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proof of divergence of harmonic series (by grouping terms)

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The harmonic series can be shown to diverge by a simple argument involving grouping terms. Write

$$\sum_{n=1}^{2^M} \frac{1}{n} = \sum_{m=1}^{M} \sum_{n=2^{m-1}+1}^{2^m} \frac{1}{n}.$$

Since $1/n \geq 1/N$ when $n \leq N$, we have

$$\sum_{n=2^{m-1}+1}^{2^m} \frac{1}{n} \geq \sum_{n=2^{m-1}+1}^{2^m} 2^{-m} = (2^m - 2^{m-1})2^{-m} = \frac{1}{2}$$

Hence,

$$\sum_{n=1}^{2^M} \frac{1}{n} \geq \frac{M}{2}$$

so the series diverges in the limit $M \to \infty$.