CBCS SCHEME

USN

18MAT31

Third Semester B.E. Degree Examination, July/August 2021

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find $L[t\,e^{-2t}\sin 4t]$. **(06 Marks)**

b. A periodic function of period $2\pi/\omega$ is defined by $f(t)=\begin{cases} E\sin\omega t, & 0\le t<\pi/\omega \\ 0, & \pi/\omega\le t<2\pi/\omega \end{cases}$. Where E and ω are constants. **(07 Marks)**

c. Solve : $y''(t)+k^2y(t)=0$; $y(0)=0$ and $y'(0)=1$ by Laplace transformation. **(07 Marks)**

2 a. Find : i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ ii) $L^{-1}\left[\cot^{-1}\left(\frac{S}{2}\right)\right]$. **(06 Marks)**

b. Find the inverse Laplace transform of $\frac{1}{(s-1)(s^2+1)}$ by using convolution theorem. **(07 Marks)**

c. Express the following function in terms of Heaviside step function and hence find its Laplace transform where $f(t)=\begin{cases} t^2, & 0<t\le 2 \\ 4t, & t>2 \end{cases}$. **(07 Marks)**

3 a. Expand $f(x)=x(2\pi-x)$ as a Fourier series in $[0,2\pi]$. **(06 Marks)**

b. Obtain Fourier series for the function f(x) given by $f(x)=\begin{cases} 1+\frac{2x}{\pi}, & -\pi\le x\le 0 \\ 1-\frac{2x}{\pi}, & 0\le x\le \pi \end{cases}$. **(07 Marks)**

c. Find the half range sine series of $f(x)=\frac{e^{ax}}{\sinh a\pi}$ in $(0,\pi)$. **(07 Marks)**

4 a. Find the Fourier series expansion of f(x) given by $f(x)=\begin{cases} 1 & 0<x<1 \\ 2 & 1<x<3 \end{cases}$. **(06 Marks)**

b. Find the half range sine series for x^2 in $(0,\pi)$. **(07 Marks)**

c. The values of x and the corresponding values of f(x) over a period T are given below. Show that $f(x)=0.75+0.37\cos\theta+1.004\sin\theta$ where $\theta=\frac{2\pi x}{T}$. **(07 Marks)**

x	0	T/6	T/3	T/2	2T/3	5T/6	T
f(x)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

5 a. State: i) Initial and final value theorems ii) Find the Z –transform of $\cos\left(\frac{n\pi}{2}+\frac{\pi}{4}\right)$. **(06 Marks)**

b. Find the complex Fourier transform of the function $f(x)=\begin{cases} 1 & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$.

Hence evaluate $\int_0^\infty\left(\frac{\sin x}{x}\right)dx$. **(07 Marks)**

c. Compute the inverse Z-transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$. **(07 Marks)**

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6 a. Find the Fourier cosine transform of $f(x)=\begin{cases} x, & 0<x<2 \\ 0, & \text{else where} \end{cases}$. **(06 Marks)**

b. Find the Z-transform of $2n+\sin\frac{n\pi}{4}+1$. **(07 Marks)**

c. Solve the difference equation : $u_{n+2}-3u_{n+1}+2u_n=0$, with $u_0=0$ and $u_1=-1$. **(07 Marks)**

7 a. Find by Taylor's series method the value of y at x = 0.1 to five places of decimals from $\frac{dy}{dx}=x^2y-1, y(0)=1$. **(06 Marks)**

b. Use fourth order Runge-Kutta method to solve $(x+y)\frac{dy}{dx}=1$, $y(0.4)=1$ at $x=0.5$ correct to four decimal places. **(07 Marks)**

c. If $\frac{dy}{dx}=2e^x-y$, $y(0)=2$, $y(0.1)=2.010$, $y(0.2)=2.040$ and $y(0.3)=2.090$, find $y(0.4)$ correct to four decimal places by using Milne's predictor – corrector method and applying corrector formula twice. **(07 Marks)**

8 a. Using modified Euler's formula compute y(1.1) correct to five decimal places given that $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x^2}$ and $y=1$ at $x=1$. [taking $h=0.1$]. **(06 Marks)**

b. Employ Taylor's series method to find y at x = 0.1 and 0.2 correct to four places of decimal. Given $\frac{dy}{dx}-2y=3e^x$, $y(0)=0$. **(07 Marks)**

c. Solve the differential equation $y'+y+xy^2=0$ with the initial values of y : $y_0=1$, $y_1=0.9008$, $y_2=0.8066$, $y_3=0.722$ corresponding to the values of x : $x_0=0$, $x_1=0.1$, $x_2=0.2$, $x_3=0.3$ by computing the value of y corresponding to $x=0.4$ applying Adams – Bashforth predictor and corrector formula. **(07 Marks)**

9 a. Given $y''-xy'-y=0$ with the initial conditions $y(0)=1$, $y'(0)=0$, compute $y(0.2)$ using fourth order Runge-Kutta method. **(06 Marks)**

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. **(07 Marks)**

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary. **(07 Marks)**

10 a. Apply Milne's method to compute y(0.8) given that $y''=1-2yy'$ and the following table of initial values. **(07 Marks)**

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

b. Prove that the geodesics on a plane are straight line. **(06 Marks)**

c. Find the extremal of the functional : $\int_{x_0}^{x_1}(y^2+y'^2-2y\sin x)dx$. **(07 Marks)**

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18MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2021

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Laplace transform of $\cos t \, \cos 2t \, \cos 3t$. (06 Marks)

b. If $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$, show that $L\{f(t)\} = \frac{1}{s^2} \tan h \left(\frac{as}{2}\right)$. (07 Marks)

c. Find the Inverse Laplace transforms of :
i) $\frac{2s+1}{s^2+6s+13}$ ii) $\frac{1}{3}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$. (07 Marks)

OR

2 a. Express the function f(t) in terms of unit step function and find its Laplace transform, where
$f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$. (06 Marks)

b. Find the Inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$ using Convolution theorem. (07 Marks)

c. Solve by the method of Laplace transforms, the equation
$y'' + 4y' + 3y = e^{-t}$ given $y(0) = 0$, $y'(0) = 0$. (07 Marks)

Module-2

3 a. Obtain the Fourier series of the function $f(x) = x^2$ in $-\pi \le x \le \pi$. (06 Marks)

b. Obtain the Fourier series expansion of
$f(x) = \begin{cases} x & , \; 0 < x < \pi \\ x - 2\pi & , \; \pi < x < 2\pi \end{cases}$. (07 Marks)

c. Find the Cosine half range series for $f(x) = x(\ell - x)$, $0 \le x \le \ell$. (07 Marks)

OR

4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-\ell, \ell)$. (06 Marks)

b. Find the sine half range series for
$f(x) = \begin{cases} x & , \; 0 < x < \pi/2 \\ \pi - \pi & , \; \pi/2 < x < \pi \end{cases}$. (07 Marks)

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. (07 Marks)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Module-3

5 a. If $f(x) = \begin{cases} 1 - x^2 & , \; |x| < 1 \\ 0 & , \; |x| \ge 1 \end{cases}$. Find the Fourier transform of f(x) and hence find value of
$\int_0^\infty \frac{x\cos x - \sin x}{x^3} dx$. (06 Marks)

b. Find the Fourier Cosine transform of
$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0, & x > 4 \end{cases}$. (07 Marks)

c. Find the Z – transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (07 Marks)

OR

6 a. Solve the Integral equation
$\int_0^\infty f(\theta)\cos\alpha\theta \, d\theta = \begin{cases} 1 - \alpha & , \; 0 \le \alpha \le 1 \\ 0 & , \; \alpha > 1 \end{cases}$ hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. (06 Marks)

b. Find the Inverse Z – transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. (07 Marks)

c. Using the Z – transform, solve $Y_{n+2} - 4Y_n = 0$, given $Y_0 = 0$, $Y_1 = 2$. (07 Marks)

Module-4

7 a. Using Taylor's series method, solve the Initial value problem
$\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider upto 4^{th} degree term. (06 Marks)

b. Use modified Euler's method to compute y(0.1), given that $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ by taking $h = 0.05$. Consider two approximations in each step. (07 Marks)

c. Given that $\frac{dy}{dx} = x - y^2$, find y at $x = 0.8$ with

x :	0	0.2	0.4	0.6
y :	0	0.02	0.0795	0.1762

By applying Milne's method. Apply corrector formula once. (07 Marks)

OR

8 a. Solve the following by Modified Euler's method
$\frac{dy}{dx} = x + |\sqrt{y}|$, $y(0) = 1$ at $x = 0.4$ by taking $h = 0.2$. Consider two modifications in each step. (06 Marks)

b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute y(0.2) by taking $h = 0.2$ using Runge – Kutta method of order IV. (07 Marks)

c. Given $\frac{dy}{dx} = (1+y)x^2$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, determine y(1.4) by Adam's Bashforth method. Apply corrector formula once. (07 Marks)

Module-5

9 a. Given $y'' - xy' - y = 0$ with $y(0) = 1$, $y'(0) = 0$. Compute y(0.2) using Runge – Kutta method. (06 Marks)

b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (07 Marks)

c. Prove that the geodesics on a plane are straight lines. (07 Marks)

OR

10 a. Find the curve on which functional
$\int_0^1 [(y')^2 + 12xy]dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized. (06 Marks)

b. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data. Apply corrector formula once. (07 Marks)

x :	1	1.1	1.2	1.3
y :	2	2.2156	2.4649	2.7514
y' :	2	2.3178	2.6725	3.0657

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is Catenary $y = c\cosh\left(\frac{x+a}{c}\right)$. (07 Marks)