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power tower sequence

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Defines	power tower sequence

For positive values of  $a$ , the *power tower sequence*

$$a, a^a, a^{a^a}, a^{a^{a^a}}, \dots$$

is convergent if and only if

$$\frac{1}{e^e} \leq a \leq e^{\frac{1}{e}},$$

approximately

$$0.065989 \leq a \leq 1.444667.$$

The limit of the sequence is the least real <http://planetmath.org/Equationroot> of the equation

$$a^x = x.$$

The proof is found in [1].

## References

- [1] E. LINDELÖF: *Differentiali- ja integralilasku ja sen sovellutukset III. Toinen osa.* Mercatorin Kirjapaino Osakeyhtiö, Helsinki (1940).