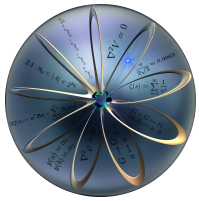

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arithmetic series

Canonical name	ArithmeticSeries
Date of creation	2013-03-22 16:17:58
Last modified on	2013-03-22 16:17:58
Owner	georgiosl (7242)
Last modified by	georgiosl (7242)
Numerical id	10
Author	georgiosl (7242)
Entry type	Definition
Classification	msc 40A05

An *arithmetic series* is the series, $\sum_{i=1}^{n} a_i$, in which each real term has the form $a_i = a_{i-1} + d$ for $i = 2, \ldots, n$ where d is constant. The sum of the sequence is given by the following $\frac{1}{2}n[2a_1 + d(n-1)]$. In order to find the formula above firstly we express the terms of the sequence, $a_2, \ldots, a_n$ in terms of a_1 and the constant d. In this case we get $a_2 = a_1 + d, a_3 = a_2 + 2d, \ldots, a_n = a_1 + (n-1)d$. Now we express the sum of the sequence by developing the series forward and we have:

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_1 + d + \cdots + a_1 + (n-2)d + a_1 + (n-1)d$$

Reversely, we develop the series backwards and we get

$$S_n = a_n - d + a_n - 2d + \cdots + a_n - (n-1)d$$

It is easily seen that by adding the two expressions we get

$$2S_n = n(a_1 + a_n) \tag{1}$$

$$S_n = \frac{1}{2}n(a_1 + a_n) \tag{2}$$

Hence, by substituting $a_n = a_1 + (n-1)d$ we get the first formula.