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alternating series test

Canonical name	AlternatingSeriesTest
Date of creation	2013-03-22 12:27:09
Last modified on	2013-03-22 12:27:09
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	18
Author	Koro (127)
Entry type	Theorem
Classification	msc 40A05
Classification	msc 40-00
Synonym	Leibniz's theorem
Synonym	Leibniz test
Related topic	AlternatingSeries

The **alternating series test**, or the **Leibniz's Theorem**, states the following:

**Theorem** [?, ?] Let  $(a_n)_{n=1}^{\infty}$  be a non-negative, non-increasing sequence of real numbers such that  $\lim_{n \rightarrow \infty} a_n = 0$ . Then the infinite series  $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$  converges.

This test provides a necessary and sufficient condition for the convergence of an alternating series, since if  $\sum_{n=1}^{\infty} a_n$  converges then  $a_n \rightarrow 0$ .

**Example:** The series  $\sum_{k=1}^{\infty} \frac{1}{k}$  does not converge, but the alternating series  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$  converges to  $\ln(2)$ .

## References

- [1] W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill Inc., 1976.
- [2] E. Kreyszig, *Advanced Engineering Mathematics*, John Wiley & Sons, 1993, 7th ed.