# finite changes in convergent series

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The following theorem means that at the beginning of a convergent series, one can remove or attach a finite amount of terms without influencing on the convergence of the series – the convergence is determined alone by the infinitely long "tail" of the series. Consequently, one can also freely change the  of a finite amount of terms.

**Theorem.** Let $k$ be a natural number. A series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges iff the series $\displaystyle\sum_{n=k+1}^{\infty} a_n$ converges. Then the sums of both series are  with

$$\sum_{n=k+1}^{\infty} a_n \;=\; \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{k} a_n. \tag{1}$$

*Proof.* Denote the $k$th partial sum of $\sum_{n=1}^{\infty} a_n$ by $S_k$ and the $n$th partial sum of $\sum_{n=k+1}^{\infty} a_n$ by $S'_n$. Then we have

$$S'_n \;=\; \sum_{n=k+1}^{k+n} a_n \;=\; S_{k+n} - S_k. \tag{2}$$

1°. If $\sum_{n=1}^{\infty} a_n$ converges, i.e. $\lim_{n\to\infty} S_n := S$ exists as a finite number, then (2) implies

$$\lim_{n\to\infty} S'_n \;=\; \lim_{n\to\infty} S_{k+n} - \lim_{n\to\infty} S_k \;=\; S - S_k.$$

Thus $\sum_{n=k+1}^{\infty} a_n$ converges and (1) is true.

2°. If we suppose $\sum_{n=k+1}^{\infty} a_n$ to be convergent, i.e. $\lim_{n\to\infty} S'_n := S'$ exists as finite, then (2) implies that

$$\lim_{n\to\infty} S_n \;=\; \lim_{n\to\infty} S_{k+n} \;=\; \lim_{n\to\infty} (S_k + S'_n) \;=\; S_k + S'.$$

This means that $\sum_{n=1}^{\infty} a_n$ is convergent and $S = S_k + S'$, which is (1), is in .