



Math for the people, by the people.

$(1 + \frac{\alpha}{n})^n$ is monotone for large n

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Lemma. *Let α be a real number. The sequence $(1 + \frac{\alpha}{n})^n$ is monotone increasing for all $n > |\alpha|$.*

Proof. Let $n > |\alpha|$. We want to prove the following inequality:

$$\left(1 + \frac{\alpha}{n}\right)^n \leq \left(1 + \frac{\alpha}{n+1}\right)^{n+1}$$

Since both sides are positive, this follows by taking the $(n+1)$ -th root and using the arithmetic-geometric-harmonic means inequality:

$$\sqrt[n+1]{\left(1 + \frac{\alpha}{n}\right)^n} = \sqrt[n+1]{\underbrace{1 \cdot \left(1 + \frac{\alpha}{n}\right) \cdots \left(1 + \frac{\alpha}{n}\right)}_{n+1 \text{ elements}}} \leq \frac{1 + n \left(1 + \frac{\alpha}{n}\right)}{n+1} = 1 + \frac{\alpha}{n+1}$$

□