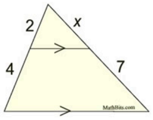
The "Side Splitter" Theorem says that if a line intersects two sides of a triangle and is parallel to the third side of the triangle, it divides those two sides proportionally.

Example:

Find x.



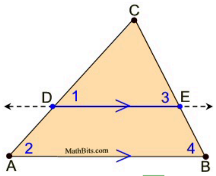
Apply the Side Splitter Theorem:
$\frac{2}{4} = \frac{x}{7}$ (form a proportion using the side lengths)

Solve the proportion for x:
$4x = (2)(7)$
$4x = 14$
$x = 3.5$ (Answer)

THEOREM: (**Side Splitter Theorem**): If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.

Given: ΔABC; $\overline{DE} \parallel \overline{AB}$

Prove: $\frac{CD}{DA} = \frac{CE}{EB}$

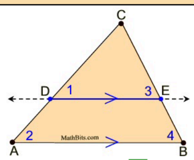


While this theorem may look somewhat like the "mid-segment" theorem, the segment $\overline{DE}$ in this theorem does not necessarily connect the "midpoints" of the sides.

THEOREM: (**Side Splitter Theorem**): If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.

Given: ΔABC; $\overline{DE} \parallel \overline{AB}$

Prove: $\frac{CD}{DA} = \frac{CE}{EB}$



While this theorem may look somewhat like the "mid-segment" theorem, the segment $\overline{DE}$ in this theorem does not necessarily connect the "midpoints" of the sides.

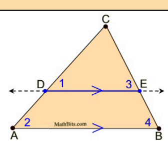
Proof:

Statements	Reasons
1. Given: ΔABC; $\overline{DE} \parallel \overline{AB}$	1. Given
2. $\angle 1 \cong \angle 2; \angle 3 \cong \angle 4$	2. If 2 || lines are cut by a transversal, the corresponding angles are congruent.
3. $\Delta ABC \sim \Delta DEC$	3. (AA) If two $\angle$s of one Δ are congruent to the corresponding $\angle$s of another Δ, the Δs are similar.
4. $\frac{CD}{CA} = \frac{CE}{CB}$	4. Corresponding sides of similar triangles are in proportion.
5. $CA = CD + DA$ $CB = CE + EB$	5. Segment Addition Postulate (or whole quantity equals the sum of its parts)
6. $\frac{CD}{CD+DA} = \frac{CE}{CE+EB}$	6. Substitution
7. $CD \bullet (CE+EB) = CE \bullet (CD+DA)$	7. In a proportion, the product of the means = the product of the extremes.
8. $\boxed{CD \bullet CE} + CD \bullet EB =$ $\boxed{CE \bullet CD} + CE \bullet DA$	8. Distributive property
9. $CD \bullet EB = CE \bullet DA$	9. Subtraction
10. $\frac{CD}{DA} = \frac{CE}{EB}$	10. In a proportion, the product of the means = the product of the extremes.

THEOREM: Converse (**Side Splitter Theorem**): If a line intersects two sides of a triangle and divides the sides proportionally, the line is parallel to the third side of the triangle.

Given: ΔABC; $\overleftrightarrow{DE}$; $\frac{CD}{DA} = \frac{CE}{EB}$

Prove: $\overline{DE} \parallel \overline{AB}$



Proof:

Statements	Reasons
1. ΔABC; $\overleftrightarrow{DE}$; $\frac{CD}{DA} = \frac{CE}{EB}$	1. Given
2. $CD \bullet EB = CE \bullet DA$	2. In a proportion, the product of the means = the product of the extremes.
3. $CD \bullet CE = CD \bullet CE$	3. Reflexive (Identity)
4. $\boxed{CD \bullet CE} + CD \bullet EB =$ $\boxed{CE \bullet CD} + CE \bullet DA$	4. Addition
5. $CD \bullet (CE+EB) = CE \bullet (CD+DA)$	5. Distributive property
6. $\frac{CD}{CD+DA} = \frac{CE}{CE+EB}$	6. In a proportion, the product of the means = the product of the extremes.
7. $CA = CD + DA$ $CB = CE + EB$	7. Segment Addition Postulate (or whole quantity equals the sum of its parts)
8. $\frac{CD}{CA} = \frac{CE}{CB}$	8. Substitution
9. $\angle C \cong \angle C$	9. Reflexive (Identity)
10. $\Delta ABC \sim \Delta DEC$	10. (SAS for Similarity). In two triangles, if two sets of corresponding sides are proportional and the included angle is congruent, the triangles are similar.
11. $\angle 1 \cong \angle 2$	11. Corresponding angles of similar triangles are congruent.
12. $\overline{DE} \parallel \overline{AB}$	12. If 2 lines are cut by a transversal and the corresponding angles are congruent, the lines are parallel.

The term "mean proportional" may also be referred to as "geometric mean".

The term "mean", when used alone, or in a context such as "mean, median, and mode", refers to finding the "average" and is known as the *Arithmetic Mean*.
Mean proportional, or *geometric mean*, is not the same as the *Arithmetic Mean*. While an arithmetic mean deals with addition, a geometric mean deals with multiplication.

Definition: The **mean proportional, or geometric mean,** of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. When solving, $x = \sqrt{a \bullet b}$.

Notice that the x value appears TWICE in the "means" positions of the proportion.

Note:* The mean proportional (geometric mean), along with the values of a and b, are positive.

Mean Proportional or Geometric Mean	$\frac{a}{x} = \frac{x}{b}$ In a "mean proportional", or "geometric mean", both "means" (x) are the exact same value.	$\frac{extreme}{mean} = \frac{mean}{extreme}$ **Remember the rule:** In a proportion, the product of the means equals the product of the extremes ("cross multiply").

Examples:

1. Find the mean proportional between 2 and 18?

$\frac{2}{x} = \frac{x}{18}$; $x^2 = 36$; $x = \pm 6$

While algebraically there are two solutions for x, the mean proportional, by definition, is positive.
Answer: $x = 6$

Check answer:
$\frac{2}{6} = \frac{6}{18} \rightarrow 36 = 36$

---

2. What is the geometric mean of 6 and 12?

$\frac{6}{x} = \frac{x}{12}$; $x^2 = 72$; $x = \pm\sqrt{72} = \pm 6\sqrt{2}$

Remember, the geometric mean of two positive numbers is positive.
Answer: $\sqrt{72} = 6\sqrt{2}$

Check answer:
$\frac{6}{\sqrt{72}} = \frac{\sqrt{72}}{12} \rightarrow 72 = 72$

You can see from Example 2 that it will be necessary to remember your skills for working with radicals. If you need a refresher, go to the "Radicals" section under Algebra 1.

---

3. The mean proportional between two values is 8. If one of the values is 16, find the other value.

$\frac{a}{8} = \frac{8}{16}$; $16a = 64$; $a = 4$
Answer: $a = 4$

Check the answer:
$\frac{4}{8} = \frac{8}{16} \rightarrow 64 = 64$

*** FYI:** (More information about the **DEFINITON** of geometric mean or mean proportional)

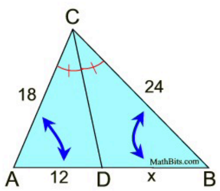
In the most general sense, a *geometric mean* is a type of mean which indicates the central tendency, or "typical value", of a set of numbers by using the "product" of the values in the set. If all of the numbers in the set are replaced by the same

The "Angle Bisector" Theorem says that an angle bisector of a triangle will divide the opposite side into two segments that are proportional to the other two sides of the triangle.

BEWARE Be sure to set up the proportion correctly. While proportions can be re-written into various forms, be sure to start with a correct arrangement. For example, in the diagram shown below, a correct proportion may be:

$$\frac{AD}{AC}=\frac{DB}{BC} \text{ or } \frac{AD}{DB}=\frac{AC}{BC} \text{ but } \cancel{\frac{AD}{DB}=\frac{BC}{AC}} \text{ nor } \cancel{\frac{AD}{BC}=\frac{DB}{AC}}.$$

Find x.



It may be helpful to remember that the segments formed by the angle bisector will form a correct proportion when paired with their adjacent triangle sides, such as:

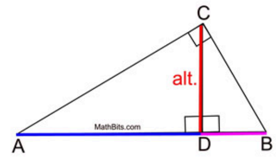
$$\frac{AD}{AC}=\frac{DB}{BC} \qquad \cancel{\frac{AD}{BC}=\frac{DB}{AC}}$$

$$\frac{12}{18}=\frac{x}{24} \qquad \frac{12}{24}=\frac{x}{18}$$

$$18x=288 \qquad 24x=216$$

$x=16$ (Answer) $\qquad$ $x \neq 9$ (NOT Answer)

THEOREM: The **altitude** to the hypotenuse of a right triangle is the **mean proportional** between the segments into which it divides the hypotenuse.



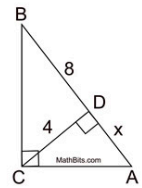
$$\frac{AD}{CD}=\frac{CD}{DB}$$

Altitude Rule:

$$\frac{\text{part of hyp}}{\text{altitude}}=\frac{\text{altitude}}{\text{other part hyp}}$$

Notice the triangle used with this rule! It is the same diagram used in the first theorem on this page - a right triangle with an altitude drawn to its hypotenuse.

Find x.



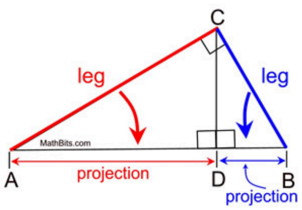
Solution:
Examine the diagram to see what information is given. In this problem, the "legs" are NOT labeled, but the "altitude" is labeled.

This is the "altitude rule".

$$\frac{8}{4}=\frac{4}{x}$$
$$8x=16$$
$$x=2$$

THEOREM: The **leg** of a right triangle is the **mean proportional** between the hypotenuse and the projection of the leg on the hypotenuse.

The "*projection*" of a leg is that segment of the hypotenuse which is attached to (adjacent to) the leg. A projection is formed by dropping a perpendicular from the end of the segment (leg) to the hypotenuse. Think of a projection as a "shadow" -- $\overline{AB}$ is the ground with the sun directly overhead and the shadow of $\overline{AC}$ is $\overline{AD}$.



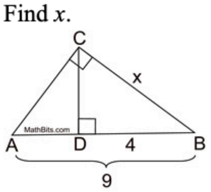
$$\frac{AB}{AC}=\frac{AC}{AD} \text{ or } \frac{AB}{CB}=\frac{CB}{DB}$$

Leg Rule:

$$\frac{\text{hypotenuse}}{\text{leg}}=\frac{\text{leg}}{\text{projection}}$$

Notice the triangle used with this rule! It is the same diagram used in the first theorem on this page - a right triangle with an altitude drawn to its hypotenuse. (Also same diagram as the Altitude Rule.)

Find x.



Solution:
Look carefully at the diagram. In this problem, the "altitude" is NOT labeled, but a "leg" is labeled.

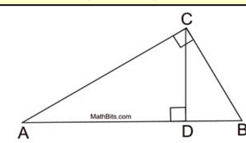
This is the "leg rule".

$$\frac{9}{x}=\frac{x}{4}$$
$$x^2=36$$
$$x=\pm 6$$
$$x=6$$

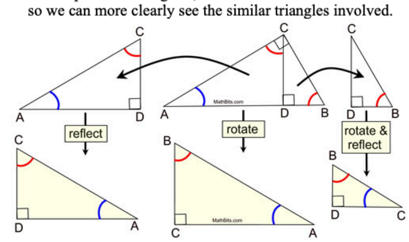
The length of the side of a triangle is a positive value.

THEOREM: The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

If right ΔABC, altitude $\overline{CD}$,
then $\Delta ADC \sim \Delta CDB$,
$\Delta ACB \sim \Delta ADC$, and
$\Delta ACB \sim \Delta CDB$.



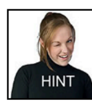
Let's separate the diagram, and move the sections around so we can more clearly see the similar triangles involved.



- $\Delta ACB \sim \Delta ADC$ by AA (Angle Angle Postulate) - each Δ has a right angle and share $\angle A$.
- $\Delta ACB \sim \Delta CDB$ by AA (Angle Angle Postulate) - each Δ has a right angle and share $\angle B$.
- We can establish that $\angle B \cong \angle ACD$ because they are each complementary to $\angle DCB$. $\Delta ADC \sim \Delta CDB$ by AA - each Δ has a right angle and $\angle B \cong \angle ACD$.

Since these triangles are similar, we can establish a series of proportions relating their corresponding sides. Two valuable theorems are formed using 3 of these proportions:

$$\frac{AB}{AC}=\frac{AC}{AD};\quad \frac{AB}{CB}=\frac{CB}{DB};\quad \frac{AD}{CD}=\frac{CD}{DB}$$



Remember the "look" of the given diagram for this theorem. If you "forget" the rules stated in the following theorems, you can simply recall this original diagram and set up the corresponding sides of the three similar triangles.

Once the triangles are similar ...

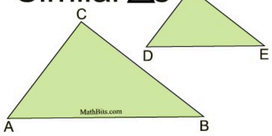
the remaining sets of angles will be congruent and the remaining corresponding sides will be in proportion. Just be sure the triangles are similar before using the following theorem.

THEOREM: The corresponding sides of similar triangles are in proportion.

Similar∆s

If: $\Delta ABC \sim \Delta DEF$

Then: $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$



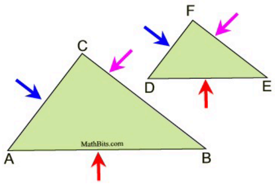
SSS

To prove two triangles are similar, it is sufficient to show that the three sets of corresponding sides are in proportion.

THEOREM: If the three sets of corresponding sides of two triangles are in proportion, the triangles are similar.

If: $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

Then: $\Delta ABC \sim \Delta DEF$



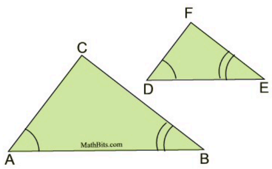
AA

To prove two triangles are similar, it is sufficient to show that two angles of one triangle are congruent to the two corresponding angles of the other triangle.

THEOREM: If two angles of one triangle are congruent to the corresponding angles of another triangle, the triangles are similar. (proof of this theorem is shown below)

If: $\angle A \cong \angle D$ and $\angle B \cong \angle E$

Then: $\Delta ABC \sim \Delta DEF$



SAS

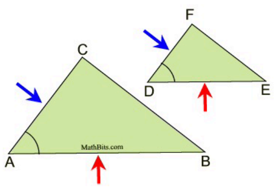
To prove two triangles are similar, it is sufficient to show that two sets of corresponding sides are in proportion and the angles they include are congruent.

THEOREM: If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

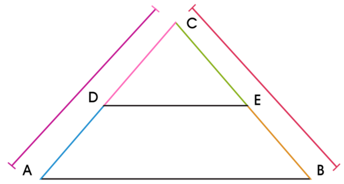
If: $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

and $\angle A \cong \angle D$

Then: $\Delta ABC \sim \Delta DEF$



1. If a segment is parallel to one side of a triangle and intersects the other two sides, then the triangle formed is similar to the original and the segment that divides the two sides it intersects is proportional.

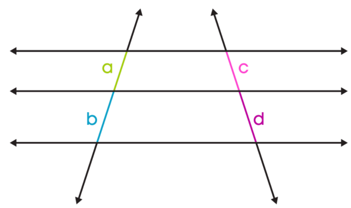


$\dfrac{DC}{DA} = \dfrac{EC}{EB}$ or $\dfrac{DC}{AC} = \dfrac{EC}{BC}$

Calcworkshop.com

Proportional Segment Theorem

2. If three parallel lines intersect two transversals, then they divide the transversals proportionally.

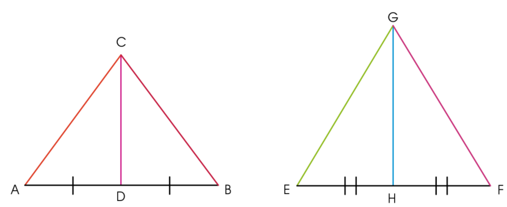


$\dfrac{a}{b} = \dfrac{c}{d}$

Calcworl

Proportional Transversal Theorem

3. The corresponding medians are proportional to their corresponding sides.

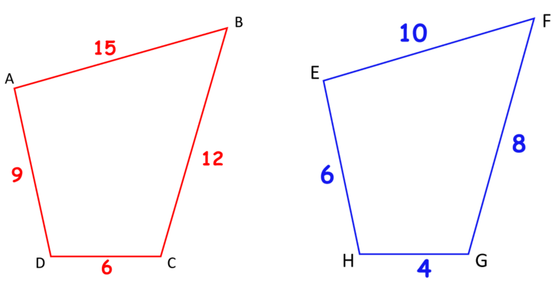


$\dfrac{CD}{GH} = \dfrac{AC}{EG}$ or $\dfrac{CD}{GH} = \dfrac{BC}{FG}$

Calcworkshop.co

Corresponding Medians

5. The perimeters of similar polygons are proportional to their corresponding sides.

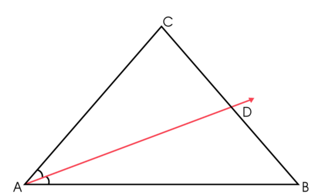


Scale Factor = $\dfrac{3}{2}$ and $\dfrac{P_{ABCD}}{P_{EFGH}} = \dfrac{42}{28} = \dfrac{3}{2}$

Calcworkshop.

Perimeter of Similar Polygons

4. If a ray bisects an angle or a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



If $\overrightarrow{AD}$ bisects $\angle CAB$, then $\dfrac{DC}{CA} = \dfrac{DB}{BA}$

Calcworkshop.

Ray Bisecting a Triangle Creating Proportional Sides