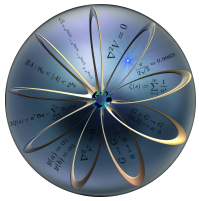

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proof of divergence of harmonic series (by splitting odd and even terms)

Canonical name	ProofOfDivergenceOfHarmonicSeriesbySplittingOddAndEvenTerms
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Suppose that the series $\sum_{n=1}^{\infty} 1/n$ converged. Since all the terms are positive, we could regroup them as we please, in particular, split the series into two series, that of even terms and that of odd terms:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{2n} + \sum_{n=1}^{\infty} \frac{1}{2n-1}$$

Since $\sum_{n=1}^{\infty} 1/n = 2\sum_{n=1}^{\infty} 1/(2n)$, we would conclude that

$$\sum_{n=1}^{\infty} \frac{1}{2n} = \sum_{n=1}^{\infty} \frac{1}{2n-1}.$$

But $2n-1 < 2n$, hence $1/(2n) < 1/(2n-1)$, so we would also have

$$\sum_{n=1}^{\infty} \frac{1}{2n} < \sum_{n=1}^{\infty} \frac{1}{2n-1},$$

which contradicts the previous conclusion. Thus, the assumption that the series converged is untenable.