planetmath.org

Math for the people, by the people.

proof of Weierstrass' criterion of uniform convergence

Canonical name	ProofOfWeierstrassCriterionOfUniformConvergence
Date of creation	2013-03-22 16:26:28
Last modified on	2013-03-22 16:26:28
Owner	argerami (15454)
Last modified by	argerami (15454)
Numerical id	4
Author	argerami (15454)
Entry type	Proof
Classification	msc 40A30
Classification	msc 26A15

The assumption that $|f_n(x)| \leq M_n$ for every x guarantees that each numerical series $\sum_n f_n(x)$ converges absolutely. We call the limit $f(x)$.

To see that the convergence is uniform: let $\epsilon > 0$. Then there exists K such that $n > K$ implies $\sum_{n>K} M_n < \epsilon$. Now, if $k > K$,

$$|f(x) - \sum_{n=1}^{k} f_n(x)| = |\sum_{n>k} f_n(x)| \leq \sum_{n>k} |f_n(x)| \leq \sum_{n>k} M_n < \epsilon$$

The ϵ does not depend on x, so the convergence is uniform.