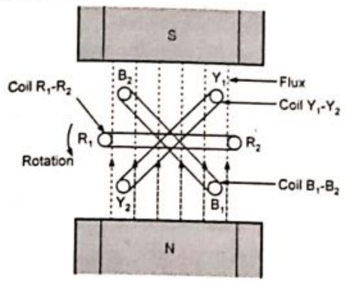


Fig. 4.3.1 Generation of 3 phase

- The ends of each coil are brought out through the slipring and brush arrangement to collect the induced e.m.f.
- Let e_R, e_Y and e_B be the three independent voltages induced in coils R_1R_2, Y_1-Y_2 and B_1-B_2 respectively.
- All are alternating voltages having same magnitude and frequency as they are rotated at uniform speed.
- All of them will be displaced from one other by 120°.
- Suppose e_R is assumed to be the reference and is zero for the instant shown in the Fig. 4.3.2.

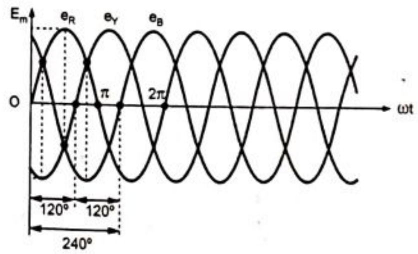


Fig. 4.3.2 Waveforms of 3 phase voltages

- At the same instant e_Y will be displaced by 120° from e_R and will follow e_R while e_B will be displaced by 120° from e_Y and will follow e_Y i.e. if e_R is reference then e_Y will attain its maximum and minimum position 120° later than e_R and e_B will attain its maximum and minimum position 120° later than e_Y i.e. 120° + 120° = 240° later with respect to e_R.
- All coils together represent three phase supply system. The waveforms are shown in the Fig. 4.3.3.
- The equations for the induced voltages are :

$$e_R = E_m \sin(\omega t)$$
$$e_Y = E_m \sin(\omega t - 120°)$$
$$e_B = E_m \sin(\omega t - 240°)$$
$$= E_m \sin(\omega t + 120°)$$

- The phasor diagram of these voltages can be shown as in the Fig. 4.3.3.

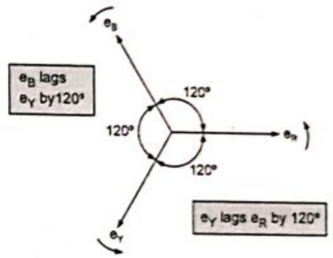


Fig. 4.3.3 Phasor diagram

- As phasors rotate in anticlockwise direction, we can say that e_Y lags e_R by 120° and e_B lags e_Y by 120°.
- If we add three voltages vectorially, it can be observed that the sum of the three voltages at any instant is zero.
- Mathematically this can be shown as :

$$e_R + e_Y + e_B = E_m \sin\omega t + E_m \sin(\omega t - 120°)$$
$$+ E_m \sin(\omega t + 120°)$$
$$= E_m [\sin\omega t + \sin\omega t \cos 120°$$
$$- \cos\omega t \sin 120°$$
$$= + \sin\omega t \cos 120° + \cos\omega t \sin 120°$$

$$= E_m [\sin \omega t + 2 \sin \omega t \cos 120°]$$

$$= E_m \left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$$

$\therefore \quad \boxed{\bar{e}_R + \bar{e}_Y + \bar{e}_B = 0}$

- The phasor addition of all the phase voltages at any instant in three phase system is always zero.

4.4 Important Definitions Related to Three Phase System

➢ ***Define phase sequence of 3 phase alternating supply ?*** VTU : Jan. 04, 07, 15 Marks 2

➢ ***Define symmetrical three phase system.***

1) Symmetrical system : It is possible in polyphase system that magnitudes of different alternating voltages are different. But a three phase system in which the three voltages are of same magnitude and frequency and displaced from each other by 120° phase angle is defined as **symmetrical system**.

2) Phase sequence : The sequence in which the voltages in three phases reach their maximum positive values is called **phase sequence**. Generally the phase sequence is R-Y-B.

- The phase sequence is important in determining direction of rotation of a.c. motors, parallel operation of alternators etc.

- The phase sequence of a three phase system can be changed by interchanging any two terminals out of R, Y and B. If three phase supply is given to a three phase motor with a phase sequence of RYB then by interchanging any two terminals of RYB the phase sequence can be reversed. Due to this the direction of motor gets reversed.

4.5 Three Phase Supply Connections

- In single phase system, two wires are sufficient for transmitting voltage to the load i.e. phase and neutral.
- But in case of three phase system, two ends of each phase i.e. R_1-R_2, Y_1-Y_2, and B_1-B_2 are available to supply voltage to the load.
- Using these six connections, two different three phase connections can be obtained which are called star connection and delta connection.

4.5.1 Star Connection

➢ ***Explain the star connection of a three phase system.***

- The star connection is formed by connecting starting or terminating ends of all the three windings together. The ends R_1 - Y_1 - B_1 are connected or ends R_2 - Y_2 - B_2 are connected together. This common point is called **Neutral Point.**
- The remaining three ends are brought out for connection purpose. These ends are generally referred as R-Y-B, to which load is to be connected.
- The star connection is shown in the Fig. 4.5.1.

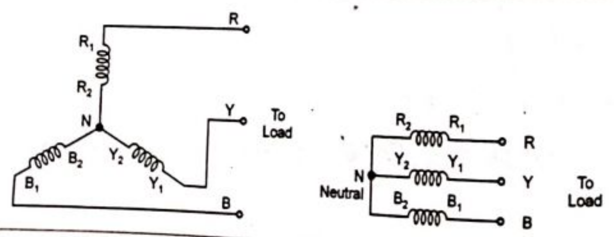


Fig. 4.5.1 Star connection

4.5.2 Delta Connection

➢ ***Explain the delta connection of a three phase system.***

- The delta is formed by connecting one end of winding to starting end of other and connections are continued to form a closed loop.
- The supply terminals are taken out from the three junction points. Delta connection always forms a closed loop.
- The delta connection is shown in the Fig. 4.5.2.

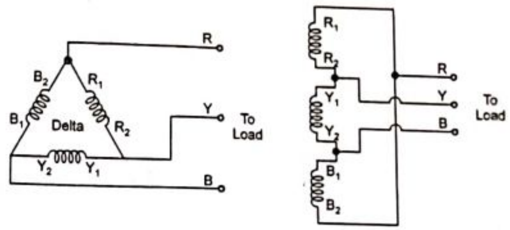


Fig. 4.5.2 Delta connection

4.6 Concept of Line Voltages and Line Currents

➢ ***Explain the concept of line voltages and line currents.***

- The potential difference between any two lines of supply is called **line voltage** and current passing through any line is called **line current.**
- Consider a star connected system as shown in the Fig. 4.6.1.
- Line voltages are denoted by V_L. These are V_{RY}, V_{YB} and V_{BR}.
- Line currents are denoted by I_L. These are I_R, I_Y and I_B.
- Similarly for delta connected system we can show the line voltages and line currents as in the Fig. 4.6.2.
- Line voltages V_L are V_{RY}, V_{BR}, V_{YB}.
- While line currents I_L are I_R, I_Y and I_B.

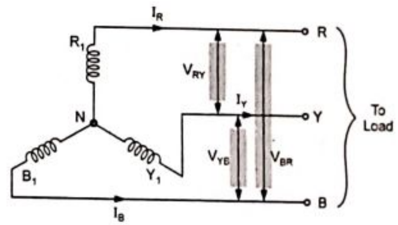


Fig. 4.6.1 Star connection

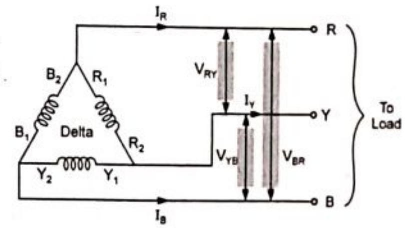


Fig. 4.6.2 Delta connection

4.7 Concept of Phase Voltages and Phase Currents

➢ ***Explain the concept of phase voltages and phase currents.***

- To define the phase voltages and phase currents let us see the connections of the three phase load to the supply lines.
- The load can be connected in two ways, i) Star connection, ii) Delta connection
- The **three phase load** is nothing but three different impedances connected together in star or delta fashion.

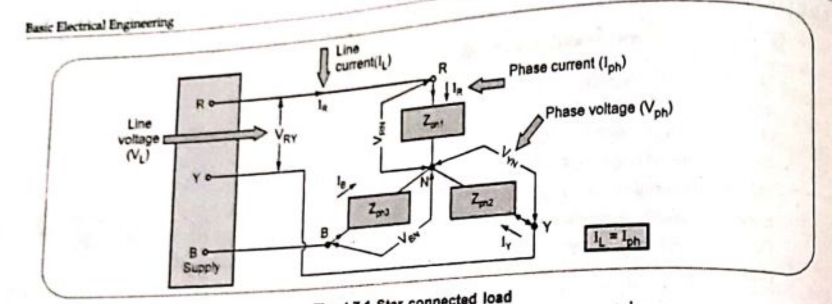


Fig. 4.7.1 Star connected load

i) Star connected load : There are three different impedances and are connected such that one end of each is connected together and other three are connected to supply terminal is R-Y-B. This is shown in the Fig. 4.7.1.

- The voltage across any branch of the three phase load i.e. across Z_{ph1}, Z_{ph2} or Z_{ph3} is called **phase voltage** and current passing through any branch of the three phase load is called **phase current.**

- In the diagram shown in the Fig. 4.7.1 V_{RN}, V_{YN} and V_{BN} are phase voltages while I_R, I_Y and I_B as shown in the Fig. 4.7.1 are phase currents.
- The phase voltages are denoted as V_{ph} while the phase currents are denoted as I_{ph}.
- Generally suffix N is not indicated for phase voltages in star connected load.

So $V_{ph} = V_R = V_Y = V_B$

- It can be seen from the Fig. 4.7.1 that,

$I_{ph} = I_R = I_Y = I_B$

- But same are the currents flowing through the three lines also. Thus we can conclude that for star connection $I_L = I_{ph}$.

$\therefore$ $\boxed{I_L = I_{ph} \quad \text{for star connection}}$

ii) Delta connected load : If the three impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are connected such that starting end of one is connected to terminating end of other, to form a closed loop it is called delta connection of load. The junction points are connected to supply terminals R-Y-B. This is shown in the Fig. 4.7.2.

- The current I_{RY}, I_{YB} and I_{BR} flowing through the various branches of the load are **phase currents.** The lines currents are I_R, I_Y, I_B flowing through supply lines. **Thus in delta connection of load, line and phase currents are different.**
- In the Fig. 4.7.2, the voltage across Z_{ph1} is V_{RY}, across Z_{ph2} is V_{YB} and across Z_{ph3} is V_{BR} and all are phase voltages.

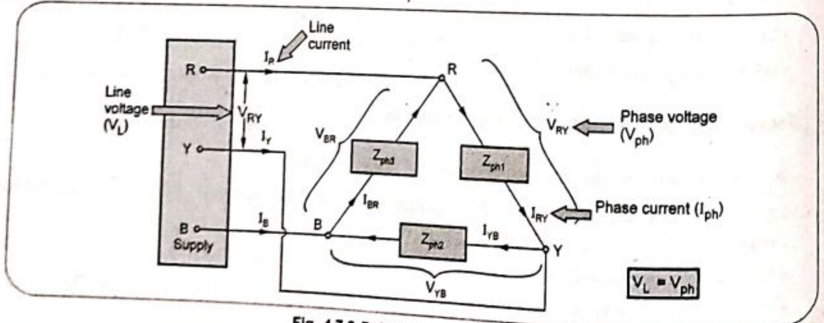


Fig. 4.7.2 Delta connected load

$$V_{ph} = V_{RY} = V_{YB} = V_{BR}$$

- But as per definition of line voltages, same are the voltages across supply lines also. Thus it can be concluded that in delta connection $V_L = V_{ph}$.

$\therefore$ $\boxed{V_L = V_{ph} \quad \text{for delta connection}}$

4.8 Balanced Load

> ***Define balanced and unbalanced load.*** **VTU : Jan.-04, 07, Marks 2**

- The load is said to be balanced when **magnitude** of all the impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are **equal** and the **phase angles** of all of them are equal and of **same nature** either all inductive or all capacitive or all resistive.

- In such case all phase voltages have equal magnitude and are displaced from each other by 120° while all phase currents also have equal magnitude and are displaced from each other by 120°.
- The same is true for all the line voltages and line currents.

- The load is said to be unbalanced when **magnitude** of all the impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are **unequal** and the **phase angles** of all of them are **unequal.** In such a case the phase voltages are unequal and not displaced from each other by 120°. Same is true for phase currents.

4.9 Relations for Star Connected Load

> ***Obtain the relationship between line and phase values of current in a three phase balanced star connected system. Also derive equation of power.***

VTU : Feb.-05, July-05, 07, 09, 16,17; June-10, 12, 13; Jan.-11, 14, 15, Dec.-11, Marks 8

- Consider the balanced star connected load as shown in the Fig. 4.9.1.

Line voltages, $V_L = V_{RY} = V_{YB} = V_{BR}$ and

Line currents, $I_L = I_R = I_Y = I_B$

Phase voltages, $V_{ph} = V_R = V_Y = V_B$ and

Phase currents, $I_{ph} = I_R = I_Y = I_B$

- As seen earlier, $I_L = I_{ph}$ for star connected load.
- To derive relation between V_L and V_{ph}, consider line voltage V_{RY}. From the Fig. 4.9.1 we can write,

$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY}$ But $\bar{V}_{NY} = -\bar{V}_{YN}$

(Generally suffix N is not used for phase voltages)

Hence $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$ ...(4.9.1)

Similarly, $\bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \bar{V}_{YN} - \bar{V}_{BN}$

$= \bar{V}_Y - \bar{V}_B$ ...(4.9.2)

and $\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$ ...(4.9.3)

- The three phase voltage are displaced by 120° from each other.

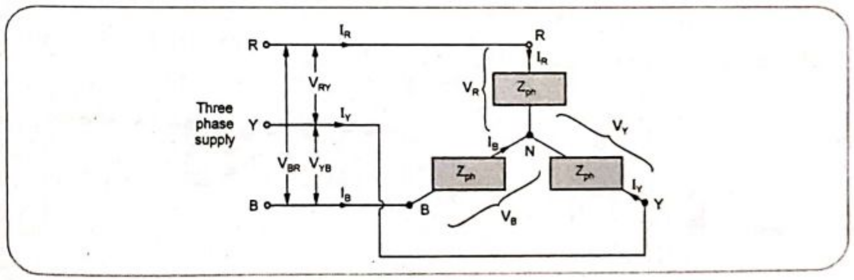


Fig. 4.9.1 Star connected load

- The phasor diagram to get V_{RY} is shown in the Fig. 4.9.2. The V_Y is reversed to get $-V_Y$ and then it is added to V_R to get V_{RY}.

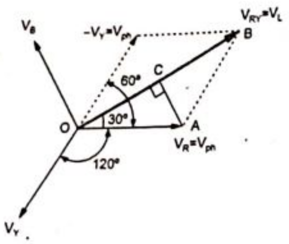


Fig. 4.9.2

- The perpendicular is drawn from point A on vector OB representing V_L.
- In triangle OAB, the sides OA and AB are same as phase voltages. Hence OB bisects angle between V_R and $-V_Y$.

$\therefore \angle BOA = 30°$

- And perpendicular AC bisects the vector OB hence

$OC = CB = \frac{V_L}{2}$

- From triangle OAB, $\cos 30° = \frac{OC}{OA} = \frac{(V_{RY}/2)}{V_R}$

i.e. $\frac{\sqrt{3}}{2} = \frac{(V_L/2)}{V_{ph}}$

$\therefore$ $V_L = \sqrt{3}\ V_{ph}$ for star connection and $I_L = I_{ph}$

Thus line voltage is $\sqrt{3}$ times the phase voltage in star connection.

- The lagging or leading nature of current depends on per phase impedance.
- If Z_{ph} is inductive i.e. $R + j\ X_L$ then current I_{ph} lags V_{ph} by angle ϕ where ϕ is $\tan^{-1}(X_L/R)$.
- If Z_{ph} is capacitive i.e. $R - j\ X_C$ then I_{ph} leads V_{ph} by angle ϕ. If Z_{ph} is resistive i.e. $R + j\,0$ then I_{ph} is in phase with V_{ph}.

- Remember that $\mathbf{Z_{ph}}$ relates $\mathbf{I_{ph}}$ and $\mathbf{V_{ph}}$ hence angle ϕ is always between $\mathbf{I_{ph}}$ and $\mathbf{V_{ph}}$ and not between the line values and $|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|}$. The line values do not decide the impedance angle or power factor angle.

- The complete phasor diagram for lagging power factor load is shown in the Fig. 4.9.3.

$$Z_{ph} = R_{ph} + j\ X_{Lph} = |Z_{ph}| \angle \phi\ \ \Omega$$

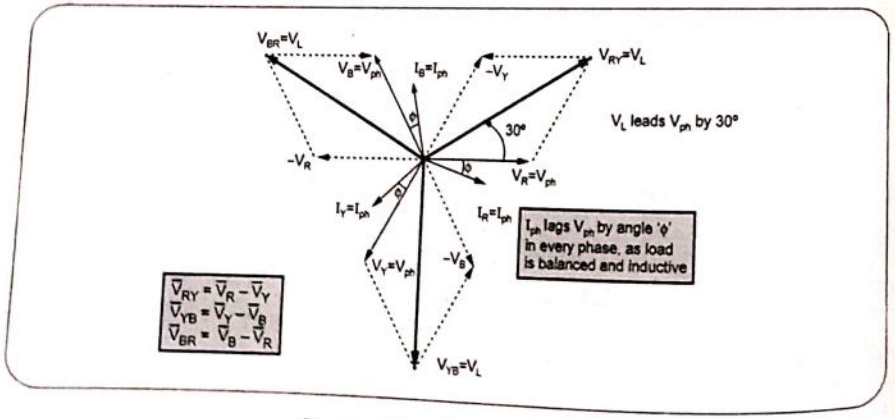


Fig. 4.9.3 Star and lagging p.f. load

- Each I_{ph} lags corresponding V_{ph} by angle ϕ.
- All line voltages are also displaced by 120° from each other.

- Every line voltage leads the respective phase voltage by 30°.

Power : The power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph}\ I_{ph} \cos\phi$$

- For balanced load, all phase powers are equal.
- Hence total three phase power consumed is,

$$P = 3 P_{ph} = 3\ V_{ph}\ I_{ph} \cos\phi$$

$$= 3\ \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$\therefore \quad \boxed{P = \sqrt{3}\ V_L I_L \cos\phi \quad \text{watts}}$$

For star connection, to draw phasor diagram, use $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$, $\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$ and $\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$

Ex. 4.9.1 *A balanced 3-phase star connected load of 150 kW takes a leading current of 100 A, with a line voltage of 1,100 V, 50 Hz. Find the circuit constants of the load per phase.*

VTU : Aug.-05, Feb.-06, July-15, Marks 10, Jan.-18, Marks 6

Ans. :

$P = 150$ kW, $I_L = 100$ A, star, $V_L = 1100$ V

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

$$\therefore \quad 150 \times 10^3 = \sqrt{3} \times 1100 \times 100 \times \cos\phi$$

$$\therefore \quad \cos\phi = 0.7873 \quad \text{i.e. } \phi = 38.066°$$

But as current is leading, ϕ must be negative in Z_{ph}.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = 635.085\ \text{V},\ I_{ph} = I_L = 100\ \text{A (star)}$$

$$\therefore |Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{635.085}{100} = 6.3508\ \Omega$$

$$\therefore \quad Z_{ph} = |Z_{ph}| \angle -\phi = 6.3508 \angle -38.066° \ \dots \text{Leading}$$

$$\therefore \quad Z_{ph} = 5 - j\ 3.9157\ \Omega = R_{ph} - j\ X_{Cph}$$

$\therefore R_{ph} = 5\ \Omega$, $X_{Cph} = 3.9157 = \frac{1}{2\pi f C_{ph}}$

$\therefore C_{ph} = \mathbf{812.906\ \mu F}$

Ex. 4.9.2 *A balanced star connected load of (8 + j6) Ω per phase is connected to a three phase, 230 V supply. Find the line current, power factor, power, reactive voltampere and total voltampere.*

VTU : Aug.-05, Feb.-06, July-15, Marks 10

Ans. : Balanced star connected load of 8 + j6 Ω per phase, $V_L = 230$ V.

i) Impedance (Z) = 8 + j6 = 10 ∠36.86° Ω.

$\therefore V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79$ volts ... Star connection

$\therefore$ Phase current $(I_{ph}) = \frac{V_{ph}}{|Z_{ph}|} = \frac{132.79}{10}$

$= 13.279$ Amp

$\therefore$ As $I_L = I_{ph}$, $I_L = \mathbf{13.279\ Amp}$

ii) Power factor $= \frac{R}{|Z|} = \frac{8}{10} = 0.8$ and $\phi = \cos^{-1} 0.8$

$= 36.86°$ hence $\sin\phi = 0.6$

iii) Power in $3\phi = \sqrt{3}\ V_L\ I_L \cos\phi$

$= \sqrt{3} \times 230 \times 13.279 \times 0.8$

$= \mathbf{4231.98\ W}$

iv) Reactive voltampere $= \sqrt{3}\ V_L\ I_L \sin\phi$

$= \sqrt{3} \times 230 \times 13.279 \times 0.6$

$= \mathbf{3173.98\ VAR}$

v) Total voltampere $= \sqrt{3}\ V_L\ I_L = \mathbf{5289.97\ VA}$

Ex. 4.9.3 *Calculate the active and reactive components of each phase of Y connected 10 kV, 3 ϕ alternator supplying 5 MW at 0.8 p.f. If the total current remains the same, when load p.f. is raised to 0.9, calculate the new output and its active and reactive components per phase.*

VTU : Aug.-08, Marks 10

Sol. :

$$P_T = 5 \text{ MW}, \cos\phi_1 = 0.8 \text{ p.f.},$$

$$V_L = 10 \text{ kV, star}$$

$$P_T = \sqrt{3} V_L I_L \cos\phi_1$$

i.e. $$5\times10^6 = \sqrt{3}\times10\times10^3\times I_L\times0.8 \quad \text{... Star connection}$$

$$\therefore \quad I_L = 360.844 \text{ A} = I_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{10\times10^3}{\sqrt{3}} = 5773.5027 \text{ V}$$

Active component of each phase $= V_{ph} I_{ph} \cos\phi_1 = \mathbf{1.666\ MW}$

Reactive component of each phase $= V_{ph} I_{ph} \sin\phi_1 = \mathbf{1.25\ MVAR}$

Now current remains same but $\cos\phi_2 = 0.9$

$$\therefore \quad P_T = \sqrt{3} V_L I_L \cos\phi_2 = \sqrt{3}\times10\times10^3\times360.844\times0.9 = \mathbf{5.625\ MW} \quad \text{... New output}$$

New active component of each phase $= V_{ph} I_{ph} \cos\phi_2 = \mathbf{1.875\ MW}$

New reactive component of each phase $= V_{ph} I_{ph} \sin\phi_2 = \mathbf{0.9081\ MVAR}$

Ex. 4.9.4 *Three 50 Ω resistors are connected in star across 400 V 3-ph supply :*
i) Find phase current, line current and power drawn from supply.
ii) What would be the above values if one of the resistors were disconnected ?

VTU : Aug.-05, Feb.-06, July-15, Marks 10, Jan.-16, Marks 6

Sol. : $R = 50\ \Omega$, $V_L = 400$ V

i) Star connection as shown in the Fig. 4.9.4 (a).

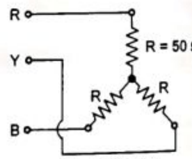


Fig. 4.9.4 (a)

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{R_{ph}} = \frac{230.94}{50} = 4.6188 \text{ A}$$

$$I_L = I_{ph} = 4.6188 \text{ A}, \cos\phi = 1, \text{ as pure resistive}$$

$$\therefore \quad P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3}\times400\times4.6188 = \mathbf{3200\ W}$$

ii) One resistor disconnected as shown in the Fig. 4.9.4 (b).

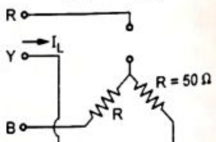


Fig. 4.9.4 (b)

The equivalent circuit across Y and B is shown in the Fig. 4.9.4 (c).

$$\therefore \quad I_L = \frac{400}{50+50} = 4 \text{ A}$$

$$\therefore \quad I_{ph} = I_L = 4 \text{ A}$$

$$\therefore \quad P = I_{ph}^2\, R + I_{ph}^2\, R = 4^2\times50 + 4^2\times50 = \mathbf{1600\ W}$$

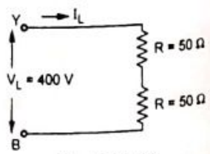


Fig. 4.9.4 (c)

Ex. 4.9.5 *Three coils each having resistance of 10 Ω and inductance of 0.02 H are connected in star across 440 V, 50 Hz, 3ϕ supply. Calculate the line current and total power consumed.*

VTU : Aug.-05, Feb.-06, July-15, Jan.-16, Marks 6, Jan.-17, Marks 5

Ans. :

$$Z_{ph} = R + j X_L = R + j\, 2\pi f L,$$

$$R = 10\ \Omega,\ L = 0.02 \text{ H}$$

$$\therefore \quad Z_{ph} = 10 + j\,(2\pi\times50\times0.02)$$

$$= 10 + j\, 6.2832 = 11.81 \angle 32.14°\ \Omega$$

For star, $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.034 \text{ V}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.034}{11.81}$$

$$= \mathbf{21.51\ A} \text{ (Magnitude)}$$

$$\therefore \quad I_L = I_{ph} = \mathbf{21.51\ A}$$

$$\cos\phi = \cos(32.14°) = 0.8467 \text{ lag}$$

$$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3}\times 440\times 21.51\times 0.8467$$

$$= \mathbf{13.88\ kW}$$

4.10 Relations for Delta Connected Load

➤ ***Obtain the relationship between line and phase values of current in a three phase balanced delta connected system. Derive equation of power.*** **VTU : Jan.-03; July-04, 05, 07,18, Marks 8; July-08, 15, 16, Marks 10; Jan.-10, 16, Dec.-11, Marks 6**

- Consider the balanced delta connected load as shown in the Fig. 4.10.1.
- Line voltages : $V_L = V_{RY} = V_{YB} = V_{BR}$
  Line currents : $I_L = I_R = I_Y = I_B$
- Phase voltages : $V_{ph} = V_{RY} = V_{YB} = V_{BR}$
  Phase currents : $I_{ph} = I_{RY} = I_{YB} = I_{BR}$
- As seen earlier, $V_{ph} = V_L$ for delta connected load.
- To derive the relation between I_L and I_{ph}, apply the KCL at the node R of the load shown in the Fig. 4.10.1.

$$\sum I_{entering} = \sum I_{leaving} \text{ at node R}$$

$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY} \quad \text{i.e.} \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \quad \text{...(4.10.1)}$$

Applying KCL at node Y and B, we can write equations for line currents I_Y and I_B as,

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \quad \text{...(4.10.2)}$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} \quad \text{...(4.10.3)}$$

- The phasor diagram to obtain line current I_R by carrying out vector subtraction of phase currents I_{RY} and I_{YB} is shown in the Fig. 4.10.2.

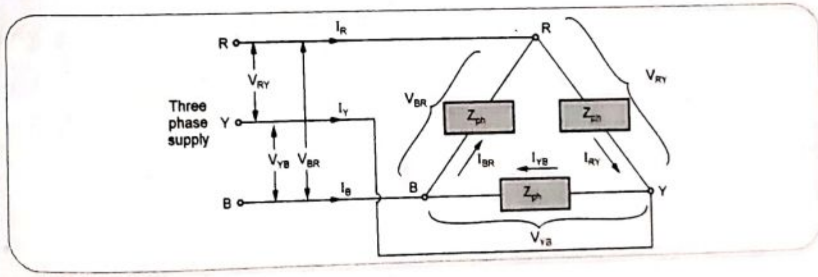


Fig. 4.10.1 Delta connected load

- The three phase currents are displaced from each other by 120°.
- I_{BR} is reversed to get $-I_{BR}$ and then added to I_{RY} to get I_R.
- The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L.
- Similarly OB bisects angle between $-I_{BR}$ and I_{RY} which is 60°.

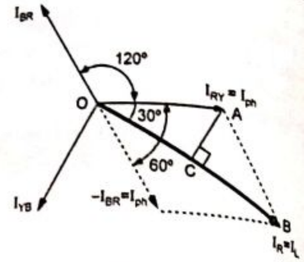


Fig. 4.10.2

$\therefore$ $\angle BOA = 30°$ and $OC = CB = \frac{I_L}{2}$

- From triangle OAB,
- $\cos 30° = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$ i.e. $\frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$

$\therefore$

$I_L = \sqrt{3}\ I_{ph}$ and $V_{ph} = V_L$ ... **for delta connection**
Thus line current is $\sqrt{3}$ times the phase current in delta connection.

- Again Z_{ph} decides whether I_{ph} has to lag, lead or remain in phase with V_{ph}. **Angle between V_{ph} and I_{ph} is ϕ.**

$\therefore$

Thus for delta connection, to draw phasor diagram, use
$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$, $\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$ and $\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$

- The complete phasor diagram for $\cos\phi$ lagging power factor load is shown in the Fig. 4.10.3.

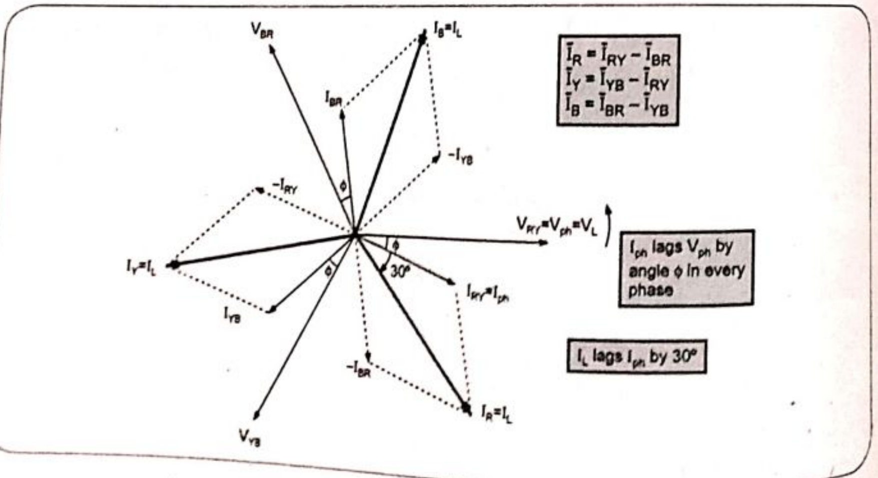


Fig. 4.10.3

$$Z_{ph} = R_{ph} + j\ X_{Lph} = |Z_{ph}| \angle \phi\ \Omega$$

- Each I_{ph} lags respective V_{ph} by angle ϕ

• Every line current lags the respective phase current by 30°.

Power : Power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph}\ I_{ph}\ \cos\phi$$

- Total power $P = 3P_{ph} = 3V_{ph} I_{ph} \cos\phi = 3V_L \frac{I_L}{\sqrt{3}} \cos\phi$

$\therefore$ $\boxed{P = \sqrt{3}\ V_L I_L \cos\phi \quad \text{watts}}$

• The expression for power is same but values of line currents are different in star and delta connected load which must be correctly determined to obtain power.

Ex. 4.10.1 *A three phase load of three equal impedances connected in delta across a balanced 400 V supply takes a line current of 10 A a power factor of 0.7 lagging. Calculate from the first principles :*

i) The phase current ii) The total power iii) The total reactive kVA.

If the windings are connected in star, what will be the new value of phase current and the total power ?

VTU : Feb.-08, Marks 7

Sol. : $\cos\phi = 0.7$ lagging,

$$\phi = 45.57299°$$

For the delta connected,

$V_L = V_{ph}$ and $I_L = \sqrt{3}\ I_{ph}$

i) $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.7735$ A

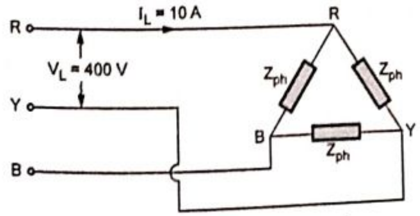


Fig. 4.10.4 Delta connected load

ii) $P_T = \sqrt{3}\, V_L I_L \cos\phi$

$\therefore$ $P_T = \sqrt{3} \times 400 \times 10 \times 0.7$

$= \mathbf{4849.7422\ W}$

iii) Reactive volt-amp $= \sqrt{3}\, V_L I_L \sin\phi$

$= \sqrt{3} \times 400 \times 10 \times \sin(45.57299°)$

$= \mathbf{4947.7267\ VAR = 4.94772\ kVAR}$

Now the windings are connected in star.

For star connection,

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9401$ V and $I_L = I_{ph}$

Note : I_L will be no longer 10 A when load connection is changed from delta to star.

From **delta connection results,**

$$|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|} = \frac{400}{5.7735} = 69.28206\ \Omega$$

This Z_{ph} remains same but now star connected.

New $|I_{ph}| = \frac{|V_{ph}|}{|Z_{ph}|} = \frac{230.9401}{69.28206} = \mathbf{3.3333\ A}$

... New phase current

And $I_L = I_{ph} = 3.333$ A

The $\cos\phi$ also remains same as winding is same as before

$\therefore$ $P_T = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 3.3333 \times 0.7$

$= \mathbf{1616.5645\ W}$

Note that total power in star connection is one third of the total power in delta connection.

4.11 Power Triangle for Three Phase Load

➤ ***Explain the power triangle for the three phase load.***

Total apparent power

$$S = 3 \times \text{Apparent power per phase}$$

$\therefore \quad S = 3\ V_{ph} I_{ph} = 3\ \frac{V_L}{\sqrt{3}} I_L = 3\ V_L \frac{I_L}{\sqrt{3}}$

Total apparent power $S = \sqrt{3}\ V_L I_L$	**volt-amperes (VA) or kVA**
Total active power	$P = \sqrt{3}\ V_L I_L \cos\phi$ watts (W) or kW
Total reactive power (VAR) or kVAR	$Q = \sqrt{3}\ V_L I_L \sin\phi$ reactive volt amperes

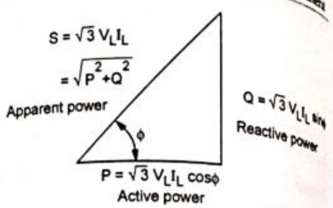


Fig. 4.11.1 Power triangle

- Hence power triangle is as shown in the Fig. 4.11.1.

While solving three phase problems :

1) Given supply voltages are always line voltages.
2) Determine phase voltage depending on whether load is star or delta connected.
3) Then determine phase curent,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

4) Determine line current depending on whether load is star or delta connected.
5) ϕ is angle between V_{ph} and I_{ph}. Value can be obtained from given Z_{ph}.
6) The total power consumed is $\sqrt{3}\ V_L I_L \cos\phi$.

Ex. 4.11.1 *A 3-phase delta connected load consumes a power of 60 kW taking a lagging current of 200 A at a line voltage of 400 V, 50 Hz. Find the parameters of each phase. What would be the power consumed, if the load were connected in star ?* **VTU : Dec.-11, Jan.-15, Marks 6**

Ans. : Delta, P = 60 kW, I_L = 200 A, V_L = 400 V

$$P = \sqrt{3}\ V_L I_L \cos\phi \quad \text{i.e.} \quad 60 \times 10^3 = \sqrt{3} \times 400 \times 200 \times \cos\phi$$

$$\therefore \quad \cos\phi = 0.433 \quad \text{i.e.} \quad \phi = 64.341°, \quad I_{ph} \text{ lags } V_{ph}$$

For delta, $V_{ph} = V_L = 400$ V, $I_{ph} = \frac{I_L}{\sqrt{3}} = 115.47$ A

$$\therefore \quad |Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{400}{115.47} = 3.4641\ \Omega$$

$$\therefore \quad Z_{ph} = |Z_{ph}| \angle \phi = 3.4641 \angle 64.341°\ \Omega = 1.5 + j\ 3.1225\ \Omega$$

$= R_{ph} + jX_{Lph}$ hence R_{ph} = **1.5 Ω**, X_{Lph} = **3.1225 Ω**

$X_{Lph} = 2\pi f L_{ph}$ i.e. $L_{ph} = \frac{3.1225}{2\pi \times 50}$ = **9.9392 mH**

If connected in star, $V_{ph} = \frac{V_L}{\sqrt{3}} = 230.94$ V

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{3.4641} = 66.667\ \text{A} = I_L,\ \cos\phi = 0.433$$

$$\therefore \quad P = \sqrt{3}\ V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 66.667 \times 0.433 = \mathbf{20\ kW}$$

Ex. 4.11.2 *The three arms of a three-phase load each comprise an inductor of resistance 25 Ω and of inductance 0.15 H in series with a 120 μF capacitor. The supply voltage is 415 V, 50 Hz. Calculate the line current and total power in watts, when the three arms are connected in delta.* **VTU : June-12, July-17, Marks 6**

Sol. : Each phase of the load having impedance of,

$Z_{ph} = R + j\ X_L - j\ X_C$, L = 0.15 H, C = 120 μF

$\therefore X_L = 2\pi fL = 47.1238\ \Omega$, $X_C = \frac{1}{2\pi fC} = 26.5258\ \Omega$

$\therefore Z_{ph} = 25 + j47.1238 - j26.5258 = 25 + j20.598\ \Omega$

$= 32.392 \angle 39.48°\ \Omega$

For delta, $V_{ph} = V_L = 415$ V

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415 \angle 0°}{32.392 \angle +39.48°}$$

$$= 12.8118 \angle -39.48°\ \text{A}$$

$$\therefore \quad I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 12.8118$$

$= \mathbf{22.1906\ A}$ ... Line current

$\phi = 39.48°$ lag i.e. $\cos\phi = 0.7718$ lag

$$\therefore \quad P = \sqrt{3}\ V_L I_L \cos\phi$$

$$= \sqrt{3} \times 415 \times 22.1906 \times 0.7718$$

$$= \mathbf{12.3114\ kW}$$

Ex. 4.11.3 *A 3 phase 230 V supply is given to balanced load which is Δ connected. Impedance in each phase of the load is 8+j6 Ω. Determine the phase current and the total power consumed.* **VTU : Jan.-13, Marks 6**

Sol. : $V_L = 230$ V, $Z_{ph} = 8 + j6\ \Omega = 10 \angle 36.869°\ \Omega$

$V_L = V_{ph} = 230$ V ... Delta connection

Let V_{ph} be reference hence,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230 \angle 0°}{10 \angle 36.869°}$$

$= 23 \angle -36.869°$ A

$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 23$

$= 39.8371$ A ... Magnitude

$\cos\phi = \cos(36.869°) = 0.8$ lagging

$$\therefore \quad P = \sqrt{3}\ V_L I_L \cos\phi$$

$$= \sqrt{3} \times 230 \times 39.8371 \times 0.8$$

$$= \mathbf{12.696\ kW}$$

Ex. 4.11.4 *A three phase load of three equal impedances connected in delta across a balanced 400 V supply, takes a line current of 10 A at a power factor of 0.7 lagging. Calculate : i) the phase current ii) the total power iii) the total reactive volt amperes.* **VTU : Jan.-16, Marks 4**

Sol. : Delta, V_L = 400 V, I_L = 10 A, $\cos\phi$ = 0.7 lagging

i) $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}}$ = **5.773 A**

ii) $P = \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 10 \times 0.7$ = **4849.742 W**

iii) $Q = \sqrt{3}\ V_L I_L \sin\phi$

$= \sqrt{3} \times 400 \times 10 \times 0.7141$

= **4947.43 VAR**

4.12 Three Phase Power Measurement

- In three phase circuits whether load is star connected or delta connected, total three phase power is given by $\sqrt{3}\ V_L I_L \cos\phi$.

- The ϕ is the angle between V_{ph} and I_{ph}.

- In practice, the problems in measuring three phase power occur as power factor cos φ for different types of loads may not be known to us.
- Not only this but power factor of induction motor, synchronous motor may vary depending on different load conditions. It is very difficult to notice such on line changes in the value of power factor and then using it to calculate the power.

- Hence it is absolute necessity to use some device which will sense the power factor and will give the wattage reading directly.
- Such a device which senses voltage, current and (power factor) angle between voltage and current to give power reading in watts directly is called **wattmeter**.

4.13 Wattmeter

➤ ***Explain the wattmeter connections. When wattmeter reads phase power ? Why two wattmeter method is necessary ?***

- It is a device which gives power reading, when connected in single phase or three phase system, directly in watts.
- It consists of two coils.

i) Current coil : This senses the current and always to be connected in series with the load. Similar to ammeter, the resistance of this coil is as small as possible and hence its cross-sectional area is large and it has less number of turns.

ii) Voltage coil : This is also called **pressure coil**. This senses the voltage and always to be connected across the supply terminals. Similar to voltmeter, the resistance of this coil is very large and hence its cross-sectional area is small and it has large number of turns.

- It is important to note that wattmeter senses the angle between current phasor which is sensed by its current coil and voltage phasor which is sensed by its voltage coil.

- It will not read phase angle 'ϕ' all the time. It depends on how we connect its current and voltage coils in the system.

- As 'ϕ' is the angle between V_{ph} and I_{ph}, if wattmeter has to sense this, its current coil must carry phase current I_{ph} and its voltage coil must sense phase voltage V_{ph}.

- In general if I_c is the current through its current coil (may be phase or line depends on its connection) and V_{pc} is voltage across its pressure coil (may be phase or line depends on its connection) then wattmeter reading is,

$$\boxed{W = V_{pc} \times I_c \times \cos\left(I_c \wedge V_{pc}\right) \quad \text{watts}}$$

- Angle between V_{pc} and I_c is to be decided from the phasor diagram.

- If $I_c = I_{ph}$ and $V_{pc} = V_{ph}$ then $I_c \wedge V_{pc} = I_{ph} \wedge V_{ph} = \phi$ and then only wattmeter reads per phase power which is $V_{ph} I_{ph} \cos\phi$.

- A wattmeter can be represented symbolically as shown in Fig. 4.13.1.

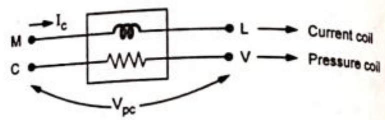


Fig. 4.13.1

- The terminologies used to denote current and pressure coil are,

M = From mains, L = To load = For current coil

C = Common, V = Voltage = For voltage coil

- The terminals M and C are generally connected together.
- To sense the phase voltage by pressure coil and the phase current by the current coil, it must be connected as shown in the Fig. 4.13.2 (a) and (b).
- But connecting wattmeter to measure phase power is not always possible because many times neutral point of star connected load is not available. Similarly in delta connected load it is necessary to open delta load to insert current coil of the wattmeter, which is not practicable.
- Hence the best method of measuring power whether load is star or delta connected, balanced or unbalanced, neutral is available or not is, using only two wattmeters which is called Two Wattmeter Method.

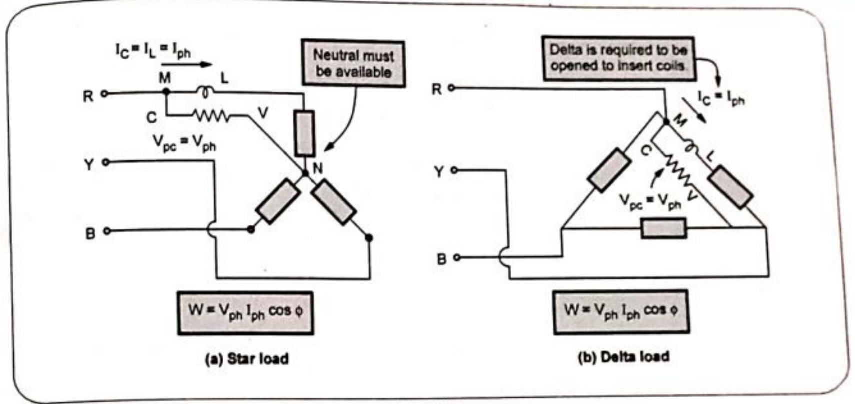


Fig. 4.13.2

4.14 Two Wattmeter Method

➤ ***Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.*** VTU : Jan.-03, 04, 06, 08, 09, 10, 16, 18, July-03, 15, 16, Marks 8

- The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeter is connected between its own current coil terminal and the line without a current coil.
- For example, the current coils are inserted in the lines R and Y then the pressure coils are connected between R - B for one wattmeter and Y- B for other wattmeter, as shown in the Fig. 4.14.1.
- The connections are same for star or delta connected load.
- It can be shown that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power dissipated in the three phase circuit.
- If W_1 and W_2 are the two wattmeter readings then total power

$$\boxed{W = W_1 + W_2 = \text{Three phase power} = \sqrt{3}\, V_L I_L \cos\phi}$$

Proof of Two Wattmeter Method for Star Connected Load :

- Consider star connected load and two wattmeters connected as shown in the Fig. 4.14.1.

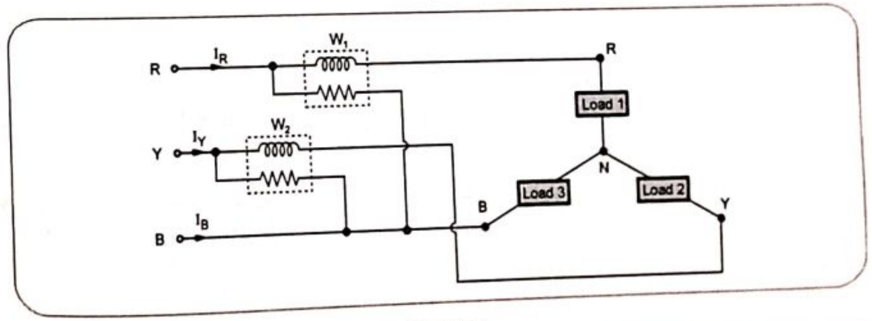


Fig. 4.14.1

- Let us consider the r.m.s. values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.

$$W_1 = I_R \times V_{RB} \times \cos\left(I_R \,^\wedge V_{RB}\right) \quad \text{and}$$

$$W_2 = I_Y \times V_{YB} \times \cos\left(I_Y \,^\wedge V_{YB}\right)$$

- To find angle between (I_R and V_{RB}) and (I_Y and V_{YB}) let us draw phasor diagram. (Assuming load p.f. be cos φ lagging)

$$\bar{V}_{RB} = \bar{V}_R - \bar{V}_B$$

and $$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$$

$$V_R \,^\wedge I_R = \phi \text{ and } V_Y \,^\wedge I_Y = \phi$$

$$V_R = V_Y = V_B = V_{ph} \quad \text{and} \quad V_{RB} = V_{YB} = V_L$$

$$I_R = I_Y = I_L = I_{ph} \text{ (star)}$$

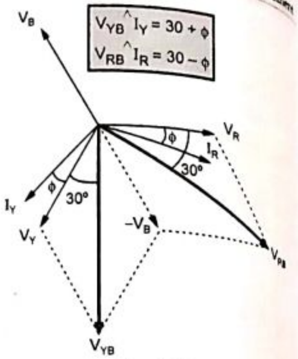


Fig. 4.14.2

- From Fig. 4.14.2, $I_R \,^\wedge V_{RB} = 30 - \phi$ and $I_Y \,^\wedge V_{YB} = 30 + \phi$

∴ $W_1 = I_R V_{RB} \cos(30-\phi)$ i.e $\boxed{W_1 = V_L I_L \cos(30-\phi)}$

and $W_2 = I_Y V_{YB} \cos(30+\phi)$ i.e $\boxed{W_2 = V_L I_L \cos(30+\phi)}$

$$\therefore \quad W_1 + W_2 = V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= 2 V_L I_L \cos 30 \cos\phi = 2 V_L I_L \frac{\sqrt{3}}{2} \cos\phi$$

$$\therefore \quad \boxed{W_1 + W_2 = \sqrt{3}\, V_L\, I_L \cos\phi = \text{Total 3 phase power}}$$

Proof of Two Wattmeter Method for Delta Connected Load :

Consider delta connected balanced load, as shown in the Fig. 4.14.3.

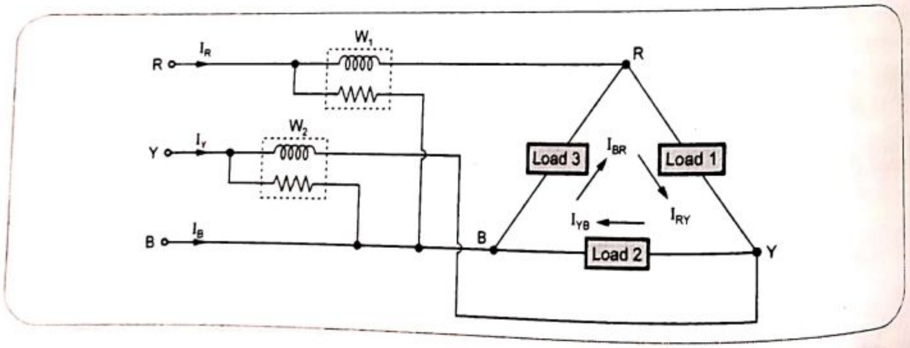


Fig. 4.14.3

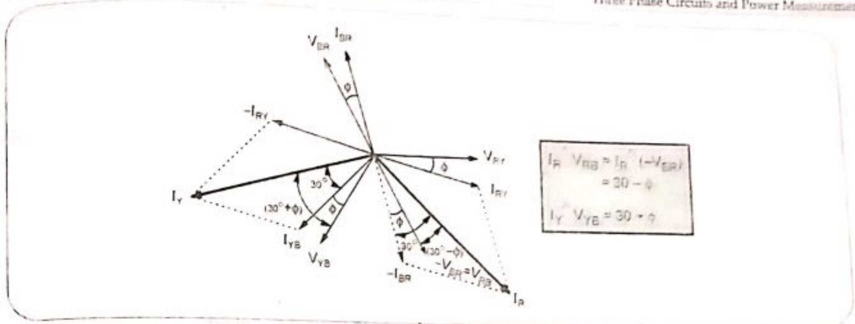


Fig. 4.14.4 Delta connected load, lagging p.f

For W_1, $\quad I_c = I_R$ and $V_{pc} = V_{RB}$

and W_2, $\quad I_c = I_Y$ and $V_{pc} = V_{YB}$

$$\therefore \quad W_1 = I_R V_{RB} \cos\left(I_R \,^\wedge V_{RB}\right) \quad \text{and} \quad W_2 = I_Y V_{YB} \cos\left(I_Y \,^\wedge V_{YB}\right)$$

- To find $I_R \,^\wedge V_{RB}$ and $I_Y \,^\wedge V_{YB}$ let us draw phasor diagram. Assume load having cos φ lagging p.f.

$$V_{RB} = V_{YB} = V_{phase} = V_{line} \text{ (Delta)} \quad \text{and} \quad I_R = I_Y = I_{line}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \quad \text{and} \quad \bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$$

$$\therefore \quad W_1 = I_R V_{RB} \cos(30-\phi) = I_L V_L \cos(30-\phi)$$

$$W_2 = I_Y V_{YB} \cos(30+\phi) = I_L V_L \cos(30+\phi)$$

$$\therefore \quad W_1 + W_2 = V_L I_L [\cos(30-\phi) + \cos(30+\phi)] = \sqrt{3} V_L I_L \cos\phi$$

∴ $\boxed{W_1 + W_2 = \text{Total power consumed by three phase load.}}$

- It can be observed that whether load is star of delta, the expressions for W_1 and W_2 remain same.

- For load having leading power factor, I_{ph} will lead V_{ph} by angle φ and hence W_1 reading and W_2 reading will get interchanged as 30 – φ will become 30 + φ and viceversa.

In case of leading power factor loads , readings of W_1 and W_2 are interchanged compared to lagging power factor load.

For star or delta lagging p.f. load, $W_1 = V_L I_L \cos(30-\phi)$ and $W_2 = V_L I_L \cos(30+\phi)$

For star or delta leading p.f. load, $W_1 = V_L I_L \cos(30-\phi)$ and $W_2 = V_L I_L \cos(30-\phi)$

For star or delta unity p.f. load, $\cos\phi = 1$ and $\phi = 0°$, $W_1 = W_2 = V_L I_L \cos 30°$

Ex. 4.14.1 *A 3 φ 400 V, motor takes an input of 40 kW at 0.45 p.f. lag. Find the reading of each of the two single phase wattmeters connected to measure the input.* **VTU : Aug.-14, Jan.-17, Marks 6**

Sol. : $V_L = 400$ V, $P_{in} = 40$ kW, $\cos\phi = 0.45$ lag

$$P_{in} = \sqrt{3}\, V_L I_L \cos\phi \quad \text{i.e.} \quad 40 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.45$$

$$I_L = 128.3 \text{ A}$$

$\therefore \quad V_L I_L = 51320$ and $\phi = \cos^{-1} 0.45 = 63.256°$

$$\therefore \quad W_1 = V_L I_L \cos(30 - \phi)$$

$$= 51320 \cos(30° - 63.256°)$$

$$= 42915.2 \text{ W}$$

$$\therefore \quad W_2 = V_L I_L \cos(30 + \phi)$$

$$= 51320 \cos(30° + 63.256°)$$

$$= -2915.2 \text{ W}$$

Cross-check : $W_1 + W_2 = 40 \text{ kW} = P_{in}$

Ex. 4.14.2 *Three similar choking coils each having resistance 10 Ω and reactance 10 Ω are connected in star across a 440 V, 3 phase supply. Find line current and reading of each of two wattmeters connected to measure power.* **VTU : Feb.-07, Jan.-14, Marks 6**

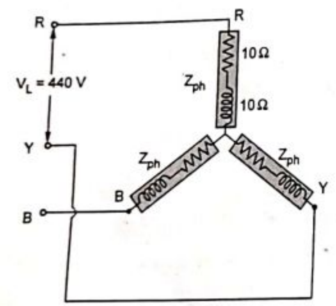


Fig. 4.14.5

Sol. :

In star connection,

$$V_{ph} = \frac{V_L}{\sqrt{3}} \text{ and } I_{ph} = I_L$$

Now $\quad V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254 \text{ V}$

$$Z_{ph} = 10 + j\,10\,\Omega$$

$$= 14.1421 \angle 45°\,\Omega$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254 \angle 0°}{14.1421 \angle +45°}$$

$$= 17.9605 \angle -45° \text{ A}$$

$\therefore \quad I_L = I_{ph} = 17.9605$ ... Line current

$\phi = 45°$ Lagging as negative

For lagging p.f. angle,

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$= 440 \times 17.9605 \times \cos(30 - 45)$$

$$= 7.6333 \text{ kW.}$$

and $\quad W_2 = V_L I_L \cos(30 + \phi)$

$$= 440 \times 17.9605 \times \cos(30 + 45)$$

$$= 2.0453 \text{ kW.}$$

Cross check : $W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$

$$= \mathbf{9.6787 \text{ kW.}}$$

Ex. 4.14.3 *Three similar coils each having resistance of 10 Ω and reactance of 8 Ω are connected in star, across 400 V, 3 phase supply. Determine*
i) line current, ii) total power,
iii) reading of each of two wattmeter connected to measure power. **VTU : July-11, June-16, Marks 6**

Sol. : $Z_{ph} = 10 + j\,8\,\Omega = 12.806 \angle 38.66°\,\Omega$,

$$V_L = 400 \text{ V}$$

i) $\quad V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94 \text{ V}$

$\therefore \quad I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230.94}{12.806} = 18.033 \text{ A (magnitude)}$

$$\therefore \quad I_L = I_{ph} = \mathbf{18.033 \text{ A}}$$

ii) $\quad P = \sqrt{3}\, V_L \; I_L \cos\phi$

$$= \sqrt{3} \times 400 \times 18.033 \times \cos(38.66°) = \mathbf{9.756 \text{ kW}}$$

iii) $\quad \phi = 38.66°$

$$\therefore \quad W_1 = V_L I_L \cos(30 - \phi)$$

$$= 400 \times 18.033 \times \cos(30° - 38.66°)$$

$$= \mathbf{7.131 \text{ kW}}$$

$$\therefore \quad W_2 = V_L I_L \cos(30 + \phi)$$

$$= 400 \times 18.033 \times \cos(30° - 38.66°) = \mathbf{2.625 \text{ kW}}$$

Cross check : $P = W_1 + W_2$

4.15 Power Factor Calculation by Two Wattmeter Method

> ***How power factor can be obtained from two wattmeter readings ?*** **VTU : Jan.-02, 05, 06, Marks 5; July-03, 07, 08, 16, Marks 6**

- In case of balanced load, the p.f. can be calculated from W_1 and W_2 readings.
- For balanced, lagging p.f. load, $W_1 = V_L I_L \cos(30 - \phi)$ and $W_2 = V_L I_L \cos(30 + \phi)$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad \text{...(4.15.1)}$$

$$W_1 - W_2 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi$$

$$- \cos 30 \cos\phi + \sin 30 \sin\phi]$$

$$= V_L I_L [2 \sin 30 \sin\phi]$$

$$= V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right]$$

$$\therefore \quad W_1 - W_2 = V_L I_L \sin(\phi) \quad \text{...(4.15.2)}$$

- Taking ratio of equations (4.15.1) and (4.15.2),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

i.e. $\quad \tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$

$$\therefore \quad \phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\therefore \quad \boxed{\text{p.f. } \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}}$$

- For **leading p.f.** we get **tan φ negative.** But cosine of negative angle is positive.

> - **The power factor cos φ is always positive but its nature must be determined by observing sign of tan φ.**

Ex. 4.15.1 *The power flowing in a 3φ, 3-wire balanced load system is measured by two wattmeter method. The reading in wattmeter A is 750 watts and wattmeter B is 1500 watts. What is the power factor of the system and load current per phase ?* **VTU : Aug.-07, Marks 8**

Sol. : $\quad W_1 = 750 \text{ W}$ and $W_2 = 1500 \text{ W}$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(750 - 1500)}{(750 + 1500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(-0.5773)\}$$

$$= \cos\{-30°\} = 0.866 \text{ leading}$$

Nature is leading as φ is negative.

$$P = \sqrt{3}\, V_L I_L \cos\phi = W_1 + W_2$$

... Total power

$$\therefore \quad (750 + 1500) = \sqrt{3}\, V_L I_L \times 0.866$$

$$\therefore \quad V_L I_L = 1500 \text{ VA}$$

If line voltage is known, I_L can be obtained and hence load current per phase can be obtained.

Ex. 4.15.2 *A balanced three phase star connected load draws power from 440 V supply. The two wattmeters connected indicate W_1 = 5 kW and W_2 = 1.2 kW. Calculate power, power factor and current in the circuit.* **VTU : Aug.-09; Jan.-11, 18, July-17, Marks 8**

Sol. : $\quad V_L = 440 \text{ V}, \; W_1 = 5 \text{ kW}, \; W_2 = 1.2 \text{ kW}$

$$P = W_1 + W_2 = 5 + 1.2 = 6.2 \text{ kW}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(5 - 1.2)}{(5 + 1.2)}\right]\right\}$$

$$= \cos\{\tan^{-1}(1.06157)\}$$

$$= 0.6856 \text{ lagging}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

i.e. $6.2 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.6856$

$\therefore \quad I_L = 11.866$ A

Ex. 4.15.3 *Three similar impedances are connected in delta across a 3 φ supply. The two wattmeters connected to measure the input power indicate 12 kW and 7 kW. Calculate: i) Power input ii) Power factor of the load.*

VTU : Feb.-10, Jan.-16, Marks 4

Sol. : $W_1 = 12$ kW, $W_2 = 7$ kW

i) $P_{in} = W_1 + W_2 = 12 + 7 = 19$ kW

ii) $\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$

$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(12-7)}{(12+7)}\right]\right\}$

$= \cos[\tan^{-1}(0.4558)]$

$= \cos[24.5036°] = 0.9099$

... Power factor

4.16 Effect of P.F. on Wattmeter Readings

> ***Discuss the effect of variation of power factor on wattmeter readings.***

VTU : July-02, 07, 16, Jan.-10, Marks 6; June-10, Marks 5

- For a lagging p.f. $W_1 = V_L I_L \cos(30-\phi)$
  and $W_2 = V_L I_L \cos(30+\phi)$
- Consider different cases,

Case i) $\cos\phi = 0 \quad \phi = 90°$

$\therefore \quad W_1 = V_L I_L \cos(30-90) = +\frac{1}{2} V_L I_L$

and $\quad W_2 = V_L I_L \cos(30+90) = -\frac{1}{2} V_L I_L$

i.e. $W_1 + W_2 = 0$

$|W_1| = |W_2| \quad$ but $\quad W_2 = -W_1$

Note Wattmeter can not show negative reading as it has only positive scale. Indication of negative reading is that pointer tries to deflect in negative direction i.e. to the left of zero. In such case, reading can be converted to positive by interchanging either pressure coil connections i.e. $(C \leftrightarrow V)$ or by interchanging current coil connections $(M \leftrightarrow L)$. This is shown in the Fig. 4.16.1.

- Remember that interchanging connections of both the coils will have no effect on the wattmeter reading.

- Such a reading obtained by interchanging connections of either of the two coils will be positive on wattmeter but must be taken as negative for calculations.

Case ii) $\cos\phi = 0.5, \ \phi = 60°$

$\therefore \quad W_1 = V_L I_L \cos(30-60)$

$= V_L I_L \cos 30$ and

$W_2 = V_L I_L \cos(30+60) = 0$

$\therefore \quad W_1 + W_2 = W_1 =$ Total power.

- **One wattmeter shows zero reading for** $\cos\phi = 0.5$.
- For all power factors between 0 to 0.5 W_2 shows negative and W_1 shows positive, **for lagging p.f.**

Case iii) $\cos\phi = 1, \ \phi = 0°$

$\therefore \quad W_1 = V_L I_L \cos(30+0) = V_L I_L \cos 30$

and $\quad W_2 = V_L I_L \cos(30-0) = V_L I_L \cos 30$

- Both W_1 and W_2 are equal and positive.

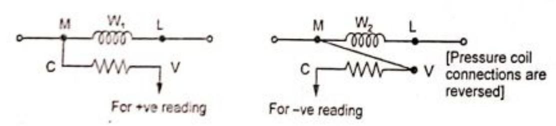


Fig. 4.16.1 Negative reading on wattmeter

- For all power factors between 0.5 to 1 both wattmeter gives +ve reading.
- In short, the result can be summarised as,

Range of p.f.	Range of 'φ'	W_1 sign	W_2 sign	Remark
$\cos\phi = 0$	$\phi = 90°$	positive	negative	$\lvert W_1 \rvert = \lvert W_2 \rvert$
$0 \angle \cos\phi \angle 0.5$	$90° \angle \phi \angle 60°$	positive	negative	
$\cos\phi = 0.5$	$\phi = 60°$	positive	0	
$0.5 \angle \cos\phi \angle 1$	$60° \angle \phi \angle 0°$	positive	positive	
$\cos\phi = 1$	$\phi = 0°$	positive	positive	$W_1 = W_2$

Table 4.16.1

The Table 4.16.1 is applicable for lagging power factors but same table is applicable for leading power factors by interchanging columns of W_1 and W_2.

Ex. 4.16.1 *Estimate the power factor in each of the following cases of two Wattmeter method of measuring three phase power;*
i) Wattmeter readings are equal
ii) Wattmeter readings are equal and opposite
iii) Wattmeter readings are in the ratio 1:2. iv) One Wattmeter reads zero.

VTU : Feb.-05, Marks 6

Sol. : If is known that the p.f. from 2 wattmeter method is,

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Case 1 : $W_1 = W_2$

$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}(0)\right\} = \cos\{90°\} = 1 \qquad ... W_1 - W_2 = 0$

Case 2 : $W_1 = -W_2$

$\therefore \quad \cos\phi = \cos\left\{\tan^{1}(\infty)\right\} = \cos\{90°\} = 0 \qquad ... W_1 + W_2 = 0$

Case 3 : $W_2 = 2W_1$

$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - 2W_1)}{3W_1}\right]\right\} = \cos\left\{\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right\}$

$= \cos\{(-30°)\} = 0.866$

Case 4 : $W_2 = 0$

$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}W_1}{W_1}\right]\right\} = \cos(60°) = 0.5$

Ex. 4.16.2 *Power is measured in a 3 phase balanced load using two wattmeters. The line voltage is 400 V. The load and its p.f. is so adjusted that the line current is always 10 A. Find the reading of the wattmeters when the p.f. is i) unity ii) 0.866 iii) 0.5 and iv) zero.*

VTU : Aug.-05, Marks 8

Sol. : $V_L = 400$ V, $I_L = 10$ A, $V_L I_L = 4000$.

The wattmeter readings are,

$$W_1 = V_L I_L \cos(30 - \phi) \text{ and } W_2 = V_L I_L \cos(30 + \phi)$$

i) $\cos\phi = 1, \phi = 0°$

$\therefore$ $W_1 = 4000 \cos 30° = \mathbf{3461.1\ W}$

and $W_2 = 4000 \cos 30° = \mathbf{3464.1\ W}$

ii) $\cos\phi = 0.866, \phi = 30°$

$\therefore$ $W_1 = 4000 \cos(30°-30°) = \mathbf{4000\ W}$

and $W_2 = 4000 \cos(60°) = \mathbf{2000\ W}$

iii) $\cos\phi = 0.5, \phi = 60°$

$\therefore$ $W_1 = 4000 \cos(30° - 60°) = \mathbf{3464.1\ W}$

and $W_2 = 4000 \cos(30° + 60°) = \mathbf{0\ W}$

iv) $\cos\phi = 0, \phi = 90°$

$\therefore$ $W_1 = 4000 \cos(30°-90°) = \mathbf{2000\ W}$

and $W_2 = 4000 \cos(30°+90°) = \mathbf{-2000\ W}$

Ex. 4.16.3 *The input power to a 3 - phase induction motor running on 400 V, 50 Hz supply was measured by two wattmeter method and readings were 3000 W and −1000 W. Calculate i) Total power input ii) Power factor iii) Line current.*

VTU : Jan.-15, Marks 6

Sol. : $W_1 = 3000$ W, $W_2 = -1000$ W, $V_L = 400$ V

i) $W = W_1 + W_2 = 3000 - 1000 = \mathbf{2000\ W}$

ii) $$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$= \cos(73.897°) = \mathbf{0.2773}$

iii) $W = \sqrt{3} V_L I_L \cos\phi$

i.e. $$I_L = \frac{2000}{\sqrt{3} \times 400 \times 0.2773} = \mathbf{10.408\ A}$$

4.17 Reactive Volt-Amperes by Two Wattmeter Method

➤ ***How to find reactive volt-amperes by two wattmeter method ?***

- We have seen that, $W_1 - W_2 = V_L I_L \sin\phi$
- The total reactive volt-amperes for a 3 phase circuit is given by,

$$Q = \sqrt{3} V_L I_L \sin\phi$$
$$= \sqrt{3}(W_1 - W_2) \quad \text{VAR}$$

- Thus reactive volt-amperes of a 3 phase circuit can be obtained by multiplying the difference of two wattmeter readings by $\sqrt{3}$.

Q = Reactive power $= \sqrt{3} V_L I_L \sin\phi = \sqrt{3}(W_1 - W_2)$	VAR or kVAR

4.18 Advantages of Two Wattmeter Method

➤ ***State the various advantages of two wattmeter method.***

1. The method is applicable for balanced as well as unbalanced loads.
2. Neutral point for star connected load is not necessary to connect the wattmeters.
3. The delta connected load, need not be opened for connecting the wattmeters.
4. Only two wattmeters are sufficient to measure total 3 phase power.
5. If the load is balanced not only the power but power factor also can be determined.
6. Total reactive volt amperes can be obtained using two wattmeter readings for balanced loads.

4.19 Disadvantages of Two Wattmeter Method

➤ ***State the disadvantages of two wattmeter method.***

1. Not applicable for three phase, 4 wire system.
2. The signs of W_1 and W_2 must be identified and noted down correctly otherwise it may lead to the wrong results.

4.20 University Questions with Answers

Jan. - 2010

Q.1 *Obtain the relationship between line and phase values of current in a three phase balanced delta connected system.* **[Refer section 4.10] [6]**

Q.2 *Discuss the effect of variation of power factor on wattmeter readings.* **[Refer section 4.16] [6]**

June - 2010

Q.3 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

Q.4 *Discuss the effect of variation of power factor on wattmeter readings.* **[Refer section 4.16] [5]**

Dec. - 2011

Q.5 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

Q.6 *Obtain the relationship between line and phase values of current in a three phase balanced delta connected system.* **[Refer section 4.10] [6]**

Jan. - 2011

Q.7 *List the advantages of three phase system over single phase system.* **[Refer section 4.2] [4]**

Q.8 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

June - 2012

Q.9 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

Jan. - 2013

Q.10 *Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.* **[Refer section 4.14] [8]**

June - 2013

Q.11 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

Jan. - 2014

Q.12 *List the advantages of three phase system over single phase system.* **[Refer section 4.2] [4]**

Q.13 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

Jan. - 2015

Q.14 *List the advantages of three phase system over single phase system.* **[Refer section 4.2] [4]**

Q.15 *Define phase sequence of 3 phase alternating supply ?* **[Refer setion 4.4] [2]**

Q.16 *Obtain the relationship between line and phase values of current in a three phase balanced star connected system.* **[Refer section 4.9] [8]**

July - 2015

Q.17 *Explain the generation of three phase voltages in an alternator.* **[Refer section 4.3] [4]**

Q.18 *Obtain the relationship between line and phase values of current in a three phase balanced delta connected system.* **[Refer section 4.10] [6]**

Q.19 *Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.* **[Refer section 4.14] [8]**

CBCS scheme

Jan. - 2016

Q.20 List the advantages of 3-ph system over 1-ph system. **[Refer section 4.2]** [6]

Q.21 Mention the advantages of three phase system over single phase system. **[Refer section 4.2]** [5]

Q.22 Derive a relation between line current and phase current in case of 3-ph delta connected load. **[Refer setion 4.10]** [6]

Q.23 With the help of a circuit diagram and vector diagram, show that two wattmeters are sufficient to measure total power and power factor in a balanced three phase circuit. **[Refer setion 4.14]** [8]

Q.24 Three similar coils are connected in delta across a 3-ph supply. The two wattmeters connected to measure the input power indicate 12 kW and 7 kW. Calculate : i) Power input, ii) Power factor of the load. **[Refer setion 4.15]** [4]

July - 2016

Q.25 In a three phase star connection, find the relation between line and phase values of currents and voltages. Also derive the equation for three phase power. **[Refer section 4.9]** [5]

Q.26 Show that the two wattmeters are sufficient to measure three phase power. Also derive an expression for the power factor in terms of wattmeter reading. **[Refer sections 4.14 and 4.15]** [6]

Q.27 In a 3 phase delta connection, find the relation between line and phase values of currents and voltages. Also derive and equation for three phase power. **[Refer section 4.10]** [5]

Q.28 Explain the effect or power factor on the two wattmeter readings connected to measure three phase power. **[Refer section 4.16]** [6]

Jan. - 2017

Q.29 Mention the advantages of three phase system over single phase system. **(Refer section 4.2)** [5]

July - 2017

Q.30 Mention advantages of three phase system over 1 phase system. **(Refer section 4.2)** [5]

Q.31 Establish the relationship between line and phase voltages and currents in a 3ϕ star connected balanced circuit. Shown the vector diagram neatly. **(Refer section 4.9)** [8]

Jan. - 2018

Q.32 Obtain the relationship between the phase and line values of voltage and currents in a delta connected system and also derive the expression for three phase power. **(Refer section 4.9)** [5]

Q.33 Show that two watt-meters are sufficient to measure power in three phase balanced star connected system with the aid of neat circuit diagram and phasor diagram. **(Refer section 4.15)** [8]

❑❑❑

MODULE - 3

5

Single Phase Transformers

Syllabus

Necessity of transformer, Principle of operation, Types and construction of transformers. emf equation, losses, variation of losses with respect to load, efficiency, Condition for maximum efficiency.

Contents

5.1 Necessity of Transformer

➤ ***What is transformer ? What are its functions ? Mention its application in a.c. transmission.*** VTU : July 06, Marks 5

- Alternating voltages can be raised or lowered as per the requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called **transformer.**
- *We can define transformer as below :*

The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.

- Thus the transformer is used to increase or decrease the voltage as per the requirement.
- The use of transformers in a.c. transmission system is shown in the Fig. 5.1.1.

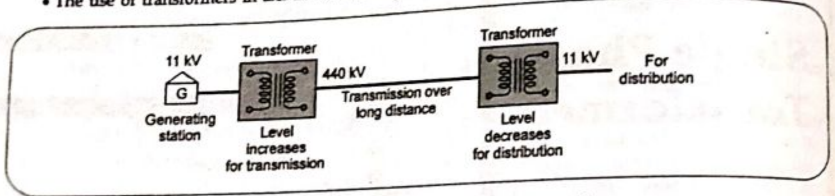


Fig. 5.1.1 Use of transformers in transmission system

5.2 Principle of Operation

➤ ***Explain the principle of operation of single phase transformer.*** VTU : Jan.-03, 08, 13, 17; July-03, 04, 08, 15, Dec. 11, June-12, 13 Marks 5

- The transformer works on the principle of **mutual induction which states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil.**
- In its elementary form, it consists of two inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig. 5.2.1.
- One of the two coils is connected to a source of alternating voltage. This coil in which electrical energy is fed with the help of source is called **primary winding (P).**
- The other winding is connected to load. The electrical energy transformed to this winding is connected to the load. This winding is called **secondary winding (S).**
- The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns.
- When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (ϕ) which completes its path through common magnetic core as shown dotted in the Fig. 5.2.1. Thus an alternating flux links with the secondary winding.

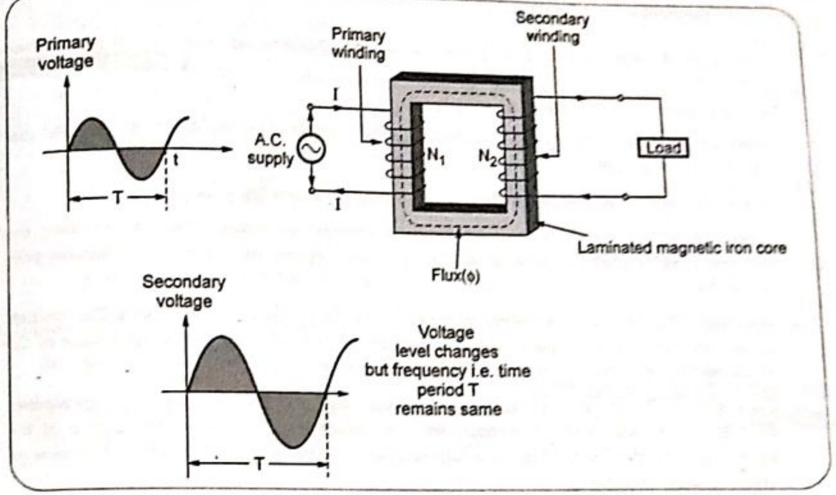


Fig. 5.2.1 Basic transformer

- As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding.
- Symbolically the transformer is indicated as shown in the Fig. 5.2.2.

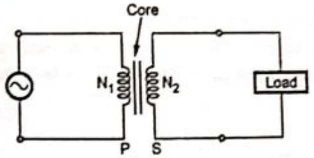


Fig. 5.2.2 Symbolic representation

- P is primary winding while S is secondary winding.
- The two parallel lines in between two windings represents the common magnetic core.
- Thus though there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary.

➤ *Can d.c. supply be used for transformers ?*

- The d.c. supply can not be used for the transformers.
- The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.
- Practically winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So total impedance of winding is very low for d.c.
- Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.
- There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c.
- Thus d.c. supply should not be connected to the transformers.

5.3 Parts of Transformer and Construction

➤ ***What are the main parts of transformer ? What is the function and main material of construction of these parts.***

VTU : Mar.-02; Jan.-06, Marks 8

The various parts of transformer are :

1. **Core** : It is made up of high grade silicon steel laminations as its function is to carry the flux produced by the winding.
2. **Limb** : It is vertical portion of the core and its function is to carry the windings.
3. **Yoke** : The top and bottom horizontal portion of the core is called **yoke.** Its function is to carry the flux produced by one winding to reach to the other winding and provide the low reluctance path to the flux.
4. **Windings :** The coils used are wound on the limbs and are insulated from each other. The function of the windings is to carry the current and produce the flux necessary for the functioning of the transformer.
5. **Conservator :** The oil in the transformer expands when temperature inside the transformer increases due to heat while it contracts when the temperature decreases. The function of the conservator is to take up the expansion and contraction of the oil without allowing it to come in contact with the ambient air.
6. **Breather :** Smaller transformers are not fully filled with oil and some space remains between oil level and tank. The tank is connected to atmosphere by vent pipe. When oil expands air goes out while when oil contracts the air is taken in. The breather is a device which extracts the moisture from the air when the air is taken in and does not allow oil to come in contact with the moisture. The breathers contain the silica gel crystals which immediately absorb the atmospheric moisture.
7. **Explosion vent :** It is a bent pipe fitted on the main tank which acts as a relief valve. It uses nonmetallic diaphragm which bursts when pressure inside the transformer becomes excessive which releases the pressure and protects the transformer.
8. **Buchholz relay :** It is a safety gas operated relay connected to transformer. When the fault gets developed inside the transformer, the gases are released. The Buchholz relay is operated with these gases and trips the circuit breaker to protect the device.

5.3.1 Construction

- There are two basic parts of a transformer i) Magnetic core ii) Winding or coils.
- The core of the transformer is either square or rectangular in size. It is further divided into two parts. The vertical portion on which coils are wound is called **limb** while the top and bottom horizontal portion is called **yoke** of the core. These parts are shown in the Fig. 5.3.1 (a).

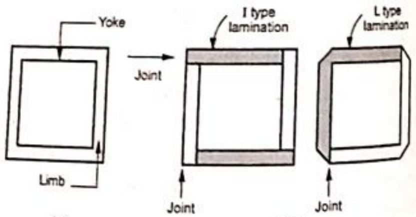


Fig. 5.3.1 Construction of transformer

- Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimized.
- Generally, high grade silicon steel laminations [0.3 to 0.5 mm thick] are used. These laminations are insulated from each other by using insulation like varnish.
- Laminations are overlapped so that to avoid the air gap at the joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the Fig. 5.3.1 (b).
- The cross-section of the limb depends on the type of coil to be used either circular or rectangular.
- The coils used are wound on the limbs and are insulated from each other. To decrease the leakage flux and to have high mutual inductance, the two windings are split into number of coils and are wound adjacent to each other on the same limb.
- The coils are made up of conducting material like copper.
- The core provides the low reluctance path to the flux produced by the primary while the windings carry the currents necessary for the functioning of the transformer.

5.4 Types of Single Phase Transformers

➤ ***With neat sketch explain the constructional details of core and shell type transformers.***

VTU : Mar.-01; July-03, 07, 08, 15; Dec.-11, Marks 8

➤ ***Mention the types of transformers.***

VTU : Jan-13, Marks 2

The various types based on the construction of single phase transformers are,

1. Core type and 2. Shell type

1. Core type transformer : It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core.

- The Fig. 5.4.1 (a) shows the schematic representation of the core type transformer while the Fig. 5.4.1 (b) shows the view of actual construction of the core type transformer.
- The coils used are of cylindrical type, wound in helical layers with different layers insulated from each other by paper or mica.
- Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil.
- Core is made up of large number of thin laminations.
- As the windings are uniformly distributed over the two limbs, the natural cooling is more effective.

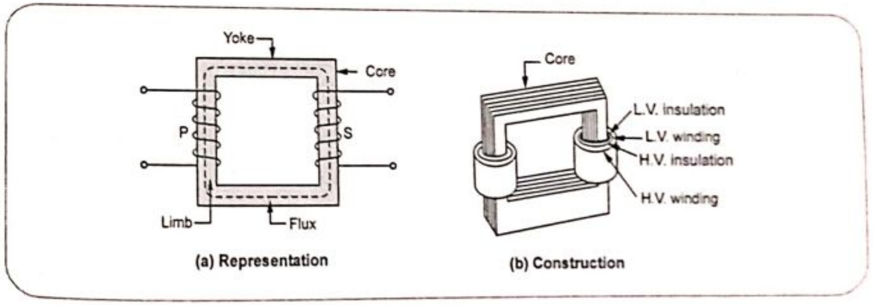


Fig. 5.4.1 Core type transformer

- The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

2. Shell type transformer : It has a double magnetic circuit. The core has three limbs.

- Both the windings are placed on the central limb. The core encircles most part of the windings.
- The coils used are generally multilayer disc type or sandwich coils.
- The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core.
- Generally for very high voltage transformers, the shell type construction is preferred.
- As the windings are surrounded by the core, the natural cooling does not exist.
- For removing any winding for maintenance, large number of laminations are required to be removed.
- The Fig. 5.4.2 (a) shows the schematic representation while the Fig. 5.4.2 (b) shows the outway view of the construction of the shell type transformer.

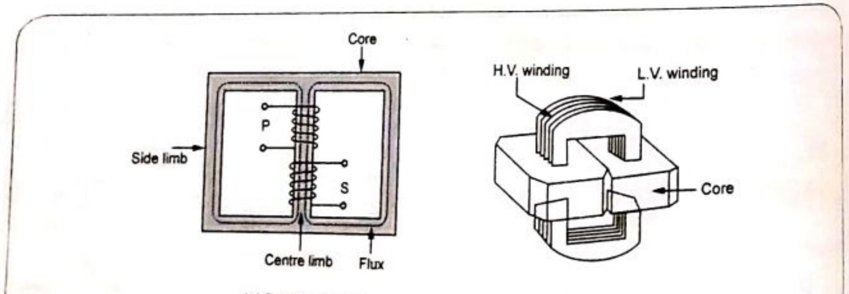
5.4.1 Comparison of Core and Shell Type Constructions

➤ ***Compare core type and shell type transformers.***

- The comparison of core type and shell type transformers is given in the Table 5.4.1.

Sr. No.	Core type	Shell type
1.	The winding encircles the core.	The core encircles most part of the windings.
2.	The cylindrical type of coils are used.	Generally, multilayer disc type or sandwich coils are used.
3.	As windings are distributed, the natural cooling is more effective.	As windings are surrounded by the core, the natural cooling does not exist.
4.	The coils can be easily removed from maintenance point of view.	For removing any winding for the maintenance, large number of laminations are required to be removed. This is difficult.
5.	The construction is preferred for low voltage transformers.	The construction is used for very high voltage transformers.
6.	It has a single magnetic circuit.	It has a double magnetic circuit.
7.	In a single phase type, the core has two limbs.	In a single phase type, the core has three limbs.

Table 5.4.1



(a) Representation **(b) Construction**

Fig. 5.4.2 Shell type transformer

5.5 E.M.F. Equation of a Transformer

➤ ***Derive the e.m.f. equation of a transformer.***

VTU : Jan. 03, 08, 14, 15,18, July-03, 04, 08, 15, 16,17, June 13, Marks 6

- When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux ϕ.
- The primary winding has N_1 number of turns. The alternating flux ϕ linking with the primary winding itself induces an e.m.f. in it denoted as E_1.
- The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f.
- The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of ϕ_m as shown in the Fig. 5.5.1.

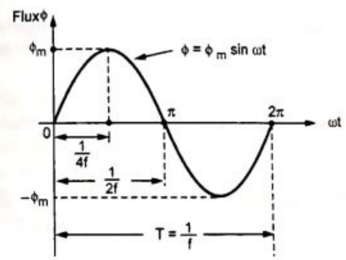


Fig. 5.5.1 Sinusoidal flux

- The various quantities which affect the magnitude of the induced e.m.f. are :

ϕ = Flux and ϕ_m = Maximum value of flux

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_1 = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

- From Faraday's law of electromagnetic induction the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

∴ Average e.m.f. per turn = Average rate of change of flux $= \frac{d\phi}{dt}$

Now, $\frac{d\phi}{dt} = \frac{\text{Change in flux}}{\text{Time required for change in flux}}$

- Consider the $1/4^{th}$ cycle of the flux as shown in the Fig. 9.5.1. Complete cycle gets completed in 1/f seconds. In $1/4^{th}$ time period, the change in flux is from 0 to ϕ_m.

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi_m - 0}{\left(\frac{1}{4f}\right)} = 4 f \phi_m \text{ Wb/sec}$$

as dt for $1/4^{th}$ time period is 1/4f seconds

∴ **Average e.m.f. per turn** $= 4 f \phi_m$ volts

- As ϕ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature.

For sinusoidal quantity,

$$\text{Form Factor} = \frac{\text{R.M.S. value}}{\text{Average value}} = 1.11$$

∴ R.M.S. value = 1.11 × Average value

∴ R.M.S. value of induced e.m.f. per turn $= 1.11 \times 4 f \phi_m = 4.44 f \phi_m$

- There are N_1 number of primary turns hence the R.M.S. value of induced e.m.f. of primary denoted as E_1 is,

$$E_1 = N_1 \times 4.44 f \phi_m \text{ volts}$$

- While as there are N_2 number of secondary turns the R.M.S. value of induced e.m.f. of secondary denoted E_2 is,

$$E_2 = N_2 \times 4.44 f \phi_m \text{ volts}$$

- The expressions of E_1 and E_2 are called e.m.f. equations of a transformer.

$$\boxed{\begin{aligned} E_1 &= 4.44 f \phi_m N_1 \text{ volts} \\ E_2 &= 4.44 f \phi_m N_2 \text{ volts} \end{aligned}} \quad \begin{aligned} &\ldots (5.5.1) \\ &\ldots (5.5.2) \end{aligned}$$

5.5.1 Concept of Ideal Transformer

➤ ***What is ideal transformer ?***

- A transformer is said to be ideal if it satisfies following properties :

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.

iv) Permeability of core is so high that negligible current is required to establish the flux in it.

5.5.2 Ratios of a Transformer

➤ ***Explain the various transformation ratios of a transformer.***

➤ ***Show that voltage ratio of the primary and secondary winding is the same as their turns ratio.*** **VTU : July-12, Marks 4**

- Consider a transformer shown in Fig. 5.5.2 indicating various voltages and currents.

1. Voltage ratio

- We know from the e.m.f. equations,

$E_1 = 4.44\, f\, \phi_m\, N_1$ and

$E_2 = 4.44\, f\, \phi_m\, N_2$

- Taking ratio of the two equations we get,

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$

- **This ratio of secondary induced e.m.f. to primary induced e.m.f. is known as voltage transformation ratio denoted as K.**

Thus, $$\boxed{E_2 = K\, E_1 \quad \text{where } K = \frac{N_2}{N_1}}$$

1. If $N_2 > N_1$ i.e. $K > 1$, we get $E_2 > E_1$ then the transformer is called **step-up transformer.**
2. If $N_2 < N_1$ i.e. $K < 1$, we get $E_2 < E_1$ then the transformer is called **step-down transformer.**
3. If $N_2 = N_1$ i.e. $K = 1$, we get $E_2 = E_1$ then the transformer is called **isolation transformer or 1:1 transformer.**

2. Current ratio

- For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $V_1\, I_1$ = Input VA

and $V_2\, I_2$ = Output VA

- For an ideal transformer, $V_1\, I_1 = V_2\, I_2$

$$\therefore \quad \boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

- **Hence the currents are in the inverse ratio of the voltage transformation ratio.**

5.5.3 Volt-Ampere Rating

➤ ***What is volt-ampere rating of a transformer ? Why is it specified in volt-ampere and not in watts ?***

- When electrical power is transferred from primary winding to secondary there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device.
- This temperature must be maintained below certain limiting value as it is always harmful from insulation point of view.
- The copper loss (I^2R) in the transformer depends on the current 'I' through the winding while the iron or core loss depends on the voltage 'V' as frequency of operation is constant.

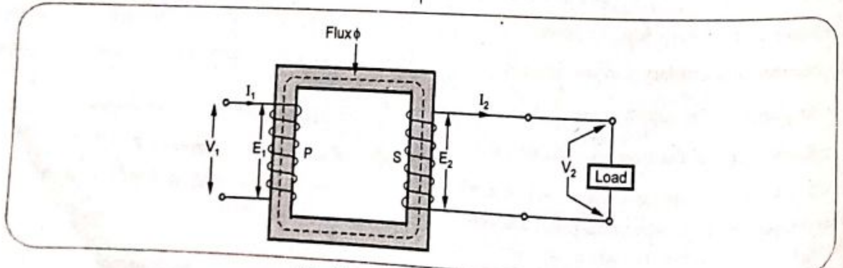


Fig. 5.5.2 Ratios of transformer

- None of these losses depend on the power factor (cos ϕ) of the load. Hence losses decide the temperature rise and hence the rating of the transformer.
- As losses depend on V and I only, the rating of the transformer is specified as a product of these two parameters V×I. Hence the transformer rating is specified as the product of voltage and current are called **VA rating.**
- On both sides, primary and secondary VA rating remains same. This rating is generally expressed in kVA (kilo volt amperes rating).

$$\boxed{\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000} \qquad \text{...1000 to express in kVA}}$$

5.5.4 Full Load Currents

➤ ***How to obtain full load currents of a transformer from its rating ?***

- If V_1 and V_2 are the terminal voltages of primary and secondary then from specified kVA rating we can decide full load currents of primary and secondary, I_1 and I_2.
- This is the **safe maximum current limit** which may carry, keeping temperature rise below its limiting value.

$$\boxed{\begin{aligned} I_1 \text{ full load} &= \frac{\text{kVA rating} \times 1000}{V_1} \\ I_2 \text{ full load} &= \frac{\text{kVA rating} \times 1000}{V_2} \end{aligned}} \qquad \text{... (1000 to convert kVA to VA)}$$

- These values indicate, how much maximum load can be connected to a given transformer of a specified kVA rating.

> The full load primary and secondary currents indicate the safe maximum values of currents which transformer windings can carry.

Ex. 5.5.1 *Find the number of turns on the primary and secondary side of a 440/230 V, 50 Hz single phase transformer, if the net area of cross section of the core is $30\ cm^2$ and the flux density is $1\ Wb/m^2$.* **VTU : June-16, Marks 6**

Sol. :

$E_1 = 440$ V, $E_2 = 230$ V, $f = 50$ Hz, $B_m = 1\ Wb/m^2$

$B_m = \frac{\phi_m}{a}$ i.e. $\phi_m = 1 \times 30 \times 10^{-4}$ Wb ... $a = 30\ cm^2$

$E_1 = 4.44\, \phi_m f N_1$ i.e. $N_1 = \frac{440}{4.44 \times 30 \times 10^{-4} \times 50} = 660.50$

$\therefore$ $N_1 \approx 661$ ...Primary number of turns

$E_2 = 4.44\, \phi_m f N_2$ i.e. $N_2 = \frac{230}{4.44 \times 30 \times 10^{-4} \times 50} = 34.534$

$\therefore$ $N_2 \approx 35$ ...Secondary number of turns

Ex. 5.5.2 *Find the number of turns on the primary and secondary side of a 440/230 V, 50 Hz single phase transformer, if the net area of cross section of the core is 30 cm² and the flux density is 1Wb/m².*

VTU : Feb.-05; July-09,16,; Aug.-05, Marks 6

Sol. : $B_m = 1\ Wb/m^2,\ a = 30\ cm^2,\ E_1 = 440\ V$

$B_m = \frac{\phi_m}{a}$ i.e. $\phi_m = 1 \times 30 \times 10^{-4}$ Wb

$E_1 = 4.44\ \phi_m\ f\ N_1$ ...f = 50 Hz

$\therefore$ $440 = 4.44 \times 30 \times 10^{-4} \times 50 \times N_1$

i.e. $N_1 = 6606.6 \approx 6607$

$\frac{E_1}{E_2} = \frac{N_1}{N_2}$ i.e. $N_2 = \frac{230 \times 6607}{440} = 3453.66$

$\therefore$ $N_2 = 3454$

Ex. 5.5.3 *A single phase. 20 kVA transformer has 1000 primary turns and 2500 secondary turns. The net cross sectional area of the core is 100 cm². When the primary winding is connected to 500 V, 50 Hz supply, calculate i) The maximum value of the flux density in the core ii) The voltage induced in the secondary winding and iii) The primary and secondary full load currents.*

VTU : June-13, 16, Marks 6

Sol. : $N_1 = 1000,\ N_2 = 2500,\ a = 100\ cm^2,\ A = 50\ Hz,\ E_1 = 500\ V.$

i) $E_1 = 4.44\ \phi_m f T_1$ i.e. $500 = 4.44 \times \phi_m \times 50 \times 1000$

$\therefore \phi_m = 2.2522$ mWb i.e. $B_m = \frac{\phi_m}{A} = \frac{2.2522 \times 10^{-3}}{100 \times 10^{-4}} = \mathbf{0.2252\ Wb/m^2}$

ii) $\frac{E_1}{E_2} = \frac{N_1}{N_2}$ i.e. $\frac{500}{E_2} = \frac{1000}{2500}$ i.e. $E_2 = \mathbf{1250\ V}$

iii) $I_1(F.L.) = \frac{VA}{E_1} = \frac{20 \times 10^3}{500} = \mathbf{40\ A}$ ... 20 kVA given

$I_2(F.L.) = \frac{VA}{E_2} = \frac{20 \times 10^3}{1250} = \mathbf{16\ A}$

Ex. 5.5.4 *A single phase transformer has 400 turns primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm². The primary winding is connected to a 500 V. 50 Hz supply. Find :*
i) Peak value of flux density.
ii) emf induced in the secondary winding.

VTU : July-15, Jan.-17, Marks 6

Sol. : $N_1 = 400,\ N_2 = 1000,\ a = 60\ cm^2,\ E_1 = 500\ V,\ f = 50\ Hz$

i) $E_1 = 4.44\ \phi_m f\ N_1$ i.e. $500 = 4.44 \times \phi_m \times 50 \times 400$

$\therefore$ $\phi_m = 5.6306$ mWb i.e. $B_m = \frac{\phi_m}{a} = \frac{5.6306 \times 10^{-3}}{60 \times 10^{-4}} = \mathbf{0.9384\ T\ i.e.\ Wb/m^2}$

ii) $\frac{E_1}{E_2} = \frac{N_1}{N_2}$ i.e. $E_2 = \frac{1000}{400} \times 500 = \mathbf{1250\ V}$

Ex. 5.5.5 *A 200 KVA, 10000V/400V, 50Hz single phase transformer has 100 turns on the secondary. Calculate : i) The primary and secondary currents*
ii) The number of primary turns
iii) The maximum value of flux.

VTU : Jan.-18, Marks 8

Ans. : $V_1 = 10000\ V,\ V_2 = 400\ V,\ N_2 = 100,$
$f = 50\ Hz,\ 200\ kVA$

i) $I_1(FL) = \frac{VA}{V_1} = \frac{200 \times 10^3}{10000} = \mathbf{20\ A}$

$I_2(FL) = \frac{VA}{V_2} = \frac{200 \times 10^3}{400} = \mathbf{500\ A}$

ii) $\frac{N_2}{N_1} = \frac{V_2}{V_1}$ i.e. $N_1 = \frac{100 \times 10000}{400} = \mathbf{2500}$

iii) $V_1 = E_1 = 4.44\ \phi_m\ f\ N_1$

$\phi_m = \frac{10000}{4.44 \times 50 \times 2500} = \mathbf{18.018\ mWb}$

5.6 Ideal Transformer on No Load

➤ ***Explain the operation of an ideal transformer on no load.***

- Consider an ideal transformer on no load as shown in the Fig. 5.6.1. The supply voltage is V_1 and as it is an no load the secondary current $I_2 = 0$.

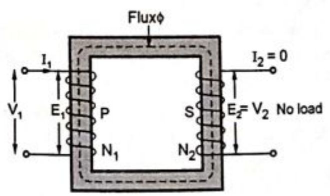


Fig. 5.6.1 Ideal transformer on no load

- The primary draws a current I_1 which is just necessary to produce flux in the core. As it is magnetizing the core, it is called **magnetizing current** denoted as I_m.
- As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature.
- The magnetizing current I_m is very small and lags V_1 by 90° as the winding is purely inductive. This I_m produces an alternating flux ϕ which is in phase with I_m.
- The flux links with both the winding producing the induced e.m.f.s E_1 and E_2, in the primary and secondary windings respectively.
- According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 is in antiphase with V_1 but equal in magnitude.
- The induced E_2 also opposes V_1 hence in antiphase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.
- The phasor diagram for the ideal transformer on no load is shown in the Fig. 5.6.2.

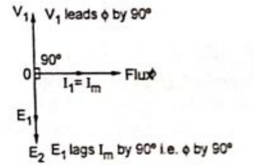


Fig. 5.6.2 Phasor diagram for ideal transformer on no load

- It can be seen that flux ϕ is reference.
- I_m produces ϕ hence in phase with ϕ. V_1 leads I_m by 90° as winding is purely inductive so current has to lag voltage by 90°.
- E_1 and E_2 are in phase and both opposing supply voltage V_1.

5.7 Practical Transformer on No Load

➤ ***Explain the operation of transformer on no load giving its vector diagram.***

VTU : Jan.-07, July-09, Marks 6

➤ ***Give reason : In transformer there is small primary current though the load is not connected to it.***

VTU : Jan.-02, 06, 07; July-05, Marks 3

- Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux.
- Practically, primary winding has certain resistance hence there are small primary copper loss present.

- Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I_0.
- Now the no load input current I_0 has two components :

1. A purely reactive component I_m called magnetizing component of no load current required to produce the flux. This is also called **wattless component**.
2. An active component I_c which supplies total losses under no load condition called **power component** of no load current. This is also called **wattful component** or **core loss component** of I_0.

- The total no load current I_0 is the vector addition of I_m and I_c.

$$\bar{I}_0 = \bar{I}_m + \bar{I}_c \quad \dots (5.7.1)$$

- In practical transformer, due to winding resistance, no load current I_0 is no longer at 90° with respect to V_1. But it lags V_1 by angle ϕ_0 which is less than 90°. Thus $\cos \phi_0$ is called **no load power factor** of practical transformer.
- The phasor diagram is shown in the Fig. 5.7.1.

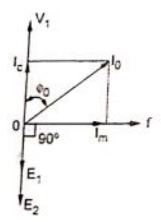


Fig. 5.7.1 Practical transformer on no load

- It can be seen that the two components of I_0 are,

$$I_m = I_0 \sin \phi_0, \text{ magnetizing component lagging } V_1 \text{ exactly by } 90° \quad \dots (5.7.2)$$

$$I_c = I_0 \cos \phi_0, \text{ core loss component which is in phase with } V_1 \quad \dots (5.7.3)$$

- The magnitude of the no load current is given by,

$$I_0 = \sqrt{I_m^2 + I_c^2} \quad \dots (5.7.4)$$

while ϕ_0 = No load primary power factor angle

- The total power input on no load is denoted as W_0 and is given by,

$$W_0 = V_1 I_0 \cos \phi_0 = V_1 I_c \quad \dots (5.7.5)$$

- It may be noted that the current I_0 is very small, about 3 to 5 % of the full load rated current. Hence the primary copper loss is negligibly small.
- Hence power input W_0 on no load always represents the iron losses, as copper loss is negligibly small. **The iron losses are denoted as P_i and are constant for all load conditions.**

$$\therefore \quad W_0 = V_1 I_0 \cos \phi_0 = P_i = \text{Iron loss} \quad \dots (5.7.6)$$

5.8 Transformer on Load (M.M.F. Balancing on Load)

> ***Explain the working of transformer on load.*** VTU : June-10, Marks 4

> ***Give reason : In transformer there is an inrush of primary current when secondary is loaded.*** VTU : Jan.-02, 06; July-05, Marks 3

- When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load.
- If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load, I_2 is in phase with V_2.
- There exists a secondary m.m.f. $N_2 I_2$ due to which secondary current sets up its own flux ϕ_2.
- This flux opposes the main flux ϕ which is produced in the core due to magnetizing component of no load current. Hence the m.m.f. $N_2 I_2$ is called **demagnitizing ampere-turns**. This is shown in the Fig. 5.8.1 (a).
- The flux ϕ_2 momentarily reduces the main flux ϕ, due to which the primary induced e.m.f. E_1 also reduces. Hence the vector difference $\bar{V}_1 - \bar{E}_1$ increases due to which **primary draws more current from the supply.**

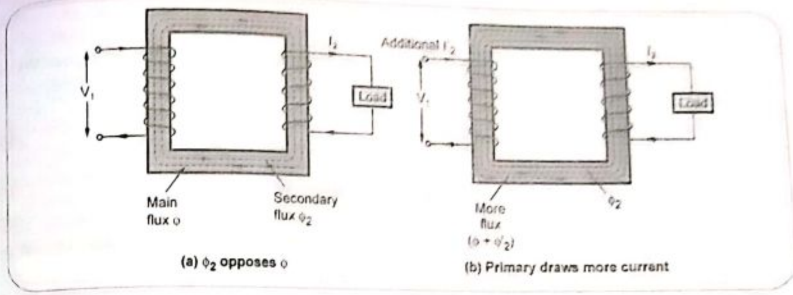


Fig. 5.8.1 Transformer on load

- This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig. 5.8.1 (b).
- This current I_2' is in antiphase with I_2. The current I_2' sets up its own flux ϕ_2' which opposes the flux ϕ_2 and helps the main flux ϕ. This flux ϕ_2' neutralizes the flux ϕ_2 produced by I_2. The m.m.f. i.e. ampere turns $N_1 I_2'$ balances the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.

- Thus for any load condition, no load to full load the flux in the core is practically constant.

The load component current I_2' always neutralizes the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called **constant flux machine**.

- As the ampere turns are balanced we can write,

$$N_2 I_2 = N_1 I_2' \quad \text{i.e.} \quad I_2' = \frac{N_2}{N_1} I_2 = K I_2 \quad \dots (5.8.1)$$

- Thus when transformer is loaded, the primary current I_1 has two components :

1. The no load current I_0 which lags V_1 by angle ϕ_0. It has two components I_m and I_c.
2. The load component I_2' which is in antiphase with I_2. And phase of I_2 is decided by the load.

- Hence primary current I_1 is vector sum of I_0 and I_2'.

$$\therefore \quad \bar{I}_1 = \bar{I}_0 + \bar{I}_2' \quad \dots (5.8.2)$$

5.9 Equivalent Resistance of Transformer

> ***How to calculate equivalent resistance referred to primary and secondary in a transformer ?***

Let R_1 = Primary winding resistance in ohms

R_2 = Secondary winding resistance in ohms

- The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer.
- The total copper loss due to both the resistances can be obtained as,

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 \left[R_1 + \frac{I_2^2}{I_1^2} R_2\right]$$

$$= I_1^2 \left[R_1 + \frac{1}{K^2} R_2\right] \quad \left(\frac{I_2}{I_1} = \frac{1}{K}\right) \quad \dots (5.9.1)$$

- Now the expression (5.9.1) indicates that the total copper loss can be expressed as $I_1^2 R_1 + I_1^2 \cdot \frac{R_2}{K^2}$. This means $\frac{R_2}{K^2}$ is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with I_2.
- This value of resistance R_2/K^2 which is the value of R_2 referred to primary is called **equivalent resistance of secondary referred to primary**. It is denoted as R_2'.

$$\therefore \quad \boxed{R_2' = \frac{R_2}{K^2}} \quad \text{... (5.9.2)}$$

- Hence the **total resistance referred to primary** is the addition of R_1 and R_2' called equivalent resistance of transformer referred to primary and denoted as R_{1e}.

$$\therefore \quad \boxed{R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}} \quad \text{... (5.9.3)}$$

- Similarly the **equivalent resistance of primary referred to secondary is,**

$$\therefore \quad R_1' = K^2 R_1 \quad \text{... (5.9.4)}$$

- Hence the total resistance referred to secondary is the addition of R_2 and R_1' called **equivalent resistance of transformer referred to secondary** and denoted as R_{2e}.

$$\therefore \quad \boxed{R_{2e} = R_2 + R_1' = R_2 + K^2 R_1} \quad \text{... (5.9.5)}$$

High voltage side → Low current side → High resistance side
Low voltage side → High current side → Low resistance side

5.10 Magnetic Leakage in a Transformer

➤ ***Explain the magnetic leakage and its effect in a transformer.***

- In practice the part of the primary flux as well as the secondary flux completes the path through air and links with the respecting winding only. Such a flux is called **leakage flux.**
- Thus there are two leakage fluxes present as shown in the Fig. 5.10.1.
- The flux ϕ_{L1} is the primary leakage flux which is produced due to primary current I_1. It is in phase with I_1 and links with primary only.
- The flux ϕ_{L2} is the secondary leakage flux which is produced due to current I_2. It is in phase with I_2 and links with the secondary winding only.

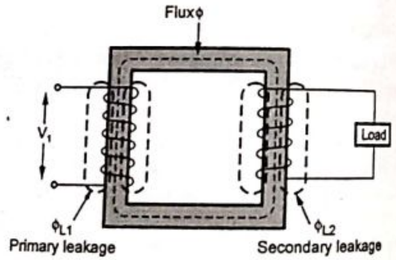


Fig. 5.10.1 Leakage fluxes

- Due to leakage flux ϕ_{L1} there is self induced e.m.f. e_{L1} in primary. While due to leakage flux ϕ_{L2} there is self induced e.m.f. e_{L2} in secondary. The primary voltage V_1 has to overcome this voltage e_{L1} to produce E_1 while induced e.m.f. E_2 has to overcome e_{L2} to produce terminal voltage V_2.
- Thus the self induced e.m.f.s are treated as the voltage drops across the fictitious reactances placed in series with the windings. These reactances are called leakage reactances of the winding.

So, $\quad X_1$ = Leakage reactance of primary winding.

and $\quad X_2$ = Leakage reactance of secondary winding.

- Leakage fluxes link with the respective windings only and not to both the windings. To reduce the leakage, as mentioned, in the construction both the winding's are placed on same limb rather than on separate limbs.

5.10.1 Equivalent Leakage Reactance

- Similar to the resistances, the leakage reactances also can be transferred from primary to secondary or viceversa. The relation through K^2 remains same for the transfer of reactances as it is studied earlier for the resistances.
- Let X_1 is leakage reactance of primary and X_2 is leakage reactance of secondary.
- Then the total leakage reactance referred to primary is X_{1e} given by,

$$\boxed{X_{1e} = X_1 + X_2' \quad \text{where} \quad X_2' = \frac{X_2}{K^2}}$$

- While the total leakage reactance referred to secondary is X_{2e} given by,

$$\boxed{X_{2e} = X_2 + X_1' \quad \text{where} \quad X_1' = K^2 X_1} \quad \text{... } K = \frac{N_2}{N_1}$$

5.11 Equivalent Impedance

- The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reactance X_2.
- Thus we can say that the total impedance of primary winding is Z_1 which is,

$$Z_1 = R_1 + j X_1 \ \Omega \quad \text{... (5.11.1)}$$

- And the total impedance of the secondary winding is Z_2 which is ,

$$Z_2 = R_2 + j X_2 \ \Omega \quad \text{... (5.11.2)}$$

- Let $\quad Z_{1e}$ = Total equivalent impedance referred to primary then,

$$\boxed{Z_{1e} = R_{1e} + j X_{1e}}$$

$$\therefore \quad Z_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2} \quad \text{... (5.11.3)}$$

- Similarly Z_{2e} = Total equivalent impedance referred to secondary then,

$$\boxed{Z_{2e} = R_{2e} + j X_{2e}}$$

$$\therefore \quad Z_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1 \quad \text{... (5.11.4)}$$

- The magnitudes of Z_{1e} and Z_{2e} are,

$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2}$$

and $$Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} \quad \text{... (5.11.5)}$$

- It can be noted that,

$$\boxed{Z_{2e} = K^2 Z_{1e} \quad \text{and} \quad Z_{1e} = \frac{Z_{2e}}{K^2}} \quad \text{... (5.11.6)}$$

Ex. 5.11.1 *A 220/110 V, 50 Hz, 1.5 kVA transformer has primary and secondary winding resistances of 1 Ω and 2 Ω while reactances of 3 Ω and 5 Ω respectively. Find the total resistance, equivalent reactance and equivalent impedance referred to primary and secondary.*

Sol. : $R_1 = 1\ \Omega,\ R_2 = 2\ \Omega,\ X_1 = 3\ \Omega,\ X_2 = 5\ \Omega$

$$K = \frac{V_2}{V_1} = \frac{110}{220} = 0.5$$

$$\therefore \quad R_{1e} = R_1 + R_2'$$

$$= R_1 + \frac{R_2}{K^2} = 1 + \frac{2}{(0.5)^2} = 9\ \Omega$$

$$\therefore \quad X_{1e} = X_1 + X_2'$$

$$= X_1 + \frac{X_2}{K^2} = 3 + \frac{5}{(0.5)^2} = 23\ \Omega$$

$$\therefore \quad Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} = \sqrt{9^2 + 23^2}$$

$$= 24.6981\ \Omega$$

$$\therefore \quad R_{2e} = K^2 R_{1e} = 2.25\ \Omega,$$

$$X_{2e} = K^2 X_{1e} = 5.75\ \Omega$$

$$\therefore \quad Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} = K^2 Z_{1e}$$

$$= 6.1745\ \Omega$$

5.12 Voltage Regulation of Transformer

➢ ***What is the regulation of transformer. State its importance.***

VTU : Jan.-04, 11, 16; July-04, 09, Marks 6

- Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load value (E_2) to load value (V_2) as load and load current increases.
- This decrease in the secondary terminal voltage expressed as a fraction of the no load secondary terminal voltage is called regulation of a transformer.
- The regulation is defined as change in the magnitude of the secondary terminal voltage, when full load i.e. rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage.
- Let E_2 = Secondary terminal voltage on no load

  V_2 = Secondary terminal voltage on given load

then mathematically voltage regulation at given load can be expressed as,

$$\boxed{\%\ \text{Voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100}$$

- The ratio $((E_2 - V_2)/V_2)$ is called **per unit regulation.**
- The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If V_2 is determined for full load and specified power factor condition the regulation is called full load regulation.
- The regulation depends on the power factor of the load.
- In case of lagging power factor $V_2 < E_2$ and we get positive voltage regulation, while for leading power factor $E_2 < V_2$ and we get negative voltage regulation. This is shown in the Fig. 5.12.1.

- The voltage drop should be as small as possible hence less the regulation better is the performance of a transformer.

5.12.1 Expression for Voltage Regulation

➢ ***State the expression for calculating the regulation of a transformer.***

- The voltage regulation is defined as,

$$\% R = \frac{E_2 - V_2}{V_2} \times 100$$

$$= \frac{\text{Total voltage drop}}{V_2} \times 100$$

- The regulation can be expressed as,

$$\boxed{\% R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100}$$

where I_2 = Secondary current on load

V_2 = Secondary voltage on load

R_{2e} = Equivalent resistance referred to secondary

X_{2e} = Equivalent reactance referred to secondary

$\cos\phi$ = Load power factor

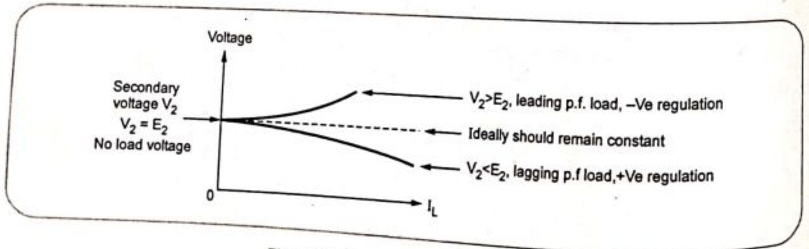


Fig. 5.12.1 Regulation characteristics

∴ **+ sign for lagging power factors while – sign for leading power factor loads.**

- The regulation can be further expressed interms of I_1, V_1, R_{1e} and X_{1e} as,

∴ $$\boxed{\% R = \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100}$$

5.13 Losses in a Transformer

➢ ***Explain the various losses in a transformer and how to minimize them ? On what factors they depend ? Give the equations for these losses.***

VTU : Feb.-05; Jan.-07, 08, 09, 10, 11, 16,18,; July-08, 11, 16, Marks 6

- In a transformer, there exists two types of losses.

i) The core gets subjected to an alternating flux, causing **core losses.**

ii) The windings carry currents when transformer is loaded, causing **copper losses.**

1. Core or Iron losses

- Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, $\boxed{\text{Hysteresis loss} = K_h B_m^{1.67} f\, v \ \text{watts}}$

K_h = Hysteresis constant depends on material and B_m = Maximum flux density

f = Frequency and v = Volume of the core.

- The induced e.m.f. in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by,

$$\boxed{\text{Eddy current loss} = K_e B_m^2 f^2 t^2 \ \text{watts/unit volume}}$$

where K_e = Eddy current constant and t = Thickness of the core.

- As seen earlier, the flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence, the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called **constant losses.** The iron losses are denoted as P_i.
- The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loop and by manufacturing the core in the form of laminations.

2. Copper losses

- The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.
- Total Cu loss = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1') = I_1^2 R_{1e} = I_2^2 R_{2e}$.
- The copper losses are denoted as P_{Cu}. If the current through the windings is full load current, we get copper losses at full load. If the load on transformer is half then we get copper losses at half load which are less than full load copper losses. Thus copper losses are called **variable losses.**

- Copper losses are proportional to the square of the current and square of the kVA rating as voltage is constant.

So, $\boxed{P_{Cu} \propto I^2 \propto (kVA)^2}$

- Thus for a transformer,

$\boxed{\text{Total losses} = \text{Iron losses} + \text{Copper losses} = P_i + P_{Cu}}$

- The copper losses are kept minimum by designing the windings with low resistance values.

5.14 Efficiency of a Transformer

> ***Define efficiency of a transformer. How to obtain efficiency at different loads ?*** **VTU : Jan.-03; July-05, Marks 6**

- Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

$\therefore$ Power output = Power input – Total losses

$\therefore$ Power input = Power output + Total losses = Power output + $P_i + P_{Cu}$.

- The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expressed as,

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power output}}{\text{Power output} + P_i + P_{Cu}}$$

Now power output = $V_2 I_2 \cos\phi$ where $\cos\phi$ = Load power factor.

- The transformer supplies full load of current I_2 and with terminal voltage V_2.

P_{Cu} = Copper losses on full load = $I_2^2 R_{2e}$

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{Cu}}$$

But $V_2 I_2$ = VA rating of a transformer

$$\therefore \quad \boxed{\% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + P_{Cu}} \times 100}$$

- This is full load percentage efficiency with I_2 = Full load secondary current.
- But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.
- Let $\boxed{n = \text{Fraction by which load is less than full load} = \frac{\text{Actual load}}{\text{Full load}}}$
- For example, if transformer is subjected to half load then, $n = \frac{\text{Half load}}{\text{Full load}} = \frac{(1/2)}{1} = 0.5$
- When load changes, the load current changes by same proportion.

$\therefore$ $\boxed{\text{New } I_2 = n\,(I_2)\text{F.L.}}$

- Similarly the output $V_2 I_2 \cos\phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

- Similarly as copper losses are proportional to square of current then,

New $P_{Cu} = n^2 (P_{Cu})$F.L.

- **The copper losses get reduced by n^2 while iron losses remain same.**
- In general for fractional load the efficiency is given by,

$$\boxed{\% \eta = \frac{n\,(\text{VA rating}) \cos\phi}{n\,(\text{VA rating}) \cos\phi + P_i + n^2 (P_{Cu})\text{F.L.}} \times 100}$$

where n = Fraction by which load is less than full load.

- For all types of load power factors lagging, leading and unity the efficiency expression does not change and remains same.

5.14.1 Condition for Maximum Efficiency

> ***Derive the condition for the maximum efficiency for a transformer.*** **VTU : Mar.-01; Jan.-03, 14, 15, 16, July-17, Marks 6**

- The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_{max}.
- The efficiency is a function of load i.e. load current I_2 assuming $\cos\phi_2$ constant. The secondary terminal voltage V_2 is also assumed constant.
- So for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0 \quad \text{while} \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e})(V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2)(V_2 \cos\phi_2 + 2 I_2 R_{2e}) = 0$$

- Cancelling $(V_2 \cos\phi_2)$ from both the terms we get,

$$V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{2e} = 0 \quad \text{i.e.} \quad P_i - I_2^2 R_{2e} = 0$$

$$\therefore \quad \boxed{P_i = I_2^2 R_{2e} = P_{Cu}}$$

So condition to achieve maximum efficiency is that,

$\boxed{\text{Copper losses} = \text{Iron losses} \quad \text{i.e.} \quad P_i = P_{Cu}}$

5.14.2 Load Current I_{2m} at Maximum Efficiency

- For η_{max}, $I_2^2 R_{2e} = P_i$ but $I_2 = I_{2m}$ i.e. $I_{2m}^2 R_{2e} = P_i$

$$\therefore \quad I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$ ... This is the load current at η_{max}

- Let (I_2)F.L. = Full load current,

$$\therefore \quad \frac{I_{2m}}{(I_2)\text{F.L.}} = \frac{1}{(I_2)\text{F.L.}}\sqrt{\frac{P_i}{R_{2e}}} \quad \text{i.e.} \quad \frac{I_{2m}}{(I_2)\text{F.L.}} = \sqrt{\frac{P_i}{[(I_2)\text{F.L.}]^2 R_{2e}}} = \sqrt{\frac{P_i}{(P_{Cu})\text{F.L.}}}$$

$$\therefore \quad \boxed{I_{2m} = (I_2)_{F.L.} \sqrt{\frac{P_i}{(P_{Cu})_{F.L.}}}}$$

- This is the load current at η_{max} interms of full load current.

5.14.3 kVA Supplied at Maximum Efficiency

- For constant V_2 the kVA supplied is function of load current.

$$\therefore \quad \text{kVA at } \eta_{max} = I_{2m} V_2 = V_2 (I_2)_{F.L.} \times \sqrt{\frac{P_i}{(P_{Cu})_{F.L.}}} = (\text{kVA rating}) \times \sqrt{\frac{P_i}{(P_{Cu})_{F.L.}}}$$

- Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as,

$$\boxed{\% \eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} \times 100 \quad \text{as } P_{Cu} = P_i}$$

Ex. 5.14.1 *A transformer is rated at 100 kVA. At full load its copper loss is 1200 W and its iron loss is 960 W. Calculate :*
i) The efficiency at full load, unity power factor ii) The efficiency at half load, 0.8 p.f.
iii) The load kVA at which maximum efficiency will occur
iv) Maximum efficiency at 0.85 p.f.

VTU : June-12, Jan.-14, Marks 8

Sol. : $P_{cu}(FL) = 1200$ W, $P_i = 960$ W, 100 kVA

i) $$\% \eta_{FL} = \frac{VA \cos\phi}{VA \cos\phi + P_i + P_{cu}(FL)} \times 100 = \frac{100\times10^3\times1}{100\times10^3\times1+960+1200}\times100$$

$$= \mathbf{97.88\ \%} \qquad \ldots \cos\phi = 1$$

ii) $$\% \eta_{HL} = \frac{nVA \cos\phi}{nVA \cos\phi + P_i + [n^2 P_{cu}(FL)]} \times 100 \qquad \ldots n = 0.5, \cos\phi = 0.8$$

$$= \frac{0.5\times100\times10^3\times0.8}{0.5\times100\times10^3\times0.8+960+[0.5^2\times1200]}\times100 = \mathbf{96.946\ \%}$$

iii) $$\text{kVA for } \eta_{max} = \text{kVA}\sqrt{\frac{P_i}{P_{cu}(FL)}} = 100\sqrt{\frac{960}{1200}} = \mathbf{89.4427\ kVA}$$

iv) For η_{max}, $P_{cu} = P_i = 960$ W, $\cos\phi = 0.85$

$$\therefore \quad \% \eta_{max} = \frac{(VA \text{ for } \eta_{max}) \cos\phi}{(VA \text{ for } \eta_{max}) \cos\phi + 2P_i}\times100 = \frac{89.4427\times10^3\times0.85}{89.4427\times10^3\times0.85+2\times960}\times100 = \mathbf{97.536\ \%}$$

Ex. 5.14.2 *The maximum efficiency at full load and unity power factor of a single-phase 25 kVA, 500/1000 V, 50 Hz, transformer is 98 %. Determine its efficiency at,*
i) 75 % load, 0.9 p.f. and ii) 50 % load, 0.8 p.f. iii) 25 % load, 0.6 p.f.

VTU : Feb.-06; Aug.-11; July-15, Jan.-16, Marks 8

Sol. : $\eta_{max} = 98$ % at full load, $\cos\phi = 1$, S = 25 kVA

$$\% \eta_{max} = \frac{VA \cos\phi}{VA\cos\phi + 2P_i}\times100 \qquad \ldots P_i = [P_{Cu}]_{F.L.} \text{ for } \eta_{max}$$

$$\therefore \quad 0.98 = \frac{25\times10^3\times1}{25\times10^3\times1+2P_i} \quad \text{i.e.} \quad P_i = 255.102 \text{ W} = [P_{Cu}]_{F.L.}$$

i) n = 0.75 for 75 % load, $\cos\phi = 0.9$

$$\therefore \quad \%\eta = \frac{n\,VA\cos\phi}{n\,VA\cos\phi + P_i + n^2[P_{Cu}]_{F.L.}}\times100$$

$$= \frac{0.75\times25\times10^3\times0.9}{0.75\times25\times10^3\times0.9+255.102+(0.75)^2\times255.102}\times100 = \mathbf{97.69\ \%}$$

ii) n = 0.5 for 50 % load, $\cos\phi = 0.8$

$$\therefore \quad \%\eta = \frac{0.5\times25\times10^3\times0.8}{0.5\times25\times10^3\times0.8+255.102+(0.5)^2\times255.102}\times100 = \mathbf{96.909\ \%}$$

iii) n = 0.25, $\cos\phi = 0.6$

$$\%\eta = \frac{0.25\times25\times10^3\times0.6}{0.25\times25\times10^3\times0.6+255.102+[(0.25)^2\times255.102]}\times100$$

$$= 93.26\ \%$$

Ex. 5.14.3 *A 600 kVA transformer has an efficiency of 92 % at full load, unity p.f. and at half load, 0.9 p.f. Determine its efficiency at 75 % of full load and 0.9 p.f.*

VTU : Jan.-03, 15, 16, June-10, Marks 6

Sol. : S = 600 kVA , η = 92 % on full load and half load

On F.L., $$\%\eta = \frac{VA\times\cos\phi}{VA\times\cos\phi+[P_{Cu}]_{F.L.}+P_i}\times100 \qquad \ldots \cos\phi = 1 \text{ given}$$

$$\therefore \quad 92 = \frac{600\times10^3\times1}{600\times10^3\times1+[P_{Cu}]_{F.L.}+P_i}\times100$$

$$\therefore \quad [P_{Cu}]_{F.L.}+P_i = 52173.913 \qquad \ldots (1)$$

On H.L., $$\%\eta = \frac{n\times VA\cos\phi}{n\times VA\cos\phi+n^2[P_{Cu}]_{F.L.}+P_i}\times100 \qquad \ldots n = 0.5, \cos\phi = 0.9$$

$$\therefore \quad 92 = \frac{0.5\times600\times10^3\times0.9}{0.5\times600\times10^3\times0.9+(0.5)^2[P_{Cu}]_{F.L.}+P_i}\times100$$

$$\therefore \quad 0.25[P_{Cu}]_{F.L.}+P_i = 23478.26087 \qquad \ldots (2)$$

Subtracting equation (2) from equation (1),

$$0.75[P_{Cu}]_{F.L.} = 28695.652$$

$$\therefore \quad [P_{Cu}]_{F.L.} = 38260.86 \text{ W} \qquad \ldots \text{Full load copper loss}$$

and $$P_i = 13913.04 \text{ W} \qquad \ldots \text{Iron loss}$$

Now for 75 % of full load, n = 0.75 and cos ϕ = 0.9,

$$\%\eta = \frac{n\,VA\cos\phi}{n\,VA\cos\phi + n^2[P_{Cu}]F.L. + P_i} \times 100$$

$$= \frac{0.75\times 600\times 10^3\times 0.9}{0.75\times 600\times 10^3\times 0.9 + [(0.75)^2\times 38260.86] + 13913.04}\times 100 = \mathbf{91.9545\ \%}$$

Ex. 5.14.4 *Find the efficiency of 150 kVA, single phase transformer at i) Full load upf; ii) 50 % of full load at 0.8 p.f. If the copper loss at full load is 1600 watts and iron loss is 1400 watts.* VTU : Jan.-07, 09,11,17, Marks 5

Sol. :

$$P_{cu}\,(FL) = 1600\ W,\ P_i = 1400\ W,\ 150\ kVA$$

i) Full load, cos ϕ = 1

$$\%\,\eta_{FL} = \frac{VA\cos\phi}{VA\cos\phi + P_i + P_{cu}(FL)}\times 100 = \frac{150\times 10^3\times 1}{150\times 10^3\times 1 + 1600 + 1400}\times 100 = 98.04\%$$

ii) 50% of full load, n = 0.5, cos ϕ = 0.8

$$\therefore \quad \%\,\eta_{HL} = \frac{nVA\cos\phi}{n\,VA\cos\phi + P_i + [n^2 P_{cu}(FL)]}\times 100$$

$$= \frac{0.5\times 150\times 10^3\times 0.8}{0.5\times 150\times 10^3\times 0.8 + 1400 + [0.5^2\times 1600]}\times 100 = \mathbf{97.08\%}$$

Ex. 5.14.5 *A single phase 25 kVA, 1000/2000 V, 50 Hz transformer has maximum efficiency of 98 % at full load u.p.f. Determine its efficiency at :*
i) 3/4 full load u.p.f. ii) 1/2 full load 0.8 p.f iii) 1.25 full load 0.9 p.f. VTU : July-07, Marks 9; Feb.-08, Jan.-18

Sol. : Refer similar Ex. 5.14.2 for the procedure and verify the answers as :

i) **97.918 %** ii) **96.909 %** iii) **97.728 %**

Ex. 5.14.6 *A 10 kVA, 1 ϕ transformer has a primary winding of 300 turns and secondary winding of 750 turns, cross sectional area of core is 64 cm^2. If primary voltagae is 440 V at 50 Hz find maximum flux density in the core, emf induced in secondary of transformer. At 0.8 lag p.f., calculate the efficiency of transformer if full load copper loss is 400 W and iron-loss is 200 W.* VTU : July-06,17, Jan.-13, Marks 5

Ans. : 10 kVA, N_1 = 300, N_2 = 750, A = 64 cm^2, E_1 = 440 V, P_{cu} (FL) = 400 W, P_i = 200 W, cos ϕ = 0.8, f = 50 Hz.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{i.e.} \quad \frac{440}{E_2} = \frac{300}{750} \quad \text{i.e.}\ E_2 = \mathbf{1100\ V}$$

$$E_1 = 4.44\,\phi_m f N_1 \quad \text{i.e.}\ 440 = 4.44\ \phi_m\times 50\times 300$$

$$\therefore \quad \phi_m = 6.606\ mWb$$

$$\therefore \quad B_m = \frac{\phi_m}{A} = \frac{6.606\times 10^{-3}}{64\times 10^{-4}}$$

$$= \mathbf{1.032\ Wb/m^2}$$

$$\%\,\eta_{FL} = \frac{VA\cos\phi}{VA\cos\phi + P_{cu}(FL) + P_i}\times 100$$

$$= \frac{10\times 10^3\times 0.8}{10\times 10^3\times 0.8 + 400 + 200}\times 100$$

$$= 93.023\ \%$$

Ex. 5.14.7 *A 25 kVA transformer has an efficiency of 94 % at full load unity p.f. and at half full load, 0.9 p.f. Determine the iron loss and full load copper loss.* VTU : Feb.-09, 10, Marks 6

Solution : Refer Ex. 5.14.3 for the procedure and verify the answers as

$\therefore$ (P_{Cu})F.L. = **1170.2126 W**, P_i = **425.5319 W**

Ex. 5.14.8 *A 50 kVA, 400/200 V, single phase transformer has an efficiency of 98 % at full-load and 0.8 p.f., while its efficiency is 96.9 % at 25 % of full-load and unity p.f. Determine the iron and full load Cu-losses and voltage regulation, if the terminal voltage on full-load if 195 V.* VTU : Dec.-11, Marks 6

Sol. : % η_{FL} = 98 %, cos ϕ = 0.8, % η_L = 96.9 %, cos ϕ = 1

$$\%\,\eta_{FL} = \frac{VA\cos\phi}{VA\cos\phi + P_i + (P_{Cu})FL}\times 100 \quad \text{i.e.}\ 0.98$$

$$= \frac{50\times 10^3\times 0.8}{50\times 10^3\times 0.8 + P_i + (P_{Cu})FL}$$

$$\therefore \quad P_i + (P_{Cu})\ FL = 816.3265 \qquad \dots (1)$$

For 25 % of full load, n = 0.25

$$\therefore \%\,\eta_L = \frac{nVA\cos\phi}{nVA\cos\phi + P_i + n^2(P_{Cu})FL}\times 100 \quad \text{i.e.}\ 0.969$$

$$= \frac{0.25\times 50\times 10^3\times 1}{0.25\times 50\times 10^3\times 1 + P_i + 0.25^2(P_{Cu})FL}$$

$$\therefore P_i + 0.25^2\ (P_{Cu})\ FL = 399.8968 \qquad \dots (2)$$

Solving, P_i = Iron loss = **372.1348 W**

(P_{Cu}) FL = Full load copper loss = **444.191 W**

V_{NL} = 200 V, V_{FL} = 195 V

$$\therefore \quad \%\,R = \frac{V_{NL}-V_{FL}}{V_{FL}}\times 100 = \frac{200-195}{195}\times 100$$

$$= \mathbf{2.564\ \%}$$

Ex. 5.14.9 *In a 25 kVA, 2000 / 200 V single phase transformer, the iron and full load copper losses are 350 watts and 400 watts respectively. Calculate the efficiency at unity power factor on i) Full load ii) Half full load.* VTU : Jan.-15, Marks 8

Sol. : Refer example 5.14.1 for the procedure and verify the answers as :

i) **97.08 %** and ii) **96.525 %**

5.15 University Questions with Answers

Jan. - 2010

Q.1 *Explain the various losses in a transformer and how to minimize them ? On what factors they depend ? Give the equations for these losses.* **[Refer section 5.13]** **[6]**

June - 2010

Q.2 *Explain the working of transformer on load.* **[Refer section 5.8]** **[4]**

Jan. - 2011

Q.3 *What is the regulation of transformer. State its importance.* **[Refer section 5.12]** **[6]**

Q.4 *Explain the various losses in a transformer and how to minimize them ? On what factors they depend ? Give the equations for these losses.* **[Refer section 5.13]** **[6]**

July - 2011

Q.5 *Explain the various losses in a transformer and how to minimize them ? On what factors they depend ? Give the equations for these losses.* **[Refer section 5.13]** **[6]**

Dec. - 2011

Q.6 *Explain the principle of operation of single phase transformer.* **[Refer section 5.2]** [4]

Q.7 *With neat sketch explain the constructional details of core and shell type transformers.* **[Refer section 5.4]** [8]

June - 2012

Q.8 *Explain the principle of operation of single phase transformer.* **[Refer section 5.2]** [4]

Q.9 *Show that voltage ratio of the primary and secondary winding is the same as their turns ratio.* **[Refer section 5.5]** [4]

Jan. - 2013

Q.10 *Explain the principle of operation of single phase transformer.* **[Refer section 5.2]** [4]

Q.11 *Mention the types of transformers.* **[Refer section 5.4]** [2]

June - 2013

Q.12 *Explain the principle of operation of single phase transformer.* **[Refer section 5.2]** [4]

Q.13 *Derive the e.m.f. equation of a transformer.* **[Refer section 5.5]** [6]

Jan. - 2014

Q.14 *Derive the e.m.f. equation of a transformer.* **[Refer section 5.5]** [6]

Q.15 *Derive the condition for the maximum efficiency for a transformer.* **[Refer section 5.14.1]** [4]

Jan. - 2015

Q.16 *Derive the e.m.f. equation of a transformer.* **[Refer section 5.5]** [6]

Q.17 *Derive the condition for the maximum efficiency for a transformer.* **[Refer section 5.14.1]** [4]

July - 2015

Q.18 *Explain the principle of operation of single phase transformer.* **[Refer section 5.2]** [4]

Q.19 *With neat sketch explain the constructional details of core and shell type transformers.* **[Refer section 5.4]** [8]

CBCS Scheme

Jan. - 2016

Q.20 *Explain various losses in transformer. How these losses can be minimized ?* **[Refer section 5.13]** [5]

Q.21 *Define the voltage regulation of a transformer. What is its importance ?* **[Refer section 5.12]** [4]

Q.22 *Derive the condition for which the efficiency of a transformer is maximum.* **[Refer section 5.14]** [6]

July - 2016

Q.23 *Derive the EMF equation of a transformer.* **[Refer section 5.5]** [5]

Q.24 *Explain the different losses occurring in a transformer.* **[Refer section 5.13]** [5]

Jan. - 2017

Q.25 *Explain the working principle of single phase transformer.* **[Refer section 5.2]** [5]

July - 2017

Q.26 *Derive the condition for maximum efficiency of a transformer.* **(Refer section 5.14.1)** [5]

Q.27 *Derive emf equation of a transformer.* **(Refer section 5.5)** [5]

Jan. - 2018

Q.28 *Explain the various power losses in a transformer.* **(Refer section 5.13)** [4]

Q.29 *Derive an expression for the emf induced in the secondary winding of a transformer.* **(Refer section 5.5)** [6]

MODULE - 3

6

Domestic Wiring

Syllabus

Service mains, meter board and distribution board. Brief discussion on concealed conduit wiring. Two - way and three - way control. Elementary discussion on Circuit protective devices : fuse and Miniature Circuit Breaker (MCB's). Electric shock, precautions against shock. Earthing : Pipe and Plate earthing

Contents

6.1 Domestic Wiring Installation

➤ ***Draw and explain domestic consumer panel wiring scheme.***

- The a.c. supply is supplied by electric companies to the consumer.
- The supply is brought to local distribution stations from the generating stations where its voltage level is reduced to 400 V between the lines and 230 V between a line and a neutral.
- The small cables used between the distribution stations and consumer premises are called **service mains.**
- The supply is given first to a **meter board** which consists of energy meter, service cable, sealing end box, bus bar arrangment, service fuse and neutral link. All this is the property of supply company and its installation and maintenance is sole responsibility of supply company.
- From meter board, the supply is taken to main switch board where consumer's fuse is installed alongwith the main switch.
- Then supply is given to the **main distribution board** from where it is given to the number of different subcircuits. Now a days instead of fuse, Miniature Circuit Breakers (MCB) are used.
- The various loads such as fans, tube lights and other electric appliances are connected in parallel across the subcircuit.
- The 3 pin, 5A socket outlets are used for all light and fan subcircuits while 3 pin, 15 A socket outlets are used for all the power circuits which are used to provide supply to the loads like mixers, electric ovens, refrigerators etc.
- The Fig. 6.1.1 shows a typical layout of domestic consumer panel wiring scheme.
- Thus the domestic wiring installation consists of distribution of electrical energy from meter board to the various subcircuits connected to electrical appliances through protective devices and network of wires.

6.2 Factors Affecting the Selection of Wiring Method

➤ ***Describe the factors affecting the selection of wiring method.*** VTU : Jan. 08, Marks 5

- Depending upon the power, current and voltage ratings of various appliances, it is necessary to select proper type of wiring scheme for the domestic purpose.
- The method of wiring to be adopted depends on various factors such as :

a) Durability : The wiring scheme selected must be sufficiently durable.

b) Safety : As there is a danger of loosing life in case of electric shocks, the safety must be observed strictly while choosing wiring scheme. The wiring should be fully shock proof and leakage proof.

c) Appearance : Due to wiring scheme the beauty of the house and premises should not get spoiled.

d) Cost : The funds made available by the consumer should be considered before recommending particular type of wiring scheme.

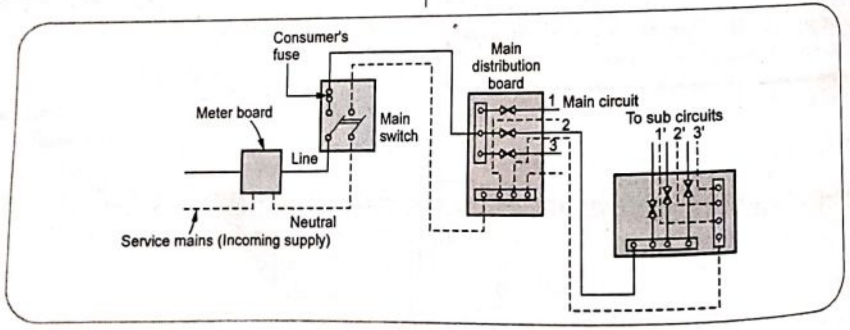


Fig. 6.1.1 Domestic consumer panel wiring scheme

e) Maintenance : The maintenance of the wiring should be as minimum as possible. There should be scope for further extension of the wiring. Renewal of the wiring should be easily possible. The fuses, plugs should be easily accessible, if at all maintenance is required.

6.3 Types of Wiring Systems

➤ ***Mention and describe in brief various types of wiring systems.*** VTU : Jan.-03, 06, 08, 10, 11; July-03, 04, 06, 07, 08; Feb.-05, June-12, 13, Marks 5

- Depending upon the various factors various types of wiring used in practice are:

1. Cleat wiring 2. Casing capping
3. Surface wiring 4. Conduit wiring
5. Metal sheathed wiring

6.3.1 Cleat Wiring

- In this type wires are clamped between porcelain cleats.
- The cleats are made up of two halves.
- One half is grooved through which wire passes while the other fits over the first.
- The whole assembly is then mounted on the wall or wooden beam with the help of screws.
- The lower half of the porcelain cleat is known as base having grooves for conductors while the upper half is known as cap which is shown in the Fig. 6.3.1.

Advantages :

1. This method is one of the cheapest method. 2. It is most suitable for temporary work.
3. It can be very quickly installed.

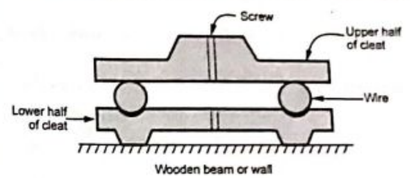


Fig. 6.3.1 Cleat wiring

4. It can be recovered without any damage of material.
5. Inspection and changes can be made very easily.
9. Erection does not require skilled labour.

Disadvantages :

1. This method does not give attractive appearance.
2. After some time due to sagging at some places, it looks shabby.
3. Dust and dirt collects on the cleats.
4. The wires are directly exposed to atmospheric conditions like moisture, chemical fumes etc. hence dangerous.
5. Maintenance cost is very high.

6.3.2 Casing Capping

- In this method, casing is a rectangular strip made from teak wood or now a days made up of P.V.C.
- It has two grooves into which the wires are laid.
- Then casing is covered with a rectangular strip of wood or P.V.C. of the same width, called **capping.** The capping is screwed into **casing** by means of screws fixed at every 15 cm.
- The casing is fixed to the walls and apart from it by 3.5 mm with the help of porcelain discs or cleats.
- This type of wiring is normally adopted for voltages upto 250 V.

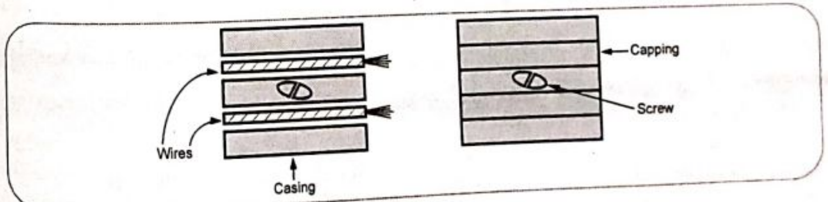


Fig. 6.3.2 Casing capping

- It is most commonly used in domestic wiring installations.

Advantages :

1. Good protection to the conductors from dangerous atmospheric conditions.
2. Neat and clean appearance.
3. Its installation is easy compared to some other methods of wiring.

Disadvantages :

1. In case of wooden casing capping, there is high risk of fire.
2. The requirement of skilled labour for the installation.
3. The method is costly.

6.3.3 Surface Wiring

- In this type, the wooden battens are fixed on the surface of the wall, by means of screws and rawl plugs.
- The metal clips are provided with the battens at regular intervals.

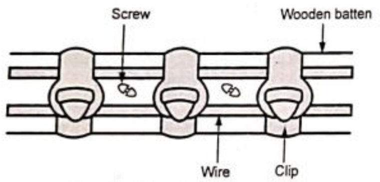


Fig. 6.3.3 Wooden batten wiring

- The wire runs on the batten and is clamped on the batten using the metal clips.
- If the wire used is tough rubber sheathed then it is called **T.R.S. wiring** while if the wire used is cab tyre sheathed then it is called **C.T.S. wiring**.
- This type of wiring is cheap and can be used in residential wiring. .

Advantages :

1. A good appearance. 2. Simple to erect.
3. Cheaper in cost.
4. Provides protection against fire, high temperature etc.

Disadvantages :

1. Wires are always exposed to atmosphere and hence subjected to dust, dirt, fumes and other dangerous atmospheric conditions.
2. Bending and breaking of batten and hence of the wires may occur after some time.

6.3.4 Conduit Wiring

- In this method, metallic tubes called as conduits are used to run the wires.
- It gives full mechanical protection to the wires. This is most desirable for workshops and public buildings.
- Depending on whether the conduits are laid inside the walls or supported on the walls, there are two types of conduitwiring which are :
  1) Surface conduit wiring.
  2) Concealed conduit wiring.
- The conduits are made up of mild steel which is annealed so that it can be bent without breaking. The standard length of conduit is generally 4 m.
- The conduits are threaded at both ends with one coupler attached. Based on the outer diameter, various sizes of conduits from 12 mm to 63 mm are available. The conduits are supplied with black enamel coating on its internal and external surface.
- The conduits are to be erected completely before laying any cable in it. The rigid conduits are always terminated at outlets into a box which may be round, square or octagonal.
- These boxes are used to provide connections for lights, fans, heaters etc. and are called outlet boxes.
- Inspection boxes are used to facilitate the pulling of conductors while junction boxes are used to house the junctions of the conductors.

Advantages :

1. The beauty of the premises is maintained due to conduit wiring.
2. It is durable.
3. It has a long life.
4. It requires very less maintenance.
5. It protects the wires from mechanical shocks, moisture and fire hazards.

6. Proper earthing of conduits makes the method electrical shock proof.

Disadvantages :

1. The repairs are very difficult in case of concealed conduit wiring.
2. This method is most costly. 3. The erection requires highly skilled labour.
4. In concealed conduit wiring, keeping conduit at earth potential is must. Without proper earthing there is danger of electric shocks.
5. If the manufacturing of conduits is not proper then sharp edges of the metal conduits can cause damage of the insulation of the wires.

6.3.5 Metal Sheathed Wiring

- In this type of wiring, Vulcanised India Rubber (V.I.R.) conductors covered with lead alloy sheath are used. It is similar to C.T.S. or T.R.S. wiring.
- The insulated conductor is covered with a metal sheath which protects the wiring system from mechanical injury and atmospheric conditions.
- The wires with metal sheath on it are run on the wooden batten which is fixed on the wall with the help of screws. The wire is clamped on the batten using metal clips.
- This type of wiring is suitable for places which are exposed to sun or rain but unsuitable for corrosive environment.
- This is used for low voltage installations. But the overall cost of wiring is more as the cost of this wire is more due to metal sheath.

Advantages :

1. It is durable type of wiring which is suitable for damp places or places exposed to sun or rain.
2. It protects the conductors from mechanical injury.
3. It provides better appearance.

Disadvantages :

1. It is expensive as compared to C.T.S. or T.R.S. wiring.
2. It is unsuitable in corrosive environment.

6.4 Wiring Schemes

- In the domestic wiring, the various appliances and lamps are connected in parallel but for the various advantages the various combinations of switches and lamps are also used. Such combinations are called wiring systems or schemes.

➤ ***With the help of circuit diagram, explain the two way and three way control of lamps.*** VTU : Jan.-03,06,08,10,13; July-03,04,06,07,08; Feb.-05, Marks 5

➤ ***With diagrams, explain three way control of lamp.*** VTU : Dec.-11, Jan.-16,17,18, July-15,16,17, Marks 4

➤ ***Explain the staircase wiring.*** VTU : Jan.-14, Marks 4

➤ ***Explain two way control of lamps with truth table and connection diagram.*** VTU : Jan.-16, Marks 5

6.4.1 Control of One Lamp from One Switch

- For a lamp, one live and one neutral is necessary.
- To control the supply to the lamp, switch is introduced in the live wire and neutral is directly connected to the lamp.

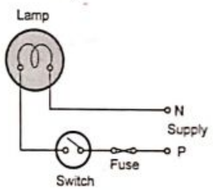


Fig. 6.4.1

- When switch is ON, a full voltage gets applied to the lamp and it glows.
- When the switch is turned off, the circuit gets opened and lamp gets switched off. This is **controlling of one lamp by one switch.**
- The number of lamps can be connected in parallel and can be controlled by a separate individual switch by this method, as shown in the Fig. 6.4.2.

6.4.2 Two Way Control of Lamps

- This is also called as **staircase wiring** as it is commonly used for stair cases and corridor lighting.
- It consist of two way switches. A two way switch operates always in one of the two possible positions.

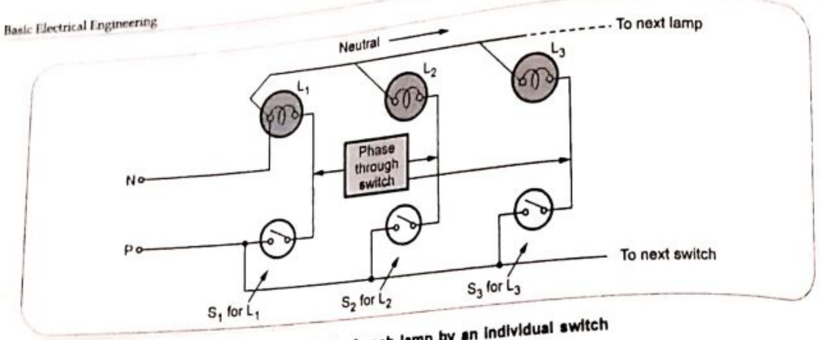


Fig. 6.4.2 Control of each lamp by an individual switch

- The circuit is shown in the Fig. 6.4.3.

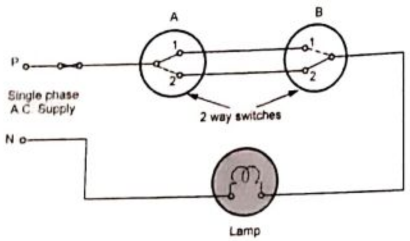


Fig. 6.4.3 Control of one from two points

- Assume that lamp is on first floor. Switch A is on first floor and B is on second floor. In the position shown in the Fig. 6.4.3, the lamp is OFF.
- When person changes position of switch A from (1) to (2) then lamp gets phase through switches A and B and it gets switched ON.
- This is shown in Fig. 6.4.4.

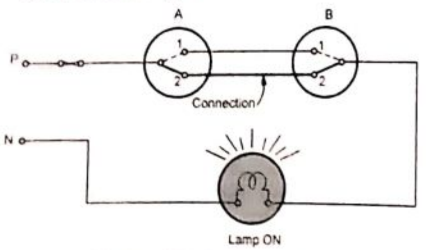


Fig. 6.4.4 'ON' Position of lamp

- When person reaches on second floor, the lamp is required to be switched OFF.
- So person will change switch B from (2) to (1), due to which phase connection reaching to the lamp gets opened and lamp will be switched OFF.
- This is shown in the Fig 6.4.5.

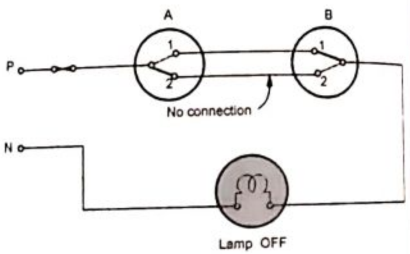


Fig. 6.4.5 'OFF' position of lamp

> • Thus ON and OFF of one lamp can be controlled from two positions with the help of two way switches.

Switching table :

Switch →	A	B	LAMP
Positions →	1	1	ON
	1	2	OFF
	2	1	OFF
	2	2	ON

6.4.3 Three Way Control of Lamps

- This is also a type of staircase wiring.
- It consist of two way switches A and B and one intermediate switch C.
- The circuit used to have three way control of lamps is shown in the Fig. 6.4.6.
- The intermediate switch can have positions to connect points 1-4, 3-2, as shown or 1-2 and 3-4 shown dotted. The switch A is on first floor and switch B is on third floor say.
- In the position shown in Fig. 6.4.6, the lamp is ON.
- When person from floor 2 changes switch C position to have connections 1-2, and 3-4 then there is open circuit in the connection and lamp gets switched OFF.
- Now if the person from third floor changes the position of switch B from 1 to 2, then again Lamp gets supply through position 2 of A, 3-4 of C and 2 of B. The lamp gets switched ON.
- Again if switch A position is changed, lamp gets switched OFF.

> • Thus we have the control of lamp from three different positions which is called three way control of lamps.

Switching table :

Switch →	A	C	B	LAMP
Positions →	1	1-2, 3-4	1	ON
	1	1-4, 2-3	1	OFF
	1	1-2, 3-4	2	OFF
	1	1-4, 2-3	2	ON
	2	1-2, 3-4	1	OFF
	2	1-4, 2-3	1	ON
	2	1-2, 3-4	2	ON
	2	1-4, 2-3	2	OFF

- This is most suitable for staircases and corridors.

6.5 Necessity of Protective Devices

➤ ***Explain the necessity of protective devices.***

- In electrical and electronic circuits many types of faults occur which may damage the entire equipments and other devices connected to the faulty equipments.
- Excessive high currents in such equipments can cause overheating, short circuiting, firing and damaging of equipments.
- Excessive high voltages can cause damage of the equipments and danger to the operators.
- The voltage fluctuations can cause interrupted operation of the devices, partial or permanent damage to the equipments and loss or corruption of data due to unexpected shutdowns.

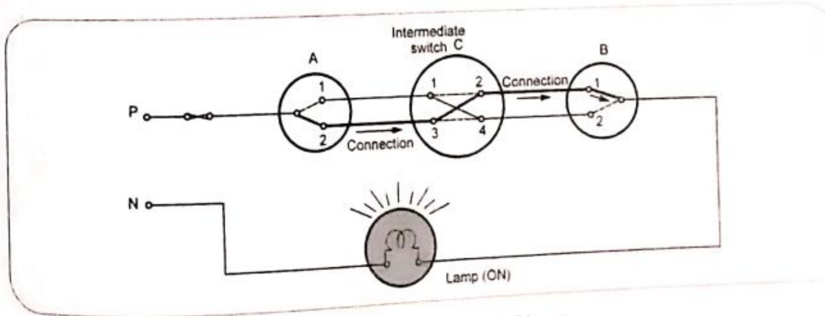


Fig. 6.4.6 Three way control of lamps

- The failure of insulation can cause severe shock to the operators which are life threatening.
- The atmospheric lightening strokes can cause fire and damage to the buildings and structures.
- Due to all these reasons there is need for various protection methods and protective devices.
- The various types of protective devices used in domestic wiring are,
  1. Fuse
  2. Miniature Circuit Breaker [MCB]
  3. Earth Leakage Circuit Breaker [ELCB]
  4. Resitual Current Circuit Breaker [RCCCB]
  5. Earthing

6.6 Introduction to Fuse

➤ ***What is fuse ? Why is it used in the electric circuits ?*** **VTU : Feb.-05; Jan.-07; July-09, Marks 6**

- The **fuse** is a protecting device of simplest form.
- It consists of a small piece of metal. When excessive current flows through it, the metal element melts and the current is interrupted and circuit gets disconnected from the supply. A small piece of metal used in a fuse is called **fusing element**.
- Thus the fuse protects the circuit from dangerous excessive current. So fuse is used to interrupt a fault current.
- It is used for overload and short circuit protection in medium voltage range upto 66 kV.
- The fuse is always connected in series with the circuit or appliance to be protected.
- The fusing element carries the normal working current safely but melts due to excessive current under abnormal conditions like overload and short circuit. As it is in series, melting of fusing element causes current interruption and breaking of the circuit, protecting the equipment from excessive current.
- Either copper or lead-tin alloy is generally used as a fuse wire.
- Instead of connecting fuse wire directly in series with live wire, a fuse top is used which is having porcelain base. The porcelain structure containing fuse wire is bridged to the base by fitting top into the base. Such arrangement is called **kit-kat type of fuse unit.** The Fig. 6.6.1 shows such arrangement.

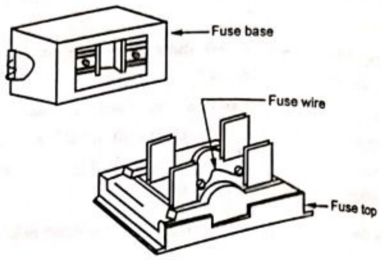


Fig. 6.6.1 Kit-kat type of fuse

- This is commonly used in domestic wiring and is connected in series with the circuit to be protected.
- The phase or incoming wire is inserted from one side and connected to terminal while outgoing connection is taken from other terminal. The two terminals inside the base are connected through fuse wire.
- Fuses cannot be used for large currents as they bear low breaking capacity. The other limitation of this type of fuse is possibility of rewiring of fuse with wrong size or material.
- The kit-kat type is of semienclosed type of fuse or also called as **rewirable fuse.** The main advantage of this type is if it is blown off, top can be taken out and fuse wire can be changed. Tin-lead alloy fuse wires are used upto 30 A.

➤ ***List the desirable characteristics of fuse element.***

The desirable characteristics of any fuse element are,
1. Low melting point
2. High conductivity
3. Free from deterioration due to oxidation
4. Low cost.

6.6.1 Definitions

➤ ***Define the following : i) Fusing current ii) Rated current iii) Fusing factor*** **VTU : July-08, Marks 5**

1. Fuse : The fuse is a device which consists of small piece of metal, which is connected in series circuit. When current through it increases beyond some predetermined value, the metal melts to interrupt the circuit current, which protects the circuit from excessive high current.

2. Fuse element : The part of the fuse which melts when excessive current flows through it is called fuse element or fuse wire.

3. Rated current of fuse : It is that maximum current which fusing element can normally carry without any undue overheating or melting.

- It depends on,
  1. Temperature rise of fuse contacts of fuse holder
  2. Fusing element material
  3. Deterioration of fuse due to oxidation.

4. Fusing current : The minimum value of the current at which the fuse element melts to interrupt the circuit current is called fusing current. Its value is always more than the current rating of the fuse.

5. Fusing factor : The ratio of the minimum fusing current and the current rating of the fuse is called the fusing factor. As minimum fusing current is more than the current rating, the fusing factor is always greater than one.

$$\boxed{\text{Fusing factor} = \frac{\text{Minimum fusing current}}{\text{Current rating of fuse}}}$$

- The small the value of fusing factor, greater it is difficult to avoid the deterioration on due to oxidation and overheating at the rated current.
- For a household fuse which is semiclosed type using copper wire, this factor is generally 2.

6.6.2 Advantages of Fuse

➤ ***State the advantages of using fuse as a protecting device.***

1. It is simplest and cheapest form of protecting device.
2. It requires no maintenance.
3. The operation of fuse is automatic while circuit breaker needs a tripping circuit to operate for its operation.
4. The minimum operating time can be made much smaller than that of circuit breaker.
5. Inverse time-current characteristic enables it to use for the overload protection.
9. With the help of a fuse, heavy currents can be interrupted without noise, smoke, gas and flame.
7. The fuse can produce a current limiting effect under short circuit conditions.

6.6.3 Disadvantages of Fuse

➤ ***State the various disadvantages of a fuse.***

1. The fuse is required to be replaced or rewired after its operation.
2. The replacement or rewiring of fuse takes a lot of time.
3. Discrimination between fuses in series cannot be obtained unless there is much difference in relative sizes of the fuses.
4. The current-time characteristics cannot be always corelated with that of the protected equipment.
5. It is not possible to provide secondary protection to fuses.

6.7 Miniature Circuit Breaker (MCB)

➤ ***What is MSB ? Explain its need and features.***

- A miniature circuit breaker is an electromechanical device which makes and breaks the circuit in normal operation and disconnects the circuit under the abnormal condition when current exceeds a preset value.
- MCB is a high fault capacity current limiting, trip free, automatic switching device with thermal and magnetic operation to provide protection against overload and short circuit.
- It is necessary to use MCB because of its following features,
  1. Its operation is very fast and opens in less than one millisecond.
  2. No tripping circuit is necessary and the operation is automatic.

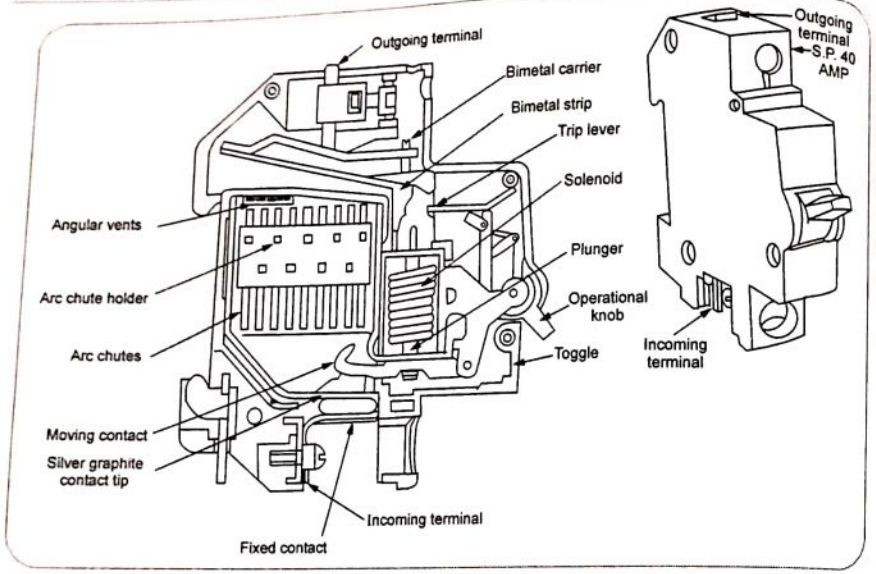


Fig. 6.7.1 Construction and appearance of MCB

3. Provides protection against overload and short circuit without noise, smoke or flame.
4. It can be reset very quickly after correcting the fault, just by switching a button. No rewiring is required.
5. It can not be reclosed if fault persists.
6. The mechanical life is upto or more than one lakh operating cycle.

- Hence now a days MCBs are used rather than rewirable fuse.
- Generally MCBs are rated for a.c. voltage of 240 V for single phase, 415 V for three phase or 220 V d.c. The current rating available is from 0.5 A to 63 A. It is available as Single Pole (SP), Double Pole (DP), Tripple Pole (TP) with short circuit breaking capacity from 1 kA to 10 kA with a rated frequency.
- A typical cross-section view of MCB and its practical appearance is shown in the Fig. 6.7.1.

6.7.1 Comparison of MCB and Fuse

➤ ***Compare MCB with fuse.***

Sr. No.	Fuse	MCB
1.	The operation of fuse is highly dependent on selection of its proper rating. If fuse wire is not selected properly then it results in non operation of fuse even in case of short circuit.	MCB instantly disconnects the supply automatically in the event of short circuit or overload. It thus eliminates the risk of fire and prevents damage to wiring system.
2.	If the fuse wire after operation is replaced with a newer one but go loose then it may be dangerous. Also to replace a blown fuse in between current carrying points is dangerous specially in dark.	Restarting power supply after tripping due to overload or short is easy.
3.	During replacement of fuse wire, the exact size of fuse wire may not be available. Also for replacement a kit of hand tools has to be kept ready	No maintenance and repairs is required for MCB. The distribution system employing MCB provides satisfactory operation and lasts for years.
4.	The board employing fuse is not compact	The board employing MCBs give beautiful look as it is compact and elegent.

6.8 Earthing

➤ ***What is earthing ?*** VTU : Jan.-16,17, July-17, Marks 2

- For all practical purposes, the earth's potential is taken zero. Almost all the machinery, electric poles, towers, neutral wires are connected to earth. The neutral wire of an a.c. supply and middle wire of three wire d.c. distribution system are always earthed.
- The connection of electrical machinery to the general mass of earth, with a conducting material of very low resistance is called **earthing** or **grounding.**
- **The earthing of electrical equipment bring the equipments to zero potential and avoid the shock to the operator, under any fault conditions.**

6.8.1 Necessity of Earthing

➤ ***Explain the necessity of earthing.*** VTU : Jan.-04, 07, 11, 15, Feb.-05, July-05, 09, May-10, Marks 4

- Consider a single phase machine which is not earthed carrying a current.
- The potential between line and neutral is V volts. The resistance between the windings and the frame is say R_i called **insulation resistance.** And R_{body} be the **resistance of the body** of a person who happens to touch to the machine.
- Neutral is generally earthed at supply system as shown in the Fig. 6.8.1.
- Let I_m = Machine current and I_{body} = Current passing through body of the person.
- When a person, standing on the earth touches the machine, current I gets an alternative path through

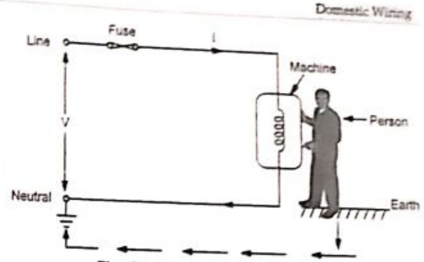


Fig. 6.8.1 Machine is not earthed

the body of the person to earth from the insulation resistance, finally to the neutral of the supply.

$$I_{body} = \frac{V}{R_i + R_{body} + R_E} \quad \ldots(6.8.1)$$

- When the insulation of the machine is perfect, the insulation resistance is of the order of few mega ohms and practically can be considered as infinity.

$$\text{So } R_i = \infty \quad \ldots \text{ Insulation perfect}$$

$$\therefore \quad I_{body} = \frac{V}{R_E + \infty + R_{body}} = 0 \text{ A} \quad \ldots(6.8.2)$$

- So in normal operating conditions, there is no current passing through the body of the person and hence there is no danger of the shock.
- But when the insulation becomes weak or defective or if one of the windings is touching to the frame directly due to some fault then R_i i.e. insulation resistance becomes almost zero. Now resistance of body and earth are not very high and hence I_{body} increases to such a high value that the person receives a fatal shock. Such a current is called a **leakage current.** Hence when the machine is not earthed, there is always a danger of the shock, under certain fault conditions.
- Let us see now, what happens due to earthing. In case of earthing, the frame of the machine is earthed as shown in the Fig. 6.8.2.
- The resistance of the path from frame to earth is very very low.
- When the person touches the frame, and if there is a leakage due to fault condition, due to earthing a leakage current takes a low resistance path i.e. path from frame to earth, bypassing the person.

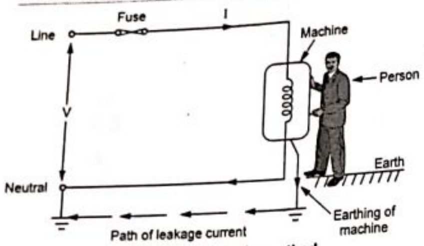


Fig. 6.8.2 Machine is earthed

- So body of the person carries very low current which is not sufficient to cause any shock.
- As earthing resistance is very very low compared to the body of the person, current prefers low resisstance path. Thus I_{body} is negligibly small compared to earth current. So entire leakage current passes through the earthing contact bypassing the body of the person. The value of I_{body} is not sufficient to cause any shock to the person.
- Not only this but the earthing current, is high due to which fuse blows off and thus it helps to isolate the machine from the electric supply.

6.8.2 Uses of Earthing

➤ ***State the uses of earthing.***

- Apart from basic use of earthing discussed above, the other uses can be stated as
  1) To maintain the line voltage constant.
  2) To protect tall buildings and structures from atmospheric lightening strikes.
  3) To protect all the machines, fed from overhead lines, from atmospheric lightening.
  4) To serve as the return conductor for telephone and traction work. In such case, all the complications in laying a separate wire and the actual cost of the wire, is thus saved.
  5) To protect the human being from disability or death from shock in case the human body comes into the contact with the frame of any electrical machinery, appliance or component, which is electrically charged due to leakage current or fault.

6.9 Methods of Earthing

➤ ***State the various methods of earthing.***

- Earthing is achieved by connecting the electrical appliances or components to earth by employing a good conductor called **'Earth Electrode'**.
- This ensures very low resistance path from appliance to the earth.
- The various methods of earthing are
  i) Plate earthing
  ii) Pipe earthing
  iii) Earthing through water main
  iv) Horizontal strip earthing
  v) Rod earthing
- Let us discuss in detail, the two methods of earthing which are commonly used in practice.

6.9.1 Plate Earthing

➤ ***Explain the plate earthing alongwith a neat diagram.***

VTU : Jan.-04,07,14,17,18; Feb.-05; July-05,09,16,17; May-10, Dec.-11, Marks 6

- The earth connection is provided with the help of copper plate or Galvanized Iron (G.I.) plate. The copper plate size is 60 cm × 60 cm × 3.18 mm while G.I. plate size is not less than 60 cm × 60 cm × 6.3 mm. The G.I. plates are commonly used now-a-days. The plate is embedded 3 meters (10 feet) into the ground. The plate is kept with its face vertical.
- The plate is surrounded by the alternate layer of coke and salt for minimum thickness of about 15 cm. The earth wire is drawn through G.I. pipe and is perfectly bolted to the earth plate. The nuts and bolts must be of copper plate and must be of galvanized iron for G.I. plate.
- The earth lead used must be G.I. wire or G.I. strip of sufficient cross-sectional area to carry the fault current safely. The earth wire is drawn through G.I. pipe of 19 mm diameter, at about 60 cm below the ground.
- The G.I. pipe is fitted with a funnel on the top. In order to have an effective earthing, salt water is poured periodically through the funnel.

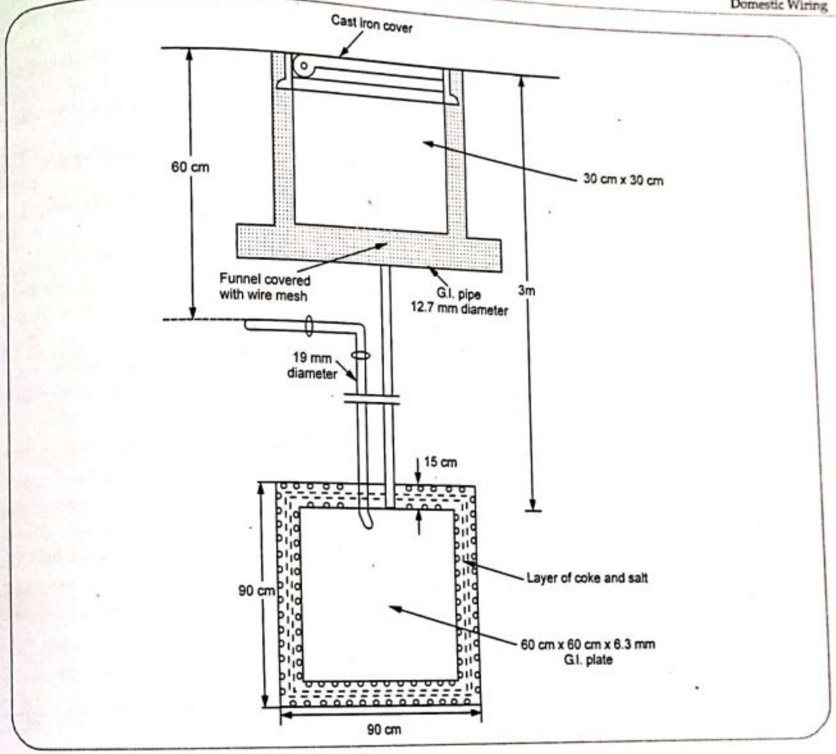


Fig. 6.9.1 Plate earthing

- The earthing efficiency, increases with the increases of the plate area and depth of embedding. If the resistivity of the soil is high, then it is necessary to embed the plate vertically at a greater depth into the ground.
- The only disadvantage of this method is that the discontinuity of the earth wire from the earthing plate below the earth can not be observed physically. This may cause misleading and may result into heavy losses under fault conditions.
- The schematic arrangement of plate earthing is shown in the Fig. 6.9.1.

6.9.2 Pipe Earthing

➤ ***Explain the pipe earthing alongwith a neat diagram.***

VTU : July-03; Jan.-06, 09, 11, 16, June-10, July-15, Marks 8

- In this method of earthing a G.I. pipe of 38 mm diameter and 2 meter (7 feet) length is embedded vertically into the ground. This pipe acts as an earth electrode. The depth depends on the condition of the soil.
- The earth wires are fastened to the top section of the pipe above the ground level with nut and bolts.
- The pit area around the pipe is filled with salt and coal mixture for improving the condition of the soil and earthing efficiency. The schematic arrangement of pipe earthing system is shown in the Fig. 6.9.2.

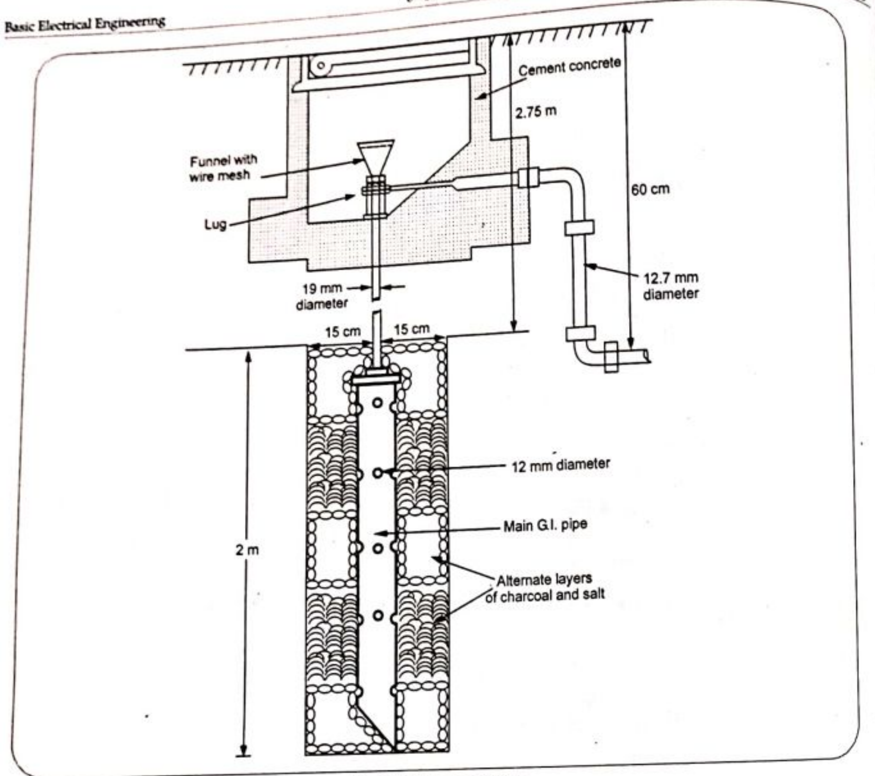


Fig. 6.9.2 Pipe earthing

- The contact surface of G.I. pipe with the soil is more as compared to the plate due to its circular section and hence can handle heavier leakage current for the same electrode size.
- According to Indian standard, the pipe should be placed at a depth of 4.75 m. Impregnating the coke with salt decreases the earth resistance. Generally alternate layers of salt and coke are used for best results.
- In summer season, soil becomes dry. In such case salt water is poured through the funnel connected to the main G.I. pipe through 19 mm diameter pipe. This keeps the soil wet.
- The earth wires are connected to the G.I. pipe above the ground level and can be physically inspected from time to time. These connections can be checked for performing continuity tests. This is the important advantage of pipe earthing over the plate earthing. The earth lead used must be G.I. wire of sufficient cross-sectional area to carry fault current safely. It should not be less than electrical equivalent of copper conductor of 12.97 mm^2 cross-sectional area.
- The only disadvantage of pipe earthing is that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of high order. This increases the excavation work and hence increased cost. In ordinary soil condition the range of the earth resistance should be 2 to 5 ohms.

- In the places where rocky soil earth bed exists, horizontal strip earthing is used. This is suitable as soil excavation required for plate or pipe earthing is difficult in such places. For such soils earth resistance is between 5 to 8 ohms.

6.10 Earth Leakage Circuit Breaker (ELCB)

> ***Explain the necessity of ELCB.*** VTU : Jan. 15, Marks 2

- There are certain situations where leakage current flows through the metal bodies of appliances. Thus person touching to such appliances may get a shock.
- There is risk of fire due to such leakage current flowing to the earth.
- The MCB and fuse can not provide protection against earth leakage currents.
- Hence there is need of a device which can directly detect the earth leakage currents and cut the supply if such currents exceed a preset value. Such a device is called Earth Leakage Circuit Breaker (ELCB).
- There are two types of ELCBs which are,

i) Voltage operated generally called ELCB.

ii) Current operated generally called Residual Current Circuit Breaker (RCCB).

6.10.1 Operation of ELCB

> ***Explain the operation of ELCB.*** VTU : Jan. 15, Marks 4

- Basic ELCB is a voltage operated device.
- It detects the rise in potential between the protected device or installation and an earth reference electrode.
- Such a rise in voltage is possible due to the touching of phase wire to metal part of the device or due to failure of insulation of the device.
- For giving protection against such a condition, the earth circuit is modified using ELCB.
- The connection to earth reference electrode is passed through ELCB, by connecting two earth terminals of ELCB as shown in the Fig. 6.10.1.
- When the voltage between metal body part of the device and earth electrode rises beyond 50 V then ELCB circulates current though the relay coil which opens the main circuit breaker to isolate the supply from the faulty device.

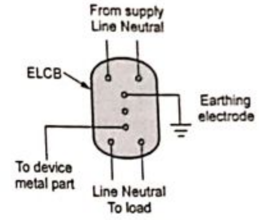


Fig. 6.10.1 ELCB

- The ELCB remains off till manually reset.
- The main disadvanage of voltage operated ELCB is that if the fault is between live and some other earth (like a person or a metal pipe) then it will not disconnect the supply and does not provide protection.

6.11 Residual Current Circuit Breaker (RCCB)

> ***With a neat diagram explain the working of RCCB.*** VTU : Jan. 16, Marks 6

- Residual current circuit breaker is nothing but the current operated ELCB.
- The schematic of RCCB is shown in the Fig. 6.11.1.
- RCCB consists of a small current transformer surrounding live and neutral wire. The sensing coil on current transformer is connected to a tripping coil of circuit breaker.
- Under normal condition the current in line conductor I_L is same as the current in neutral conductor I_N. Hence $(I_L - I_N)$ is zero hence two fluxes produce by I_L and I_N cancel each other and sensing coil does not sense any imbalance.
- If there is a fault, the fault current I_f flows through the earth conductor hence there is difference between the currents I_L and I_N. The difference $I_L - I_N$ is called residual current.
- The fluxes produced by I_L and I_N are no longer same under fault condition, producing flux in the core.
- Due to the residual flux, e.m.f. gets induced in the sensing coil which circulates current through the tripping coil of the circuit breaker.

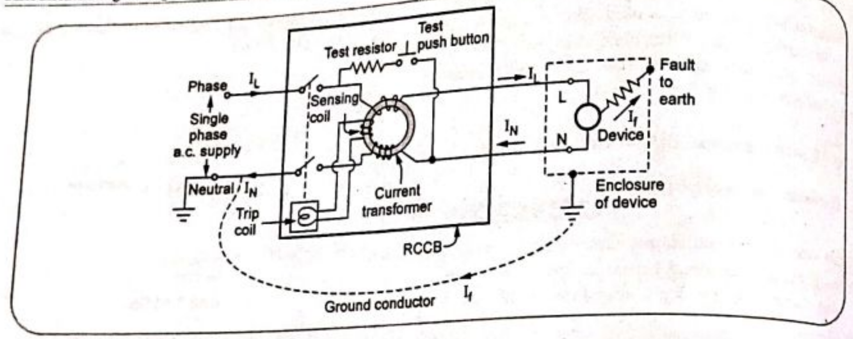


Fig. 6.11.1 RCCB

- This operates the circuit breaker and disconnects the supply from the device.
- As the action of trip coil depends on the residual current, the device is called residual current circuit breaker.
- The properly connected RCCB can detect small currents in milliampers and operates in around 25 msec.

6.12 Electric Shock

➤ ***What is an electric shock ? State the factors on which severity of the shock depends.*** **VTU : July-07, Marks 6**

- A sudden agitation of the nervous system of a body, due to the passage of an electric current is called an **electric shock.**
- The factors affecting the severity of the shock are,
  1. Magnitude of current through the body.
  2. Path of the current through the body.
  3. Time for which the current is passed through the body.
  4. Frequency of the current.
  5. Physical and psychological condition of the person.
- The Table 6.12.1 shows an electric shock effect chart.

250 V supply current	Resistance of body	Condition of body	Effect due to shock
About 2.5 mA	10 - 600 kΩ	Dry skin	Mild shock
About 25 mA	1 kΩ	Wet skin	Strong painful shock, stoppage of breathing, possible death
More than 25 mA	Few hundred ohms	Wet skin making fair contact with earth	Ventricular fibrillation, stoppage of breathing, muscular contraction, death

Table 6.12.1

- Thus it is necessary to avoid the electric shocks.

6.12.1 Elementary First Aid against Shock

➤ ***Write about elementary first aid against shock.*** **VTU : Mar.-01; Jan.-04; July-06, Marks 5**

- The first aid can save the life and reduce severity of the accidents. Hence elementary first aid is important. The first aid against an electric shock involves following steps,
  1. Do not panic.
  2. Carry the affected person and lay him in a comfortable position and call the doctor immediately.
  3. Look for stoppage of breathing.
  4. Start giving him artificial respiration if breathing is stopped.
  5. Never give anything to the person to drink when the person is unconscious.
  6. The artificial respiration should be continued for longer time.
  7. The burns caused due to electric flashes should be covered with sterile dressing and then bandaged.
  8. Do not make crowd round and let patient get fresh air.

6.12.2 Safety Precautions against Electric Shock

➤ ***Write about precautionary measures taken against electric shock.*** **VTU : July-07, 11, June-12, Jan.-13, 15, Marks 4**

- It is necessary to observe some safety precautions while using the electric supply to avoid serious problems like shocks and fire hazards.
- Some of the safety precautions are as follows :

1) Insulation of the conductors used must be proper and in good condition. If it is not so the current carried by the conductors may leak out. The person coming in contact with such faulty insulated conductors may receive a shock.
2) Megger tests should be conducted and insulation must be checked. With the help of megger all the tests discussed above must be performed, on the new wiring before starting use of it.
3) Earth connection should be always maintained in proper condition.
4) Make the mains supply switch off and remove the fuses before starting work with any installation.
5) Fuses must have correct ratings.
6) Use rubber soled shoes while working. Use some wooden supper under the feet. this removes the contact with the earth.
7) Use rubber gloves while touching any terminals or removing insulation layer from a conductor.
8) Use a line tester to check whether a 'live' terminal carries any current still better method is to use a test lamp.
9) Always use insulated screw drivers, pliers, line testers etc.
10) Never touch two different terminals at the same time.
11) Never remove the plug by pulling the wires connected to it.
12) The sockets should be fixed at a height beyond the reach of the children.

6.13 University Questions with Answers

Jan. - 2010

Q.1 *Mention and describe in brief various types of wiring systems.* **[Refer section 6.3]** **[5]**

Q.2 *With the help of circuit diagram, explain the two way and three way control of lamps.* **[Refer section 6.4]** **[5]**

May - 2010

Q.3 *Explain the necessity of earthing.* **[Refer section 6.8.1]** **[4]**

Q.4 *Explain the plate earthing alongwith a neat diagram.* **[Refer section 6.9.1]** **[6]**

June - 2010

Q.5 *Explain the pipe earthing alongwith a neat diagram.* **[Refer section 6.9.2]** **[8]**

Jan. - 2011

Q.6 *Mention and describe in brief various types of wiring systems.* **[Refer section 6.3]** **[5]**

Q.7 *Explain the necessity of earthing.* **[Refer section 6.8.1]** **[4]**

Q.8 *Explain the pipe earthing alongwith a neat diagram.* **[Refer section 6.9.2]** **[8]**

July - 2011

Q.9 *Write about precautionary measures taken against electric shock.* **[Refer section 6.12.2]** **[4]**

Dec. - 2011

Q.10 *With diagrams, explain three way control of lamp.* **[Refer section 6.4]** **[4]**

Q.11 *Explain the plate earthing alongwith a neat diagram.* **[Refer section 6.9.1]** **[6]**

June - 2012

Q.12 *Mention and describe in brief various types of wiring systems.* **[Refer section 6.3]** **[5]**

Q.13 *Write about precautionary measures taken against electric shock.* **[Refer section 6.12.2]** **[4]**

Jan. - 2013

Q.14 *With the help of circuit diagram, explain the two way and three way control of lamps.* **[Refer section 6.4]** **[5]**

Q.15 *Write about precautionary measures taken against electric shock.* **[Refer section 6.12.2]** **[4]**

June - 2013

Q.16 *Mention and describe in brief various types of wiring systems.* **[Refer section 6.3]** **[5]**

Jan. - 2014

Q.17 *Explain the staircase wiring.* **[Refer section 6.4]** **[4]**

Q.18 *Explain the plate earthing alongwith a neat diagram.* **[Refer section 6.9.1]** **[6]**

Jan. - 2015

Q.19 *Explain the necessity of earthing.* **[Refer section 6.8.1]** **[4]**

Q.20 *Explain the necessity of ELCB.* **[Refer section 6.10]** **[2]**

Q.21 *Explain the operation of ELCB.* **[Refer section 6.10.1]** **[4]**

Q.22 *Write about precautionary measures taken against electric shock.* **[Refer section 6.12.2]** **[4]**

July - 2015

Q.23 *With diagrams, explain three way control of lamp.* **[Refer section 6.4]** **[4]**

Q.24 *Explain the pipe earthing alongwith a neat diagram.* **[Refer section 6.9.2]** **[8]**

Jan. - 2016

Q.25 *Explain the working of three-way control of lamp with the help of switching table.* **[Refer section 6.4.3]** **[6]**

Q.26 *With a neat diagram explain the working of RCCB.* **[Refer section 6.11]** **[6]**

CBCS Scheme

Jan. - 2016

Q.27 *Explain two way control of lamps with truth table and connection diagram.* **[Refer section 6.4.2]**. **[5]**

Q.28 *What is earthing ? Explain any one type of earthing with neat figure.* **[Refer sections 6.8 and 6.9]** **[6]**

July - 2016

Q.29 *With a circuit diagram, explain the working of a three way control of lamp.* **[Refer section 6.4.3]** **[5]**

Q.30 *With a neat figure explain plate earthing.* **[Refer section 6.9.1]** **[6]**

Jan. - 2017

Q.31 *With a neat sketch, explain 3-way control of lamp.* **(Refer section 6.4.3)** **[6]**

Q.32 *Define earthing. Explain any one type of earthing with a neat diagram.* **(Refer section 6.9.1)** **[5]**

July - 2017

Q.33 *With a neat connection diagram and functional table, explain three way control of a lamp.* **(Refer section 6.4.3)** **[5]**

Q.34 *What is earthing ? Explain any one type with neat diagram.* **(Refer sections 6.8 and 6.9.1)** **[6]**

❑❑❑

Notes

MODULE - 4

7

D.C. Generators

Syllabus

Principle of operation, Construction of D.C. Generators. Expression for induced e.m.f, Types of D.C. Generators, Relation between induced emf and terminal voltage.

Contents

7.1 Introduction to D.C. Machines

- An electrical machine, deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form. This process is called **electromechanical energy conversion.**
- An electrical machine which converts mechanical energy into an electrical energy is called an **electric generator.**
- While an electrical machine which converts an electrical energy into the mechanical energy is called an **electrical motor.**
- The d.c. machines are thus classified as,
  1. **D.C. Generators :** These machines convert mechanical input power into d.c. electrical power.
  2. **D.C. Motors :** These machines convert d.c. electrical power into mechanical power.
- The construction of both the types of d.c. machines basically remains same.

7.2 Principle of Operation of D.C. Generator

➤ ***Explain the working principle of d.c. generator.***
VTU : Jan.-04, 08, Marks 6

- All generators work on the principle of dynamically induced e.m.f.
- This principle is nothing but the Faraday's law of electromagnetic induction. It states that, 'whenever the number of magnetic lines of force i.e. flux linking with a conductor or a coil changes, an electromotive force is set up in that conductor or coil.'
- The magnitude of induced e.m.f. in a conductor is proportional to the rate of change of flux associated with the conductor. This is mathematically given by,

$$e \text{ (magnitude)} \propto \frac{d\phi}{dt}$$

- The relative motion can be achieved by rotating conductor with respect to flux or by rotating flux with respect to a conductor.
- So a voltage gets generated in a conductor, as long as there exists a relative motion between conductor and the flux.
- Such an induced e.m.f. which is due to physical movement of coil or conductor with respect to flux or movement of flux with respect to coil or conductor is called **dynamically induced e.m.f.**

- So a generating action requires following basic components to exist, i) The conductor or a coil ii) The flux iii) The relative motion between conductor and flux.

- To have a large voltage as the output, the number of conductors are connected together in a specific manner, to form a winding. This winding is called **armature winding** of a d.c. machine.
- The part on which this winding is kept is called **armature** of a d.c. machine.
- To have the rotation of conductors, the conductors placed on the armature are rotated with the help of some external device. Such an external device is called a **prime mover.**
- The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc.
- The necessary magnetic flux is produced by current carrying winding which is called **field winding.**
- The direction of the induced e.m.f. can be obtained by using Fleming's right hand rule.
- If angle between the plane of rotation and the plane of the flux is 'θ' as measured from the axis of the plane of flux then the induced e.m.f. is given by,

$$\boxed{E = B\,l\,(v \sin \theta) \text{ volts}}$$

- where $v \sin \theta$ is the component of velocity which is perpendicular to the plane of flux and hence responsible for the induced e.m.f.
- If the plane of rotation is parallel to the plane of the flux, $\theta = 0°$ hence induced e.m.f. is zero.
- If the plane of rotation is perpendicular to the plane of the flux, $\theta = 90°$ hence induced e.m.f. is maximum.
- From the equation of the induced e.m.f., it can be seen that the basic nature of the induced e.m.f. in a d.c. generator is purely sinusoidal i.e. alternating. To have d.c. voltage, a device is used in a d.c. generator to convert the alternating e.m.f. to

unidirectional e.m.f. This device is called **commutator.**

7.3 Constructional Details of a D.C. Machine

➤ ***Draw the neat sketch representing the cut section of a d.c. machine. Explain the important features of different parts involved there on.***
VTU : Jan.-03,16,17,18, July-04, 06, 08, 11,16 Marks 6, June-10, Marks 8

➤ ***What are the functions of yoke, armature, poles and brushes in d.c. generator ?***
VTU : Dec.-11, Marks 4

➤ ***With a neat sketch explain the construction of a d.c. machine.***
VTU : Jan.-13, Marks 6, July 15, Marks 4

- Whether a machine is **d.c. generator or a motor** the construction basically remains the same as shown in the Fig. 7.3.1.
- It consists of the following parts :

7.3.1 Yoke

a) Functions :

1. It serves the purpose of outermost cover of the d.c. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.
2. It provides mechanical support to the poles.
3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

b) Choice of material : It is prepared by using cast iron because it is cheapest and provides low reluctance path. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

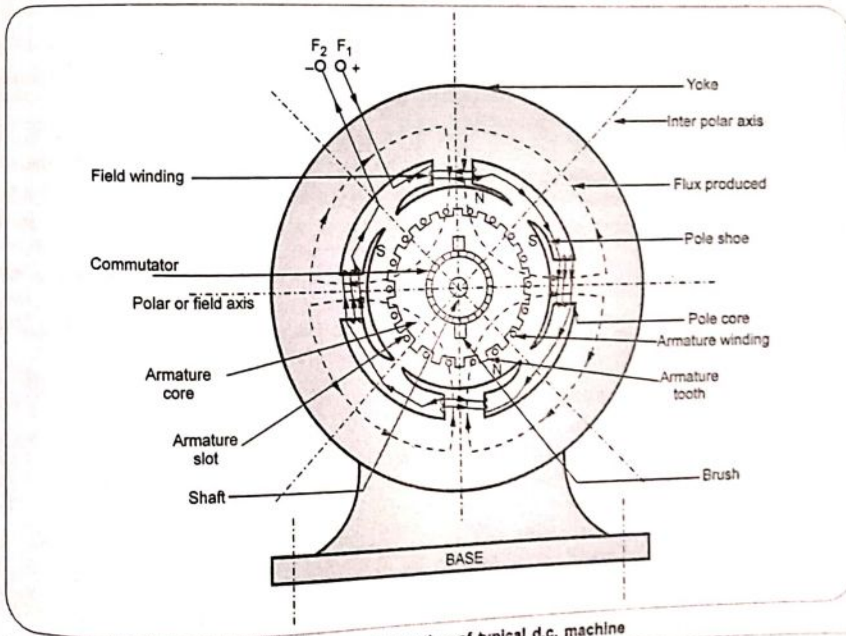


Fig. 7.3.1 A cross-section of typical d.c. machine

7.3.2 Poles

➢ ***Why pole shoe has been given a particular shape ?***

- Each pole is divided into two parts namely, I) Pole core and II) Pole shoe
- This is shown in the Fig. 7.3.2.

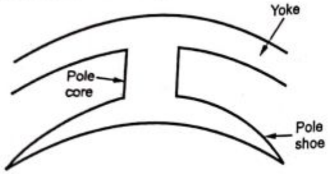


Fig. 7.3.2 Pole structure

a) Functions of pole core and pole shoe :

1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core, to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoe has been given a particular shape.

b) Choice of material : It is made up of magnetic material like cast iron or cast steel.

- As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

7.3.3 Field Winding (F1 - F2)

➢ ***What are the functions of field winding ? State the choice of material for the field winding.***

- The field winding is wound on the pole core with a definite direction.

a) Functions : To carry current due to which pole core, on which the field winding is placed behaves as an electromagnet, producing necessary flux.

- As it helps in producing the magnetic field i.e. exciting the pole as an electromagnet it is called **Field winding** or **Exciting winding**.

b) Choice of material : It has to carry current hence obviously made up of some conducting material. So aluminium or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

- Field winding is divided into various coils called field coils. These are connected in series with each other and wound in such a direction around pole cores, such that alternate 'N' and 'S' poles are formed.
- The total number of poles is denoted as P.

7.3.4 Armature

- The armature is further divided into two parts namely,

I) Armature core and II) Armature winding

I) Armature core : Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

a) Functions :

1. Armature core provides house for armature winding i.e. armature conductors.
2. To provide a path of low reluctance to the magnetic flux produced by the field winding.

b) Choice of material : As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

- It is made up of laminated construction to keep eddy current loss as low as possible. A single circular lamination used for the construction of the armature core is shown in the Fig. 7.3.3

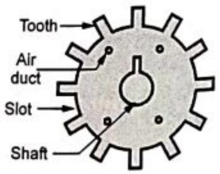


Fig. 7.3.3 Single circular lamination of armature core

II) Armature winding : Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery.

- When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f. gets induced in them.

a) Functions :

1. Generation of e.m.f. takes place in the armature winding in case of generators.
2. To carry the current supplied in case of d.c. motors.
3. To do the useful work in the external circuit.

b) Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

7.3.5 Commutator

➢ ***Explain the construction of commutator in brief.*** VTU : Feb.-05, July-05, 06, Marks 2

- The basic nature of e.m.f. induced in the armature conductors is alternating. This needs rectification in case of d.c. generator, which is possible by a device called commutator.

a) Functions :

1. To facilitate the collection of current from the armature conductors.
2. To convert internally developed alternating e.m.f. to unidirectional (d.c.) e.m.f.
3. To produce unidirectional torque in case of motors.

b) Choice of material : As it collects current from armature, it is also made up of copper segments.

- It is cylindrical in shape and is made up of wedge shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layer of mica.
- Each commutator segment is connected to the armature conductor by means of copper lug or strip. This construction is shown in the Fig. 7.3.4

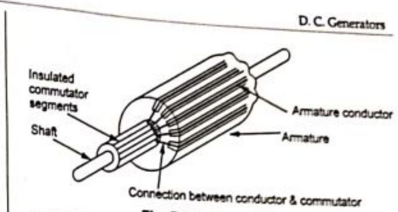


Fig. 7.3.4 Commutator

7.3.6 Brushes and Brush Gear

➢ ***Explain brushes of a d.c. machine.*** VTU : Feb.-05, July-05, 06, Marks 2

- Brushes are stationary and resting on the surface of the commutator.

a) Function : To collect current from commutator and make it available to the stationary external circuit.

b) Choice of material : Brushes are normally made up of soft material like carbon.

- To avoid wear and tear of commutator, the brushes are made up of soft material like carbon.

7.3.7 Bearings

- Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

7.4 Types of Armature Winding

➢ ***Explain the armature winding of a d.c. machine.*** VTU : Mar.-01; Feb.-05, Marks 2

- The number of armature conductors are connected in a specific manner to give armature winding.
- According to way of connecting the conductors, the armature winding has two types, a) Lap and b) Wave.
- In lap type, the connections overlap each other as the winding proceeds as shown in the Fig. 7.4.1.

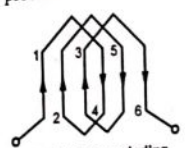


Fig. 7.4.1 Lap winding

- Due to the this, number of parallel paths in which conductors are divided is P where P = Number of poles in the machine.

$\therefore$ $\boxed{A = P = \text{Number of parallel paths for lap}}$

- Large number of parallel paths indicate high current capacity of machine hence lap winding is preferred for high current rating generators.

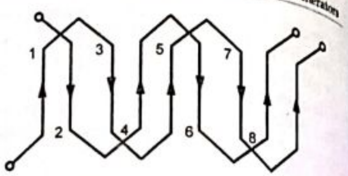


Fig. 7.4.2 Wave winding

- In wave type, the winding travels ahead avoiding the overlapping as shown in the Fig. 7.4.2 in a progressive fashion.
- Due to this, the armature conductors always get divided into two parallel paths, irrespective of number of poles.

$\therefore$ $\boxed{A = 2 = \text{Number of parallel paths for wave}}$

➤ ***Compare lap and wave type armature winding.***

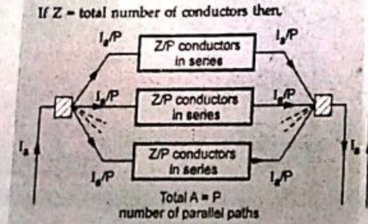
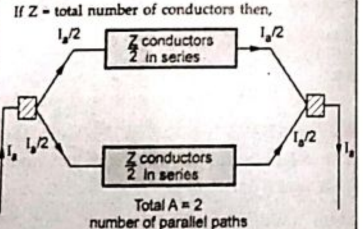
Sr. No.	Lap winding	Wave winding
1.	Number of parallel paths (A) = poles (P)	Number of parallel paths (A) = 2 (always)
2.	Number of brush sets required is equal to number of poles.	Number of brush sets required is always equal to two.
3.	Preferable for high current, low voltage capacity generators	Preferable for high voltage, low current capacity generators.
4.	Normally used for generators of capacity more than 500 A.	Preferred for generators of capacity less than 500 A.
5.	If Z = total number of conductors then, 	If Z = total number of conductors then, 

Table 7.4.1

7.4.1 Winding Terminologies

a) **Conductor :** It is the actual armature conductor which is under the influence of the magnetic field, placed in the armature slot.

b) **Turn :** The two conductors placed in different slots when connected together, forms a turn. While describing armature winding the number of turns may be specified from which, the number of conductors can be decided.

$Z = 2 \times \text{Number of turns}$

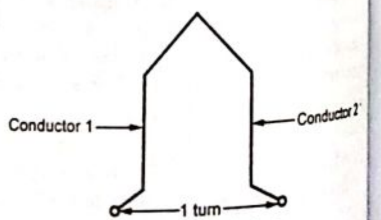


Fig. 7.4.3 Single turn

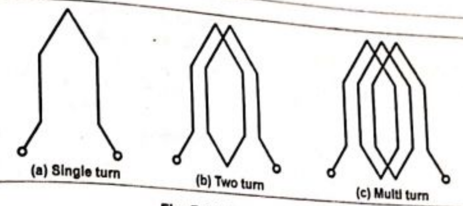


Fig. 7.4.4 Armature coils

c) **Coil :** For simplicity of connections, the turns are grouped together to form a coil.

- If coil contains only one turn it is called single turn coil while coil with more than one turn is called multiturn coil.
- Hence if number of coils, alongwith number of turns per coil are specified, it is possible to determine the total number of turns and hence total number of armature conductors 'Z' required to calculate generated e.m.f.

d) **Pole-pitch :** The distance between the two adjacent poles is called a **pole pitch.** It is measured interms of number of slots. Thus total slots along the periphery of armature divided by the total number of poles is called a pole pitch.

7.5 E.M.F. Equation of D.C. Generator

➤ ***With usual notations derive the e.m.f. equation of a d.c. generator.***

VTU : Jan.-04, 06, 15, 16, 17, 18; Aug.-05, June-12, July- 16, 17; Marks 7

Let P = Number of poles of the generator

ϕ = Flux produced by each pole in webers (Wb)

N = Speed of armature in r.p.m.

Z = Total number of armature conductors

A = Number of parallel paths in which the 'Z' number of conductors are divided

- So A = P for **lap** type of winding and A = 2 for **wave** type of winding
- Induced e.m.f. gets induced in the conductor according to Faraday's law of electromagnetic induction. Hence average value of e.m.f. induced in each armature conductor is,

$$e = \text{Rate of cutting the flux} = \frac{d\phi}{dt}$$

- Consider one revolution of conductor. In one revolution, conductor will cut total flux produced by all the poles i.e. $\phi \times P$. While time required to complete one revolution is $\frac{60}{N}$ seconds as speed is N r.p.m.

$$\therefore \quad e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

... The e.m.f. induced in one conductor

- Now the conductors in one parallel path are always in series. There are total Z conductors with A parallel paths, hence $\frac{Z}{A}$ number of conductors are always in series and e.m.f. remains same across all the parallel paths.

$\therefore$ Total e.m.f. can be expressed as, $E = \phi P \frac{N}{60} \times \frac{Z}{A}$ volts

- This is nothing but the e.m.f. equation of a d.c. generator.

So, $\boxed{E = \frac{\phi PNZ}{60A} \quad \text{e.m.f. equation with A} = \text{P for Lap and A} = 2 \text{ for Wave}}$

Ex. 7.5.1 *A 4 pole, 1500 r.p.m. d.c. generator has a lap wound armature having 24 slots with 10 conductors per slot. If the flux per pole is 0.04 Wb, calculate the e.m.f. generated in the armature. What would be the generated e.m.f. if the winding is wave connected ?*

VTU : Jan.-09, 18, Marks 6

Sol. : P = 4, N = 1500 r.p.m., Lap i.e. A = P,

ϕ = 0.04 Wb

$$Z = \text{Slots} \times \text{Conductors / Slot}$$

$$= 24 \times 10 = 240$$

$$\therefore \quad E_g = \frac{\phi PNZ}{60 A} = \frac{0.04 \times 4 \times 1500 \times 240}{60 \times 4}$$

$$= \mathbf{240\ V}$$

If winding is wave connected, A = 2

$$\therefore \quad E_g = \frac{0.04 \times 4 \times 1500 \times 240}{60 \times 2} = \mathbf{480\ V}$$

Ex. 7.5.2 *A 4 pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 weber. At what speed must the armature rotate to give an induced emf of 220 V ? What will be the voltage developed if the winding is lap and the armature ratates at the same speed ?*

VTU : Jan.-04, 13, Marks 6; July 15

Sol. : P = 4, wave hence A = 2, 51 slots, 24 conds/slot ϕ = 0.01 Wb, E_g = 220 V

$$\therefore \quad E_g = \frac{\phi PNZ}{60A} \quad \text{where } Z = 51 \times 24 = 1224$$

$$\therefore \quad 220 = \frac{0.01 \times 4 \times N \times 1224}{60 \times 2}$$

i.e. $\quad N$ = **539.2156 r.p.m.** ... Speed for 220 V

For lap wound,

A = P = 4 and N = 539.2156 r.p.m.,

$$\therefore \quad E_g' = \frac{\phi PNZ}{60A} = \frac{0.01 \times 4 \times 539.2156 \times 1224}{60 \times 4} = \mathbf{110\ V}$$

Ex. 7.5.3 *An 8-pole, lap-connected armature has 40 slots with 12 conductors per slot, generates a voltage of 500 V. Determine the speed at which it is running if the flux per pole is 50 mWb.*

VTU : June-12, Marks 6

Sol. : P = 8, Lap A = P, E_g = 500 V,

ϕ = 50 mWb

$$Z = \text{No. of slots} \times \text{conductors/slot}$$

$$= 40 \times 12 = 360$$

$$E_g = \frac{\phi\ PNZ}{60\ A} \quad \text{i.e.} \quad 500 = \frac{50 \times 10^{-3} \times 8 \times N \times 360}{60 \times 8}$$

$\therefore \quad N$ = **1666.67 r.p.m.** ...Speed

Ex. 7.5.4 *An 8-pole generator has 500 armature conductors and has a useful flux per pole of 0.065 Wb. What will be the e.m.f. generated if it is lap connected and runs at 1000 r.p.m. ? What must be the speed at which it is to be driven to produce the same e.m.f if it is wave wound ?*

VTU : Jan.-14, 16, Marks 8

Sol. : P = 8, Z = 500, ϕ = 0.065 Wb, lap A = P,

N = 1000 r.p.m.

$$\therefore \quad E_g = \frac{\phi\ PNZ}{60\ A} = \frac{0.065 \times 8 \times 1000 \times 500}{60 \times 8} = \mathbf{541.667\ V}$$

Now it is wave connected, A = 2, E_g = 541.667 V

$$\therefore \quad 541.667 = \frac{0.065 \times 8 \times N' \times 500}{60 \times 2}$$

i.e. $\quad N'$ = **250 r.p.m.**

Ex. 7.5.5 *A 4-pole, lap connected D.C. generator has 600 armature conductors and runs at 1200 rpm. If the flux per pole is 0.06 Wb, calculate the e.m.f. induced. Also find the speed at which it should be driven to produce same e.m.f. when wave connected.*

VTU : Jan.-17, Marks 5

Sol. : P = 4, Lap, A = P, Z = 600, N = 1200 rpm,

ϕ = 0.06 Wb

$$E_g = \frac{\phi PNZ}{60A} = \frac{0.06 \times 4 \times 1200 \times 600}{60 \times 4} = \mathbf{720\ V}$$

To find N, for same E_g and wave connection

i.e. A = 2

$$\therefore 720 = \frac{0.06 \times 4 \times N \times 600}{60 \times 2} \quad \text{i.e. } N = \mathbf{600\ r.p.m.}$$

7.6 Symbolic Representation of D.C. Generator

- The armature is denoted by a circle with two brushes. Mechanically it is connected to another device called prime mover. The two ends of armature are denoted as A_1-A_2.
- The field winding is shown near armature and the two ends are denoted as F_1- F_2. The representation of field vary little bit, depending on the type of generator.

- The symbolic representation is shown in the Fig. 7.6.1. Many times an arrow (↑) is indicated near armature.

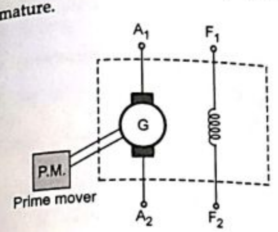


Fig. 7.6.1 Symbolic representation of D.C. generator

- This arrow denotes the direction of current which induced e.m.f. will set up, when connected to an external load.
- Every practical generator needs a **prime mover** to rotate its armature. Hence to avoid complexity of the diagram, prime mover need not be included in the symbolic representation of generator.

7.7 Types of D.C. Generators

➤ ***Explain the classification of d.c. generators.***

VTU : Jan.-18, Marks 4

- The field winding is also called **exciting winding** and current carried by the field winding is called an **exciting current.**
- Thus supplying current to the field winding is called excitation and the way of supplying the exciting current is called **method of excitation.**
- Depending on the method of excitation used, the d.c. generators are classified as,

1. Separately excited generator
2. Self excited generator

- In **separately excited generator**, a separate external d.c. supply is used to provide exciting current through the field winding.
- The d.c. generator produces d.c. voltage. If this generated voltage itself is used to excite the field winding of the same d.c. generator, it is called **self excited generator.**
- The Fig. 7.7.1 shows the various types of d.c. generators.

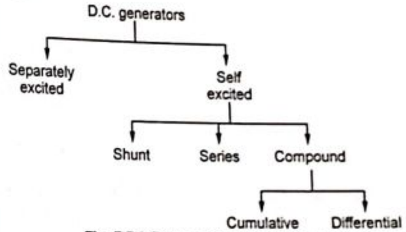


Fig. 7.7.1 Types of d.c. generators

7.8 Separately Excited Generator

➤ ***Draw the schematic representation of a seperately excited d.c. generator. State its voltage and current equations.***

- When the field winding is supplied from external, separate d.c. supply i.e. excitation of field winding is separate then the generator is called separately excited generator.
- The schematic representation of this type is shown in the Fig. 7.8.1.
- The field winding of this type of generator has large number of turns of thin wire. So length of

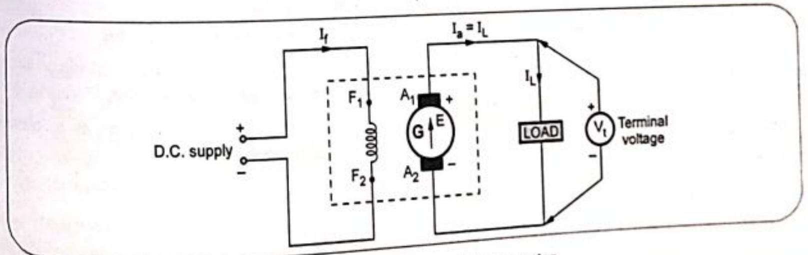


Fig. 7.8.1 Separately excited generator

such winding is more with less cross sectional area. So resistance of this field winding is high in order to limit the field current.

- In the terminology of a d.c. machine the various currents are denoted as,

I_a = Armature current

I_L = Load current

I_f = Field current

Voltage and Current Relations :

- The field winding is excited separately, so the field current depends on supply voltage and resistance of the field winding.
- For armature side, we can see that it is supplying a load, demanding a load current of I_L at a voltage of V_t which is called **terminal voltage**.

Now, $\boxed{I_a = I_L}$

- The internally induced e.m.f. E is supplying the voltage to the load hence terminal voltage V_t is a part of E.
- But E is not equal to V_t while supplying a load. This is because when armature current I_a flows through armature winding, due to armature winding resistance R_a ohms, there is a voltage drop across armature winding equal to $I_a R_a$ volts. The induced e.m.f. has to supply this drop, along with the terminal voltage V_t. To keep $I_a R_a$ drop to minimum, the resistance R_a is designed to be very very small.
- In addition to this drop, there is some voltage drop at the contacts of the brush called brush contact drop. But this drop is negligible and hence generally neglected.
- When armature carries current, it produces its own flux which distorts the main flux. Due to this, there is small voltage drop called armature reaction drop. But as small, this drop is also practically neglected.
- So in all, induced e.m.f. E has components namely,

i) Terminal voltage V_t

ii) Armature resistance drop $I_a R_a$

iii) Brush contact drop V_{brush}

iv) Armature reaction drop

- So voltage equation for separately excited generator can be written as,

$$\boxed{E = V_t + I_a R_a + V_{brush} + \text{Armature reaction drop}}$$

where $E = \frac{\phi P N Z}{60 A}$ = Generated e.m.f.

- Generally V_{brush} is taken as 1 V per brush but many times it is neglected.

7.9 Self Excited Generator

> ***State the causes of failure of self excited generator.***

- When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator.
- Unless generator produces some voltage, such generator can not be excited. This is possible because of **residual magnetism** in case of self excited generators.
- Practically though the generator is not working, without any current through field winding, the field poles possess some magnetic flux. This is called **residual flux** and the property is called **residual magnetism.**
- Thus when the generator is started, due to such residual flux, it develops a small e.m.f. which now drives a small current through the field winding. This tends to increase the flux produced. This inturn increases the induced e.m.f. This further increases the field current and the flux. The process is cumulative and continues till the generator develops rated voltage across its armature. This is **voltage building process** in self excited generators.
- The following are the causes of failure of self excited generators,

1. Absence of residual magnetism.
2. Wrong connections of field winding so as to cancel the residual magnetism.
3. Driven in opposite direction so as to cancel the residual magnetism.

- Based on how field winding is connected to the armature to derive its excitation, this type is further divided into following three types :

i) Shunt generator

ii) Series generator

iii) Compound generator

7.10 Shunt Generator

> ***Draw the circuit diagram of a d.c. generator. State its voltage and current equations.***

VTU : Jan.-04, 06, Marks 2

- When the field winding is connected in parallel with the armature and the combination across the load then the generator is called **shunt generator.**
- The Fig. 7.10.1 shows the symbolic representation of d.c shunt generator.

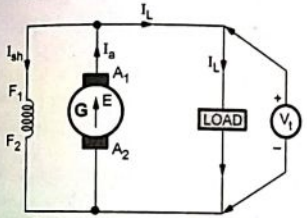


Fig. 7.10.1 D.C. shunt generator

- The field winding has large number of turns of thin wire so it has high resistance denoted as R_{sh}.
- From the Fig. 7.10.1, we can write

$$\boxed{I_a = I_L + I_{sh}}$$

- Now voltage across load is V_t which is same across field winding as both are in parallel with each other.

$$\therefore \quad \boxed{I_{sh} = \frac{V_t}{R_{sh}}}$$

- While induced e.m.f. E, still requires to supply voltage drop $I_a R_a$ and brush contact drop.

$$\therefore \quad \boxed{E = V_t + I_a R_a + V_{brush}}$$

- Armature reaction drop is practically neglected.

Power developed :

- The power developed by armature is given by the product of induced e.m.f. E and armature current I_a.

$$\therefore \quad \boxed{\text{Power developed in armature} = E I_a \text{ W}}$$

- While power available to the load is $V_t I_L$.

Ex. 7.10.1 *The emf generated in the armature of a shunt generator is 625 volts, when delivering its full load current of 400 A to the external circuit. The field current is 6 amp and the armature resistance is 0.06 Ω. What is the terminal voltage ?*

VTU : June-13, Marks 5

Sol. : The generator is shown in the Fig. 7.10.2.

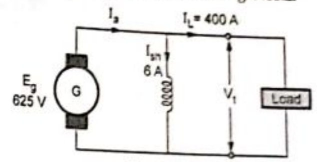


Fig. 7.10.2

$$I_L = 400 \text{ A}, I_{sh} = 6 \text{ A},$$

$$I_a = I_L + I_{sh} = 406 \text{ A}$$

$$R_a = 0.06 \ \Omega$$

$$\therefore \quad V_t = E_g - I_a R_a = 625 - 406 \times 0.06$$

$$= 600.64 \text{ V}$$

Ex. 7.10.2 *A 30 kW, 300 V DC shunt generator has armature and field resistances of 0.05 ohm and 100 ohm respectively. Calculate the total power developed by the armature when it delivers full output power.*

VTU : Jan. 15, July-17, Marks 8

Sol. : The generator is shown in the Fig. 7.10.3

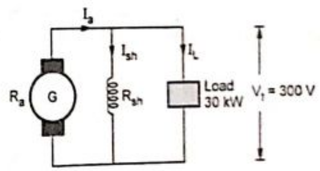


Fig. 7.10.3

$$I_L = \frac{P_{out}}{V_t} = \frac{30 \times 10^3}{300} = 100 \text{ A}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{300}{100} = 3 \text{ A},$$

$I_a = I_L + I_{sh} = 103$ A

$E = V_t + I_a R_a = 300 + 103 \times 0.05 = 305.15$ V

Total power developed by armature

$= E \times I_a = 305.15 \times 103 = 31.4304$ kW

7.11 Series Generator

> ***Draw the circuit diagram of d.c. series generator. State its voltage and current equations.*** VTU : Jan.-04, 06, Marks 2

- When the field winding is connected in series with the armature winding while supplying the load then the generator is called **series generator.**
- It is shown in the Fig. 7.11.1.

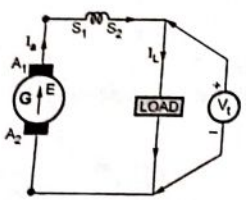


Fig. 7.11.1 Series generator

- The field winding, in this case is denoted as S_1 and S_2.
- The resistance of series field winding is very small and hence naturally it has less number of turns of thick cross-section wire as shown in the Fig. 7.11.1
- Let R_{se} be the resistance of the series field winding.
- As all armature, field and load are in series they carry the same current.

∴ $\boxed{I_a = I_{se} = I_L}$ where I_{se} = Current through series field winding.

- Now in addition to drop $I_a R_a$, induced e.m.f. has to supply voltage drop across series field winding too. This is $I_{se} R_{se}$ i.e. $I_a R_{se}$ as $I_a = I_{se}$. So voltage equation can be written as,

∴ $$\boxed{E = V_t + I_a R_a + I_a R_{se} + V_{brush}}$$

Ex. 7.11.1 *A d.c. series generator has armature resistance of 0.5 Ω and series field resistance of 0.03 Ω. It drives a load of 50 A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500 r.p.m., calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2 mWb and total brush drop as 2 V.*

Sol. : Consider the series generator as shown in Fig. 7.11.2.

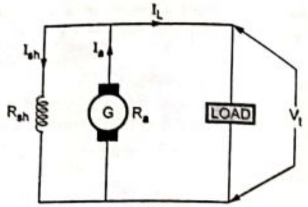


Fig. 7.11.2

$R_a = 0.5\ \Omega$, $R_{se} = 0.03\ \Omega$,

$V_{brush} = 2$ V, N = 1500 r.p.m.

Total coils are 540 with 6 turns/coil.

i.e. Total turns = 540 × 6 = 3240

∴ Total conductors Z = 2 × Turns = 2 × 3240 = 6480

∴ $$E = \frac{\phi P N Z}{60 A}$$

For lap type, A = P

and $\phi = 2$ mWb $= 2 \times 10^{-3}$ Wb

∴ $$E = \frac{2 \times 10^{-3} \times 1500 \times 6480}{60} = 324 \text{ V}$$

$$E = V_t + I_a (R_a + R_{se}) + V_{brush}$$

... Total V_{brush} given

where $I_a = I_L = 50$ A

∴ $324 = V_t + 50\,(0.5 + 0.03) + 2$

i.e. $V_t = 295.5$ V

7.12 Compound Generator

> ***What is compound generator ? State its two types.***

- In this type, the part of the field winding is connected in parallel with armature and part in series with the armature.
- Both series and shunt field windings are mounted on the same poles.
- Depending upon the connection of shunt and series field winding, compound generator is further classified as :

i) Long shunt compound generator and

ii) Short shunt compound generator

7.12.1 Long Shunt Compound Generator

> ***Draw the circuit diagram of long shunt compound generator. Write its voltage and current equations.*** VTU : Jan.-04, 06, Marks 2

- In this type, shunt field winding is connected across the entire series combination of armature and series field winding as shown in the Fig. 7.12.1.

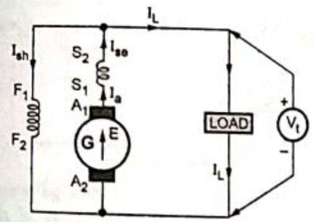


Fig. 7.12.1 Long shunt compound generator

- From the Fig. 7.12.1, $I_a = I_{se}$.

$$\boxed{I_a = I_{sh} + I_L}$$

Voltage across shunt field winding is V_t.

$$\boxed{I_{sh} = \frac{V_t}{R_{sh}} \text{ where } R_{sh} = \text{Resistance of shunt field winding}}$$

- And voltage equation is,

∴ $$\boxed{E = V_t + I_a R_a + I_a R_{se} + V_{brush}}$$ where R_{se} = Resistance of series field winding

7.12.2 Short Shunt Compound Generator

> ***Draw the circuit diagram of short shunt compound generator. Write its voltage and current equations.*** VTU : Jan.-04, 06, Marks 2

- In this type, shunt field winding is connected, only across the armature, excluding series field winding as shown in the Fig. 7.12.2.

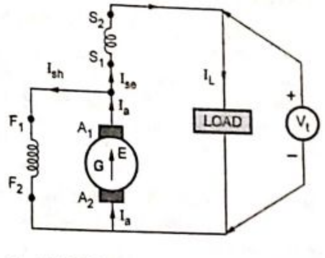


Fig. 7.12.2 Short shunt compound generator

- For the Fig. 7.12.2, $I_a = I_{se} + I_{sh}$

and $I_{se} = I_L$

∴ $$\boxed{I_a = I_L + I_{sh}}$$

- The drop across shunt field winding is drop across the armature only and not the total V_t, in this case.
- So drop across shunt field winding is $E - I_a R_a$.

∴ $$\boxed{I_{sh} = \frac{E - I_a R_a}{R_{sh}}}$$

- Now the voltage equation is $E = V_t + I_a R_a + I_{se} R_{se} + V_{brush}$

Now, $$\boxed{\begin{aligned} & I_{se} = I_L \\ & \text{hence } E = V_t + I_a R_a + I_L R_{se} + V_{brush} \end{aligned}}$$

- Neglecting V_{brush}, we can write,

$E = V_t + I_a R_a + I_L R_{se}$ i.e. $E - I_a R_a = V_t + I_L R_{se}$

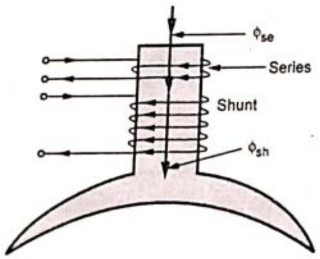
$$\therefore \quad \boxed{I_{sh} = \frac{V_t + I_L R_{se}}{R_{sh}}}$$

- Any of the two above expressions of I_{sh} can be used, depending on the quantities known while solving the problems.

7.12.3 Cumulative and Differential Compound Generator

➢ ***What are cumulative and differential compound generators ?***

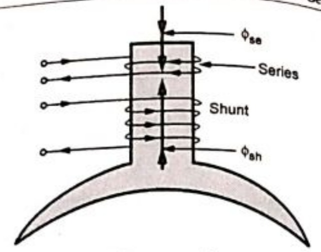
- The two windings, shunt and series field are wound on the same poles.
- Depending on the direction of winding on the pole, two fluxes produced by shunt and series field may help or may oppose each other. This fact decides whether generator is cumulative or differential compound.
- If the two fluxes help each other as shown in Fig. 7.12.3 (a) the generator is called **cumulative compound generator.**



(a) Cumulative compound generator

Fig. 7.12.3

- If the two windings are wound in such a direction that the fluxes produced by them oppose each other then the generator is called **differential compound generator.** This is shown in the Fig. 7.12.3 (b)



(b) Differential compound generator

Fig. 7.12.3

Ex. 7.12.1 *A long shunt d.c. compound generator drives 20 lamps, all are connected in parallel. Terminal voltage is 550 V with each lamp resistance as 500 Ω. If R_{sh} = 25 Ω, R_a = 0.06 Ω and R_{se} = 0.04 Ω, calculate the armature current and the generated e.m.f.*

Sol. : Consider the arrangement as shown in the Fig. 7.12.4 (a).

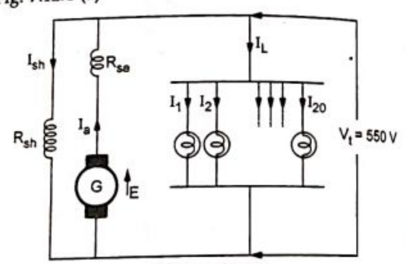


Fig. 7.12.4 (a)

As all lamps are in parallel, the voltage across all of them is same which is terminal voltage of generator V_t = 550 V.

Consider only one lamp as shown in the Fig. 7.12.4 (b).

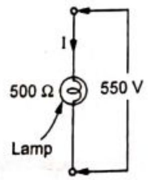


Fig. 7.12.4 (b)

So current drawn by each lamp is

$$I = \frac{V_t}{R_{lamp}} = \frac{550}{500} = 1.1 \text{ A}$$

Such 20 lamps are used as a load.

$\therefore \quad I_L = 20 \times I_{lamp} = 20 \times 1.1 = 22$ A

Now $\quad I_{sh} = \frac{V_t}{R_{sh}} = \frac{550}{25} = 22$ A

$\therefore \quad I_a = I_L + I_{sh} = \mathbf{44\ A}$

$\therefore \quad E = V_t + I_a R_a + I_a R_{se}$

$= 550 + 44 \times 0.06 + 44 \times 0.04$

$= \mathbf{554.4\ V}$

Ex. 7.12.2 *A short shunt compound d.c. generator supplies a current of 75 A at a voltage of 225 V. Calculate the generated voltage if the resistance of armature, shunt field and series field windings are 0.04 Ω, 90 Ω and 0.02 Ω respectively.*

Sol. : Consider a short shunt generator as shown in the Fig. 7.12.5.

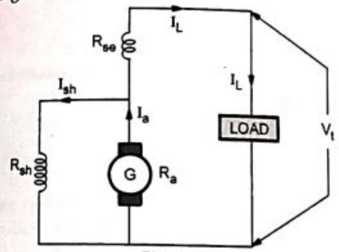


Fig. 7.12.5

$R_a = 0.04\ \Omega, \quad R_{sh} = 90\ \Omega,$

$R_{se} = 0.02\ \Omega$

$V_t = 225$ V

$I_L = 75$ A

$I_a = I_L + I_{sh}$

Now $\quad E = V_t + I_a R_a + I_L R_{se}$

and drop across armature terminals is,

$E - I_a R_a = V_t + I_L R_{se}$

$= 225 + 75 \times 0.02 = 226.5$ V

$\therefore \quad I_{sh} = \frac{E - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}}$

$= \frac{226.5}{90} = 2.5167$ A

$\therefore \quad I_a = I_L + I_{sh} = 75 + 2.5167 = 77.5167$ A

$\therefore \quad E = V_t + I_a R_a + I_L R_{se}$

$= 225 + 77.5167 \times 0.04 + 75 \times 0.02$

$= 229.6$ V

7.13 Applications of Various Types of D.C. Generators

➢ ***State the applications of various types of d.c. generators.***

Separately excited generators :

- As a separate supply is required to excite field, the use is restricted to some special applications like electro-plating, electro-refining of materials etc.

Shunt generators :

- Commonly used in battery charging and ordinary lighting purposes.

Series generators :

- Commonly used as boosters on d.c. feeders, as a constant current generators for welding generator and arc lamps.

Cumulatively compound generators :

- These are used for domestic lighting purposes and to transmit energy over long distance.

Differential compound generator :

- The use of this type of generators is very rare and it is used for special application like electric arc welding.

7.14 Efficiency of a D.C. Machine

➢ ***Define and give the expressions for electrical, mechanical and commercial efficiencies for a d.c. generator.***

- The following efficiencies can be defined for a d.c. generator.

1. Mechanical efficiency :

- It is defined as the ratio of total watts generated $(E_g I_a)$ in the armature to the total mechanical power supplied by the prime mover.

- The power supplied by the prime mover is the output of the driving machine.

$$\% \text{ Mechanical efficiency } (\eta_m) = \frac{E_g\ I_a}{\text{Output of driving machine}} \times 100$$

2. Electrical efficiency :

- It is defined as the ratio of total watts available in the load circuit ($V_t\ I_L$) to the total watts generated in the armature ($E_g\ I_a$).

$$\% \text{ Electrical efficiency } (\eta_e) = \frac{V_t\ I_L}{E_g\ I_a} \times 100$$

3. Commercial efficiency :

- It is defined as the ratio of total watts available in the load circuit ($V_t\ I_L$) to the total mechanical power supplied by the prime mover.

$$\% \text{ Commercial efficiency } (\eta_c) = \frac{V_t\ I_L}{\text{Output of the driving machine}} \times 100$$

- The overall efficiency is $\eta_c = \eta_e \times \eta_m$.
- Practically the efficiency of a generator is nothing but the commercial or overall efficiency and can also be expressed as,

$$\% \eta = \frac{\text{Total output}}{\text{Total input}} \times 100$$

7.15 University Questions with Answers

June - 2010

Q.1 *Draw the neat sketch representing the cut section of a d.c. machine. Explain the important features of different parts involved there on.* **[Refer section 7.3]** **[5]**

July - 2011

Q.2 *Draw the neat sketch representing the cut section of a d.c. machine. Explain the important features of different parts involved there on.* **[Refer section 7.3]** **[5]**

Dec. - 2011

Q.3 *What are the functions of yoke, armature, poles and brushes in d.c. generator ?* **[Refer section 7.3]** **[4]**

June - 2012

Q.4 *With usual notations derive the e.m.f. equation of a d.c. generator.* **[Refer section 7.5]** **[7]**

Jan. - 2015

Q.5 *With usual notations derive the e.m.f. equation of a d.c. generator.* **[Refer section 7.5]** **[7]**

July - 2015

Q.6 *With a neat sketch explain the construction of a d.c. machine.* **[Refer section 7.3]** **[8]**

Jan. - 2016

Q.7 *With a neat diagram showing important parts of DC machine and explain important features of the parts shown.* **[Refer section 7.3]** **[8]**

Q.8 *Derive the emf equation of a DC generator.* **[Refer section 7.5]** **[8]**

CBCS scheme

Jan. - 2016

Q.9 *Derive the EMF equation of DC generator.* **[Refer section 7.5]** **[4]**

July - 2016

Q.10 *With a neat sketch, explain the construction of the various parts of a D.C. generator.* **[Refer section 7.3]** **[5]**

Q.11 *Derive the EMF equation of a D.C. generator.* **[Refer section 7.5]** **[5]**

Jan. - 2017

Q.12 *With a neat sketch, explain the construction of various parts of a D.C. machine.* **(Refer section 7.3)** **[5]**

Q.13 *Derive e.m.f. equation for D.C. generator.* **(Refer section 7.5)** **[6]**

July - 2017

Q.14 *Derive emf equation for a DC Generator.* **(Refer section 7.5)** **[5]**

Jan. - 2018

Q.15 *Draw a neat cross section of a d.c. machine and explain the functions of each part.* **(Refer section 7.3)** **[8]**

Q.16 *Derive emf equation of a d.c. generator and mention the classification of d.c. generators.* **(Refer sections 7.5 and 7.7)** **[8]**

❑❑❑

Notes

MODULE - 4

8

D.C. Motors

Syllabus

Principle of operation, Back emf, Torque equation, Types of dc motors, Characteristics of dc motors (shunt and series motors only) and Applications.

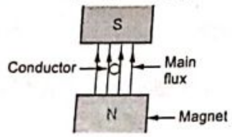
Contents

8.1 Principle of Operation of a D.C. Motor

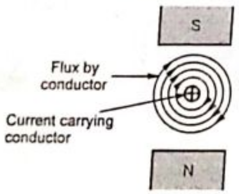
➤ ***Explain the working principle of d.c. motor.*** VTU : Jan.-14, Marks 4

➤ ***Explain how torque is produced in a d.c. motor.*** VTU : June-12, Marks 4

- The principle of operation of a d.c. motor can be stated in a single statement as 'when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force'.
- In a practical d.c. motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductors and hence armature conductors experience a force.
- Consider a single conductor placed in a magnetic field as shown in the Fig. 8.1.1 (a).



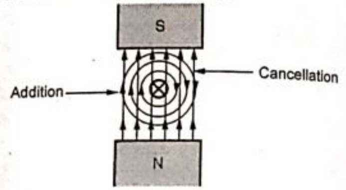
(a) Conductor in a magnetic field



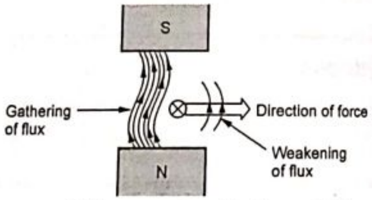
(b) Flux produced by current carrying conductor

Fig. 8.1.1

- Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an observer as shown in the Fig. 8.1.1 (b).
- Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered, the direction of flux around a conductor is clockwise. For simplicity of understanding, the main flux produced by the permanent magnet is not shown in the Fig. 8.1.1(b).
- Now there are two fluxes present,
  1. The flux produced by the permanent magnet called main flux.
  2. The flux produced by the current carrying conductor.
- These are shown in the Fig. 8.1.2 (a).
- From this, it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case, on the left of the conductor there is gathering of the flux lines as two fluxes help each other.
- As against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened. So on the left, there exists high flux density area while on the right of the conductor there exists low flux density area as shown in the Fig. 8.1.2 (b).



(a) Interaction of two fluxes



(b) Force experienced by the conductor

Fig. 8.1.2

- This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right for the case considered as shown in the Fig. 8.1.2.(b).
- In the practical d.c. motor, the permanent magnet is replaced by a field winding which produces the required flux called main flux and all the armature conductors, mounted on the periphery of the armature drum, get subjected to the mechanical force.

- Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.
- The magnitude of the force experienced by the conductor in a motor is given by,

$$\boxed{F = B\,l\,I \quad \text{Newtons (N)}}$$

B = Flux density due to the flux produced by the field winding.

l = Active length of the conductor.

I = Magnitude of the current passing through the conductor.

- The direction of such force i.e. the direction of rotation of a motor can be determined by **Fleming's left hand rule.**
- To reverse the direction of rotation of a d.c. motor, either direction of main field produced by the field winding is reversed or direction of the current passing through the armature is reversed.

8.2 Back E.M.F. in a D.C. Motor

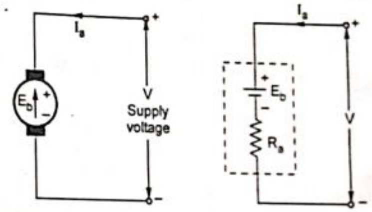
➤ ***What is back e.m.f. ? State its significance.*** VTU : Feb.-05, Jan.-11, July-07, 08, 11, 15, 16, Marks 6

- It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f. gets induced in the conductor. In a d.c. motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. So is there a generating action existing in a motor after motoring action.
- There is an induced e.m.f. in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced e.m.f. in the armature always acts in the opposite direction to the supply voltage. This is according to the **Lenz's law** which states that the direction of the induced e.m.f. is always so as to oppose the cause producing it.
- In a d.c. motor, electrical input i.e. the supply voltage is the cause for the armature current and the motoring action and hence this induced e.m.f. opposes the supply voltage. This e.m.f. tries to set up a current through the armature which is in the opposite direction to that, which supply voltage is forcing through the conductor.
- As this e.m.f. always opposes the supply voltage, it is called **back e.m.f.** and denoted as E_b. Though it is denoted as E_b, basically it gets generated by the generating action which we have seen earlier in case of generators. So its magnitude can be determined by the e.m.f. equation which is derived earlier. So,

$$\boxed{E_b = \frac{\phi P N Z}{60\,A} \quad \text{volts}}$$

where all symbols carry the same meaning as seen earlier in case of generators.

- This e.m.f. is shown schematically in the Fig. 8.2.1 (a). So if V is supply voltage in volts and R_a is the value of the armature resistance, the equivalent electric circuit can be shown as in the Fig. 8.2.1 (b).



(a) Back e.m.f. in a d.c. motor (b) Equivalent circuit

Fig. 8.2.1

8.2.1 Voltage Equation of D.C. Motor

- From the equivalent circuit, the voltage equation for a d.c. motor can be obtained as,

$$\boxed{V = E_b + I_a R_a + \text{Brush drop}}$$

- The brush drop is practically negelected.
- Hence the armature current I_a can be expressed as,

$$\boxed{I_a = \frac{V - E_b}{R_a}}$$

8.2.2 Significance of Back E.M.F.

- Due to the presence of back e.m.f. the d.c. motor becomes a regulating machine i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand.
- The basic principle of this fact is that the back e.m.f. is proportional to speed, $E_b \propto N$.

- When load is suddenly put on to the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f. also decreases. So the net voltage across the armature ($V - E_b$) increases and motor draws more armature current.
- Due to the increased armature current, force experienced by the conductors and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand.
- When load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f. increases. This causes ($V - E_b$) to reduce which eventually reduces the current drawn by the armature. The motor speed stops increasing when the armature current is just enough to produce the less torque required by the new load.
- So back e.m.f. regulates the flow of armature current and it automatically alters the armature current to meet the load requirement. This is the practical significance of the back e.m.f.
- **At start the speed N of the motor is zero hence the back e.m.f. is also zero.**

Ex. 8.2.1 *A 220 V, d.c. motor has an armature resistance of 0.75 Ω. It is drawing an armature current of 30 A, driving a certain load. Calculate the induced e.m.f. in the motor under this condition.*

Sol. : $V = 200$ V, $I_a = 30$ A, $R_a = 0.75\ \Omega$ are the given values.

For a motor, $V = E_b + I_a R_a$ i.e. $220 = E_b + 30 \times 0.75$

$\therefore \quad E_b = $ **197.5 volts**

This is the induced e.m.f. called back e.m.f. in a motor.

Ex. 8.2.2 *Find the useful flux per pole on no-load of a 250 V, 6 pole shunt motor having a two circuit connected armature winding with 220 conductors. At normal working temperature, the overall armature resistance including brushes is 0.2 Ω. The armature current is 13.3 A at the no-load speed of 908 r.p.m.*

VTU : Feb.-99, Marks 6

Sol. : $V = 250$ V, $P = 6$, $Z = 220$, $A = 2$ as two circuit armature

$$R_a = 0.2\ \Omega,\ I_a = 13.3\ \text{A},\ N = 908\ \text{r.p.m.}$$

For a d.c. shunt motor,

$$V = E_b + I_a R_a$$

$$\therefore \quad 250 = E_b + 13.3 \times 0.2 \quad \text{i.e. } E_b = 247.34\ \text{V}$$

Back e.m.f. E_b is given by,

$$E_b = \frac{\phi P N Z}{60\,A}$$

$$\text{i.e.} \quad 247.34 = \frac{\phi \times 6 \times 908 \times 220}{60 \times 2}$$

$$\therefore \quad \phi = \mathbf{24.76\ \ mWb}$$

8.3 Power Equation of a D.C. Motor

➤ ***State the power equation of a d.c. motor.***

- The voltage equation of a d.c. motor is given by,

$$V = E_b + I_a R_a$$

- Multiplying both sides of the above equation by I_a we get,

$$\boxed{VI_a = E_b I_a + I_a^2 R_a}$$

- This equation is called **power equation** of a d.c. motor.

VI_a = Net electrical power input to the armature measured in watts.

$I_a^2 R_a$ = Power loss due to the resistance of the armature called **armature copper loss.**

- So difference between VI_a and $I_a^2 R_a$ i.e. input - losses gives the output of the armature.

So $E_b I_a$ is called **electrical equivalent of gross mechanical power developed** by the armature. This is denoted as P_m.

∴ Power input to the armature – Armature copper loss = Gross mechanical power developed in the armature.

8.4 Torque Equation of a D.C. Motor

➤ ***Derive the expression of armature torque developed in a d.c. motor.***

VTU : Jan.-03, 05, 09, 13, 14, 15, July-03, 07, 16, Marks 6, Dec. 11

- The turning or twisting force about an axis is called torque.

- Consider a wheel of radius R meters acted upon by a circumferential force F newtons as shown in the Fig. 8.4.1.

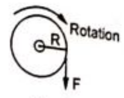


Fig. 8.4.1

- The wheel is rotating at a speed of N r.p.m. then its angular speed is,

$$\omega = \frac{2\pi N}{60} \ \text{rad/sec}$$

- So workdone in one revolution is,

$$W = F \times \text{Distance travelled in one revolution}$$

$$= F \times 2\pi R \ \text{Joules}$$

$$P = \text{Power developed} = \frac{\text{Workdone}}{\text{Time}}$$

$$= \frac{F \times 2\pi R}{\text{Time for 1 rev}} = \frac{F \times 2\pi R}{\left(\frac{60}{N}\right)}$$

$$= (F \times R) \times \left(\frac{2\pi N}{60}\right)$$

$$\therefore \quad \boxed{P = T \times \omega \ \ \text{Watts}}$$

Where T = Torque in Nm and ω = Angular speed in rad/sec.

- Let T_a be the gross torque developed by the armature of the motor. It is also called **armature torque.**
- The gross mechanical power developed in the armature is $E_b I_a$, as seen from the power equation.
- So if speed of the motor is N r.p.m. then,

Power in armature = Armature torque × ω

$$\text{i.e.} \quad E_b I_a = T_a \times \frac{2\pi N}{60}$$

- But E_b in a motor is given by, $E_b = \dfrac{\phi P N Z}{60\,A}$

$$\therefore \quad \frac{\phi P N Z}{60\,A} \times I_a = T_a \times \frac{2\pi N}{60}$$

$$\therefore \quad \boxed{T_a = \frac{1}{2\pi} \phi I_a \times \frac{PZ}{A} = 0.159\, \phi\, Ia \,.\, \frac{PZ}{A}\ \text{Nm}}$$

- This is the **torque equation** of a d.c. motor.

8.4.1 Types of Torque in the Motor

- The mechanical power developed in the armature is transmitted to the load through the shaft of the motor.
- It is impossible to transmit the entire power developed by the armature to the load. This is because while transmitting the power through the shaft, there is a power loss due to the friction, windage and the iron loss.
- The torque required to overcome these losses is called **lost torque,** denoted as T_f. These losses are also called **stray losses.**
- The torque which is available at the shaft for doing the useful work is known as **load torque** or **shaft torque** denoted as T_{sh}.

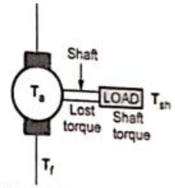


Fig. 8.4.2 Types of torque

$$\therefore \quad \boxed{T_a = T_f + T_{sh}}$$

- The shaft torque magnitude is always less than the armature torque, ($T_{sh} < T_a$).
- The speed of the motor remains same all along the shaft say N r.p.m. Then the product of shaft torque T_{sh} and the angular speed ω rad/sec is called power available at the shaft i.e. net output of the motor. The maximum power a motor can deliver to the load safely is called **output rating** of a motor. Generally it is expressed in H.P. It is called **H.P. rating** of a motor.

$$\boxed{\text{Net output of motor} = P_{out} = T_{sh} \times \omega}$$

8.4.2 No Load Condition of a Motor

- On no load, the load requirement is absent. So $T_{sh} = 0$.
- This does not mean that motor is at halt. The motor can rotate at a speed say N_0 r.p.m. on no load. The motor draws an armature current of I_{a0}.

$$\boxed{I_{a0} = \frac{V - E_{b0}}{R_a}}$$

where E_{b0} is back e.m.f. on no load, proportional to speed N_0.

- Now armature torque T_a for a motor is,

$$T_a \propto \phi I_a$$

- As flux is present and armature current is present, hence T_{a0} i.e. armature torque exists on no load.
- Now $T_a = T_f + T_{sh}$ but on no load, $T_{sh} = 0$

$$\therefore \quad \boxed{T_{a0} = T_f}$$

- So on no load, motor produces a torque T_{a0} which satisfies the friction, windage and iron losses of the motor.

$\therefore$ Power developed $(E_{b0} \times I_{a0})$ = Friction, windage and, iron losses.

where E_{b0} = Back e.m.f. on no load.

and I_{a0} = Armature current drawn on no load.

- This component of stray losses i.e. $E_{b0}I_{a0}$ is practically assumed to be constant though the load on the motor is changed from zero to the full capacity of the motor.

Ex. 8.4.1 *A 4 pole d.c. motor takes a 50 A armature current. The armature has lap connected 480 conductors. The flux per pole is 20 m Wb. Calculate the gross torque developed by the armature of the motor.*

Sol. : $P = 4,\ A = P = 4,\ Z = 480$

$\phi = 20 \text{ mWb} = 20 \times 10^{-3} \text{ Wb},\ I_a = 50 \text{ A}$

Now
$$T_a = 0.159 \times \phi I_a . \frac{PZ}{A}$$
$$= 0.159 \times 20 \times 10^{-3} \times 50 \times \frac{4 \times 480}{4}$$
$$= 76.394 \text{ N-m}$$

Ex. 8.4.2 *A 500 V shunt motor has 4 poles and wave connected winding with 492 conductors. The flux per pole is 0.05 Wb. The full laod current is 20 Amps. The armature and shunt field resistances are 0.1 Ω and 250 Ω respectively. Calculate the speed and the developed torque.*

VTU : Aug.-03, Marks 6

Sol. : $V = 500$ V, $P = 4$, wave, $Z = 492$, $\phi = 0.05$ Wb, $I_L = 20$ A.

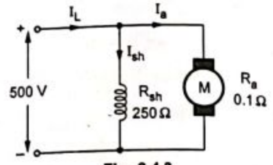


Fig. 8.4.3

$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2 \text{ A}$$

$\therefore \quad I_a = I_L - I_{sh} = 20 - 2 = 18$ A

$\therefore \quad E_b = V - I_a R_a = 500 - 18 \times 0.1 = 498.2$ V

But $\quad E_b = \dfrac{\phi PNZ}{60A}$ where A = 2 for wave type

$\therefore \quad N = \dfrac{60 \times 2 \times 498.2}{0.05 \times 4 \times 492} = 607.56$ **r.p.m.** ...Speed

$$T_a = 0.159\, \phi I_a \left[\frac{ZP}{A}\right]$$
$$= 0.159 \times 0.05 \times 18 \times 492 \times \frac{4}{2}$$

$= 140.8104$ **Nm** ... Torque developed

8.5 Types of D.C. Motors

➤ ***State the various types of d.c. motors.***

- Similar to the d.c. generators, the d.c. motors are classified depending upon the way of connecting the field winding with the armature winding.

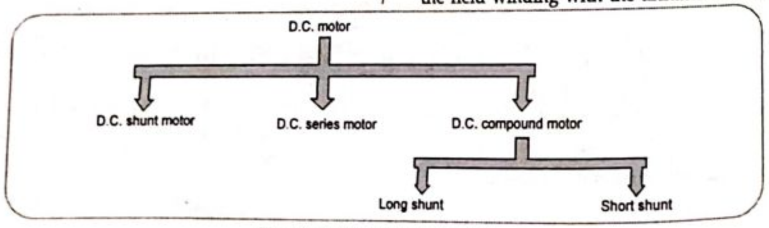


Fig. 8.5.1 Types of d.c. motors

8.5.1 D.C. Shunt Motor

➤ ***Draw the circuit diagram of d.c. shunt motor and give its voltage and current relation.***

- In this type, the field winding is connected across the armature winding and the combination is connected across the supply, as shown in the Fig. 8.5.2.

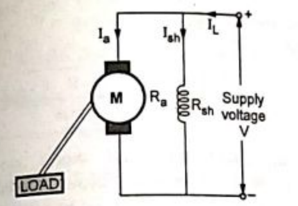


Fig. 8.5.2 D.C. shunt motor

- Let R_{sh} be the resistance of shunt field winding and R_a be the resistance of armature winding.
- The value of R_a is very small while R_{sh} is quite large. Hence shunt field winding has more number of turns with less cross-sectional area.

Voltage and Current Relationship :

- The voltage across armature and field winding is same **equal** to the supply voltage V.
- The total current drawn from the supply is denoted as line current I_L.

$$I_L = I_a + I_{sh} \quad \text{and} \quad I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a R_a + V_{brush}$$

- Now flux produced by the field winding is proportional to the current passing through it i.e. I_{sh}.

$$\boxed{\phi \propto I_{sh}}$$

- As long as supply voltage is constant, which is generally so in practice, the flux produced is constant. Hence ***d.c. shunt motor is called constant flux motor.***

8.5.2 D.C. Series Motor

➤ ***Draw the circuit diagram of d.c. series motor and give its voltage and current relations.***

- In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the Fig. 8.5.3.

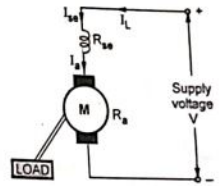


Fig. 8.5.3 D.C. series motor

- Let R_{se} be the resistance of the series field winding then the value of R_{se} is very small and it is made of small number of turns having large cross-section area.

Voltage and Current Relationship :

- Let I_L be the total current drawn from the supply.

So $\quad I_L = I_{se} = I_a$ and

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$
$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

- Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.
- In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to the armature current.

$$\boxed{\phi \propto I_{se} \propto I_a \quad \text{for series motor}}$$

8.5.3 D.C. Compound Motor

- The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature. It is further classified as short shunt compound and long shunt compound motor.

1. Long Shunt Compound Motor :

➤ ***Draw the diagram of long shunt compound d.c. motor and give its voltage and current relations.***

- In this type, the shunt field winding is connected across the combination of armature and the series field winding as shown in the Fig. 8.5.4.

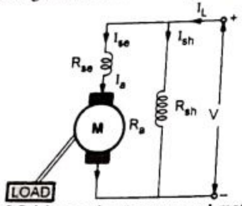


Fig. 8.5.4 Long shunt compound motor

Voltage and Current Relationship :

- Let R_{se} be the resistance of series field and R_{sh} be the resistance of shunt field winding.
- The total current drawn from supply is I_L.

So $\quad I_L = I_{se} + I_{sh}$

But $\quad I_{se} = I_a$ i.e. $I_L = I_a + I_{sh}$ and $I_{sh} = \dfrac{V}{R_{sh}}$

and $\quad V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

but as $\quad I_{se} = I_a$,

$\therefore \quad V = E_b + I_a (R_a + R_{se}) + V_{brush}$

2. Short Shunt Compound Motor :

➤ ***Draw the diagram of short shunt compound d.c. motor and give its voltage and current relations.***

- In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination shown in the Fig. 8.5.5 (a).

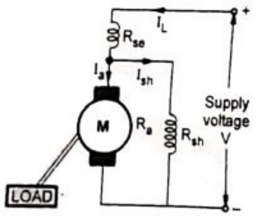


Fig. 8.5.5 (a) Short shunt compound motor

- The entire line current is passing through the series field winding.

$$I_L = I_{se} \quad \text{and} \quad I_L = I_a + I_{sh}$$

- Now the drop across the shunt field winding is to be calculated from the voltage equation.

So $\quad V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$ but $I_{se} = I_L$

$\therefore \quad V = E_b + I_L R_{se} + I_a R_a + V_{brush}$

- Drop across shunt field winding $= V - I_L R_{se} = E_b + I_a R_a + V_{brush}$

$$\therefore \quad \boxed{I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} = \frac{E_b + I_a R_a + V_{brush}}{R_{sh}}}$$

- **A long shunt compound motor can be of cumulative or differential type. Similarly short shunt compound motor can be cumulative or differential type.**

Ex. 8.5.1 *A 4 pole, 220 V, lap connected, D.C. shunt motor has 36 slots, each slot containing 16 conductors. It draws a current of 40 A from the supply. The field resistance and armature resistance are 110 Ω, 0.1 Ω respectively. The motor develops an output power of 6 kW. The flux per pole is 40 mWb. Calculate,*
a) The speed b) The torque developed by the armature and c) The shaft torque. **VTU : Jan. 04, 17; Aug.-04, Marks 7**

Sol. : P = 4, V = 220 V, Lap hence A = P = 4, 36 slots, 16 conds/slot.

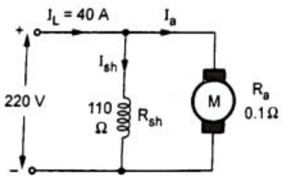


Fig. 8.5.6

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2\text{A}$$

$\therefore \quad I_a = I_L - I_{sh} = 40 - 2 = 38$ A

$P_{out} = 6$ kW, $\phi = 40$ mWb ...Given

$\therefore \quad E_b = V - I_a R_a = 220 - 38 \times 0.1 = 216.2$ V

a) $E_b = \dfrac{\phi PNZ}{60A}$ where $Z = 36 \times 16 = 576$

$\therefore \quad 216.2 = \dfrac{40 \times 10^{-3} \times 4 \times N \times 576}{60 \times 4}$

$\therefore \quad N = \mathbf{563.02}$ **r.p.m.** ... Speed

b) $T_a = 0.159\, \phi I_a \left[\dfrac{PZ}{A}\right]$

$= 0.159 \times 40 \times 10^{-3} \times 38 \times 4 \times \dfrac{576}{4}$

$= \mathbf{139.207}$ **Nm** ...Armature torque

c) $T_{sh} = \dfrac{P_{out}}{\omega} = \dfrac{P_{out}}{\left(\dfrac{2\pi N}{60}\right)} = \dfrac{6 \times 10^3}{\left(\dfrac{2\pi \times 563.02}{60}\right)}$

$= \mathbf{101.7325}$ **Nm** ...Shaft torque

Ex. 8.5.2 *A 200 V, 4 pole, lap wound, d.c. shunt motor has 800 conductors on its armature. The resistance of the armature winding is 0.5 Ω and that of shunt field winding is 200 Ω. The motor takes a current of 21 A, the flux/pole is 30 mWb. Find the speed and gross torque developed in the motor.* **VTU : Jan. 10, 15, 16, 18, Marks 6**

Sol. : V = 200 V, P = 4, Lap i.e. A = P = 4, Z = 800, Shunt motor $R_a = 0.5$ Ω, $R_{sh} = 200$ Ω, $I_L = 21$ A, $\phi = 30$ mWb.

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{200} = 1 \text{ A}$$

$I_L = I_{sh} + I_a$ i.e. $I_a = I_L - I_{sh}$

$\therefore \quad I_a = 21 - 1 = 20$ A

$E_b = V - I_a R_a = 200 - 20 \times 0.5 = 190$ V

But $E_b = \dfrac{\phi PNZ}{60\text{ A}}$

i.e. $190 = \dfrac{30 \times 10^{-3} \times 4 \times N \times 800}{60 \times 4}$

$\therefore \quad N = \mathbf{475}$ **r.p.m.**

Ex. 8.5.3 *A 4 pole DC shunt motor takes 22 A from 220 V supply. The armature and field resistances are respectively 0.5 Ω and 100 Ω respectively. The armature is lap connected with 300 conductors. If the flux per pole is 20 mWb, calculate the speed and gross torque.* **VTU : June-11, Marks 6**

Sol : P = 4, $I_L = 22$ A, V = 220 V, $R_a = 0.5$ Ω, $R_{sh} = 100$ Ω

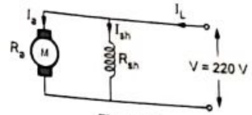


Fig. 8.5.7

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{100} = 2.2 \text{ A}$$

$\therefore \quad I_a = I_L - I_{sh} = 19.8$ A

$\therefore \quad E_b = V - I_a R_a = 210.1$ V

$E_b = \dfrac{\phi PNZ}{60\text{ A}}$ where Z = 300,

A = P = 4 as lap, $\phi = 20$ mWb

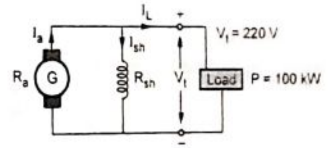
$\therefore \quad 210.1 = \dfrac{20 \times 10^{-3} \times 4 \times N \times 300}{60 \times 4}$

i.e. $\quad N = \mathbf{2101}$ **r.p.m.**

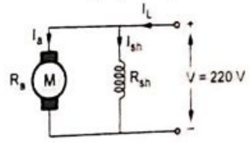
$T_a = 0.159\, \phi I_a \left[\dfrac{PZ}{A}\right] = \mathbf{18.89}$ **Nm.**

Ex. 8.5.4 *A 100 kW belt driven shunt generator running at 300 r.p.m. on 220 V bus-bars, continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed ? Given $R_a = 0.025$ Ω, $R_{sh} = 60$ Ω, BCD = 1 V per brush and ARD = 0.* **VTU : Dec.-11, Marks 6**

Sol. :



(a) N_g = 300 r.p.m.



(b) N_m = Motor speed

Fig. 8.5.8

For generator, $I_{sh} = \frac{V_t}{R_{sh}} = \frac{220}{60} = 3.667$ A, BCD = Brush contact drop

$$I_L = \frac{P}{V_t} = \frac{100\times10^3}{220} = 454.5454 \text{ A}$$

$$\therefore \quad I_a = I_L + I_{sh} = 458.2121 \text{ A}$$

$$\therefore \quad E_g = V_t + I_a R_a + BCD \times 2$$

$$= 220 + 458.2121 \times 0.025 + 2 = 233.455 \text{ V}$$

For motor, $P_{in} = 10$ kW, $I_L = \frac{P_{in}}{V} = \frac{10\times10^3}{220}$

$$= 45.4545 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = 3.667 \text{ A}, \ I_a = I_L - I_{sh}$$

$$= 41.7878 \text{ A}$$

$$\therefore \quad E_b = V - I_a R_a - BCD \times 2$$

$$= 220 - 41.7878 \times 0.025 - 2$$

$$= 216.9553 \text{ V}$$

$E \propto N$ as ϕ is constant hence $\frac{E_g}{E_b} = \frac{N_g}{N_m}$

$$\therefore \quad \frac{233.455}{216.9553} = \frac{300}{N_m}$$

i.e. $N_m = 278.7971$ r.p.m. ...Speed of as motor

Ex. 8.5.5 *A 220 volts series motor is taking a current of 40 amperes. Resistance of armature 0.5 Ω, resistance of series field is 0.25 Ohm.*
Calculate i) Voltage at the brushes ii) Back emf
iii) Power wasted in armature
iv) Power wasted in series field. **VTU : June-13, Marks 8**

Sol. : The motor is shown in the Fig. 8.5.9.

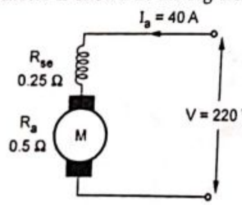


Fig. 8.5.9

i) Voltage at the brushes = $V - I_a R_{se}$

$$= 220 - 40 \times 0.25 = \mathbf{210 \ V}$$

ii) $E_b = V - I_a (R_a + R_{se})$

$$= 220 - 40(0.5 + 0.25) = \mathbf{190 \ V}$$

iii) Power wasted in armature = $I_a^2 R_a$ = **800 W**

iv) Power wasted in field = $I_a^2 R_{se}$ = **400 W**

Ex. 8.5.6 *4 pole DC shunt motor takes 22.5 A from a 250 V supply. $R_a = 0.5\Omega$ and $R_{sh} = 125\ \Omega$. The armature is wave wound with 300 conductors. If the flux per pole is 0.02 Wb. Calculate : i) Speed, ii) Torque developed, iii) Power developed.* **VTU : July-15, 17, Marks 9**

Sol. : $I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2$ A

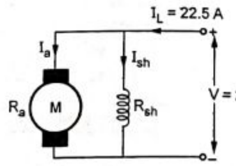


Fig. 8.5.10

$$I_a = I_L - I_{sh} = 20.5 \text{ A}$$

Wave wound A = 2, Z = 300, P = 4

$$E_b = V - I_a R_a = 250 - 20.5 \times 0.5$$

$$= 239.75 \text{ V}, \ \phi = 0.02 \text{ Wb}$$

(i) $E_b = \frac{\phi PNZ}{60 A}$

$$\therefore \quad N = \frac{239.75 \times 60 \times 2}{0.02 \times 4 \times 300} = \mathbf{1198.75 \ r.p.m.}$$

(ii) $T = 0.159 \phi I_a \left[\frac{PZ}{A}\right]$

$$= 0.159 \times 0.02 \times 20.5 \times \left[\frac{4 \times 300}{2}\right] = 39.114 \text{ Nm}$$

(iii) Power developed = $T \times \omega = T \times \frac{2\pi N}{60}$ = 4912.066 W

The power developed is also given by $E_b I_a$.

Ex. 8.5.7 *The field current in a d.c. shunt machine is 2 A and the line current is 20 A at 200 V. Calculate : i) The generated e.m.f. when working as generator. ii) Torque in Nm when running at 1500 r.p.m. as motor. Take the armature resistance as 0.5 ohm.* **VTU : July-06, Marks 8**

Sol. : i) As a Generator :

V_t = 200 V and Line current = 20 A

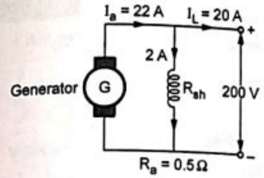


Fig. 8.5.11 (a)

$$E_g = V_t + I_a R_a$$

$$E = 200 + 22 \times 0.5 = 200 + 11$$

$$= \mathbf{211 \ volts}$$

ii) Torque when running as motor

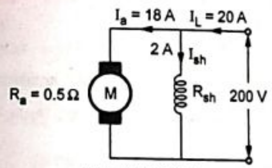


Fig. 8.5.11 (b)

$$E_b = V - I_a R_a = 200 - 18 \times 0.5$$

$$E_b = 191 \text{ volts}$$

P_m = Mechanical power developed

$$= E_b I_a = 191 \times 18$$

$$= 3438 \text{ W}$$

$$\therefore \quad T = \frac{P_m}{\omega} = \frac{P_m}{\frac{2\pi N}{60}} = \frac{3438}{\frac{2\pi \times 1500}{60}} = 21.887 \text{ Nm}$$

8.6 Torque and Speed Equations

- Before analyzing the various characteristics of motors, let us revise the torque and speed equations as applied to various types of motors.

$$\boxed{T \propto \phi I_a}$$ ...From torque equation.

- This is because, $0.159 \frac{PZ}{A}$ is a constant for a given motor.
- Now ϕ is the flux produced by the field winding and is proportional to the current passing through the field winding.

$$\phi \propto I_{field}$$

- For a d.c. shunt motor, I_{sh} is constant as long as supply voltage is constant. Hence ϕ flux is also constant.

$$\therefore \quad \boxed{T \propto I_a \quad \text{...For shunt motors}}$$

- For a d.c. series motor, I_{se} is same as I_a. Hence flux ϕ is proportional to the armature current I_a.

$$\therefore \quad \boxed{T \propto I_a \phi \propto I_a^2 \quad \text{...For series motors}}$$

- Similarly as $E_b = \frac{\phi PNZ}{60 A}$, we can write the speed equation as,

$$\therefore \quad \boxed{E_b \propto \phi N \quad \text{i.e.} \quad N \propto \frac{E_b}{\phi}}$$

$$\therefore \quad E_b = V - I_a R_a \text{ ...Neglecting brush drop}$$

- So for shunt motor as flux ϕ is constant.

$$\therefore \quad \boxed{N \propto V - I_a R_a}$$

- While for series motor, flux ϕ is proportional to I_a.

$$\therefore \quad \boxed{N \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}}$$

- These relations play an important role in understanding the various characteristics of different types of motors.

Ex. 8.6.1 *A 440 V d.c. shunt motor takes an armature current of 20 A and runs at 500 r.p.m. The armature resistance is 0.6 Ω. If the flux is reduced by 30 % and the torque is increased by 40 %. What are the new values of armature current and speed ?*
VTU : Aug.-05, Marks 6; June-10, Jan-16, July-07, Marks 8

Sol. : V = 440 V, I_{a1} = 20 A, N_1 = 500 r.p.m.

ϕ_1 = Original flux, $\phi_2 = \phi_1 - 30\ \% \ \phi_1 = 0.7\ \phi_1$,

$$\frac{\phi_2}{\phi_1} = 0.7$$

T_1 = Original torque, $T_2 = T_1 + 40\ \%\ T_1 = 1.4\ T_1$,

$$\frac{T_1}{T_2} = \frac{1}{1.4}$$

Now $T \propto \phi I_a$ i.e. $\frac{T_1}{T_2} = \frac{\phi_1}{\phi_2}\times\frac{I_{a1}}{I_{a2}}$

$$\therefore \quad \frac{1}{1.4} = \frac{1}{0.7}\times\frac{20}{I_{a2}}$$

$\therefore \quad I_{a2} = 40$ A ...New armature current

$N \propto \frac{E_b}{\phi}$ i.e. $\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}\times\frac{\phi_2}{\phi_1}$

$$\therefore \quad \frac{500}{N_2} = \left[\frac{V-I_{a1}R_a}{V-I_{a2}R_a}\right]\times 0.7$$

$$\text{i.e.} \quad \frac{500}{N_2} = \left[\frac{440-20\times 0.6}{440-40\times 0.6}\right]\times 0.7$$

$\therefore \quad N_2 =$ **694.259 r.p.m.** ... New speed

Ex. 8.6.2 *A 6 pole, d.c. shunt motor has a lap-connected armature with 492 conductors. The resistance of the armature is 0.2 Ω and the flux per pole is 50 mWb. The motor runs at 20 revolutions per second when it is connected to a 500 V supply for a particular load. What will be the speed of the motor when the load is reduced by 50 %. Neglect contact drop and magnetic saturation.* **VTU : Feb.-06, Marks 8**

Sol. : P = 6, Lap i.e. A = P = 6, Z = 492, R_a = 0.2 Ω,

ϕ = 50 mWb

$$N_1 = 20 \text{ rev per sec} = 20\times 60 \text{ r.p.m.} = 1200 \text{ r.p.m.}$$

$$E_{b1} = \frac{\phi P N_1 Z}{60A} = \frac{50\times10^{-3}\times 6\times 1200\times 492}{60\times 6} = 492 \text{ V}$$

$$E_{b1} = V - I_{a1}R_a$$

$$\therefore \quad I_{a1} = \frac{V-E_{b1}}{R_a} = \frac{500-492}{0.2} = 40 \text{ A}$$

Now load is reduced by 50 % i.e. $T_2 = 0.5\ T_1$.

$T \propto \phi I_a \propto I_a$ ... ϕ constant for shunt motor

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}} \quad \text{i.e.} \quad \frac{1}{0.5} = \frac{40}{I_{a2}}$$

$\therefore \quad I_{a2} = 20$ A

$\therefore \quad E_{b2} = V - I_{a2}R_a = 500 - 20\times 0.2 = 496$ V

Now $N \propto \frac{E_b}{\phi} \propto E_b$ i.e. $\frac{N_1}{N_2} = \frac{Eb_1}{Eb_2}$

$$\therefore \quad N_2 = \frac{E_{b2}}{E_{b1}}\times N_1 = \frac{496}{492}\times 1200 = \textbf{1209.756 r.p.m.}$$

$$\therefore \quad N_2 = \frac{1209.756}{60} = \textbf{20.1626 revolutions per second}$$

Ex. 8.6.3 *A 230 V DC shunt motor takes a no load current of 3 A and runs at 1100 rpm. If the full load current is 41 A, find the speed on full load. Assume armature resistance 0.25Ω and shunt filed resistance 230 Ω.* **VTU : Jan 16, Marks 6**

Sol. : The motor is shown in the Fig. 8.6.1.

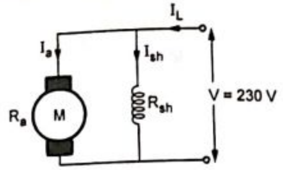


Fig. 8.6.1

N_0 = 1100 r.p.m.

I_{L0} = No load current = 3 A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1 \text{ A}$$

$\therefore I_{a0} = I_{L0} - I_{sh} = 3 - 1 = 2$ A ... No load armature current

$\therefore E_{b0} = V - I_{a0} R_a = 230 - 2\times 0.25 = 229.5$ V

On full load, I_{FL} = 41 A, I_{sh} = 1 A (same)

$\therefore I_{aFL} = I_{FL} - I_{sh} = 41 - 1 = 40$ A

$\therefore E_{bFL} = V - I_{aFL}\ R_a = 230 - 40\times 0.25 = 220$ V

$N \propto \frac{E_b}{\phi} \propto E_b$ ... ϕ constant as I_{sh} is constant

$$\therefore \frac{N_0}{N_{FL}} = \frac{E_{b0}}{E_{bFL}} \text{ i.e. } N_{FL} = \frac{1100\times 220}{229.5} = \textbf{1054.467 r.p.m.}$$

Ex. 8.6.4 *A series motor runs at 600 r.p.m. when taking 110 A from a 250 V supply. The resistance of the armature circuit is 0.12 Ω, and that of series winding is 0.03 Ω. The useful flux per pole for 110 A is 0.024 Wb, and that for 50 A is 0.0155 Wb. Calculate the speed when the current has fallen to 50 A.* **VTU : Jan.-08, Marks 8**

Sol. : N_1 = 600 r.p.m., I_{a1} = 110 A, V = 250 V, R_a = 0.12 Ω, R_{se} = 0.03 Ω

For I_{a1} = 110 A, ϕ_1 = 0.024 Wb and I_{a2} = 50 A, ϕ_2 = 0.0155 Wb

$$E_{b1} = V - I_{a1}(R_a + R_{se}) = 250 - 110\ (0.12 + 0.03) = 233.5 \text{ V}$$

$$E_{b2} = V - I_{a2}(R_a + R_{se}) = 250 - 50\ (0.12 + 0.03) = 242.5 \text{ V}$$

Now, $N \propto \frac{E_b}{\phi}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}\times\frac{\phi_2}{\phi_1}$$

$$\text{i.e.} \quad \frac{600}{N_2} = \frac{233.5}{242.5}\times\frac{0.0155}{0.024}$$

$$\therefore \quad N_2 = \frac{600\times 242.5\times 0.024}{233.5\times 0.0155} = \textbf{964.8407 r.p.m.}$$

Ex. 8.6.5 *The current drawn from the mains by a 220 V d.c. shunt motor is 4 A on no-load. The resistance field and armature windings are 110 ohms and 0.2 ohm respectively. If the line current on full load is 40 A at speed of 1500 r.p.m. find the no-load speed.* **VTU : Jan.-09, Marks 6**

Sol. : I_{L0} = 4 A, V = 220 V, R_{sh} = 110 Ω, R_a = 0.2 Ω, I_{FL} = 40 A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2 \text{ A}$$

$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 4 - 2 = 2$ A

$\therefore \quad E_{b0} = V - I_{a0}R_a = 220 - 2\times 0.2 = 219.6$ V

$I_{aFL} = I_{FL} - I_{sh} = 40 - 2 = 38$ A ... I_{sh} is constant

$\therefore \quad E_{bFL} = V - I_{aFL}\ R_a = 220 - 38\times 0.2 = 212.4$ V

$N \propto \frac{E_b}{\phi} \propto E_b$ ... ϕ is constant

$$\therefore \quad \frac{N_0}{N_{FL}} = \frac{E_{b0}}{E_{bFL}} \text{ i.e. } \frac{N_0}{1500} = \frac{219.6}{212.4} \quad \dots N_{FL} = 1500 \text{ r.p.m.}$$

$\therefore \quad N_0$ = **1550.8474 r.p.m.** ... No load speed.

Ex. 8.6.6 *A d.c. series motor is running with a speed of 1000 r.p.m., while taking a current of 22 amps from the supply. If the load is changed such that the current drawn by the motor is increased to 55 amps, calculate the speed of the motor on new load. The armature and series winding resistances are 0.3 Ω and 0.4 Ω respectively. Assume supply voltage as 250 V.* **VTU : July-09, Marks 8**

Sol. : N_1 = 1000 r.p.m., I_{a1} = 22 A, I_{a2} = 55 A

R_a = 0.3 Ω, R_{se} = 0.4 Ω, V = 250 V

$$E_{b1} = V - I_{a1}(R_a + R_{se}) = 250 - 22\times(0.3 + 0.4) = 234.6 \text{ V}$$

$$E_{b2} = V - I_{a2}(R_a + R_{se}) = 250 - 55\times(0.3 + 0.4) = 211.5 \text{ V}$$

For series motor, $\phi \propto I \propto I_a$ ... $I = I_a = I_{se}$

$$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_a}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}\times\frac{I_{a2}}{I_{a1}} \quad \text{i.e.} \quad \frac{1000}{N_2} = \frac{234.6}{211.5}\times\frac{55}{22}$$

i.e. N_2 = 360.6138 r.p.m.

8.7 D.C. Motor Characteristics

➤ ***Which characteristics are important for d.c. motors ?***

- The performance of a d.c. motor under various conditions can be judged by the following characteristics.

i) Torque - Armature current characteristics (T Vs I_a) :

- The graph showing the relationship between the torque and the armature current is called a torque - armature current characteristics or torque-load characteristics. These are also called electrical characteristics.

ii) Speed - Armature current characteristics (N Vs I_a) :

- The graph showing the relationship between the speed and armature current characteristics or speed-load characteristics.

iii) Speed - Torque characteristics (N Vs T) :

- The graph showing the relationship between the speed and the torque of the motor is called speed - torque characteristics of the motor. These are also called mechanical characteristics.

8.8 Characteristics of D.C. Shunt Motor

➤ ***Explain the various characteristics of d.c. shunt motor.***

VTU : Jan.-04, 06, 07, 08, 10, 16, Feb.-05, July-05, Marks 5

i) Torque - Armature current characteristics :

- For a constant values of R_{sh} and supply voltage V, I_{sh} is also constant and hence flux is also constant.

$\therefore$ $$\boxed{T_a \propto I_a}$$

- The equation represents a straight line, passing through the origin, as shown in the Fig. 8.8.1.

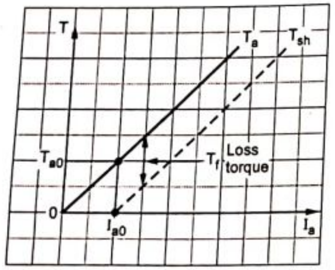


Fig. 8.8.1 T Vs I_a for shunt motor

- Torque increases linearly with armature current. So as load increases, armature current increases, increasing the torque developed linearly.
- On no load $T_{sh} = 0$ but armature torque is present which is just enough to overcome stray losses shown as T_{a0}. The current required is I_{a0} on no load to produce T_{a0} and hence T_{sh} graph has an intercept of I_{a0} on the current axis.

II) Speed - Armature current characteristics :

- From the speed equation we get,

$N \propto V - I_a R_a$ as ϕ is constant.

- So as load increases, the armature current increases and hence drop $I_a R_a$ also increases.
- Hence for constant supply voltage, $V - I_a R_a$ decreases and hence speed reduces.
- But as R_a is very small, for change in I_a from no load to full load, drop $I_a R_a$ is very small and hence drop in speed is also not significant from no load to full load.

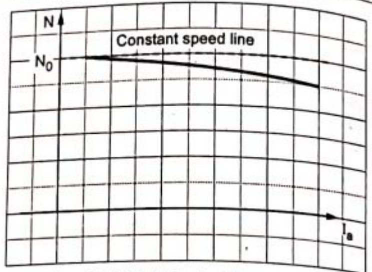


Fig. 8.8.2 N Vs I_a for shunt motor

- But for all practical purposes these type of motors are considered to be a **constant speed motors.**

III) Speed - Torque characteristics :

- These characteristics can be derived from the above two characteristics.
- This graph is similar to speed-armature current characteristics as torque is proportional to the armature current.
- This curve shows that the speed almost remains constant though torque changes from no load to full load conditions. This is shown in the Fig. 8.8.3.

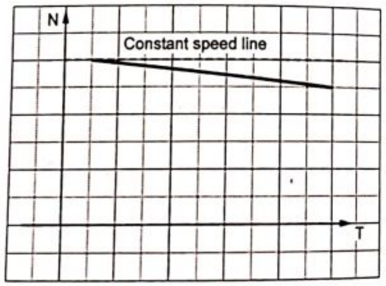


Fig. 8.8.3 N Vs T for shunt motor

8.9 Characteristics of D.C. Series Motor

➤ ***Explain the various characteristics of d.c. series motor.***

VTU : Jan.-04,07,08,10,16; Feb.-05; July-05,15,17, Marks 5

I) Torque - Armature current characteristics :

- For the series motor the series field winding is carrying the entire armature current hence,

$$\boxed{T_a \propto \phi I_a \propto I_a^2}$$

- Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature as shown in the Fig. 8.9.1.

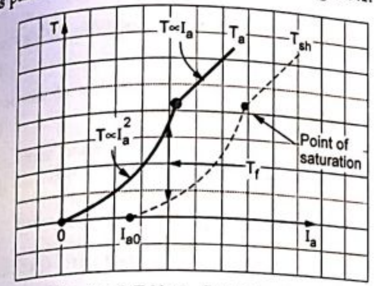


Fig. 8.9.1 T Vs I_a for series motor

- As load increases, armature current increases and torque produced increases proportional to the square of the armature current upto a certain limit.
- As the entire I_a passes through the series field, there is a property of an electromagnet called **saturation,** may occur.
- Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristics take the shape of straight line as flux becomes constant, as shown.
- The difference between T_a and T_{sh} is loss torque T_f which is also shown in the Fig. 8.9.1.

II) Speed - Armature current characteristics

From the speed equation we get,

$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}$$

as $\phi \propto I_a$ in case of series motor

- The values of R_a and R_{se} are so small that the effect of change in I_a on speed overrides the effect of change in $V - I_a R_a - I_a R_{se}$ on the speed.
- Hence in the speed equation, $E_b \cong V$ and can be assumed constant.
- So speed equation reduces to,

$$\boxed{N \propto \frac{1}{I_a}}$$

- So speed - armature current characteristics is rectangular hyperbola type as shown in the Fig. 8.9.2

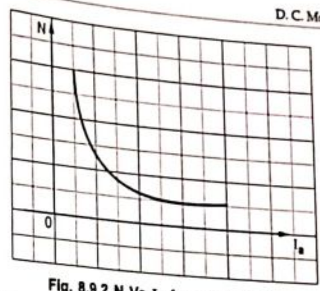


Fig. 8.9.2 N Vs I_a for series motor

III) Speed - Torque characteristics

- In case of series motors,

$$T \propto I_a^2 \quad \text{and} \quad N \propto \frac{1}{I_a}$$

- Hence we can write,

$$\boxed{N \propto \frac{1}{\sqrt{T}}}$$

- Thus as torque increases when load increases, the speed decreases.
- On no load, torque is very less and hence speed increases to dangerously high value.
- Thus the nature of the speed - torque characteristics is similar to the nature of the speed - armature current characteristics.
- The speed - torque characteristics of a series motor is shown in the Fig. 8.9.3

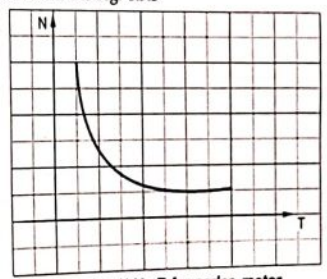


Fig. 8.9.3 N Vs T for series motor

➤ ***Why series motor is never started on no load ?***

VTU : Feb.-05, Jan.-06, Marks 5

- It is seen earlier that motor armature current is decided by the load.
- On light load or no load, the armature current drawn by the motor is very small.

- In case of a d.c. series motor, $\phi \propto I_a$ and on no load as I_a is small hence flux produced is also very small.
- According to speed equation, $N \propto \frac{1}{\phi}$ as E_b is almost constant.
- So on very light load or no load as flux is very small, the motor tries to run at dangerously high speed which may damage the motor mechanically.
- This can be seen from the speed - armature current and the speed-torque characteristics that on low armature current and low torque condition motor shows a tendency to rotate with dangerously high speed.
- This is the reason why series motor should never be started on light loads or no load conditions.

8.10 Applications of D.C. Motors

➤ ***State the applications of d.c. shunt and series motors.***

VTU : Jan.-04, 06, 07, 08, 10, 16, Feb.-05, July-05, Marks 2

Types of motor	Characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1) Blowers and fans 2) Centrifugal and reciprocating pumps 3) Lathe machines 4) Machine tools 5) Milling machines 6) Drilling machines
Series	High starting torque. No load condition is dangerous. Variable speed.	1) Cranes 2) Hoists, Elevators 3) Trolleys 4) Conveyors 5) Electric locomotives
Cumulative compound	High starting torque. No load condition is allowed.	1) Rolling mills 2) Punches 3) Shears 4) Heavy planers 5) Elevators
Differential compound	Speed increases as load increases.	Not suitable for any practical application.

Table 8.10.1 Applications of d.c. motors

8.11 University Questions with Answers

Jan. - 2010

Q.1 *Explain the various characteristics of d.c. shunt motor.* **[Refer section 8.8]** [5]

Q.2 *Explain the various characteristics of d.c. series motor.* **[Refer section 8.9]** [6]

Q.3 *State the applications of d.c. shunt and series motors.* **[Refer section 8.10]** [2]

Jan. - 2011

Q.4 *What is back e.m.f. ? State its significance.* **[Refer section 8.2]** [6]

July - 2011

Q.5 *What is back e.m.f. ? State its significance.* **[Refer section 8.2]** [6]

Dec. - 2011

Q.6 *Derive the expression of armature torque developed in a d.c. motor.* **[Refer section 8.4]** [6]

June - 2012

Q.7 *Explain how torque is produced in a d.c. motor.* **[Refer section 8.1]** [4]

Jan. - 2014

Q.8 *Explain the working principle of d.c. motor.* **[Refer section 8.1]** [4]

Q.9 *Derive the expression of armature torque developed in a d.c. motor.* **[Refer section 8.4]** [6]

Jan. - 2015

Q.10 *Derive the expression of armature torque developed in a d.c. motor.* **[Refer section 8.4]** [6]

July - 2015

Q.11 *What is back e.m.f. ? State its significance.* **[Refer section 8.2]** [6]

Q.12 *Explain the various characteristics of d.c. series motor.* **[Refer section 8.9]** [6]

CBCS scheme

Jan. - 2016

Q.13 *Sketch torque versus armature current and speed versus armature current characteristics of a D.C. shunt motor and mention its applications.* **[Refer sections 8.8 and 8.10]** [6]

July - 2016

Q.14 *What is the significance of back EMF in a D.C. motor ?* **[Refer section 8.2]** [5]

Q.15 *Derive an equation for the torque developed in the armature of a D.C. motor.* **[Refer section 8.4]** [5]

July - 2017

Q.16 *Discuss about various characteristics of a DC series motor with neat diagrams.* **(Refer section 8.9)** [6]

❑❑❑

Notes

MODULE - 5

9

Three Phase Synchronous Generators (Alternators)

Syllabus

Principle of operation, Constructional details, Synchronous speed, Frequency of generated voltage, emf equation, Concept of winding factor (excluding the derivation and calculation of distribution and pitch factors).

Contents

9.1 Introduction

- It is known that the electric supply used, now a days for commercial as well as domestic purposes, is of alternating type.
- The machines generating a.c. e.m.f. are called **alternators. The alternators work at a specific constant speed called synchronous speed** and hence in general called **synchronous generators.**
- The main difference between d.c. generators and alternators is that in alternators the field is rotating while armature is stationary and the commutator is absent.

9.2 Concept of Slip Rings and Brush Assembly

- In alternators, the armature in which e.m.f. gets induced is rotating while the field to which d.c. supply is given is stationary.
- The load to which generated a.c. e.m.f. is to be supplied is always stationary.
- The arrangement which is used to collect an induced e.m.f. from the **rotating armature** and make it available to the **stationary load circuit** is called **slip ring and brush assembly.**
- The armature winding is three phase and connected in generally in star. The three end terminals of armature winding are brought out.
- The slip rings, made up of conducting material are mounted on the shaft. Each terminal of armature winding is connected to an individual slip ring, permanently.
- Hence three phase supply generated in the aramture is now available across the rotating slip rings.
- The brushes are resting on the slip rings, just making contact and are stationary

> • The brushes are stationary. Hence as brushes make contact with the slip rings, the three phase supply is now available across the brushes which are stationary.

- Hence any stationary load can then be connected across these stationary terminals available from the brushes.
- The schematic arrangement is shown in the Fig. 9.2.1.

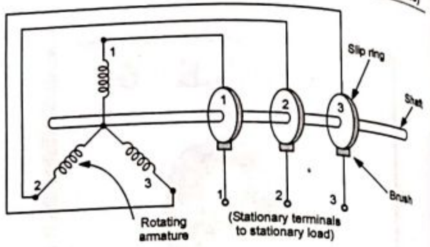


Fig. 9.2.1 Arrangement of slip rings

9.3 Advantages of Rotating Field over Rotating Armature

➢ ***State the advantages of rotating field over rotating armature used in alternators.*** VTU : July-05, 06, 09, 17, Jan.-07, 08; Dec.-11 Jan.-16, Marks 6

- Practically most of the alternators prefer rotating field type construction with stationary armature due to certain advantages which are,

1) As everywhere a.c. is used, the generation level of a.c. voltage is higher about 11 kV to 33 kV. This gets induced in the armature. For stationary armature large space can be provided to accommodate large number of conductors and the insulation.
2) It is always better to protect high voltage winding from the centrifugal forces caused due to the rotation. So high voltage armature is generally kept stationary. This avoids the interaction of mechanical and electrical stresses.
3) It is easier to collect larger currents at very high voltages from a stationary member than from the slip ring and brush assembly. The voltage required to be supplied to the field is very low (110 V to 220 V d.c.) and hence can be easily supplied with the help of slip ring and brush assembly by keeping it rotating.
4) The problem of sparking at the slip rings can be avoided by keeping field rotating which is low voltage circuit and high voltage armature as stationary.
5) Due to low voltage level on the field side, the insulation required is less and hence field system has very low inertia. It is always better to rotate low inertia system than high inertia, as efforts required to rotate low inertia system are always less.
6) Rotating field makes the overall construction very simple.
7) If field is rotating, to excite it by an external d.c. supply two slip rings are enough. One each for positive and negative terminals. As against this, in three phase rotating armature, the minimum number of slip rings required are three and can not be easily insulated due to high voltage levels.
8) The ventilation arrangement for high voltage side can be improved if it is kept stationary.

9.4 Construction of Alternators

➢ ***By means of a neat diagram, describe the construction details of an alternator with their functions.*** VTU : Jan.-14, 17, Marks 8

- In alternators the stationary winding is called 'Stator' while the rotating winding is called 'Rotor'.

> • Most of the alternators have **stator as armature** and **rotor as field,** in practice.

9.4.1 Stator

➢ ***Explain the construction of stator of an alternator.*** VTU : June-13, Marks 4

- The stator is a stationary armature. This consists of a core and the slots to hold the armature winding similar to the three phase induction motor.
- The stator core uses a laminated construction. It is built up of special steel stampings insulated from each other with varnish or paper. The laminated construction is basically to keep down eddy current losses. Generally choice of material is steel to keep down hysteresis losses.
- The entire core is fabricated in a frame made of steel plates. The core has slots on its periphery for housing the armature conductors. Frame does not carry any flux and serves as the support to the core.
- Ventilation is maintained with the help of holes casted in the frame.
- The section of an alternator stator is shown in the Fig. 9.4.1.

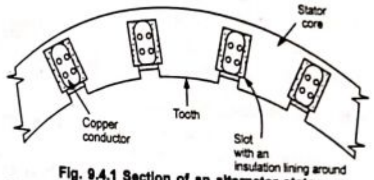


Fig. 9.4.1 Section of an alternator stator

9.4.2 Rotor

➢ ***Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.*** VTU : Jan.-03, 04, 06, 09, 10, 13; July-03, 04, 08, 10, 11, June-13, Marks 8

➢ ***With neat sketches, explain the construction of salient pole alternator.*** VTU : Jan.-16, 18, Marks 4

- There are two types of rotors used in alternators,
  - i) Salient pole or projected pole type and
  - ii) Smooth cylindrical or non salient type.

1. Salient pole type rotor :

- This is also called projected pole type as all the poles are projected out from the surface of the rotor.
- The poles are built up of thick steel laminations. The poles are bolted to the rotor as shown in the Fig. 9.4.2.

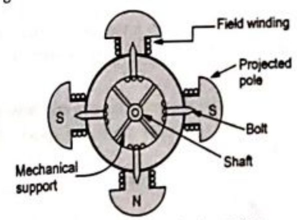


Fig. 9.4.2 Salient pole type rotor

- The pole face has been given a specific shape as discussed earlier in case of d.c. generators. The field winding is provided on the pole shoe.

- These rotors have large diameters and small axial lengths.
- As mechanical strength of salient pole type is less, this is preferred for low speed alternators ranging from 125 r.p.m. to 500 r.p.m..
- The prime movers used to drive such rotor are generally water turbines and I.C. engines.

2. Smooth cylindrical type rotor :

- This is also called non salient type or non-projected pole type of rotor.
- The Fig. 9.4.3 shows smooth cylindrical type of rotor.

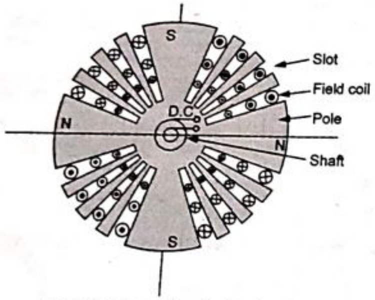


Fig. 9.4.3 Smooth cylindrical rotor

- The rotor consists of smooth solid steel cylinder, having number of slots to accommodate the field coil. The slots are covered at the top with the help of steel or manganese wedges.
- The unslotted portions of the cylinder itself act as the poles. The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air gap between stator and the rotor.
- These rotors have small diameters and large axial lengths. This is to keep peripheral speed within limits.
- The main advantage of this type is that these are mechanically very strong and thus preferred for high speed alternators ranging between 1500 to 3000 r.p.m.. Such high speed alternators are called **'turboalternators'**.
- The prime movers used to drive such type of rotors are generally steam turbines, electric motors.

9.4.3 Difference between Salient and Cylindrical Type of Rotor

➤ ***Distinguish between salient and nonsalient type of alternator rotors.*** VTU : Dec.-11, June-12, Marks 4

Sr. No.	Salient or projected pole type	Smooth cylindrical or nonsalient type
1.	Poles are projecting out from the surface.	Unslotted portion of the cylinder acts as poles hence poles are non-projecting.
2.	Air gap is non-uniform.	Air gap is uniform due to smooth cylindrical periphery.
3.	Diameter is high and axial length is small.	Small diameter and large axial length is the feature.
4.	Mechanically weak.	Mechanically robust.
5.	Preferred for low speed alternators.	Preferred for high speed alternators i.e. for turbo alternators.
6.	Prime mover used are water turbines, I.C. engines.	Prime movers used are steam turbines, electric motors.
7.	For same size, the rating is smaller than cylindrical type.	For same size, rating is higher than salient pole type.
8.	Separate damper winding is provided.	Separate damper winding is not necessary.

9.5 Working Principle of Alternator

➤ ***Explain the principle of operation of an alternator.*** VTU : Jan.-03, 09, July-04, June-13, Marks 6

- The alternators work on the principle of **electromagnetic induction**. When there is a relative motion between the conductors and the flux, e.m.f. gets induced in the conductors. Though in an alternator the conductors are stationary and field is rotating, for understanding purpose we can always consider relative motion of conductors with respect to the flux produced by the field winding.
- Consider a relative motion of a single conductor under the magnetic field produced by two stationary poles. The magnetic axis of the two poles

produced by field is vertical, shown dotted in the Fig. 9.5.1.

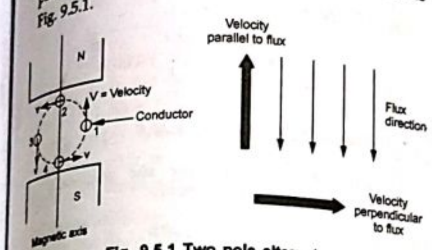


Fig. 9.5.1 Two pole alternator

- Let conductor starts rotating from position 1. At this instant, the entire velocity component is **parallel** to the flux lines. Hence there is no cutting of flux lines by the conductor. Hence induced e.m.f. in the conductor is also zero.
- As the conductor moves from position 1 towards position 2, the part of the velocity component becomes perpendicular to the flux lines and proportional to that, e.m.f. gets induced in the conductor. The magnitude of such an induced e.m.f. increases as the conductor moves from position 1 towards 2.
- At position 2, the entire velocity component is **perpendicular** to the flux lines. Hence there exists maximum cutting of the flux lines. And at this instant, the induced e.m.f. in the conductor is at its maximum.
- As the position of conductor changes from 2 towards 3, the velocity component perpendicular to the flux starts decreasing and hence induced e.m.f. magnitude also starts decreasing. At position 3, again the entire velocity component is parallel to the flux lines and hence at this instant induced e.m.f. in the conductor is zero.
- As the conductor moves from position 3 towards 4, the velocity component perpendicular to the flux lines again starts increasing. But the direction of velocity component now is opposite to the direction of velocity component existing during the movement of the conductor from position 1 to 2. Hence an induced e.m.f. in the conductor increases but in the opposite direction.
- At position 4, it achieves maxima in the opposite direction, as the entire velocity component becomes **perpendicular** to the flux lines.
- Again from position 4 to 1, induced e.m.f. decreases and finally at position 1, again becomes zero. This cycle continues as conductor rotates at a certain speed.
- So if we plot the magnitudes of the induced e.m.f. against the time, we get an alternating nature of the induced e.m.f. as as shown in the Fig. 9.5.2.

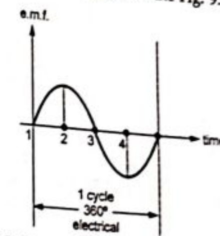


Fig. 9.5.2 Alternating nature of the induced e.m.f.

- **Thus for 2 pole alternator, one mechanical revolution corresponds to one electrical cycle i.e. 360° electrical of an induced e.m.f.**

9.5.1 Mechanical and Electrical Angle

➤ ***Derive the relation between mechanical and electrical angle in an alternator.***

- Consider 4 pole alternator i.e. the field winding is designed to produce 4 poles.
- Due to 4 poles, the magnetic axis exists diagonally shown dotted in the Fig. 9.5.2(a).

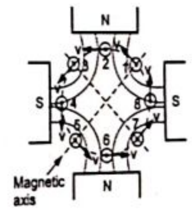


Fig. 9.5.2 (a) 4 Pole alternator

- Now in position 1 of the conductor, the velocity component is parallel to the flux lines while in position 2, there is gathering of flux lines and entire

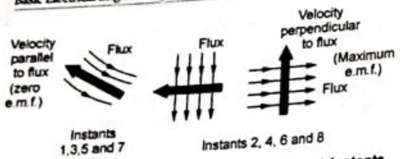


Fig. 9.5.2 (b) Velocity components at different instants

velocity component is perpendicular to the flux lines. So at position 1, the induced e.m.f. in the conductor is zero while at position 2, it is maximum.

- Similarly as conductor rotates, the induced e.m.f. will be maximum at positions 4, 6 and 8 and will be zero at positions 3, 5 and 7.
- So during one complete revolution of the conductor, induced e.m.f. will experience four times maxima, twice in either direction and four times zero. This is because of the distribution of flux lines due to existence of four poles.
- So if we plot the nature of the induced e.m.f ; for one revolution of the conductor, we get the two electrical cycles of the induced e.m.f., as shown in the Fig. 9.5.3

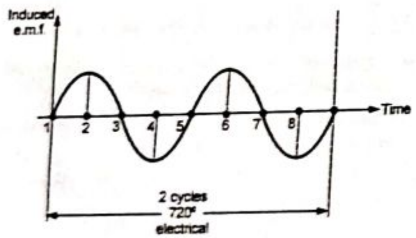


Fig. 9.5.3 Nature of the induced e.m.f.

> - Thus the degrees electrical of the induced e.m.f. i.e. number of cycles of the induced e.m.f. depends on the number of poles of an alternator.

- So for a four pole alternator we can write,

  360° mechanical = 720° electrical

- From this we can establish the general relation between degrees mechanical and degrees electrical as,

$$360° \text{ mechanical} = 360° \times \frac{P}{2} \text{ electrical}$$

where P = Number of poles

i.e. $$\boxed{1° \text{ Mechanical} = \left(\frac{P}{2}\right)^{\circ} \text{ Electrical.}}$$

9.5.2 Frequency of Induced E.M.F.

➤ ***Obtain the expression for the frequency of induced e.m.f. in an alternator.***

Let P = Number of poles

and N = Speed of the rotor in r.p.m.

f = Frequency of the induced e.m.f.

- One mechanical revolution of rotor = $\frac{P}{2}$ cycles of e.m.f. electrically
- Thus there are P/2 electrical cycles per revolution.
- As speed is N r.p.m., in one second, rotor will complete $\left(\frac{N}{60}\right)$ revolutions.
- But electrical cycles/sec = Frequency = f

∴ Frequency f = (No. of electrical cycles per revolution) × (No. of revolutions per second)

$$\therefore \quad f = \frac{P}{2} \times \frac{N}{60}$$

$$\therefore \quad \boxed{f = \frac{PN}{120} \text{ Hz (cycles per sec).}}$$

- So there exists a fixed relationship between three quantities, the number of poles P, the speed of the rotor N in r.p.m. and f the frequency of an induced e.m.f. in Hz (hertz).

9.5.3 Synchronous Speed (N_s)

➤ ***Why a.c. generators are called synchronous generators ?*** VTU : Jan.-07, Marks 4

- For fixed number of poles, alternator has to be rotated at a particular speed to keep the frequency of the generated e.m.f. constant at the required value. Such a speed is called synchronous speed of the alternator denoted as N_s.

So $$\boxed{N_s = \frac{120 f}{P}}$$

where f = Required rated frequency

- In our nation, the frequency of an alternating e.m.f. is standard equal to 50 Hz.
- To get 50 Hz frequency, for different number of poles, alternator must be driven at different speeds called synchronous speeds.
- Following table gives the values of the synchronous speeds for the alternators having different number of poles.

Number of poles P	2	4	8	12	24
Synchronous speed N_s in r.p.m.	3000	1500	750	500	250

Table 9.5.1

- From the table, it can be seen that minimum number of poles for an alternator can be two hence maximum value of synchronous speed possible in our nation i.e. for frequency of 50 Hz is 3000 r.p.m.

> - Such a machine bearing a fixed relationship between P, N and f is called **synchronous machine** and hence alternators are also called **synchronous generators.**

Ex. 9.5.1 *For an alternator find the different possible synchronous speeds for choice of poles from 2 to 10 for a frequency of 50 Hz.* VTU : Aug.-07, Marks 4

Sol. :

Sr. No.	Frequency f Hz	Number of poles P	Synchronous speed $N_s = \frac{120 f}{P}$ r.p.m.
1.	50	2	3000
2.	50	4	1500
3.	50	6	1000
4.	50	8	750
5.	50	10	600

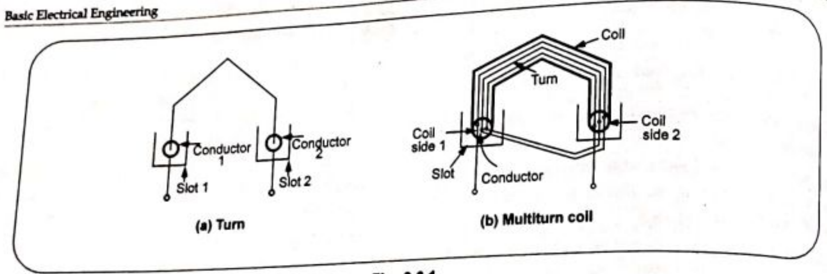
9.6 Armature Winding

- Basically three phase alternators carry three sets of windings arranged in the slots in such a way that there exists a phase difference of 120° between the induced e.m.f.s in them.
- In three phase alternators, the six terminals are brought out which are finally connected in star or delta and then the three terminals are brought out.
- Each set of windings represents winding per phase and induced e.m.f. in each set is called induced e.m.f. per phase denoted as E_{ph}.
- All the coils used for one phase must be connected in such a way that their e.m.f.s help each other. And overall design should be in such a way that the waveform of an induced e.m.f. is almost **sinusoidal** in nature.

9.6.1 Winding Terminology

1) **Conductor :** The part of the wire, which is under the influence of the magnetic field and responsible for the induced e.m.f. is called active length of the conductor. The conductors are placed in the armature slots.
2) **Turn :** A conductor in one slot, when connected to a conductor in another slot forms a turn. So two conductors constitute a turn. This is shown in Fig. 9.6.1 (a).
3) **Coil :** As there are number of turns, for simplicity the number of turns are grouped together to form a coil. Such a coil is called multiturn coil. A coil may consist of single turn called single turn coil. The Fig. 9.6.1 (b) shows a multiturn coil.
4) **Coil side :** Coil consists of many turns. Part of the coil in each slot is called coil side of a coil as shown in the Fig. 9.6.1 (b).
5) **Pole pitch :** It is centre to centre distance between the two adjacent poles. **1 pole is responsible for 180° electrical of induced e.m.f.**

- So 180° electrical is also called **one pole pitch.**
- Practically how many slots are under one pole which are responsible for 180° electrical, are measured to specify the pole pitch.



(a) Turn (b) Multiturn coil

Fig. 9.6.1

- e.g. Consider 2 pole, 18 slots armature of an alternator. Then under 1 pole there are $\frac{18}{2}$ i.e. 9 slots. So pole pitch is 9 slots or 180° electrical. This means 9 slots are responsible to produce a phase difference of 180° between the e.m.f.s induced in different conductors.
- This number of slots/pole is denoted as 'n'.

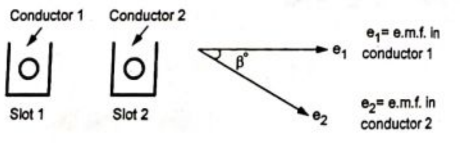
$\therefore$ $\boxed{\text{Pole pitch} = 180° \text{ Electrical } = \text{Slots per pole (no. of slots/P)} = n}$

6) **Slot angle (β) :** The phase difference contributed by one slot in degrees electrical is called slot angle β.

- As slots per pole contributes 180° electrical which is denoted as 'n', we can write,

$$\therefore \quad \boxed{\beta = 1 \text{ slot angle} = \frac{180°}{n}}$$

- In the above example, $n = \frac{18}{2} = 9$, while $\beta = \frac{180°}{n} = 20°$
- This means that if we consider an induced e.m.f. in the conductors which are placed in the slots which are adjacent to each other, there will exist a phase difference of β° in between them. While if e.m.f. induced in the conductors which are placed in slots which are 'n' slots distance away, there will exist a phase difference of 180° in between them.



(a) Adjacent slots (b) Indication of phase difference

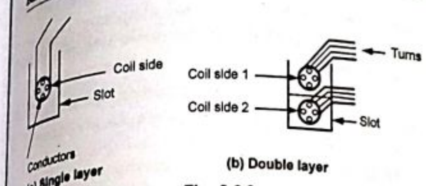
Fig. 9.6.2 Concept of slot angle

9.6.2 Types of Armature Windings

1) Single layer and double layer winding :

➤ ***What is single layer and double layer winding ?***

- If a slot consists of only one coil side, winding is said to be single layer.
- This is shown in the Fig. 9.6.3 (a).
- While there are two coil sides per slot, one at the bottom and one at the top the winding is called double layer as shown in the Fig. 9.6.3 (b).



(a) Single layer (b) Double layer

Fig. 9.6.3

- A lot of space gets wasted in single layer hence in practice generally double layer winding is preferred.

2) Full pitch and short pitch winding :

➤ ***What is full pitch and short pitch winding ?***

- One pole pitch is 180° electrical. The value of 'n', slots per pole indicates how many slots are contributing 180° electrical phase difference.

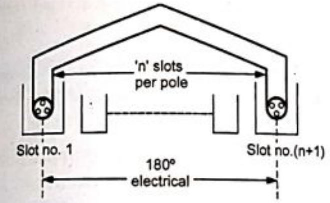


Fig. 9.6.4 Full pitch coil

- So if coil side in one slot is connected to a coil side in another slot which is one pole pitch distance away from first slot, the winding is said to be **full pitch winding** and coil is called **full pitch coil.**

Coil span : It is the distance on the periphery of the armature between two coil sides of a coil. It is usually expressed interms of number of slots or degrees electrical.

- If coil span is 'n' slots i.e. 180° electrical then the coils are full pitch coils.
- If coils are used in such a way that coil span is slightly less than a pole pitch i.e. less than 180° electrical, the coils are called, **short pitched coils or fractional pitched coils.** Generally coils are shorted by one or two slots.

Advantages of short pitch or fractional pitched coils :

➤ ***State the advantages of fractional pitched coils.***

1) The length required for the end connections of coils is less i.e. inactive length of winding is less. So less copper is required. Hence economical.
2) Short pitching eliminates high frequency harmonics which distort the sinusoidal nature of e.m.f.. Hence waveform of an induced e.m.f. is more sinusoidal due to short pitching.
3) As high frequency harmonics get eliminated, eddy current and hysteresis losses which depend on frequency also get minimised. This increases the efficiency.

3) Concentrated and distributed winding :

➤ ***Why distributed winding is preferred ?*** VTU : July-05, Marks 3

- In three phase alternators, there are three different sets of windings, each for a phase.
- So depending upon the total number of slots and number of poles, we have certain slots per phase available under each pole. This is denoted as 'm'.
- m = Slots per pole per phase = n/number of phases = n/3 (For 3 phase machine)
- For example in 18 slots, 2 pole alternator we have,

$$n = \frac{18}{2} = 9 \quad \text{and} \quad m = \frac{9}{3} = 3$$

- So we have 3 slots per pole per phase available.
- Now let 'x' number of conductors per phase are to be placed under one pole. And we have 3 slots per pole per phase available.
- But if all 'x' conductors per phase are placed in one slot keeping remaining slots per pole per phase empty then the winding is called **concentrated winding.**
- **So in concentrated winding all conductors or coils belonging to a phase are placed in one slot under every pole.**
- But in practice, an attempt is always made to use all the 'm' slots per pole per phase available for distribution of the winding.
- So if 'x' conductors per phase are distributed amongst the 'm' slots per phase available under every pole, the winding is called **distributed winding.**

- So in distributed type of winding all the coils belonging to a phase are well distributed over the 'm' slots per phase, under every pole.
- Distributed winding makes the waveform of the induced e.m.f. more sinusoidal in nature.
- Also in concentrated winding due to large number of conductors per slot, heat dissipation is poor.
- **So in practice, double layer, short pitched and distributed type of armature winding is preferred for the alternators.**

9.7 E.M.F. Equation of an Alternator

➤ ***Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.***

VTU : Feb.-05; July-08, 10,12, 15, Jan.-09, 11, 13, 16, Marks 6

- Let ϕ = Flux per pole, in Wb

  P = Number of poles

  N_s = Synchronous speed in r.p.m.

  f = Frequency of induced e.m.f. in Hz

  Z = Total number of conductors

  Z_{ph} = Conductors per phase

  $\therefore \quad Z_{ph} = \frac{Z}{3}$ as number of phases = 3.

- Consider a single conductor placed in a slot.
- The average value of e.m.f. induced in a conductor $= \frac{d\phi}{dt}$
- For one revolution of a conductor,

$$e_{avg} \text{ per conductor} = \frac{\text{Flux cut in one revolution}}{\text{Time taken for one revolution}}$$

- Total flux cut in one revolution is $\phi \times P$.
- Time taken for one revolution is $\frac{60}{N_s}$ seconds.

$$\therefore \quad e_{avg} \text{ per conductor} = \frac{\phi P}{\left(\frac{60}{N_s}\right)} = \phi \frac{PN_s}{60} \quad \dots (9.7.1)$$

But $\quad f = \frac{PN_s}{120} \quad$ i.e. $\quad \frac{PN_s}{60} = 2f$

- Substituting in equation (9.7.1),

e_{avg} per conductor = 2 f ϕ volts

- Assume full pitch winding for simplicity i.e. this conductor is connected to a conductor which is 180° electrical apart.
- So these two e.m.f.s will try to set up a current in the same direction i.e. the two e.m.f.s are helping each other and hence resultant e.m.f. per turn will be twice the e.m.f. induced in a conductor.

$$\therefore \quad \text{e.m.f. per turn} = 2 \times (\text{e.m.f. per conductor})$$
$$= 2 \times (2 f \phi) = 4 f \phi \text{ volts.}$$

- Let T_{ph} be the total number of turns per phase connected in series.
- Assuming concentrated winding, we can say that all are placed in single slot per pole per phase. So induced e.m.f.s in all turns will be in phase as placed in single slot. Hence net e.m.f. per phase will be algebraic sum of the e.m.f.s per turn.

$$\therefore \quad \text{Average } E_{ph} = T_{ph} \times (\text{Average e.m.f. per turn})$$
$$= T_{ph} \times 4 f \phi$$

- But in a.c. circuits R.M.S. value of an alternating quantity is used for the analysis. The form factor is 1.11 of sinusoidal e.m.f.

$$K_f = \frac{\text{R.M.S.}}{\text{Average}} = 1.11 \quad \dots \text{For sinusoidal}$$

$$\therefore \text{R.M.S. value of } E_{ph} = K_f \times \text{Average value}$$
$$= 1.11 \times 4 f \phi T_{ph}$$

$$\boxed{E_{ph} = 4.44 f \phi T_{ph} \text{ volts}} \quad \dots (9.7.2)$$

- This is the basic e.m.f. equation for an induced e.m.f. per phase for full pitch, concentrated type of winding where T_{ph} = Number of turns per phase.

$$\boxed{T_{ph} = \frac{Z_{ph}}{2}} \quad \dots \text{As 2 conductors constitute 1 turn}$$

- But as mentioned earlier, the winding used for the alternators is distributed and short pitch hence e.m.f. induced slightly gets affected. This effect is considered by the factors called winding factors.

1) Pitch factor or coil span factor (K_c) :

➤ ***What is pitch factor ? State its expression.***

VTU : Jan.-06, 08, 10, Marks 2

- In practice short pitch coils are preferred.

- So coil is formed by connecting one coil side to another which is less than one pole pitch away. So actual coil span is less than 180°.
- The angle by which coils are short pitched is called **angle of short pitch** denoted as 'α'.
- α = Angle by which coils are short pitched.

$$\therefore \quad \alpha = \beta \times \text{Number of slots by which coils are short pitched.}$$
$$\text{or} \quad \alpha = 180° - \text{Actual coil span of the coils.}$$

- The factor by which, induced e.m.f. gets reduced due to short pitching is called **pitch factor or coil span factor** denoted by K_c.
- It is given by the expression,

$$\boxed{K_c = \cos\left(\frac{\alpha}{2}\right) \quad \text{where } \alpha = \text{Angle of short pitch}}$$

2) Distribution Factor (K_d)

➤ ***What is distribution factor ? State its expression.***

VTU : Jan.-06, 08, 10, Marks 2

- In practice distributed winding is used using all the slots available per pole per phase.
- Due to this, the total e.m.f. is not the algebraic sum of the e.m.f.s in various conductors but it is the vector sum and hence the total induced e.m.f. decreases.
- The factor by which there is a reduction in the e.m.f. due to distribution of coils is called **distribution factor** denoted as K_d.
- It is defined as the ratio of phasor sum of e.m.f.s induced in the coils to the arithmetic sum of e.m.f.s induced in the coils.
- It is given by the expression,

$$\boxed{K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)}}$$

where m = Slots per pole per phase, β = Slot angle $= \frac{180°}{n}$, n = Slots per pole.

- For short pitch and distributed winding K_c and K_d are always less than unity.
- So generalized expression for e.m.f. equation considering the winding factors is given by,

$$\boxed{E_{ph} = 4.44 K_c K_d f \phi T_{ph} \text{ volts}}$$

- For full pitch coil, $K_c = 1$ and for concentrated winding $K_d = 1$.
- For a star connected alternator the terminal voltage is $\sqrt{3}$ times the phase voltage hence.

$$\boxed{E_{line} = \sqrt{3} \times E_{ph} = \sqrt{3} \times 4.44 K_c K_d f \phi T_{ph} \text{ volts (For star connected)}}$$

- The product of K_d and K_c is called winding factor denoted as K_w. Thus $K_w = K_c \times K_d$.

Ex. 9.7.1 *A 3 φ 4 pole, 50 Hz star connected alternator has 36 slots and 30 conductors per slot. The useful flux per pole is 0.05 Wb. Find synchronous speed and line voltage on no-load. Assume winding factor of 0.96.* **VTU : Jan.-03,04,09,11, July-04,07,09,11,17, Marks 5.**

Sol. : $P = 4$, $f = 50$ HZ, 36 slots,

30 conductors per slot, $\phi = 0.05$ Wb, $K_d = 0.96$

Assume full pitch winding, $K_c = 1$

$$N_s = \frac{120\ f}{P} = \frac{120 \times 50}{4} = \textbf{1500 r.p.m.}$$

$$Z = \text{Slots} \times \text{Conductors / slot} = 36 \times 30 = 1080$$

$$Z_{ph} = \frac{Z}{3} = \frac{1080}{3} = 360$$

$$T_{ph} = \frac{Z_{ph}}{2} = 180 \quad \text{... 2 conductors - 1 turn}$$

$$\therefore\ E_{ph} = 4.44\ K_c K_d\ \phi\ f\ T_{ph}$$

$$= 4.44 \times 1 \times 0.96 \times 0.05 \times 50 \times 180$$

$$\therefore\ E_{ph} = 1918.08 \text{ V}$$

$$\therefore\ E_{line} = \sqrt{3} \times E_{ph} = \textbf{3.322 kV}$$

Ex. 9.7.2 *A 24-pole turbo alternator has a star connected armature winding with 144 slots and 10 conductors per slot. It is derived by a low speed Kaplan turbine at a speed of 250 revolutions per minute. The winding has full-pitched coils with a distribution factor of 0.966. The flux per pole is 67.3 mWb. Determine,*

i) The frequency and the magnitude of the line voltage

ii) The output kVA of the machine if the total current in each phase is 50 . **VTU : Feb.-06, Marks 8**

Sol. : $P = 24$, Slots = 144, 10 Conductors/slot,

$N_s = 250$ r.p.m., Full pitch coils $K_c = 1$,

$K_d = 0.966$, $\phi = 67.3$ mWb.

i) $$N_s = \frac{120 f}{P} \quad \text{i.e.} \quad f = \frac{N_s \times P}{120} = \frac{250 \times 24}{120} = \textbf{50 Hz}$$

$$Z = \text{Slots} \times \text{Conductors/slot} = 144 \times 10 = 1440$$

$$Z_{ph} = \frac{Z}{3} = \frac{1440}{3} = 480 \quad \text{i.e.} \quad T_{ph} = \frac{Z_{ph}}{2} = 240$$

$$\therefore\ E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$$

$$= 4.44 \times 1 \times 0.966 \times 673 \times 10^{-3} \times 50 \times 240$$

$$= 3463.8287 \text{ V}$$

$$\therefore\ E_{line} = \sqrt{3}\ E_{ph} = 5999.527 \text{ V} \approx \textbf{6 kV}$$

ii) $$\text{Output kVA} = \sqrt{3}\ V_L\ I_L/1000$$

.. 1/1000 For expressing in kVA

For star $I_L = I_{ph} = 50$ A given, $V_L = E_{line} = 6$ kV

$$\therefore\ \text{Output kVA} = \frac{\sqrt{3} \times 6 \times 10^3 \times 50}{10^3} = \textbf{519.615 kVA}$$

Ex. 9.7.3 *A 2 pole, 3 phase alternator running at 3000 r.p.m. has 42 armature slots with 2 conductors in each slot. Calculate the flux/pole required to generate a line voltage of 2300 V. Distribution factor is 0.952 and the pitch factor is 0.956.*

VTU : Aug.-06, Marks 8; July-08; June-10, Jan.-16,18, Marks 6

Sol. : Given, $P = 2$, $N_s = 3000$ r.p.m., $E_{line} = 2300$ V, $K_d = 0.952$, $K_c = 0.956$

$$N_s = \frac{120\ f}{P} \quad \text{i.e.} \quad f = \frac{2 \times 3000}{120} = 50 \text{ Hz}$$

$$E_{ph} = \frac{E_{line}}{\sqrt{3}} = \frac{2300}{\sqrt{3}} = 1327.9056 \text{ V}$$

... Assume star

Total slots = 42, 2 Conductors/ slot

$$\therefore\ Z = \text{Slots} \times \text{Conductors/slot}$$

$$= 2 \times 42 = 84$$

$$\therefore\ Z_{ph} = \frac{Z}{3} = \frac{84}{3} = 28$$

$$\text{i.e.}\quad T_{ph} = \frac{Z_{ph}}{2} = \frac{28}{2} = 14$$

$$E_{ph} = 4 \cdot 44\ \phi\ f\ K_c\ K_d\ T_{ph}$$

$$\text{i.e.}\quad 1327.9056 = 4.44 \times \phi \times 50 \times 0.952 \times 0.956 \times 14$$

$$\therefore\ \phi = \textbf{0.4694 Wb}$$

Ex. 9.7.4 *A 4 - pole, 1500 r.p.m star connected alternator has 9 slots / pole and 8 conductors per slot. Determine the flux per pole to give a terminal voltage of 3300 V. Take winding factor and pitch factor as unity.* **VTU : Jan.-15, Marks 8**

Sol. : $P = 4$, $N_s = 1500$ r.p.m., 9 slots/pole = n, 8 conductors/slot

$$\therefore\ n = 9,\ \text{Slots} = n \times P = 9 \times 4 = 36,\ V_L = 3300 \text{ V}$$

$$\therefore\ Z = \text{Slots} \times \text{Conductors/Slot} = 36 \times 8 = 288$$

$$Z_{ph} = \frac{Z}{3} = 96,\ T_{ph} = \frac{Z_{ph}}{2} = 48,$$

$$E_{ph} = \frac{V_L}{\sqrt{3}} = 1905.255 \text{ V}$$

$$N_s = \frac{120\ f}{P}$$

$$\text{i.e.}\quad f = \frac{1500 \times 4}{120} = 50 \text{ Hz},\ K_c = K_d = 1$$

$$\therefore\ E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$$

$$\text{i.e.}\quad 1905.2558 = 4.44 \times \phi \times 50 \times 48$$

$$\therefore\ \phi = \textbf{0.1788 Wb}$$

Ex. 9.7.5 *A 6 pole, 3 phase, star connected alternator has an armature with 90 slots and 12 conductors per slot. It revolves at 1000 rpm, the flux per pole being 0.5 web. Calculate the emf generated, if the winding factor is 0.97 and all the conductors in each are in series. The coil is full pitched.* **VTU : Dec.-11, June-12, July-16, Jan.-17, Marks 6**

Sol. : $P = 6$, slots = 90, 12 conductors/slot,

$N_s = 1000$ r.p.m

$\phi = 0.5$ Wb, $K_d = 0.97$, $K_c = 1$ (full pitch)

$$N_s = \frac{120 f}{P} \quad \text{i.e.} \quad f = \frac{1000 \times 6}{120} = 50 \text{ Hz}$$

$$Z = \text{Slots} \times \text{Conductors/Slot} = 90 \times 12 = 1080$$

$$\therefore\ Z_{ph} = \frac{Z}{3} = 360,\ T_{ph} = \frac{Z_{ph}}{2} = 180$$

...2 conductors → 1 turn

$$\therefore\ E_{ph} = 4.44\ K_c K_d\ \phi f T_{ph}$$

$$= 4.44 \times 0.97 \times 1 \times 0.5 \times 50 \times 180$$

$$= \textbf{19.3806 kV}$$

$$\therefore\ E_{line} = \sqrt{3}\ E_{ph} = \sqrt{3} \times 19.3806 = \textbf{33.568 kV}$$

Ex. 9.7.6 *A 12 pole 500 rpm star connected alternator has 48 slots with 15 conductors per slot. The flux per pole is 0.02 web. and is distributed sinusoidally. The winding factor is 0.97 and pitch factor is 0.98. Calculate the line emf.* **VTU : Jan.-13, 15, 16, Marks 6**

Sol. : $P = 12$, $N_s = 500$ r.p.m., $\phi = 0.02$ Wb, 48 slots, 15 Conductors/slot, $K_d = 0.97$, $K_c = 0.98$

$$N_s = \frac{120\ f}{P} \quad \text{i.e.} \quad f = \frac{500 \times 12}{120} = 50 \text{ Hz}$$

$$Z = 48 \times 15 = 720,\ Z_{ph} = \frac{Z}{3} = 240,$$

$$T_{ph} = \frac{Z_{ph}}{2} = 120$$

$$\therefore\ E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph} = 506.48 \text{ V}$$

$$\therefore\ E_{line} = \sqrt{3}\ E_{ph} = 877.248 \text{ V}$$

... Star

Ex. 9.7.7 *A 3-phase, 6-pole, star-connected alternator revolves at 1000 r.p.m. The stator has 90 slots and 8 conductors per slot. The flux per pole is 0.05 Wb. (sinusoidally distributed). Calculate the voltage generated by the machine if the winding factor is 0.96, line and phase value.* **VTU : Jan.-14, Marks 8**

Sol. : $P = 6$, $N_s = 1000$ r.p.m., 90 slots, 8 Conductors / slot

$$\phi = 0.05 \text{ Wb},\ K_d = 0.96,\ K_c = 1$$

$$Z = \text{Slots} \times \text{Conductors / slot} = 90 \times 8 = 720$$

$$\therefore\ Z_{ph} = \frac{Z}{3} = 240,\quad T_{ph} = \frac{Z_{ph}}{2} = 120$$

$$n = \frac{\text{Slots}}{\text{Pole}} = \frac{90}{6} = 15,$$

$$m = \text{Slots / pole / ph} = \frac{n}{3} = 5$$

$$\beta = \frac{180^\circ}{n} = 12^\circ,$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m \sin\frac{\beta}{2}} = 0.9566 \approx 0.96 \text{ (given)}$$

$$\therefore\ N_s = \frac{120 f}{P} \quad \text{i.e.} \quad f = \frac{1000 \times 6}{120} = 50 \text{ Hz}$$

$$\therefore\ E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph} = \textbf{1278.72 V}$$

$$E_{line} = \sqrt{3}\ E_{ph} = \textbf{2214.808 V} \quad \text{...Star connection}$$

Ex. 9.7.8 *A 3 - phase, 50 Hz, 16 pole generator with star connected winding has 144 slots with conductor/slot is 10. The flux per pole is 24.8 Wb is sinusoidally distributed. The coils are full pitched.*

Find : i) Speed ii) The line emf. **VTU : July-15, Jan.-17, Marks 8**

Sol. : P = 16 , 144 slots , 10 Conductors/slot , ϕ = 24.8 mWb

$$n = \frac{Slots}{P} = 9,\ \beta = \frac{180^\circ}{n} = 20^\circ,$$

$$m = \frac{n}{3} = 3, \quad f = 50 \text{ Hz}$$

$$\therefore \quad K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = 0.9598,$$

$$Z = \text{Slots} \times \text{Conductors/Slot} = 1440$$

$$Z_{ph} = \frac{Z}{3} = 480,$$

$$T_{ph} = \frac{Z_{ph}}{2} = 240 \quad \text{... 2 conductors 1 turn}$$

i) $N_s = \frac{120f}{P} = \frac{120 \times 50}{16} = 375 \text{ r.p.m}$

ii) $E_{ph} = 4.44\, K_c K_d\, \phi f T_{ph}$

$= 1268.226 \text{ V}$ ... $K_c = 1$ full pitch

$\therefore E_{line} = \sqrt{3}\, E_{ph} = 2196.6318 \text{ V}$ ... Star connection

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Ex. 9.7.9 *A 2-pole, 3-ph alternator running at 3000 rpm has 42 armature slots with 2 conductors in each slot. Calculate the flux/pole required to generate a phase voltage of 1100 V. Assume K_d = 0.97, and full pitch winding.* **VTU : Jan.-16, Marks 5**

Sol. : Refer similar example 9.7.3 and verify the answer as : ϕ = **0.3648 Wb**

Ex. 9.7.10 *A 6 pole, 3 phase, star connected alternator has an armature with 90 slots and 12 conductors per slot. It revolves at 1000 rpm, the flux per pole being 0.5 web. Calculate the emf generated, if the winding factor is 0.97 and all the conductors in each are in series. The coil is full pitched* **VTU : July-16, Marks 5**

Sol. : Refer similar example 9.7.1 and verify the answers : For full pitch winding K_c = 1,

$$E_{ph} = \mathbf{19.3806\ kV},$$

$$E_{line} = \sqrt{3}\, E_{ph} = \mathbf{33.568\ kV}$$

Ex. 9.7.11 *A 6 pole, 3 phase, 50 Hz alternator has 12 slots per pole and 4 conductors per slot. The winding is 5/6 full pitched. A flux of 25 mWb is sinusoidally distributed along the air gap. Determine the line emf, if the alternator is star connected.* **VTU : July-16, Marks 5**

Sol. : P = 6, f = 50 Hz, ϕ = 25 mWb, Star

$$Z = \text{Slots} \times (\text{Conductors/Slot}) = 12 \times 4 = 48$$

$$\therefore \quad Z_{ph} = \frac{Z}{3} = 16, \quad T_{ph} = \frac{Z_{ph}}{2} = 8$$

Winding is $\frac{5}{6}$ of full pitch. One full pitch = 180° coil span.

$$\therefore \quad \text{Coil span} = \frac{5}{6} \times 180^\circ = 150^\circ$$

$$\therefore \quad \alpha = 180^\circ - \text{Actual coil span} = 180^\circ - 150^\circ = 30^\circ$$

$$\therefore \quad K_c = \cos\frac{\alpha}{2} = \cos 15^\circ = 0.9659$$

$$n = 12 \text{ slots/pole},$$

$$m = \frac{12}{3} = 4 \text{ slots/pole/ph}, \qquad \beta = \frac{180^\circ}{n} = 15^\circ$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{4 \times 15^\circ}{2}\right)}{4\sin\left(\frac{15^\circ}{2}\right)} = 0.9576$$

$$\therefore \quad E_{ph} = 4.44 K_c K_d\, \phi f T_{ph} = 4.44 \times 0.9659 \times 0.9576 \times 25 \times 10^{-3} \times 50 \times 8 = \mathbf{41.067V}$$

$$\therefore \quad E_{line} = \sqrt{3}\, E_{ph} = \mathbf{71.1311V}$$

9.8 University Questions with Answers

Jan. - 2010

Q.1 *Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.* **[Refer section 9.4.2]** **[8]**

Q.2 *What is pitch factor ? State its expression.* **[Refer section 9.7]** **[2]**

Q.3 *What is distribution factor ? State its expression.* **[Refer section 9.7]** **[2]**

Q.4 *Explain voltage regulation of an alternator and state its significance.* **[Refer section 9.9]** **[6]**

July - 2010

Q.5 *Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.* **[Refer section 9.4.2]** **[8]**

Q.6 *Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.* **[Refer section 9.7]** **[6]**

Jan. - 2011

Q.7 *Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.* **[Refer section 9.7]** **[6]**

July - 2011

Q.8 *Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.* **[Refer section 9.4.2]** **[8]**

Jan. - 2012

Q.9 *State the advantages of rotating field over rotating armature used in alternators.* **[Refer section 9.3]** **[6]**

Q.10 *Distinguish between salient and nonsalient type of alternator rotors.* **[Refer section 9.4.2]** **[4]**

June - 2012

Q.11 *Distinguish between salient and nonsalient type of alternator rotors.* **[Refer section 9.4.2]** **[4]**

Q.12 *Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.* **[Refer section 9.7]** **[6]**

Jan. - 2013

Q.13 *Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.* **[Refer section 9.4.2]** **[8]**

Q.14 *Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.* **[Refer section 9.7]** **[6]**

June - 2013

Q.15 *Explain the construction of stator of an alternator.* **[Refer section 9.4.1]** **[4]**

Q.16 *Discuss the different types of rotors used in the alternators. Mention their characteristic features and applications.* **[Refer section 9.4.2]** **[8]**

Q.17 *Explain the principle of operation of an alternator.* **[Refer section 9.5]** **[5]**

Jan. - 2014

Q.18 *By means of a neat diagram, describe the main parts of an alternator with their functions.* **[Refer section 9.4]** **[8]**

July - 2015

Q.19 *Derive the e.m.f. equation of an alternator. Explain the significance of winding factors.* **[Refer section 9.7]** **[6]**

CBCS Scheme

Jan. - 2016

Q.20 *What are the advantages of rotating field type alternator ?* **[Refer section 9.3]** **[6]**

Q.21 *With neat sketches, explain the construction of salient pole alternator.* **[Refer section 9.4]** **[4]**

Q.22 *Derive an e.m.f. equation of alternator.* **[Refer section 9.7]** **[6]**

Jan. - 2017

Q.23 *With a neat sketch, explain the constructional details of alternator.* **(Refer section 9.4)** **[5]**

Q.24 *A 3 ϕ 16 pole alternator has a star connected winding with 144 slots and 10 conductor per slots. The flux per pole is 30 mWb. Find the phase and line voltages, if the speed is 375 rpm.* **(Refer example 9.7.10)** **[5]**

July - 2017

Q.25 *Mention the advantages of stationary armature of an alternator.* **(Refer section 9.3)** **[5]**

Jan. - 2018

Q.26 *With a neat diagram, explain the constructional features of three phase alternator (consider salient pole rotor).* **(Refer section 9.4.2)** **[6]**

□□□

MODULE - 5

10

Three Phase Induction Motors

Syllabus

Principle of Operation, Generation of rotating magnetic field, Construction and working of three-phase induction motor, Slip and its significance. Necessity of starter, star-delta starter.

Contents

10.1 Introduction

- An electric motor which operates on a.c. supply is called a.c. motor.
- As a.c. supply is commonly available, the a.c. motors are very popularly used in practice.
- The a.c. motors are classified as single and three phase induction motors, synchronous motors and some special purpose motors.
- Out of all these types, three phase induction motors are widely used for various industrial applications.
- The important advantages of three phase induction motors over other types are self starting property, no need of starting device, higher power factor, good speed regulation and robust construction.

10.2 Rotating Magnetic Field

➤ ***What is rotating magnetic field ? What is the speed of rotating magnetic field ?*** **VTU : Jan.-03, 05, 07, 10; July-03, 05, 06, 08, 15, Marks 4**

➤ ***With diagrams, explain the concept of rotating magnetic field.*** **VTU : Dec.-11, July-17, Marks 6**

➤ ***Prove that a rotating magnetic field of constant magnitude is produced when the stator windings of a polyphase induction motor are energized by a balanced 3 phase supply.*** **VTU : Jan.-13, Marks 8**

- The stator of a three phase induction motor carries a three phase star or delta connected winding, to which three phase a.c. supply is given.

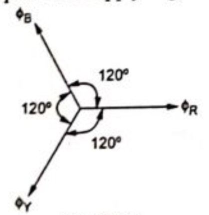


Fig. 10.2.1

- The three phase currents flow simultaneously through the windings and are displaced by 120° from each other.
- If the phase sequence is RYB, the three phase currents produce the three fluxes ϕ_R, ϕ_Y and ϕ_B which are displaced by 120° from each other.
- Let the magnitude of each flux is ϕ_m.
- The Fig. 10.2.1 shows the phasor diagram with ϕ_R as reference. The directions shown are the **assumed positive directions** of the three fluxes. The flux in opposite direction to the directions shown in treated as negative.
- The equations of the three fluxes are,

$$\boxed{\begin{aligned}\phi_R &= \phi_m \sin\theta,\\ \phi_Y &= \phi_m \sin(\theta - 120°),\\ \phi_B &= \phi_m \sin(\phi - 240°)\end{aligned}}$$

- The total flux ϕ_T is the vector sum of ϕ_R, ϕ_Y and ϕ_B for various values of θ.

Case 1 : θ = 0° and use in the flux equations
∴ $\phi_R = 0$, $\phi_Y = -0.866\ \phi_m$, $\phi_B = +0.866\ \phi_m$

- The phasor addition is shown in the Fig. 10.2.2.
- The negative ϕ_Y is indicated in opposite direction to the assumed positive direction of ϕ_Y, shown in Fig. 10.2.1.

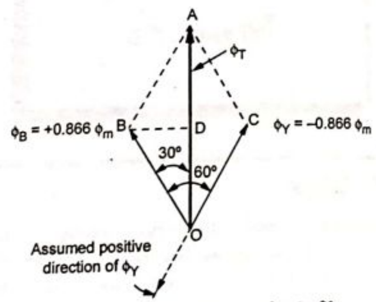


Fig. 10.2.2 Vector diagram for θ = 0°

- BD is perpendicular drawn from B on ϕ_T which bisects ϕ_T.

∴ $OD = DA = \frac{\phi_T}{2}$ and in Δ OBD,

$\angle BOD = 30°$

∴ $\cos 30° = \frac{OD}{OB} = \frac{\phi_T/2}{0.866\,\phi_m}$

i.e. $\phi_T = 1.5\ \phi_m$ ...cos 30° = 0.866

- Thus $\phi_T = 1.5\ \phi_m$ in magnitude and its position is vertically upwards at θ = 0°.

Case 2 : θ = 60° and use in the flux equations
∴ $\phi_R = 0.866\ \phi_m$, $\phi_Y = -0.866\ \phi_m$, $\phi_B = 0$

- Thus ϕ_R is positive and ϕ_Y is negative hence phasor diagram is as shown in the Fig. 10.2.3.

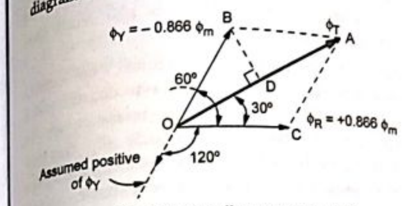


Fig. 10.2.3 Vector diagram for θ = 60°

- By same geometrical construction, it can be seen that $\phi_T = 1.5\ \phi_m$.
- But though magnitude of ϕ_T is same, **it is rotated through 60° in space in clockwise direction.**
- Similarly if phasor diagram is drawn for various values of θ, it can be seen that the magnitude of ϕ_T is always 1.5 ϕ_m but it rotates in space. Such a magnetic field is called **rotating magnetic field.**
- Thus though supply is stationary, windings are stationary, the resultant flux produced is rotating in space with constant mgnitude and speed.

> *This shows that when a three phase stationary windings are excited by balanced three phase a.c. supply then the resulting field produced is rotating magnetic field. Though nothing is physically rotating, the field produced is rotating in space having constant amplitude.*

- There exists a fixed relation between frequency f of a.c. supply to the windings, the number of poles P for which winding is wound and speed N r.p.m. of rotating magnetic field.
- For a standard frequency whatever speed of R.M.F. results is called **synchronous speed,** in case of induction motors. It is denoted as N_s.

$$\therefore \quad \boxed{N_s = \frac{120f}{P} = \textbf{Speed of R.M.F.}}$$

where f = Supply frequency in Hz

P = Number of poles for which winding is wound.

- The direction of rotating magnetic field depends on the phase sequence of the three phase supply.

> - By interchanging any two terminals of three phase winding while connecting it to three phase a.c. supply, direction of rotation of R.M.F. gets reversed.

- Thus by changing the supply phase sequence, the direction of three phase induction motor can be reversed.

10.3 Construction

- Basically, the induction motor consists of two main parts, namely
  1. The part i.e. three phase windings, which is stationary called stator.
  2. The part which rotates and is connected to the mechanical load through shaft called rotor.
- The conversion of electrical power to mechanical power takes place in a rotor. Hence rotor develops a driving torque and rotates.

10.3.1 Stator

➤ ***Discuss the construction of stator of three phase induction motor.***

- The stator has a laminated type of construction made up of stampings which are 0.4 to 0.5 mm thick.
- The stampings are slotted on its periphery to carry the stator winding. The stampings are insulated from each other. Such a construction essentially keeps the iron losses to a minimum value.

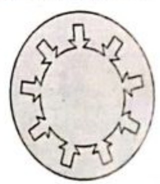


Fig. 10.3.1 Stator lamination

- The number of stampings are stamped together to build the stator core.
- The built up core is then fitted in a casted or fabricated steel frame. The choice of material for the

stampings is generally silicon steel, which minimises the hysteresis loss.

- The slots on the periphery of the stator core carries a **three phase winding**, connected either in star or delta. This three phase winding is called **stator winding**. It is wound for definite number of poles.
- The radial ducts are provided for the cooling purpose. The Fig. 10.3.1 shows a stator lamination.

10.3.2 Rotor

➤ ***Discuss the important features of squirrel cage and phase wound rotor constructions in an induction motor.***

VTU : Jan.-03, 05, Marks 6; Jan.-08, July-17, Marks 8

- The rotor is placed inside the stator. The air gap between stator and the rotor is 0.4 mm to 4 mm.
- The two types of rotor constructions which are used for induction motors are,
  1. Squirrel cage rotor and
  2. Slip ring or phase wound rotor

1. Squirrel Cage Rotor

- The rotor core is cylindrical and slotted on its periphery.
- The rotor consists of uninsulated copper or aluminium bars called rotor conductors. The bars are placed in the slots.
- These bars are permanently shorted at each end with the help of conducting copper ring called **end ring**. The bars are usually brazed to the end rings to provide good mechanical strength.
- The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel cage rotor. The construction is shown in the Fig. 10.3.2.
- As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor is also called **short circuited rotor.**
- As rotor itself is short circuited, no external resistance can have any effect on the rotor resistance. Hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly is not required for this rotor. Hence the construction of this rotor is very simple.
- Fan blades are generally provided at the ends of the rotor core. This circulates the air through the machine while operation, providing the necessary cooling.
- In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in the Fig. 10.3.3.

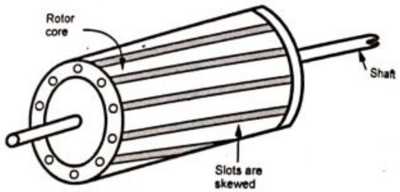
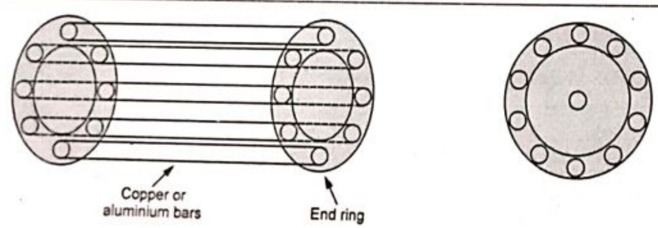


Fig. 10.3.3 Skewing in rotor construction

➤ ***State the advantages of skewing.***

- The advantages of skewing are,

1. A magnetic hum i.e. noise gets reduced due to skewing hence skewing makes the motor operation quiter.
2. It makes the motor operation smooth.
3. The stator and rotor teeth may get magnetically locked. Such a tendency of magnetic locking gets reduced due to skewing.
4. It increases the effective transformation ratio between stator and rotor.



(a) Cage type structure of rotor

(b) Symbolic representation

Fig. 10.3.2 Squirrel cage rotor

2. Slip Ring Rotor or Phase Wound Rotor

- In this type of construction, rotor winding is exactly similar to the stator. The rotor carries a three phase star or delta connected, distributed winding, wound for same number of poles as that of stator.
- The rotor construction is laminated and slotted. The slots contain the rotor winding.
- The three ends of three phase winding, available after connecting the winding in star or delta, are permanently connected to the slip rings.
- With the help of slip rings, the external resistances can be added in series with each phase of the rotor winding. This arrangement is shown in the Fig. 10.3.4.

> This way the value of rotor resistance per phase can be controlled. This helps us to control some of the important characteristics of the motor like starting torque, speed etc.

- In the running condition, the slip rings are shorted.
- The possibility of addition of an external resistance in series with the rotor, with the help of slip rings is the main feature of this type of rotor.

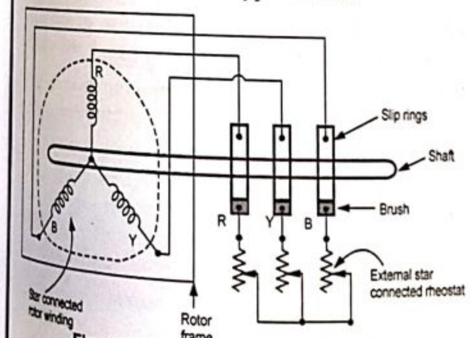


Fig. 10.3.4 Slip rings or wound rotor

10.3.3 Comparison of Squirrel Cage and Wound Rotor

➤ ***Compare squirrel cage and phase wound type of rotors.***

VTU : Jan.-16, Marks 6

Sr. No.	Wound or slip ring rotor	Squirrel cage rotor
1.	Rotor consists of a three phase winding similar to the stator winding.	Rotor consists of bars which are shorted at the ends with the help of end rings.
2.	Construction is complicated.	Construction is very simple.
3.	Resistance can be added externally.	As permanently shorted, external resistance cannot be added.
4.	Slip rings and brushes are present to add external resistance.	Slip rings and brushes are absent.
5.	The construction is delicate and due to brushes, frequent maintenance is necessary.	The construction is robust and maintenance free.
6.	The rotors are very costly.	Due to simple construction, the rotors are cheap.
7.	Only 5 % of induction motors in industry use slip ring rotor.	Very common and almost 95 % induction motors use this type of rotor.
8.	High starting torque can be obtained.	Moderate starting torque which cannot be controlled.
9.	Rotor resistance starter can be used.	Rotor resistance starter cannot be used.
10.	Rotor must be wound for the same number of poles as that of stator.	The rotor automatically adjusts itself for the same number of poles as that of stator.
11.	Speed control by rotor resistance is possible.	Speed control by rotor resistance is not possible.
12.	Rotor copper losses are high hence efficiency is less.	Rotor copper losses are less hence have higher efficiency.
13.	Used for lifts, hoists, cranes, elevators, compressors etc.	Used for lathes, drilling machines, fans, blowers, water pumps, grinders, printing machines etc.

10.4 Principle of Operation

> ***Explain the working principle of three phase induction motor.*** VTU : Jan.-07,09,10,11,13,14,17,18; July-03, 04, 05, 06, 07, 08, 11, June-12, Marks 6

- Induction motor works on the principle of electromagnetic induction.
- When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced. The speed of this rotating magnetic field is synchronous speed, N_s r.p.m.

$$N_s = \frac{120\,f}{P} = \text{Speed of rotating magnetic field.}$$

- This rotating field produces an effect of rotating poles around a rotor.
- Let direction of rotation of this rotating magnetic field is clockwise as shown in the Fig. 10.4.1 (a).
- Now at this instant rotor is **stationary** and stator flux R.M.F. is **rotating**. So its obvious that there exists a relative motion between the R.M.F. and rotor conductors.
- Whenever conductor cuts the flux, e.m.f. gets induced in it. So e.m.f. gets induced in the rotor conductors called **rotor induced e.m.f.** This is electro-magnetic induction.
- As rotor forms closed circuit, induced e.m.f. circulates current through rotor called **rotor current** as shown in the Fig. 10.4.1 (b). Let direction of this current is going into the paper denoted by a cross as shown in the Fig. 10.4.1 (b).
- Any current carrying conductor produces its own flux. So rotor produces its flux called **rotor flux**. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown in the Fig. 10.4.1 (c).
- Both the fluxes interact with each as shown in the Fig. 10.4.1 (d).

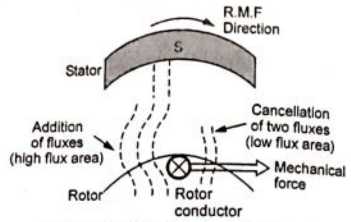


Fig. 10.4.1 (d) Interaction of fluxes

- On left of rotor conductor, two fluxes are in same direction hence add up to get high flux area.
- On right side, two fluxes cancel each other to produce low flux area.
- As flux lines act as stretched rubber band, high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case, as shown in the Fig. 10.4.1 (d), due to **interaction of the two fluxes.**
- As all the rotor conductors experience a force, the overall rotor experiences a torque and starts rotating. So **interaction of the two fluxes is very essential for a motoring action.**

> ***What is the direction of rotation of rotor ?***

- According to Lenz's law the direction of induced current in the rotor is so as to oppose the cause producing it.

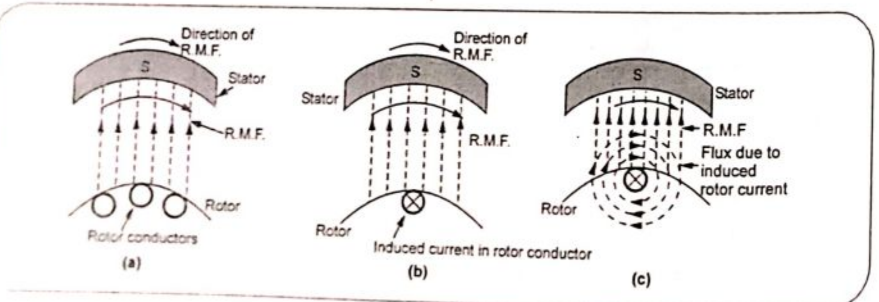


Fig. 10.4.1

- The cause of rotor current is the induced e.m.f. which is induced because of relative motion present between the rotating magnetic field and the rotor conductors.
- Hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of rotating magnetic field.
- N_s = Speed of rotating magnetic field in r.p.m.
  N = Speed of rotor i.e. motor in r.p.m.
  $N_s - N$ = Relative speed between the two, rotating magnetic field and the rotor
- Thus rotor always rotates in same direction as that of R.M.F.

10.4.1 Can $N = N_s$?

> ***Why induction motor can not run at synchronous speed ?*** VTU : Jan.-06, 09, Marks 3

> ***Explain synchronous speed, slip speed and motor speed in case of three phase induction motor. Why slip is never zero in an induction motor ?*** VTU : Jan.-04, 08; July-05, 09, 10, 15, Marks 6

> ***What is the significance of slip in an induction motor ?*** VTU : Dec.-11, Marks 4

> ***What is meant by the slip of the induction motor ? Under what circumstances the slip is i) Unity and ii) Zero.*** VTU : June-13, Jan.-16, July-16, Marks 8

- When rotor starts rotating, it tries to catch the speed of rotating magnetic field.
- If it catches the speed of the rotating magnetic field, the relative motion between rotor and the rotating magnetic field will vanish ($N_s - N = 0$).
- In fact the relative motion is the main cause for the induced e.m.f. in the rotor. So induced e.m.f. will vanish and hence there cannot be rotor current and the rotor flux which is essential to produce the torque on the rotor.
- Eventually motor will stop. But immediately there will exist a relative motion between rotor and rotating magnetic field and it will start.
- But due to inertia of rotor, this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of the rotating magnetic field in the steady state.
- The induction motor never rotates at synchronous speed.
- The speed at which it rotates is hence called **subsynchronous speed** and motor sometimes called **asynchronous motor.**

$$\therefore \quad N < N_s$$

- So it can be said that rotor slips behind the rotating magnetic field produced by stator.
- The difference between the two is called **slip speed** of the motor.

$$N_s - N = \text{Slip speed of the motor in r.p.m.}$$

- This speed decides the magnitude of the induced e.m.f. and the rotor current, which in turn decides the torque produced.

10.4.2 Slip of Induction Motor

- The slip speed ($N_s - N$) is generally expressed as the percentage of the synchronous speed.
- Slip of the induction motor is defined as the difference between the synchronous speed (N_s) and actual speed of rotor i.e. motor (N) expressed as a fraction of the synchronous speed (N_s). This is also called **absolute slip** or **fractional slip** and is denoted as 's'.

$$\text{Thus} \quad \boxed{s = \frac{N_s - N}{N_s}} \quad \text{... (Absolute slip)}$$

- The percentage slip is expressed as,

$$\boxed{\%\,s = \frac{N_s - N}{N_s} \times 100} \quad \text{... (Percentage slip)}$$

- In terms of slip, the actual speed of motor (N) can be expressed as,

$$\boxed{N = N_s\,(1 - s)} \quad \text{... (From the expression of slip)}$$

- At start, motor is at rest and hence its speed N is zero.

$$\therefore \quad \boxed{s = 1 \text{ at start}}$$

- This is maximum value of slip s possible for induction motor which occurs at start.
- While s = 0 gives us N = N_s which is not possible for an induction motor. So **slip of induction motor cannot be zero** under any circumstances.
- Practically motor operates in the slip range of 0.01 to 0.05 i.e. 1 % to 5 %. The slip corresponding to full load speed of the motor is called **full load slip.**

Ex. 10.4.1 *A 3 phase, 50 Hz, 6 pole induction motor has a full load percentage slip of 3 %. Find i) Synchronous speed and ii) Actual speed.* **VTU : Jan.-03, Marks 4**

Sol. : f = 50 Hz, P = 6, s = 3 % = 0.03.

i) $N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = \textbf{1000 r.p.m.}$

... Synchronous speed

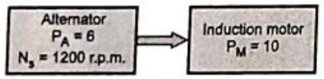
ii) $\% s = \frac{N_s - N}{N_s} \times 100$

i.e. $N = N_s(1 - s) = 1000\,(1 - 0.03)$

$= \textbf{970 r.p.m.}$ ... Actual speed

Ex. 10.4.2 *A 10 pole induction motor supplied by a 6 pole alternator, which is driven at 1200 rpm. If the motor runs at slip of 3 %, what is its speed ?* **VTU : Aug.-06; June-10, Marks 6; July-08, Jan.-17, Marks 5**

Sol. : For alternator, $N_s = \frac{120\,f}{P_A}$



$\therefore \quad f = \frac{1200 \times 6}{120} = 60 \text{ Hz}$

For induction motor,

$\therefore \quad \% s = \frac{N_s - N}{N_s}$ and $N_s = \frac{120\,f}{P_M}$

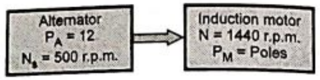
$N_s = \frac{120 \times 60}{10} = 720 \text{ r.p.m.}$

$\therefore \quad 3 = \frac{720 - N}{720} \times 100$ i.e. **N = 698.4 r.p.m**

Ex. 10.4.3 *A 12 pole, 3 phase alternator is coupled to an engine running at 500 r.p.m. It supplies an induction motor, which has a full load speed of 1440 r.p.m. Find the percentage slip and the number of poles of the motor.* **VTU : Feb.-07, Marks 6, Aug.-05; Feb.-05; Aug.-11, Marks 6**

Sol. : For an alternator, $N_s = \frac{120\,f}{P_A}$

$f = \frac{500 \times 12}{120} = 50 \text{ Hz}$



For an induction motor, speed is 1440 r.p.m. and practically actual speed is slightly less than the synchronous speed. Hence the synchronous speed for an induction motor has to be 1500 r.p.m.

$N_s = 1500$ r.p.m. ... For motor

But $N_s = \frac{120\,f}{P_M}$

$\therefore \quad P_M = \frac{120 \times 50}{1500} = 4$ ... Poles of motor

$\% s = \frac{N_s - N}{N_s} \times 100 = \frac{1500 - 1440}{1500} \times 100 = \textbf{4 \%}$

Ex. 10.4.4 *An 8-pole alternator runs at 750 r.p.m and supplies power to a 6-pole induction motor which runs at 970 r.p.m. What is the slip of the induction motor ?* **VTU : Jan.-15, Marks 6**

Sol. : P = 8, N_s = 750 r.p.m. alternator

P = 6, N = 970 r.p.m. induction motor

For alternator, $N_s = \frac{120\,f}{P}$

i.e. $f = \frac{750 \times 8}{120} = 50 \text{ Hz}$

For induction motor, $N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6}$

$= 1000$ r.p.m.

$\therefore$ Slip $s = \frac{N_s - N}{N_s} \times 100 = \frac{1000 - 970}{1000} \times 100 = \textbf{3 \%}$

10.5 Effect of Slip on the Rotor Frequency

> ***Derive the expression for the slip and frequency of rotor currents.*** **VTU : Jan.-09, 16, July-07, 11, 16, June-12, Marks 6**

- In case of induction motor, the speed of rotating magnetic field is,

$$N_s = \frac{120\,f}{P} \quad \text{... (10.5.1)}$$

- At start when N = 0, s = 1 and stationary rotor has maximum relative motion with respect to R.M.F. Hence maximum e.m.f. gets induced in the rotor at start.
- **The frequency of this induced e.m.f. at start is same as that of supply frequency.**
- As motor actually rotates with speed N, the relative speed of rotor with respect R.M.F. decreases and becomes equal to slip speed of $N_s - N$.
- The induced e.m.f. in rotor depends on rate of cutting flux i.e. relative speed $N_s - N$.
- Hence in running condition magnitude of induced e.m.f. decreases so as its frequency.
- **The rotor is wound for same number of poles as that of stator i.e. P.**
- If f_r is the frequency of rotor induced e.m.f. and rotor currents, in running condition at slip speed $N_s - N$ then there exists a fixed relation between $(N_s - N)$, f_r and P similar to equation (10.5.1).
- So we can write for rotor in running condition,

$$(N_s - N) = \frac{120\,f_r}{P},$$

Rotor poles = Stator pole = P ... (10.5.2)

- Dividing equation (10.5.2) by equation (10.5.1) we get,

$$\frac{N_s - N}{N_s} = \frac{(120 f_r / P)}{(120 f / P)} \quad \text{but } \frac{N_s - N}{N_s} = \text{Slip s}$$

i.e. $s = \frac{f_r}{f}$

$$\therefore \quad \boxed{f_r = s\,f}$$

- Thus frequency of rotor induced e.m.f. in running condition (f_r) is slip times the supply frequency (f).
- As slip of the induction motor is in the range 0.01 to 0.05, rotor frequency is very small inthe running condition.

Ex. 10.5.1 *A 3ϕ 4 pole, 400 V, 50 Hz induction motor runs with a slip of 4 %, find rotor speed and frequency.* **VTU : Feb. 06, Marks 8**

Sol. : P = 4, f = 50 Hz, s = 4 %

$\therefore \quad N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = \textbf{1500 r.p.m.}$

$\therefore \quad \% s = \frac{N_s - N}{N_s}$ i.e. $4 = \frac{1500 - N}{1500} \times 100$

$\therefore \quad$ **N = 1440 r.p.m.**

$f_r = sf = 0.04 \times 50 = \textbf{2 Hz}$

Ex. 10.5.2 *A 4 pole, 3-phase, 50 Hz induction motor runs at a speed of 1,470 rpm. Find the synchronous speed, the slip and frequency of the induced emf in the rotor under this condition.* **VTU : Jan. 18, Marks 8**

Ans. : f = 50 Hz, P = 4, N = 1470 r.p.m.

$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = \textbf{1500 r.p.m.}$

$\% s = \frac{N_s \times N}{N_s} \times 100 = \frac{1500 - 1470}{1500} \times 100 = \textbf{2 \%}$

$f_r = s\,f = 0.02 \times 50 = \textbf{1 Hz}$

Ex. 10.5.3 *An 8 pole alternator runs at 750 r.p.m. supplies power to 4 pole induction motor. The frequency of rotor is 1.5 Hz. What is the speed of the motor ? What is the slip ?* **VTU : Aug.-07, Jan.-15, Marks 4**

Sol. : Alternator supplies induction motor,

Alternator	Induction motor
P_A = 8, N_s = 750 r.p.m.	P_M = 4, f_r = 1.5 Hz

For alternator,

$N_s = \frac{120\,f}{P_A}$ i.e. $750 = \frac{120\,f}{8}$

$\therefore \quad f = 50$ Hz

For an induction motor, $f_r = s\,f$ i.e. $1.5 = s \times 50$

$\therefore \quad s = 0.03$ i.e. 3 % ... Slip

$N_s = \frac{120\,f}{P_M} = \frac{120 \times 50}{4} = 1500$ r.p.m.

$N = N_s(1 - s) = 1500(1 - 0.03)$

$= \textbf{1455 r.p.m.}$ ... Speed of motor

Ex. 10.5.4 *If a six pole induction motor supplied from a three phase 50 Hz supply has a rotor frequency of 2.3 Hz. Calculate : i) The percentage slip ii) The speed of the motor*

VTU : Feb.-08; Jan.-11, 16, Marks 6

Sol. : $P = 6,\ f = 50\ Hz,\ f_r = 2.3\ Hz$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

i) $f_r = s\,f$ i.e. $2.3 = s \times 50$

$\therefore$ $s = \frac{2.3}{50} = 0.046$ i.e. **4.6 %** ...Slip

ii) $N = N_s\,(1 - s) = 1000\,(1 - 0.046)$

$= \mathbf{954\ r.p.m.}$ ...Speed of motor

Ex. 10.5.5 *A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1 % at no load, and 3% at full load. Determine : i) Synchronous speed; ii) No-load speed; iii) Full-load speed; iv) Frequency of rotor current at stand still; v) Frequency of rotor current at full load.*

VTU : Jan.-14, Marks 5

Sol. : $P = 6,\ f = 50\ Hz,\ s_0 = 1\ \%,\ s_f = 3\ \%$

i) $N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = \mathbf{1000\ r.p.m.}$

ii) $N_0 = N_s\,(1 - s_0) = 1000\,(1 - 0.01) = \mathbf{990\ r.p.m.}$

iii) $N_f = N_s\,(1 - s_f) = 1000\,(1 - 0.03) = \mathbf{970\ r.p.m.}$

iv) At standstill, $f_r = f = \mathbf{50\ Hz}$

v) At full load, $f_r = s_f \times f = 0.03 \times 50 = \mathbf{1.5\ Hz}$

Ex. 10.5.6 *The frequency of the e.m.f. in the stator of 4 pole induction motor is 50 Hz and that in the rotor is 1.5 Hz. What is the slip and at what speed is the motor is running ?*

VTU : Dec.-11, July-15, Marks 6

Sol. : $P = 4,\ f = 50\ Hz,\ f_r = 1.5\ Hz$

$\therefore$ $f_r = sf$ i.e. $1.5 = 5 \times 50$

i.e. $s = \mathbf{0.03}$ i.e. **3 %**

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

$\therefore$ $N = N_s\,(1 - s) = 1500\,(1 - 0.03)$

$= \mathbf{1455\ r.p.m.}$

Ex. 10.5.7 *A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1 % at no load, and 3 % at full load. Determine : i) Synchronous speed, ii) No-load speed, iii) Full load speed, iv) Frequency of rotor at standstill, v) Frequency of rotor current at full load.*

VTU : July-12, Jan.-16, Marks 6

Sol. : i) $P = 6,\ f = 50$ Hz so $N_s = 120\ f/P = \mathbf{1000\ r.p.m.}$

ii) On no load, $s = 1\ \% = 0.01$

$\therefore$ $N = (1 - s)\,N_s = (1 - 0.01) \times 1000 = \mathbf{990\ r.p.m.}$

iii) On full load, $s = 3\ \% = 0.03$

$\therefore$ $N = (1 - s)\,N_s = (1 - 0.3) \times 1000 = \mathbf{970\ r.p.m.}$

iv) At standstill, $f_r = f = \mathbf{50\ Hz}$

v) On full load, $f_r = s\,f = 0.03 \times 50 = \mathbf{1.5\ Hz}$

Ex. 10.5.8 *A 3-ph IM with 4-pole is supplied from an alternator having 6-poles and running at 1000 rpm. Calculate :*

i) The synchronous speed of IM,

ii) Its speed when slip is 0.04.

iii) Frequency of the rotor emf when the speed is 600 rpm

VTU : Jan.-16, Marks 6

Sol. :

i) For alternator, $N_s = \frac{120f}{P_A}$ i.e. $f = \frac{1000 \times 6}{120} = 50$ Hz

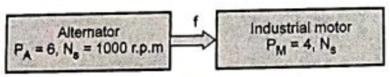


Fig. 10.5.1

$\therefore$ For I.M., $N_s = \frac{120\,f}{P_M} = \frac{120 \times 50}{4} = \mathbf{1500\ r.p.m.}$

ii) $N = N_s\,(1 - s) = 1500\,(1 - 0.04) = \mathbf{1440\ r.p.m.}$

iii) $N = 600$ r.p.m. i.e. $s = \frac{N_s - N}{N_s} = \frac{1500 - 600}{1500} = 0.6$

$\therefore$ $f_r = s \times f = 0.6 \times 50 = \mathbf{30\ Hz}$

Ex. 10.5.9 *Define slip speed and slip. What is the slip speed, slip and at what speed rotor runs if the frequency of the emf in the stator of a 4 pole, 3 φ IM is 50 Hz and in the rotor is 1.5 Hz ?*

VTU : July-17, Marks 5

Sol. : $P = 4,\ f = 50\ Hz,\ f_r = 1.5\ Hz,\ f_r = sf$

i.e. $s = \frac{f_r}{f} = \frac{1.5}{50} = \mathbf{0.03}$ i.e. **3 %**

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

$N = N_s\,(1 - s) = 1500\,(1 - 0.03) = \mathbf{1455\ r.p.m}$

Slip Speed $= N_s - N = 1500 - 1455 = \mathbf{45\ r.p.m.}$

10.6 Applications

➤ ***What are the applications of three phase induction motor ?*** **VTU : July-09, Marks 4**

i) Squirrel cage type of motors having moderate starting torque and constant speed characteristics preferred for driving fans, blowers, water pumps, grinders, lathe machines, printing machines, drilling machine.

ii) Slip ring induction motors can have high starting torque as high as maximum torque. Hence they are preferred for lifts, hoists, elevators, cranes, compressors.

10.7 Advantages of Induction Motors

➤ ***State the various advantages of induction motor.***

1. Cost is low compared to other types of motors.
2. Maintenance is less as robust and rugged .
3. Simple in construction.
4. Efficiency is high.
5. Power factor is better.
6. The starting torque can be controlled in slip ring type.

10.8 Limitations of Induction Motors

➤ ***State the various limitations of induction motor.***

1. The starting torque is low and can not be adjusted in squirrel cage type.
2. The speed control is difficult.
3. The various parameters like speed, power factor, efficiency etc. vary as load condition changes.

10.9 Necessity of Starter in Induction Motor

➤ ***Explain why an induction motor needs starter ?***

VTU : Jan.-04, 06, 16; July-03, 04, 05, 06, 10, Dec.-11, Marks 4

➤ ***Why induction motor draws high starting current ?***

VTU : Jan.-07; July-08, Marks 6

- In a three phase induction motor, magnitude of induced e.m.f. in the rotor circuit depends on the slip of the induction motor.
- At start the value of slip is at its maximum equal to unity.
- The rotor current at start is given by,

$$\boxed{I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{at start as } s = 1}$$

where E_2 = Rotor induced e.m.f. per phase at start

- The magnitude of induced e.m.f. at start is maximum as slip speed i.e. relative speed between rotor and the rotating magnetic field is maximum.

- Hence at start, large e.m.f. gets induced in the rotor.

- As rotor conductors are short circuited in most of the motors, due to squirrel cage construction, this e.m.f. circulates very high current through rotor at start.
- The induction motors acts as a transformer having short circuited secondary, at start.

- Hence as rotor current is high at start, consequently stator draws a very high current of the order of 5 to 8 times full load current at start.

- Due to such high current at start there is possibility of damage of the motor winding.
- Similarly due to sudden in rush of current, other appliances connected to the same line may be subjected to voltage spikes which may affect their working.

- To avoid such effects it is necessary to limit current drawn by the motor at start. Hence starter is necessary for an induction motor.
- In the running condition, the relative speed of rotor with respect to rotating magnetic field becomes slip speed which is very small.
- Hence the magnitude of the induced e.m.f. in the rotor also reduces by slip times the magnitude of induced e.m.f. at standstill condition.
- Hence in the running condition, the rotor current is not very high.
- Starters not only limit the starting current but also provide protection to the induction motor against over loading and low voltage conditions. The starters also provide single phasing protection too.

10.10 Star-Delta Starter

➤ ***With a neat circuit diagram explain a star-delta starter for a 3 phase induction motor.***

VTU : July-03, 04, 05, 06, 16; June-13, Jan.-04, 06, 14, Marks 8

- This is the cheapest starter of all.
- It uses TPDT [Tripple Pole Double Throw Switch] which connects the stator winding in star at start and then in delta while normal running. Hence this starter is suitable only for those motors designed to run with the delta connected stator winding.
- The arrangement is shown in the Fig. 10.10.1.
- Initially when switch is in the START position, the stator winding gets connected in star.

Hence phase voltage gets reduced by the factor $\frac{1}{\sqrt{3}}$.

$$\boxed{V_{ph} = \frac{V_L}{\sqrt{3}}}$$

- Due to this, starting current also gets reduced by factor $1/\sqrt{3}$.
- When motor attains 50 to 60 % of normal speed, switch is thrown in the RUN position.
- Hence winding gets connected in delta.

$$\boxed{V_{ph} = V_L = \text{Rated voltage}}$$

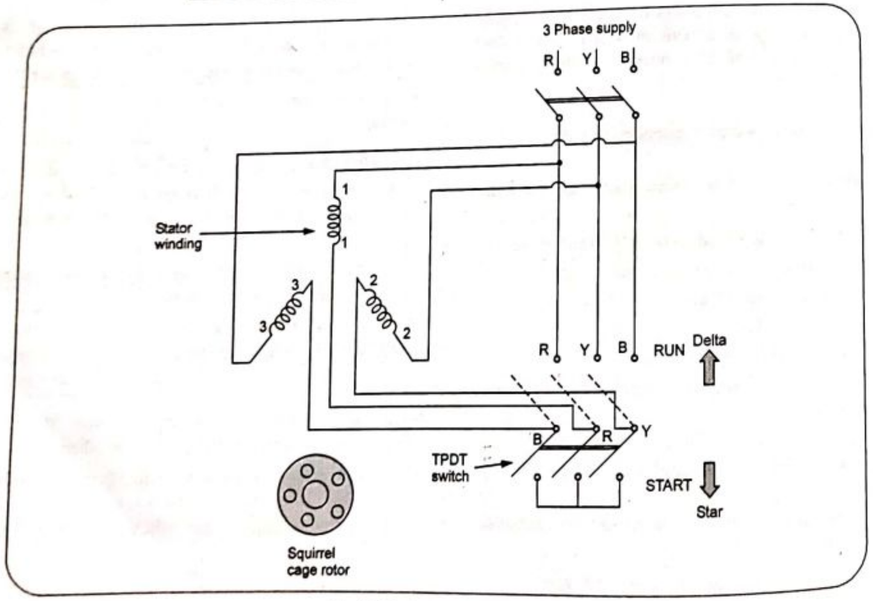


Fig. 10.10.1 Star-delta starter

- Each phase of the winding gets rated voltage.
- The operation of switch can be automatic by using relays which ensures that motor will not start with switch in RUN position.
- As this is cheap and maintenance free is frequently used.
- The limitations of this starter are that it is suitable for normal delta connected motors and factor by which voltage changes is $1/\sqrt{3}$ and cannot be changed.

10.11 University Questions with Answers

Jan. - 2010

Q.1 *What is rotating magnetic field ? What is the speed of rotating magnetic field ?* **[Refer section 10.2]** **[4]**

Q.2 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

July - 2010

Q.3 *Explain synchronous speed, slip speed and motor speed in case of three phase induction motor. Why slip is never zero in an induction motor ?* **[Refer section 10.4.1]** **[6]**

Q.4 *Explain why an induction motor needs starter ?* **[Refer section 10.9]** **[4]**

Jan. - 2011

Q.5 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

July - 2011

Q.6 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

Q.7 *Derive the expression for the slip and frequency of rotor currents.* **[Refer section 10.5]** **[6]**

Dec. - 2011

Q.8 *With diagrams, explain the concept of rotating magnetic field.* **[Refer section 10.2]** **[8]**

Q.9 *What is the significance of slip in an induction motor ?* **[Refer section 10.4.1]** **[4]**

Q.10 *Explain why an induction motor needs starter ?* **[Refer section 10.9]** **[4]**

June - 2012

Q.11 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

Q.12 *Derive the expression for the slip and frequency of rotor currents.* **[Refer section 10.5]** **[6]**

Jan. - 2013

Q.13 *Prove that a rotating magnetic field of constant magnitude is produced when the stator windings of a polyphase induction motor are energized by a balanced 3 phase supply.* **[Refer section 10.2]** **[8]**

Q.14 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

June - 2013

Q.15 *What is meant by the slip of the induction motor ? Under what circumstances the slip is i) unity and ii) zero.* **[Refer section 10.4.1]** **[8]**

Q.16 *With a neat circuit diagram explain a star-delta starter for a 3 phase induction motor.* **[Refer section 10.10]** **[8]**

Jan. - 2014

Q.17 *Explain the working principle of three phase induction motor.* **[Refer section 10.4]** **[6]**

Q.18 *With a neat circuit diagram explain a star-delta starter for a 3 phase induction motor.* **[Refer section 10.10]** **[8]**

July - 2015

Q.19 *What is rotating magnetic field ? What is the speed of rotating magnetic field ?* **[Refer section 10.2]** **[4]**

Q.20 *Explain synchronous speed, slip speed and motor speed in case of three phase induction motor. Why slip is never zero in an induction motor ?* **[6]**
[Refer section 10.4.1]

CBCS Scheme

Jan. - 2016

Q.21 *List the differences between squirrel cage and wound rotor induction motor.* **[4]**
[Refer section 10.3.3]

Q.22 *Explain the necessity of starters in 3-ph induction motor.* **[Refer section 10.9]** **[4]**

Q.23 *In a 6 pole, induction motor supplied from a three phase 50 Hz supply has a rotor frequency 2.3 Hz, Calculate : i) The percentage slip, ii) The speed of motor.* **[Refer section 10.5.3]** **[4]**

Q.24 *Define slip. Derive an expression for frequency of rotor current.*
[Refer setions 10.4.2 and 10.5] **[5]**

July - 2016

Q.25 *Define the slip of an induction motor and derive the relation between the supply frequency and rotor current frequency.*
[Refer sections 10.4.1 and 10.5] **[5]**

Q.26 *With a circuit diagram, explain the working of a star-delta starter for a three phase induction motor.* **[Refer section 10.10]** **[5]**

Jan. - 2017

Q.27 *Explain the working principle of an 3 ϕ induction motor with a neat sketch.*
[Refer section 10.4] **[5]**

July - 2017

Q.28 *Explain with neat vector diagrams, the concept of rotating magnetic field theory.*
[Refer section 10.2] **[6]**

Q.29 *Define slip speed and slip.*
[Refer section 10.4] **[5]**

Q.30 *With neat diagrams, explain construction of types of rotors of 3ϕ induction motor.*
[Refer section 10.3.2] **[6]**

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