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limit comparison test

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The following theorem is a powerful test for convergence of series.

Theorem 1 (Limit). *Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be two series of positive numbers.*

1. *If the limit*

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$$

exists and $L \neq 0$ is a non-zero finite number, then both series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converge or both diverge.

2. *If $L = 0$ and $\sum_{n=0}^{\infty} b_n$ converges then $\sum_{n=0}^{\infty} a_n$ converges as well. If $L = 0$ and $\sum_{n=0}^{\infty} a_n$ diverges then $\sum_{n=0}^{\infty} b_n$ diverges as well.*
3. *Similarly, if the limit is infinite (" $L = \infty$ ") and $\sum_{n=0}^{\infty} a_n$ converges then $\sum_{n=0}^{\infty} b_n$ converges as well. If $L = \infty$ and $\sum_{n=0}^{\infty} b_n$ diverges then $\sum_{n=0}^{\infty} a_n$ diverges as well.*