series

Canonical name	Series
Date of creation	2013-03-22 12:41:35
Last modified on	2013-03-22 12:41:35
Owner	mathwizard (128)
Last modified by	mathwizard (128)
Numerical id	7
Author	mathwizard (128)
Entry type	Definition
Classification	msc 40-00
Related topic	AbsoluteConvergence
Related topic	HarmonicNumber
Related topic	CompleteUltrametricField
Related topic	Summation
Related topic	PrimeHarmonicSeries

Given a sequence of numbers (real or complex) $\{a_n\}$ we define a sequence of *partial sums* $\{S_N\}$, where $S_N = \sum_{n=1}^{N} a_n$. This sequence is called the *series* with *terms* a_n. We define the *sum of the series* $\sum_{n=1}^{\infty} a_n$ to be the limit of these partial sums. More precisely

$$\sum_{n=1}^{\infty} a_n = \lim_{N \to \infty} S_n = \lim_{N \to \infty} \sum_{n=1}^{N} a_n.$$

In a context where this distinction does not matter much (this is usually the case) one identifies a series with its sum, if the latter exists.

Traditionally, as above, series are infinite sums of real numbers. However, the formal constraints on the terms $\{a_n\}$ are much less strict. We need only be able to add the terms and take the limit of partial sums. So in full generality the terms could be complex numbers or even elements of certain rings, fields, and vector spaces.