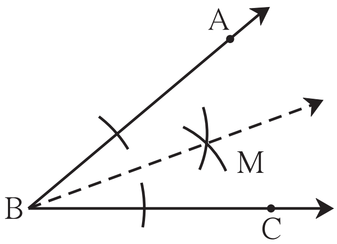
1 Geometrical Constructions

Part One

Let's recall.

In previous classes, we have learnt about the line, line segment, angle, angle bisector, etc. We measure an angle in degrees. If $\angle ABC$ measures 40°, we write it as $m\angle ABC = 40°$.

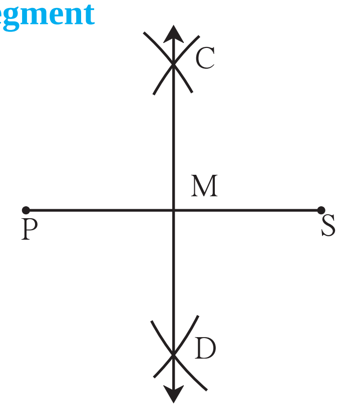


Angle Bisector

You see the figure of $\angle ABC$ alongside.
An angle bisector divides an angle into two equal parts. Is ray BM the bisector of $\angle ABC$?

Perpendicular Bisector of a Line Segment

Draw a line segment PS of length 4 cm and draw its perpendicular bisector.
Name it line CD.



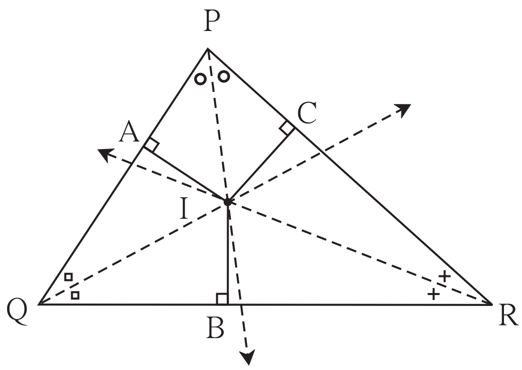
- How will you verify that CD is the perpendicular bisector?
  $m\angle CMS = \boxed{}°$
- Is $l(PM) = l(SM)$?

Let's learn.

The Property of the Angle Bisectors of a Triangle

Activity

1. Draw any ΔPQR.
2. Use a compass to draw the bisectors of all three of its angles. (Extend the bisectors, if necessary, so that they intersect one another.)
3. These three bisectors pass through the same point. That is, they are **concurrent**.
   Name the point of concurrence 'I'. Note that the point of concurrence of the angle bisectors of a triangle is in the interior of the triangle.



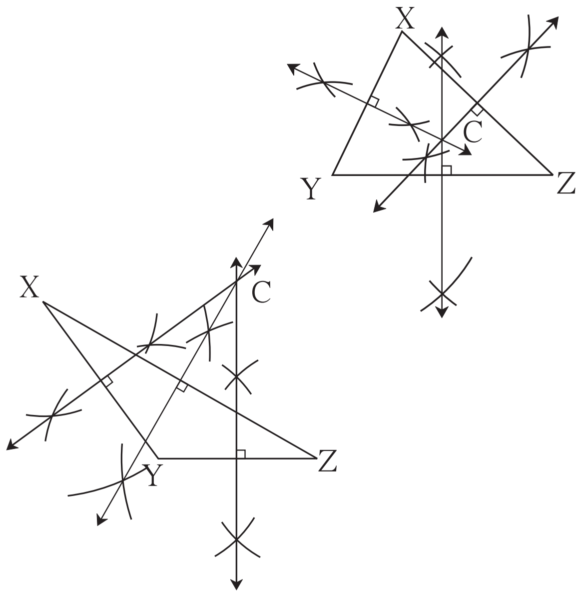
4. Draw perpendiculars IA, IB and IC respectively from I on to the sides of the triangle PQ, QR and PR. Measure the lengths of these perpendiculars. Note that IA = IB = IC.

The Property of Perpendicular Bisectors of the Sides of a Triangle

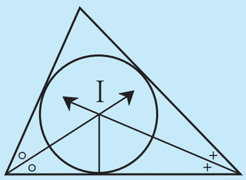
Activity

1. Use a ruler to draw an acute-angled triangle and an obtuse-angled triangle. Draw the perpendicular bisectors of each side of the two triangles.
2. In each triangle, note that the perpendicular bisectors of the sides are concurrent.
3. Name their point of concurrence 'C'. Measure the distance between C and the vertices of the triangle.

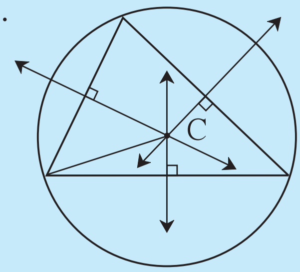
   Note that CX = CY = CZ
4. Observe the location of the point of concurrence of the perpendicular bisectors.



✸ Something more



(1) The angle bisectors of a triangle are **concurrent**. Their point of concurrence is called the **incentre**, and is shown by the letter 'I'.



(2) The perpendicular bisectors of the sides of a triangle are **concurrent**. Their point of concurrence is called the **circumcentre** and is shown by the letter 'C'.

Practice Set 1

1. Draw line segments of the lengths given below and draw their perpendicular bisectors.

   (1) 5.3 cm (2) 6.7 cm (3) 3.8 cm
2. Draw angles of the measures given below and draw their bisectors.

   (1) 105° (2) 55° (3) 90°
3. Draw an obtuse-angled triangle and a right-angled triangle. Find the points of concurrence of the angle bisectors of each triangle. Where do the points of concurrence lie?
4. Draw a right-angled triangle. Draw the perpendicular bisectors of its sides. Where does the point of concurrence lie?

5*. Maithili, Shaila and Ajay live in three different places in the city. A toy shop is equidistant from the three houses. Which geometrical construction should be used to represent this? Explain your answer.

Let's learn.

Construction of a Triangle

Activity

Let us see if we can draw the triangles when the measures of some sides and angles, are given.

Draw ΔABC such that $l(AB) = 4$ cm, and $l(BC) = 3$ cm.

- Can this triangle be drawn?
- A number of triangles can be drawn to fulfil these conditions. Try it out.
- Which further condition must be placed if we are to draw a unique triangle using the above information?

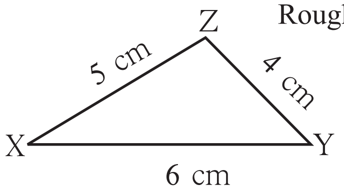
Before constructing any building, the structure of the building is first drawn on paper. You might have even seen a small model of some such building. It becomes easier to construct the building from a drawing. In a similar way, a rough sketch of a geometrical construction makes it easier to draw the required figure. It helps to plan the sequence of the steps in the construction.

(I) To construct a triangle given the lengths of its three sides

Example Draw ΔXYZ such that $l(XY) = 6$ cm, $l(YZ) = 4$ cm, $l(XZ) = 5$ cm

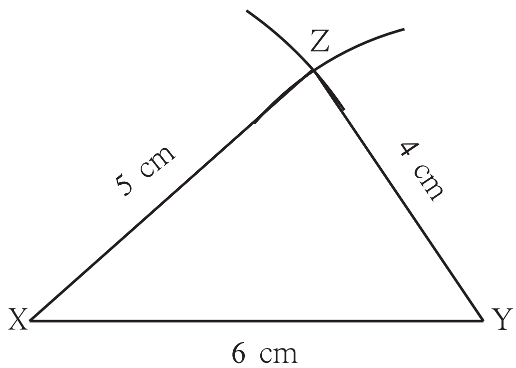
- Let us draw a rough figure quickly and show the given information in it as accurately as possible. For example, side XY is the longest, so, in the rough figure, too, XY should be the longest side.

Steps

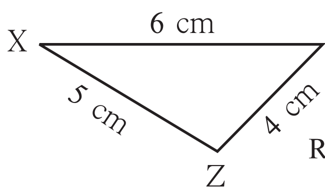


Rough figure

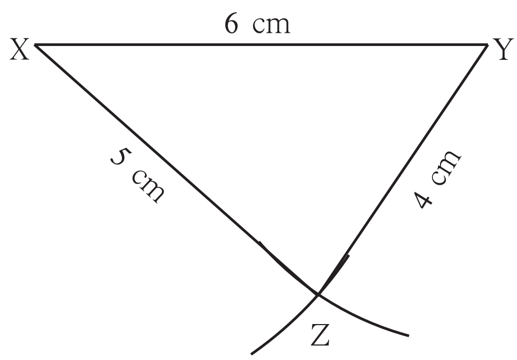
1. According to the rough figure, segment XY of length 6 cm is drawn as the base.
2. As $l(XZ)$ is 5 cm, draw an arc on one side of seg XY with the compass opened to 5 cm and with its point at X.
3. Next, with the point at Y and the compass opened to 4 cm, draw an arc to cut the first arc at Z. Draw segs XY and YZ.



A similar construction can be drawn on the other side of the base as shown below.



Rough figure



Practice Set 2

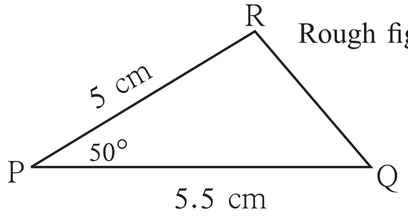
1. Draw triangles with the measures given below.
   (a) In ΔABC, $l(AB) = 5.5$ cm, $l(BC) = 4.2$ cm, $l(AC) = 3.5$ cm
   (b) In ΔSTU, $l(ST) = 7$ cm, $l(TU) = 4$ cm, $l(SU) = 5$ cm
   (c) In ΔPQR, $l(PQ) = 6$ cm, $l(QR) = 3.8$ cm, $l(PR) = 4.5$ cm
2. Draw an isosceles triangle with base 5 cm and the other sides 3.5 cm each.
3. Draw an equilateral triangle with side 6.5 cm.
4. Choose the lengths of the sides yourself and draw one equilateral, one isosceles and one scalene triangle.

(II) To construct a triangle given two sides and the angle included by them

Example Draw ΔPQR such that $l(PQ) = 5.5$ cm, $m\angle P = 50°$, $l(PR) = 5$ cm.

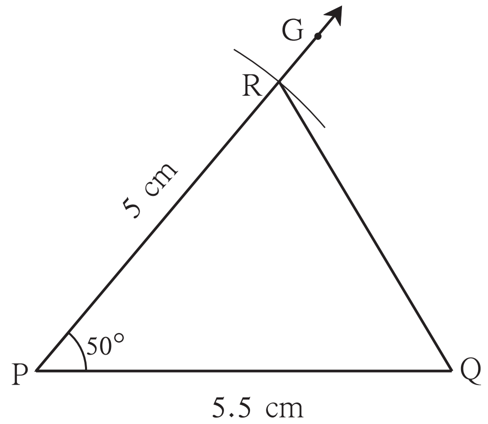
(A rough figure has been drawn showing the given information. $\angle P$ is an acute angle and that is shown in the rough figure, too.)

Rough figure



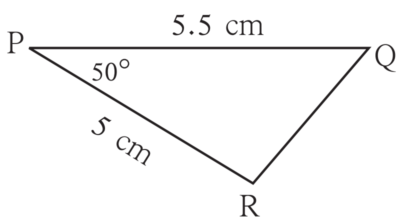
Steps

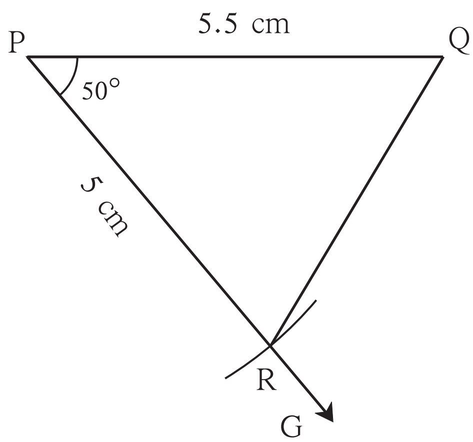
1. According to the rough figure, seg PQ forms the base of length 5.5 cm.
2. Ray PG is drawn so that $m\angle GPQ = 50°$
3. Open the compass to 5 cm. Placing the compass point on P, draw an arc to cut ray PG at R. Join points Q and R. ΔPQR is the required triangle.



The ray PG may be drawn on the other side of the seg PQ. Its rough figure will be as shown below.

Rough figure





Practice Set 3

- Draw triangles with the measures given below.

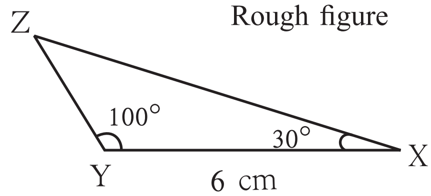
1. In ΔMAT, l(MA) = 5.2 cm, $m\angle$A = 80°, l(AT) = 6 cm
2. In ΔNTS, $m\angle$T = 40°, l(NT) = l(TS) = 5 cm
3. In ΔFUN, l(FU) = 5 cm, l(UN) = 4.6 cm, $m\angle$U = 110°
4. In ΔPRS, l(RS) = 5.5 cm, l(RP) = 4.2 cm, $m\angle$R = 90°

(III) To construct a triangle given two angles and the included side

Example Construct ΔXYZ such that l(YX) = 6 cm, $m\angle$ZXY = 30°, $m\angle$XYZ = 100°

$\angle$XYZ is an obtuse angle and that is shown in the rough figure.

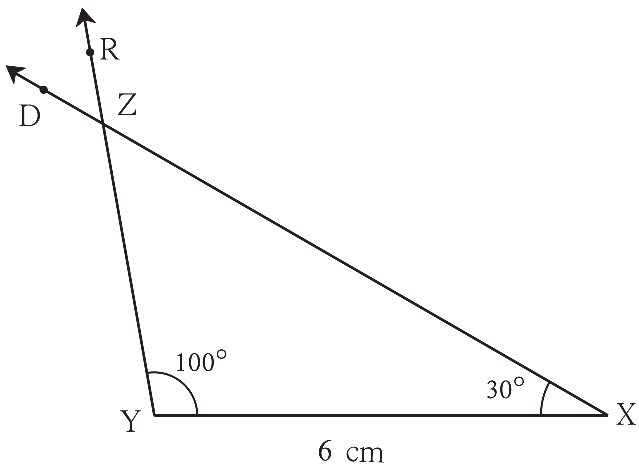
Rough figure



Steps

1. According to the rough figure, draw seg YX as base of length 6 cm.
2. Draw ray YR such that $m\angle$XYR = 100°
3. On the same side of seg YX as point R, draw ray XD so that $m\angle$YXD = 30°. Name the point of intersection of rays YR and XD, Z.

   ΔXYZ is the required triangle.
4. See how an identical triangle can be drawn on the other side of the base.



Use your brain power.

Example In ΔABC, $m\angle$A = 60°, $m\angle$B = 40°, l(AC) = 6. Can you draw ΔABC? What further information is required before it can be drawn? Which property can be used to get it? Draw the rough figure to find out.

Recall the property of the sum of the angles of a triangle. Using that property, can we find the measures of two angles and an included side AC ?

Practice Set 4

⊙ Construct triangles of the measures given below.

1. In ΔSAT, $l(AT) = 6.4$ cm, $m\angle A = 45^\circ$, $m\angle T = 105^\circ$

2. In ΔMNP, $l(NP) = 5.2$ cm, $m\angle N = 70^\circ$, $m\angle P = 40^\circ$

3. In ΔEFG, $l(EG) = 6$ cm, $m\angle F = 65^\circ$, $m\angle G = 45^\circ$

4. In ΔXYZ, $l(XY) = 7.3$ cm, $m\angle X = 34^\circ$, $m\angle Y = 95^\circ$

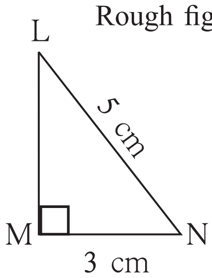
(IV) To construct a right-angled triangle given the hypotenuse and one side

We know that a triangle with a right angle is called a right-angled triangle. In such a triangle, the **side opposite the right angle** is called the **hypotenuse**.

Example Draw ΔLMN such that $m\angle LMN = 90^\circ$, hypotenuse = 5 cm, $l(MN) = 3$ cm.

Let us draw the rough figure using the given information.

Rough figure



As $m\angle LMN = 90^\circ$, we draw a right-angled triangle approximately and mark the right angle. Thus we show the given information in the rough figure.

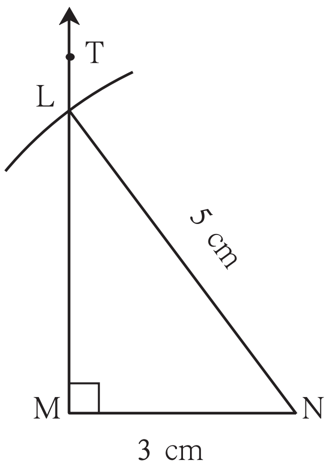
Steps

1. As shown in the rough figure, draw the base seg MN of length 3 cm.

2. At point M of seg MN, draw ray MT to make an angle of 90° to seg MN.

3. Opening the compass to 5 cm and with the point at N, draw an arc to cut seg MT at L. ΔLMN is the required triangle.

4. Note that a similar figure can be drawn on the other side of the base.



Practice Set 5

⊙ Construct triangles of the measures given below.

1. In ΔMAN, $m\angle MAN = 90^\circ$, $l(AN) = 8$ cm, $l(MN) = 10$ cm.

2. In the right-angled ΔSTU, hypotenuse SU = 5 cm and $l(ST) = 4$ cm.

3. In ΔABC, $l(AC) = 7.5$ cm, $m\angle ABC = 90^\circ$, $l(BC) = 5.5$ cm.

4. In ΔPQR, $l(PQ) = 4.5$ cm, $l(PR) = 11.7$ cm, $m\angle PQR = 90^\circ$.

5. Students should take examples of their own and practise construction of triangles.

Activity

Try to draw triangles with the following data.

1. ΔABC in which $m\angle A = 85°$, $m\angle B = 115°$ $l(AB) = 5$ cm
2. ΔPQR in which $l(QR) = 2$ cm, $l(PQ) = 4$ cm, $l(PR) = 2$ cm

Could you draw these triangles? If not, look for the reasons why you could not do so.

✸ An activity for learning something more

Example Draw ΔABC such that $l(BC) = 8$ cm, $l(CA) = 6$ cm, $m\angle ABC = 40°$.

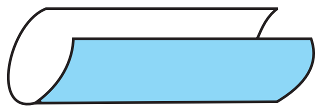
Draw a ray to make an angle of 40° with the base BC, $l(BC) = 8$ cm.

We have to obtain point 'A' on the ray. With 'C' as the centre, draw an arc of radius 6 cm to do so. What do we observe? The arc intersects the ray in two different points. Thus, we get two triangles of two different shapes having the given measures.

Can a triangle be drawn if the three angles are given, but not any side? How many such triangles can be drawn?

Let's learn.

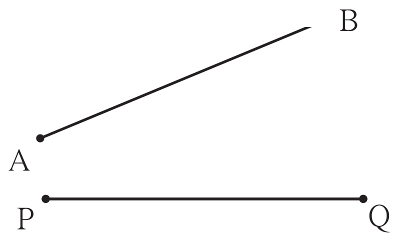
Congruence of Segments



Activity I Take a rectangular paper. Place two opposite sides one upon the other. They coincide exactly.

Activity II Using the ruler measure the lengths of seg AB and seg PQ.

$l(AB) = $ .... $l(PQ) = $ ....



Are they of the same length? You cannot pick up and place one segment over the other. Trace the seg AB along with the names of the points on a sheet of transparent paper. Place this new segment on seg PQ. Verify that if point A is placed on point P, then B falls on Q. It means that seg AB is **congruent** with seg PQ.

We can infer from this that if two line segments have the same lengths, they coincide exactly with each other. That is, they are **congruent**. If seg AB and seg PQ are congruent, it is written as seg AB $\cong$ seg PQ.

Now I know!

If given line segments are equal in length, they are congruent.

- If seg AB $\cong$ seg PQ it means that seg PQ $\cong$ seg AB.
- Note also that if seg AB $\cong$ seg PQ and seg PQ $\cong$ seg MN, then seg AB $\cong$ seg MN.

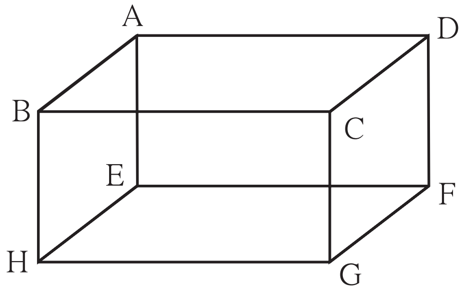
In other words, if one line segment is congruent to another and that segment is congruent to a third, then the first segment is also congruent to the third.

Activity I

Take any box. Measure the lengths of each of its edges. Which of them are congruent?

Activity II

From the shape shown below, write the names of the pairs of congruent line segments.



(1) seg AB ≅ seg DC

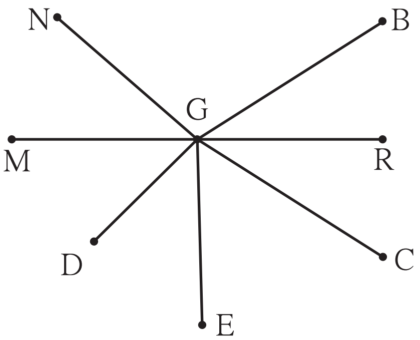
(2) seg AE ≅ seg BH

(3) seg EF ≅ seg ....

(4) seg DF ≅ seg ....

Practice Set 6

1. Write the names of pairs of congruent line segments. (Use a divider to find them.)



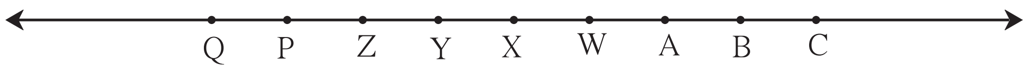
(i) ....

(ii) ....

(iii) ....

(iv) ....

2. On the line below, the distance between any two adjoining points shown on it is equal. Hence, fill in the blanks.

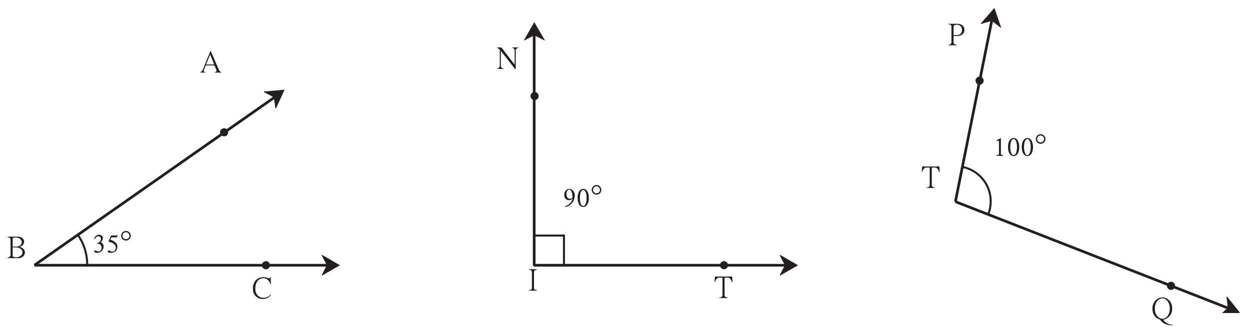


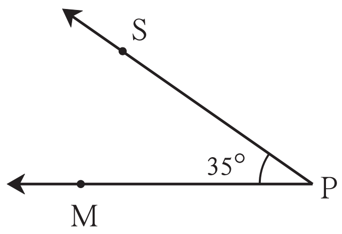
(i) seg AB ≅ seg.......   (ii) seg AP ≅ seg.......   (iii) seg AC ≅ seg.......

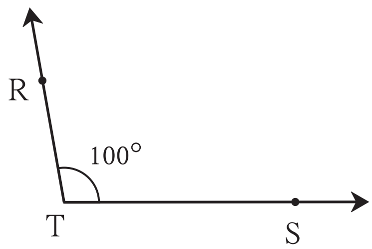
(iv) seg....... ≅ seg BY   (v) seg....... ≅ seg YQ   (vi) seg BW ≅ seg.......

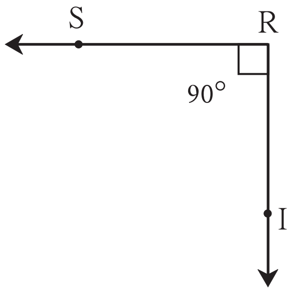
Let's learn. **Congruence of Angles**

Observe the given angles and write the names of those having equal measures.

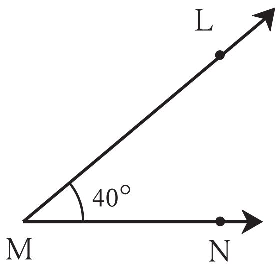


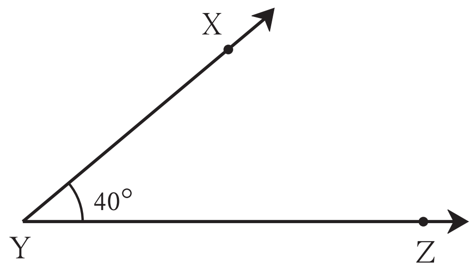






Activity





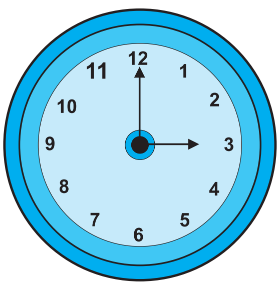
Draw two angles ∠LMN and ∠XYZ of 40° each as shown in the figure. Trace the arms of ∠LMN and the names of the points on a transparent paper. Now lift the transparent paper and place the angle you obtain on ∠XYZ. Observe that if point M is placed on Y and ray MN on ray YZ, then ray ML falls on ray YX. We can infer that angles of equal measure are congruent. The congruence of angles does not depend on the length of their arms. It depends upon the measures of those angles. That ∠LMN is congruent with ∠XYZ is written as ∠LMN ≅ ∠XYZ.

Now I know !

Two angles with equal measures are congruent to each other.

- If ∠LMN ≅ ∠XYZ then ∠XYZ ≅ ∠LMN.
- If ∠LMN ≅ ∠ABC and ∠ABC ≅ ∠XYZ then ∠LMN ≅ ∠XYZ.

Let's discuss.



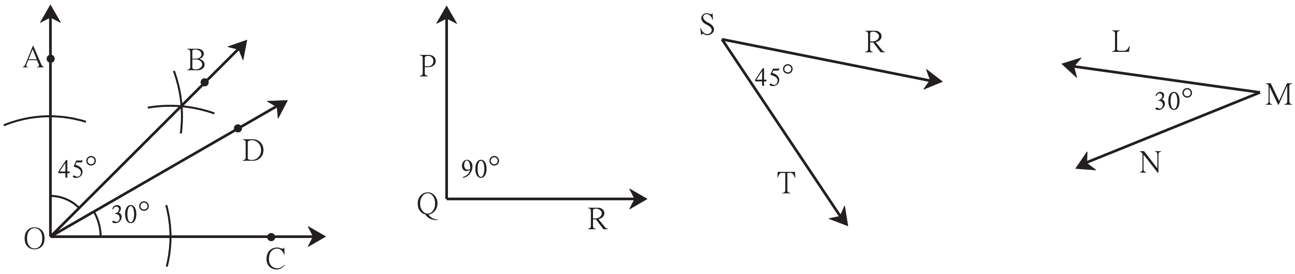
(1) What time does this clock show? $\boxed{}$

(2) What is the measure of the angle between its two hands? $\boxed{}$

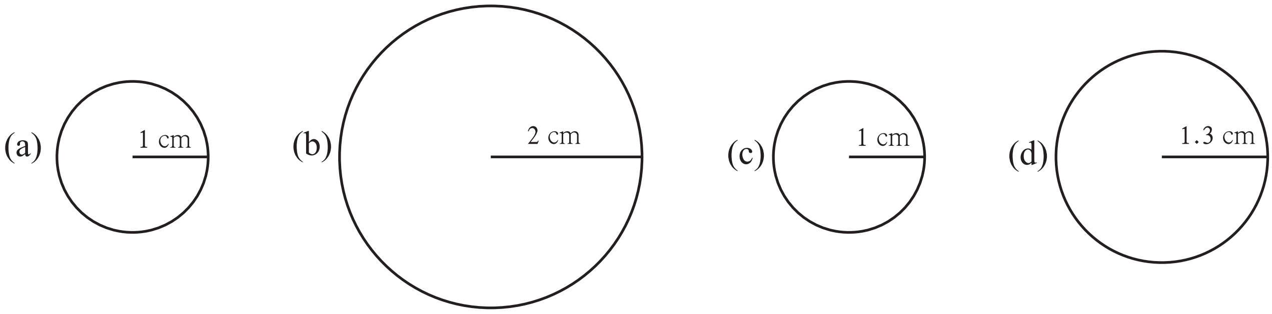
(3) At which other times is the angle between the hands congruent with this angle? $\boxed{}$

Practice Set 7

- Some angles are given below. Using the symbol of congruence write the names of the pairs of congruent angles in these figures.



Let's learn. **Congruence of Circles**



Activity I

Observe the circles in the figures above. Draw similar circles of radii 1 cm, 2 cm, 1 cm and 1.3 cm on a paper and cut out these circular discs. Place them one upon the other to find out which ones coincide exactly.

Observations: 1. The circles in (a) and (c) coincide.

2. Circles in fig (b) and (c) and in fig (a) and (d) do not coincide. Are there other pairs like these ?

Circles which coincide exactly are said to be **congruent circles.**

Activity II Get bangles of different sizes but equal thickness and find the congruent ones among them.

Activity III Find congruent circles in your surroundings.

Activity IV Take some round bowls and plates. Place their edges one upon the other to find pairs of congruent edges.

Now I know!

Circles of equal radii are congruent circles.

ICT Tools or Links

Use the construction tools in the Geogebra software to draw triangles and circles.