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Proof of Stolz-Cesaro theorem

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From the definition of convergence , for every $\epsilon > 0$ there is $N(\epsilon) \in \mathbb{N}$ such that $(\forall)n \geq N(\epsilon)$, we have :

$$l - \epsilon < \frac{a_{n+1} - a_n}{b_{n+1} - b_n} < l + \epsilon$$

Because b_n is strictly increasing we can multiply the last equation with $b_{n+1} - b_n$ to get :

$$(l - \epsilon)(b_{n+1} - b_n) < a_{n+1} - a_n < (l + \epsilon)(b_{n+1} - b_n)$$

Let $k > N(\epsilon)$ be a natural number . Summing the last relation we get :

$$(l - \epsilon) \sum_{i=N(\epsilon)}^{k} (b_{i+1} - b_i) < \sum_{i=N(\epsilon)}^{k} (a_{n+1} - a_n) < (l + \epsilon) \sum_{i=N(\epsilon)}^{k} (b_{i+1} - b_i) \Rightarrow$$

$$(l - \epsilon)(b_{k+1} - b_{N(\epsilon)}) < a_{k+1} - a_{N(\epsilon)} < (l + \epsilon)(b_{k+1} - b_{N(\epsilon)})$$

Divide the last relation by $b_{k+1} > 0$ to get :

$$(l - \epsilon)(1 - \frac{b_{N(\epsilon)}}{b_{k+1}}) < \frac{a_{k+1}}{b_{k+1}} - \frac{a_{N(\epsilon)}}{b_{k+1}} < (l + \epsilon)(1 - \frac{b_{N(\epsilon)}}{b_{k+1}}) \Leftrightarrow$$

$$(l - \epsilon)(1 - \frac{b_{N(\epsilon)}}{b_{k+1}}) + \frac{a_{N(\epsilon)}}{b_{k+1}} < \frac{a_{k+1}}{b_{k+1}} < (l + \epsilon)(1 - \frac{b_{N(\epsilon)}}{b_{k+1}}) + \frac{a_{N(\epsilon)}}{b_{k+1}}$$

This means that there is some K such that for $k \geq K$ we have :

$$(l - \epsilon) < \frac{a_{k+1}}{b_{k+1}} < (l + \epsilon)$$

(since the other terms who were left out converge to 0)

This obviously means that :

$$\lim_{n \to \infty} \frac{a_n}{b_n} = l$$

and we are done .