hw2

Coding the Matrix, Summer 2013

Please fill out the stencil file named "`hw2.py`". While we encourage you to complete the Ungraded Problems, they do not require any entry into your stencil file.

Vector Comprehension and Sum

Problem 1:

1. Write and test a procedure `vec_select` using a comprehension for the following computational problem:
   - *input:* a list `veclist` of vectors over the same domain, and an element k of the domain
   - *output:* the sublist of `veclist` consisting of the vectors `v` in `veclist` where `v[k]` is zero
2. Write and test a procedure `vec_sum` using the built-in procedure `sum(·)` for the following:
   - *input:* a list `veclist` of vectors, and a set `D` that is the common domain of these vectors
   - *output:* the vector sum of the vectors in `veclist`.

   Your procedure must work even if `veclist` has length 0.

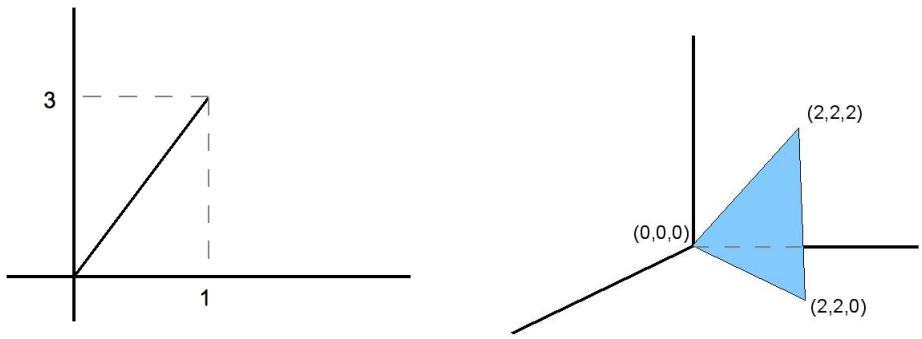
   Hint: Recall from the Python Lab that `sum(·)` optionally takes a second argument, which is the element to start the sum with. This can be a vector.

   Disclaimer: The `Vec` class is defined in such a way that, for a vector `v`, the expression `0 + v` evaluates to `v`. This was done precisely so that `sum([v1,v2,... vk])` will correctly evaluate to the sum of the vectors when the number of vectors is nonzero. However, this won't work when the number of vectors is zero.
3. Put your procedures together to obtain a procedure `vec_select_sum` for the following:
   - *input:* a set `D`, a list `veclist` of vectors with domain `D`, and an element k of the domain
   - *output:* the sum of all vectors `v` in `veclist` where `v[k]` is zero

Problem 2: Write and test a procedure `scale_vecs(vecdict)` for the following:

- *input:* A dictionary `vecdict` mapping positive numbers to vectors (instances of `Vec`)
- *output:* a list of vectors, one for each item in `vecdict`. If `vecdict` contains a key k mapping to a vector $\boldsymbol{v}$, the output should contain the vector $(1/k)\boldsymbol{v}$

Sets of Linear Combinations and Geometry



(a) (b)

Figure 1: Linear Combinations

Ungraded Problem: Express the line segment in Figure 1(a) using a set of linear combinations. Do the same for the plane containing the triangle in Figure 1(b).

Ungraded Problem: Let a, b be real numbers. Consider the equation $z = ax + by$. Prove that there are two 3-vectors $\boldsymbol{v}_1, \boldsymbol{v}_2$ such that the set of points $[x, y, z]$ satisfying the equation is exactly the set of linear combinations of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. (Hint: Specify the vectors using formulas involving a, b.)

Ungraded Problem: Let a, b, c be real numbers. Consider the equation $z = ax + by + c$. Prove that there are three 3-vectors $\boldsymbol{v}_0, \boldsymbol{v}_1, \boldsymbol{v}_2$ such that the set of points $[x, y, z]$ satisfying the equation is exactly

$$\{\boldsymbol{v}_0 + \alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2 \ : \ \alpha_1 \in \mathbb{R}, \alpha_2 \in \mathbb{R}\}$$

(Hint: Specify the vectors using formulas involving a, b, c.)

Constructing the Span of Given Vectors Over $GF(2)$

Problem 3: Write a procedure `GF2_span` with the following spec:

- *input:* a set D of labels and a list L of vectors over $GF(2)$ with label-set D
- *output:* the list of all linear combinations of the vectors in L

(Hint: use a loop (or recursion) and a comprehension. Be sure to test your procedure on examples where L is an empty list.)

Identifying Vector Spaces

Problem 4: For each of the following definitions of $\mathcal{V}$, say whether $\mathcal{V}$ is a vector space.

1. $\mathcal{V} = \{[x, y, z] \in \mathbb{R}^3 \ : \ x + y + z = 0\}$
2. $\mathcal{V} = \{[x, y, z] \in \mathbb{R}^3 \ : \ x + y + z = 1\}$

Problem 5: For each of the following definitions of $\mathcal{V}$, say whether $\mathcal{V}$ is a vector space.

1. $\mathcal{V} = \{[x_1, x_2, x_3, x_4, x_5] \in \mathbb{R}^5 \ : x_2 = 0$ and $x_5 = 0\}$
2. $\mathcal{V} = \{[x_1, x_2, x_3, x_4, x_5] \in \mathbb{R}^5 \ : x_2 = 0$ or $x_5 = 0\}$

Problem 6: For each of the following definitions of $\mathcal{V}$, say whether $\mathcal{V}$ is a vector space.

1. $\mathcal{V}$ is the set of 5-vectors over $GF(2)$ that have an even number of 1's.
2. $\mathcal{V}$ is the set of 5-vectors over $GF(2)$ that have an odd number of 1's.