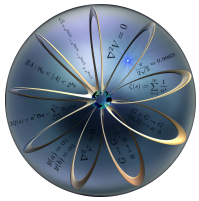

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trick to sum all the reciprocal triangular numbers

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The following trick to sum all the reciprocals of the triangular numbers is funny:

$$\begin{aligned}
\sigma &= 1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\frac{1}{28}+\frac{1}{36}+\frac{1}{45}+\frac{1}{55}+\frac{1}{66}+\frac{1}{78}+... \\
&= 1+\left(\frac{1}{3}+\frac{1}{6}\right)+\left(\frac{1}{10}+\frac{1}{15}\right)+\left(\frac{1}{21}+\frac{1}{28}\right)+\left(\frac{1}{36}+\frac{1}{45}\right)+\left(\frac{1}{55}+\frac{1}{66}\right)+... \\
&= 1+\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{10}+\frac{1}{2}\cdot\frac{1}{15}+... \\
&= 1+\frac{1}{2}\left(1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+...\right)
\end{aligned}$$

which implies $\sigma = 1+\sigma/2$ and hence

$$1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\frac{1}{28}+\frac{1}{36}+\frac{1}{45}+\frac{1}{55}+\frac{1}{66}+\frac{1}{78}+... = 2$$