Module 2

Know Your Data

Getting to Know Your Data

- *Data Objects and Attribute Types*

- *Basic Statistical Descriptions of Data*

- *Measuring Data Similarity and Dissimilarity*

Data Objects

- Data sets are made up of data objects.
- A **data object** represents an entity.
- Examples:
  - **sales database: customers, store items, sales**
  - **medical database: patients, treatments**
  - **university database: students, professors, courses**
- Also called samples , examples, instances, data points, objects, tuples.
- Data objects are described by **attributes**.
- Database rows -> data objects; columns ->attributes.

Attributes

- **Attribute** (or **dimensions, features, variables**): A data field, representing a characteristic or feature of a data object.
  - **e.g., customer _ID, name, address**
- **Observations**: observed values for a given attribute
- **Attribute vector/feature vector:** A set of attributes that define an obje

- **Types:**
  - **Nominal**
  - **Ordinal**
  - **Interval-scaled**
  - **Ratio-scaled**

Attribute Types – Categorical/Qualitative

1. **Nominal:** categories, states, or “names of things”

   - **Hair_color = {auburn, black, blond, brown, grey, red, white}**
   - **marital status, occupation, ID numbers, zip codes**

- In the cases of nominal attributes with numeric values e.g. Cust_ID, the numbers are not intended to be used quantitatively.

- Also in case of numeric nominal attributes , values do not have any meaningful order about them.

- We can also represent values of nominal attributes(symbols or names) by numbers. However these numbers are not intended to be used quantitatively.

Attribute Types – Categorical/Qualitative

2. Binary attributes

- **Nominal attribute with only 2 categories/states (0 or 1)**
  - 0: attribute is absent
  - 1: attribute is present
- **Symmetric binary: both outcomes equally important**
  - e.g., gender
- **Asymmetric binary: outcomes not equally important.**
  - e.g., medical test (positive vs. negative)
  - Convention: **assign 1 to most important outcome** (e.g., HIV positive)

- If two states are True and False ,then called as Boolean Attribute

Attribute Types – Categorical/Qualitative

3. Ordinal Attributes:

- **Values have a meaningful order or a ranking among them but magnitude between successive values is not known.**

  Ex: Size = {small, medium, large}, grades, professor rankings

▪Useful for registering subjective assessments of qualities that cannot be measured objectively; thus often used in surveys for ratings.

E.g Customer satisfaction survey

▪We can compute mean and median but not mode for the ordinal attributes .

Note:nominal, binary, and ordinal attributes are qualitative attributes.

Numeric/Quantitative Attribute

4. Numeric Attributes /Quantitative Attributes

- Represents measurable quantity in integer or real values.

4.1 Interval scaled attributes

- Measured on a scale of **equal-sized units**
- Values have order and can be positive, 0, or negative
- E.g., temperature in C˚or F˚, calendar dates
- We can obtain a ranking of objects by ordering the values.
- Also allow us **to compare and quantify** the difference between values.

  e.g 20 C is 5 C less than 25C. or Year 2002 and 2010 are 8 years apart.

- **No true zero-point** -We can not speak of values in terms of ratio or multiples

  e.g without a true zero point, we can’t say that 10 C˚ is twice as warm as 5C˚. 0C doesnt mean no temp, Year 0 does not correspond to the beginning of a time.

  Mean ,Median and Mode can be calculated as numeric values.

Numeric/Quantitative Attribute

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- **No true zero-point** -We can not speak of values in terms of ratio.

  e.g without a true zero point, we can't say that 10 C˚ is twice as warm as 5C˚.

- Mean ,Median and Mode

Numeric Attribute Types

4.2 Ratio scaled attributes

- Inherent **zero-point**
- The values are ordered, and we can also compute the difference between values, as well as the mean, median,and mode
- Examples: Count attributes such as years of experience and number of words attribute for a document
- attributes to measure age, weight, height and monetary quantities (e.g., you are 100 times richer with $100 than with $1).

Discrete vs. Continuous Attributes

- **Discrete Attribute**

Discrete attributes have a **finite** or **countable** number of distinct values.

They typically represent **whole numbers** and cannot take fractional values.

- **Example:**
  - Number of students in a class (e.g., 30, 31, 32…)
  - Number of cars in a parking lot
  - Roll numbers of students
  - Binary values (e.g., 0 and 1 for gender or Yes/No answers)

- **Note: Binary attributes are a special case of discrete attributes**

Discrete vs. Continuous Attributes

- **Continuous Attribute**

Continuous attributes have an **infinite** number of possible values within a given range.

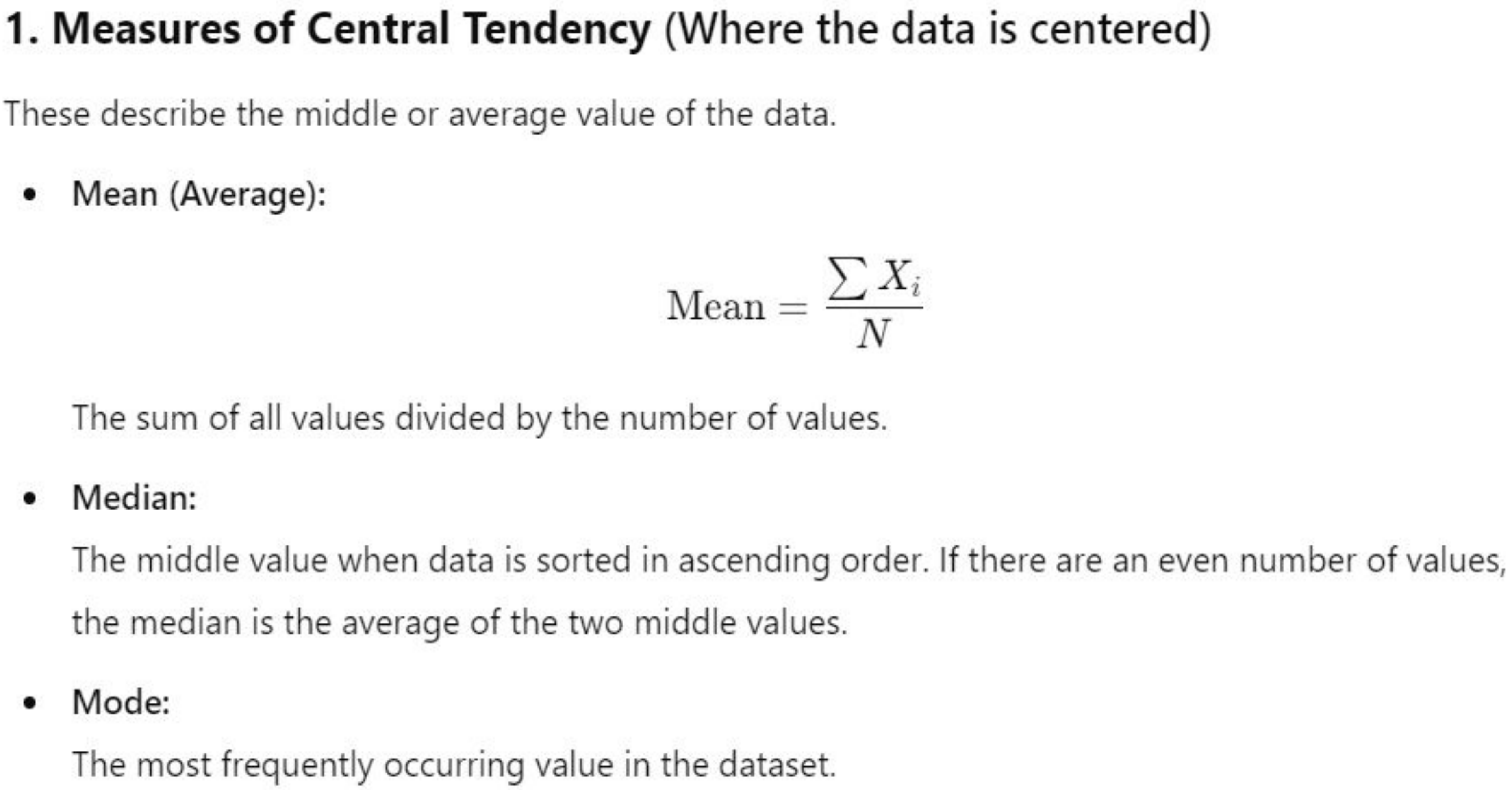
They can take **fractional** or **decimal** values and are typically measured rather than counted.

- **Example:**
  - Temperature (e.g., 25.3°C, 26.7°C)
  - Height of a person (e.g., 5.8 ft, 6.1 ft)
  - Weight of an object (e.g., 70.5 kg, 80.2 kg)
  - Time taken to complete a task (e.g., 4.56 seconds)

Basic Statistical Descriptions of Data

- Used to identify properties and identify which data values can be treated as noise or outliers.

- 3 areas of statistical descriptions:
  - **Measuring central tendencies**
  - **Measuring dispersion of data(How data spread out?)**
  - **Graphic display of basic statistical description**



1. Measures of Central Tendency (Where the data is centered)

These describe the middle or average value of the data.

- **Mean (Average):**

  $$\text{Mean} = \frac{\sum X_i}{N}$$

  The sum of all values divided by the number of values.

- **Median:**

  The middle value when data is sorted in ascending order. If there are an even number of values, the median is the average of the two middle values.

- **Mode:**

  The most frequently occurring value in the dataset.

Ii each data value occurs only once, then there is no mode.

- **Midrange:**
  - The **midrange** can also be used to assess the central tendency of a numeric data set.
  - It is the average of the largest and smallest values in the set.

Measuring the Central Tendency:Mean

Mean (algebraic measure) (sample vs. population):

- If we were to plot the observations for attribute, where would most of the values fall?
- The most common and effective numeric measure of the "center" of a set of data is the (arithmetic) mean**.**
- Let x_1, x_2, x_3, x_4, **…..** x_N be a set of N values or observations, such as for some numeric attribute X, like salary.

The **mean** of this set of values is

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N}.$$

Weighted mean is :

$$\bar{x} = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}$$

- **Problem with mean is its sensitivity to extreme values.**

- **For skewed (asymmetric) data, a better measure of the center of data is the median**

Measuring the Central Tendency:Median

- ***<u>Median:</u>***
  - ***Middle value if odd number of values, or average of the middle two values otherwise***
  - ***The median is expensive to compute when we have a large number of observations.***
  - ***Estimated by interpolation (for grouped data):***

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c}{f} \right] \times h$$

- l = lower limit of median class
- n = total number of observations
- c = cumulative frequency of the preceding class
- f = frequency of each class
- h = class size

Marks	Number of students	Cumulative frequency	
0 - 20	6	0 + 6	6
20 - 40	20	6 + 20	26
40 - 60	37	26 + 37	63
60 - 80	10	63 + 10	73
80 - 100	7	73 + 7	80

Solution:

We need to calculate the cumulative frequencies to find the median.

N = sum of f = 80, N/2 = 80/2 = 40

Since n is even, we will find the average of the $n/2^{th}$ and the $(n/2 +1)^{th}$ observation i.e. the cumulative frequency greater than 40 is 63 and the class is 40 - 60. Hence, the median class is 40 - 60.

l = 40, f = 37, c = 26, h = 20

Median = l + [(n/2−c)/f] × h == 40 + [(37 - 26)/40] × 20 == 40 + (11/40) × 20 = = 40 + (220/40) = 40 + 5.5 == 45.5

Measuring the Central Tendency:Mode and midrange

- *Mode*
  - ***Value that occurs most frequently in the data***
  - ***Unimodal, bimodal, trimodal,multimodal***
  - At the other extreme, if each data value occurs only once, then there is no mode.

- Midrange
  - The **midrange** can also be used to assess the central tendency of a numeric data set.
  - It is the average of the largest and smallest values in the set.

Measuring the Central Tendency: Example

Suppose we have the following values for *salary* (in thousands of dollars), shown in increasing order: 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110.
Claculate mean, median,mode and midrange

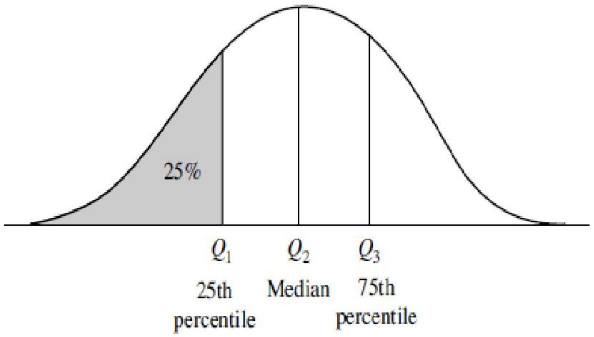
Mean=58
Median=54
Modes = bimodal. The two modes are 52 and 70
Midrange=70

Measuring the Dispersion of Data

(Range, Quartiles, Variance ,Standard Deviation, and Interquartile Range)

- **Range, Quartiles, and Interquartile Range**
  - The **range** of the data set is the difference between the largest (max()) and smallest (min()) values.
  - **Quantiles** are points taken at regular intervals of a data distribution, dividing it into essentially equal size consecutive sets.
  - The ***kth q-quantile*** for a given data distribution is the value *x* such that at most k/q of the data values are less than x and at the most *(q-k)/q* of the data values are more than *x*, where *k* is an integer such that 0 < *k* < *q*. There are *q*-1 *q*-quantiles.
  - The **2-quantile** is the data point dividing the lower and upper halves of the data distribution.It corresponds to the median
  - The **4-quantiles** are the three data points that split the data distribution into four equal parts; each part represents one-fourth of the data distribution. They are more commonly referred to as **quartiles**
  - The **100-quantiles** are more commonly referred to as **percentiles**; they divide the data distribution into 100 equal-sized consecutive sets.
  - The median, quartiles, and percentiles are the most widely used forms of quantiles.

Quartiles:The quartiles are the three values that split the sorted data set into four equal parts



A plot of the data distribution for some attribute X. The quantiles plotted are quartiles. The three quartiles divide the distribution into four equal-size consecutive subsets. The second quartile corresponds to the median.

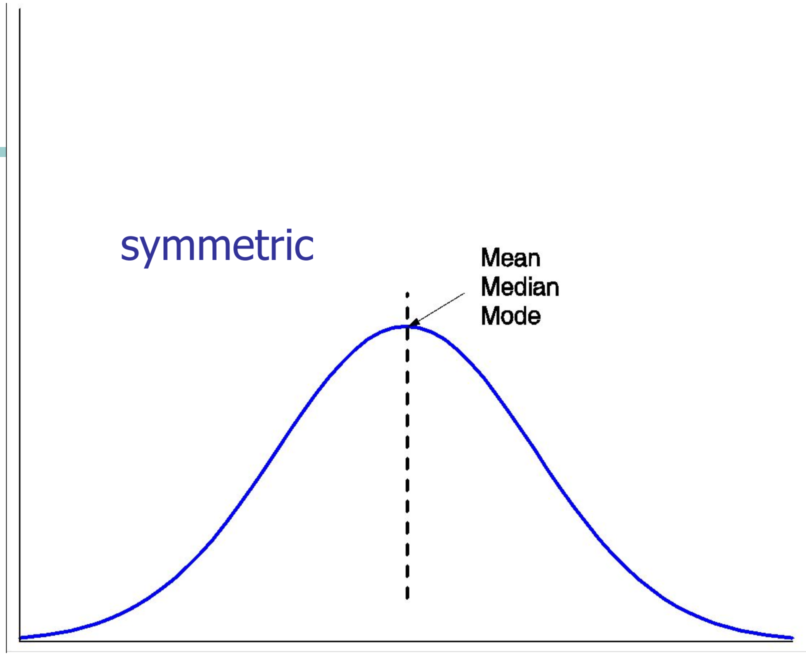
- The quartiles give an indication of a distribution’s center, spread, and shape
- *Q*1, is the 25th percentile
- *Q*3, is the 75th percentile
- The second quartile Q2 is, the 50th percentile. As the median, it gives the center of the data distribution.

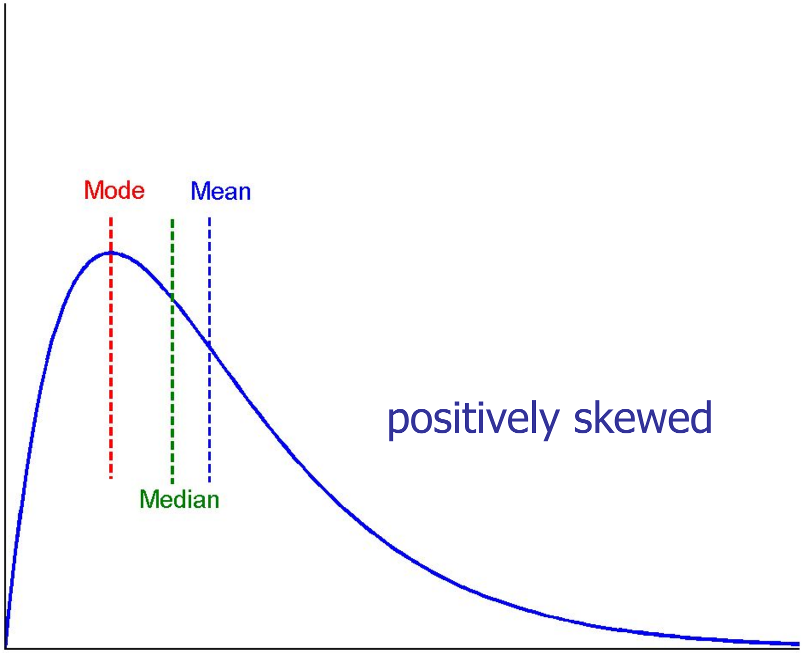
- **InterQuartile Range** (**IQR**) :The distance between the first and third quartiles is a simple measure of spread that gives the range covered by the middle half of the data.
  IQR = *Q*3 -*Q*1.

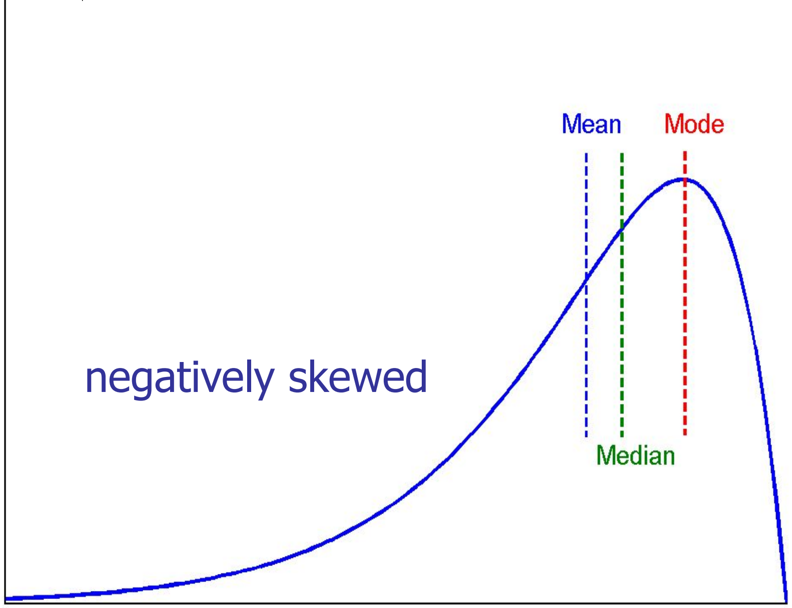
Ex: 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110
the quartiles for this data are the third, sixth, and ninth values respectively, in the sorted list
Therefore, *Q*1 =47 and *Q*3 =63. Thus, the IQR=63-47 =16

Median, mean and mode of symmetric, positively and negatively skewed data

- In the symmetric distribution, the median (and other measures of central tendency) splits the data into equal-size halves.



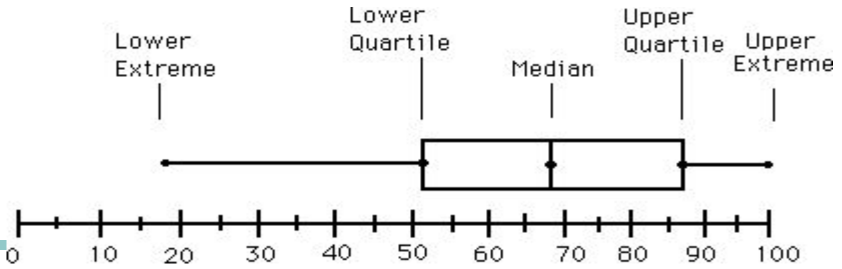




Five Number Summary

- ***Five-number summary** of a distribution* (*Minimum, Q1, Median, Q3, Maximum*)
  - It is more informative to also provide the two quartiles Q1 and Q3, along with the median
  - A common rule of thumb for identifying suspected **outliers** is to single out values falling at least 1.5*IQR* above the third quartile or below the first quartile.
  - Because *Q*1, the median, and *Q*3 together contain no information about the endpoints (e.g., tails) of the data, a fuller summary of the shape of a distribution can be obtained by providing the lowest and highest data values as well. This is known as the *five-number summary.*
  - The **five-number summary** of a distribution consists of the median (*Q*2), the quartiles *Q*1 and *Q*3, and the smallest and largest individual observations, written in the order of *Minimum*, *Q*1, *Median*, *Q*3, *Maximum*.

Boxplot Analysis



- **Boxplots** are a popular way of visualizing a distribution.
- A boxplot incorporates the five-number summary as follows:
  - Typically, the ends of the box are at the quartiles so that the box length is the interquartile range.
  - The median is marked by a line within the box.
  - Two lines (called *whiskers*) outside the box extend to the smallest (*Minimum*) and largest (*Maximum*) observations.
  - When dealing with a moderate number of observations, it is worthwhile to plot potential outliers individually. To do this in a box plot, the whiskers are extended to the extreme low and high observations *only if* these values are less than 1.5*IQR* beyond the quartiles. Otherwise, the whiskers terminate at the most extreme observations occurring within 1.5*IQR* of the quartiles. The remaining cases are plotted individually.

Find the first, second, and third quartiles for the data 8, 5,15, 20, 18, 30, 40, 25

Step 1: *Sort the given data in the ascending order*

5, 8, 15, 18, 20, 25, 30, 40.

Step 2: *Find all Quartiles step by step*

First Quartile= {(n + 1)/4}th term

Here n = 8 because there are total 8 numbers in the given data.

⇒ First Quartile = {(8 + 1)/4}th term
⇒ First Quartile= {9/4})th term
⇒ First Quartile= 2.25th term

Thus, 2.25th Term = 2nd term + (0.25)(3rd term - 2nd term)

⇒ 2.25th Term = 8+(0.25)(15-8) = 9.75

First Quartile value is 9.75

Find the first, second, and third quartiles for the data 8, 5,15, 20, 18, 30, 40, 25

Second Quartile = {(n + 1)/2}th term

⇒ Second Quartile = (9 + 1)/2}th term
⇒ Second Quartile = {10/2}th term
⇒ Second Quartile = 5th term

5th term is 20

So the second Quartile value is 20.

Third Quartile = 3(n + 1)/4th term

⇒ Third Quartile = (3(8 + 1)/4)th term
⇒ Third Quartile = (27/4)th term
⇒ Third Quartile = 6.75th term

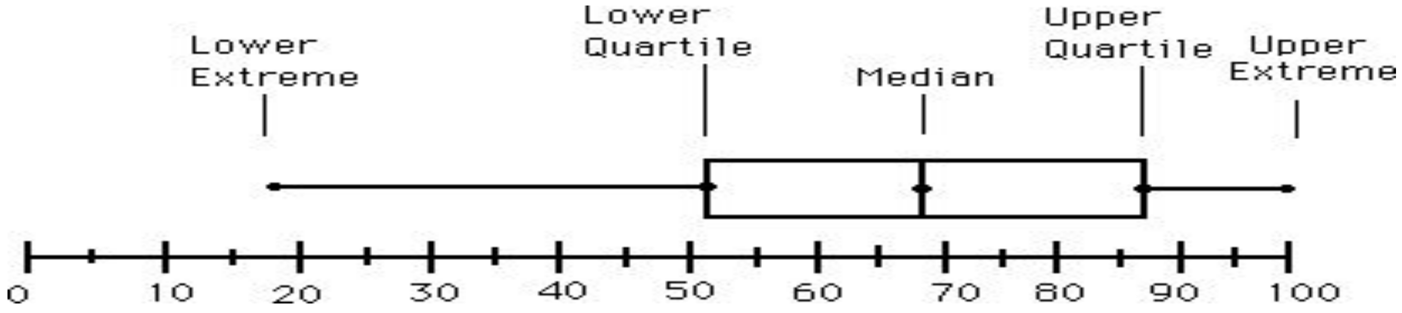
Thus, 6.75th = 6th term +(0.75)(7th -6th)

⇒ 6.75th = 25+ (0.75)(5)= 28.75

So the third Quartile value is 28.75

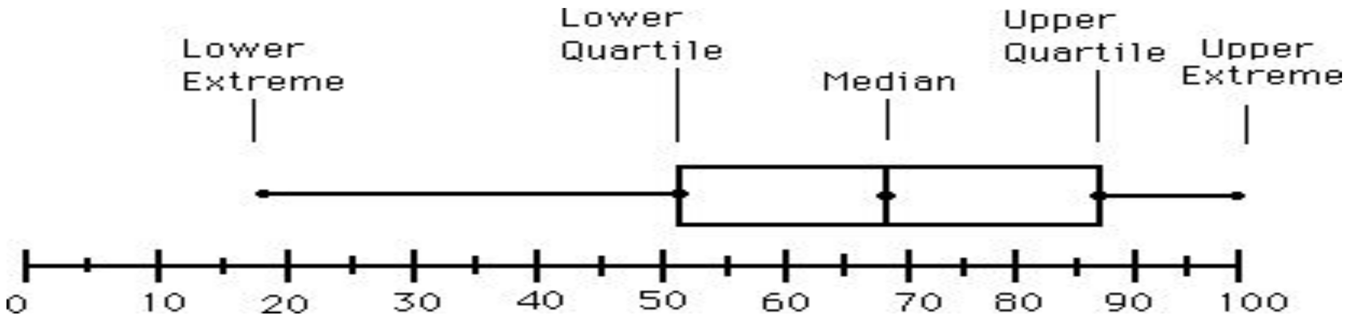
Boxplot Analysis

- A **boxplot** (also called a **box-and-whisker plot**) is a graphical representation of a dataset's five-number summary.
- It helps in visualizing the **distribution, spread, and outliers** of the data.
- **Components of a Boxplot**
  1. **Minimum (Lower Bound):** The smallest data point within 1.5 × IQR from Q1.
  2. **First Quartile (Q1):** The 25th percentile (lower quartile).
  3. **Median (Q2):** The 50th percentile (middle value).
  4. **Third Quartile (Q3):** The 75th percentile (upper quartile).
  5. **Maximum (Upper Bound):** The largest data point within 1.5 × IQR from Q3.
- **Outliers:** Data points that lie beyond 1.5 × IQR from Q1 or Q3 are plotted as individual points.



How to interpret Boxplot?

- **Box (Interquartile Range, IQR):** Represents the middle 50% of the data.
- **Whiskers:** Extend from the box to the minimum and maximum values within 1.5 × IQR range.
- **Outliers:** Any data points beyond the whiskers are considered potential outliers.
- **Median Line:** A line inside the box represents the median (Q2).
- **Skewness:**
  - If the **median** is in the center, the data is **symmetrical**.
  - If the **median** is closer to Q1, the data is **right-skewed (positive skew)**.
  - If the **median** is closer to Q3, the data is **left-skewed (negative skew)**.



Example of Boxplot?

For a dataset: **{2, 5, 7, 10, 15, 18, 22, 25, 30}**

- **Min:** 2
- **Q1:** 6
- **Median (Q2):** 15
- **Q3:** 23.5
- **Max:** 30

Example of Boxplot

For a dataset: **10, 5, 12, 8, 15, 7, 9, 11, 6, 13 . Draw Boxplot.**

Example of Boxplot

For a dataset: **10, 5, 12, 8, 15, 7, 9, 11, 6, 13 . Draw Boxplot.**

1. Order the data: 5, 6, 7, 8, 9, 10, 11, 12, 13, 15

2. Find the median: Median = (9 + 10) / 2 = 9.5

3. Find the quartiles:

- **Q1 (first quartile):** The median of the lower half of your data.
  - Lower half: 5, 6, 7, 8, 9 Q1 = 7 (the middle value)
- **Q3 (third quartile):** The median of the upper half of your data.
  - Upper half: 10, 11, 12, 13, 15 Q3 = 12 (the middle value)

4. **Calculate the Interquartile Range (IQR):**

IQR = Q3 - Q1 = 12 - 7 = 5

5. **Determine outliers (if any):**

Outliers are data points that fall significantly outside the rest of the data. A common rule is to consider values as outliers if they are:

- Less than Q1 - 1.5 * IQR → Lower bound: 7 - 1.5 * 5 = -0.5
- Greater than Q3 + 1.5 * IQR → Upper bound: 12 + 1.5 * 5 = 19.5

In our example, all data points are within this range, so there are no outliers.

```
      |---Box (IQR)-|
Min   Q1   Q2     Q3     Max
 |-----|--------|---------|----------|
5      7      9.5      12          15
```

6. Draw the boxplot:

- **Draw a number line:** Make sure it covers the range of your data.
- **Draw the box:**
  - The left edge of the box is at Q1.
  - The right edge of the box is at Q3.
  - Draw a vertical line inside the box at the median.
- **Draw the whiskers:**
  - Extend lines (whiskers) from the box to the minimum and maximum values within the outlier bounds.
  - **Lower whisker:** The smallest value in the dataset that is greater than or equal to the lower bound. In this case, it's 5.
  - **Upper whisker:** The largest value in the dataset that is less than or equal to the upper bound. In this case, it's 15.
- **Plot outliers (if any):**
  - Mark any outliers as individual points beyond the whiskers.

Example 2: Dataset: 2, 5, 7, 8, 9, 10, 11, 12, 14, 25 Draw Boxplot.

Dataset: 2, 5, 7, 8, 9, 10, 11, 12, 14, 25

1. Order the data:

2, 5, 7, 8, 9, 10, 11, 12, 14, 25

2. Find the median:

Median = (9 + 10) / 2 = 9.5

3. Find the quartiles:

- Q1 = 7
- Q3 = 12

4. Calculate the IQR:

IQR = 12 - 7 = 5

5. Determine outliers:

- Lower bound: 7 - 1.5 * 5 = -0.5
- Upper bound: 12 + 1.5 * 5 = 19.5

The data point 25 is greater than the upper bound, so it's considered an outlier.

6. Draw the boxplot:

- **Number line:** Covers the range of the data (2 to 25).
- **Box:** Left edge at Q1 (7), right edge at Q3 (12), vertical line at the median (9.5).
- **Whiskers:**
  - Lower whisker extends to the minimum value 2.
  - Upper whisker extends to the largest value *within the outlier bounds*, which is 14.
- **Outlier:** The outlier (25) is plotted as an individual point beyond the upper whisker.

Variance and Standard Deviation

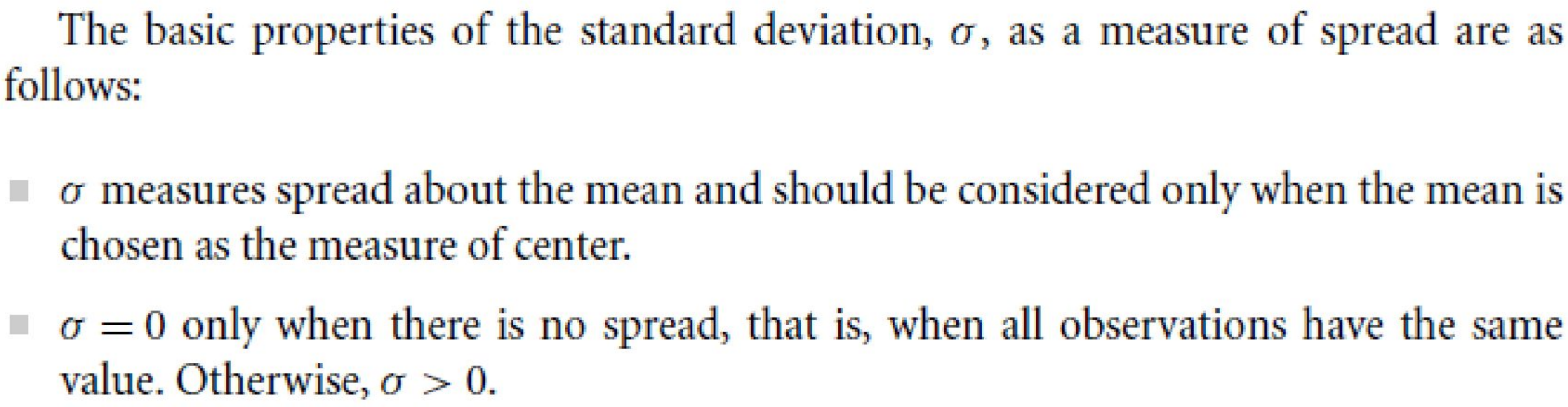
(how spread out a data distribution is)

- The **standard deviation**, σ of the observations is the square root of the variance.

- A low standard deviation means that the data observations tend to be very close to the mean,
- A high standard deviation indicates that the data are spread out over a large range of values.

- The **variance** of *N* observations, *x*1,*x*2, : : : ,*xN*, for a numeric attribute *X* is

$$\sigma^2 = \frac{1}{N}\sum^{N}(x_i - \bar{x})^2 = \left(\frac{1}{N}\sum^{N} x_i^2\right) - \bar{x}^2,$$



The basic properties of the standard deviation, σ, as a measure of spread are as follows:

- σ measures spread about the mean and should be considered only when the mean is chosen as the measure of center.
- $\sigma = 0$ only when there is no spread, that is, when all observations have the same value. Otherwise, $\sigma > 0$.

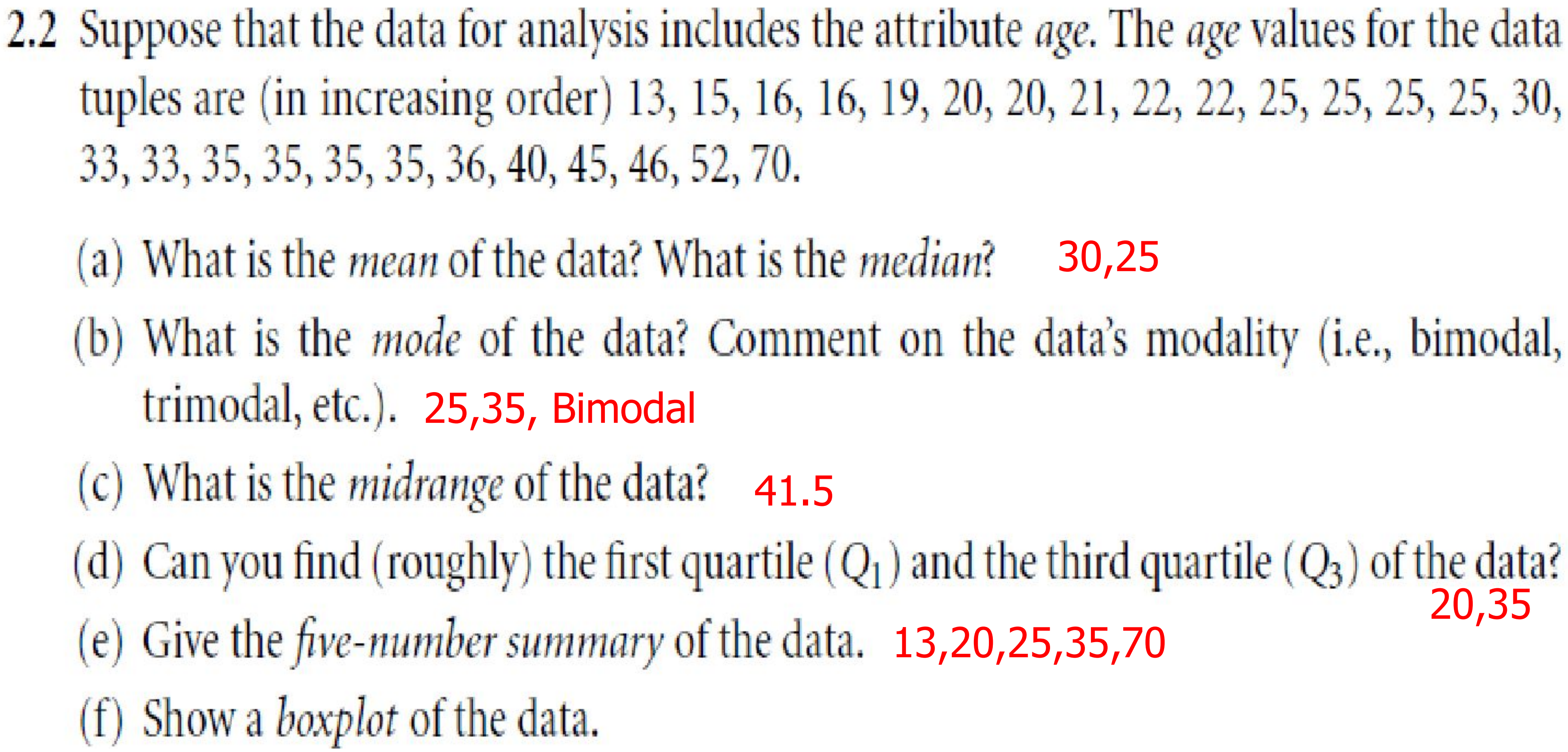
Measuring the Central Tendency

Suppose we have the following values for *salary* (in thousands of dollars), shown in increasing order: 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110.
Claculate mean,median,modes,midrange,variance and standard deviation.

Mean=58
Median=54
Modes = bimodal. The two modes are 52 and 70
Midrange=70

Variance= 379.17
Std. Deviation= 19.47

Problems



2.2 Suppose that the data for analysis includes the attribute *age*. The *age* values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70.

(a) What is the *mean* of the data? What is the *median*? 30,25

(b) What is the *mode* of the data? Comment on the data's modality (i.e., bimodal, trimodal, etc.). 25,35, Bimodal

(c) What is the *midrange* of the data? 41.5

(d) Can you find (roughly) the first quartile (Q_1) and the third quartile (Q_3) of the data? 20,35

(e) Give the *five-number summary* of the data. 13,20,25,35,70

(f) Show a *boxplot* of the data.

Suppose a hospital tested the age and body fat data for 18 randomly selected adults with the following result

age	23	23	27	27	39	41	47	49	50	52	54	54	56	57	58	58	60	61
%fat	9.5	26.5	7.8	17.8	31.4	25.9	27.4	27.2	31.2	34.6	42.5	28.8	33.4	30.2	34.1	32.9	41.2	35.7

(a) Calculate the mean, median and standard deviation of *age* and *%fat*.

(b) Draw the boxplots for *age* and *%fat*.

(c) Draw a *scatter plot* and a *q-q plot* based on these two variables.

Measuring Data Similarity and Dissimilarity

Proximity(Similarity and Dissimilarity)

Similarity

- Numerical measure of how alike two data objects are.
- Value is higher when objects are more alike
- Often falls in the range [0,1]

Dissimilarity (e.g., distance)

- Numerical measure of how different two data objects are.
- Lower when objects are more alike.
- Minimum dissimilarity is often 0.
- Upper limit varies

Data Matrix and Dissimilarity Matrix

Data matrix *n* x *p* matrix (*n* objects *p* attributes) : 2 mode matrix.

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

Data Matrix and Dissimilarity Matrix

Dissimilarity matrix : n X n Matrix

- In general, *d*(*i*, *j)* is a non-negative number that is close to 0 when objects *i* and *j* are highly similar or "near" each other, and becomes larger the more they differ.
- Note that *d(i*, *i*)= 0; i.e. the difference between an object and itself is 0. Furthermore, *d*(*i*, *j*) = *d*(*j*, *i)*

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \ldots & \ldots & 0 \end{bmatrix}$$

- Measures of similarity can often be expressed as a function of measures of dissimilarity.
- For example, for nominal data, *sim*(*i*, *j*) = 1 – *d(i*, *j*) where *sim(i*, *j*) is the similarity between objects *i* and *j*.
- Also called as **one-mode** matrix

Proximity Measure for Nominal Attributes

- A nominal attribute Can take 2 or more states,

- “How is dissimilarity computed between objects described by nominal attributes?”

Method 1: Simple matching :The dissimilarity between two objects i and j can be computed based on the ratio of mismatches:

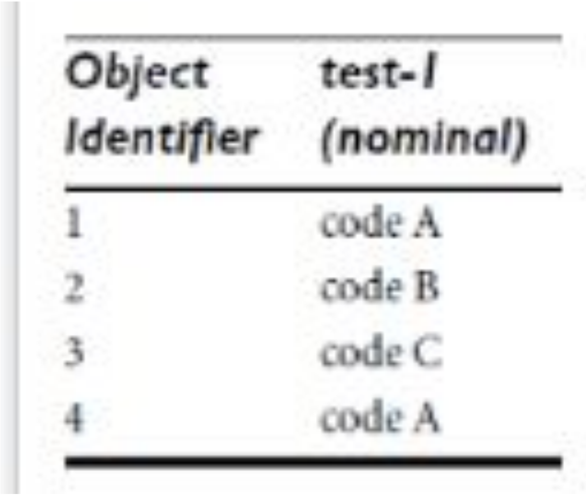
$$d(i,j)=\frac{p-m}{p}$$

Where

- p is the total number of attributes describing the objects.
- m is the number of matches (i.e., the number of attributes for which i and j are in the same state)

Proximity Measure for Nominal Attributes Example

: Dissimilarity between nominal attributes.



Object Identifier	test-1 (nominal)
1	code A
2	code B
3	code C
4	code A

Here p= 1
d(*i*, *j*) evaluates to 0 if objects *i* and *j* match, and 1 if the objects differ.

$$d(i,j)=\frac{p-m}{p}$$

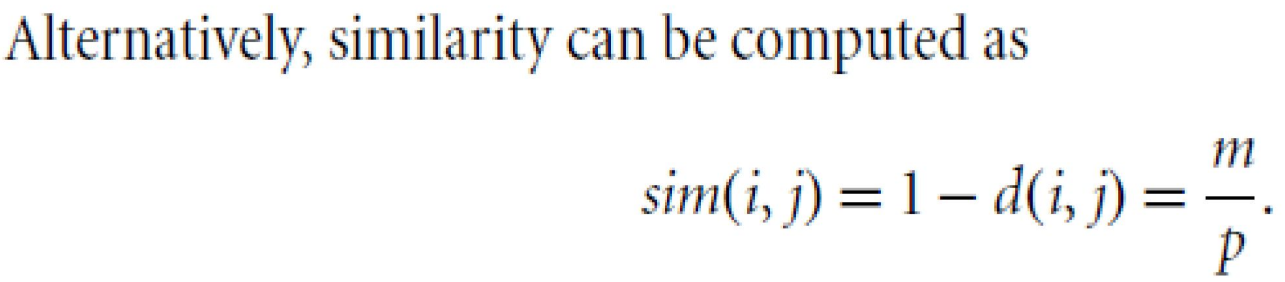
$$\begin{bmatrix} 0 & & & \\ d(2,1) & 0 & & \\ d(3,1) & d(3,2) & 0 & \\ d(4,1) & d(4,2) & d(4,3) & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

From this, we see that all objects are dissimilar except objects 1 and 4 (i.e.,d(4,1)= 0.

Proximity Measure for Nominal Attributes Example

: Dissimilarity between nominal attributes.



Alternatively, similarity can be computed as

$$sim(i, j) = 1 - d(i, j) = \frac{m}{p}.$$

Method 2: Use a large number of binary attributes

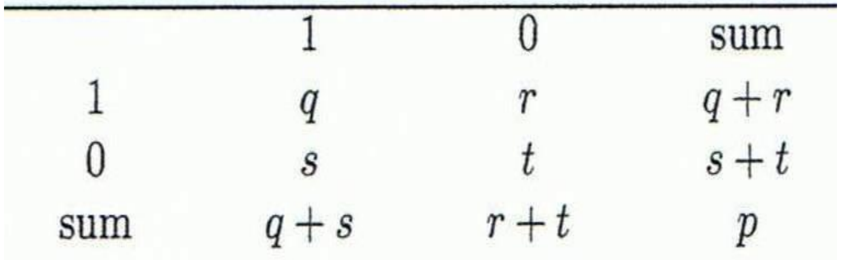
creating a new binary attribute for each of the M nominal states

how can we compute the dissimilarity between two binary attributes?

- One approach involves computing a dissimilarity matrix from the given binary data.
- If all binary attributes are thought of as having the same weight, we have the 2 x 2 contingency table as shown below

Object *j*

Object *i*



	1	0	sum
1	q	r	$q+r$
0	s	t	$s+t$
sum	$q+s$	$r+t$	p

Where *q* = the number of attributes that equal 1 for both objects *i* and *j*,

r = the number of attributes that equal 1 for object *i* but equal 0 for object *j*,

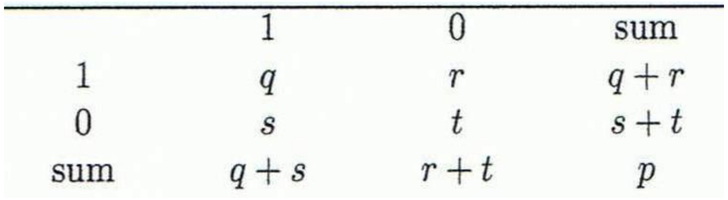
s = the number of attributes that equal 0 for object *i* but equal 1 for object *j*,

t = the number of attributes that equal 0 for both objects *i* and *j*.

p = The total number of attributes = *q* + *r* + *s* + *t* .

Proximity Measure for Binary Attributes

A contingency table for binary data



Object *j*

		1	0	sum
	1	q	r	$q+r$
Object *i*	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

Distance measure for symmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s+t}$$

Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s}$$

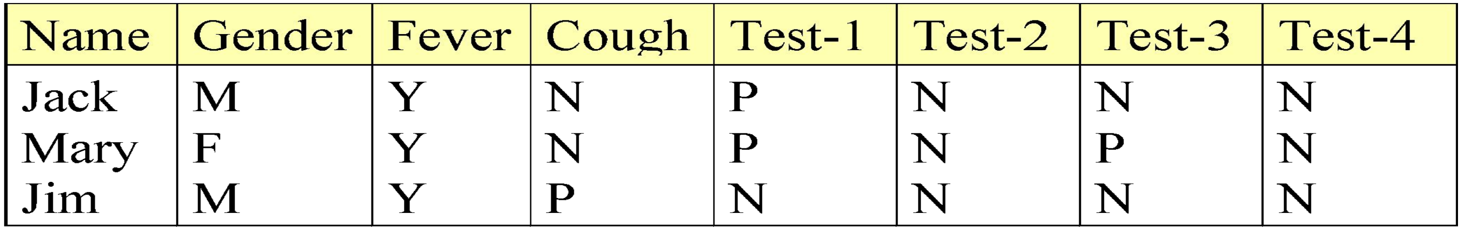
Jaccard coefficient (similarity measure for asymmetric binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$

= 1- d(i,j)

Dissimilarity between Binary Attributes

Example



Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

Dissimilarity between Binary Attributes

Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- *Gender is a symmetric attribute*
- *The remaining attributes are asymmetric binary*
- *Let the values Y and P be 1, and the value N be 0.*

Suppose distance is computed based on only asymmetric attributes

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

$$d(i, j) = \frac{r+s}{q+r+s}$$

- These measurements suggest that
  - Jim and Mary are unlikely to have a similar disease because they have the highest dissimilarity value among the three pairs.
  - Of the three patients, Jack and Mary are most likely to have a similar disease

Dissimilarity of Numeric Data:

Euclidean distance: The most popular distance measure

$$d(i, j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{ip} - x_{jp})^2}.$$

where i = (x_{i1}, x_{i2}, …, x_{ip}) and j = (x_{j1}, x_{j2}, …, x_{jp}) are two objects described by numeric attributes.

Manhattan (or city block) distance**:** named so because it is the distance in blocks between any two points in a city (such as 2 blocks down and 3 blocks over for a total of 5 blocks).

- The distance between two points measured along axes at right angles

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|.$$

Dissimilarity of Numeric Data

(distance measures that are commonly used for computing the dissimilarity of objects described by numeric attributes)

- Euclidean distance: The most popular distance measure

$$d(i, j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{ip} - x_{jp})^2}.$$

where i = (x_{i1}, x_{i2}, …, x_{ip}) and j = (x_{j1}, x_{j2}, …, x_{jp}) are two objects described by numeric attributes.

- Manhattan (or city block) distance: The distance between two points measured along axes at right angles

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|.$$

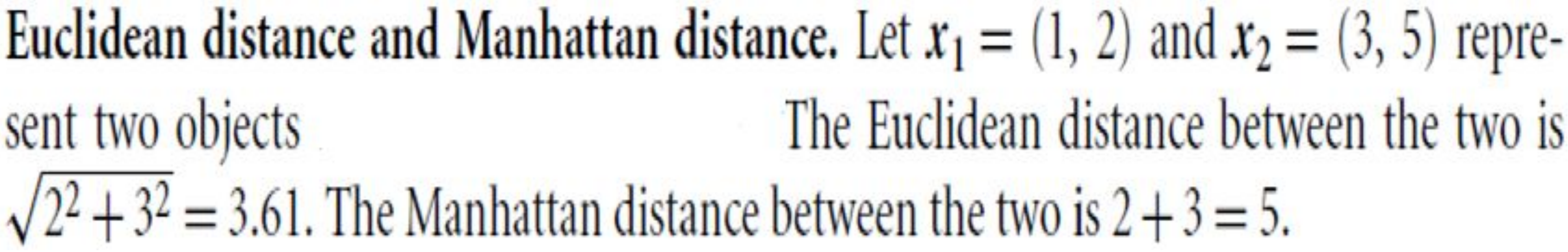
Dissimilarity of Numeric Data: Minkowski Distance

The Euclidean and the Manhattan distance satisfy the following mathematical properties:

- **Non-negativity:** $d(i, j) >= 0$: Distance is a non-negative number.
- **Identity of indiscernibles:** $d(i, j) = 0$:The distance of an object to itself is 0.
- **Symmetry:** $d(i, j) = d(j, i)$: Distance is a symmetric function.
- **Triangle inequality:** $d(i, j) <= d(i, k)+d(k,j)$: Going directly from object *i* to object *j* in space is no more than making a detour over any other object *k*.

- A measure that satisfies these conditions is known as **metric**.

Example: Euclidean and Manhattan distance



Euclidean distance and Manhattan distance. Let $x_1 = (1, 2)$ and $x_2 = (3, 5)$ represent two objects The Euclidean distance between the two is $\sqrt{2^2 + 3^2} = 3.61$. The Manhattan distance between the two is $2 + 3 = 5$.

$$d(i, j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{ip} - x_{jp})^2}.$$

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|.$$

Dissimilarity of Numeric Data: Minkowski Distance

Minkowski distance: A generalization of Euclidean and Manhattan distances

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where i = (x_{i1}, x_{i2}, …, x_{ip}) and j = (x_{j1}, x_{j2}, …, x_{jp}) are two objects described by p numeric attributes and h is a real number such that h >= 1.

- Also called as L_p norm where p refers to h.

Special Cases of Minkowski Distance

- *h = 1: Manhattan (city block, L_1 norm) distance*

$$d(i,j)=|x_{i_1}-x_{j_1}|+|x_{i_2}-x_{j_2}|+...+|x_{i_p}-x_{j_p}|$$

- *h = 2: (L_2 norm) Euclidean distance*

$$d(i,j)=\sqrt{(|x_{i_1}-x_{j_1}|^2+|x_{i_2}-x_{j_2}|^2+...+|x_{i_p}-x_{j_p}|^2)}$$

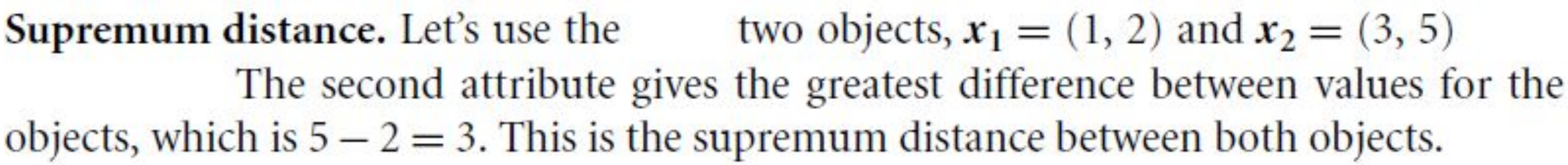
- *h → ∞. "supremum" (L_{max} norm, L_∞ norm,Chebyshev distance) distance.*

To compute it, we find the attribute *f* that gives the maximum difference in values between the two objects.

This difference is the supremum distance, defined more formally as:

$$d(i,\,j) = \lim_{h\to\infty}\left(\sum_{f=1}^{p}|x_{if}-x_{jf}|^h\right)^{\frac{1}{h}} = \max_{f}^{p}|x_{if}-x_{jf}|$$

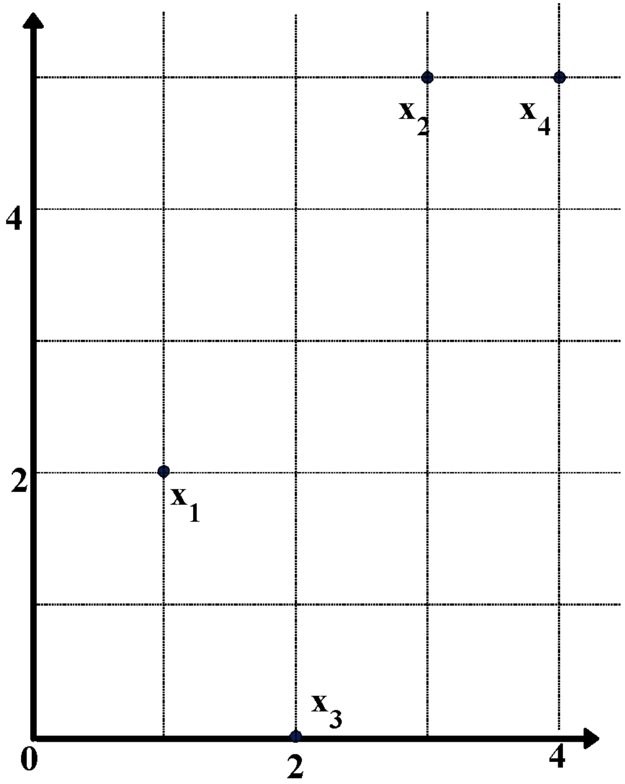
Example:



Supremum distance. Let's use the two objects, $\boldsymbol{x_1} = (1, 2)$ and $\boldsymbol{x_2} = (3, 5)$ The second attribute gives the greatest difference between values for the objects, which is $5 - 2 = 3$. This is the supremum distance between both objects.

Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



Dissimilarity Matrices

Manhattan (L_1)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean L_2)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Supremum

L_∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

Ordinal Variables

- Suppose that f is an attribute from a set of ordinal attributes describing n objects. The dissimilarity computation with respect to f involves the following 3 steps:

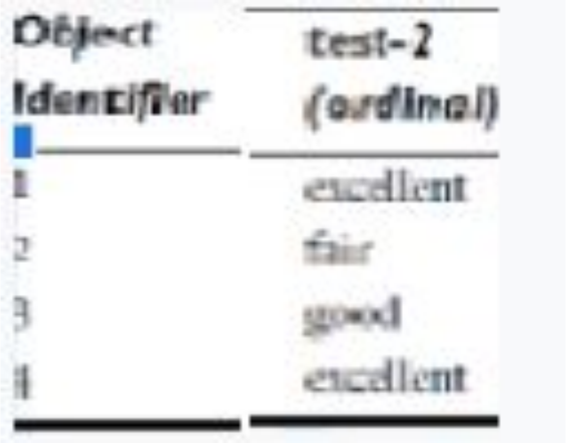
Step 1. Let the value of f for the i-th object is x_{if} , and f has M_f ordered states representing the ranking: 1, 2, …. , M_f . Replace each x_{if} by its corresponding rank, $r_{if} = \{1,2 ,.... , M_f \}$

Step 2. Map the range of each attribute onto [0.0, 1.0] so that each attribute has equal weight. Perform such data normalization by replacing the rank r_{if} of the i-th object in the f th attribute by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}.$$

Step 3. Dissimilarity can then be computed using any of the distance measures for numeric attributes, using z_{if} to represent the f value for the i-th object.

Example: Dissimilarity for Ordinal Variables



Object Identifier	test-2 (ordinal)
1	excellent
2	fair
3	good
4	excellent

Step 1: There are three states for test-2: fair, good, excellent, that is, Mf = 3.
Replace each value for test-2 by its rank, So objects 1 to 4 will be assigned ranks 3, 1, 2, and 3

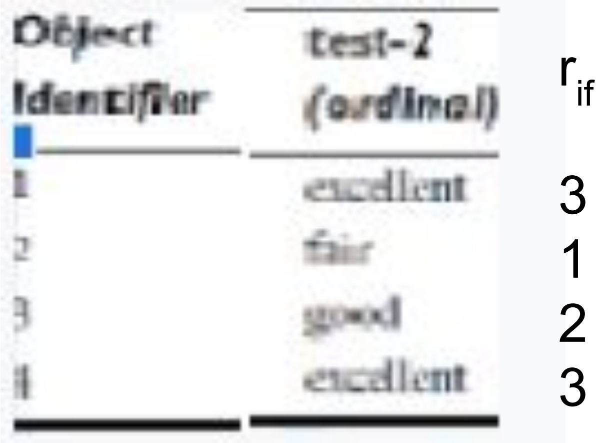
Step 2: Normalize the ranking by mapping rank 1 to 0.0, rank 2 to 0.5, and rank 3 to 1.0.
Step 3: we can use, say, the Euclidean distance , which results in the following dissimilarity matrix:

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & 1.0 & 0 & & \\ 3 & 0.5 & 0.5 & 0 & \\ 4 & 0 & 1.0 & 0.5 & 0 \end{array}$$

- Objects 1 and 2 are the most dissimilar, as are objects 2 and 4 (i.e., d(2,1)=1.0 and d(4,2)= 1.0).

- sim(i, j)= 1 - d(i,,j).

Example: Dissimilarity for Ordinal Variables



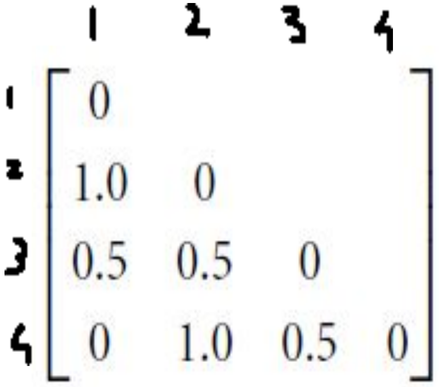
Object Identifier	test-2 (ordinal)	r_{if}	z_{if}
1	excellent	3	1
2	fair	1	0
3	good	2	0.5
4	excellent	3	1

Step 1: There are three states for test-2: fair, good, excellent, So $M_f=3$ Replace each value for test-2 by its rank, So objects 1 to 4 will be assigned ranks 3, 1, 2, and 3

Step 2: Normalize the ranking by mapping rank 1 to 0.0, rank 2 to 0.5, and rank 3 to 1.0.

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}.$$

Step 3: we can use, say, the Euclidean distance , which results in the following dissimilarity matrix:

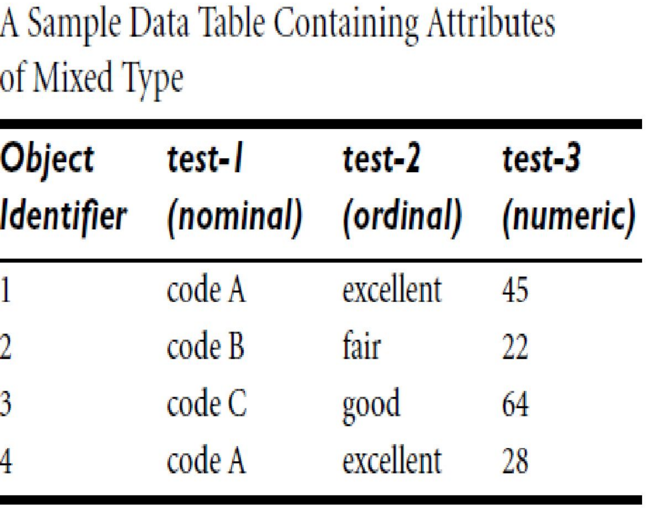


$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & 1.0 & 0 & & \\ 3 & 0.5 & 0.5 & 0 & \\ 4 & 0 & 1.0 & 0.5 & 0 \end{array}$$

- Objects 1 and 2 are the most dissimilar, Also objects 2 and 4. (i.e., d(2,1)=1.0 and d(4,2)= 1.0).

- sim(i, j)= 1 - d(i,,j).

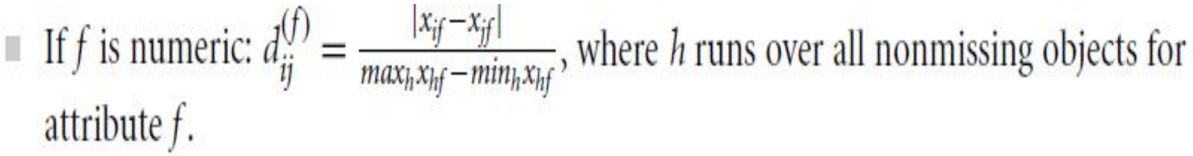
Example: Dissimilarity for numeric attributes



A Sample Data Table Containing Attributes of Mixed Type

Object Identifier	*test-1* (nominal)	*test-2* (ordinal)	*test-3* (numeric)
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

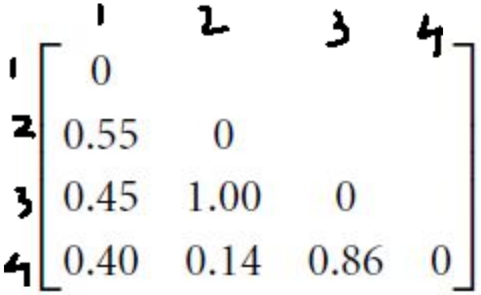
Let us compute the dissimilarity matrix for the third attribute,test-3 (which is numeric).



- If f is numeric: $d_{ij}^{(f)} = \frac{|x_{if}-x_{jf}|}{max_h x_{hf} - min_h x_{hf}}$, where h runs over all nonmissing objects for attribute f.

Using the above case of numeric attributes, let $max_h x_h = 64$ and $min_h x_h = 22$.

The difference between the two is used below to normalize the values of the dissimilarity matrix. The resulting dissimilarity matrix for test-3 is

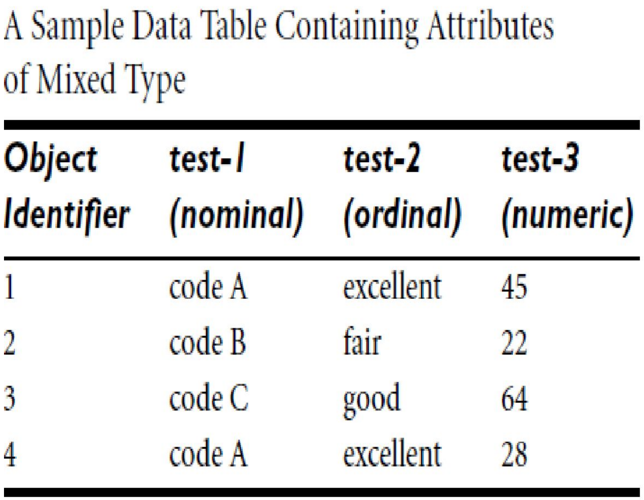


$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & 0.55 & 0 & & \\ 3 & 0.45 & 1.00 & 0 & \\ 4 & 0.40 & 0.14 & 0.86 & 0 \end{array}$$

Dissimilarity matrix for test-3 attribute

Example: Dissimilarity between attributes of mixed type

$d_{ij}^{(3)}$ $d_{ij}^{(3)}$



A Sample Data Table Containing Attributes of Mixed Type

Object Identifier	test-1 (nominal)	test-2 (ordinal)	test-3 (numeric)
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

We have computed Dissimilarity matrix for test-1 (which is nominal) and test-2 (which is ordinal),
Let us compute the dissimilarity matrix for the third attribute, test-3 (which is numeric).That is, we must compute $d_{ij}^{(3)}$

- If f is numeric: $d_{ij}^{(f)} = \frac{|x_{if} - x_{jf}|}{max_h x_{hf} - min_h x_{hf}}$, where h runs over all nonmissing objects for attribute f.

Using the above case of numeric attributes, let maxhxh = 64 and minhxh = 22.

The difference between the two is used below to normalize the values of the dissimilarity matrix. The resulting dissimilarity matrix for test-3 is

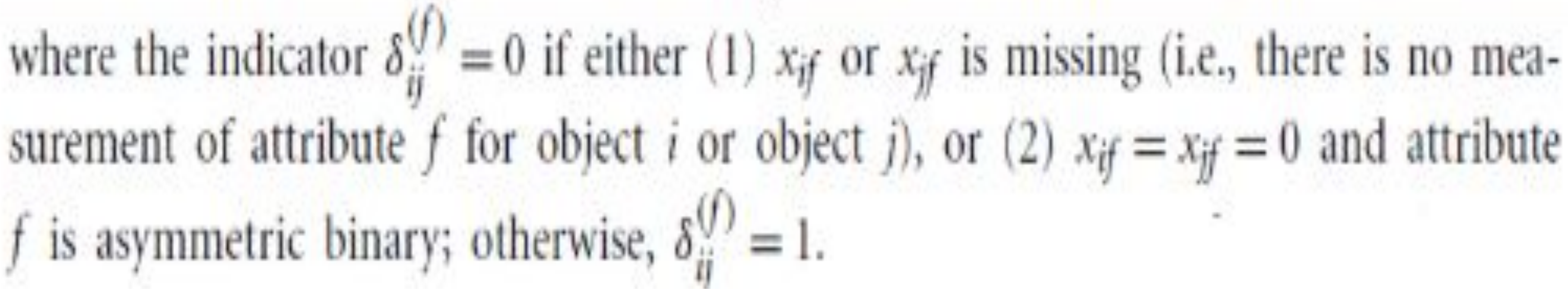
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & 0.55 & 0 & & \\ 3 & 0.45 & 1.00 & 0 & \\ 4 & 0.40 & 0.14 & 0.86 & 0 \end{array}$$

Dissimilarity matrix for test-3 attribute

Attributes of Mixed Type

Suppose that the data set contains p attributes of mixed type. The dissimilarity d(i, j) between objects i and j is defined as

$$d(i,j)=\frac{\sum_{f=1}^{p}\delta_{ij}^{(f)}d_{ij}^{(f)}}{\sum_{f=1}^{p}\delta_{ij}^{(f)}}$$

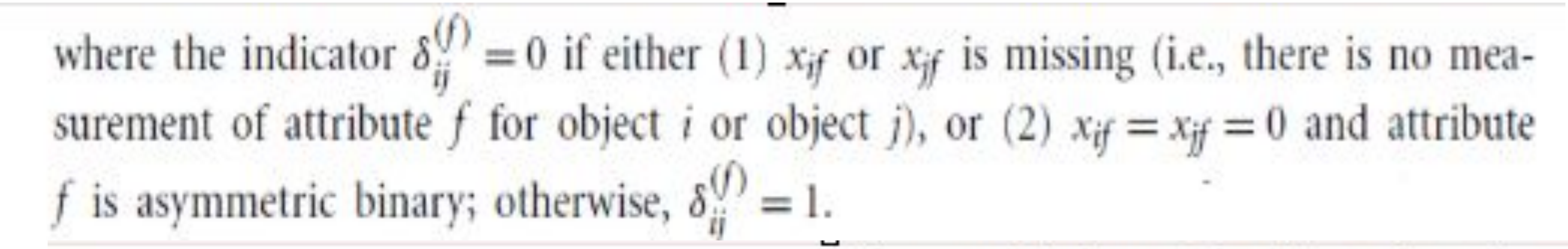


where the indicator $\delta_{ij}^{(f)} = 0$ if either (1) x_{if} or x_{jf} is missing (i.e., there is no measurement of attribute f for object i or object j), or (2) $x_{if} = x_{jf} = 0$ and attribute f is asymmetric binary; otherwise, $\delta_{ij}^{(f)} = 1$.

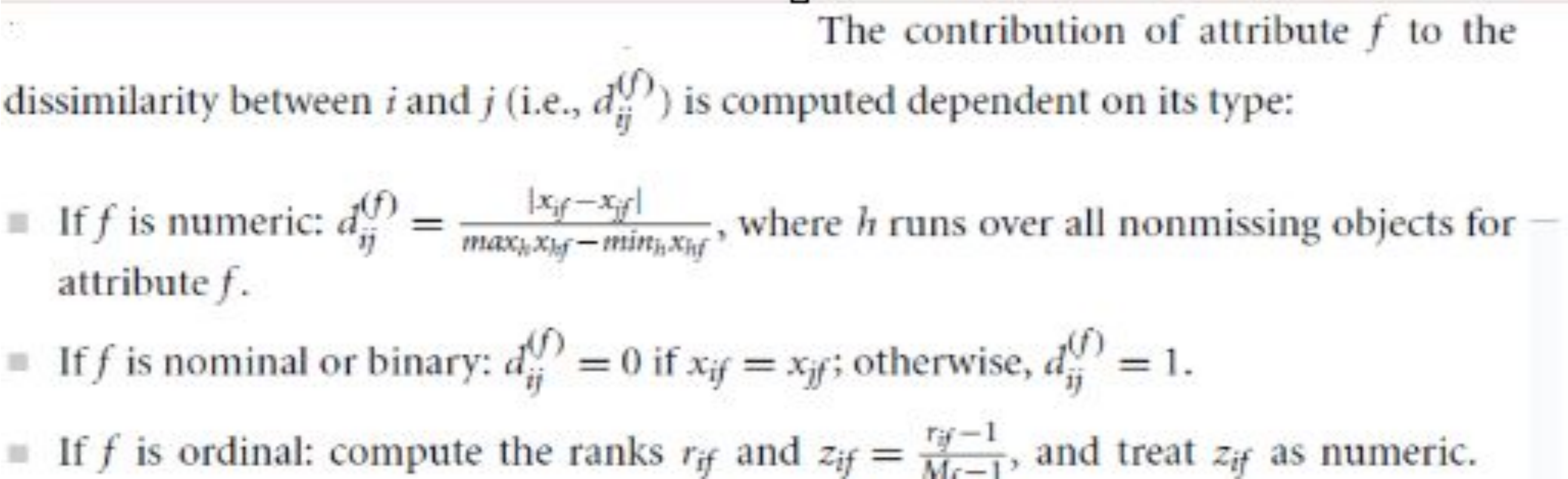
Attributes of Mixed Type

Suppose that the data set contains p attributes of mixed type. The dissimilarity d(i, j) between objects i and j is defined as

$$d(i,j)=\frac{\sum_{f=1}^{p}\delta_{ij}^{(f)}d_{ij}^{(f)}}{\sum_{f=1}^{p}\delta_{ij}^{(f)}}$$



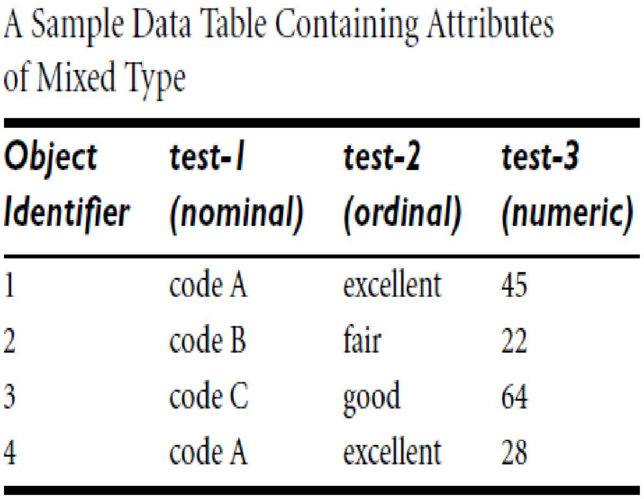
where the indicator $\delta_{ij}^{(f)}=0$ if either (1) x_{if} or x_{jf} is missing (i.e., there is no measurement of attribute f for object i or object j), or (2) $x_{if}=x_{jf}=0$ and attribute f is asymmetric binary; otherwise, $\delta_{ij}^{(f)}=1$.



The contribution of attribute f to the dissimilarity between i and j (i.e., $d_{ij}^{(f)}$) is computed dependent on its type:

- If f is numeric: $d_{ij}^{(f)}=\frac{|x_{if}-x_{jf}|}{max_h x_{hf}-min_h x_{hf}}$, where h runs over all nonmissing objects for attribute f.
- If f is nominal or binary: $d_{ij}^{(f)}=0$ if $x_{if}=x_{jf}$; otherwise, $d_{ij}^{(f)}=1$.
- If f is ordinal: compute the ranks r_{if} and $z_{if}=\frac{r_{if}-1}{M_f-1}$, and treat z_{if} as numeric.

Attributes of Mixed Type



A Sample Data Table Containing Attributes of Mixed Type

Object Identifier	test-1 (nominal)	test-2 (ordinal)	test-3 (numeric)
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

$\delta_{ij}^{(f)} = 0$ for all three attributes

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Test1 (Nominal)

$$\begin{bmatrix} 0 & & & \\ 1.0 & 0 & & \\ 0.5 & 0.5 & 0 & \\ 0 & 1.0 & 0.5 & 0 \end{bmatrix}$$

Test2 (Ordinal)

$$\begin{bmatrix} 0 & & & \\ 0.55 & 0 & & \\ 0.45 & 1.00 & 0 & \\ 0.40 & 0.14 & 0.86 & 0 \end{bmatrix}$$

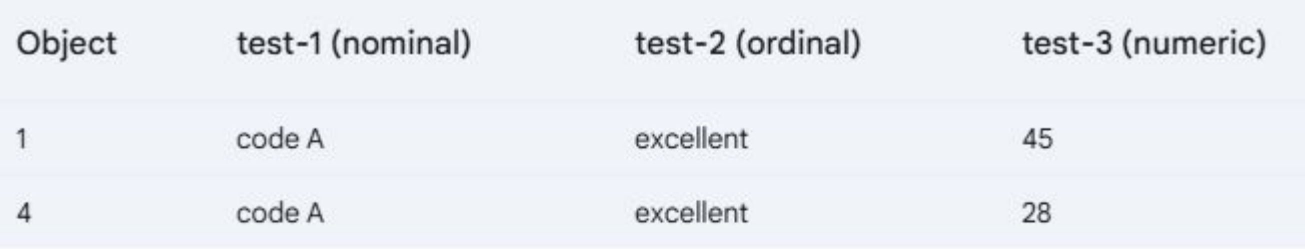
Test3 (Numeric)

Dissimilarity matrix =

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & & & \\ 2 & 0.85 & 0 & & \\ 3 & 0.65 & 0.83 & 0 & \\ 4 & 0.13 & 0.71 & 0.79 & 0 \end{array}.$$

for object 1 and object 4,

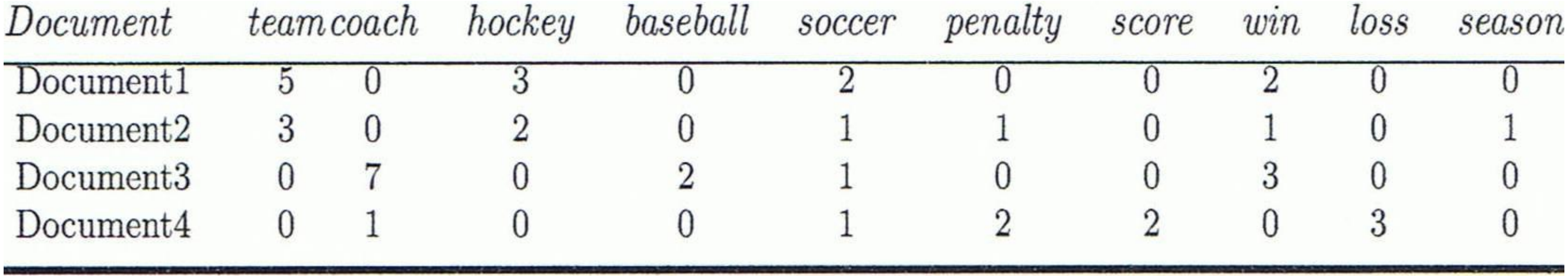
Dissimilarity = d(1, 4) = ((1 * 0) + (1 * 0) + (1 * 0.4048)) / (1 + 1 + 1)
d(1, 4) = 0.40 / 3 ≈ 0.1349



Object	test-1 (nominal)	test-2 (ordinal)	test-3 (numeric)
1	code A	excellent	45
4	code A	excellent	28

Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the frequency of a particular word (such as keywords) or phrase in the document.



Document	*team*	*coach*	*hockey*	*baseball*	*soccer*	*penalty*	*score*	*win*	*loss*	*season*
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If x and y are two vectors (e.g., term-frequency vectors), then

  cos(x, y) = (x • y) /||x|| ||y|| ,

  where dot (•)indicates vector dot product,

  ||x||: the Euclidean norm of vector x, defined as

  $\sqrt{x_1^2 + x_2^2 + \cdots + x_p^2}$. **= length of vector x**

- The Cosine measure computes the cosine of the angle betweer vectors $\boldsymbol{x}$ and $\boldsymbol{y}$.

- A cosine value of 0 means that the two vectors are at 90 degrees to each other (orthogonal) and have no match.

- The closer the cosine value to 1, the smaller the angle and the greater the match between vectors.

Example: Cosine Similarity

cos(x, y) = = (x • y) / ||x|| ||y|| ,

where • indicates vector dot product, ||d|: the length of vector d

Ex: Find the **similarity** between documents 1 and 2.

x = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)
y = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

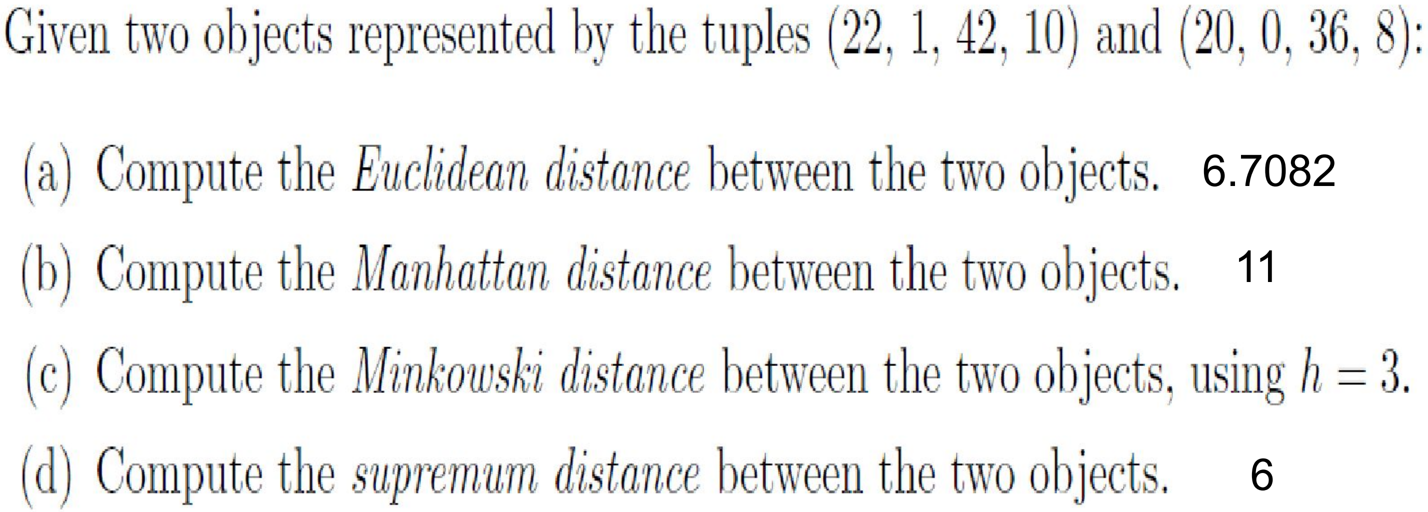
x•y = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25

||x||= $(5*5+0*0+3*3+0*0+2*2+0*0+0*0+2*2+0*0+0*0)^{0.5}=(42)^{0.5}$ = 6.481

||y||= $(3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5}=(17)^{0.5}$ = 4.12

cos(x, y) = 0.94 -----Quite similar

Problem 1



Given two objects represented by the tuples (22, 1, 42, 10) and (20, 0, 36, 8):

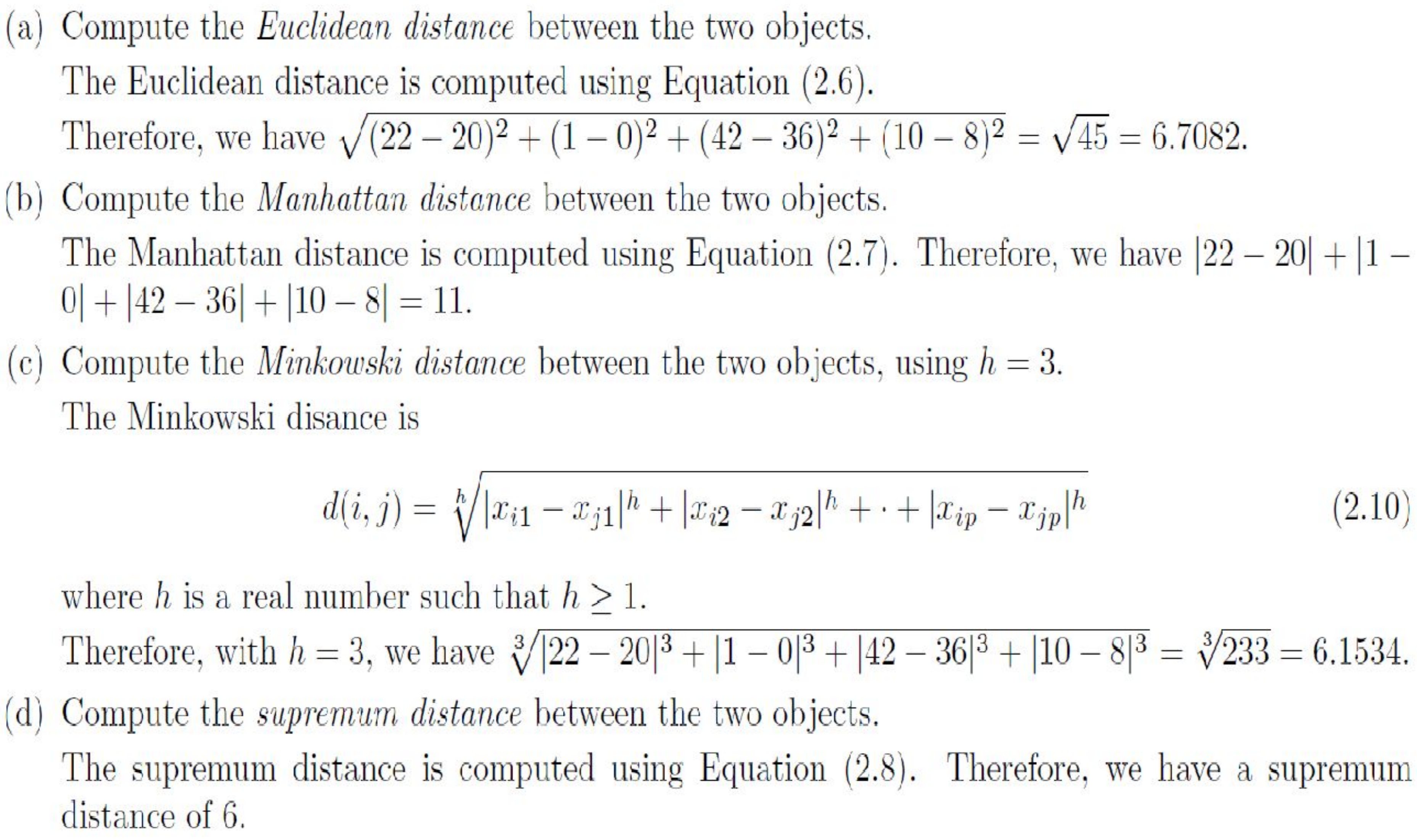
(a) Compute the *Euclidean distance* between the two objects. 6.7082

(b) Compute the *Manhattan distance* between the two objects. 11

(c) Compute the *Minkowski distance* between the two objects, using $h = 3$. 6.1534

(d) Compute the *supremum distance* between the two objects. 6

Solution-Problem 1



(a) Compute the *Euclidean distance* between the two objects.

The Euclidean distance is computed using Equation (2.6).

Therefore, we have $\sqrt{(22-20)^2+(1-0)^2+(42-36)^2+(10-8)^2} = \sqrt{45} = 6.7082$.

(b) Compute the *Manhattan distance* between the two objects.

The Manhattan distance is computed using Equation (2.7). Therefore, we have $|22-20|+|1-0|+|42-36|+|10-8| = 11$.

(c) Compute the *Minkowski distance* between the two objects, using $h = 3$.

The Minkowski disance is

$$d(i,j) = \sqrt[h]{|x_{i1}-x_{j1}|^h + |x_{i2}-x_{j2}|^h + \cdot + |x_{ip}-x_{jp}|^h} \qquad (2.10)$$

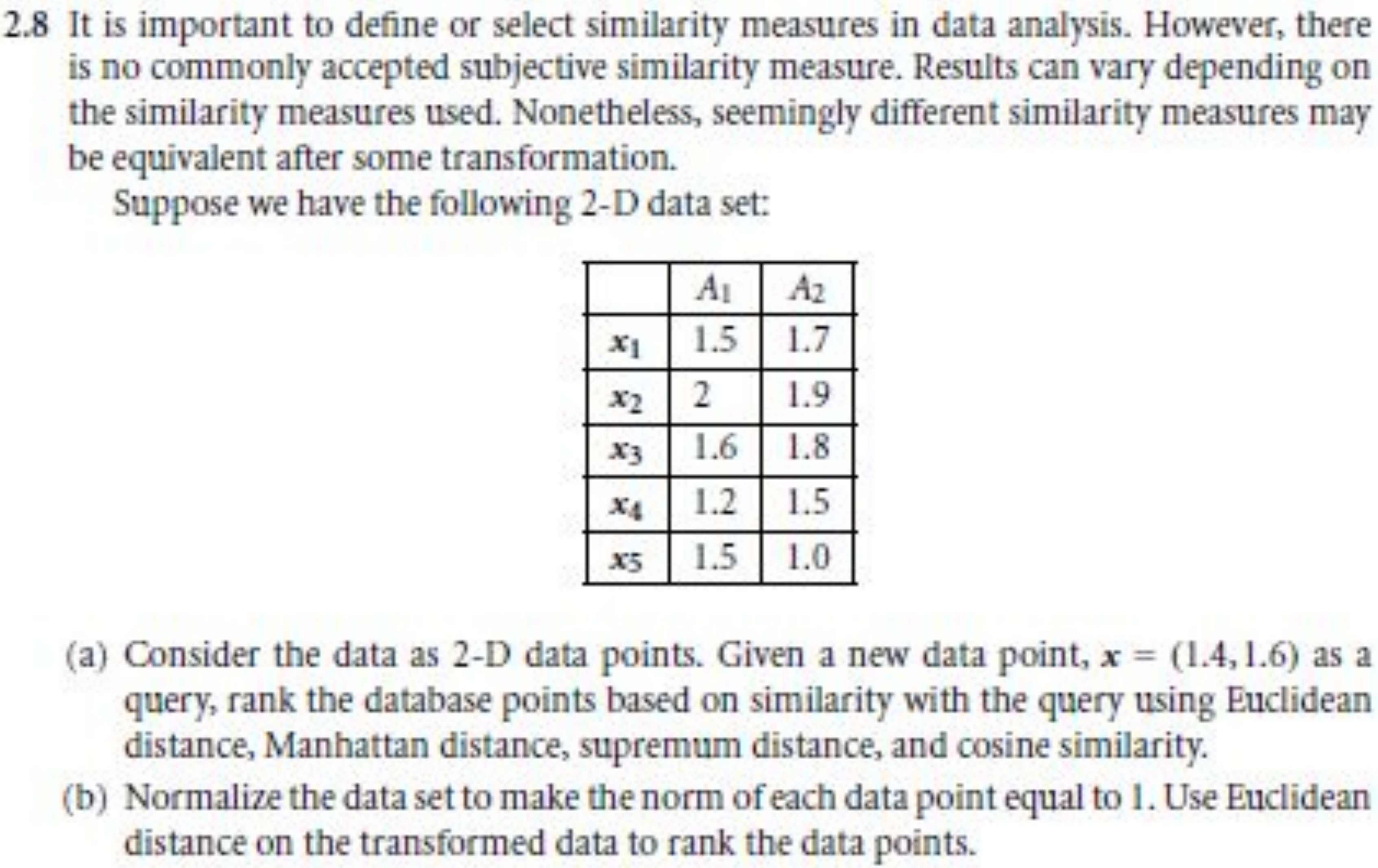
where h is a real number such that $h \geq 1$.

Therefore, with $h = 3$, we have $\sqrt[3]{|22-20|^3+|1-0|^3+|42-36|^3+|10-8|^3} = \sqrt[3]{233} = 6.1534$.

(d) Compute the *supremum distance* between the two objects.

The supremum distance is computed using Equation (2.8). Therefore, we have a supremum distance of 6.

Problem 2



2.8 It is important to define or select similarity measures in data analysis. However, there is no commonly accepted subjective similarity measure. Results can vary depending on the similarity measures used. Nonetheless, seemingly different similarity measures may be equivalent after some transformation.

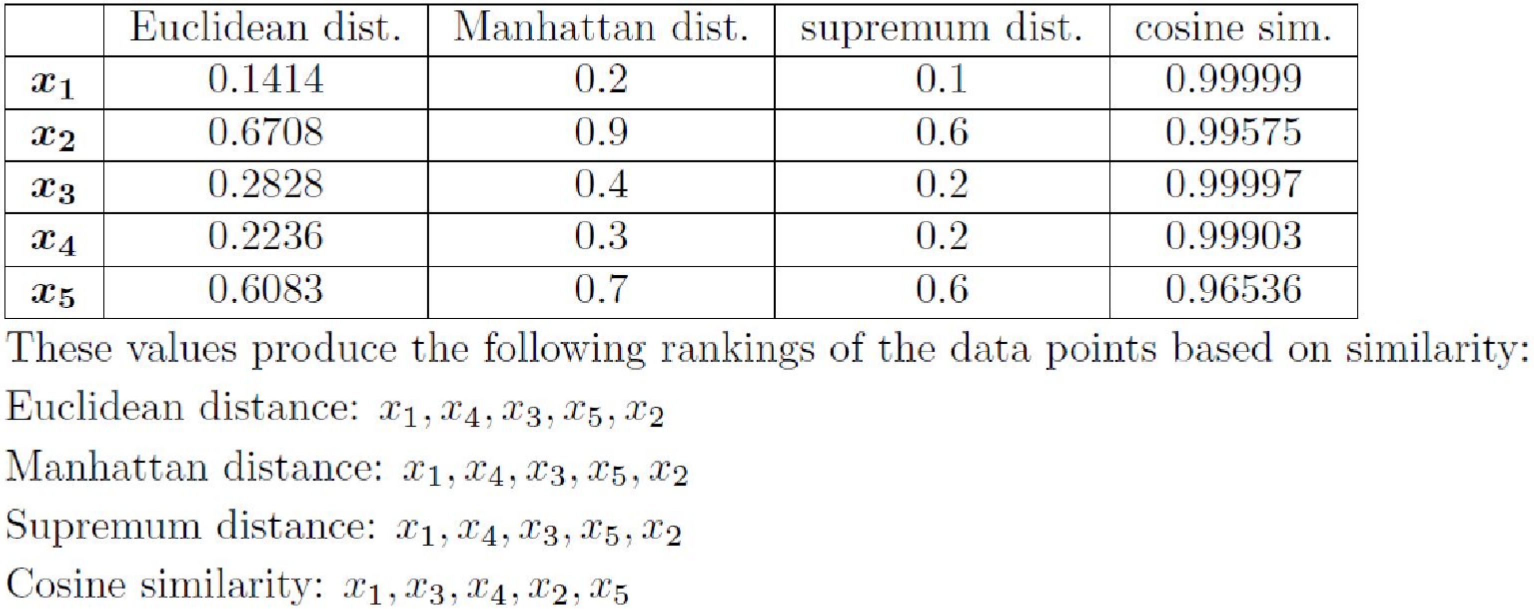
Suppose we have the following 2-D data set:

	A_1	A_2
x_1	1.5	1.7
x_2	2	1.9
x_3	1.6	1.8
x_4	1.2	1.5
x_5	1.5	1.0

(a) Consider the data as 2-D data points. Given a new data point, $\boldsymbol{x} = (1.4, 1.6)$ as a query, rank the database points based on similarity with the query using Euclidean distance, Manhattan distance, supremum distance, and cosine similarity.

(b) Normalize the data set to make the norm of each data point equal to 1. Use Euclidean distance on the transformed data to rank the data points.

Solution –Problem2(a)



	Euclidean dist.	Manhattan dist.	supremum dist.	cosine sim.
$\boldsymbol{x}_1$	0.1414	0.2	0.1	0.99999
$\boldsymbol{x}_2$	0.6708	0.9	0.6	0.99575
$\boldsymbol{x}_3$	0.2828	0.4	0.2	0.99997
$\boldsymbol{x}_4$	0.2236	0.3	0.2	0.99903
$\boldsymbol{x}_5$	0.6083	0.7	0.6	0.96536

These values produce the following rankings of the data points based on similarity:

Euclidean distance: x_1, x_4, x_3, x_5, x_2

Manhattan distance: x_1, x_4, x_3, x_5, x_2

Supremum distance: x_1, x_4, x_3, x_5, x_2

Cosine similarity: x_1, x_3, x_4, x_2, x_5

Solution-Problem 2(b)

(b) The normalized query is (0.65850, 0.75258). The normalized data set is given by the following table

	A_1	A_2
x_1	0.66162	0.74984
x_2	0.72500	0.68875
x_3	0.66436	0.74741
x_4	0.62470	0.78087
x_5	0.83205	0.55470

Recomputing the Euclidean distances as before yields

	Euclidean dist.
x_1	0.00415
x_2	0.09217
x_3	0.00781
x_4	0.04409
x_5	0.26320

which results in the final ranking of the transformed data points: x_1, x_3, x_4, x_2, x_5