zero sequence

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Let a field k be equipped with a rank one valuation $|.|$. A sequence

$$\langle a_1,\, a_2,\, \ldots\rangle \tag{1}$$

of elements of k is called a *zero sequence* or a *null sequence*, if $\lim_{n\to\infty} a_n = 0$ in the metric induced by $|.|$.

If k together with the metric induced by its valuation $|.|$ is a complete ultrametric field, it's clear that its sequence (1) has a limit (in k) as soon as the sequence

$$\langle a_2 - a_1,\, a_3 - a_2,\, a_4 - a_3,\, \ldots\rangle$$

is a zero sequence.

If k is not complete with respect to its valuation $|.|$, its http://planetmath.org/Completioncon can be made as follows. The Cauchy sequences (1) form an integral domain D when the operations "+" and "·" are defined componentwise. The subset P of D formed by the zero sequences is a maximal ideal, whence the quotient ring D/P is a field K. Moreover, k may be isomorphically embedded into K and the valuation $|.|$ may be uniquely extended to a valuation of K. The field K then is complete with respect to $|.|$ and k is dense in K.