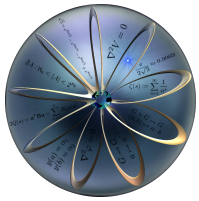

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proof of Cauchy's root test

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If for all $n \geq N$

$$\sqrt[n]{a_n} < k < 1$$

then

$$a_n < k^n < 1.$$

Since $\sum_{i=N}^{\infty} k^i$ converges so does $\sum_{i=N}^{\infty} a_n$ by the comparison test. If $\sqrt[n]{a_n} > 1$ then by comparison with $\sum_{i=N}^{\infty} 1$ the series is divergent. Absolute convergence in case of nonpositive a_n can be proven in exactly the same way using $\sqrt[n]{|a_n|}$.