boundedness of terms of power series

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Theorem. If the set

$$\{a_0,\ a_1c,\ a_2c^2,\ \ldots\}$$

of the of a power series

$$\sum_{n=0}^{\infty} a_n z^n$$

at the point $z = c$ is http://planetmath.org/BoundedIntervalbounded, then the power series converges, http://planetmath.org/AbsoluteConvergenceabsolutely, for any value z which satisfies

$$|z| < |c|.$$

Proof. By the assumption, there exists a positive number M such that

$$|a_n c^n| < M \quad \forall\, n\, =\, 0,\, 1,\, 2,\, \ldots$$

Thus one gets for the coefficients of the series the estimation

$$|a_n| < \frac{M}{|c|^n}.$$

If now $|z| < |c|$, one has

$$|a_n z^n| < M \left|\frac{z}{c}\right|^n,$$

and since the geometric series $\displaystyle\sum_{n=0}^{\infty} \left|\frac{z}{c}\right|^n$ is convergent, then also the real series $\displaystyle\sum_{n=0}^{\infty} |a_n z^n|$ converges.