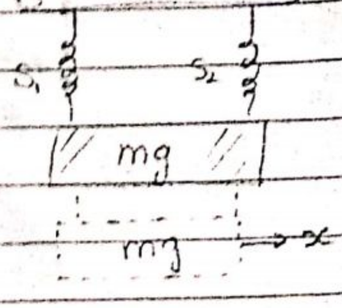
WAVES AND OSCILLATIONS



$F + mg = 0$

$-kx + mg$

$mg = kx$

$K_{eq} = \frac{mg}{x} \Rightarrow k_1 = \frac{mg}{x}$

$k_2 = \frac{mg}{x}$ } same load

where x is the elongation of spring S_1 & S_2

Restoring force $S_1 = k_1 x$

$S_2 = k_2 x$

$\therefore K_{eq} = S_1 + S_2$

multiply x $\rightarrow xK_{eq} = k_1 x + k_2 x$

$\boxed{K_{eq} = k_1 + k_2}$

* Dimension for Force constant

$N = kg/ms^2$

$F = -ky$ (negligible)

$F = ky$

$k = \frac{F}{y} \Rightarrow \frac{kg/\cancel{m}s^2}{\cancel{m}}$

$\Rightarrow kg/s^2$

The motion of a physical system can be classified into two broad categories:

i) Translatory motion

ii) Vibratory Motion

i) Translatory motion: If the motion of a particle varies linearly with time t, its motion is translatory.

Eg: Train moving on a straight track.

To understand SHM, two types of motion is required.

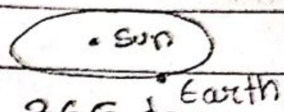
a) Periodic Motion

b) Oscillatory Motion

a) Periodic Motion: A motion which repeats itself after regular intervals of time. Eg: The motion of a hand of a clock

If we consider point A, then the minute hand repeats its motion about pt. A. After every hour (60 min)
Hence, motion of hand of a clock is periodic in nature.

Eg 2: Earth revolving around the sun comes back to same position after 365 days.

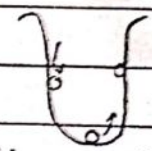


Hence, motion of earth is periodic nature.

b) Oscillatory Motion: If a body in periodic motion executes to & fro motion about a fixed reference point is said to be at oscillatory Motion.

Eg: The motion of a bob in a pendulam.

The motion of a ball in potential well.



Note: All periodic motion are not oscillatory but all oscillatory motion may be periodic

The cause of oscillation in a system
The oscillation of a system results from two basic property of a system mainly elasticity & inertia.

Elasticity: The tendency of a body to retain its original shape & size (position), when the external force is removed.

Inertia: Every body continues in its state of rest are of uniform motion in a straight line unless its compelled to change that state by an external unbalanced force.

Consider a body (an oscillator) is displaced by its mean position of equilibrium (o) by an application of force i.e. by doing work on it by a distance x. When it is released,

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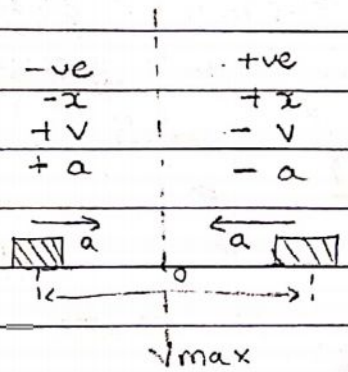
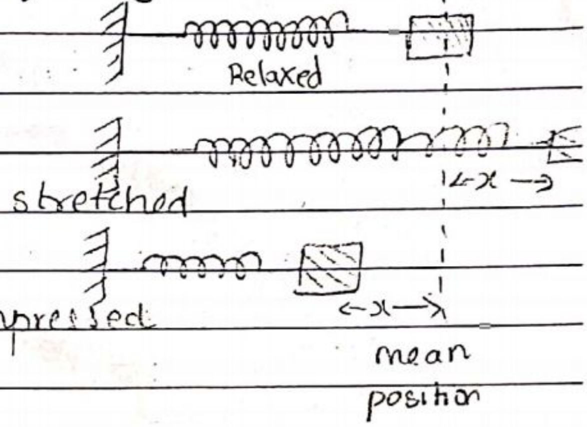
the restoring force comes into play, which tends to restore 'x' to its original value (0), by imparting an appropriate (-ve) velocity $\left(\frac{dx}{dt}\right)$. (As according to Hooks law, the restoring force is proportional to its displacement (x) & depends upon the elastic properties ie Elasticity of the system)





Inertia on the other hand tries to oppose any change in velocity when the body reaches its equillibrium position $(x=0)$, the velocity is max., which produces a -ve displacement. The body then overshoots its equillibrium position. The restoring force now becomes +ve, thus, helps ↑ in x, & must know overcome the inertia of its -ve velocity.

Thus, velocity keeps on ↓ & ultimately becomes zero, when x is -ve. This process of the restoring force trying to bring x to 0 by imparting a velocity & inertia preserving the velocity & making x over shoot, repeat itself & body oscillates about a fixed point (0).

The term oscillatory motion is not only restricted to displacement of a mechanical oscillator, but it may be any physical quantity.

Eg: In electrical system, the oscillatory variation of charge, current or voltage takes place. The atom in solid vibrate, the electron in radiating / recieving antena are in oscilation state. A tunned circuit in a radio can oscillate electromagnetically. Radio waves, micro waves & light waves are simply oscillating

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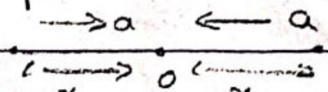
in electric & magnetic fields. Thus, the study of oscillation is essential for the understanding of various physical system - Mechanical, Acoustical, Electrical & Atomic.
In the module, we will study a simplest & smootest type of oscillatory motion, namely: SHM.

follow 1st pg → SHM: It's a type of oscillation in which the body oscillates such that its acc^n is always directed towards to a certain fixed pt. (o) & its magnitude is α to the displacement of the particle from the point (o), the fixed pt. is called centre of oscillation.



Thus, a SHM posses the foll. characteristics:
(i) The motion should be periodic.
(ii) When displaced from mean (eq^m position) a restoring force tends to bring it to the mean position & directed towards mean position must act on the body.
(iii) The restoring force should be α to the displacement of the body from its mean position.
• The direction of acc is opposite to that of the displacement $F \alpha -x$

Differencial eq^n of SHM

Consider a particle of mass 'm' executing SHM. If 'y' be the displacement of the particle from eq^m position at any instant 't', the restoring force 'F' is given by:

$$F \alpha y$$
$$F = -ky \quad \text{--- (1)}$$

where k is the force constant of proportionality or stiffness constant. The -ve sign indicates that the direction of force is opp to the displacement.
Force constant k is defined as the restoring force per unit displacement: $K = \frac{F}{y}$ SI unit N/m

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If $\frac{d^2y}{dt^2}$ is the accⁿ of the particle at time t, then:

$$ma = -ky$$

$$m\frac{d^2y}{dt^2} = -ky$$

substituting $\frac{k}{m}$ as ω^2

$$\frac{d^2y}{dt^2} = -\left(\frac{k}{m}\right)y$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\Rightarrow \frac{d^2y}{dt^2} + \omega^2 y = 0 \quad -(2)$$

(2) is the general diff eqⁿ of motion of SHM.

Solution of SHM diff eqⁿ.

The differential eqⁿ representing SHM is given by

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

multiply by $\frac{dy}{dt}$, we get

$$\frac{dy}{dt}\cdot\frac{d^2y}{dt^2} + \omega^2\cdot\frac{dy}{dt}\cdot y = 0$$

$$\frac{dy}{dt}\frac{d}{dt}\left(\frac{dy}{dt}\right) + \omega^2 y\frac{d}{dt}(y) = 0$$

Integrating

$$\frac{dy}{dt}\int\frac{d}{dt}\left(\frac{dy}{dt}\right) + \omega^2 y\int\frac{d(y)}{dt} = 0$$

$$\left(\frac{dy}{dt}\right)^2 + \omega^2 y^2 = c \quad -(2)$$

where c is the $\int$ constant.

when displacement is max. i.e $y = a$

$$\frac{dy}{dt} = 0$$

(Where a is the amplitude of oscillatory particle) i.e the particle is momentarily at rest in the extreme position & begins its journey in backward direction.

Sub. $y = a$ & $\frac{dy}{dt} = 0$ in eq. (2) we get

$$\omega^2 a^2 = c$$

Sub c in eq^n ②

$$\left(\frac{dy}{dt}\right)^2 + \omega^2 y^2 - \omega^2 a^2 = 0$$

$$\left(\frac{dy}{dt}\right)^2 + \omega^2 (y^2 - a^2) = 0$$

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2 (y^2 - a^2)$$
$$= \omega^2 (a^2 - y^2)$$

$$\frac{dy}{dt} = \omega \sqrt{a^2 - y^2}$$

cross multiply

$$\int \frac{dy}{\sqrt{a^2-y^2}} = \int \omega \, dt$$

$$\cancel{\sin^{-1}\left(\frac{a}{y}\right)} \quad \sin^{-1}\left(\frac{y}{a}\right) = \omega t + c$$

$$\frac{y}{a} = \sin(\omega t + c)$$

$$\boxed{y = a \sin(\omega t + \phi)}$$

- The term $(\omega t + \phi)$ represents the total phase of a particle at time (t) & ϕ is known as phase constant.
- If the time is recorded for that instant $y = 0$, then $\phi = \sin^{-1}(0) \Rightarrow 0$

The eq^n $y = a\sin(\omega t + \phi)$ is just one solution for the diff. eq^n $\frac{d^2y}{dt^2} + \omega^2 y = 0$

$$y = a\sin(\omega t + \phi)$$
$$y = a[\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$
$$y = A \sin \omega t + B \cos \omega t$$

where $A = a\cos\phi$ $\Rightarrow \frac{B}{A} = \tan\phi$
$B = a\sin\phi$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right)$$

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From Eq ①

$$a = \sqrt{A^2 + B^2}$$

$|||^{ly}$ by expanding

$$y = a\cos(\omega t + \phi)$$

$$y = a[\cos\omega t \overset{\cos}{\cancel{\sin}}\phi - \sin\omega t \overset{\sin}{\cancel{\cos}}\phi].$$

where $A = -a\sin\phi$ $\Rightarrow \frac{B}{A} = \frac{a\cos\phi}{-a\sin\phi} = -\tan\phi$

$B = a\cos\phi$

$$\phi = \tan^{-1}\left(\frac{-A}{B}\right)$$

Complex notation (Exponential form)

Consider, $y = a\sin(\omega t + \phi)$ — ①

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad -②$$

Eq ② can be written in operator form by substituting

$\frac{d}{dt} = D$ (or) $\frac{d^2}{dt^2} = D^2$

Sub value in eq ②

$$D^2y + \omega^2 y = 0$$

$$(D^2 + \omega^2)y = 0 \Rightarrow y = 0$$

$$D^2 = -\omega^2$$

$$D = \pm i\omega$$

Hence, the G.S of eq ② becomes

$$\boxed{y = Ae^{i\omega t} + Be^{-i\omega t}} \quad -③$$

$\because$ eq ③ solⁿ of eq ② may give a real value of y, then,
A & B must complex conjugate of each other

i.e $A = a + ib$

$B = a - ib$

$$y = (a+ib)e^{i\omega t} + (a-ib)e^{-i\omega t}$$

$$y = \underbrace{a(e^{i\omega t} + e^{-i\omega t})}_{\text{Real}} - \underbrace{ib(e^{i\omega t} + e^{-i\omega t})}_{\text{Imaginary}}$$

$y = a(e^{i\omega t} + e^{-i\omega t})$

$\Rightarrow \boxed{y = ae^{i\omega t}}$, $y = ae^{-i\omega t}$

Taken to consideration

$\boxed{y = a e^{(i(\omega t+\phi))}}$

Velocity of S.H.M oscillator.

The displacement of a SHO at any instant of time (t) is given by $y = a\sin(\omega t + \phi)$

The velocity is defined as time rate of change of displacement.

$\because$ velocity $V = \frac{dy}{dt} = a\omega\cos(\omega t + \phi)$

$= a\sin(\omega t + \phi + \frac{\pi}{2})$

$\sin(\omega t + \phi) = \frac{y}{a}$

$\sin^2\omega t + \cos^2\omega t = 1$

$\cos^2\omega t = 1 - \sin^2\omega t$

$\cos^2\omega t = 1 - \frac{y^2}{a^2}$

$\cos\omega t = \sqrt{1 - \frac{y^2}{a^2}} = \frac{\sqrt{a^2-y^2}}{a} \Rightarrow a\cos\omega t = \sqrt{a^2-y^2}$

Sub. eqn to ①

$\boxed{V = \omega\sqrt{a^2-y^2}}$

Max. velocity when $\sin(\omega t + \phi + \frac{\pi}{2}) = 1$.

$\because V_{max} = a\omega$ when $y = 0$.

i.e the particle executing SHM in its mean position.

We find that the velocity of SHO at any instant of time t leads the displacement by a phase diff. $\pi/2$ rad i.e the two are in quadrature.

Acceleration of SHM

It's defined as the time rate of change of velocity

Now velocity $V = a\omega\cos(\omega t + \phi)$ — ①

$\frac{dV}{dt} = -a\omega^2\sin(\omega t + \phi)$

$= a\omega^2 \sin(\omega t + \phi + \pi)$

The accⁿ of the oscillator is max. when $\sin(\omega t + \phi + \pi) = 1$.
& is given by $\boxed{\left.\frac{d^2y}{dt^2}\right|_{max} = a\omega^2}$

Phase relationship b/w displacement velocity accleration.

The velocity $v = a\omega \cos(\omega t + \phi)$

$\Rightarrow a\omega \sin\left(\omega t + \phi + \frac{\pi}{2}\right)$ — ①

$a = a\omega^2 (\omega t + \phi + \pi)$ — ②

Comparing ① & ② eqⁿ, we find that the accⁿ of SHO leads the velocity by $\frac{\pi}{2}$ rad · in phase.

The displacement $y = a \sin(\omega t + \phi)$ — ③

Comparing eq ② & ③, we find that accⁿ of SHO leads the displacement by π rad (180°) in phase i.e the accⁿ & displacement are in anti-phase

Graphical rep. of displacement velocity accⁿ.

Let the displacement of a particle executing SHM be $y = a \sin(\omega t + \phi)$

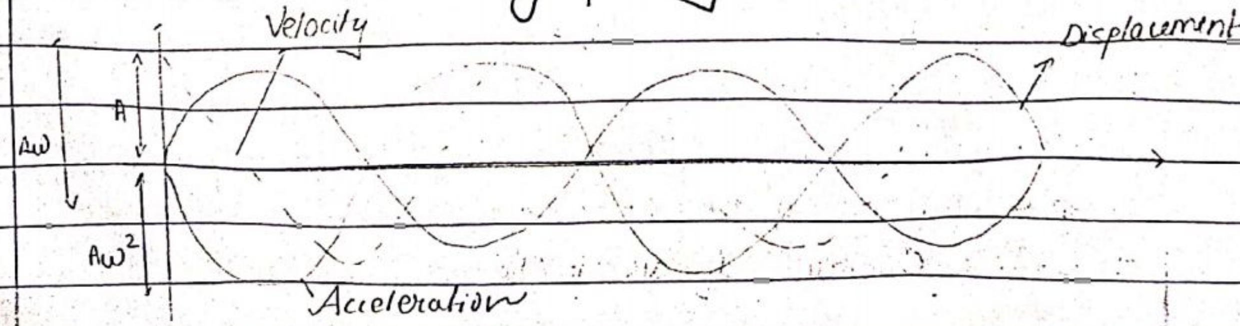
$V = A\omega \cos(\omega t + \phi)$

$a = -A\omega^2 \sin(\omega t + \phi)$

Hence for

$y = 0$	$V = A\omega$	$a = 0$
$y = A$	$V = 0$	$a = -\omega^2 A$
$y = -A$	$V = 0$	$a = \omega^2 A$

The Same as shown graphically



Period and frequency of SHM

A SHM is represented by the equation $y = a\sin(\omega t + \phi)$

In this equation, if we increase 't' by $\frac{2\pi}{\omega}$

$$y = a \sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right]$$

$$= a \sin\left[\omega t + 2\pi + \phi\right]$$

$$\Rightarrow y = a \sin(\omega t + \phi)$$

i.e the displacement of the particle after the time $t = \frac{2\pi}{\omega}$ is the same. Hence, t gives the periodic time of the SHO.

$$T = \frac{2\pi}{\omega} \Rightarrow n = \frac{1}{T} \Rightarrow \frac{\omega}{2\pi}$$

$\because \omega = 2\pi n$ of angular velocity of the SHO. The acc^n of a SHO is given by the relation

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Neglecting the -ve sign, we have : $\frac{d^2y}{dt^2} = \omega^2 y$

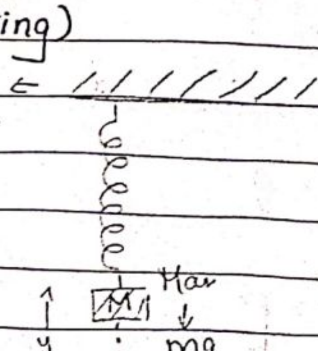
$$\omega^2 = \frac{d^2y}{dt^2} \Big/ y \Rightarrow \omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$T = \frac{2\pi}{\omega} \Rightarrow 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$n = \frac{1}{T} \Rightarrow \frac{1}{2\pi}\sqrt{\frac{\text{Accleration}}{\text{Displacement}}}$$

Mechanical SHO

i) Spring & Mass System (Loaded Spring)



Consider a spring whose upper end is fixed on a rigid support & the lower end is attached to a mass m. Suppose, the spring be extended through a distance l, due to the

weight mg of the mass, a linear restoring force $= -kl$ at once coms into play in opp. direction.
The equilibrium position is attained, when these two forces balances each other.

$$F + mg = 0$$
$$-kl + mg = 0$$
$$kl = mg$$
$$\boxed{k = \frac{mg}{l}}$$

where k is the force constant (or) Stifness of the spring.

If the mass be now pulled down through a distance y from this equillibrium position, the linear restoring force

$$f = mg - k(l+y)$$
$$F = mg - \left(\frac{mg}{l}\right)(l+y)$$
$$F = mg - \left[\frac{mg\cancel{l}}{\cancel{l}} + \frac{mgy}{l}\right]$$
$$F = -\frac{mg}{l}(y)$$
$$\boxed{F = -ky}$$

If $\frac{d^2y}{dt^2}$ be the acc^n setup in the spring,

then $ma = -ky$

$$\frac{md^2y}{dt^2} = -ky$$
$$\frac{d^2y}{dt^2} = \frac{-ky}{m}$$

sub. $\frac{k}{m}$ as μ

$$\frac{d^2y}{dt^2} = -\mu y$$

$\frac{k}{m} - \omega^2 \Rightarrow$ acc^n per unit displacement

$$\therefore \frac{d^2y}{dt^2} \propto -y$$

This is the D.E mass suspended from the spring.

Thus, linear acc^n is directly proportional to its linear displacement & is directed oppostly to it. Hence, mass m attached to a spring executes Simple harmonic motion and its time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

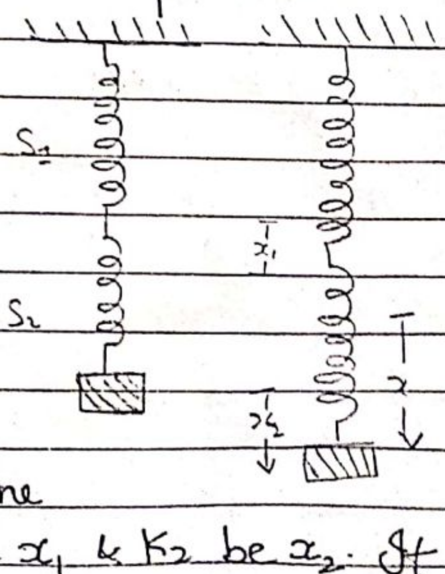
$$\boxed{\eta = \frac{1}{T} \Rightarrow \frac{1}{2\pi}\sqrt{\frac{k}{M}}}$$

The force constant is defined as the magnitude of the applied force that produces unit extension / compression in the spring while it is loaded within the elastic limit.

Physical Significance

Force constant is a measure of stiffness. In the case of springs, it rep. how much force it takes to stretch the spring over a unit length. Thus, springs with larger value for force constant will be stiffer.

Derivation for force constant for series & parallel combination of springs.



(a) Springs in series

In case, if the mass is connected to a spring which consists of two diff. springs of diff. stiffness factor.

Let the stiffness factor of spring S_1 & S_2 be K_1 & K_2 resp. & the $\uparrow$ in the length of the spring K_1 be x_1 & K_2 be x_2. If x is the total $\uparrow$ in the length of the spring system because of mass m as shown in fig.

$$x = x_1 + x_2 \quad \text{——— (1)}$$

Both the spring experiences the same force

$\because mg = k_1 x_1 \qquad mg = k_2 x_2$

$x_1 = \frac{mg}{k_1} \qquad x_2 = \frac{mg}{k_2}$

If k_{eq} is the equivalent & stiffness factor for the $comb^n$.

$\because mg = k_{eq}\, x$

$x = \frac{mg}{k_{eq}}$

Substituting the x, x_1 & x_2 value in eq ①

$$\frac{mg}{k_{eq}} = \frac{mg}{k_1} + \frac{mg}{k_2}$$

$$\frac{\cancel{mg}}{k_{eq}} = \cancel{mg}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{—— ②}$$

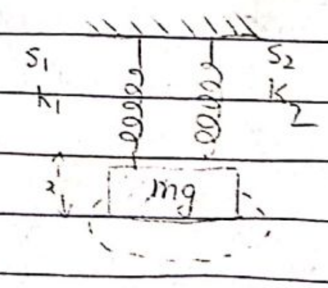
$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$\therefore$ If a no. of springs of diff. stiffness are connected in series, the multi-spring system can be regarded as single spring of equivalent stiffness factor is given by the foll. relation:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n}$$

$$\therefore k_s = \frac{k_1 k_2}{k_1 + k_2}$$

(b) Springs in parallel



Consider two springs S_1 & S_2 as shown in fig. connected in parallel.

Each spring will share the total load & will have equal elongation by x.

If k_1 & k_2 are the stiffness factor for springs S_1 & S_2 & k be the equivalent stiffness factor for the $comb^n$

Total restoring force $mg = k_{eq}\, x$

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Restoring force in Spring $S_1 = K_1 x$
Restoring force in spring $S_2 = K_2 x$

Now total restoring force = Restoring force in Spring S_1 + Restoring force in Spring S_2

$$K_{eq}\, x = K_1 x + K_2 x$$

$$K_{eq} = K_1 + K_2$$

$\therefore$ If a no. of springs are connected in $\parallel^{le}$, the equivalent stiffness factor is the sum of individual stiffness factor.

Free Oscillations.

In an ideal SHM, the displacement follows a sinusoidal curve. Here, the amplitude of the oscillation remains constant for an infinite time. This is because there is no loss of energy & Thus, the total energy becomes constant.

Such oscillations are called 'Free Oscillations'.

Eg: Simple pendulam, spring

Equation of Motion for Free Oscillation

The general solution of motion of SHM itself rep. the equation of motion for free oscillation.

Thus, if m is the mass of the oscillating body, k is the force constant & x-is the displacement at instant t of an oscillatory body, the eqn of motion for its oscillation can be written as

$$F = -kx$$

$$\frac{d^2y}{dt^2} + \mu x = 0 \quad \text{where } \mu = \frac{k}{m}$$

$$T = 2\pi\sqrt{m/k}$$

$$n = \frac{1}{T} \Rightarrow \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

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Qualitative discussion

The most common & simplest example for free oscillation is the motion of Simple pendulam which oscillates with a fixed tim period: $T = 2\pi\sqrt{\frac{l}{g}}$

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{--- (1)}$$

l = length of the pendulam.
g = Acc" due to gravity

This frequency is called the natural frequency of the pendulam.

The pendulam, whenever set to oscillate freely without any resistance, it will always oscillate with the frequency given by eq (1)

IIIly, if we consider vibration of a spring mass system, it vibrate with the natural frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

If no resistance is offered to the motion of the vibrating body by any force such as air friction or in forces on the body will keep on vibrating indefinetly & such vibrations are called free vibration.

In other words, free oscillation can be defined as the oscillation in which the body oscillates with its own natural frequency when left free to itself.

Formulas.

$T = 2\pi\sqrt{\frac{m}{k}}$ | $F = kx$ | $\omega = 2\pi f$

$v = \omega\sqrt{a^2 - y^2}$ | $\omega = \frac{2\pi}{T}$

$f = n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ | $v = \omega a$

$a = \omega^2 y \,/\, \omega^2 A$ | $\omega^2 = \frac{k}{m}$

Q A man weighing 600N steps on a Spring scale machine. The Spring in the machine is compressed by 1cm. Find force constant of the spring

$\Rightarrow$ $F = 600N$
$x = 0.01\,m$

$$k = \frac{F}{x} \Rightarrow \frac{600}{0.01} \Rightarrow 60{,}000 \Rightarrow 6 \times 10^4\ N/m$$

Q. A mass of 5 kg is suspended from the free end of a spring. When set for vertical oscillations, the system executes 100 oscillations in 40 sec. Calculate the force const. of the spring.

$\Rightarrow$ $T = 2\pi\sqrt{\frac{m}{k}}$ $\qquad T = \frac{100}{40} \Rightarrow 2.5\ sec$

$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$

$\frac{T^2}{4\pi^2} = \frac{m}{k} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$k = \frac{4 \times (3.14)^2 \times 5}{(2.5)^2} \Rightarrow 31.55\ N/m$

Q. A mass 0.5 kg causes an extension 0.03 m in a spring & the system is set of oscillations. Find:

a. Force constant k of the spring

b. Angular frequency ω,

c. Period T of the resulting oscillation.

$F = ky$

$mg = ky$

$\omega = 2\pi f$

$\omega = \frac{2\pi}{T}$

$\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow$ Given $m = 0.5\ kg$

$x = 0.03\ m$

a. $F = mg$

$= (0.5)(9.8)$

$\Rightarrow 4.9\ N$

$k = \frac{F}{x} = \frac{4.9}{0.03} \Rightarrow 163.3\ N/m$

b. $\omega = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{163.3}{0.5}}$

$\Rightarrow \sqrt{326.6} \Rightarrow 18\ rad/s$

c. $T = \frac{2\pi}{\omega} = \frac{2 \times (3.14)}{18} \Rightarrow 0.34\ sec$

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Q. An electric motor weighing 50 kg is mounted on 4 springs eac of which has a spring constant 2×10^3 N/m. The motor move only in vertical direction. Find the natural frequency of the syst

$\Rightarrow$ Given $m = 50$ kg

$k = 2\times10^3$ N/m

$K_{eq} = 4\times2\times10^3 \Rightarrow 8\times10^3$ N/m

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2(3.14)}\sqrt{\frac{8\times10^3}{50}}$$

$$0.159\sqrt{160}$$

$$\Rightarrow 2.011 \text{ Hz}$$

Q A car has a spring system that supports the inbuilt mass 1000 kg. When a person with the weight 980 N sits at the c of G the spring system sinks by 2.8 cm. When the car hits a bump it starts oscillating vertically. Find the T & f of oscillation.

$\Rightarrow$ $m = 1000$ kg

$x = 0.028$

$W = 980$ N

$$k = \frac{W}{x} = \frac{980}{0.028}$$

$$= 35\times10^4 \text{ N/m}.$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2(3.14)\sqrt{\frac{11000}{3.5\times10^4}} \Rightarrow 6.28\sqrt{0.031}$$

$T \Rightarrow 1.1$ sec.

person's mass $= \frac{F}{g}$

$$= \frac{980}{9.8} \Rightarrow 100 \text{ kg}$$

$$f = \frac{1}{T} = \frac{1}{1.1}$$

$$= 0.9 \text{ Hz}$$

Total mass $= 1100$ kg

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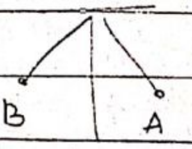
Q A body executes SHM such that its velocity at its mean position $(y=0)$ 1 m/s & acc^n at one of the extremity is 1.57 m/s^2. Calculate the time period of vibration.

⇒ $V = 1 m/s$
$(y=0)$



$a = 1.57 m/s^2 = \omega^2 y \quad (y=A)$

$V = \omega\sqrt{A^2 - y^2}$

$V = \omega A$

$1 = \omega A \quad \text{—} \; ①$

$a = \omega^2 A \quad \text{—} \; ②$

② / ① $\quad a = \dfrac{\omega^2 A}{\omega A} \qquad \omega = a = 1.57$ rad/s.

$T = \dfrac{2\pi}{(1.57)} \Rightarrow 4$ sec

Q A Spring when compressed by 10 cm develops a restoring force of 10 N. A body of mass 4 kg is attached to it. Calculate the compression of the spring due to the weight of the body & calculate the period of oscillation.

⇒ $m = 4 kg$

$F = 10 N$ [compressed]

$(x)\; d = 10 cm \Rightarrow 0.1 m$

$k = \dfrac{F}{x} = \dfrac{10}{0.1} = 100 N/m$

[not comp^n] $F = mg$

$\cancel{10} = \cancel{4} \qquad F = 4 \times 9.8$

$= 39.2 N$

$x = \dfrac{F}{K} = \dfrac{39.2}{100}$

$\Rightarrow 39.2 \times 10^{-2} m$

$T = 2\pi\sqrt{\dfrac{m}{K}}$

$= 2 \times (3.14)\sqrt{\dfrac{4}{100}} \Rightarrow 1.256$ sec

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$f = \frac{\omega}{2\pi}$ $\omega = 2\pi f$
$T = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi}{T}$

Q A spring is hanged vertically & loaded with a mass of 400g made to oscillate. Calculate: Time period, The frequency of oscillation, when the spring is loaded with 100g it extends by 5 cm.

$m_1 = 400g = 0.4\ kg$
$m_2 = 100g = 0.1\ kg$
$x = \frac{mg}{k}$

$K = \frac{mg}{x} = \frac{(0.1)(9.8)}{0.05} = 19.6\ m$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\times(3.14)\sqrt{\frac{0.1}{19.6}} \Rightarrow 0.44\ sec$

$f = \frac{1}{T} = \frac{1}{0.44} \Rightarrow 2.27\ Hz.$

Q An hydrogen atom has a mass of 1.68×10^{-27} kg. When attached with a massive molecule, it oscillates as a classical oscillator with a frequency of 10^{14} cycles/sec & with $\overset{y}{A} = 10^{-10}$ m. Calculate the force acting on the hydrogen atom.

$\Rightarrow m = 1.68\times10^{-27}\ kg$
$f = 10^{14}\ c/s$
$A\omega = 10^{-10}\ m$
$\omega = 2\pi f$
$= 2\times(3.14)\times10^{14}$
$\Rightarrow 6.28\times10^{14}$

$a = \omega^2 y$
$= (6.28\times10^{14})^2 \times 10^{-10}\ m$
$\Rightarrow 3.943\times10^{19}$

$F = ma$
$= 1.68\times10^{-27}\times3.943\times10^{19}$
$\Rightarrow 6.625\times10^{-8}\ N/m$

$y = Ae^{\alpha x}$

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Damped Oscillation

Any oscillation in which the amplitude of the oscillating system ↓ & fades out after some time is known as damped oscillation.

The oscillation dies out due to frictional restance to the motion of the system.

Hence, in another ~~verse~~ words "It's the type of motion executed by a body subjected to the combined action of both the restoring & the ~~co~~ resistive force & the motion gets terminated with the body coming at rest at the equillibrium position in a finite interval of time."

Eg. of Damped Oscillation

1) Mechanical Oscillation of Simple Pendulam.
2) Motion of a swing
3) Mass suspended to spring in air & mass suspended to Spring in liquid.

The oscillation fades quickly in a liquid compared to air (As resistive force) Hence, damping is more in liquids. When mass moves

Theory of damped vibration (Expression for the period & amplitude of damped harmonic oscillator)

or.

Differencial eqn of motion for a damped harmonic oscillator

A damped Harmonic Oscillator experences two forces.

- The restoring force proportional to displacement 'y' given by $-Ky$ where K = Spring Constant.
- A frictional (retarding) force proportional to the velocity given by $-r\frac{dy}{dt}$ where r is the proportionality constant, known as damping constant.

-ve sign is indicates retarding force also acts opp. to the

$\frac{1}{m} = 2b$..

direction of motion of the body.

Thus, if 'm' is the mass of the ascillating particle & its acc^n i $\frac{d^2y}{dt^2}$, when it has a displacement 'y' & is moving with a vel. $-ty\ \frac{dy}{dt}$, then, rest fr

$$F = F_r + F_{fr}$$

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} \quad — \div \text{ by } m$$

$$\frac{d^2y}{dt^2} = -\frac{k}{m}y - \frac{r}{m}\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{r}{m}\frac{dy}{dt} = 0 \quad \text{where } \frac{k}{m} = \omega^2$$

$$\frac{d^2y}{dt^2} + \omega^2 y + 2b\frac{dy}{dt} = 0 \quad — (1) \quad \text{Substituting } \frac{r}{m} = 2b$$

where b is the damping co-efficient & 2b gives the force per unit velocity & eq (1) is known as differential Eq^n for damping Harmonic Oscillator. [DHO]

Solution of Differencial Eq^n

To solve eq (1) Let's assume the solution is $y = Ae^{\alpha t}$ — (2)

where A & α are arbitary constants

Differenciate eq (2) w.r.t 't', we get

$$\frac{dy}{dt} = \alpha Ae^{\alpha t} \quad — (3)$$

Diff. twice

$$\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t} \quad — (4)$$

(2).

Substituting eq (3) & (4) in (1)

$$A\alpha^2 e^{\alpha t} + \omega^2(Ae^{\alpha t}) + 2bA\alpha e^{\alpha t} = 0$$

$$Ae^{\alpha t}(\alpha^2 + \omega^2 + 2b\alpha) = 0$$

$$\alpha^2 + \omega^2 + 2b\alpha = 0 \quad — (5)$$

This is a quadratic equation of α

$\alpha_1 - b \quad A_1\sqrt{b^2-\omega^2} + A_2 b - A_2\sqrt{b^2-\omega^2}$

$A_1 - b - A_2 b + A_1\sqrt{b^2-\omega^2} - A_2\sqrt{b^2-\omega^2}$

$Ae^{\alpha t} \neq 0$

$$\alpha = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{taking } a=1 \ \& \ c=\omega^2, \ b=2b$$

$$\alpha = \frac{-2b \pm \sqrt{(2b)^2-4(1)(\omega^2)}}{2(1)}$$

$$= \frac{-2b \pm \sqrt{4b^2-4\omega^2}}{2}$$

$$= -\cancel{2}\left[\frac{b \pm \sqrt{b^2-\omega^2}}{\cancel{2}}\right]$$

Hence 'α' has 2 roots : $-b+\sqrt{b^2-\omega^2} = \alpha_1$

$-b-\sqrt{b^2-\omega^2} = \alpha_2$

$\therefore$ The general solⁿ of eqⁿ ② is

$$y = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

$$y = A_1 e^{[-b+\sqrt{b^2-\omega^2}]t} + A_2 e^{[-b-\sqrt{b^2-\omega^2}]t} \quad — ⑥$$

where A_1 & A_2 are arbitrary constants

$\therefore$ The constants are determined from initial conditions

Suppose at $t=0$, $y=y_0$ & velocity $\frac{dy}{dt}=0$

If we substitute above conditions in eq ⑥ then we get

$$y_0 = A_1 + A_2$$

$$A_1 + A_2 = y_0 \quad — ⑦$$

Diff. eqⁿ ⑥ w.r.t 't'

$$\frac{dy}{dt} = A_1\left(-b+\sqrt{b^2-\omega^2}\right)e^{(-b+\sqrt{b^2-\omega^2})t} + A_2\left(-b-\sqrt{b^2-\omega^2}\right)e^{(-b-\sqrt{b^2-\omega^2})t}$$

Substitute $t=0$, $\frac{dy}{dt}=0$

$$0 = A_1\left(-b+\sqrt{b^2-\omega^2}\right) + A_2\left(-b-\sqrt{b^2-\omega^2}\right)$$

$$0 = -b(A_1+A_2) + A_1\sqrt{b^2-\omega^2} - A_2\sqrt{b^2-\omega^2}$$

$$-b(A_1+A_2) + \sqrt{b^2-\omega^2}(A_1-A_2) = 0$$

$$b(A_1+A_2) = \sqrt{b^2-\omega^2}(A_1-A_2)$$

From eq ⑦

$$A_1 - A_2 = \frac{by_0}{\sqrt{b^2-\omega^2}} \quad — ⑧$$

Adding eq ⑦ & ⑧

$$A_1 + \cancel{A_2} + A_1 - \cancel{A_2} = y_0 + \frac{by_0}{\sqrt{b^2-\omega^2}}$$

$$2A_1 = y_0\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right]$$

$$A_1 = \frac{y_0}{2}\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right] \quad — ⑨$$

Sub ⑦ & ⑧ in ⑨ ⑦ − ⑧

$$2A_2 = y_0\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right]$$

$$A_2 = \frac{y_0}{2}\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right] \quad — ⑩$$

Sub ⑩ & ⑨ in eq ⑥

$$y = \frac{y_0}{2}\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left[-b+\sqrt{b^2-\omega^2}\right]t} + \frac{y_0}{2}\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left[-b-\sqrt{b^2-\omega^2}\right]t} \quad — ⑪$$

Eqⁿ ⑪ is the general solⁿ of a damped harmonic oscillator. This eqⁿ gives the displacement 'y' of a particle of mass 'm', executing DHO. The nature of motion depends on relative values of 'b' and 'ω'. The foll. 3 imp. cases arises.

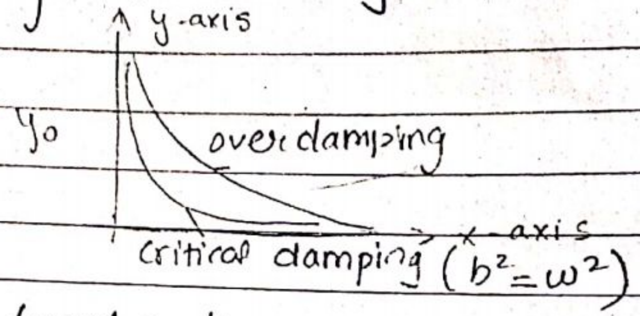
case 1 → Aperiodic or dead beat motion. ($b^2 > \omega^2$)

This is the case of heavy damping, in this case $\sqrt{b^2-\omega^2}$ is real since, $(b^2-\omega^2)$ is +ve. Eq ⑪ shows that the displacement y ↓ from its initial value y_0, and it tends to zero when $t \to \infty$.

In this case, no oscillation occurs. Hence, its called aperiodic or overdamped.

Under very heavy damping, the particle passes its eqᵐ position at the most once before returning exponentially to rest

This condition is met with a dead beat moving coil galvanometer or a pendulam vibrating in an viscous liquid like oil



Case 2: Critically damped motion

when $b^2 = \omega^2$

If we Substitute this value in eq (11) the 2nd term has the interminate (∞). Thus in this case, eq (11) does not represent the solution, i.e the Soln breaks down. Let us, however, consider the case $\sqrt{b^2-\omega^2} \neq 0$, but take a very small quantity 'h'. Thus $\sqrt{b^2-\omega^2} = h$.

$\therefore$ Sub eq (11) we get

$$y = \frac{y_0}{2}\left[\left(1 + \frac{-b}{h}\right)e^{(-b+h)t} + \left(1 - \frac{b}{h}\right)e^{(-b-h)t}\right]$$

$$y = \frac{y_0}{2} e^{-bt}\left[e^{ht} + e^{-ht}\right] + \frac{b}{h}\left[e^{ht} - e^{-ht}\right]$$

$$y = y_0 e^{-bt}\left(\frac{e^{ht}+e^{-ht}}{2}\right) + \frac{b}{h}\left[\frac{e^{ht}-e^{-ht}}{2}\right]$$ $\frac{e^{x}+e^{x}}{2} = \cos x$

$$y = y_0 e^{-bt}\left[\cosh(ht) + \frac{b}{h}\sinh(ht)\right]$$ $\frac{e^{x}-e^{x}}{2} = \sin x$

$$y = y_0 e^{-bt}\left[1 + \frac{h^2t^2}{\lfloor 2} + \frac{h^4t^4}{4} + \dots\right] + \frac{b}{h}\left[ht + \frac{h^3t^3}{\lfloor 3}\right]$$

Now since h is very small the term containing h^2 & higher powers are neglected.

$\therefore$ We'll get $y = y_0 e^{-bt}(1+bt)$ — (12)

This equation suggests that at $t=0$, $y=y_0$ & as 't' $\uparrow$ 'y' $\downarrow$ as shown in fig (a).

$i\omega'$

The motion is neither overdamped nor oscillating & is said to be critically damped. It's a transition stage b/w dead beat motion & DOM.

case 3 : **Oscillatory Damped Motion**

when $b^2 < \omega^2$ (it has light damping)

In this case, $b^2-\omega^2$ (is −ve) ie (imaginary Quantity) so ω^2-b^2 is +ve.

$\sqrt{b^2-\omega^2} = i\omega$ or $\omega' = \sqrt{\omega^2-b^2}$ where $i = \sqrt{-1}$.

Sub values in eq (11)

$$y = \frac{y_0}{2}\left[\left(1+\frac{b}{i\omega'}\right)e^{(-b+i\omega')t} + \left(1-\frac{b}{i\omega'}\right)e^{(-b-i\omega')t}\right]$$

$$y = \frac{y_0}{2}\left[e^{-bt}e^{i\omega' t} + \frac{b}{i\omega'}e^{(-bt+i\omega' t)} + e^{-bt}\cdot e^{-i\omega' t} - \frac{b}{i\omega'}e^{(-b-i\omega')t}\right]$$

$$\frac{y_0}{2}\left[e^{-bt}\left[e^{i\omega' t} + e^{-i\omega' t}\right] + \frac{b}{i\omega'}e^{-bt}\left[e^{i\omega' t} - e^{-i\omega' t}\right]\right]$$

$$y = \frac{y_0}{2}e^{-bt}\left[\left(e^{i\omega' t} + e^{-i\omega' t}\right) + \frac{b}{i\omega'}\left(e^{i\omega' t} - e^{-i\omega' t}\right)\right]$$

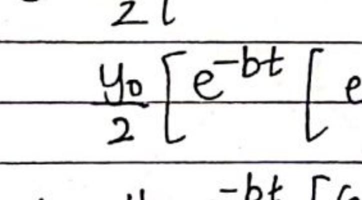
$$y = y_0 e^{-bt}\left[\left(\frac{e^{i\omega' t} + e^{-i\omega' t}}{2}\right) + \frac{b}{i\omega'}\left(\frac{e^{i\omega' t} - e^{-i\omega' t}}{2}\right)\right]$$

$$y = y_0 e^{-bt}\left[\left(\frac{e^{i\omega' t} + e^{-i\omega' t}}{2}\right) + \frac{b}{\omega'}\left(\frac{e^{i\omega' t} - e^{-i\omega' t}}{2i}\right)\right]$$

$$y = y_0 e^{-bt}\left[(\cos \omega' t) + \frac{b}{\omega'}(\sin \omega' t)\right]$$

$\cos \omega t = \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$

$\sin \omega t = \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right)$



$$\cot\phi = \frac{\cos\phi}{\sin\phi} = \frac{b}{\omega'}$$

$$\tan\phi = \frac{\omega'}{b} = \frac{\sqrt{\omega^2-b^2}}{b}$$

$$\cos\phi = \cos\omega' t = \frac{b}{\omega} \; ; \; \sin\omega' t = \frac{\omega'}{\omega}$$

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$$y = y_0 e^{-bt}\left[\cos\omega' t + \frac{\cos\phi}{\sin\phi}\sin\omega' t\right]$$

$$y = \frac{y_0 e^{-bt}}{\sin\phi}\left[\cos\omega' t \sin\phi + \cos\phi \sin\omega' t\right]$$

$$y = y_0 e^{-bt}\left(\frac{\omega}{\omega'}\right)\left[\sin(\omega' t + \phi)\right]$$

$$y = A_0 e^{-bt}\left(\sin(\omega' t + \phi)\right) \quad \text{— (13)}$$

where $A_0 = y_0\left(\frac{\omega}{\omega'}\right)$

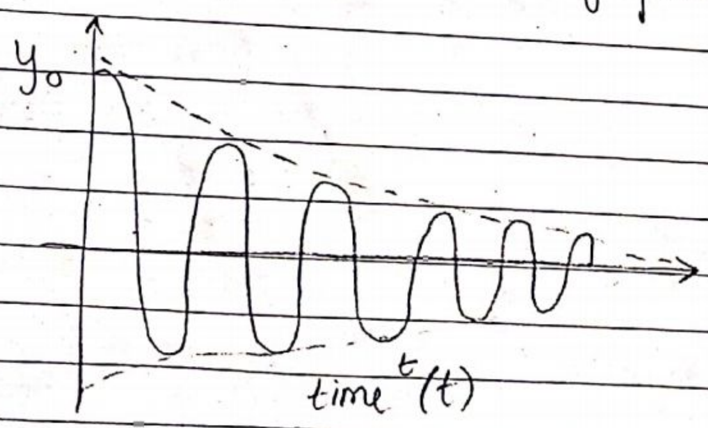
$= y_0\left(\frac{\omega}{\sqrt{\omega^2 - b^2}}\right)$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$\omega' = \sqrt{\omega^2 - b^2} = \sqrt{k/m - \frac{r^2}{(2m)^2}}$$

$$b = \frac{r}{2m} \rightarrow \boxed{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}}$$

From eq (13) the motion of particle is under DOM.



In this case, the restoring force & resistive force acting on a body are such that the body vibrate with diminishing amplitude as the time progress & ultimately comes to halt at the eq^m position. DOM

$\because$ The amplitude of SHM is $A_0 = y_0\left(\frac{\omega}{\sqrt{\omega^2 - b^2}}\right)$

$$A = A_0 e^{-bt}$$

$$= y_0\left(\frac{\omega}{\sqrt{\omega^2 - b^2}}\right)e^{-bt}$$

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$$\boxed{A = y_0\left(\frac{\omega}{\sqrt{\omega^2-b^2}}\right)e^{-bt}} \quad \text{—— (14)}$$

The amplitude A ↓ exponentially with time 't'
∵ The Motion is called Damped Oscillatory.

Time Period for DOM

$$T = \frac{2\pi}{\omega'} \Rightarrow 2\pi\sqrt{\frac{m}{K} - \frac{r^2}{4m^2}}$$

$$\boxed{\text{Frequency} : \frac{1}{T} \Rightarrow \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{4m^2}{r^2}}}$$

The frequency of natural undamped oscillation $(r \to 0)$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Thus, f is less than f_0.

Quality of factor.

The Quality factor is defined as 2π times the ratio of the energy stored in the system to the energy lost per cycle.

$$Q = 2\pi\left(\frac{\text{Energy Stored}}{\text{Energy losted per cycle}}\right).$$

The quality factor, $Q = \frac{\omega'}{2b}$ where $2b = \frac{r}{m}$ &

$$\omega' = \sqrt{\omega^2 - b^2}$$

If damping is less $\omega' = \omega$

∴ The Quality is large if damping co-eff 'b' is small, & Q is small if b is large.

∵ The Quality factor rep. the effeciency of oscillator, hence, lower the damping, higher the quality factor

When damping is small. b^2 is negligible compared to ω^2

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hence, $\omega' = \omega$

$$Q \text{ factor} = \frac{\omega}{\gamma} \times m$$

$$Q \Rightarrow \frac{m\omega}{r} \Rightarrow \frac{m}{\gamma}\sqrt{\frac{K}{m}}$$

where K is spring const.

M is mass of damped oscillator.

$$\therefore \boxed{Q = \frac{1}{2b}\sqrt{\frac{K}{m}}}$$

Forced Oscillation

When the harmonic oscillator, oscillates ~~with its~~ naturally in a medium like air, its oscillation gets damped i.e the amplitude ↓ exponentially with time 0. The other damping force experienced may be friction, viscosity, etc. If the amplitude of oscillation are to be maintained indeffinitely, Energy must be supplied externally. Such oscillations of the body under the action of external periodic force are known as forced oscillation & the oscillator is called as forced harmonic oscillator [FHO] or driven harmonic oscillator.

Eg: Oscillation of a swing which is pushed periodically by a person.

: The motion of a hammer in a calling bell.

: The vibration of ear drum caused by sound from a sounding body such as tuning fork

Theory of forced oscillation

Consider a body of mass m executing oscillation in a damping medium acted upon by an external periodic force ($F \sin pt$).

Where p is the angular frequency of the external force.

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The force acted upon the body is
Restoring force proportional to the displacement but oppositel directed, given by $(-Ky)$ where k is force constant.

A frictional force proportional to velocity, but oppositely directed $\left(-r\frac{dy}{dt}\right)$ where r is the frictional force per unit velocity.

The external periodic force, rep by $(F\sin pt)$ where F is the max. force & p is angular frequency
∴ The total forces acting on the body is given by:

$$F = -Ky - r\frac{dy}{dt} + F\sin pt$$

From Newton's II law

$$m\frac{d^2y}{dt^2} = -Ky - r\frac{dy}{dt} + F\sin pt \quad \div \text{ by } m$$

$$\frac{d^2y}{dt^2} = -\frac{K}{m}y - \frac{r}{m}\frac{dy}{dt} + \frac{F}{m}\sin pt$$

Sub $\frac{k}{m} = \omega^2$; $\frac{r}{m} = 2b$; $\frac{F}{m} = f$

$$\frac{d^2y}{dt^2} = -\omega^2 y - 2b\frac{dy}{dt} + f\sin pt$$

$$\frac{d^2y}{dt^2} + \omega^2 y + 2b\frac{dy}{dt} = f\sin pt \quad \text{— ①}$$

Eq ① is the diff eq^n of the motion of the particle
In this case, when the ~~State~~ Steady State is setup, the particle vibrates with the frequency of applied force & not with its own natural frequency.
∴ The sol^n of diff. eq^n ① must be of the type

$$y = A\sin(pt - \theta) \quad \text{— ②}$$

where A is the steady amplitude of vibration & 'θ' by which the displacement y lags behind the applied force

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$f \sin pt$. A & θ being arbitary constants.

Diff. eq ② w.r.t 't'.

$$\frac{dy}{dt} = A\cos(pt-\theta)\cdot p.$$

$$\frac{d^2y}{dt^2} = -A\sin(pt-\theta)\cdot p^2$$

Sub $\frac{d^2y}{dt^2}$ in ①

$$-A\sin(pt-\theta)p^2 + \omega^2 y + 2b\left(A\cos(pt-\theta)\cdot p\right) = f\sin pt$$

$$-A\sin(pt-\theta)p^2 + \omega^2 y + 2bp\,A\cos(pt-\theta) = f\sin pt$$

$$-Ap^2\sin(pt-\theta) + \omega^2 A\sin(pt-\theta) + 2bp\,A\cos(pt-\theta) = f\sin pt$$

$$A\sin(pt-\theta)\{(\omega^2-p^2)\} + 2bAP\cos(pt-\theta) = f\sin pt.$$

$$A\sin(pt-\theta)\{(\omega^2-p^2)\} + 2bAPs\cos(pt-\theta) = f\sin\{(pt-\theta)+\theta\}$$

$$A\sin(pt-\theta)\{(\omega^2-p^2)\} + 2bAP\cos(pt-\theta) = f\{\sin(pt-\theta)\cos\theta + \cos(pt-\theta)\sin\theta\}$$

Comparing the co-eff. $\sin(pt-\theta)$ & $\cos(pt-\theta)$

$$A(\omega^2-p^2) = f\cos\theta \quad — ③$$

Sq & Add $\quad 2bAp = f\sin\theta \quad — ④$

~~Sub~~ ③ & ④ in previous

$$A^2(\omega^2-p^2)^2 + 4b^2A^2p^2 = f^2\cos^2\theta + f^2\sin^2\theta$$

$$A^2\{(\omega^2-p^2)^2 + 4b^2p^2\} = f^2(\cos^2\theta + \sin^2\theta) \Rightarrow 1$$

$$A^2 = \frac{f^2}{(\omega^2-p^2)^2 + 4b^2p^2}$$

Sq. root on b.s.

$$A = \frac{f}{\sqrt{(\omega^2-p^2)^2 + 4b^2p^2}} \qquad \left[f = \frac{F}{m}\right]$$

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$$A = \frac{F}{m\sqrt{(\omega^2-p^2)^2+4b^2p^2}} \quad - (5)$$

$\div$ eq $\frac{(4)}{(3)}$

$$\frac{2bAp}{A(\omega^2-p^2)} = \frac{f\sin\theta}{f\cos\theta}$$

$$\frac{2b\cancel{A}P}{\cancel{A}(\omega^2-p^2)} = \tan\theta$$

$$\tan\theta = \frac{2bP}{\omega^2-p^2}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{2bP}{\omega^2-p^2}\right]} \quad - (6)$$

Eq (5) is the amplitude of forced oscillation
Eq (6) gives the phase of forced oscillation.

Given the amplitude of forced vibration & its phase depending upon the relative values of p & ω. The foll. 3 cases are possible

Case i. When driven frequency is low i.e $p \ll \omega$.

In this case, the amplitude of vibration is given by

$$A = \frac{F}{m\sqrt{(\omega^2-p^2)^2+4b^2p^2}}$$ as p^2 will be very small, $\omega^2 - p^2 \Rightarrow \omega^2$

$\therefore$ The amplitude will be:

$$A = \frac{(F/m)}{\sqrt{(\omega^2)^2}} \Rightarrow \frac{F/m}{\omega^2}$$

$$\theta = \tan^{-1}\left[\frac{2bP}{\omega^2-p^2}\right]$$

$$\theta = 0$$

This shows that, the amplitude of vibration is independent of frequency of force. This aplitude depends on the magnitude of the applied force & force constant. The force & displacem are always in phase.

se 2: When $p=\omega$, ie the frequency of the force is equal to the frequen of the body, i.e $\omega^2-p^2=0$.
In this case, the amplitude of vibration is given by:

$$A = \frac{F}{m\sqrt{(\omega^2-\omega^2)+4b^2\omega^2}} \Rightarrow \frac{F}{m\sqrt{4b^2\omega^2}}$$

$$A \Rightarrow \frac{F/m}{2b\omega}$$

Because, where $2b = \frac{r}{m}$

$$A = \left(\frac{F}{\cancel{m}}\right)\left(\frac{\cancel{m}}{\omega r}\right) \Rightarrow \frac{F}{\omega r}$$

$$\boxed{A = \frac{F}{\omega r}}$$

$$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2-\omega^2}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2b\omega}{0}\right)$$

$$\theta \Rightarrow \tan^{-1}(\infty) \Rightarrow \frac{\pi}{2}$$

Thus, the amplitude of vibration is governed by damping & for small damping forces, the amplitude of vibration can be quite large.
The displacement lags behind the force by a phase of $\frac{\pi}{2}$

e 3: When $p \gg \omega$ $(\omega = 0)$.
The frequency of force is greater than the natural frequency of the body. In this case

$$(\omega^2-p^2)^2 = (-p^2)^2 = p^4$$

$$A = \frac{F}{m\sqrt{p^4+4b^2p^2}}$$

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$$A = \frac{F}{m\sqrt{(p^2)^2 + (4b^2p^2)}}$$

As p increases, A becomes smaller & smaller, since b is very small, $4b^2p^2 << p^4$

$$A = \frac{(F/m)}{\sqrt{p^4}} \Rightarrow \frac{F}{mp^2}$$

$$\theta = \tan^{-1}\left(-\frac{2bp}{p^2}\right)$$

$$\theta = \tan^{-1}\left(-\frac{2b}{p}\right)$$

b is very small, $2b = 0$ or some -ve values $(\theta = \tan^{-1}(0))$

$\theta = \pi$

Thus, in this case the amplitude A goes on decreasing & phase difference tends towards π i.e the displacement develops a large phase that approaches the value π w.r.t the phase of the applied force.

Resonance

Consider a body of mass m vibrating in a resistive medium of a damping constant r under the influence of ext. force $f \sin pt$. If ω is the natural frequency of vibration is given by $A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$ — (1)

where $2b = \frac{r}{m}$ & $\alpha = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right)$ — (2)

The phenomenon of making a body oscillate with its natural frequency under the influence of another vibrating body with the same frequency is called resonance.

Condisions for resonance:

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For 'A' becomes max. when the denominator of the above eq. must be minimum. For a Given body, this condision can be achieved by:

1) Making $\left(b = \frac{r}{2m}\right)$ minimum, i.e., resistive force is min.

2) When $p = \omega$, i.e frequency of the applied force is equal to the natural frequency of oscillation.

The two condisions can be proved theoretically in the foll. way:

∵ from eq ①

$$\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2} = \frac{f}{A} \text{ (min.)}$$

Diff w.r.t p

$$\frac{d}{dp}(\omega^2 - p^2)^2 + 4b^2p^2 = 0$$

$$2(\omega^2 - p^2)(-2p) + 8b^2p = 0$$

$$\cancel{-}\cancel{4}p(\omega^2 - p^2) = \cancel{-}\cancel{8}b^2\cancel{p}$$

$$2b^2 = (\omega^2 - p^2)$$

$$p^2 = \omega^2 - 2b^2$$

$$\boxed{p = \sqrt{\omega^2 - 2b^2}}$$

Thus; the amplitude is max. when the frequency $\frac{p}{2\pi}$ of the impressed force becomes $\frac{\sqrt{\omega^2 - 2b^2}}{2\pi}$

This is the resonance frequency ~~bas~~

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} \Rightarrow \frac{\sqrt{\omega^2 - 2b^2}}{2\pi}$

∴ It's clear from above eqns that the denominator reaches the min. in general when the above eqn is satisfied. Further the condisions when b is negligible in $p = \sqrt{\omega^2 - 2b^2}$

$$p^2 = \omega^2$$

Then it reduces to the above condision

$$\therefore \boxed{p = \omega}$$

when $p = \omega$, from eq ① becomes

$$A_{max} = \frac{f}{2b\omega} \qquad A_{max} = \frac{f}{2bp}$$

Thus, $A_{max} \longrightarrow \infty$, as $b \to 0$.

w.k.t $p = \sqrt{\omega^2 - 2b^2} \Rightarrow p^2 = \omega^2 - 2b^2$

sub to eq ①

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}}$$

$$A_{max} = \frac{f}{\sqrt{(\omega^2 - \omega^2 - 2b^2)^2 + 4b^2(\omega^2 - 2b^2)}}$$

$$\Rightarrow \frac{f}{\sqrt{\omega^2 + \omega^2 + 4b^4 + 4b^2\omega^2 - 8b^4}}$$

$(\omega^2 - \omega^2)$ for p+q.

$$= \frac{f}{\sqrt{4b^2\omega^2 - 4b^4}}$$

$p^2 = \omega^2 - 2b^2$

$\omega^2 = p^2 + 2b^2$

$$\Rightarrow \frac{f}{2b\sqrt{\omega^2 - b^2}}$$

$$\Rightarrow \frac{f}{2b\sqrt{p^2 + 2b^2 - b^2}}$$

$$\boxed{A_{max} \Rightarrow \frac{f}{2b\sqrt{p^2 + b^2}}}$$

For low damping

$A_{max} \Rightarrow \frac{f}{2bp}$ | $A_{max} = \frac{f}{2b\omega}$

* Sharpness of Resonance

The amplitude of the forced oscillation is given by

$$A = \frac{f}{\sqrt{(\omega^2 - b^2)^2 + 4b^2p^2}} \quad —①$$

At resonance, frequency of applied force is equal to the frequency of oscillator, i.e $p = \omega$ or $p^2 = \omega^2$

∴ Amplitude of the forced vibration at resonance

$A = \frac{f}{2bp}$ where, $f = \frac{F}{m}$ applied force per unit mass

$\frac{P}{2\pi}$ = frequny of the applied force.

b = Damping co-eff.

As the frequeny of the applied force 'P' is increased or decrease from its resonant value ~~a~~ ω, the value of amplitude always decreases.

When the amplitude at resonance falls ~~r~~rapidly as the frequency 'p' of the applied force is changed slightly from its resonant value, the resonance is said to be sharp.

When the amplitude resonance falls Gradually as the frequency 'p' of the applied force is changed slightly from its resonance value, the resonance is said to be flat.

Thus, sharpness of resonance is the rate of change of amplitude w.r.t its small change in frequency of the applied force at resonance.

Sharpness of resonance = $\frac{\text{Change in amplitude}}{\text{Change in frequency}}$.

The Sharpness of resonance depend on damping.

(i) when b = 0 i.e there is no damping, the amplitude at resonance becomes ∞ as shown in curve of figure
a = ∞

(ii) when damping is small, the amplitude is sufficiently large but ~~falos~~ falls of rapidly as the frequeny of the applid force becomes slightly different from the natural frequency as shown in curve of fig-2.

(iii) As the value of b ↑, the amplitude at resonance ↓.

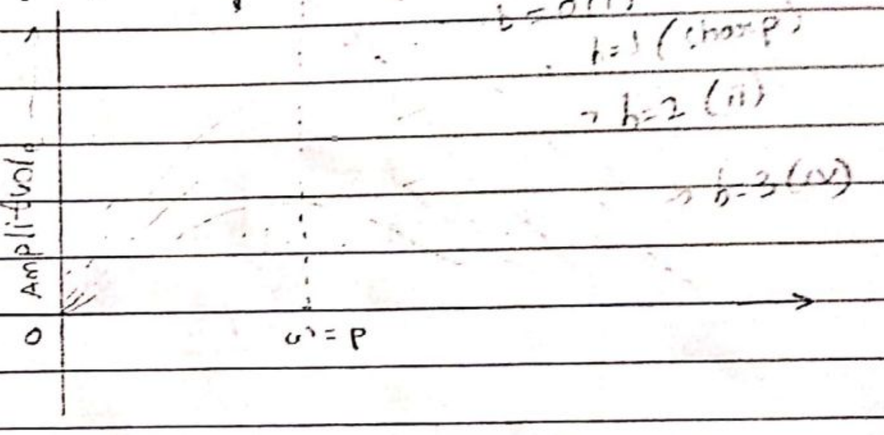
(iv) The amplitude gradually ↓ when the frequency of the applied force is diff. from the natural frequency of the body.

In other words, damping is small, resonance is sharp & when damping is large, resonance is flat.

Examples for resonance:

- Tuning of radio / transistor, when the natural frequency is so adjusted by moving the tuning knob of the receiver such that it equals the frequency of the radio waves.

- Musical instruments can be made to vibrate by bringing them in contact with vibrations which have the frequency equal to the natural frequency of the system / instruments.



Problems.

Q A mass of 25×10^{-2} kg is suspended from the lower end of a vertical spring having a force constant 25 N/m. What should be the damping constant of the system so that the motion is critically damped?

$\Rightarrow$ $m = 25 \times 10^{-2}$ kg

$k = 25$ N/m

$b = ?$

$b^2 = \omega^2$

$\left(\frac{r}{2m}\right)^2 = \omega^2$

$\left(\frac{r}{2m}\right)^2 = \frac{k}{m}$

$r^2 = 4km$

$r = 2\sqrt{km}$

$r = 2\sqrt{25 \times 10^{-2} \times 25}$

$r \Rightarrow 5$ kg/sec

$b = \frac{r}{2m} \Rightarrow \frac{5}{2(25 \times 10^{-2})}$

$\Rightarrow 10$

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Q A body of mass 500g is attached to spring & the system is driven by an external periodic force of amplitude 15 N & frequency 0.796 Hz. The spring extends by a length of 88 mm under the given load. Calculate amplitude of oscillation. If the resistance co-eff of the medium is 5.05. Ignore the mass of the spring.

$\Rightarrow$ $f = 0.796\ Hz$ $\quad A = ?$

$m = 0.5\ kg$

(applied) $F = 15\ N$

$y = 88 \times 10^{-3}\ m$

$r = 5.05\ kg/s$

$p = 2\pi f$

$\Rightarrow 2 \times (3.14) \times (0.796)$

$p \Rightarrow 5\ rad/sec.$

$$A = \frac{(F/m)}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

$$A = \frac{(15/0.5)}{\sqrt{((10.55)^2 - (5)^2)^2 + 4(5.05)^2(5)^2}}$$

$$\Rightarrow \frac{30}{\sqrt{((10.55)^2 - 25)^2 + 4(5.05)^2(25)}}$$

$$\Rightarrow \frac{30}{99.99} \Rightarrow \frac{30}{100} \Rightarrow 0.3\,m$$

$k = F/y$

$$\Rightarrow \frac{\cancel{15}\ mg}{88 \times 10^{-3}} \Rightarrow \frac{0.5 \times 9.8}{88 \times 10^{-3}} \Rightarrow 55.68\ N/m.$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{55.68}{0.5}} \Rightarrow 10.55\ rad/sec$$

$$b = \frac{r}{2m} \Rightarrow \frac{5.05}{2(0.5)} \Rightarrow 5.05\ rad/sec$$

Q A mass 25×10^{-3} kg is suspended to a lower end of a vertical spring having a force constant 25 N/m. The mechanical resistance of the system is 1.5 Ns/m. The mass is displaced vertically & released. Find whether the motion is oscillatory? If so, calculate its period of oscillation.

$\Rightarrow$ $m = 25 \times 10^{-3}\ kg$

$k = 25\ N/m$

$r = 1.5\ N\ s/m$

$$\omega^2 = \frac{k}{m} = \frac{25}{25 \times 10^{-3}} \Rightarrow 1000.$$

$$b^2 = \frac{(1.5)^2}{4 \times (25 \times 10^{-3})^2} = 900$$

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$\because b^2 < \omega^2$ $\therefore$ The motion is underdamped or is oscillatory

$$T = \frac{2\pi}{\omega}, \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}} \Rightarrow \frac{2(3.14)}{\sqrt{\cancel{25}\ 1000 - \frac{(1.5)^2}{4(25\times10^{-3})^2}}}$$

$$\Rightarrow \underline{0.628 \text{ Sec.}}$$

Q A mass of 1 kg is suspended from a spring of stiffness constant 25 N/m. If the undamped (natural) frequency is $\frac{2}{\sqrt{3}}$ times the damped frequency, calculate the damping factor.

$\Rightarrow$ $m = 1\text{kg}.$

$k = 25\ Nm^{-1}$

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}} \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Damped frequency f' = formula   Undamped frequency f_0 = formula

$$\frac{f_0}{f'} = \frac{2}{\sqrt{3}} \qquad \frac{\sqrt{k/m}}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}} = \frac{2}{\sqrt{3}}$$

$$\left(b = \frac{r}{2m} \Rightarrow r = b\right)$$

$$\Rightarrow \frac{k/m}{k/m - \frac{r^2}{4m^2}} = \frac{4}{3}$$

$$\frac{(25)/1}{25 - \frac{r^2}{4}} = \frac{4}{3} \Rightarrow \frac{25}{25 - \frac{r^2}{4}} = \frac{4}{3}$$

$$3(25) = 4\left(25 - \frac{r^2}{4}\right)$$

$$75 = 100 - r^2$$

$$r^2 = 25$$

$$\underline{r = 5\ Nm/kg}$$

Q A 20g oscillator with natural angular frequency, 10 rad/sec is vibrating in damping medium. The damping force is

5 - 9 for density

90 9 19

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proportional to the velocity of the vibrator. If the damping co-eff is 0.17, how does the oscillations decay?

$\Rightarrow$ $m = 20gm.$

$\Rightarrow 0.02 kg$

$\omega \Rightarrow 10$ rad/sec

$\gamma \Rightarrow 0.17$ kg/s

$$b^2 = \frac{r^2}{4m^2} \Rightarrow \frac{(0.17)^2}{4(0.02)^2} \Rightarrow 18.06$$

$$\omega^2 = \frac{k}{m} \Rightarrow m\omega^2 = k$$

$$(0.02)(10)^2 \Rightarrow 2 N/m$$

$\omega_0^2 > b^2$ $\because$ It's the case of underdamping

Q Calculate the peak amplitude of vibration of a system whose natural frequency is 1000 Hz when it oscillates in a resistive medium for which the value of damping per unit mass is 0.008 rad/sec under the action of an ext. periodic force per unit mass of amplitude 5 N/kg with tunable frequency.

$\Rightarrow$ $\nu = 1000 Hz$

$F = 5 N/kg$

$\gamma/m = 0.008$ rad/sec

$p = 2\pi\nu$

$\Rightarrow 2 \times 3.14 \times 1000$

$\Rightarrow 6280$

$$A = \frac{F/m}{\sqrt{(\omega^2 - p^2) + 4b^2p^2}}$$

$\omega \rightarrow \sqrt{\frac{k}{m}}$

$\omega \Rightarrow \sqrt{\frac{5 \times 10^{-3}}{0.008}}$ $k = \frac{F}{v}$

For max. amplitude

$p = \omega$ or $\omega^2 = p^2$

$$a_{max} = \frac{F/m}{\sqrt{0 + 4b^2p^2}} \Rightarrow \frac{F/m}{2bp} \Rightarrow b = 0.004 \Rightarrow \frac{5}{1000}$$

$\Rightarrow 5 \times 10^{3}$

$$a_{max} = \frac{5}{2(0.004) \times 2\pi \times 1000}$$

~~Now $P = 2\pi\nu$~~

~~$\Rightarrow 2\pi \times (10$~~

$\therefore$ The max. amplitude of vibration is $a_{max} = \frac{5}{50}$

$\Rightarrow 0.1m$

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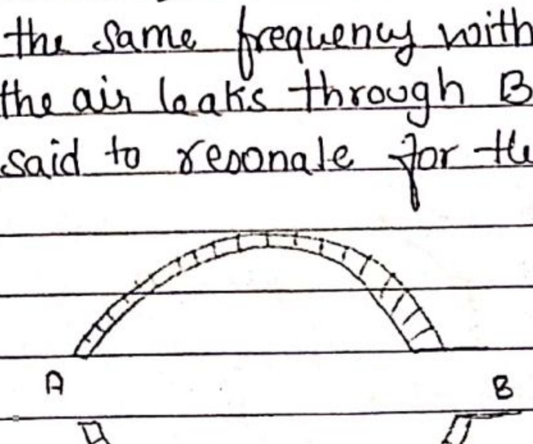
Resonator is a device to analyse a complex note sound, i.e to find out what particular frequencies are present in the given note. For this purpose it is nescessary that the resonator should exhibit a sharp resonance that is it should resound with a not of one particular frequency, namely, its own natural frequency. Thus, resonators of diff. natural frequencies comprising 7x detect the diff. frequencies comprizing the given note (musical note)

It consists of a hollow vessel with a hole called the mouth at one end, & opposite to it is a narrow opening B called the neck. The end B is held near the ear with the end A open for the entry of the air carrying musical note

The air streams in & out of A that is it vibrates through A. The air entered in such a hollow body will have definite value for its natural frequency of vibrations ω. When one of the component frequencies of a musical note carried by the incoming air same as ω, the air inside the vessel is set for vibrations of the same frequency with a large amplitude. A small part of the air leaks through B & is heard. Thus, the resonator is said to resonate for the note of that particular frequency.



If 'l' be the length of the neck

'A' be the area of cross section

'ρ' be the density of the air in the vessel.

'V' be the volume of the vessel

'K' be the volume elasticity of the air.

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∵ The air vibration executes a SHM & its time period is given by

$$T = 2\pi\sqrt{\frac{1}{\mu}} \quad \text{where } \mu = \frac{KA}{Vl\rho}$$

$$T = 2\pi\sqrt{\frac{Vl\rho}{KA}}$$. The velocity of sound in air is given by $v = \sqrt{\frac{K}{\rho}}$

∵ $T = \frac{2\pi}{v}\sqrt{\frac{Vl}{A}}$ & the frequeny of oscillation

$$\eta = \frac{1}{T} = \frac{v}{2\pi}\sqrt{\frac{A}{Vl}}$$

∵ Thus, the frequency of the resonator depends on total volume, length area of the vessel

Q Calculate the resonance frequency for a simple pendulam of length 1m

⇒ Given $L = 1m$

$v = ?$

Simple pendulam set for oscillations, oscillates with a period

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1}{9.8}} \Rightarrow 2 \text{ sec}$$

∵ Its frequency of oscillation is

$$v = \frac{1}{T} \Rightarrow \frac{1}{2} = 0.5 \text{ Hz}$$

This is its natural frequency of oscillation. Any external periodic force with this frequency causes resonant oscillation in the pendulam.

∵ Resonance frequency of the Spring,

$$v = 0.5 \text{ Hz}$$

Q Evaluate the resonance frequency of a Spring of force

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constant 1974 N/m, carrying a mass of 2 kg.

⇒ $K = 1974\ K\ N/m$

$m = 2 kg \quad \nu = ?$

W.K.T resonance frequency of any oscillation is equal to its natural frequency of oscillation.

$$\omega = \sqrt{\frac{K}{m}}, \quad \omega = 2\pi\nu$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1974}{2}}$$

$$\nu = 5 Hz.$$

∴ Resonance frequency of the Spring, $\nu = 5 Hz$

Shock Waves.

Definition of Mach number

It is defined as the ratio of the Speed of the object to the Speed of Sound in the given medium, that is.

$$\text{Mach number} = \frac{\text{Object Speed}}{\text{Speed of Sound in the medium}}$$

$M = \frac{v}{a}$ where v is the object Speed.

'a' is the Speed of Sound in the medium.

It doesn't have a unit, that indicates it is a pure number.

Classification of objects based on Mach number

Name	Mach No. RANGE
Subsonic	$M < 1$
Transonic	$0.8 < M < 1.2$
Supersonic	$M > 1$
Hypersonic	$M > 5$

∴ Resonance frequency of the Spring, $\nu = 5\,Hz$

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Subsonic	$M < 1$
Transonic	$0.8 < M < 1.2$
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Hypersonic	$M > 5$

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(i) Subsonic : If the speed of mechanical wave or body moving in the fluid is lesser than that of sound, then such a speed is referred to as subsonic & the wave is a subsonic wave. That is Mach no. < 1

Eg: (i) Speed of vehicles such as motor cars or trains.
(ii) Speed of flight of birds.

(ii) Supersonic : Supersonic waves are mechanical waves which travel with speeds greater than that of sound, i.e Mach number > 1

Eg : Supersonic flight.

(iii) Transonic : There is a speed range which overlaps on the subsonic & supersonic ranges.

Transonic ~~to~~ range for speeds $0.8 < M < 1.2$.

(iv) Hypersonic :- They travel with speeds for which Mach number > 5 & there is an overlapping area b/w supersonic & hypersonic flow.

- Today the highest speed realised in actual flight conditions is Mach 3.1.

Definition of Shock waves.

- Shock waves can be produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space.

" A shock waves is a narrow surface that manifests as a discontinuity in a fluid medium in which it is propagating with supersonic speed. The disturbance is characterized by sudden increase in pressure, temperature & density of the gas through which it propagates "

Properties of Shock waves

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1. They always travel in the medium with Mach number exceeding 1.
2. Shock waves obey the laws of fluid dynamics.
3. The effects caused by shock waves results in increase of entropy.
4. On impact, they physically travel through any medium (even in solid medium) through, the energy is dissipated fast.

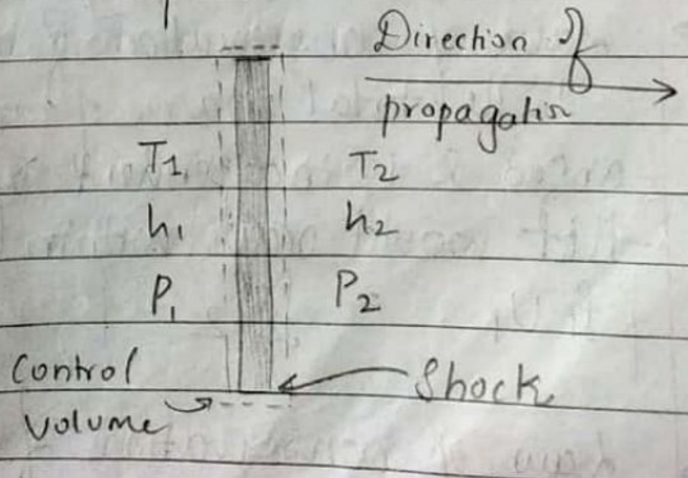
5. Shock waves exist in the medium of propagation confined to a very thin space of thickness not exceeding 1 μm within which, the medium is subjected to an increase in pressure, temp. & density in a exaggerated scale.
6. Across the shock waves, supersonic flow is decelerated into subsonic flow. This process occurs adiabatically but with a change in internal energy.

Eg: Shock waves are produced in nature during earthquakes & when lighting strikes.

Control volume

Control volume, is a model on the basis of which the shock waves are analysed.

It is an imaginary thin envelope that surrounds the shock front within which, there is a sharp increase in the pressure, temperature, & density in the compressed medium.



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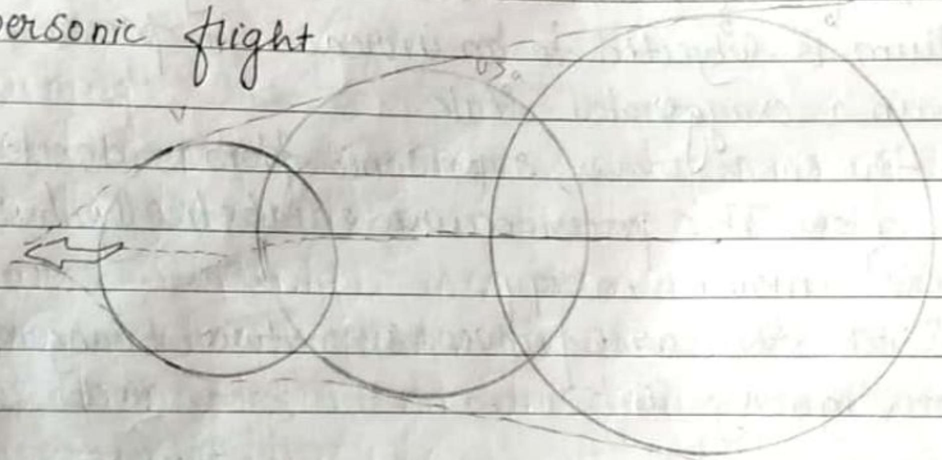
Subsonic flight

Supersonic flight



imp

Law of conservation of mass, momentum & energy.

Conservation means the maintainence of certain quantities unchanged during physical process.

Conservation laws applied to closed system. A closed system is the one that doesn't exchange any matter with the outside [surroundings]

→ Law of conservation of mass.

" The total mass of any isolated system remains unchanged & is independent of any chemical & physical changes ~~that~~ would occur within the system. It is expressed as

$$\boxed{\rho_1 v_1 = \rho_2 v_2}$$

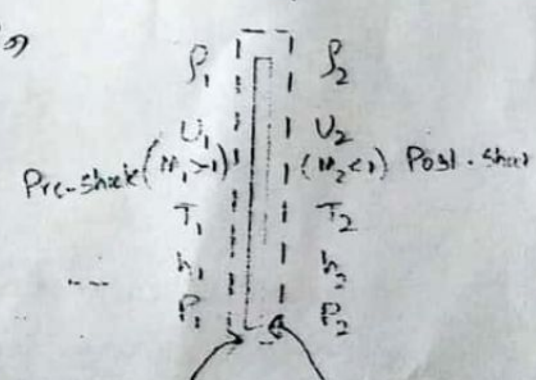
→ Law of conservation of momentum

Control volume

"Control volume is a model on the basis of which shock waves are analysed. It is an imaginary envelope that surrounds the shock front within which there is a sharp increase in pressure, temperature and density in the compressed medium".

It is a one dimensional confinement in the medium within two surfaces - pre shock and the post shock, with inner separation very small.

Let, pre shock side, density, flow velocity, temperature, pressure and enthaply are denoted by; ρ_1, u_2, T_1, P_1 and h_1 respectively. And post shock parameter ~~are~~; ρ_2, u_2, T_2, P_2 and h_2 respectively.



→ It is assumed
- heat energy remain constant (no heat leaves or gets into the volume).

→ The energy transfer is adiabatic and follows Rankine-Hugoniot eqn.

→ follows law of conservation of mass, momentum and energy.

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Basics of conservation of mass, momentum and energy:-

1. law of Conservation of mass:- (matter).

The total mass of any isolated system remains unchanged and is independent of any chemical and physical changes that could accour within the system. Given by;

$$\rho_1 u_1 = \rho_2 u_2 \quad \text{——— (1)}$$

2. law of Conservation of momentum:-

" In a closed system total momentum remains constant". Another words, " when two objects collide with each other in an isolated system, the total momentum of the object before collesion and after collesion remain same. Given by;

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \text{. ——— (2)}$$

3. law of conservation of Energy:-

' The total energy of a closed system remains constant and is independent of any change accuring within the system

Given by, $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$ ——— (3)

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Reddy Shock Tube

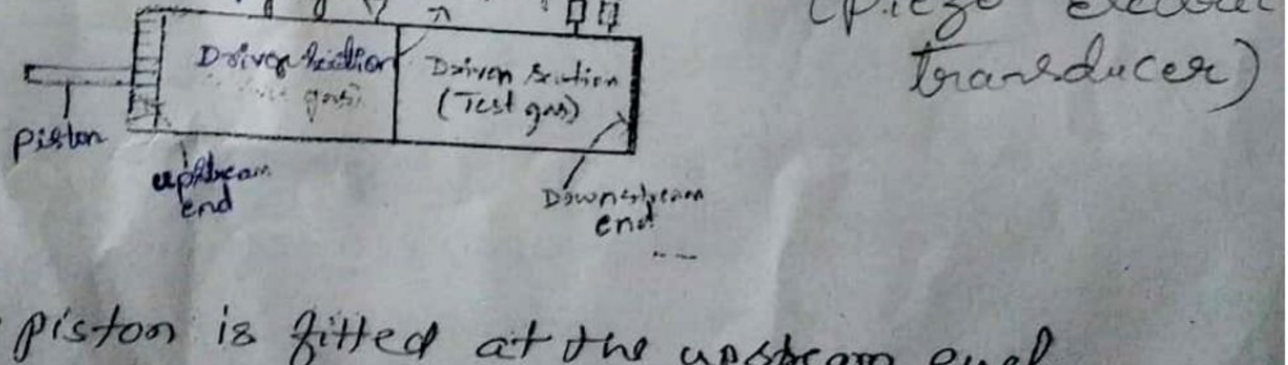
"Reddy tube is hand operated shock tube capable of producing shock waves using human energy. It is a long cylindrical tube with two sections seperated by a diaphragm. Its one end is fitted with a piston and other end is closed or open to the surroundings.

Construction:-

→ Reddy tube consists of a cylindrical stainless steel tube of about 30 mm diameter and 1 m length.

→ Its divided into two sections, the driver section and driven section of each 50cm in length.

→ The two sections are sperated by a 0.1mm thick aluminium or Mylar or paper diaphragm



→ A piston is fitted at the upstream end of the driven section, whereas far end (down stream end) of the driven section is closed.

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→ A digital pressure gauge is mounted in the driver section next to the diaphragm.

→ Two piezo-electric sensors S_1 and S_2 are (mech to elec) mounted 70 mm apart towards the close end of the shock tube.

→ A port is provided at the closed end of the shock tube (driven section) for filling test gas at required pressure.

→ The driver section is filled with gas. Known as driver gas, which is held at relatively high pressure to the compressing action of piston.

→ The gas in the driven section is known as driven gas.

<u>Working:-</u>

→ The driver gas is compressed by pushing the piston very hard into the driver tube until the diaphragm ruptures.

→ following the rupture, the driver gas rushes into the driven section, and pushes the driven gas towards the fardown stream end.

→ This amounts the moving shock

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wave that traverses the length of the driven
(travel across)
section.

→ The shock wave instantaneously raises the temperature & pressure of the driven (test) gas as the shock moves over it.

→ The propogating primary shock waves are reflected from the downstream end. After the test gas undegoes further compression which increases the test gas temperature and pressure to still higher values by the reflected shock waves.

→ This state of high values of pressure and temperature are sustained at the downstream end until the expansion waves reflected from the upstream end of the driver tube arrives there and neutralises the compression partially.

→ Expansion waves are created at the instant diaphragm ruptures and they travel is the direction opposite to the shock wave.

→ The period over which the extreme temp and pressure condition at the down-stream is sustained is of the order of milisecond.

→ The pressure rise caused by the

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primary and secondary (reflected) shock wave are sensed as the signals by the sensors S_1 and S_2 respectively. and they are recorded by the Cathode Ray oscilloscop (CRO).

→ Since the expt. involve 1 ms duration measurment, the rise time of CRO must be few μs. Hence CRO of bandwidth ≥ 1 M. is required.

→ from the recording of CRO, the shock arrival time is calculated by associate time base calculations.

→ using data so obtained, Mach no. pressure and temperature can be calculat

characteristics of Reddy tube:-

1. The Reddy tube operates on the princip. of free piston driven (hand operated) shock tube (FPST)
2. It is hand operated shock producing device.
3. It is capable of M > 1.5.
4. The rupture pressure depends on thicknes of diaphgram.
5. T > 900K can be obtained by using 'He' as driver gas and 'Ar' as driveng the temp. useful for chemical kinetic

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Methods of creating shock waves in the lab using a shock tube:-

1. Reddy shock tube.
2. Detonation.
3. Verry high pressure gas cylinder
4. Combustion.
5. Using Small charge of explosive.

Application of shock waves:-

1. Cell Information:- The DNA can be pushed (inside) to the cell, by passing shock waves of appropriate strength, however its functionality is not affected by the impact of shock wave. This find biological application.

2. Wood preservation:- By using the shock waves, the chemical preservatives in the form of solution could be pushed into the interior of wood samples. Ex. bamboo. and makes the process of introduction of preservatives into wood much faster and more efficient. The set up used in the method is known as (shock wave reactor).

3. Use in Pencil Industry:- In this industry, wood is first softened

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in a polymer at 70°c for about 8 hrs and thendried, hence take more days. today The injection of liquid into wor using shock waves almost can be done instantaneously and hence drying times gets lesser and reddy for next process.

4. Kidney stone treatment:-

shock wave is used in a therapy called "extra-corporal lithotripsy" to break the kidney stone into smaller fragments

5. Gas Dynamics study:-

The extreme conditions of P and T produced in the shock tube, enable to study high T gas dynamics. It is crucial in supersonic motion of objects & hypersonic re-entry of space vechiles into the atmosphere.

6. shock wave assisted needleless drug delivery. (Painless treatment)

The drug is filled into a cartrid which is kept pressed on the skin & shock wave is sent into the body at high The drug enters driectly through the skin throu porosity. (depth penteration → 100μ).

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7. Treatment of dry Borewells:-

In the borewells, water is available from feeder sources accumulates in the borewell through the number of seepage point which are porous.

Sometime these pores are blocked by sand particles. When the shock waves are send into such a dry borwell, it clears the blockages and rejuvenates the borewell into a water sourse.

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Problems

1) The distance between the two pressure sensors in a shock tube is 150 mm. The time taken by shock ~~waive~~ to travel this distance is 0.3 ms. If the velocity of sound under the same condition is $340\,ms^{-1}$. find the Mach no. of the shock wave.

$(M = 1.47)$

2) Estimate the speed of the sound(a) in helium gas at 350K. Given, γ for He $= 1.667$, $R = 2008\,J\,kg^{-1}K^{-1}$

$(a = 1082\,m/s)$

3) The distance between two pressure sensors in shock tube is 100mm and if the time taken to travel between these pressure sensors is $195\mu s$. Calculate the Mach no. of the shock wave

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