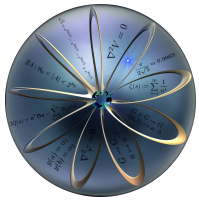

planetmath.org

Math for the people, by the people.

a series related to harmonic series

Canonical name	ASeriesRelatedToHarmonicSeries
Date of creation	2013-03-22 17:56:40
Last modified on	2013-03-22 17:56:40
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	5
Author	pahio (2872)
Entry type	Example
Classification	msc 40A05
Related topic	PTest
Related topic	RaabesCriteria

The series

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}} \tag{1}$$

is divergent. In fact, since for every positive integer n, one has $2^n > n$, i.e. $\sqrt[n]{n} < 2$, any of the series satisfies

$$\frac{1}{n\sqrt[n]{n}} > \frac{1}{2n}.$$

Because the harmonic series and therefore also $\sum_1^{\infty} \frac{1}{2n}$ diverges, the comparison test implies that the series (1) diverges.