# J. P. Morgan Quant Mentorship Program 2021

**General Instructions:**

This document contains two case studies each with equal score. You are expected to attempt both case studies and submit the solution by the deadline.

Each of these case studies introduce the basics of a problem and pose a few questions on them. A question might have multiple sub-parts, all of which need to be answered. The scores for the sub-parts have been indicated against the question. The solution format for each question has been specified alongside the question and the final submission format is described at the end of this document.

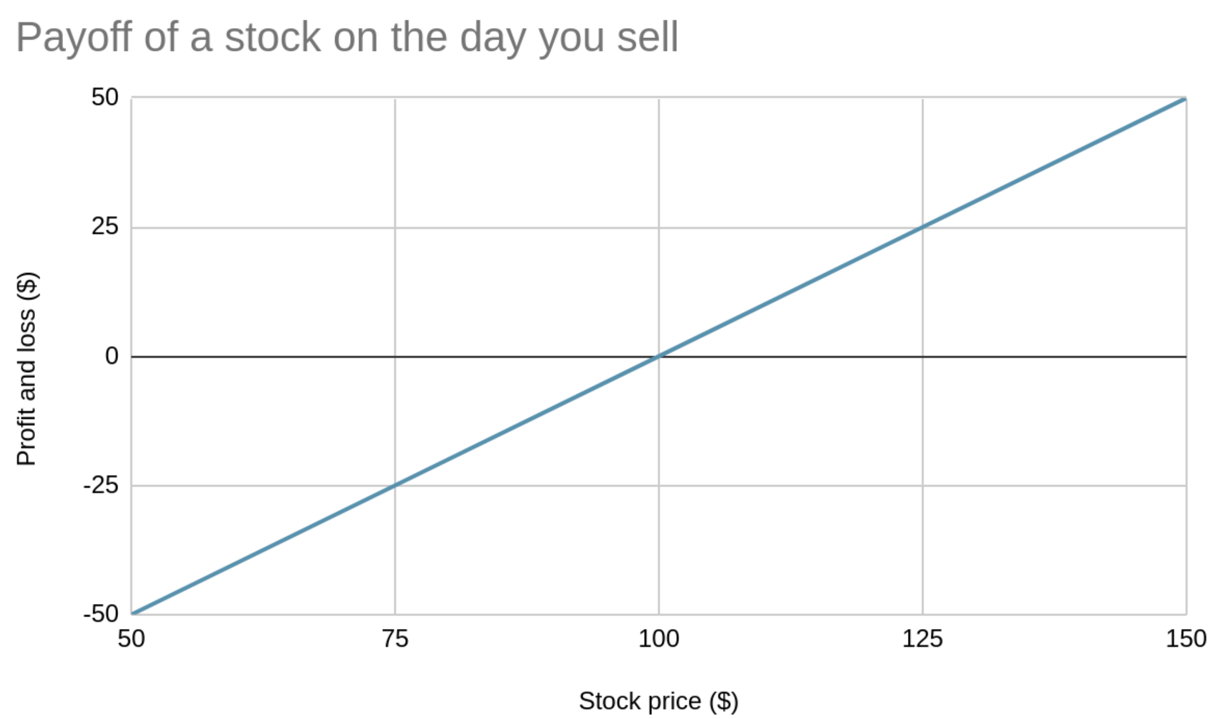
You are free to use online resources to improve your understanding of the problem statement. **Plagiarism of any kind will not be tolerated**.

## Case Study A: Options (100 points)

**Theory:**

*Stocks:*

You might have definitely come across the terms ‘Nifty’ or ‘Sensex’ by now. Have you ever wondered what those were? A short answer is that Nifty is the flagship benchmark index of the National Stock Exchange (NSE) and Sensex is the flagship benchmark index of the Bombay Stock Exchange (BSE). What is an Index, you may ask. An Index is a group of any assets. In our case, these assets are stocks. A stock (or a share) is a small ownership of a company that can be freely traded (allows you to buy and sell from other participants like you). Like any other asset, the price of the stock can fluctuate based on the forces of supply and demand. Higher the demand, greater the price; and higher the supply, lower the price. If you buy a stock at, for example, $100 today, and after 3 months, its value rises to $120, you would have made a profit of $20. Likewise, if it went down $80, you would have made a loss of $20.




Are stocks the only way you can participate in the market? Read further sections to find out.

*Derivatives (options):*

In the previous example, you have already purchased the stock, and plan to book your profit or loss after 3 months. Obviously, you have purchased it in the hope that the price rises so that you can sell at a higher price than what you bought for, but nobody can guarantee on what happens 3-months from now.
Theoretically speaking, the stock price can go to 0 at the worst case and you will lose all your money. What if there is some way which you can use to floor your losses? Let’s now understand derivatives (options).

*Call option:*

Your goal is to bet that the price of the stock goes up after 3 months, and at the same time, you want to floor your losses. A call option is a contract that gives you the right, but not obligation, which allows you to buy the stock at a pre-specified price called strike price, on a certain date in the future (3 months, in our example), irrespective of the prevailing market price on the expiry date. You can buy an option by paying a small premium to the seller of the option.

Imagine you bought a call option after paying a small premium. As the holder of the call option, you now have the right to buy the stock at $100, 3 months from now, even if the market price is greater than $100. If the stock price ends up <= $100, you can let your option expire without any action, and the maximum amount you can lose is the initial premium that you paid to buy the option.

Note that you don’t need to have the stock with you when you are buying or selling the option. So, your initial investment is very minimal unlike the previous case where you were buying the stock at the current market price today, and hoping that it would increase in price 3 months after.

‘Buying’ of an asset is termed as a ‘long position’ and ‘Selling’ of an asset is termed as a ‘short position’.

See the below diagrams to see how the payoff looks like on the date of expiry for the call option:

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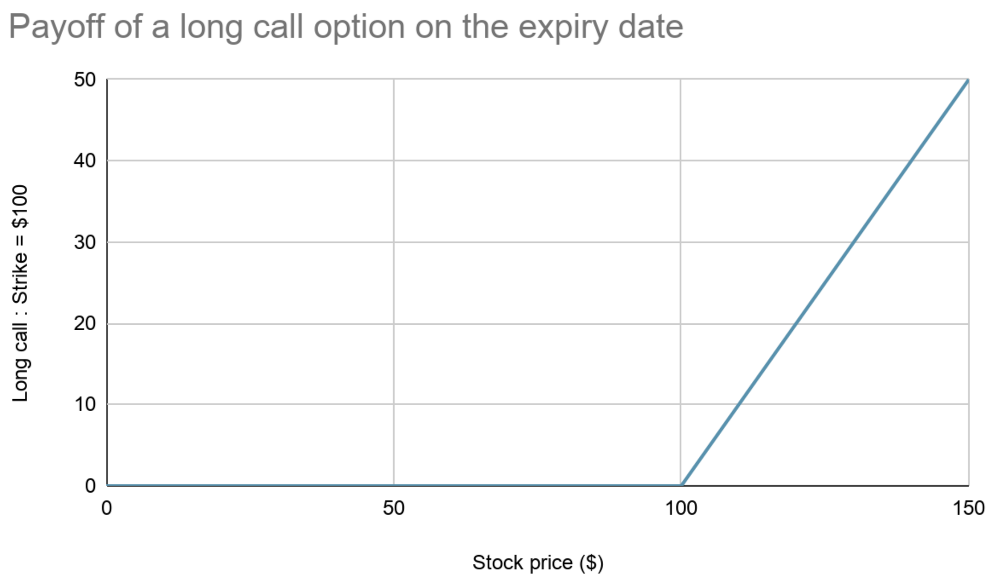
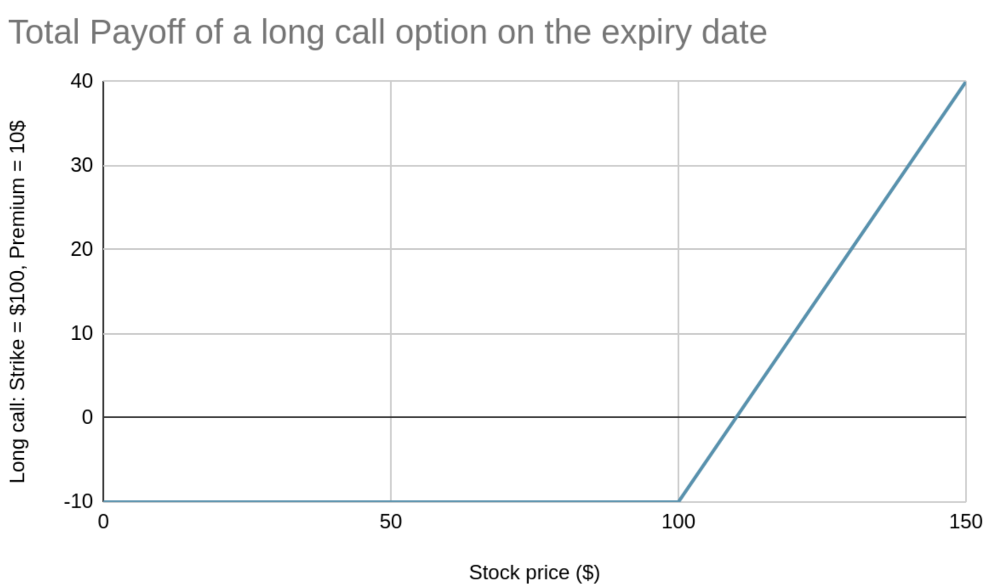
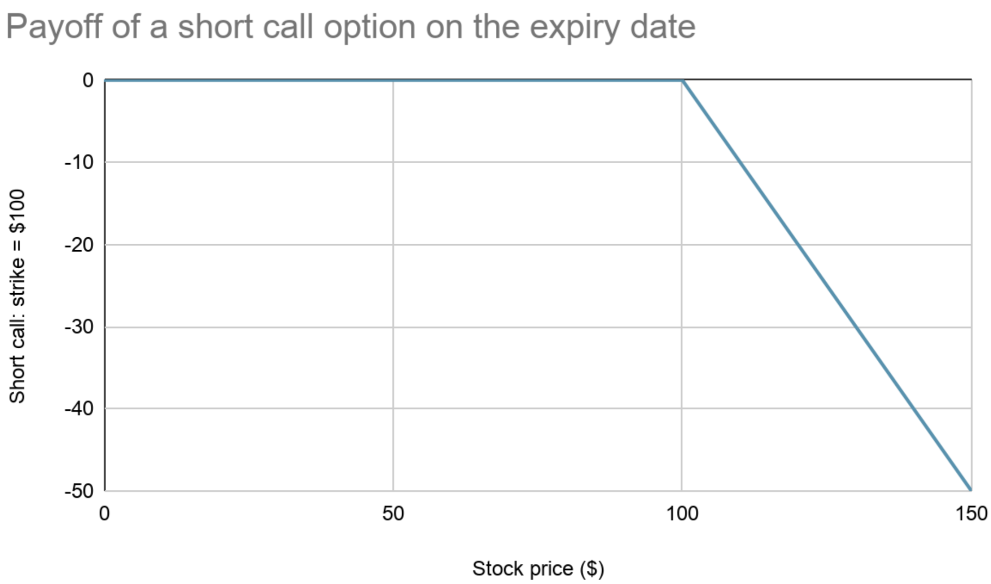
1. A long call with Strike = $100 has the payoff as C = max (stock price on expiry date - strike, 0).



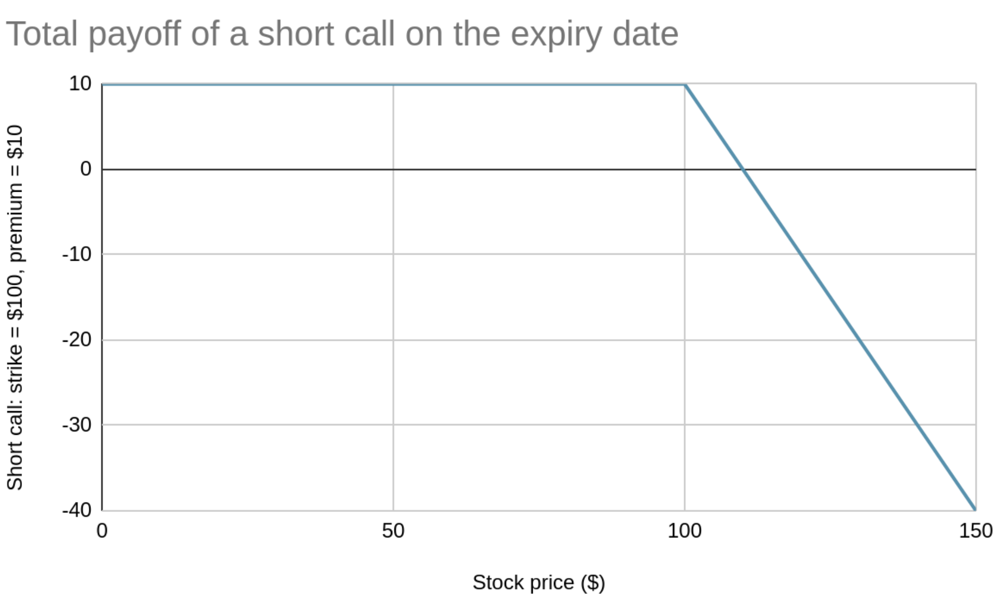

2. Total payoff for a long call with strike = $100. As you paid an initial premium (assume $10) to buy the call option, the payoff graph gets shifted down by the premium amount.



3. As you guessed, short call payoff with Strike = $100 has the payoff of long call reflected on the x-axis.




4. Total payoff for a short call option with strike = $100. As you received the initial premium (assume $10) from the buyer of the option, the payoff graph gets shifted up by the premium amount.



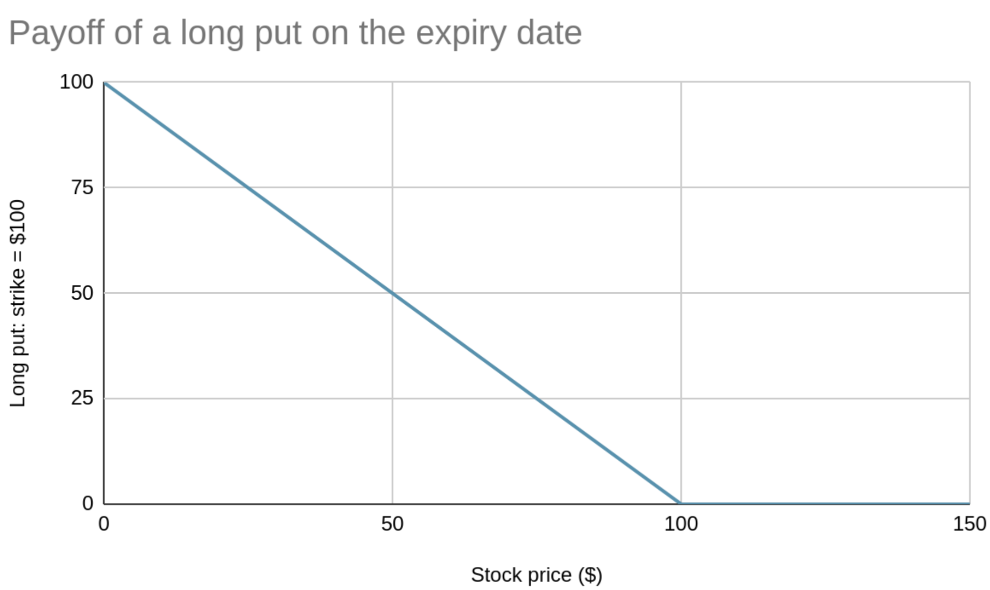
*Put option:*

Just as the call option gave the buyer of the option the right to buy the stock at a predetermined price (strike) on the expiry date, a put option will give the buyer of the option the right to sell the stock at a predetermined price (strike) on the expiry date, irrespective of the prevailing market price on the expiry date. The payoff diagrams will just reflect the other side of the deal.
You can buy an option by paying a small premium to the seller of the option.

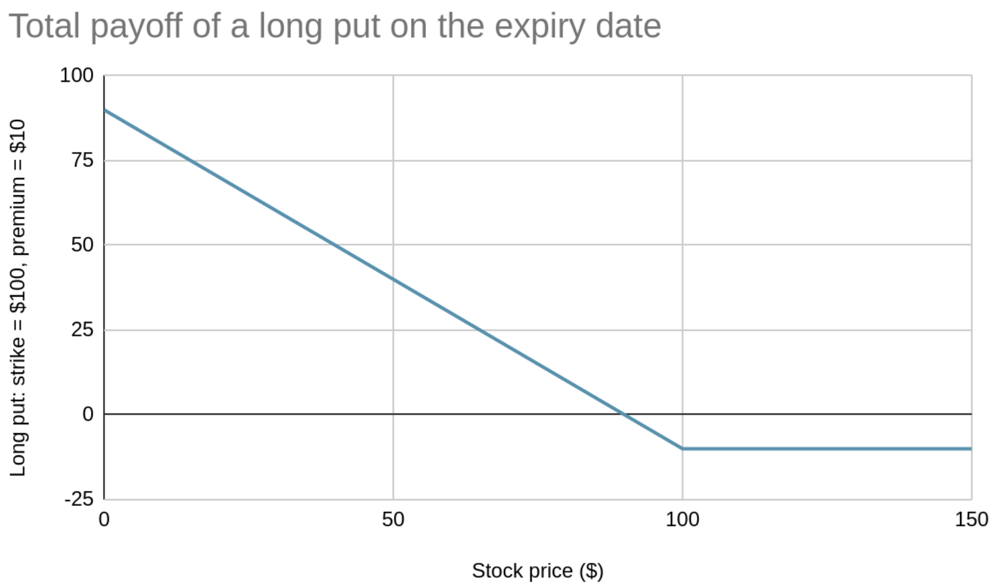
Imagine you bought a put option after paying a small premium. As the holder of the put option, you now have the right to sell the stock at $100, 3 months from now, even if the market price is < $100. If the stock price ends up >= $100, you can let your option expire without any action, and the maximum amount you can lose is the initial premium that you paid to buy the option.

See the below diagrams to see how the payoff looks like on the date of expiry for the put option:

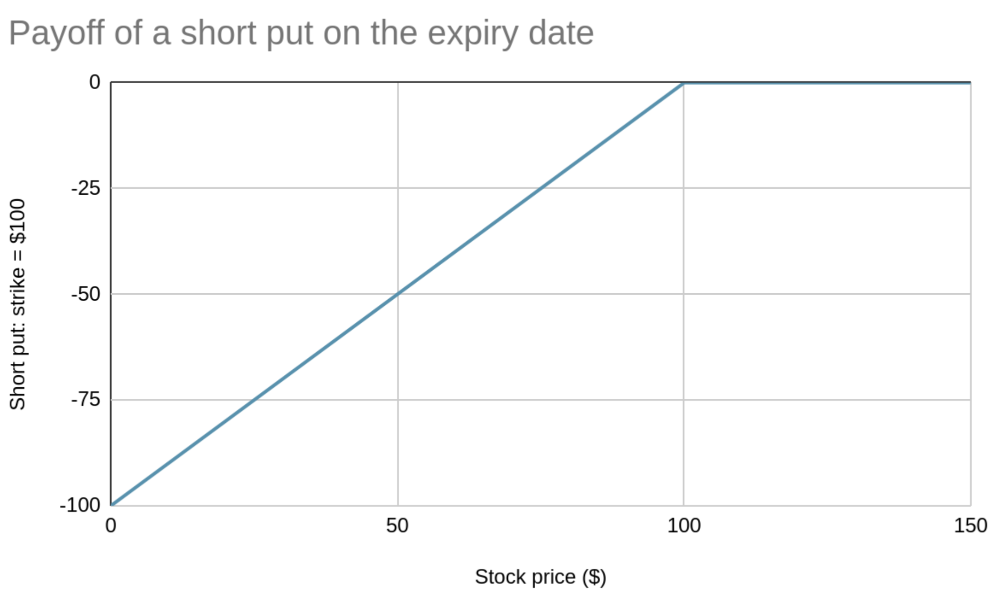
1. A long put with Strike = $100 has the payoff as P = max (strike - stock price on the expiry date, 0).



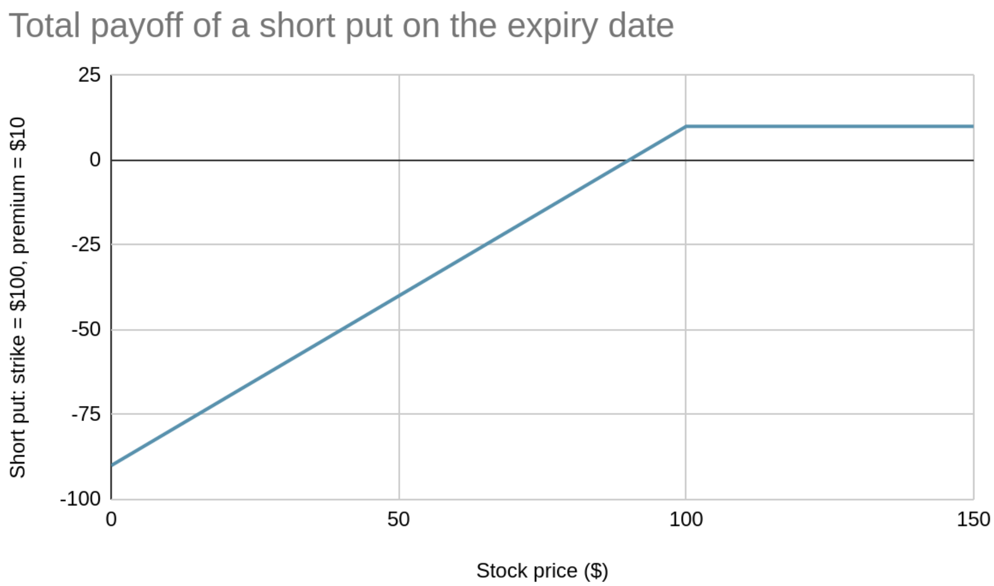
2. Total payoff for a long put with strike = $100. As you paid an initial premium (assume $10) to buy the put option, the payoff graph gets shifted down by the premium amount.




3. Short put payoff with Strike = $100 has the payoff of long put reflected on the x-axis.



4. Total payoff for a short put option with strike = $100. As you received the initial premium (assume $10) from the buyer of the option, the payoff graph gets shifted up by the premium amount.




**<u>Problem statement:</u>**

Design and implement a system to compute the payoff of a linear combination of calls and puts. You may either solve it <u>programmatically (either of Python, C, C++, Java, etc.) by writing classes/functions</u> **or** you <u>may provide the pseudocode for the design in case you are not familiar with coding</u>.

Key points to note:

1. For all the questions, **assume that the stock price ranges from $0 to $100 (both inclusive).**
2. Please use the most ubiquitous plotting library for all your plots in case you are coding the solution. There are no extra marks for exotic plots. For example, you may stick to **matplotlib** in case you are using python for your solution.
3. Assume the premium for any **call option as $4** and premium for any **put option as $2**, unless explicitly stated in the question.
4. Assume the **existence of calls and puts at all strikes in [$0, $100]**, unless explicitly stated in the question.
5. If you have written the **code**, please **provide us with all the code files.**
6. In case you are only providing us with the **pseudocode**, please explain the design and structure of your functions/classes elaborately, and **submit all your manual calculations for each problem you attempt in the answers report.**

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**Q1: a:** Show the payoff plot when you long a call and short a put, both at the strike of $50? **(5 points)**

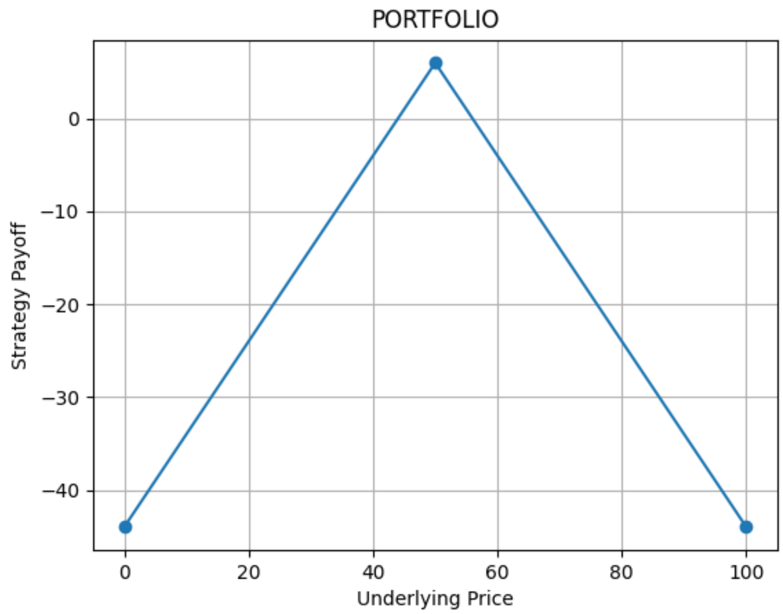
<u>Solution format:</u>
- If you have written the code, please provide a standalone function (no-arguments) that generates the payoff plot
- If there is no simple way to plot in the language of your choice, make the function return the XY- coordinates

In the answers report, show us the payoff plot, either from the function you wrote, or generate the plot in a spreadsheet using the coordinates from your function. If you have not coded the solution, then show us the full manual calculations that you performed along with the payoff plot.

**Q1: b:** Based on your understanding of the payoff plot in Q1: a: what real-world asset is the portfolio trying to mimic? **(5 points)**

<u>Solution format:</u> Describe in words.

**Q2: a:** Based on the plot below, can you replicate the payoff by a combination of calls and puts? The peak in the graph is at (50, 6) and the absolute slopes of the sides = 1. **(10 points)**




<u>Solution format:</u>
- If you have written the code, please provide a standalone function (no-arguments) that generates the plot **and** returns the portfolio of calls and puts that you have chosen to replicate the payoff.
- If there is no simple way to plot in the language of your choice, make the function return the XY- coordinates **and** the portfolio of calls and puts that you chose.

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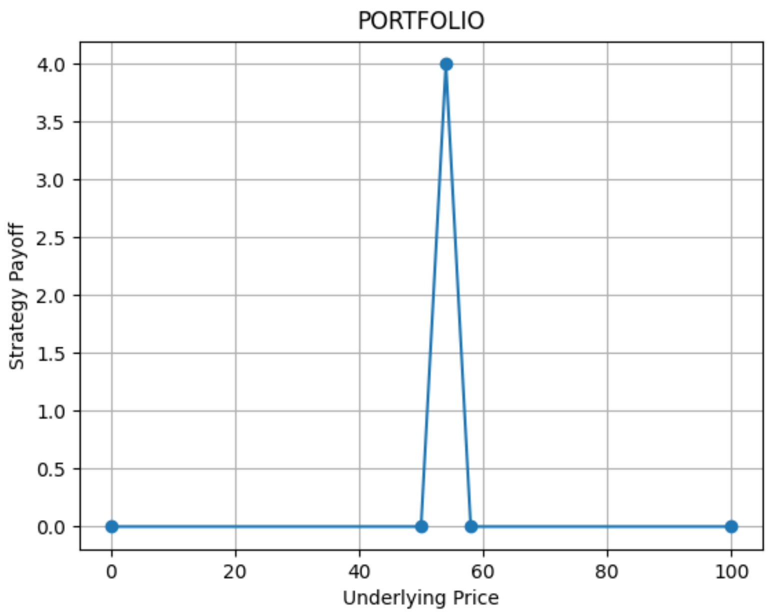
In the answers report, show us the plot, either from the function you wrote, or generate the plot in a spreadsheet using the coordinates from your function. Also mention the portfolio of calls and puts.

If you have not coded the solution, then show us the full manual calculations that you performed along with the plot.

**Q2: b:** What could be the investor's belief if she/he is targeting the payoff shown in Q2: a:? In other words, at what scenarios would such a strategy give positive returns to the investor? **(5 points)**

<u>Solution format:</u> Describe in words.

**Q3:** The investor seems to be happy with his/her strategy in Q2: a: but worried about the potential uncapped losses. How could the investor achieve the below payoff? The peak in the graph is at (54, 4) and bases of the peak are at (50, 0) and (58, 0). **(20 points)**




<u>Solution format:</u>

- If you have written the code, please provide a standalone function (no-arguments) that generates the plot **and** returns the portfolio of calls and puts that you have chosen to replicate the payoff.
- If there is no simple way to plot in the language of your choice, make the function return the XY- coordinates **and** the portfolio of calls and puts that you chose.

In the answers report, show us the plot, either from the function you wrote, or generate the plot in a spreadsheet using the coordinates from your function. Also mention the portfolio of calls and puts. If you have not coded the solution, then show us the full manual calculations that you performed along with the plot.

**Q4:** Imagine that calls and puts are available at all strikes in range [$0, $100] in multiples of $0.5 only. And also assume that the premiums of all calls and puts is $0.0. What is the **nearest possible replication** that can be done such that the portfolio behaves as a **custom call option** struck at $50 with the portfolio payoff as {1 if stock price > $50, 0 otherwise}? **(15 points)**

<u>Solution format:</u>

- If you have written the code, please provide a standalone function (no-arguments) that generates the plot **and** returns the portfolio of calls and puts that you have chosen to replicate the payoff.
- If there is no simple way to plot in the language of your choice, make the function return the XY- coordinates **and** the portfolio of calls and puts that you chose.

In the solution document, show us the plot, either from the function you wrote, or generate the plot in a spreadsheet using the coordinates from your function. Also mention the portfolio of calls and puts. If you have not coded the solution, then show us the full manual calculations that you performed along with the plot.

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<u>Probability:</u>
For the below questions, assume that you have 3 days to sell a stock that you hold, and the potential buyer is quoting you the prices each day. You can either accept the price the buyer quotes, or wait for the next day's quote by rejecting the quote. There is no chance of changing your previous decision.

**Q5: a:** The buyer is free to quote any price he/she wishes and the quotes do not appear to follow any standard probability distributions. For example, the quotes can be 10000X or 0.001X of the previous quote. What should be your strategy to optimize your chances of selling the stock at the highest possible price across the three days? **(10 points)**

<u>Solution format:</u> Describe in words.

**Q5: b:** Using your strategy in Q5: a:, what is the probability that you would sell the stock at the maximum price possible? **(5 points)**

<u>Solution format:</u> Describe in words.

**Q6: a:** Imagine that the quotes are always in multiples of 10, and always range in [110, 160] with equal probability, i.e., 1/6th chance of drawing one of {110, 120, 130, 140, 150, 160}. What strategy should you adopt to maximize your expected returns? **(10 points)**
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Solution format: Describe in words.

**Q6: b:** What is the expected return in Q6: a:? **(5 points)**

Solution format: Describe in words.

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Statistics:

Imagine two banks (A and B) in a country which are governed by a central authority, have similar customer base and offer similar products and services. Therefore, any change in the business environment will have a similar impact on both the banks. In the ideal world, if the stock price of Bank A rises by x%, then stock price of Bank B is also expected to rise, at least by y% and vice versa. But, since each bank is an independent entity, it will be also subject to its internal environment. For example, assume that there is a change in the top echelon of Bank A, leading to a negative effect on its stock price; Bank B's stock will still maintain its status quo as this event is localized to Bank A. Ceteris paribus, Bank A's stock price is expected to maintain its usual relation with Bank B in some time.

**Q7: a:** Which statistical measure would you use to find the most suitable pair of stocks that are in tandem with the above scenario and why? **(5 points)**

Solution format: Describe in words.

**Q7: b:** Explain how you can leverage in the scenario described above to gain profit in the market and how to minimize the risk associated with it? **(5 points)**

Solution format: Describe in words.
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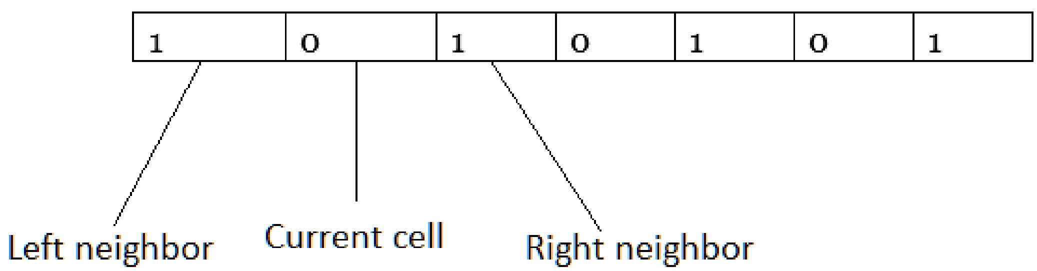
## **Case Study B:** The Automaton **[100 Points]**

A cellular automaton is a model of a system of “cell” objects with the following characteristics. The cells live on a grid. Although a cellular automaton can exist in any finite number of dimensions, we will be using a 1D grid for this problem.

Example of a simple 1 dimensional grid:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |

Each cell has a **state**. The number of state possibilities is typically finite. A simple example has the two possibilities of 1 and 0 (otherwise referred to as “on” and “off” or “alive” and “dead”). Each cell has a **neighborhood**. This can be defined in any number of ways, but it is typically a list of adjacent cells.

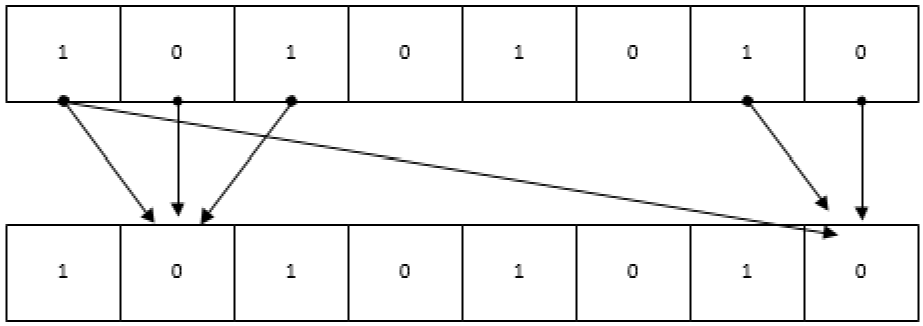



For cells at either end of the array, the cell at the opposite end of the array can be considered as one of the neighbors.

A representation of a grid containing states can be considered as a **generation** of the automaton. An important characteristic of the cellular automaton is that it changes with time. The transformation from one generation to the next is governed by a set of rules that are predefined. The rules in general, depend only on the states of cells in the current generation only.

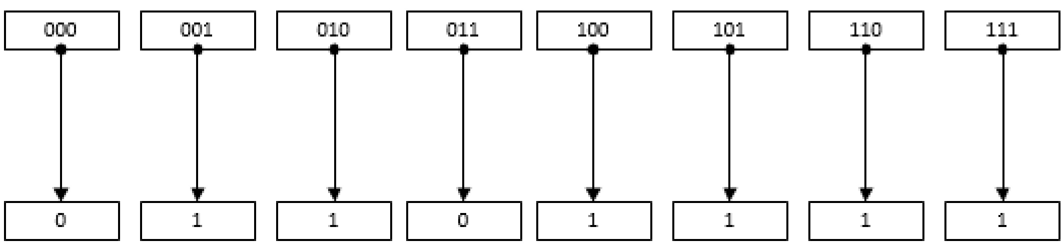
The rule set for a 1-D automaton with {0, 1} as the only valid state, is a many to one function that takes the configuration of the current cell’s parents only.

E.g. of a rule




If parents (which are defined as neighbors of the cell and the cell itself in the previous generation) are 101 (value of left neighbor, cell at that index followed by the right index) are 101, the value of that cell in the next generation will be 0.
Note: During these transformations, the length of the grid does not change.
Since there are 8 possible configurations for parents (000, 001, 010… 111) there are eight rules that need to be specified to determine the next generation.

Therefore, each rule set can be defined like so:




Which can be represented simply as:
Parents (000) -> 0
Parents (001) -> 1
And so on.

Applying this rule to the generation depicted previously yields the next generation which is identical to the parent generation (try it yourself! Hint: Only rules Parents (101)->1 and Parents (010)->1 are applied)

Since the parents are presented as a combination of zeroes and ones, we can assume that the parents are a binary representation of a decimal and therefore we can denote each parent combination as decimal numbers. i.e. (000)2 – 010, (101)2 – (5)10 etc. Here is a helpful tutorial on how to go about converting binary to decimal.

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If we replace the notation of parents with decimal numbers, the same rules transform into Parents (0) -> 0
Parents (1) ->1
…
Parents (7) -> 1

**Problem statement:**

**Q1: a:** Given the seed generation of a cellular automaton containing 4 cells, and the rules for cell transformation, determine the state of cells in the grid after 4 generations (i.e. 4 time steps).
Seed Generation (Generation 0) **[10 Points]**

| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 0 |

| Parent Configuration | Child value |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 0 |
| 3 | 0 |
| 4 | 1 |
| 5 | 0 |
| 6 | 1 |
| 7 | 0 |

Solution format:
Write the answer in the form of a 1D grid containing 4 cells.

**Q1: b:** If each generation of the cellular automaton is printed, it will form wondrous patterns. Some of these patterns might repeat, some might be completely random.
For the automaton described in Part 1, print each generation of the automaton in a new line and replace all 1s with '|' and 0s with '.' .

Solution format:

1) Write the answer in the form of a 4 lines, each containing 4 characters depicting the generation. **[5 points]**

   e.g.:
   .|..
   ||..
   ….
   .|..

2) Additionally, write a program or pseudo code to do the above **[15 points].**

What has been described previously is a deterministic cellular automata. Now, in real life, states of a cell do not change deterministically, there is a probability involved.
So the same rules defined previously, have a probability attached to them.
I.e. Parents (0) -> 0, with probability of 0.5
Parents (1) -> 1 with probability of 0.6
…
Since the only two cell states are 0 and 1, and they are mutually exclusive, we can denote the rules to represent only the probability of cell becoming 0, given the parents. (Probability of cell becoming 1 will be 1- Probability of cell becoming 0)

Therefore, the above rules can be redefined to represent the probability of each cell becoming 0, given the parent state as below:

| Parent state | Probability of child becoming 0 |
|---|---|
| 0 | 0.4 |
| 1 | 0.6 |
| 2 | 0.4 |
| 3 | 0.7 |
| 4 | 0.4 |
| 5 | 0.7 |
| 6 | 0.7 |
| 7 | 0.6 |

Note that the conditional probability of the child cell becoming 0 given the parent configuration, does not need to sum to 1, since the rules are independent of each other.

**<u>Problem statement:</u>**

**Q2: a:** Given the seed generation to be the below and using the rule set depicted above, determine the three most likely configurations of the automaton after 1 generation.

| | | | |
|---|---|---|---|
| 0 | 1 | 1 | 0 |

<u>Solution format:</u>
Expected answer is in the form of three grids, ranked by probability. You can do the above manually or via code. Both will be scored equally. **[20 points]**

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**Q2: b:** Assume the automaton with the below grid as the seed generation goes through ‘k’ transformations. (I.e. there are k generations excluding the seed)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |

What is the most likely configuration of the grid if:

i. k=5 **[ 10 points ]**

ii. k=15 **[ 15 points ]**

iii. k=infinity? Are there multiple configurations that are equally likely? **[ 25 points ]**

Write a program to compute the above. Expected output for each value of k is a grid depicting the most likely configuration along with its probability.
Alternately, Write pseudo code to compute the above. **[20 points]**

<u>Solution format:</u>
1. Program that computes the above.
Or
Pseudo code that can compute the output as described.
**Note:** Pseudo code has a lower max score than actual code for this problem.

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### Final Submission Format:

Create a well formatted word/pdf document describing your solution to each question, clearly enumerated and in the same order as the questions. Include your personal details such as name, institute name, branch and year in the first page.

For each question, if the expected solution is a written answer/ plot / table or pseudo code, include it in line in this document. Name this document in the format

{FirstName}_{LastName}_{CollegeName(short)}_JPMQuantMentorshipCaseStudy

If the expected solution is a program, state the file name of the program.

### Instructions for program submissions:

Any programming language you are comfortable with is permitted. You are permitted to use standard libraries in your chosen language. The program submitted must compulsorily have a main function that must call all the test cases described in the question when run. The program's file name must be of the format {CANDIDATE_NAME}_{QUESTION_NO_SUB_PART}_MAIN. If there are additional modules that are imported in the script with the main function, name those files as {CANDIDATE_NAME}_Module_{ SCRIPT_NAME}.

You may include a short description of your program in the solution document if you wish.

Finally, zip the solution document and code (if any) together and name it

{FirstName}_{LastName}_{CollegeName(short)}_JPMQuantMentorshipCaseStudy

Mail this compressed file to the indicated email address: jpmqrmentorship.mumbai@jpmorgan.com