



Time: 3 Hrs		Answer All Questions		Max Marks: 100
1.	a)	Find $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right]$ (ii) $L[e^{-t}\cos^2 t]$ .	7	
	b)	Express the following in terms of unit step function and hence find the Laplace transforms, $f(t) = \begin{cases} \sin t & 0 \leq t < \pi \\ \sin 2t & \pi \leq t < 2\pi \\ \sin 3t & t \geq 2\pi \end{cases}$ .	7	
	c)	If $f(t)$ is a periodic function with period $T$ , show that $L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$ .	6	
2.	a)	Find the Laplace inverse of $\frac{s}{s^4 + 4a^4}$ .	7	
	b)	Find $L^{-1}\left[s \log\left(\frac{s+4}{s-4}\right)\right]$ .	7	
	c)	Using Laplace transform method, solve the initial value problem $y'' + 3y' - 10y = 4\delta(t-2)$ ; $y(0) = 2$ , $y'(0) = -3$ .	6	
3.	a)	If $\alpha$ and $\beta$ are the roots of $J_n(ax) = 0$ , then show that $\int_0^a x J_n(\alpha x) J_n(\beta x) dx = \begin{cases} 0 & \text{if } \alpha \neq \beta \\ \frac{a^2}{2} [J_{n+1}(\alpha a)]^2 & \text{if } \alpha = \beta \end{cases}$ .	9	
	b)	Express $\int_0^1 \sqrt{-\log x} dx$ in terms of beta function and hence evaluate.	5	
	c)	Derive the relation between beta and gamma functions.	6	
4.	a)	Find constants $a$ and $b$ such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. Also find a scalar function $\phi$ such that $\vec{F} = \nabla\phi$ .	7	
	b)	A surface $S$ consists of that part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ for $y \geq 0$ and the two semi-circles of radius 3 in the planes $z = 0$ and $z = 4$ . Verify Stokes theorem where $\vec{F} = z\hat{i} + xy\hat{j} + xz\hat{k}$ .	7	
	c)	If $V$ is the region in first octant bounded by $y^2 + z^2 = 9$ , and plane $x = z$ and $\vec{F} = 2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ , then evaluate $\iiint_V \nabla \cdot \vec{F} dV$ .	6	

a)

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$$x = 1 - \frac{8}{\pi^2} \left\{ \cos \frac{\pi x}{2} + \frac{1}{3^2} \cos \frac{3\pi x}{2} + \frac{1}{5^2} \cos \frac{5\pi x}{2} + \dots \right\}.$$
$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$$