geometric series

Canonical name	GeometricSeries
Date of creation	2013-03-22 12:05:37
Last modified on	2013-03-22 12:05:37
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	16
Author	mathcam (2727)
Entry type	Definition
Classification	msc 40A05
Related topic	GeometricSequence
Related topic	ExampleOfAnalyticContinuation
Related topic	ApplicationOfCauchyCriterionForConvergence
Defines	infinite geometric series

A *geometric series* is a series of the form

$$\sum_{i=1}^{n} ar^{i-1}$$

(with a and r real or complex numbers). The partial sums of a geometric series are given by

$$s_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}. \tag{1}$$

An *infinite geometric series* is a geometric series, as above, with $n \to \infty$. It is denoted by

$$\sum_{i=1}^{\infty} ar^{i-1}$$

If $|r| \geq 1$, the infinite geometric series diverges. Otherwise it converges to

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \tag{2}$$

Taking the limit of s_n as $n \to \infty$, we see that s_n diverges if $|r| \geq 1$. However, if $|r| < 1$, s_n approaches (2).

One way to prove (1) is to take

$$s_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

and multiply by r, to get

$$rs_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n$$

subtracting the two removes most of the terms:

$$s_n - rs_n = a - ar^n$$

factoring and dividing gives us

$$s_n = \frac{a(1 - r^n)}{1 - r}$$

$\square$