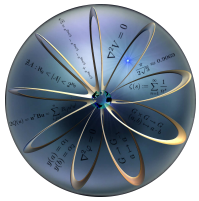

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## uniqueness of limit of sequence

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If a number sequence has a limit, then the limit is uniquely determined.

*Proof.* For an `http://planetmath.org/ReductioAdAbsurdum`indirect proof, suppose that a sequence

$$a_1,\ a_2,\ a_3,\ \ldots$$

has two distinct limits $a$ and $b$. Thus we must have both

$$|a_n - a| < \frac{|a-b|}{2} \quad \text{for all } n > \text{ some } n_1$$

and

$$|a_n - b| < \frac{|a-b|}{2} \quad \text{for all } n > \text{ some } n_2$$

But when $n$ exceeds the greater of $n_1$ and $n_2$, we can write

$$|a-b| \ = \ |a-a_n+a_n-b| \ \leqq \ |a-a_n| + |a_n-b| \ < \ \frac{|a-b|}{2} + \frac{|a-b|}{2} \ = \ |a-b|.$$

This inequality  an impossibility, whence the antithesis made in the begin is wrong and the assertion is .