radius of convergence

Canonical name	RadiusOfConvergence
Date of creation	2013-03-22 12:32:59
Last modified on	2013-03-22 12:32:59
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	13
Author	PrimeFan (13766)
Entry type	Theorem
Classification	msc 40A30
Classification	msc 30B10
Synonym	Abel's theorem on power series
Related topic	ExampleOfAnalyticContinuation
Related topic	NielsHenrikAbel

To the power series

$$\sum_{k=0}^{\infty} a_k (x - x_0)^k \tag{1}$$

there exists a number $r \in [0, \infty]$, its *radius of convergence*, such that the series converges absolutely for all (real or complex) numbers x with $|x - x_0| < r$ and diverges whenever $|x - x_0| > r$. This is known as *Abel's theorem on power series*. (For $|x - x_0| = r$ no general statements can be made.)

The radius of convergence is given by:

$$r = \liminf_{k \to \infty} \frac{1}{\sqrt[k]{|a_k|}} \tag{2}$$

and can also be computed as

$$r = \lim_{k \to \infty} \left| \frac{a_k}{a_{k+1}} \right|, \tag{3}$$

if this limit exists.

It follows from the `http://planetmath.org/WeierstrassMTest`Weierstrass M-test that for any radius r' smaller than the radius of convergence, the power series converges uniformly within the closed disk of radius r'.