harmonic series

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Defines	harmonic series of order

The *harmonic series* is

$$h = \sum_{n=1}^{\infty} \frac{1}{n}$$

The harmonic series is known to diverge. This can be proven via the integral test; compare h with

$$\int_1^{\infty} \frac{1}{x}\, dx.$$

The harmonic series is a special case of the *p-series*, h_p, which has the form

$$h_p = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

where p is some positive real number. The series is known to converge (leading to the p-series test for series convergence) iff $p > 1$. In using the comparison test, one can often compare a given series with positive terms to some h_p.

Remark 1. One could call h_p with $p > 1$ an *overharmonic series* and h_p with $p < 1$ an *underharmonic series*; the corresponding names are known at least in Finland.

Remark 2. A p-series is sometimes called *a* harmonic series, so that *the* harmonic series is a harmonic series with $p = 1$.

For complex-valued p, $h_p = \zeta(p)$, the Riemann zeta function.

A famous p-series is h_2 (or $\zeta(2)$), which converges to $\frac{\pi^2}{6}$. In general no p-series of odd p has been solved analytically.

A p-series which is not summed to ∞, but instead is of the form

$$h_p(k) = \sum_{n=1}^{k} \frac{1}{n^p}$$

is called a p-series (or a harmonic series) of order k of p.