# application of Cauchy criterion for convergence



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Without using the methods of the entry determining series convergence, we show that the real-term series

$$\sum_{n=0}^{\infty}\frac{1}{n!} \;=\; 1+\frac{1}{1!}+\frac{1}{2!}+\ldots$$

is convergent by using Cauchy criterion for convergence, being in in $\mathbb{R}$ equipped with the usual absolute value $|.|$ as `http://planetmath.org/node/1604`norm.

Let $\varepsilon$ be an arbitrary positive number. For any positive integer $n$, we have

$$\frac{1}{n!} \;\leqq\; \frac{1}{1\cdot2\cdot2\cdots2} \;=\; \frac{1}{2^{n-1}},$$

whence we can as follows.

$$\begin{aligned}
\left|\frac{1}{(n+1)!}+\ldots+\frac{1}{(n+p)!}\right| &= \frac{1}{(n+1)!}+\ldots+\frac{1}{(n+p)!}\\
&\leqq \frac{1}{2^n}+\ldots+\frac{1}{2^{n+p-1}}\\
&= \frac{1}{2^n}\left(1+\frac{1}{2}+\ldots+\frac{1}{2^{p-1}}\right)\\
&= \frac{1}{2^n}\cdot\frac{1-(1/2)^p}{1-1/2}\\
&< \frac{1}{2^{n-1}} \;<\; \varepsilon
\end{aligned}$$

The last inequality is true for all positive integers $p$, when $n \;>\; 1-\operatorname{lb}\varepsilon$. Thus the Cauchy criterion implies that the series converges.