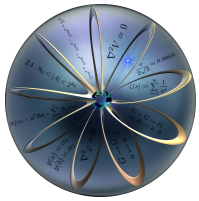

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## proof of absolute convergence theorem

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Suppose that $\sum a_n$ is absolutely convergent, i.e., that $\sum |a_n|$ is convergent. First of all, notice that

$$0 \leq a_n + |a_n| \leq 2|a_n|,$$

and since the series $\sum(a_n + |a_n|)$ has non-negative terms it can be compared with $\sum 2|a_n| = 2\sum |a_n|$ and hence converges.

On the other hand

$$\sum_{n=1}^{N} a_n = \sum_{n=1}^{N}(a_n + |a_n|) - \sum_{n=1}^{N} |a_n|.$$

Since both the partial sums on the right hand side are convergent, the partial sum on the left hand side is also convergent. So, the series $\sum a_n$ is convergent.