Control Systems

Vaishnavi K
ME18BTECH11017

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1. Problem 2.64
2. Solution
   - Finding Transfer Function
   - Finding Q_0 in Steady state
   - Verifying the result by Final Value Theorem

Problem Statement

Chapter 2: Problem 64

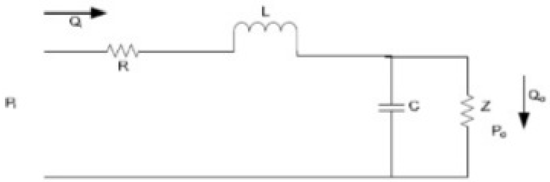
Given,

$$R_3 = 4176\Omega, C_3 = 0.98\mu F, L_3 = 140.6H, Z_3 = 308163\Omega \tag{2.1}$$

Find,

- Transfer Function $= \frac{Q_{03}(s)}{P_2(s)}$
- Q_0 in Steady state
- Verify the above results by Final Value Theorem.

Finding Transfer function



Converting the impedances to their Laplace transform equivalent and by applying the voltage divider rule we get,

$$Z \parallel \frac{1}{sC} = \frac{\frac{Z}{sC}}{Z + \frac{1}{sC}} = \frac{\frac{1}{C}}{s + \frac{1}{ZC}} \tag{3.1}$$

$$\frac{P_0}{P_i}(s) = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L} + \frac{1}{ZC}\right)s + \left(\frac{R}{ZCL} + \frac{1}{CL}\right)} \tag{3.2}$$

Since $\mathrm{Q}_0 = \frac{P_0}{Z}$,

$$\frac{Q_0}{P_i}(s) = \frac{\frac{1}{LCZ}}{s^2 + \left(\frac{R}{L} + \frac{1}{ZC}\right)s + \left(\frac{R}{ZCL} + \frac{1}{CL}\right)} \tag{3.3}$$

On simplifying and substituting the values we get,

$$\frac{Q_0}{P_i}(s) = \frac{0.0236}{s^2 + 33.0125s + 7355.9} \tag{3.4}$$

Finding Q_0 in Steady state

The steady state Circuit becomes,

Now,

$$Q_0 = \frac{P_i}{R+Z} = \frac{1}{4176+308163} = 3.2X10^{-6} \tag{3.5}$$

Verifying the result by Final Value Theorem

Applying the Final Value Theorem,

$$q_0\left(\infty\right) = \lim_{x\to\ 0}\left(s\frac{0.0236}{s^2+33.0125s+7355.9}\frac{1}{s}\right) \\ = 3.2X10^{-6} \tag{3.6}$$

Hence,
Verified that we get same result from Part a and Part b and Final value theorem.