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multiplicative cocycle

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Entry type	Definition
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Defines	multiplicative cocycle

Let  $f : M \rightarrow M$  be a measurable transformation, and let  $\mu$  be an invariant probability measure. Consider  $A : M \rightarrow GL(d, \mathbf{R})$ , a measurable transformation, where  $GL(d, \mathbf{R})$  is the space of invertible square matrices of size  $d$ . We define  $A^{-1} : M \rightarrow GL(d, \mathbf{R})$  by  $A^{-1}(x) = [A(x)]^{-1}$ . Then we define the sequence of functions:

$$\phi^n(x) = A(f^{n-1}(x)) \cdots A(f(x))A(x)$$

$$\phi^{-n}(x) = [\phi^n(f^{-n}(x))]^{-1}$$

for  $n \geq 1$  and  $x \in M$ .

It is easy to verify that:

$$\phi^{m+n}(x) = \phi^n(f^m(x))\phi^m(x)$$

for  $n, m \in \mathbf{Z}$  and  $x \in M$ .

The sequence  $(\phi^n)_n$  is called a multiplicative cocycle, or just cocycle defined by the transformation  $A$ .