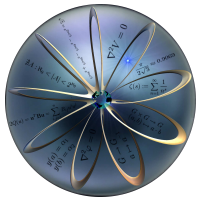

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examples using comparison test without limit

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Defines	over-harmonic series

Do the following series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n+1} \tag{1}$$

$$\sum_{n=1}^{\infty} \frac{n^3+n+1}{n^4+n+1} \tag{2}$$

The general of (1) may be estimated upwards:

$$0 < \frac{1}{n^2+n+1} < \frac{1}{n^2+0+0} = \frac{1}{n^2}$$

Because $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (an *over-harmonic series*) converges, then also (1) converges.

The general of (2) may be estimated downwards:

$$\frac{n^3+n+1}{n^4+n+1} > \frac{n^3+0+0}{n^4+n^4+n^4} = \frac{1}{3} \cdot \frac{1}{n} > 0$$

Because $\sum_{n=1}^{\infty} \frac{1}{3}\frac{1}{n}$ (the harmonic series divided by 3) diverges, then also (2) diverges.