proof of Abel's lemma (by induction)

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Proof. The proof is by induction. However, let us first recall that sum on the right side is a piece-wise defined function of the upper limit $N-1$. In other words, if the upper limit is below the lower limit 0, the sum is identically set to zero. Otherwise, it is an ordinary sum. We therefore need to manually check the first two cases. For the trivial case $N=0$, both sides equal to $a_0 b_0$. Also, for $N=1$ (when the sum is a normal sum), it is easy to verify that both sides simplify to $a_0 b_0 + a_1 b_1$. Then, for the induction step, suppose that the claim holds for some $N \geq 1$. For $N+1$, we then have

$$\begin{aligned}
\sum_{i=0}^{N+1} a_i b_i &= \sum_{i=0}^{N} a_i b_i + a_{N+1} b_{N+1} \\
&= \sum_{i=0}^{N-1} A_i (b_i - b_{i+1}) + A_N b_N + a_{N+1} b_{N+1} \\
&= \sum_{i=0}^{N} A_i (b_i - b_{i+1}) - A_N (b_N - b_{N+1}) + A_N b_N + a_{N+1} b_{N+1}.
\end{aligned}$$

Since $-A_N(b_N - b_{N+1}) + A_N b_N + a_{N+1} b_{N+1} = A_{N+1} b_{N+1}$, the claim follows. $\square$.