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Proof of Baroni's theorem

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Let $m = \inf A'$ and $M = \sup A'$. If $m = M$ we are done since the sequence is convergent and A' is the degenerate interval composed of the point $l \in \overline{\mathbb{R}}$, where $l = \lim_{n\to\infty} x_n$.

Now , assume that $m < M$. For every $\lambda \in (m, M)$, we will construct inductively two subsequences x_{k_n} and x_{l_n} such that $\lim_{n\to\infty} x_{k_n} = \lim_{n\to\infty} x_{l_n} = \lambda$ and $x_{k_n} < \lambda < x_{l_n}$

From the definition of M there is an $N_1 \in \mathbb{N}$ such that :

$$\lambda < x_{N_1} < M$$

Consider the set of all such values N_1 . It is bounded from below (because it consists only of natural numbers and has at least one element) and thus it has a smallest element . Let n_1 be the smallest such element and from its definition we have $x_{n_1-1} \leq \lambda < x_{n_1}$. So , choose $k_1 = n_1 - 1$, $l_1 = n_1$. Now, there is an $N_2 > k_1$ such that :

$$\lambda < x_{N_2} < M$$

Consider the set of all such values N_2 . It is bounded from below and it has a smallest element n_2 . Choose $k_2 = n_2 - 1$ and $l_2 = n_2$. Now , proceed by induction to construct the sequences k_n and l_n in the same fashion . Since $l_n - k_n = 1$ we have :

$$\lim_{n\to\infty} x_{k_n} = \lim_{n\to\infty} x_{l_n}$$

and thus they are both equal to λ.