Stolz-Cesaro theorem

Canonical name	StolzCesaroTheorem
Date of creation	2013-03-22 13:17:16
Last modified on	2013-03-22 13:17:16
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	9
Author	CWoo (3771)
Entry type	Theorem
Classification	msc 40A05
Related topic	CesaroMean
Related topic	ExampleUsingStolzCesaroTheorem
Related topic	KroneckersLemma

Let $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ be two sequences of real numbers. If b_n is positive, strictly increasing and unbounded and the following limit exists:

$$\lim_{n\to\infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = l$$

Then the limit:

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$

also exists and it is equal to l.

Remark. This theorem is also valid if a_n and b_n are monotone, tending to 0.