



convergence of a sequence with finite upcrossings

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The following result characterizes convergence of a sequence in terms of finiteness of numbers of upcrossings.

Theorem. *A sequence x_1, x_2, \dots of real numbers converges to a limit in the extended real numbers if and only if the number of upcrossings $U[a, b]$ is finite for all $a < b$.*

Since the number of upcrossings $U[a, b]$ differs from the number of downcrossings $D[a, b]$ by at most one, the theorem can equivalently be stated in terms of the finiteness of $D[a, b]$.