proof of Leibniz's theorem (using Dirichlet's convergence test)

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Proof. Let us define the sequence $\alpha_n = (-1)^n$ for $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$. Then

$$\sum_{i=0}^{n} \alpha_i = \begin{cases} 1 & \text{for even}\, n, \\ 0 & \text{for odd}\, n, \end{cases}$$

so the sequence $\sum_{i=0}^{n} \alpha_i$ is bounded. By assumption $\{a_n\}_{n=1}^{\infty}$ is a bounded decreasing sequence with limit 0. For $n \in \mathbb{N}$ we set $b_n := a_{n+1}$. Using Dirichlet's convergence test, it follows that the series $\sum_{i=0}^{\infty} \alpha_i b_i$ converges. Since

$$\sum_{i=0}^{\infty} \alpha_i b_i = \sum_{n=1}^{\infty} (-1)^{n+1} a_n,$$

the claim follows. $\square$