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# PES UNIVERSITY, BANGALORE

(Established under Karnataka Act 16 of 2013)

**UE18MA151**

## END SEMESTER ASSESSMENT B. TECH. II SEMESTER– May 2019

## UE18MA151 – Engineering Mathematics – II

**Time: 3 Hrs** **Answer All Questions** **Max Marks: 100**

| | | | |
|---|---|---|---|
| 1. | a) | Find the directional derivative of the function $f(x,y,z) = x^2yz + 4xz^2$ at the point P(1,-2,-1) in the direction of the line PQ, where Q is the point (2,-1,-2). In what direction it will be a maximum? Find the magnitude of this maximum and $div(grad f)$ at P and explain the significance. | 7 |
| | b) | Show that $\vec{F} = (yz-1)\vec{i} + (z + xz + z^2)\vec{j} + (y + yx + 2yz)\vec{k}$ is a conservative field. Find the scalar potential and work done in moving the object in this field from (1,2,2) to (2,3,4). | 7 |
| | c) | Using divergence theorem, evaluate the surface integral $\iint_S (x^4 + y^4 + z^4)\, ds$, where S is the sphere $x^2 + y^2 + z^2 = a^2$ | 6 |
| 2. | a) | Evaluate $\int_a^b \frac{x\, dx}{(x-a)^{\left(\frac{1}{3}\right)}(b-x)^{\left(\frac{2}{3}\right)}}$ using beta and gamma functions. | 7 |
| | b) | Establish the Jacobi series and hence show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$ where n is a positive integer. | 7 |
| | c) | Evaluate $\int x^3 J_3(x)dx$ | 6 |
| 3. | a) | Find the Laplace transform of $\left(\frac{sin2t}{\sqrt{t}}\right)^2 + t\, sin2t + 5^t + \int_0^t t\, cosht dt$ | 8 |
| | b) | State and prove the Laplace transform of a periodic function. | 5 |
| | c) | Express the following function interms of the unit step function and hence find their Laplace transforms $f(t) = \begin{cases} t, & 0 \leqslant t < 2 \\ 1, & 2 \leqslant t < 3 \\ (t-3)^3, & t \geq 3. \end{cases}$ | 7 |
| 4. | a) | Find $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$ using partial fractions. | 7 |
| | b) | Solve the differential equation $ty'' + 2y' + ty = sint, y(0) = 1, y'(0) = 0$ by using Laplace transforms | 7 |
| | c) | Evaluate $\int_0^2 x^{3/2}\,(2-x)^{5/2}dx$ using convolution theorem. | 6 |
| 5. | a) | Obtain the Fourier series expansion of $f(x) = x\, sinx$ in the interval $(0, 2\pi)$. | 7 |
| | b) | Prove that in $0 < x < l, x = \frac{l}{2} - \frac{4l}{\pi}\left[cos\frac{\pi x}{l} + \frac{1}{3^2}cos\frac{3\pi x}{l} + \frac{1}{5^2}cos\frac{5\pi x}{l} + \dots\right]$and deduce | 6 |

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that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots=\frac{\pi^4}{96}$

c) The following table gives the variations of periodic current over a period T

| $tsec$ | 0 | $\frac{T}{6}$ | $\frac{T}{3}$ | $\frac{T}{2}$ | $\frac{2T}{3}$ | $\frac{5T}{6}$ | $T$ |
|---|---|---|---|---|---|---|---|
| $Aamp$ | 1.98 | 1.30 | 1.05 | 1.30 | −0.88 | −0.25 | 1.98 |

Show that there is a direct current part 0.75 amp in the variable current and obtain the first two harmonic of the Fourier series. 7