double series

Canonical name	DoubleSeries
Date of creation	2013-03-22 16:32:54
Last modified on	2013-03-22 16:32:54
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	6
Author	PrimeFan (13766)
Entry type	Theorem
Classification	msc 40A05
Classification	msc 26A06
Synonym	double series theorem
Related topic	FourierSineAndCosineSeries
Related topic	AbsoluteConvergenceOfDoubleSeries
Related topic	PerfectPower
Defines	diagonal summing

Theorem. If the *double series*

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} = \sum_{n=1}^{\infty} a_{1n} + \sum_{n=1}^{\infty} a_{2n} + \sum_{n=1}^{\infty} a_{3n} + \ldots \tag{1}$$

converges and if it remains convergent when the of the partial series are replaced with their absolute values, i.e. if the series

$$\sum_{n=1}^{\infty} |a_{1n}| + \sum_{n=1}^{\infty} |a_{2n}| + \sum_{n=1}^{\infty} |a_{3n}| + \ldots \tag{2}$$

has a finite sum M, then the addition in (1) can be performed in reverse , i.e.

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{mn} = \sum_{m=1}^{\infty} a_{m1} + \sum_{m=1}^{\infty} a_{m2} + \sum_{m=1}^{\infty} a_{m3} + \ldots$$

Proof. The assumption on (2) implies that the sum of an arbitrary finite amount of the numbers $|a_{mn}|$ is always $\leqq M$. This means that (1) is absolutely convergent, and thus the order of summing is insignificant.

Note. The series satisfying the assumptions of the theorem is often denoted by

$$\sum_{m,n=1}^{\infty} a_{mn}$$

and this may by interpreted to an arbitrary summing . One can use e.g. the *diagonal summing*:

$$a_{11} + a_{12} + a_{21} + a_{13} + a_{22} + a_{31} + \ldots$$