examples for limit comparison test

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Example 1. Does the following series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$$

The series is similar to $\sum_{n=1}^{\infty} 1/n^2$ which converges (use p-test, for example). Next we compute the limit:

$$\lim_{n\to\infty} \frac{\frac{1}{n^2+n+1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+n+1} = 1$$

Therefore, since $1 \neq 0$, by the Limit Comparison Test (with $a_n = 1/(n^2 + n+1)$ and $b_n = 1/n^2$), the series converges.

Example 2. Does the following series converge?

$$\sum_{n=1}^{\infty} \frac{n^3+n+1}{n^4+n+1}$$

If we "forget" about the lower order terms of n:

$$\frac{n^3+n+1}{n^4+n+1} \sim \frac{n^3}{n^4} = \frac{1}{n}$$

and $\sum_{n=1}^{\infty} 1/n$ is the harmonic series which diverges (by the p-test). Thus, we take $b_n = 1/n$ and compute:

$$\lim_{n\to\infty} \frac{\frac{n^3+n+1}{n^4+n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n(n^3+n+1)}{n^4+n+1} = \lim_{n\to\infty} \frac{n^4+n^2+n}{n^4+n+1} = \lim_{n\to\infty} \frac{1+1/n^2+1/n^3}{1+1/n^3+1/n^4} = 1$$

Therefore the series diverges like the harmonic does.