almost convergent

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A real sequence (x_n) is said to be **almost convergent** to L if each Banach limit assigns the same value L to the sequence (x_n).

Lorentz [**?**] proved that (x_n) is almost convergent to L if and only if

$$\lim_{p \to \infty} \frac{x_n + \ldots + x_{n+p-1}}{p} = L$$

uniformly in n.

The above limit can be rewritten in detail as

$$(\forall \varepsilon > 0)(\exists p_0)(\forall p > p_0)(\forall n) \left| \frac{x_n + \ldots + x_{n+p-1}}{p} - L \right| < \varepsilon.$$

Almost convergence is studied in summability theory. It is an example of a summability method which cannot be represented as a matrix method.

References

[1] G. Bennett and N.J. Kalton: Consistency theorems for almost convergence. *Trans. Amer. Math. Soc.*, 198:23–43, 1974.

[2] J. Boos: *Classical and modern methods in summability.* Oxford University Press, New York, 2000.

[3] Jeff Connor and K.-G. Grosse-Erdmann: Sequential definitions of continuity for real functions. *Rocky Mt. J. Math.*, 33(1):93–121, 2003.

[4] G. G. Lorentz: A contribution to the theory of divergent sequences. *Acta Math.*, 80:167–190, 1948.