Cauchy product

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Let a_k and b_k be two sequences of real or complex numbers for $k \in \mathbb{N}_0$ ($\mathbb{N}_0$ is the set of natural numbers containing zero). The Cauchy product is defined by:

$$(a \circ b)(k) = \sum_{l=0}^{k} a_l b_{k-l}. \tag{1}$$

This is basically the convolution for two sequences. Therefore the product of two series $\sum_{k=0}^{\infty} a_k$, $\sum_{k=0}^{\infty} b_k$ is given by:

$$\sum_{k=0}^{\infty} c_k = \left(\sum_{k=0}^{\infty} a_k\right) \cdot \left(\sum_{k=0}^{\infty} b_k\right) = \sum_{k=0}^{\infty} \sum_{l=0}^{k} a_l b_{k-l}. \tag{2}$$

A sufficient condition for the resulting series $\sum_{k=0}^{\infty} c_k$ to be absolutely convergent is that $\sum_{k=0}^{\infty} a_k$ and $\sum_{k=0}^{\infty} b_k$ both converge absolutely .