First Semester B.E. Degree Examination, Dec.2018/Jan.2019 Calculus and Linear Algebra

Time: 3 hrs.

rtant Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining to the completion of Abritication, appeal to evaluator and for equations written eg. 42:48 =

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-

- a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ are intersect orthogonally. (06 Marks
 - b. Find the radius of curvature of the curve $y = a \log \sec(\frac{x}{a})$ at any point (x, y). (06 Marks
 - c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. (08 Marks)

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2 a. With usual notation, prove that $\tan \phi = r \frac{d\theta}{dr}$.

- (06 Marks)
- b. Find the pedal equation of the curve $r = ae^{0 \cot \alpha}$.
- (06 Marks)



- a. Using Maclaurin's expansion. Prove that $\sqrt{1+\sin 2x} = 1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}$. (06 Marks)
- b. Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ (67 Marks)
- Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.em. (67 Marks)

OR

- $4 \quad \text{a. If } u = f(y-z,z-x,x-y) \text{ , show that } \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \text{ .} \tag{06 Marks)}$
 - $b.\quad \text{If } u=x^2+y^2+z^2, \ \ v=xy+yz+zx, \quad w=x+y+z \ . \ \text{Find Jacobian } \ J=\frac{\widehat{\varepsilon}(u,y,w)}{\widehat{\varepsilon}(x,y,z)}. \\ (07 \ \text{Marks})$
 - c. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition x + y + z = 3a. (07 Marks)

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Module-3

- 5 a. Evaluate $\int_{0.0}^{\infty} e^{-(x^2+y^2)} dxdy$, by changing into polar coordinates. (06 Marks)
 - b. Find the volume of the tetrahedron bounded by the planes :

$$x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$
 (07 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

OR

- 6 a Evaluate $\int_{0}^{1} \int_{x}^{\sqrt{x}} xy \, dy \, dx$ by change of order of integration. (06 Marks)
 - b. Evaluate \iii \(\left(x + y + z \right) \dy dx dz \). \[\frac{\text{Big U.E. 4300ClATION 3}}{\text{VACHABLA PTIBLES A}} \] (07 Marks)



Module-4

- A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes.
 - b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. (67 Marks)
 - c. Solve $xyp^2 (x^2 + y^2)p + xy = 0$. (07 Marks)

OR

- 8 a. Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$. (06 Marks)
 - b. Show that the family of parabolas $y^2=4a(x+a)$ is self-orthogonal. (07 Marks) c. Find the general solution of the equation (px-y)(py+x)=0 by reducing into Claricut's
 - from, taking the substitution $X = x^2$, $Y = y^2$. (07 Marks)

Module-5

a. Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$

(07 Marks)

b. Solve the system of equations:

$$12x + y + z = 31$$

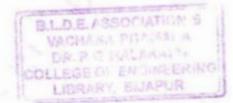
 $2x + 8y - z = 24$
 $3x + 4y + 10z = 58$

By Gauss -Siedal method.

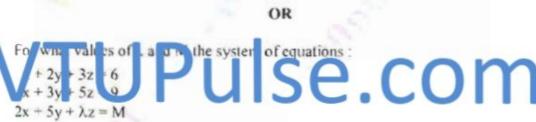
(07 Marks)

c. Diagonalize the matrix :

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$



(06 Marks)



has i) no solution ii) a unique solution iii) infinite number of solution.

(07 Marks)

b. Find the largest eigen value and the corresponding eigen vector of :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by Rayleigh's power method, use [1 1 1] as the initial eigen vector (carry out 6 iterations). (07 Marks)

c. Solve the system of equations :

$$x + y + z = 9$$

 $2x + y - z = 0$
 $2x + 5y + 7z = 52$

By Gauss elimination method.

(06 Marks)