limit of sequence as sum of series

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If U is the limit of a sequence

$$u_1, \; u_2, \; u_3, \; \ldots$$

of real or complex numbers, then U can be expressed as the series sum

$$U = u_1 + \sum_{i=1}^{\infty}(u_{i+1} - u_i).$$

Proof. Let $s_n := u_1 + \sum_{i=1}^{n-1}(u_{i+1} - u_i)$. We see that

$$s_n = u_1 + \sum_{i=1}^{n-1} u_{i+1} - \sum_{i=1}^{n-1} u_i = u_1 + \sum_{j=2}^{n} u_j - \sum_{i=1}^{n-1} u_i = u_n$$

for all $n = 1, \, 2, \, 3, \, \ldots$ Thus

$$u_1 + \sum_{i=1}^{\infty}(u_{i+1} - u_i) = \lim_{n \to \infty} s_n = \lim_{n \to \infty} u_n = U,$$

Q.E.D.