proof of ratio test

Canonical name	ProofOfRatioTest
Date of creation	2013-03-22 12:24:46
Last modified on	2013-03-22 12:24:46
Owner	vitriol (148)
Last modified by	vitriol (148)
Numerical id	6
Author	vitriol (148)
Entry type	Proof
Classification	msc 40A05
Classification	msc 26A06

Assume $k < 1$. By definition $\exists N$ such that
$n > N \implies |\frac{a_{n+1}}{a_n} - k| < \frac{1-k}{2} \implies |\frac{a_{n+1}}{a_n}| < \frac{1+k}{2} < 1$

i.e. eventually the series $|a_n|$ becomes less than a convergent geometric series, therefore a shifted subsequence of $|a_n|$ converges by the comparison test. Note that a general sequence b_n converges iff a shifted subsequence of b_n converges. Therefore, by the absolute convergence theorem, the series a_n converges.

Similarly for $k > 1$ a shifted subsequence of $|a_n|$ becomes greater than a geometric series tending to ∞, and so also tends to ∞. Therefore a_n diverges.