telescoping sum

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Defines	telescope

A *telescoping sum* is a sum in which cancellation occurs between subsequent terms, allowing the sum to be expressed using only the initial and final terms.

Formally a telescoping sum is or can be rewritten in the form

$$S = \sum_{n=\alpha}^{\beta} (a_n - a_{n+1}) = a_\alpha - a_{\beta+1}$$

where a_n is a sequence.

Example:

Define $S(N) = \sum_{n=1}^{N} \frac{1}{n(n+1)}$. Note that by partial fractions of expressions:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

and thus $a_n = \frac{1}{n}$ in this example.

$$S(N) = \sum_{n=1}^{N} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$= \left(1 - \frac{1}{2}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \cdots + \left(\frac{1}{N} - \frac{1}{N+1}\right)$$

$$= 1 + \left(-\frac{1}{2} + \frac{1}{2}\right) + \cdots + \left(-\frac{1}{n+1} + \frac{1}{n+1}\right) + \cdots - \frac{1}{N+1}$$

$$= 1 - \frac{1}{N+1}$$