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## characterization of convergence of sequences in metric spaces

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Let  $(M, d)$  be a metric space, and let  $\mathbb{N} = P_1 \cup \cdots \cup P_k$  be a partition of the set of natural numbers such that  $P_i$  is infinite for every  $i$ , that is, there is a bijection  $f_i: \mathbb{N} \rightarrow P_i$ . Then, given a sequence  $(x_n)_{n \in \mathbb{N}}$ , it converges to  $x \in M$  if and only if the subsequence

$$(x_{f_i(n)})_n$$

converges to  $x$  for every  $i = 1, \dots, k$ .

### **Examples**

If you have a sequence  $(x_n)_n$  and a natural number  $k$ , and you know that it converges to  $x$  for every corresponding subsequence over the classes of remainders modulo  $k$ , then it converges to  $x$ .