p test

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The following is an immediate corollary of the integral test.

Corollary (p**-Test).** *A series of the form* $\sum_{n=1}^{\infty} \frac{1}{n^p}$ *converges if* $p > 1$ *and diverges if* $p \leq 1$.

Proof. The case $p = 1$ is well-known, for $\sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series, which diverges (see `http://planetmath.org/ProofOfDivergenceOfHarmonicSreies`this proof). From now on, we assume $p \neq 1$ (notice that one could also use the integral test to prove the case $p = 1$). In order to apply the integral test, we need to calculate the following improper integral:

$$\int_1^{\infty} \frac{1}{x^p} dx = \lim_{n \to \infty} \left[\frac{x^{1-p}}{1-p} \right]_1^n = \lim_{n \to \infty} \frac{n^{-p+1}}{1-p} - \frac{1}{1-p}.$$

Since $\lim_{n \to \infty} n^t$ diverges when $t > 0$ and converges for $t \leq 0$, the integral above converges for $1 - p < 0$, i.e. for $p > 1$ and diverges for $p < 1$ (and also diverges for $p = 1$). Therefore, the corollary follows by the integral test. □