manipulating convergent series

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The of the series in the following theorems are supposed to be either real or complex numbers.

Theorem 1. If the series $a_1 + a_2 + \cdots$ and $b_1 + b_2 + \cdots$ converge and have the sums a and b, respectively, then also the series

$$(a_1 + b_1) + (a_2 + b_2) + \cdots \tag{1}$$

converges and has the sum $a+b$.

Proof. The n^{th} partial sum of (1) has the limit

$$\lim_{n\to\infty} \sum_{j=1}^{n} (a_j + b_j) = \lim_{n\to\infty} \sum_{j=1}^{n} a_j + \lim_{n\to\infty} \sum_{j=1}^{n} b_j = a+b.$$

Theorem 2. If the series $a_1 + a_2 + \cdots$ converges having the sum a and if c is any , then also the series

$$ca_1 + ca_2 + \cdots \tag{2}$$

converges and has the sum ca.

Proof. The n^{th} partial sum of (2) has the limit

$$\lim_{n\to\infty} \sum_{j=1}^{n} ca_j = c \lim_{n\to\infty} \sum_{j=1}^{n} a_j = ca.$$

Theorem 3. If the of any converging series

$$a_1 + a_2 + a_3 + \cdots \tag{3}$$

are grouped arbitrarily without changing their , then the resulting series

$$(a_1 + \cdots + a_{m_1}) + (a_{m_1+1} + \cdots + a_{m_2}) + (a_{m_2+1} + \cdots + a_{m_3}) + \cdots \tag{4}$$

also converges and its sum equals to the sum of (3).

Proof. Since all the partial sums of (4) are simultaneously partial sums of (3), they have as limit the sum of the series (3).