# PES University, Bengaluru

UE21MA141B

(Established under Karnataka Act No. 16 of 2013)

## JULY 2022: END SEMESTER ASSESSMENT (ESA) B TECH II SEMESTER
(Chemistry Cycle)

### UE21MA141B – ENGINEERING MATHEMATICS 2

| Time: 3 Hrs | Answer All Questions | Max Marks: 100 |
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| 1 | a) | Find the moments of inertia of the planar region bounded by the curves $y = x^2$ and $y = x + 2$, given that the density $\rho(x,y) = k$, $a\ constant$. | 7 |
| | b) | Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ using the substitution $x = au$, $y = bv\ and\ z = cw$. | 7 |
| | c) | Evaluate $\int_0^3 \int_0^{\sqrt{4-y}}(x + y)dx\,dy$ by changing the order of integration. | 6 |
| | | | |
| 2 | a) | Show that the force field given by $\vec{F} = (y^2\cos x + z^3)\hat{\imath} + (2\,y\sin x - 4)\hat{\jmath} + (3\,x\,z^2 + 2)\hat{k}$ is conservative and hence find its scalar potential $\phi$. | 7 |
| | b) | Calculate the flux of $\vec{F} = x\,\hat{\imath} + y\,\hat{\jmath} + (1 - 2z)\hat{k}$ across the surface $S$ using the outward normal, given that S is the portion of the surface that lies between $z - x^2 - y^2 = 0$ and below the plane $z = 1$ and above the x-y plane. | 7 |
| | c) | Evaluate $\int\int_S \vec{F}.\hat{n}\,ds$ over the sphere $x^2 + y^2 + z^2 = k^2$, using the divergence theorem where $\vec{F} = k[a\,x\hat{\imath} + b\,y\hat{\jmath} + c\,z\,\hat{k}]$. Is $\vec{F}$ irrotational ? | 6 |
| | | | |
| 3 | a) | i) If $L[t\,f(t)] = \frac{1}{s(s+1)}$, then find $L[f(3t)]$.<br><br>ii) Find $L\left[\int_0^t u\sin 4u\,du\right]$ | 7 |
| | b) | Find the Laplace transform of $\frac{1-\cos t}{t^2}$ | 7 |
| | c) | Determine the Laplace transform of the triangular wave function $f(t) = \begin{cases} t, & 0 < t < c \\ 2c - t, & c < t < 2c \end{cases}$, $f(t + 2c) = f(t)$. Express the solution in terms of a hyperbolic function. | 6 |
| | | | |
| 4 | a) | Solve the differential equation $\frac{d^2 i}{dt^2} + 4\,i(t) = \begin{cases} 4t, & 0 < t < 1 \\ 4, & t > 1 \end{cases}$, $i(0) = 1, i'(0) = 0$ using the Laplace transform technique. | 7 |
| | b) | Obtain the inverse Laplace transform of $\frac{1}{s^5(s+3)}$ using convolution theorem. | 7 |
| | c) | Obtain the inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2}$. | 6 |

| 5 | a) | Obtain the half range Fourier cosine series of $f(x) = x$ in $0 < x < 2$. Hence deduce the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ | 7 |
|---|---|---|---|
| | b) | Find the complex form of the Fourier series of $f(x) = \cos ax$ in $-\pi < x < \pi$. Here the parameter $'a'$ is not an integer. | 7 |
| | c) | Find the Fourier series expansion and the amplitude of the first harmonic for the function $f(x)$ described by the following table: | 6 |

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 9 | 18 | 24 | 28 | 26 | 20 | 9 |