DOMAIN BASED APPLICATIONS OF MULTIVARIABLE CALCULUS

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INTRODUCTION

Multivariable calculus or multivariate calculus is the branch of mathematics that is useful to find the relationship between the input and output variables. In mathematics, multivariable calculus is the extension of calculus that deals with multiple variables. Multivariable calculus includes two or more variables instead of a single variable.

Moreover, multivariable calculus consists of limits and derivatives, partial differentiation, multiple integrations, fundamental theorems with multivariable dimensions, vector fields, etc. Apart from this, multivariable calculus plays an essential role in many fields such as engineering, natural and social science, weather forecasting, astronomy, etc. Also, the two- dimensional(2D) and three-dimensional(3D) models are studied with the help of multivariable calculus.

Mathematically, f(z) = f(x,y)

Where, f(z) is an output function, x and y are the input variables. Also, z is dependent on x and y.

Multivariable calculus is widely accepted in many fields such as science, engineering, finance, research etc. some of the major applications of multivariable are mentioned below:

1. In a dynamic system, multivariable calculus is the core tool used which is used for optimal control.

2. Multivariable calculus is helpful in evaluating empirical data in regression analysis.

3. In the field of finance, quantitative analysts use multivariable calculus which is helpful in predicting the future trends in the stock market.

4. Multivariable calculus is also used to determine the 2D and 3D motion of fluid particles and streams in mechanical engineering.

5. Multivariable calculus is used to study the high dimensional model system which has deterministic nature in engineering as well as social science.

DOMAIN1: Application of Multivariable Calculus in Life Science

In the life sciences, multivariate calculus is used in a variety of applications, including:

- **Biomechanics:** Multivariate calculus is used to study the mechanical behavior of biological systems, such as the motion of joints and the deformation of tissues.

- **Pharmacokinetics and pharmacodynamics:** Multivariate calculus is used to model the absorption, distribution,

metabolism, and excretion of drugs in the body, as well as the relationship between drug concentration and its effect on the body.

- **Systems biology:** Multivariate calculus is used to model and analyze complex biological systems, such as the interactions between genes and proteins in a cell.

- **Medical imaging:** Multivariate calculus is used to analyze and process medical images, such as those obtained through CT, MRI, and ultrasound, in order to extract useful information about the body.

- **Cardiovascular physiology:** Multivariate calculus is used to analyze and model the cardiovascular system, including blood flow, pressure, and the electrical activity of the heart.

- **Optimal control in physiology:** Multivariate calculus is used to optimize the control of physiological systems, such as glucose control in diabetes patients.

- **Neurosciences:** Multivariate calculus is used to model the electrical activity of neurons and the dynamics of neural networks.

- **Ecology:** Multivariate calculus is used to model and analyze ecological systems, such as population dynamics and food webs.

- **Epidemiology:** Multivariate calculus is used to model the spread and dynamics of infectious diseases, taking into account multiple variables such as population density, population mobility, and vaccination rates.

DOMAIN2: Application of Multivariable Calculus in Electrical Engineering

In electrical engineering, multivariate calculus is used in a variety of applications, including:

- **Circuit analysis:** Multivariate calculus is used to analyze and design electrical circuits, such as those used in electronic devices and systems.

- **Electromagnetism:** Multivariate calculus is used to study the behavior of electromagnetic fields, including the propagation of waves and the behavior of antennas.

- **Signal processing:** Multivariate calculus is used to analyze and process signals, such as audio or video signals, in order to extract useful information.

- **Control systems:** Multivariate calculus is used to design and analyze control systems, which are used to control the behavior of mechanical or electrical systems.

- **Power systems:** Multivariate calculus is used to analyze and design power systems, such as those used to generate, transmit, and distribute electricity.

- **Optimal control:** Multivariate calculus is used to optimize the control of systems, such as in the field of power systems and robotics.

- **Robotics:** Multivariate calculus is used to model and control the movement of robots and robotic systems.

- **Communication systems**: Multivariate calculus is used to analyze and design communication systems, such as those used in wireless networks.

- **Digital signal processing:** Multivariate calculus is used to analyze and process digital signals, such as those used in audio and video compression, speech recognition, and image processing.

DOMAIN3: Application of Multivariable Calculus in Weather Forecasting In weather forecasting, multivariate calculus is used to model and analyze the behavior of the atmosphere. Some specific applications include:

- **Numerical weather prediction (NWP):** Multivariate calculus is used to model the dynamics of the atmosphere, such as the movement of air masses, the formation of clouds and precipitation, and the development of storms. NW on in the process, it's also used to optimize the numerical models used for NW predictions.

- **Data assimilation:** Multivariate calculus is used to combine observations from various sources, such as weather stations, radar, and satellites, with the predictions of NW numerical models to improve the accuracy of weather forecasts.

- **Ensemble forecasting:** Multivariate calculus is used to generate a set of possible weather forecasts by perturbing the initial conditions of the numerical models. This allows for the estimation of the uncertainty of the forecast.

- **Atmospheric circulation:** Multivariate calculus is used to study the movement of air masses and the development of large-scale weather patterns, such as high-pressure and low-pressure systems.

- **Air-sea interactions:** Multivariate calculus is used to study the interactions between the atmosphere and the ocean, such as the formation of clouds and precipitation over the ocean, and the influence of ocean currents on weather patterns.

- **Climate modeling:** Multivariate calculus is used to model the long-term behavior of the atmosphere, including the Earth's climate, and to make predictions about future climate change.

- **Severe weather forecasting:** Multivariate calculus is used to model and predict severe weather events, such as hurricanes, tornadoes, and thunderstorms.

  Overall, multivariate calculus plays a crucial role in weather forecasting by allowing for the modeling and analysis of the complex dynamics of the atmosphere, and by providing powerful tools for data assimilation, ensemble forecasting, and optimization of numerical models.

Let's take an example.

Imagine a desert island where a deadly virus takes hold. Every day, a tenth of the population dies. We might say that:

Number dying per day = 0.1 x population

Let's now write this using maths symbols:

$$\frac{dN}{dt} = 0.1xN$$

where:

$$\frac{dN}{dt} = Number of people dying per day$$

The negative sign tells us that people dying decreases the population (it's a negative change)

N = Population

WHAT WILL HAPPEN TO THE POPULATION, N ?

The population, N, will obviously decrease every day. This means that the number of people dying every day will also decrease.

For example:

On day zero the population is 10 000 and then the virus arrives

On day one the population starts at 10 0000 so 1000 die

On day two the population starts at 9000 so 900 die

On day three the population starts at 8100 so 810 die

and so on.