Assignment - 1

Q1

Proof by Induction

$\forall n \geq 1$

$$1+2+3+ \ldots + n = \frac{n(n+1)}{2}$$

Best case $(n=1)$

$LHS = 1$

$RHS = \frac{1(1+1)}{2}$

$= 1$

$LHS = RHS$

Formula holds for $n=1$

Assume for some $k \geq 1$

Induction Hypothesis

For k

$$1+2+3+ \ldots + k = \frac{k(k+1)}{2}$$

Inductive Hypothesis

For $k+1$

$$1+2+3+ \ldots + k + k+1 = \frac{k(k+1)}{2} + (k+1)$$

Factor $(k+1)$

$$1+2+3+ \ldots + k + k+1 = (k+1)\left(\frac{k}{2}+1\right)$$

$$(k+1)\left(\frac{k+2}{2}\right) = \frac{(k+1)(k+2)}{2}$$

This is the formula with $n = k+1$

Hence mathematical induction holds true for all $n \geq 1$.

Q2

a) $A = \{1, 2, 3, 4\}$

$B = \{2, 4, 1, 3\}$

Equal sets

↳ Every element of X is in Y & every element of Y is in X.

$X = Y \Leftrightarrow (x \in X \Rightarrow x \in Y)$ & $(y \in Y \Rightarrow y \in X)$

Order & repetition doesn't matter

$\{1, 2\} = \{2, 1\} = \{1, 1, 2\}$

Hence $A = B$

b) subset of set $A = \{1, 3, 5, 7\}$

Every element of S is in A

n elements $\Rightarrow 2^n$ subsets

including empty set ϕ & set itself.

$n = 4$

$2^4 = 16$ subsets

$\{\phi\}, \{1\}, \{3\}, \{5\}, \{7\}, \{1, 3\}, \{3, 5\}, \{1, 5\}, \{1, 7\}, \{3, 7\}$,
$\{5, 7\}, \{1, 3, 5\}, \{1, 3, 7\}, \{1, 5, 7\}, \{3, 5, 7\}$
$\{1, 3, 5, 7\}$

Q3

c) Write in set builder form

$\therefore \{x \mid \text{property of } x\}$

$A = \{ x \in Z \mid x \geq 1 \}$

d) $A = \{1, 3, 5, 7, 9, 11\}$

$B = \{1, 2, 3, 13\}$

$A - B = \{5, 7, 9, 11\}$

$B - A = \{2, 13\}$

e) $A \cup (B \cup C)$ where $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$C = \{1, 5, 7\}$

Associative

$A \cup (B \cup C) = (A \cup B) \cup C$

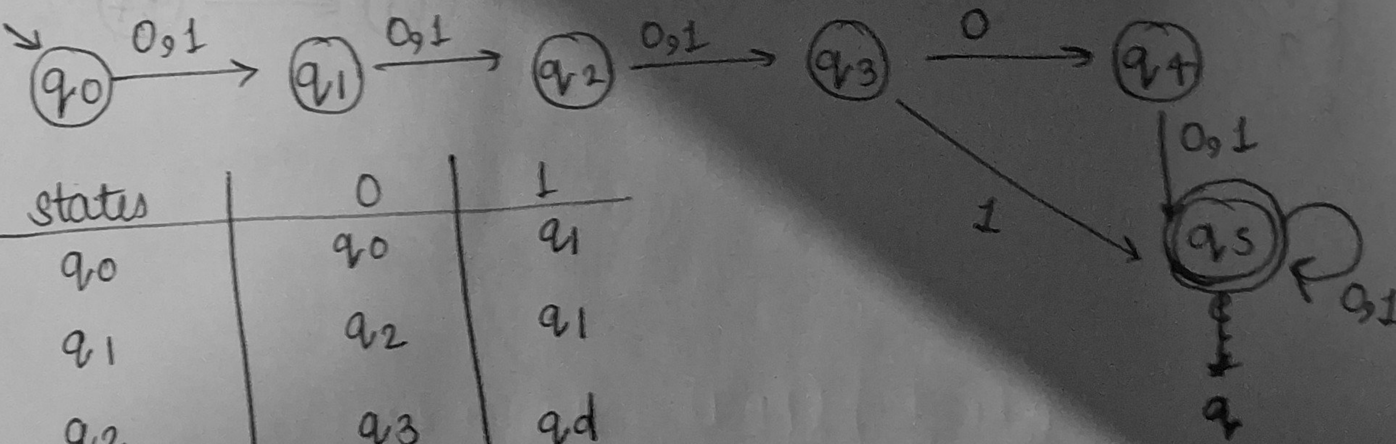
$(B \cup C) = \{2, 4, 6, 1, 5, 7\}$

$(A \cup B) = \{1, 2, 3, 4, 5, 6\}$

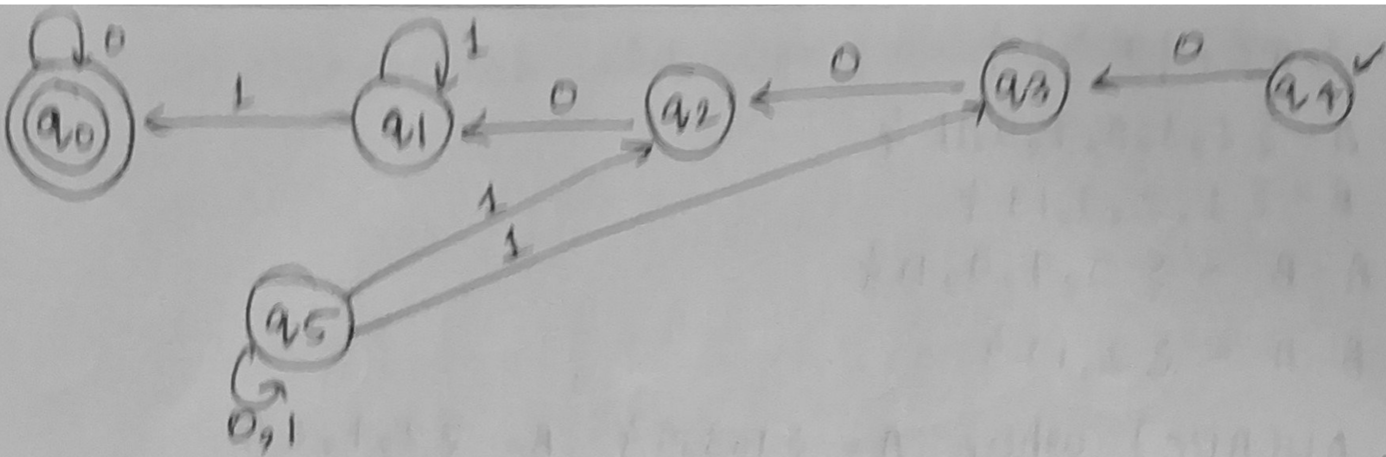
$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

Q3 $L = \{ w \mid w \in \{0,1\}^* \}$ & 4th symbol from the beginning of w is 0

i) DFA with 6 states, single final state



states	0	1
q0	q0	q1
q1	q2	q1
q2	q3	qd
q3	q4	qd
q4	q4	q4
q5	q5	q5

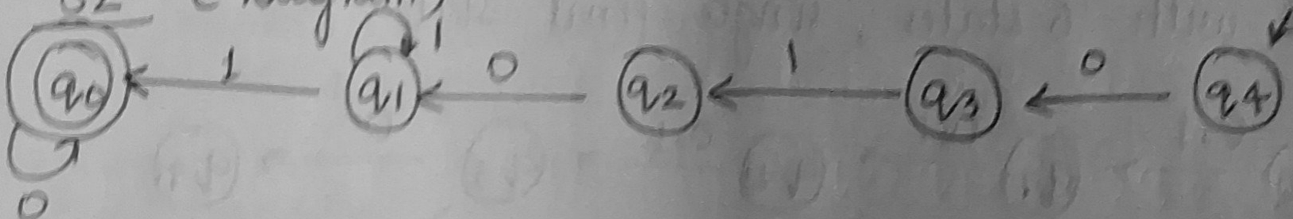


3) Yes it is a valid finite automation because it has:-
- finite number of states
- initial state q1
- final state q0

It is NFA because from q5 there are 3 possible transitions on 1. In DFA we have at most one transaction from each symbol.

4) S1 - Remove unreachable states → q5 (no incoming edge, only self loop

S2 - (Diagram)



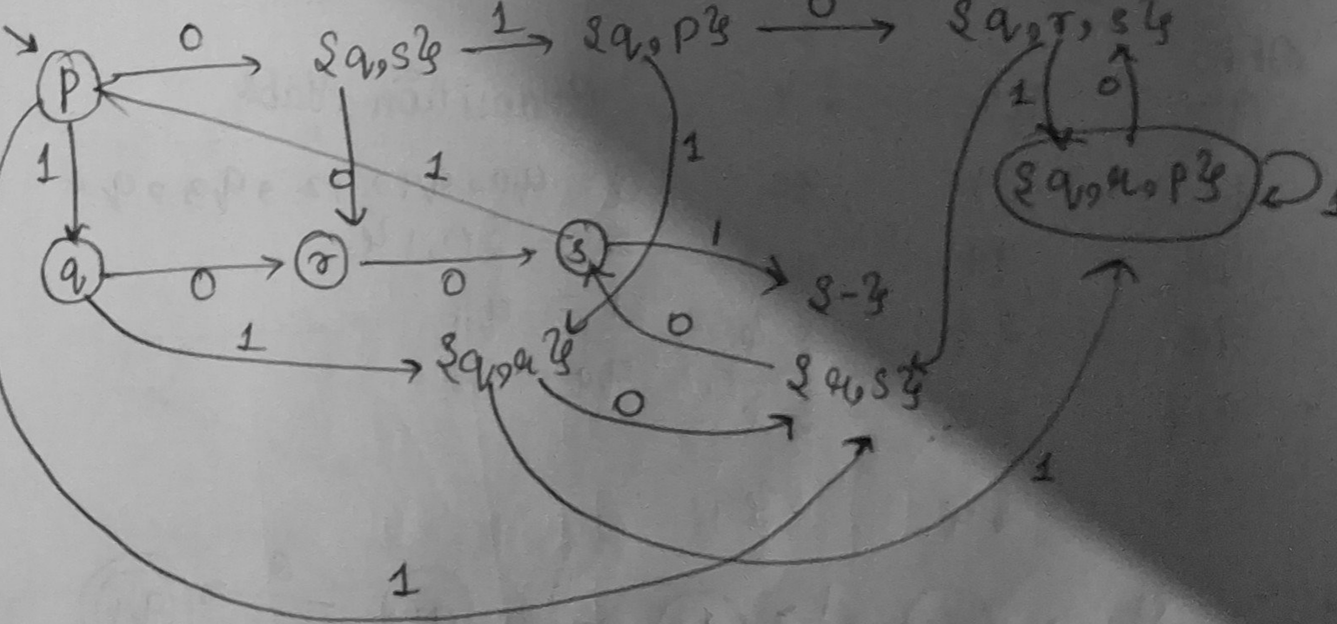
Q10

S	0	1
P	{q, s}	{q}
q	{r}	{q, r}
r	{s}	{p}
s	—	{p}

⇐ given, NFA

DFA transaction table

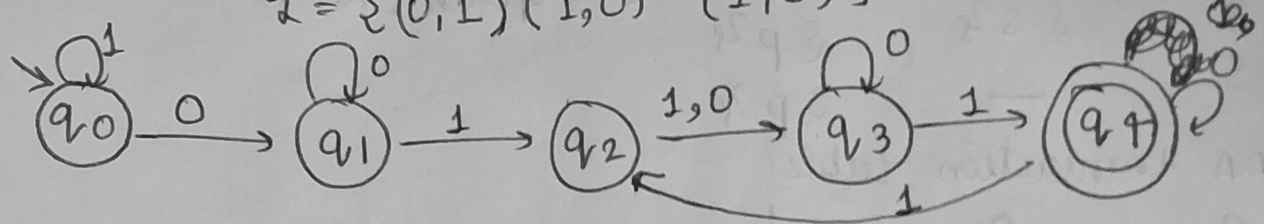
S	0	1
P	{q, s}	{q}
q	{r}	{q, r}
{q, s}	{r}	{q, p}
r	{s}	{p}
{q, r}	{r, s}	{q, r, p}
{q, p}	{r, q, s}	{q, r}
s	{-}	{p}
{r, s}	{s}	{p}
{q, r, p}	{r, s, q}	{q, r, p}
{r, q, s}	{s, r}	{p, q, r}
{-}	{-}	{ }



Q4

NFA

$L = \{(0,1)(1,0)^*(1,0)\}$



converting NFA into DFA

Transition Table :-

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_3
q_3	q_3	q_4
q_4	q_4	q_2

DFA	0	1
q_0	q_1	$\{\}$
q_1	$\{\}$	q_2
q_2	q_3	q_3
q_3	q_4	$\{\}$
q_4	$\{\}$	$\{\}$
$\{-\}$	$\{\}$	$\{\}$

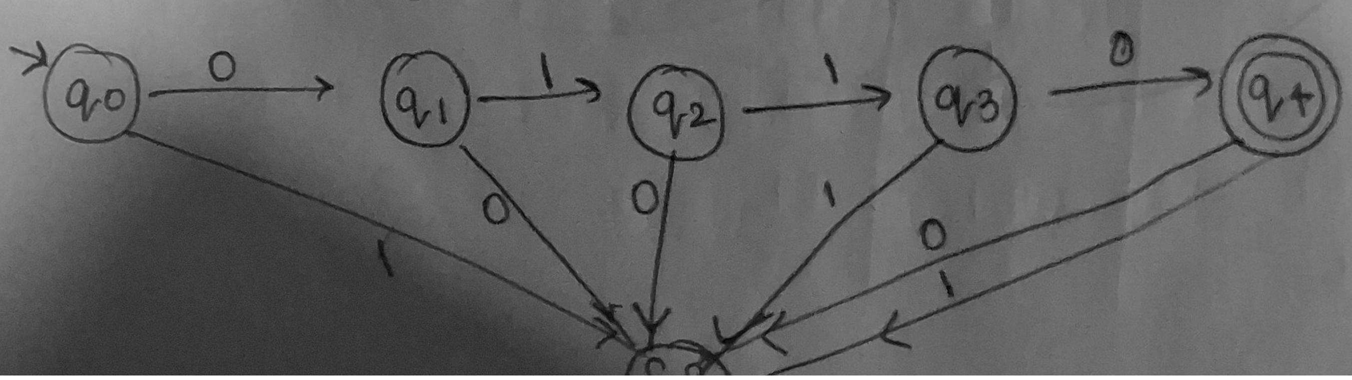
→ Transition Table

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{0, 1\}$

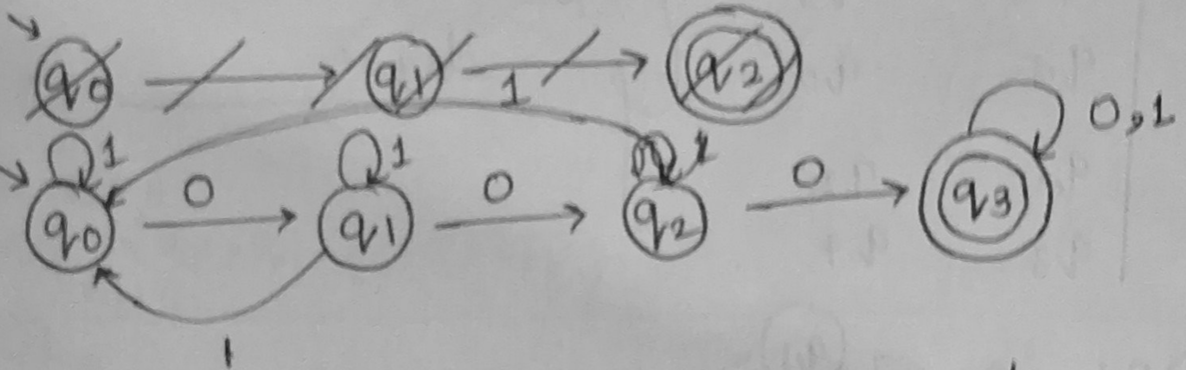
$F = q_4$

$q_0 = q_0$

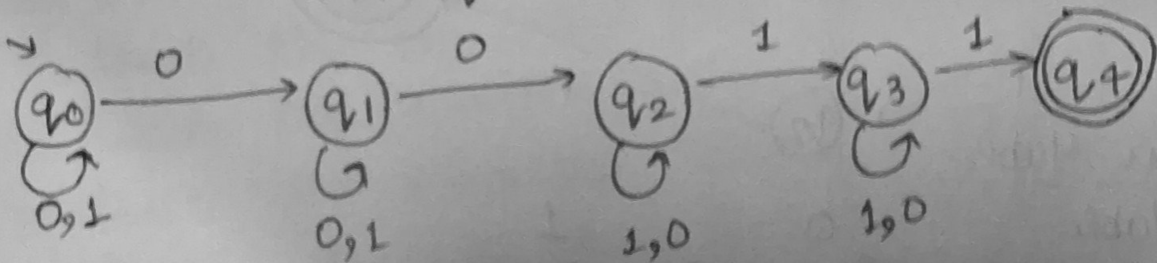


Q5 DFA for language $\{0,1\}^*$ such that it contains "000" substring.

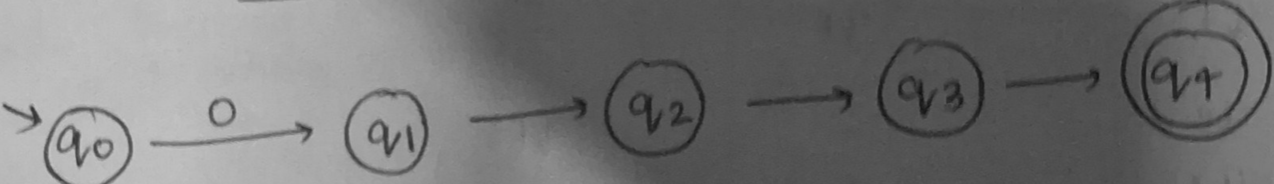
$L = \{000\}$

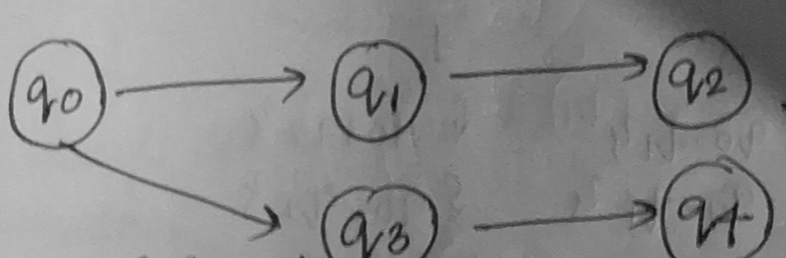


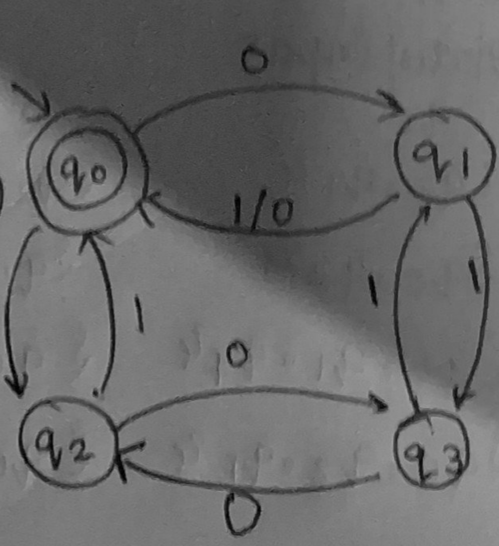
Q6 $\Sigma = \{0,1\}$ and is with even no of zeros & even number of ones



DFA







Transition Table :-

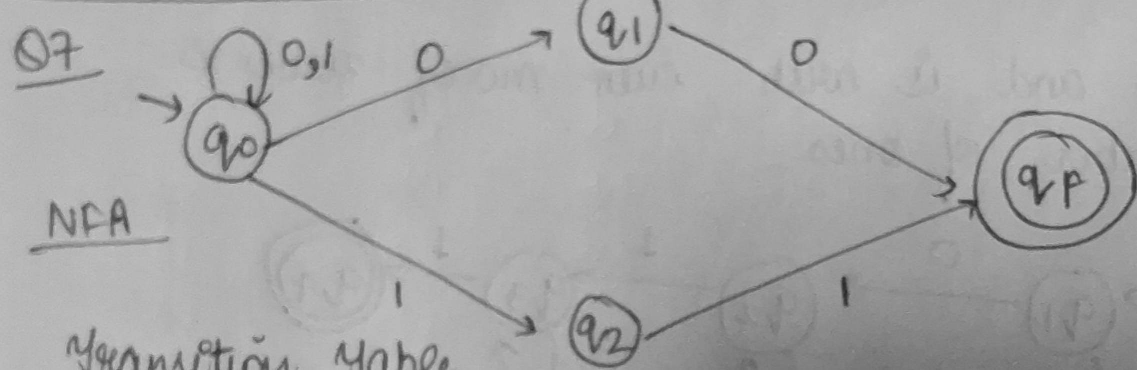
	0	1
q0	q1	q2
q1	q0	q3
q2	q3	q0
q3	q2	q1

Require DFA divisible by 5 → there will be 5 states

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

see this then draw

Q7



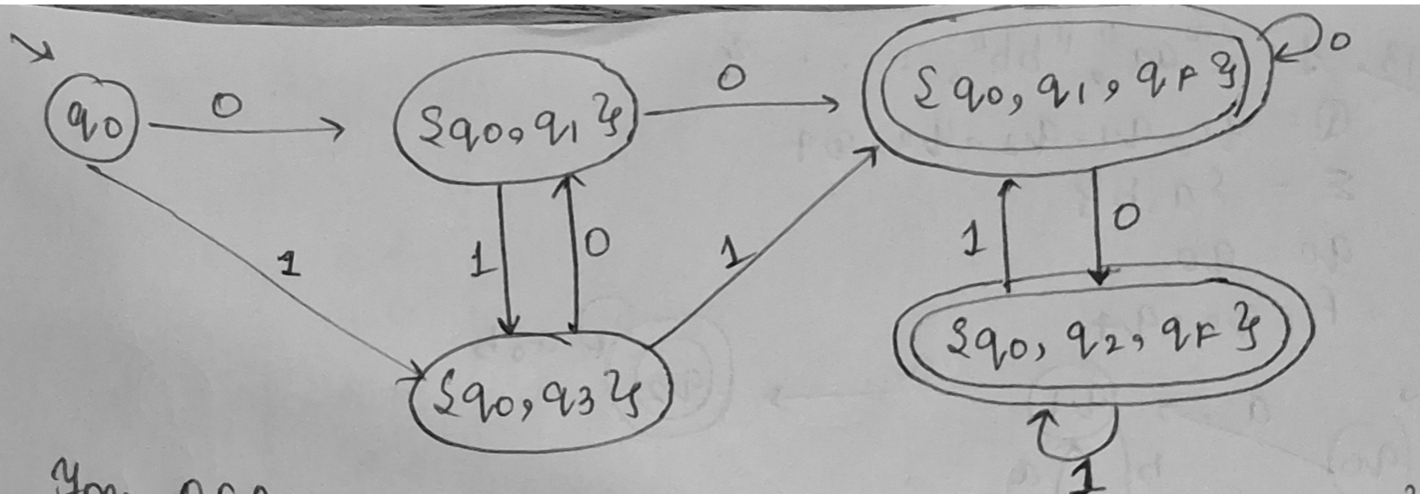
NFA

Transition Table

States/Table	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	q_F	—
q_2	—	q_F
q_F	q_F	q_F

DFA

Status/input	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$
$\{q_0, q_1, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$
$\{q_0, q_2, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$



For DFA

$Q = q_0, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$F = \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$

Q8 S1 Remove the unreachable states

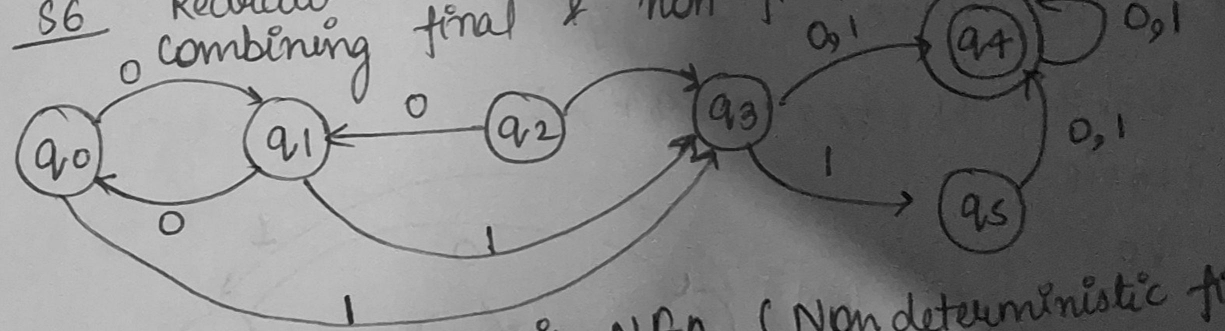
S2 Create the transition table of given DFA

S3 Create the transition table for non-final states

S4 Create the transition table for final states

S5 Remove duplicate rows from both the tables

S6 Redraw DFA with transition table formed after combining final & non-final states table



The above diagram is NFA (Non deterministic finite automata)

NFA states | input	0	1
→q0	q1	q3
q1	q0	q3
q2	q1	q3
q3	q4	{q4, q5}
(q4)	q4	q4
q5	q4	q4

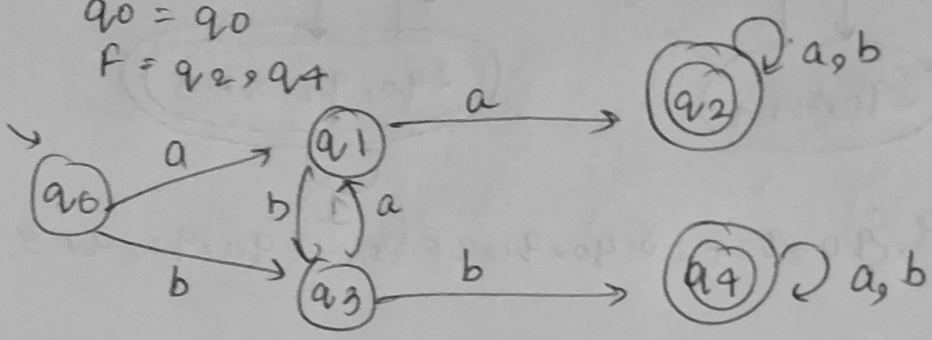
Q13. $L = \{$"aa", "bb", . . . $\}$

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$F = q_2, q_4$



Transition Table

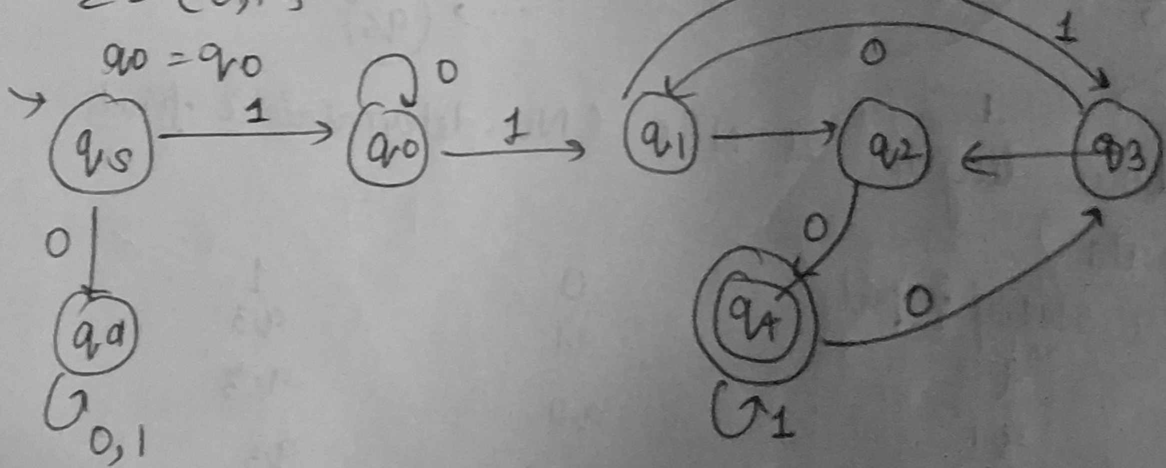
δ	a	b
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_2
q_3	q_1	q_4
q_4	q_4	q_4

Q15 $L = \{1010, 10100, - - - \}$

$Q = q_s, q_0, q_1, q_2, q_3, q_4$ ~~qd~~

$\Sigma = \{0, 1\}$

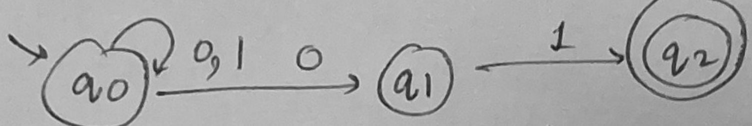
$q_0 = q_0$



Transition table

S	0	1
q_s	q_d	q_0
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4
q_d	q_d	q_d

Q15 NFA

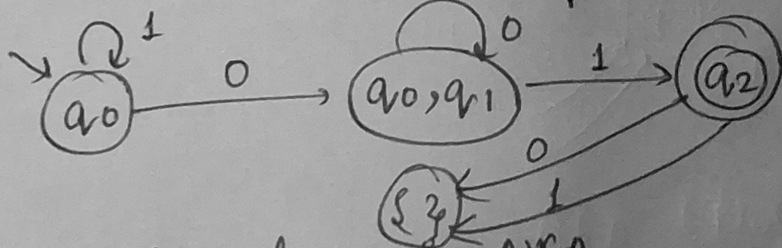


NFA

S	0	1
q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	—	$\{q_2\}$
q_2	—	—

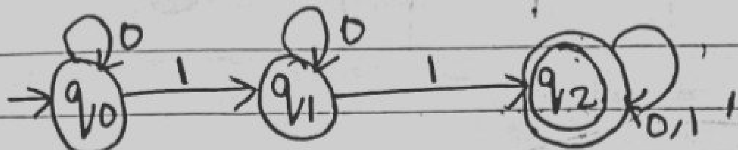
DFA

S	0	1
q_0	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_2\}$
q_2	$\{-\}$	$\{-\}$



DFA for given NFA

Ques 18 = Given,



Checking for 101101

for 1 :- $q_0 \xrightarrow{1} q_1$

for 0 : $q_1 \xrightarrow{0} q_1$

for 1 : $q_1 \xrightarrow{1} q_2$

for 1 : $q_2 \xrightarrow{1} q_2$

for 0 : $q_2 \xrightarrow{0} q_2$

for 1 : $q_2 \xrightarrow{1} q_2$

Hence, the given automata is acceptable for given string.

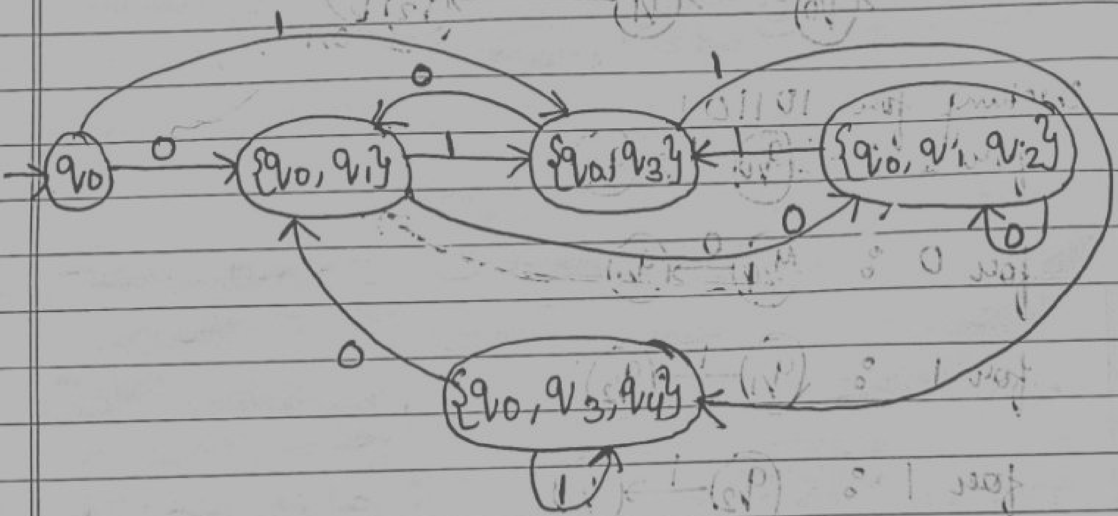
Ques 20- Given,

NFA

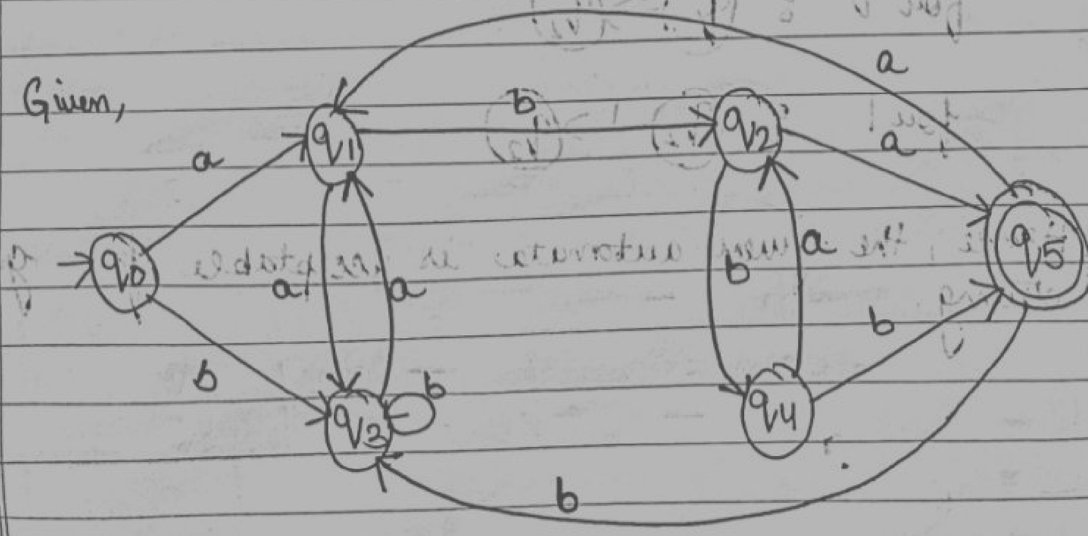
δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	$\{q_2\}$	ϕ
q_2	ϕ	ϕ
q_3	ϕ	$\{q_4\}$
q_4	ϕ	ϕ

DFA

δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$



Ques 22= Given,



Step 1 – Remove unreachable states
There are no unreachable states

Step 2 – Transition Table for given DFA.

δ	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_3	q_2
q_2	q_5	q_4
q_3	q_1	q_3
q_4	q_2	q_5
q_5	q_1	q_3

Step 3 – Non final state transition table.

δ	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_3	q_2
q_2	q_5	q_4
q_3	q_1	q_3
q_4	q_2	q_5

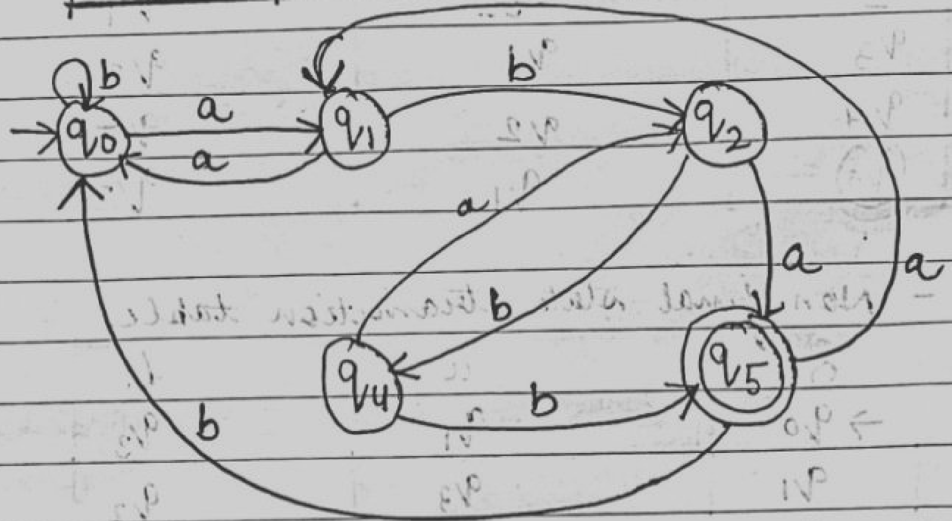
Step 4 – Final state transition table.

δ	a	b
q_5	q_1	q_3

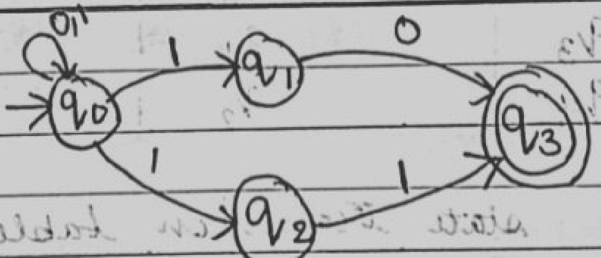
Step 5 – Remove duplicate rows.
q_0, q_3 are duplicate so we replace q_3 with q_0 and remove q_3 row.

Step 6– Final transition table

δ	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_5	q_4
q_4	q_2	q_5
q_5	q_1	q_0



Ques 22- NFA :–

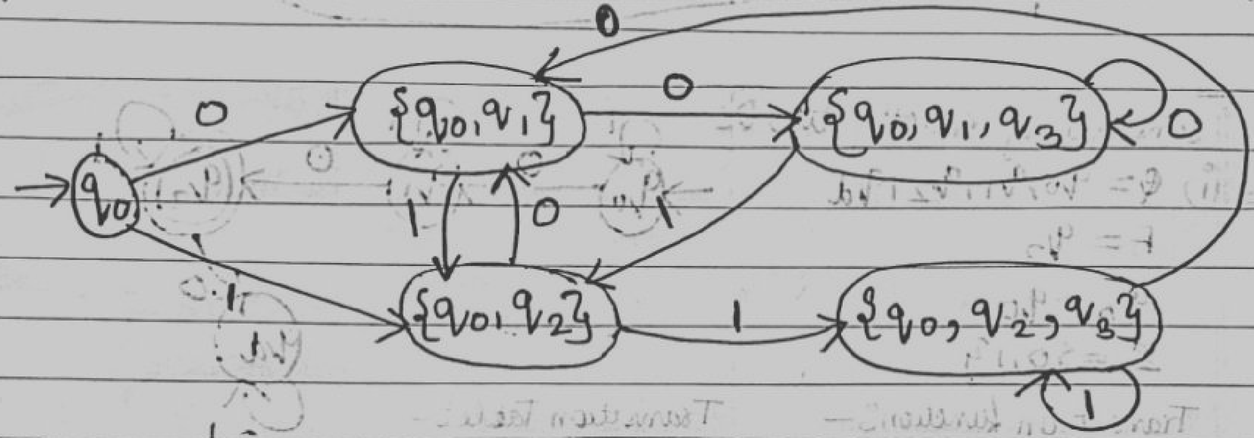


Transition Table :–

δ	0	1
$\rightarrow q_0$	q_1	$\{q_0, q_2\}$
q_1	q_3	
q_2	—	q_3
q_3	—	—

Transition Table for DFA

δ	0	1
$\to q_0$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$



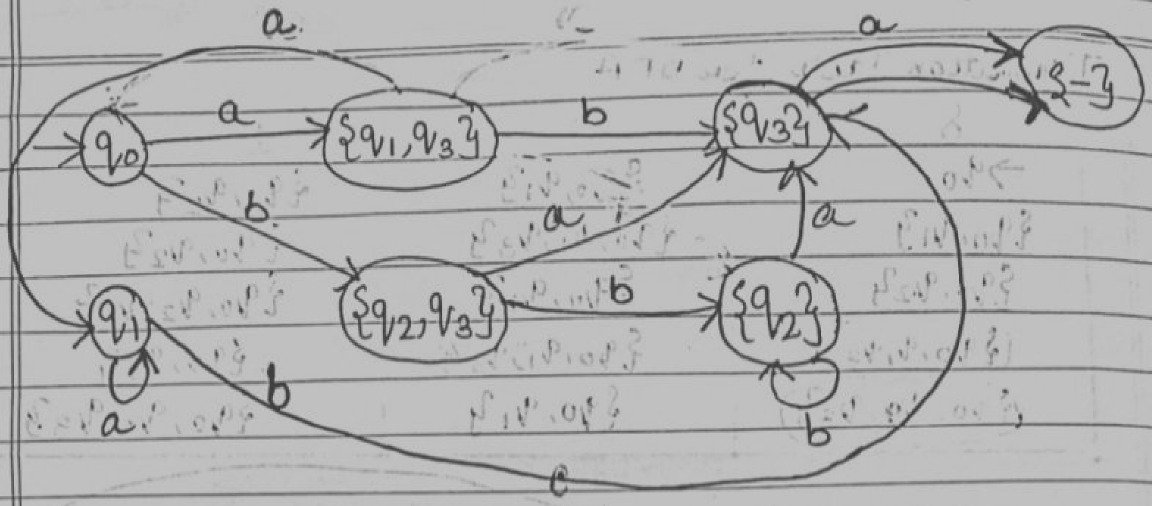
ues24=

δ	a	b
$\to q_0$	$\{q_1, q_3\}$	$\{q_2, q_3\}$
q_1	q_1	q_3
q_2	q_3	q_2
q_3	—	—

$\Leftarrow$ NFA table

Now, the DFA table.

δ	a	b
$\to \{q_0\}$	$\{q_1, q_3\}$	$\{q_2, q_3\}$
$\{q_1, q_3\}$	$\{q_1\}$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_3\}$	$\{q_2\}$
$\{q_3\}$	$\{-\}$	$\{-\}$
$\{q_2\}$	$\{q_3\}$	$\{q_2\}$
$\{q_1\}$	$\{q_1\}$	$\{q_3\}$



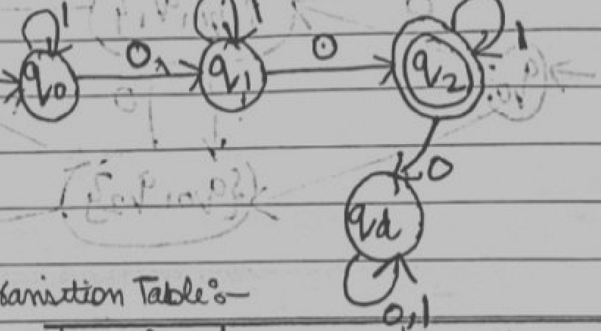
containing exactly two 0's

Qus25 = (i) $Q = q_0, q_1, q_2, q_d$

$F = q_2$

$q_0 = q_0$

$\Sigma = \{0, 1\}$



Transition function:-

$\delta : (q_0, 0) = q_1 \quad \delta : (q_0, 1) = q_0$

$\delta : (q_1, 0) = q_2 \quad \delta : (q_1, 1) = q_1$

$\delta : (q_2, 0) = q_d \quad \delta : (q_2, 1) = q_2$

Transition Table:-

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_d	q_2

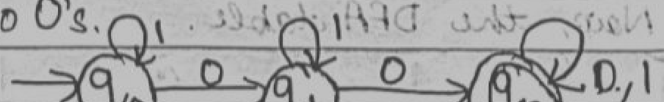
(ii) containing at least two 0's.

$Q = q_0, q_1, q_2$

$F = q_2$

$q_0 = q_0$

$\Sigma = \{0, 1\}$



Transition function:-

$\delta : (q_0, 0) = q_1 \quad \delta : (q_0, 1) = q_0$

$\delta : (q_1, 0) = q_2 \quad \delta : (q_1, 1) = q_1$

$\delta : (q_2, 0) = q_2 \quad \delta : (q_2, 1) = q_2$

Transition table:-

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_2	q_2