Furstenberg-Kesten theorem

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Consider μ a probability measure, and $f : M \to M$ a measure preserving dynamical system. Consider $A : M \to GL(d, \mathbf{R})$, a measurable transformation, where GL(d,**R**) is the space of invertible square matrices of size d. Consider the multiplicative cocycle $(\phi^n(x))_n$ defined by the transformation A.

If $\log^+ ||A||$ is integrable, where $\log^+ ||A|| = \max\{\log ||A||, 0\}$, then:

$$\lambda_{\max}(x) = \lim_n \frac{1}{n} \log ||\phi^n(x)||$$

exists almost everywhere, and $\lambda_{\max}^+$ is integrable and

$$\int \lambda_{\max} d\mu = \lim_n \frac{1}{n} \int \log ||\phi^n|| d\mu = \inf_n \frac{1}{n} \int \log ||\phi^n|| d\mu$$

If $\log^+ ||A^{-1}||$ is integrable, then:

$$\lambda_{\min}(x) = \lim_n -\frac{1}{n} \log ||\phi^{-n}(x)||$$

exists almost everywhere, and $\lambda_{\min}^+$ is integrable and

$$\int \lambda_{\min} d\mu = \lim_n -\frac{1}{n} \int \log ||\phi^{-n}|| d\mu = \sup_n -\frac{1}{n} \int \log ||\phi^{-n}|| d\mu$$

Furthermore, both $\lambda_{\min}$ and $\lambda_{\max}$ are invariant for the tranformation f, that is, $\lambda_{\min} \circ f(x) = \lambda_{\min}(x)$ and $\lambda_{\max} \circ f(x) = \lambda_{\max}(x)$, for μ almost everywhere.

This theorem is a direct consequence of Kingman's subadditive ergodic theorem, by observing that both

$$\log ||\phi^n(x)||$$

and

$$\log ||\phi^{-n}(x)||$$

are subadditive sequences.

The results in this theorem are strongly improved by Oseledet's multiplicative ergodic theorem, or Oseledet's decomposition.