

Below, we list some <http://planetmath.org/ConvergenceOfIntegralsconvergent> improper integrals.

$$\text{http://planetmath.org/areaundergaussiancurve1.} \quad \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

$$\text{http://planetmath.org/generalisationofgaussianintegral2.} \quad \int_0^\infty e^{-x^2} \cos kx dx = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{4}k^2}$$

$$\text{http://planetmath.org/usingconvolutiontofindlaplacetransform3.} \quad \int_0^\infty \frac{e^{-x^2}}{a^2+x^2} dx = \frac{\pi}{2a} e^{a^2} \operatorname{erfc} a$$

$$\text{http://planetmath.org/fresnelformulas4.} \quad \int_0^\infty \sin x^2 dx = \int_0^\infty \cos x^2 dx = \frac{\sqrt{2\pi}}{4}$$

$$\text{http://planetmath.org/sineintegralatinfinity5.} \quad \int_0^\infty \frac{\sin ax}{x} dx = (\operatorname{sgn} a) \frac{\pi}{2} \quad (a \in \mathbb{R})$$

$$\text{http://planetmath.org/twoimproperintegrals6.} \quad \int_0^\infty \left(\frac{\sin x}{x} \right)^2 dx = \frac{\pi}{2}$$

$$\text{http://planetmath.org/twoimproperintegrals7.} \quad \int_0^\infty \frac{1 - \cos kx}{x^2} dx = \frac{\pi k}{2}$$

$$\text{http://planetmath.org/usingresiduethetheoremnearbranchpoint8.} \quad \int_0^\infty \frac{x^{-k}}{x+1} dx = \frac{\pi}{\sin \pi k} \quad (0 < k < 1)$$

$$\text{http://planetmath.org/exampleofchangingvariable9.} \quad \int_{-\infty}^\infty \frac{e^{kx}}{1+e^x} dx =$$

$$\frac{\pi}{\sin \pi k} \quad (0 < k < 1)$$

$$\text{http://planetmath.org/exampleofusingresidueththeorem10.} \quad \int_0^\infty \frac{\cos kx}{x^2+1} dx = \frac{\pi}{2e^k}$$

$$\text{http://planetmath.org/laplaceintegrals11.} \quad \int_0^\infty \frac{a \cos x}{x^2+a^2} dx = \int_0^\infty \frac{x \sin x}{x^2+a^2} dx = \frac{\pi}{2e^a} \quad (a > 0)$$

$$\text{http://planetmath.org/applicationofsineintegralatinfinit12.} \quad \int_0^\infty \frac{\sin ax}{x(x^2+1)} dx = \frac{\pi}{2}(1 - e^{-a}) \quad (a > 0)$$

$$\text{http://planetmath.org/node/922313.} \quad \int_0^\infty e^{-x} x^{-\frac{3}{2}} dx = \sqrt{\pi}$$

$$\text{http://planetmath.org/laplacetransformoftnft14.} \quad \int_0^\infty e^{-x} x^3 \sin x dx = 0$$

$$\text{http://planetmath.org/node/789115.} \quad \int_0^\infty \left(\frac{1}{e^x-1} - \frac{1}{xe^x} \right) dx = \gamma$$

$$\text{http://planetmath.org/relativeofcosineintegral16.} \quad \int_0^\infty \frac{\cos ax^2 - \cos ax}{x} dx = \frac{\gamma + \ln a}{2} \quad (a > 0)$$

$$\text{http://planetmath.org/relativeofexponentialintegral17.} \quad \int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \ln \frac{b}{a} \quad (a > 0, b > 0)$$

$$\text{http://planetmath.org/integralrelatedtoarcsine18.} \quad \int_1^\infty \left(\arcsin \frac{1}{x} - \frac{1}{x} \right) dx = 1 + \ln 2 - \frac{\pi}{2}$$

$$\text{http://planetmath.org/exampleofimproperintegral19.} \quad \int_0^1 \frac{\arctan x}{x\sqrt{1-x^2}} dx = \frac{\pi}{2} \ln(1+\sqrt{2}) = \frac{\pi}{2} \operatorname{arsinh} 1$$

<http://planetmath.org/applicationoflogarithmseries>**20.** $\int_0^1 \frac{\ln(1+x)}{x} dx =$
 $\frac{\pi^2}{12}$
 21. $\int_{\frac{1}{2}}^1 \frac{\ln(1-x)}{x^2} dx = -2 \ln 2$