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## Cauchy's root test

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If  $\sum a_n$  is a series of positive real terms and

$$\sqrt[n]{a_n} < k < 1$$

for all n > N, then  $\sum a_n$  is convergent. If  $\sqrt[n]{a_n} \ge 1$  for an infinite number of values of n, then  $\sum a_n$  is divergent.

## Limit form

Given a series  $\sum a_n$  of complex terms, set

$$\rho = \limsup_{n \to \infty} \sqrt[n]{|a_n|}$$

The series  $\sum a_n$  is absolutely convergent if  $\rho < 1$  and is divergent if  $\rho > 1$ . If  $\rho = 1$ , then the test is inconclusive.