Problem 1
Consider a lending pool where one can deposit ETH and borrow USDC (exp. Compound or AAVE). A user deposits 100 ETH and uses those as collateral to borrow USDC. Let the initial price be 2000 USDC for 1 ETH, and the liquidation threshold $l = 0.8$. The user borrows USDC with the loan to value ratio $\beta = 0.75$. Calculate the liquidation price for this position.

Notation: x the amount of ETH in collateral.
In order to find the liquidation price, we need to calculate HF, for this we will use the formula:

$$HF = \frac{l * amount_{ETH} * price_{ETH}}{\beta * debt_{USD}},$$

where $debt_{USD} = 100 * 2000, amount_{ETH} = 100$
Then we solve this inequality(was in lecture 5):

$$HF < 1$$

but I will consider the case when

$$HF \leq 1$$

Let's solve this equation:

$$\frac{0.8 * 100 * x}{100 * 2000 * 0.75} \leq 1$$

after the calculations, we get:

$$\frac{x}{1875} \leq 1,$$

т.е

$$x \leq 1875$$

THE LIQUIDATION PRICE = 1875

Problem 2
Assume you have with 1 ETH in your wallet. How could you double your exposure to ETH dollar price using a lending market? Current ETH price is p, the liquidation threshold for ETH is l and max loan to value ratio for ETH is β. Provide a step by step instruction and calculate the resulting liquidation price. You may assume that all stablecoins are priced perfectly at 1 dollar and the swap fees are negligible.

The way to double ETH exposure: Берем flashloan на 1 ETH (по факту берем чуть больше 1 ETH из-за комиссий), депозируем в lending pool эти 2 ETH, берем займ (долг) в стейбле в размере 1 ETH, покупаем на них 1 ETH и возвращаем flashloan. Теперь lending pool будет депозит в размере 2 ETH -> x2 exposure

Взяли в займ в flashloan ETH, вкладываем в pool 2 ETH, 1 ETH меняем на USDC и уже на них покупаем 1 ETH. Купленный ETH возвращаем в flashloan.

$$HF = \frac{l * 1 * x}{p * 1 * 0.5} = 1$$

T.e

$$x = \frac{p}{2l}$$

The liquidation condition:

$$x = \frac{p}{2l}$$

Problem 3
Derive the expression for the impermanent loss in a standard Uniswap V2 pool with the constant product rule $XY = K$. For convenience use the notation $\gamma = \frac{p_1}{p_0}$ for the relative price change from p0 to p1.

Notation: V_{pool} is the liquidity provider's assets value in the pool, V_{hold} is the assets value outside of the pool but kept in the same proportion as in the pool initially.

$$XY = KP = \frac{X}{Y}, \gamma = \frac{P_1}{P_0}$$

$$Vpool = Y * P_1 + X * 1 = 2\sqrt{KP_1}$$

$$Vhold = Y_0 * P_0 + X_0 * 1 = \sqrt{KP_0} + \frac{\sqrt{K}P_1}{\sqrt{P_0}} = \sqrt{KP_0} * (1+\gamma)$$

$$IL = \frac{Vpool - Vhold}{Vhold} = \frac{2\sqrt{KP_1} - \sqrt{KP_0} * (1+\gamma)}{\sqrt{KP_0} * (1+\gamma)} = \frac{\sqrt{KP_0}(2\sqrt{\gamma} - (1+\gamma))}{\sqrt{KP_0} * (1+\gamma)} = \frac{2\sqrt{\gamma}}{1+\gamma} - 1$$

THE EXPRESSION FOR IMPERMANENT LOSS:

$$IL = \frac{2\sqrt{\gamma}}{1+\gamma} - 1$$

Problem 4
Consider a Uniswap V2 pool with two tokens, ETH and USDC. Let X and Y be the token amounts. The initial price is $p0$ = 2000 USDC for 1 ETH. Let a trader swap 100 ETH for USDC in the pool, the swap price is p1. Let's call $\frac{(p_0-p_1)}{p0}$ the price slippage in the pool for this swap. Calculate the token reserves in the pool to keep the slippage below 0.1 percent for such swap.

Notation: price slippage $\alpha = 0.1\%$, $y = 100$ ETH. The trader sends y ETH into the pool and gets x USDC back from the pool, p_0 is the price in the pool before swap, $p = \frac{x}{y}$ is the swap price.

let's denote Δ - how much USDC a market participant will receive. also from the condition we get: $2000x = y$ As a result, we get a system of 2 equations:

$$\begin{cases} xy = (x+100)(y-\Delta) \\ 2000x = y \end{cases} \tag{1}$$

Since price slippage should not be more than 0.1, then Δ = 100*2000*0.999 = 199800

$$\begin{cases} xy = (x+100)(y-199800) \\ 2000x = y \end{cases} \tag{2}$$

After solving the system we get x, y:

$$\begin{cases} x = 99900 \\ y = 199800000 \end{cases} \tag{3}$$

TOKEN RESERVES: x = 99900, y = 199800000

Problem 5
Consider a Uniswap V3 liquidity pool with concentrated liquidity and two tokens, ETH and USDC. Let the initial price in the pool be p. Let a liquidity provider deposit the assets into the price range [p1, p2], and let L be the liquidity parameter in the chosed price range. Assume no other liquidity providers in the range. Derive the liquidity provider's position value as a function of p.

$$(x + \frac{L}{\sqrt{p_2}})(y + L\sqrt{p_1}) = L^2$$

Пусть

$$\begin{cases} x_1 = x + \frac{L}{\sqrt{p_2}} \\ y_1 = y + L\sqrt{p_1} \end{cases} \tag{4}$$

Тогда

$$\begin{cases} x_1 y_1 = L^2 \\ x_1 \geq 0 \\ y_1 \geq 0 \end{cases} \tag{5}$$

$$\begin{cases} x_1 = \frac{L}{\sqrt{P}} \\ y_1 = L\sqrt{P} \end{cases} \tag{6}$$

$$\begin{cases} x = x_1 - \frac{L}{p_2} = L(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{p_2}}) \\ y = y_1 - L\sqrt{p_1} = L(\sqrt{P} - \sqrt{p_1}) \end{cases} \tag{7}$$

$$V = y + xP = L(\sqrt{P} - \sqrt{p_1}) + LP(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{p_2}}) = L(\sqrt{P} - \sqrt{p_1}) + L(\sqrt{P} - \frac{P}{\sqrt{p_2}})$$

T.e

$$V = 2L\sqrt{P} - L(\sqrt{p_1} + \frac{P}{\sqrt{p_2}})$$

The liquidity provider's position value:

$$V = 2L\sqrt{P} - L(\sqrt{p_1} + \frac{P}{\sqrt{p_2}})$$

or

$$V = L(\sqrt{P} - \sqrt{p_1}) + LP(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{p_2}})$$