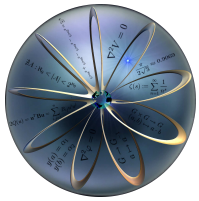

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proof of comparison test

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Assume $|a_k| \leq b_k$ for all $k > n$. Then we define

$$s_k := \sum_{i=k}^{\infty} |a_i|$$

and

$$t_k := \sum_{i=k}^{\infty} b_i.$$

Obviously $s_k \leq t_k$ for all $k > n$. Since by assumption (t_k) is `http://planetmath.org/node/601`conv
(t_k) is bounded and so is (s_k). Also (s_k) is monotonic and therefore . Therefore $\sum_{i=0}^{\infty} a_i$ is absolutely convergent.

Now assume $b_k \leq a_k$ for all $k > n$. If $\sum_{i=k}^{\infty} b_i$ is divergent then so is $\sum_{i=k}^{\infty} a_i$ because otherwise we could apply the test we just proved and show that $\sum_{i=0}^{\infty} b_i$ is convergent, which is is not by assumption.