UNIVERSIDADE FEDERAL FLUMINENSE

Programa de Mestrado e Doutorado em Engenharia de Produção

Forecasting

Lesson: Holt-Winter's Method

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Forecasting

The Holt-Winters Model is a demand forecasting method used for time series that show trend and seasonality. This method is similar to the Holt Model, however it can deal with seasonality.

- The normal level term is calculated based in an α variable where $0 \leq \alpha \leq 1$.
- The trend term is calculated based in a β variable where $0 \leq \beta \leq 1$.
- The seasonality term is calculated based in a γ variable where $0 \leq \gamma \leq 1$.

Forecasting

To demonstrate the method, let m represent the number of periods representing 1 year of the time series (Ex: $m = 4$ for quarterly data and $m = 12$ for monthly data). And that S_i represent the seasonality index of period i.

Forecasting

For additive types the first m values of S_i should be calculated as:

$$S_i = D_i - \frac{\sum_{i=1}^{m} D_i}{m}; i = 1, \dots, m$$

The first value of A_i should be calculated as:

$$A_i = D_i - S_i$$

The first value of T_i must be equal to zero (0).

Forecasting

For multiplicative types the first m values of S_i should be calculated as:

$$S_i = \frac{D_i}{\frac{\sum_{i=1}^{m} D_i}{m}} ; i = 1, \ldots , m$$

The first value of A_i should be calculated as:

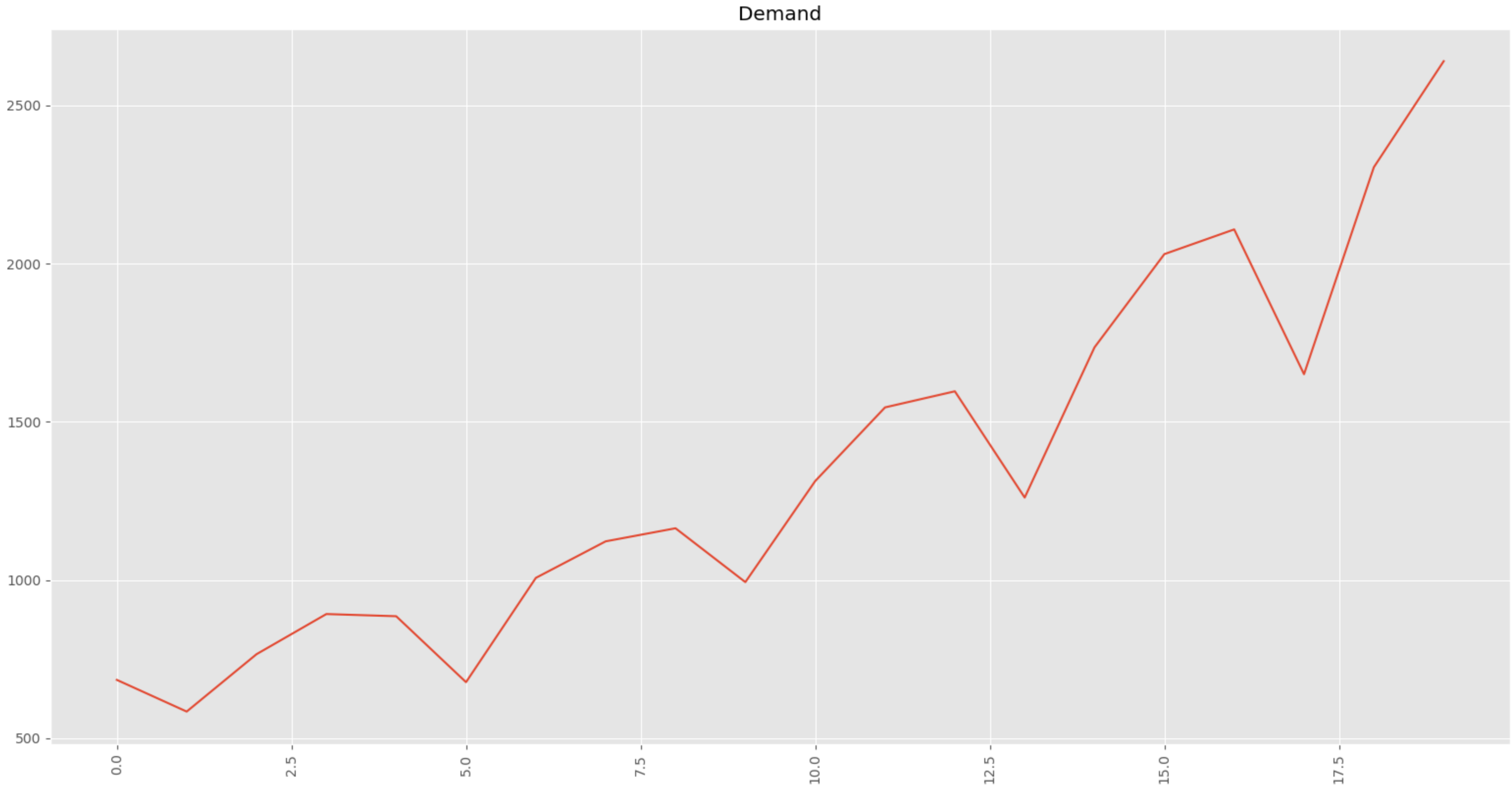
$$A_i = \frac{D_i}{S_i} = \frac{\sum_{i=1}^{m} D_i}{m}$$

The first value of T_i must be equal to one (1).

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Year	Quarter	Demand
2010	1	684.20
	2	584.10
	3	765.40
	4	892.30
2011	1	885.40
	2	677.00
	3	1006.60
	4	1122.10
2012	1	1163.40
	2	993.20
	3	1312.50
	4	1545.30
2013	1	1596.20
	2	1260.40
	3	1735.20
	4	2029.70
2014	1	2107.80
	2	1650.30
	3	2304.40
	4	2639.40


Demand
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0.0
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10.0
12.5
15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

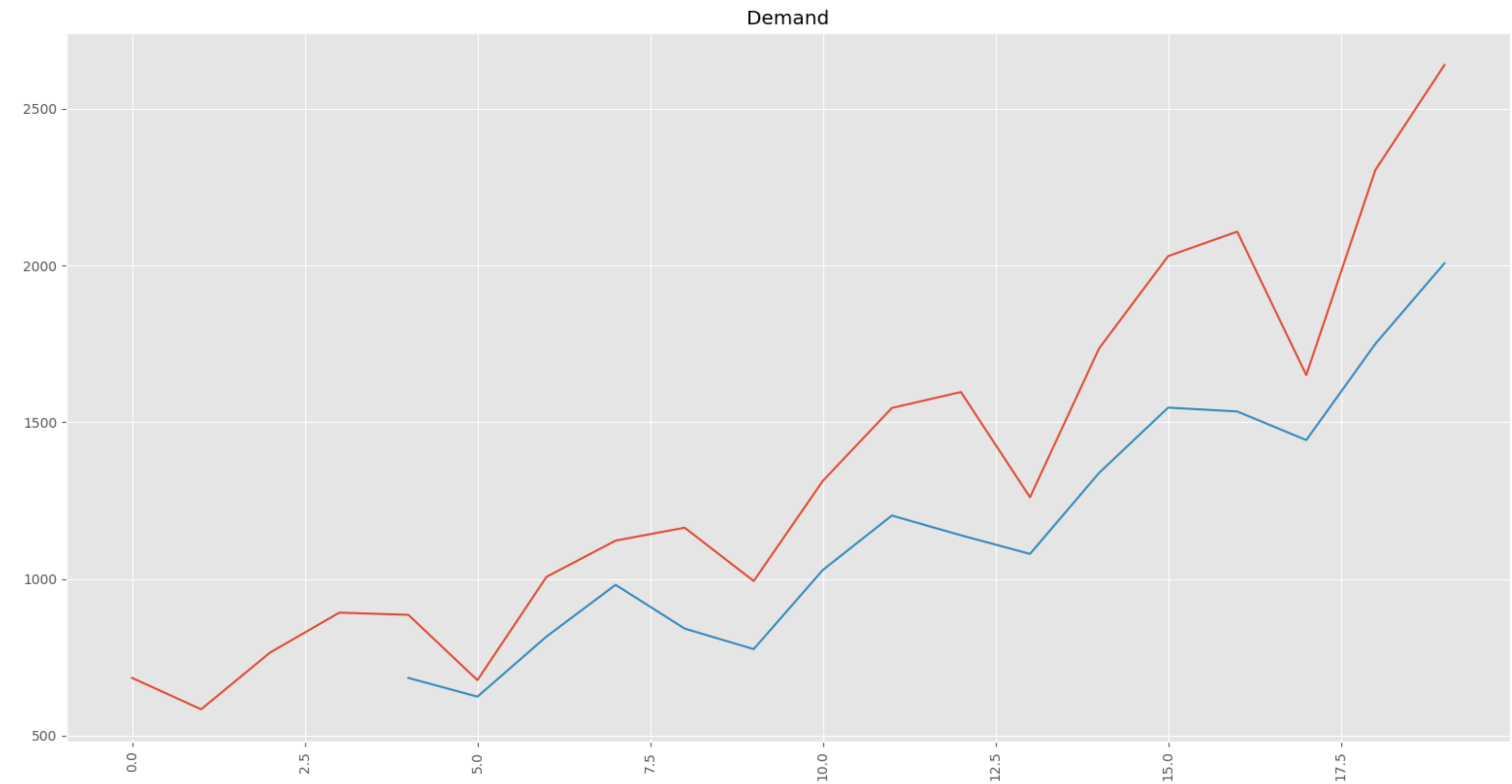
$$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$$

$$S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$$

$$F_t = A_{t-1} + S_{t-m}$$

- $alpha$ = 0.2
- $gama$ = 0.2
- $rmse$ = 367.52

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**		**160.80**	
2011	1	885.40	771.74		-7.06	684.20
	2	677.00	782.27		-136.87	624.34
	3	1006.60	820.36		71.99	816.17
	4	1122.10	848.55		188.99	981.16
2012	1	1163.40	912.93		57.32	841.49
	2	993.20	956.36		-93.44	776.06
	3	1312.50	1013.19		128.82	1028.34
	4	1545.30	1081.81		257.61	1202.18
2013	1	1596.20	1173.23		148.74	1139.14
	2	1260.40	1209.35		-57.32	1079.79
	3	1735.20	1288.76		208.22	1338.17
	4	2029.70	1385.42		354.27	1546.37
2014	1	2107.80	1500.15		263.46	1534.16
	2	1650.30	1541.64		-15.82	1442.83
	3	2304.40	1652.55		319.13	1749.87
	4	2639.40	1779.06		480.79	2006.83


Demand
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12.5
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17.5

Optimization

Forecasting

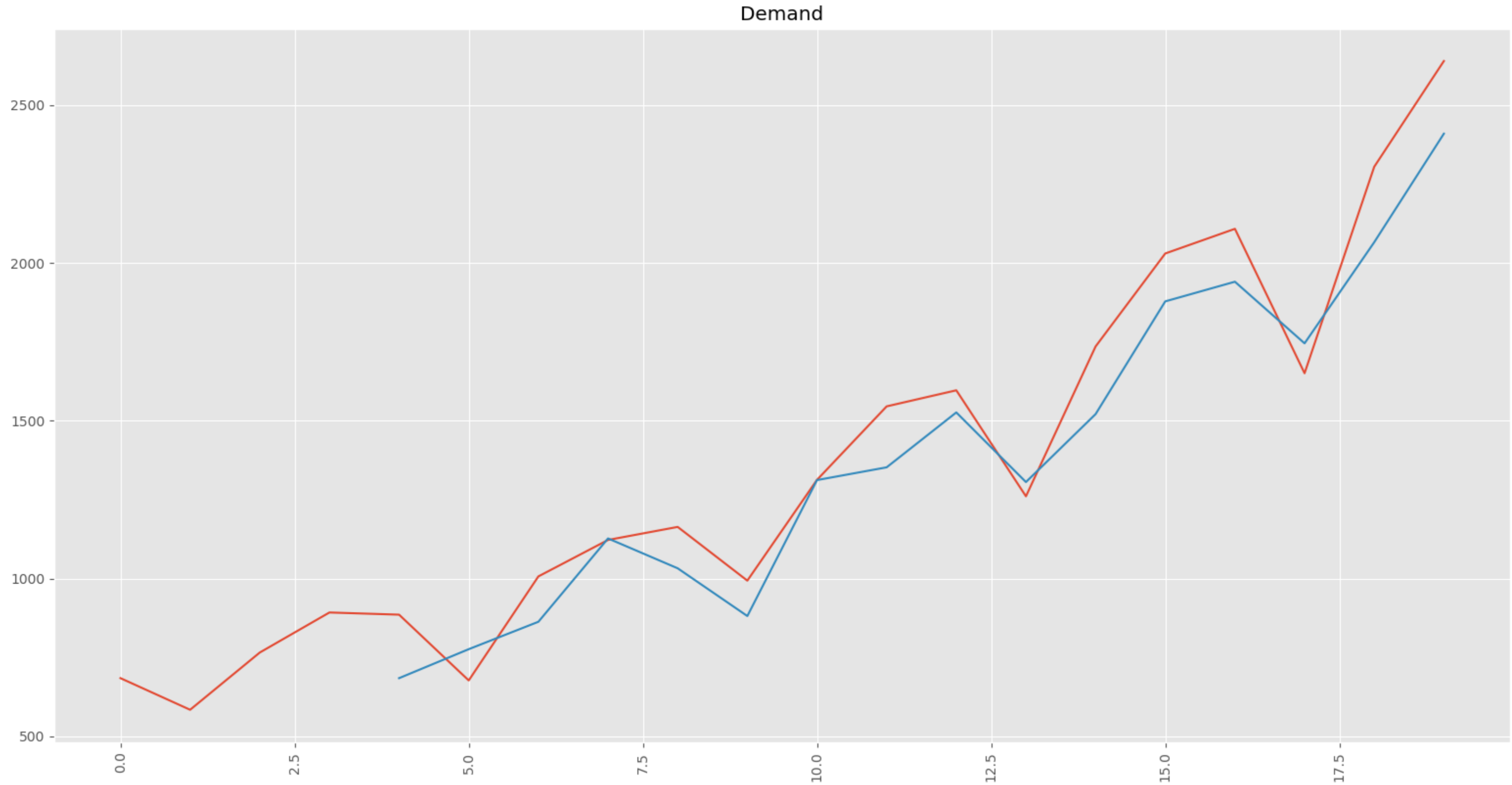
- Step 1 – Implement the Holt-Winter's Method.
- Step 2 – Choose the objective function (minimization).
- Step 3 – Choose the variable cells.
- Step 4 – Add the constraints:

$$0 \leq \alpha \leq 1; 0 \leq \gamma \leq 1$$

- Step 5 – Find the solution with GRG Non Linear the method:

- $alpha$ = 0.95
- $gama$ = 0.59
- $rmse$ = 150.28

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**		**160.80**	
2011	1	885.40	923.44		70.79	684.20
	2	677.00	828.96		-205.52	776.04
	3	1006.60	966.08		118.26	862.86
	4	1122.10	961.52		157.99	1126.88
2012	1	1163.40	1086.58		147.73	1032.31
	2	993.20	1193.56		-139.71	881.05
	3	1312.50	1194.21		118.66	1311.82
	4	1545.30	1378.42		271.32	1352.20
2013	1	1596.20	1445.25		188.84	1526.14
	2	1260.40	1402.18		-166.20	1305.54
	3	1735.20	1606.67		244.47	1520.84
	4	2029.70	1751.39		360.36	1878.00
2014	1	2107.80	1911.24		287.19	1940.23
	2	1650.30	1820.86		-221.81	1745.04
	3	2304.40	2048.93		384.78	2065.33
	4	2639.40	2268.45		495.42	2409.29



Demand
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2.5
5.0
7.5
10.0
12.5
15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

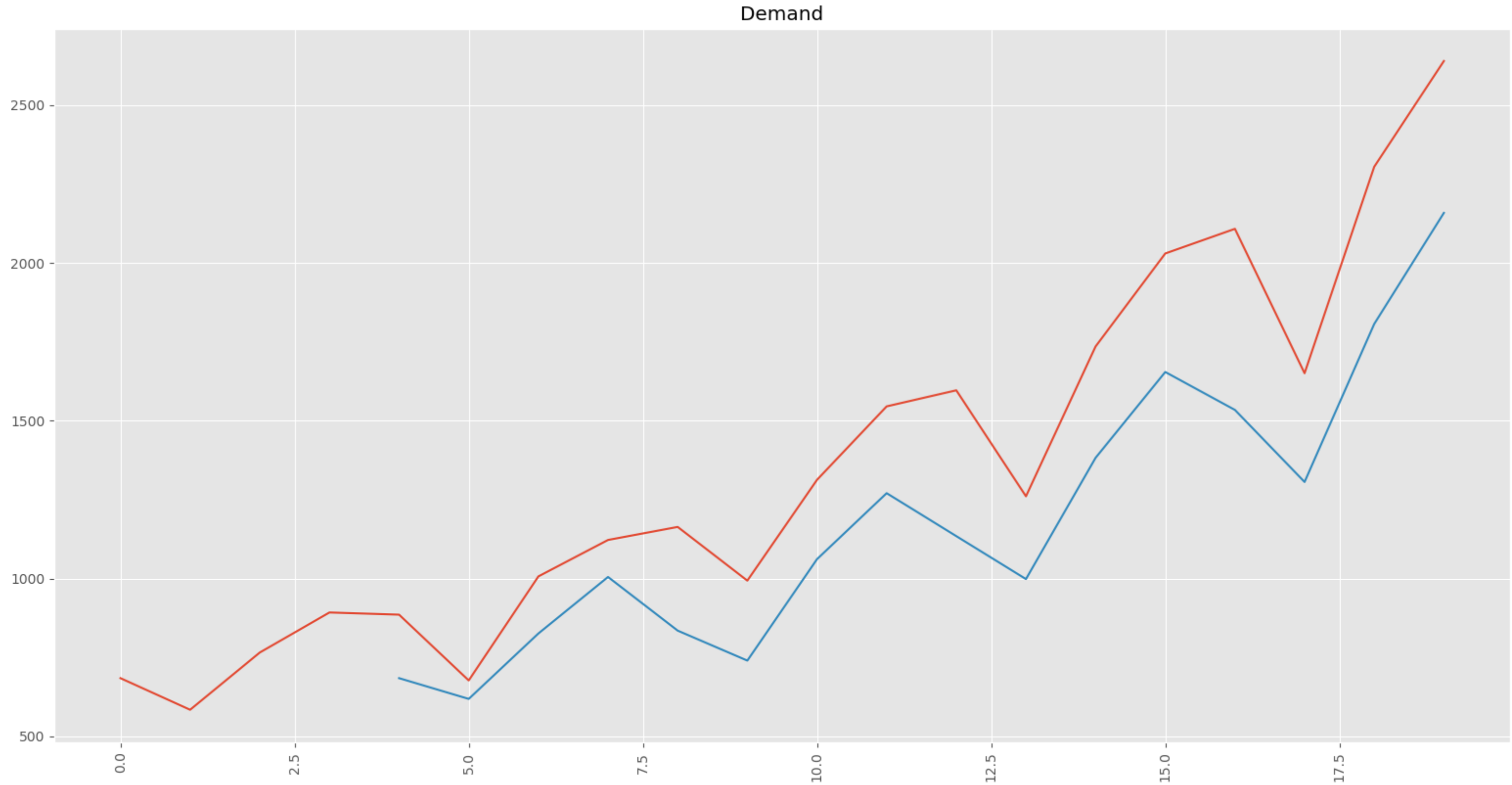
$$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$$

$$S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$$

$$F_t = A_{t-1} \times S_{t-m}$$

- $alpha$ = 0.2
- $gama$ = 0.2
- $rmse$ = 342.43

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**		**1.220**	
2011	1	885.40	774.52		0.990	684.20
	2	677.00	789.19		0.814	618.45
	3	1006.60	823.75		1.092	825.76
	4	1122.10	842.98		1.248	1004.83
2012	1	1163.40	909.33		1.068	834.84
	2	993.20	971.61		0.869	739.85
	3	1312.50	1017.63		1.144	1061.17
	4	1545.30	1061.69		1.302	1270.31
2013	1	1596.20	1148.18		1.155	1134.21
	2	1260.40	1208.52		0.915	998.16
	3	1735.20	1270.19		1.202	1382.43
	4	2029.70	1327.86		1.361	1654.22
2014	1	2107.80	1427.17		1.242	1534.11
	2	1650.30	1502.45		0.963	1305.88
	3	2304.40	1585.29		1.269	1806.38
	4	2639.40	1655.97		1.422	2158.32



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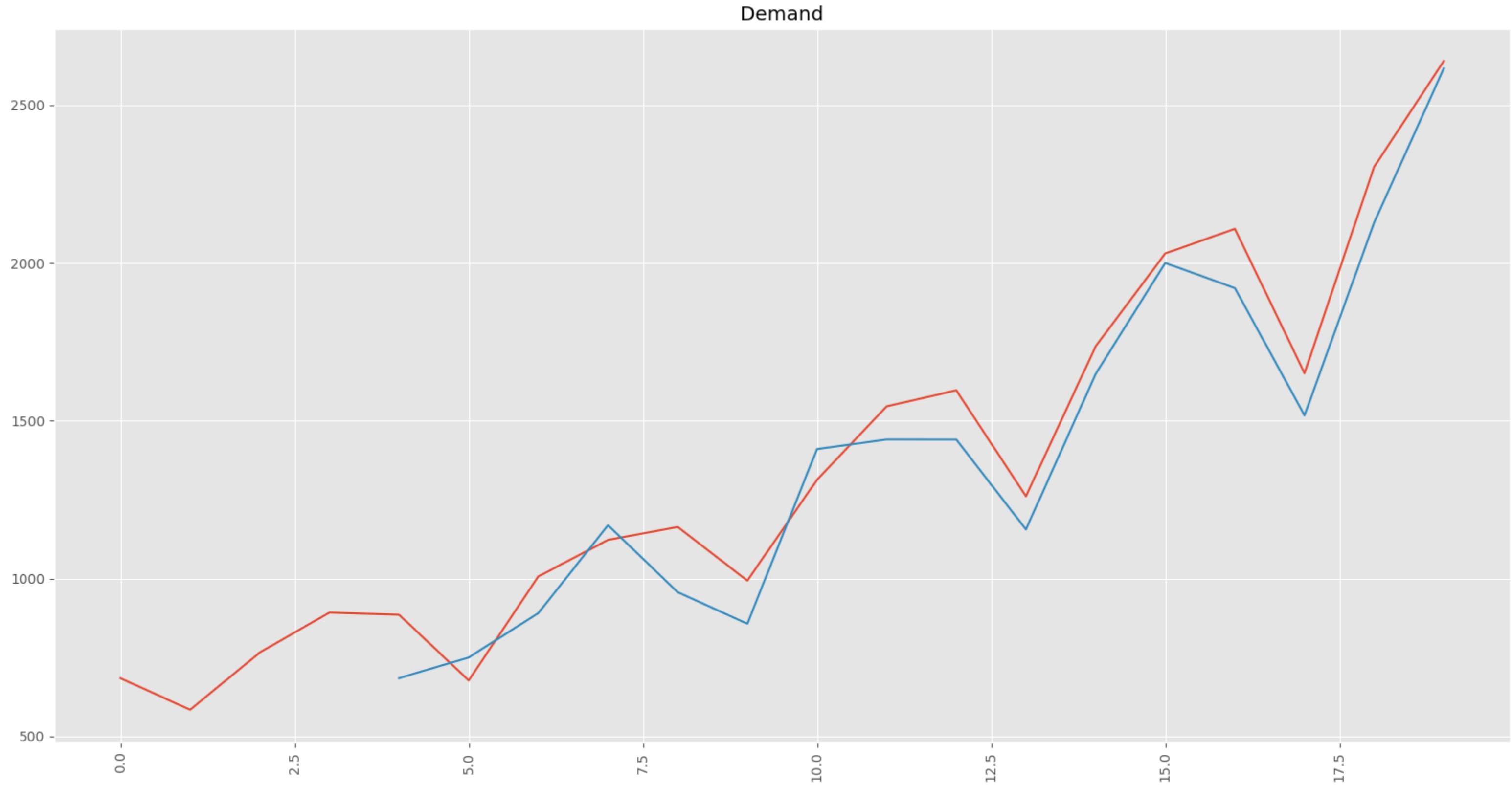
Optimization

- $alpha$ = 0.96
- $gama$ = 0.37
- $rmse$ = 130.36

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**		**1.220**	
2011	1	885.40	939.09		1.038	684.20
	2	677.00	851.03		0.769	749.86
	3	1006.60	958.14		1.098	890.47
	4	1122.10	921.23		1.202	1168.76
2012	1	1163.40	1113.33		1.123	956.68
	2	993.20	1284.67		0.815	856.59
	3	1312.50	1198.99		1.069	1409.94
	4	1545.30	1283.03		1.234	1440.66
2013	1	1596.20	1416.98		1.168	1440.38
	2	1260.40	1541.22		0.843	1155.43
	3	1735.20	1620.23		1.090	1647.67
	4	2029.70	1643.58		1.241	1999.84
2014	1	2107.80	1798.72		1.211	1920.00
	2	1650.30	1951.65		0.871	1516.68
	3	2304.40	2107.76		1.124	2128.02
	4	2639.40	2125.83		1.245	2616.16


Demand
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15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

$$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$$

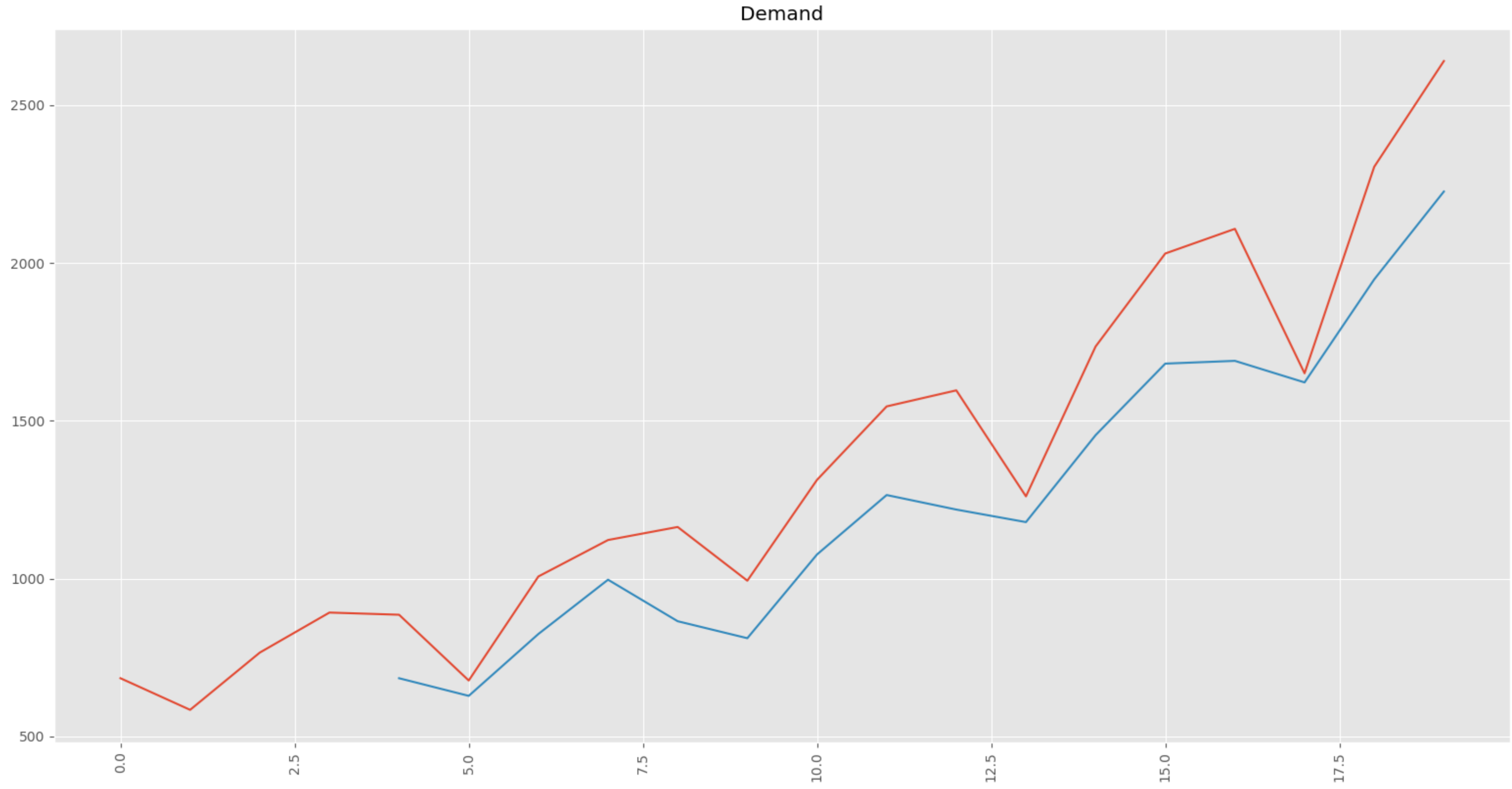
$$T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$$

$$F_t = A_{t-1} + T_{t-1} + S_{t-m}$$

- $alpha$ = 0.2
- $beta$ = 0.1
- $gama$ = 0.2
- $rmse$ = 270.69

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**	**0.00**	**160.80**	
2011	1	885.40	771.74	4.02	-7.06	684.20
	2	677.00	785.49	5.00	-137.67	628.36
	3	1006.60	826.93	8.64	70.34	824.39
	4	1122.10	860.72	11.16	185.95	996.37
2012	1	1163.40	931.59	17.13	52.66	864.81
	2	993.20	985.15	20.77	-101.24	811.04
	3	1312.50	1053.17	25.50	117.59	1076.26
	4	1545.30	1134.80	31.11	242.08	1264.61
2013	1	1596.20	1241.44	38.66	128.18	1218.57
	2	1260.40	1296.41	40.29	-84.93	1178.86
	3	1735.20	1392.88	45.91	173.77	1454.29
	4	2029.70	1508.56	52.89	311.85	1680.88
2014	1	2107.80	1645.08	61.25	211.82	1689.63
	2	1650.30	1712.11	61.83	-79.15	1621.40
	3	2304.40	1845.28	68.96	245.11	1947.71
	4	2639.40	1996.90	77.23	394.51	2226.09


Demand
2500
2000
1500
1000
500
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Optimization

Forecasting

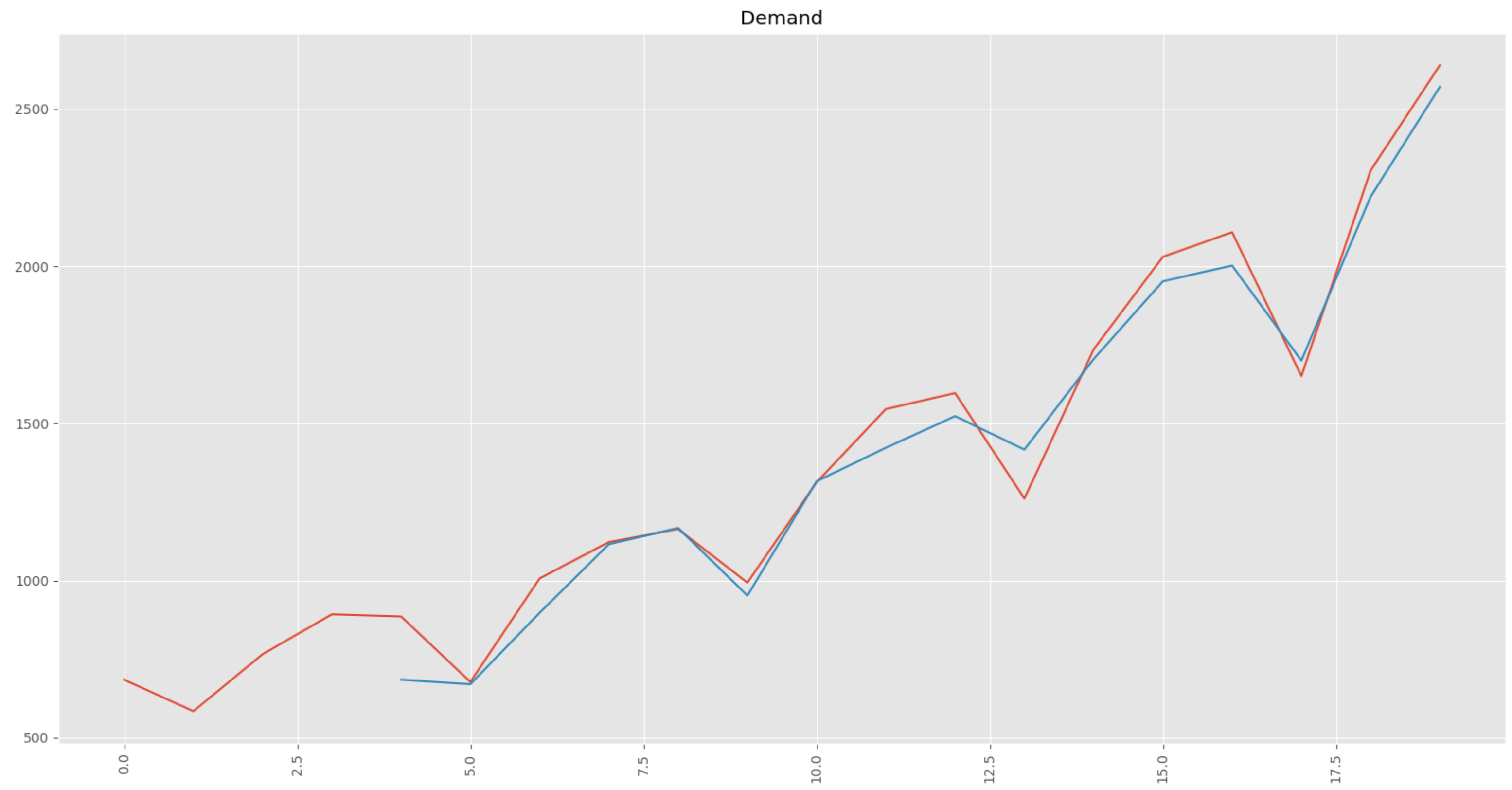
- Step 1 – Implement the Holt-Winter's Method.
- Step 2 – Choose the objective function (minimization).
- Step 3 – Choose the variable cells.
- Step 4 – Add the constraints:

$$0 \leq \alpha \leq 1; 0 \leq \beta \leq 1; 0 \leq \gamma \leq 1$$

- Step 5 – Find the solution with GRG Non Linear the method:

- $alpha$ = 0.21
- $beta$ = 1
- $gama$ = 0.94
- $rmse$ = 90.65

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**	**0.00**	**160.80**	
2011	1	885.40	774.48	42.98	141.56	684.20
	2	677.00	818.94	44.46	-140.88	670.06
	3	1006.60	886.75	67.81	136.50	897.30
	4	1122.10	956.00	69.25	167.13	1115.36
2012	1	1163.40	1024.52	68.52	138.36	1166.81
	2	993.20	1101.80	77.29	-102.35	952.16
	3	1312.50	1178.43	76.63	133.60	1315.59
	4	1545.30	1281.36	102.92	282.69	1422.19
2013	1	1596.20	1399.99	118.64	207.41	1522.65
	2	1260.40	1485.33	85.34	-248.67	1416.27
	3	1735.20	1577.28	91.95	162.63	1704.27
	4	2029.70	1685.84	108.56	355.70	1951.92
2014	1	2107.80	1817.04	131.20	306.90	2001.81
	2	1650.30	1937.72	120.68	-294.92	1699.57
	3	2304.40	2076.20	138.49	240.89	2221.03
	4	2639.40	2229.43	153.23	420.48	2570.39



Demand
2500
2000
1500
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500
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

$$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$$

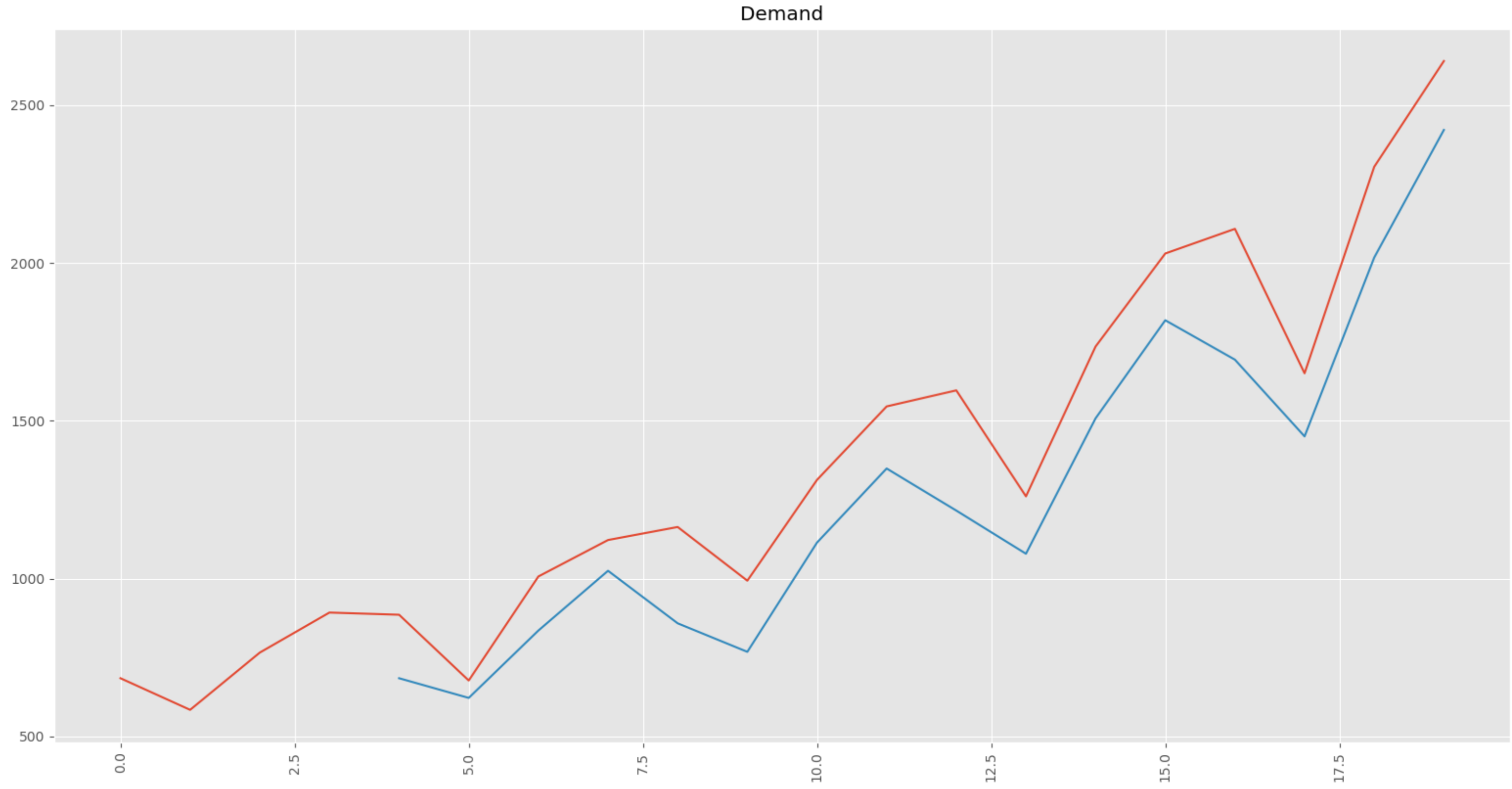
$$T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$$

$$F_t = (A_{t-1} + T_{t-1})S_{t-m}$$

- $alpha$ = 0.2
- $beta$ = 0.1
- $gama$ = 0.2
- $rmse$ = 239.90

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**	**0.00**	**1.220**	
2011	1	885.40	774.52	4.30	0.990	684.20
	2	677.00	792.63	5.68	0.813	621.89
	3	1006.60	831.05	8.96	1.089	835.31
	4	1122.10	855.98	10.55	1.243	1024.66
2012	1	1163.40	928.18	16.72	1.061	858.18
	2	993.20	1000.35	22.26	0.860	767.87
	3	1312.50	1059.08	25.91	1.128	1113.89
	4	1545.30	1116.63	29.07	1.279	1348.67
2013	1	1596.20	1217.51	36.25	1.127	1215.36
	2	1260.40	1296.01	40.48	0.889	1078.67
	3	1735.20	1376.82	44.51	1.162	1507.69
	4	2029.70	1454.39	47.82	1.309	1818.27
2014	1	2107.80	1575.73	55.17	1.182	1693.41
	2	1650.30	1675.85	59.67	0.914	1450.41
	3	2304.40	1784.99	64.61	1.195	2016.92
	4	2639.40	1882.95	67.95	1.333	2421.17



Demand
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1500
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0.0
2.5
5.0
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12.5
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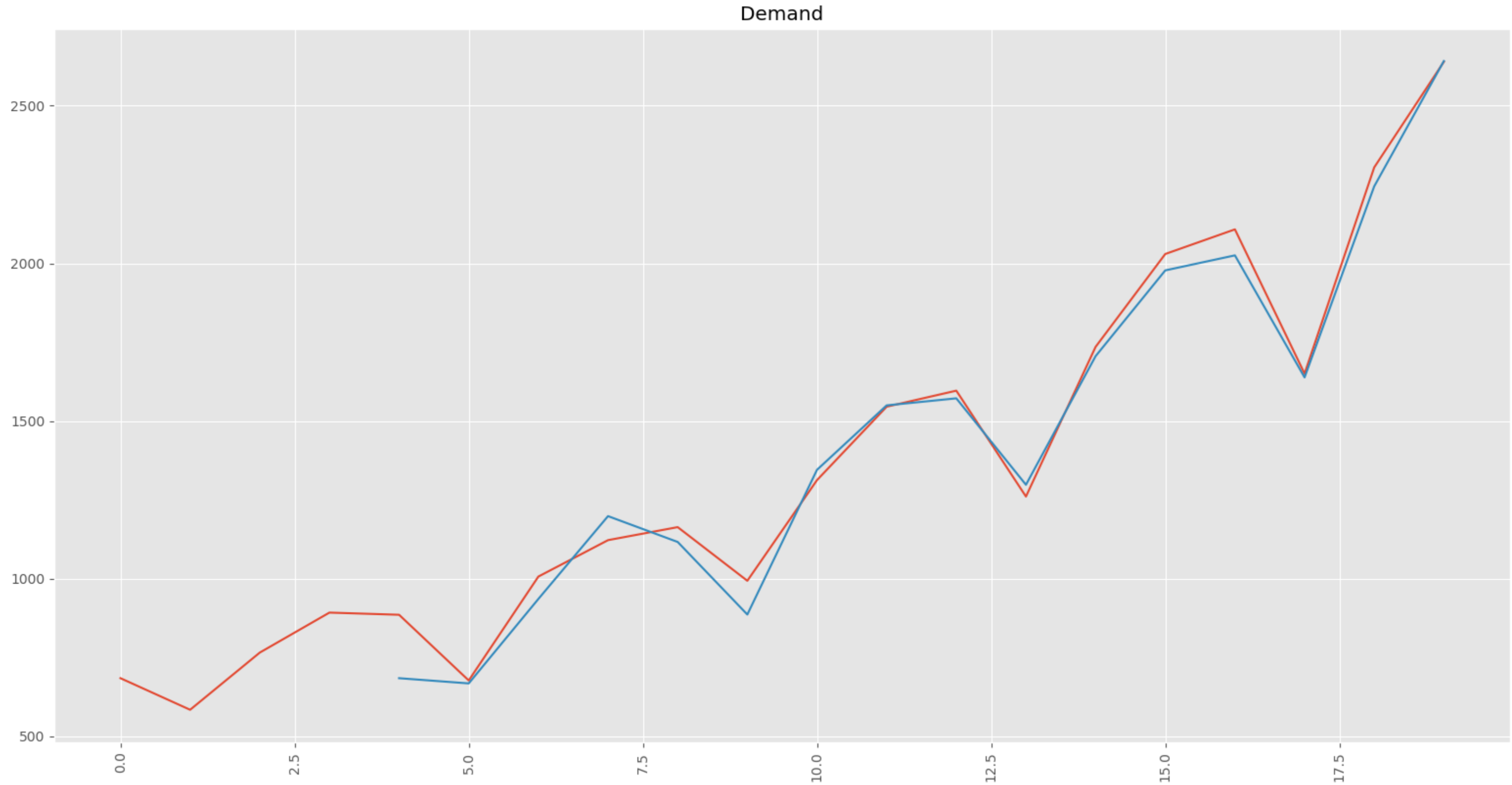
17.5

Optimization

- $alpha$ = 0.24
- $beta$ = 1
- $gama$ = 0.56
- $rmse$ = 71.71

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**	**0.00**	**1.220**	
2011	1	885.40	783.83	52.33	1.091	684.20
	2	677.00	839.00	55.17	0.805	667.67
	3	1006.60	910.68	71.68	1.091	935.62
	4	1122.10	967.16	56.48	1.176	1198.30
2012	1	1163.40	1034.14	66.98	1.116	1116.33
	2	993.20	1133.47	99.33	0.860	886.17
	3	1312.50	1225.52	92.05	1.076	1345.16
	4	1545.30	1316.70	91.17	1.174	1549.54
2013	1	1596.20	1413.17	96.47	1.126	1571.88
	2	1260.40	1499.07	85.90	0.846	1297.75
	3	1735.20	1591.64	92.56	1.087	1705.73
	4	2029.70	1694.99	103.35	1.192	1977.65
2014	1	2107.80	1816.14	121.16	1.152	2025.36
	2	1650.30	1940.74	124.60	0.849	1638.33
	3	2304.40	2078.78	138.04	1.103	2244.36
	4	2639.40	2216.34	137.56	1.191	2641.73



Demand
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0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

$$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$$

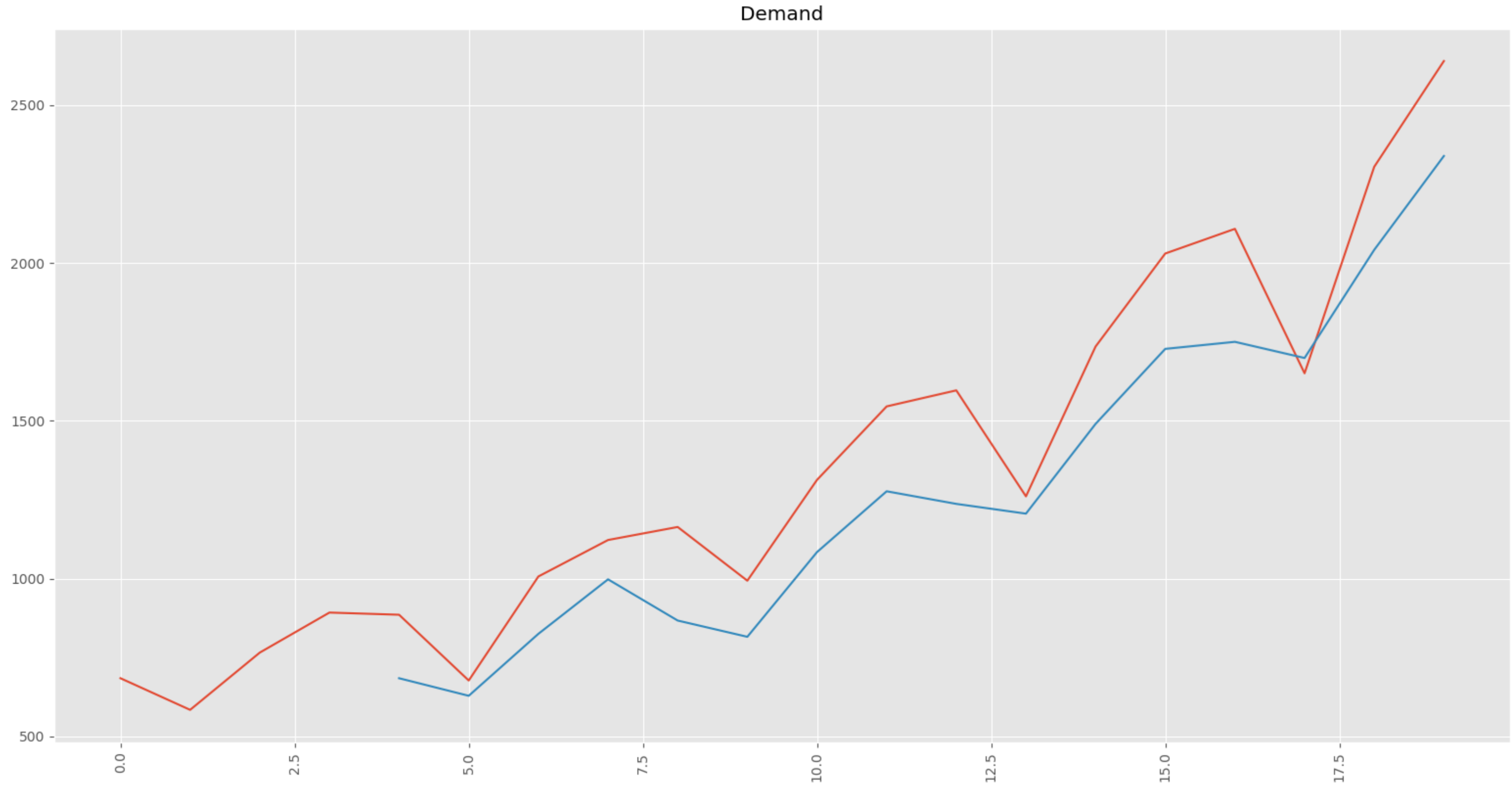
$$T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$$

$$F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$$

- $alpha$ = 0.2
- $beta$ = 0.1
- $gama$ = 0.2
- $rmse$ = 238.38

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**	**1.00**	**160.80**	
2011	1	885.40	771.74	1.01	-7.06	684.20
	2	677.00	785.67	1.01	-137.72	628.59
	3	1006.60	827.32	1.01	70.24	824.88
	4	1122.10	861.65	1.01	185.71	997.54
2012	1	1163.40	933.33	1.02	52.22	866.99
	2	993.20	988.73	1.03	-102.17	815.47
	3	1312.50	1059.28	1.03	115.99	1083.78
	4	1545.30	1144.51	1.03	239.48	1276.46
2013	1	1596.20	1256.26	1.04	124.15	1236.55
	2	1260.40	1318.79	1.04	-91.23	1205.68
	3	1735.20	1423.12	1.05	165.01	1490.09
	4	2029.70	1548.52	1.05	299.90	1727.58
2014	1	2107.80	1697.37	1.05	195.72	1749.95
	2	1650.30	1780.24	1.05	-100.91	1698.69
	3	2304.40	1928.91	1.06	217.63	2041.30
	4	2639.40	2098.85	1.06	360.07	2338.59


Demand
2500
2000
1500
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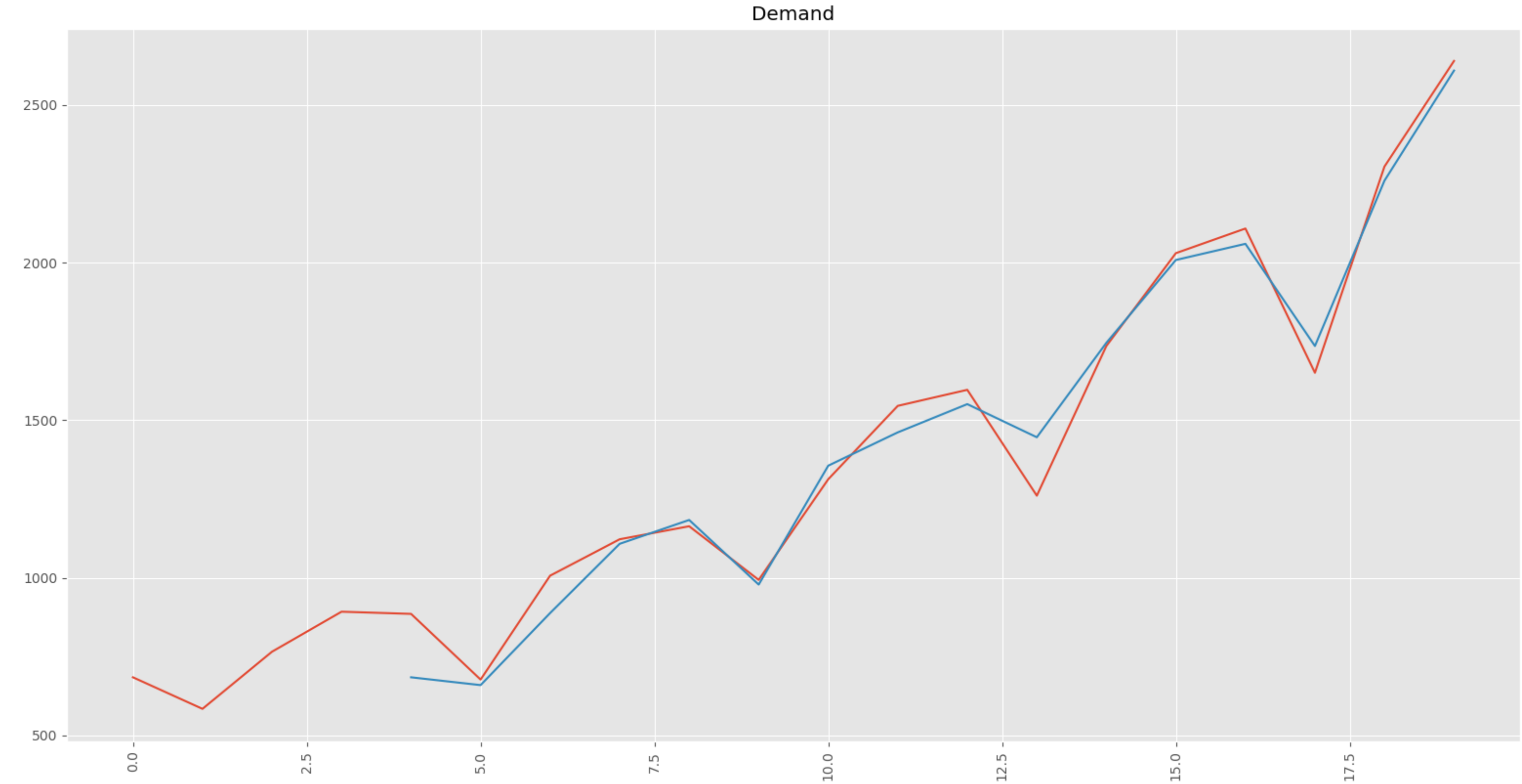
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Optimization

- $alpha$ = 0.18
- $beta$ = 1
- $gama$ = 1
- $rmse$ = 84.43

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**-47.30**	
	2	584.10			**-147.40**	
	3	765.40			**33.90**	
	4	892.30	**731.50**	**1.00**	**160.80**	
2011	1	885.40	768.25	1.05	153.90	684.20
	2	677.00	810.05	1.05	-129.84	659.44
	3	1006.60	875.78	1.08	152.48	888.02
	4	1122.10	949.49	1.08	175.25	1107.65
2012	1	1163.40	1025.76	1.08	134.00	1183.30
	2	993.20	1110.88	1.08	-114.97	978.32
	3	1312.50	1195.20	1.08	109.43	1355.54
	4	1545.30	1301.29	1.09	259.37	1461.18
2013	1	1596.20	1425.09	1.10	179.41	1550.80
	2	1260.40	1526.82	1.07	-300.26	1445.69
	3	1735.20	1633.97	1.07	99.39	1745.24
	4	2029.70	1752.61	1.07	281.05	2008.03
2014	1	2107.80	1888.73	1.08	227.94	2059.27
	2	1650.30	2019.91	1.07	-385.11	1735.15
	3	2304.40	2168.40	1.07	144.19	2259.61
	4	2639.40	2333.37	1.08	311.61	2608.84


Demand
2500
2000
1500
1000
500
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Trend × Seasonality

	N (None)	A (Additive)	M (Multiplicative)
N	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)A_{t-1}$ $S_t = \gamma(D_t - A_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)A_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times S_{t-m}$
A	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} - T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} + T_{t-1} + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} + T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} + T_{t-1})S_{t-m}$
M	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$	$A_t = \alpha(D_t - S_{t-m}) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma(D_t - A_{t-1} \times T_{t-1}) + (1-\gamma)S_{t-m}$ $F_t = (A_{t-1} \times T_{t-1}) + S_{t-m}$	$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$ $F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$

Forecasting

The model:

$$A_t = \alpha\left(\frac{D_t}{S_{t-m}}\right) + (1-\alpha)(A_{t-1} \times T_{t-1})$$

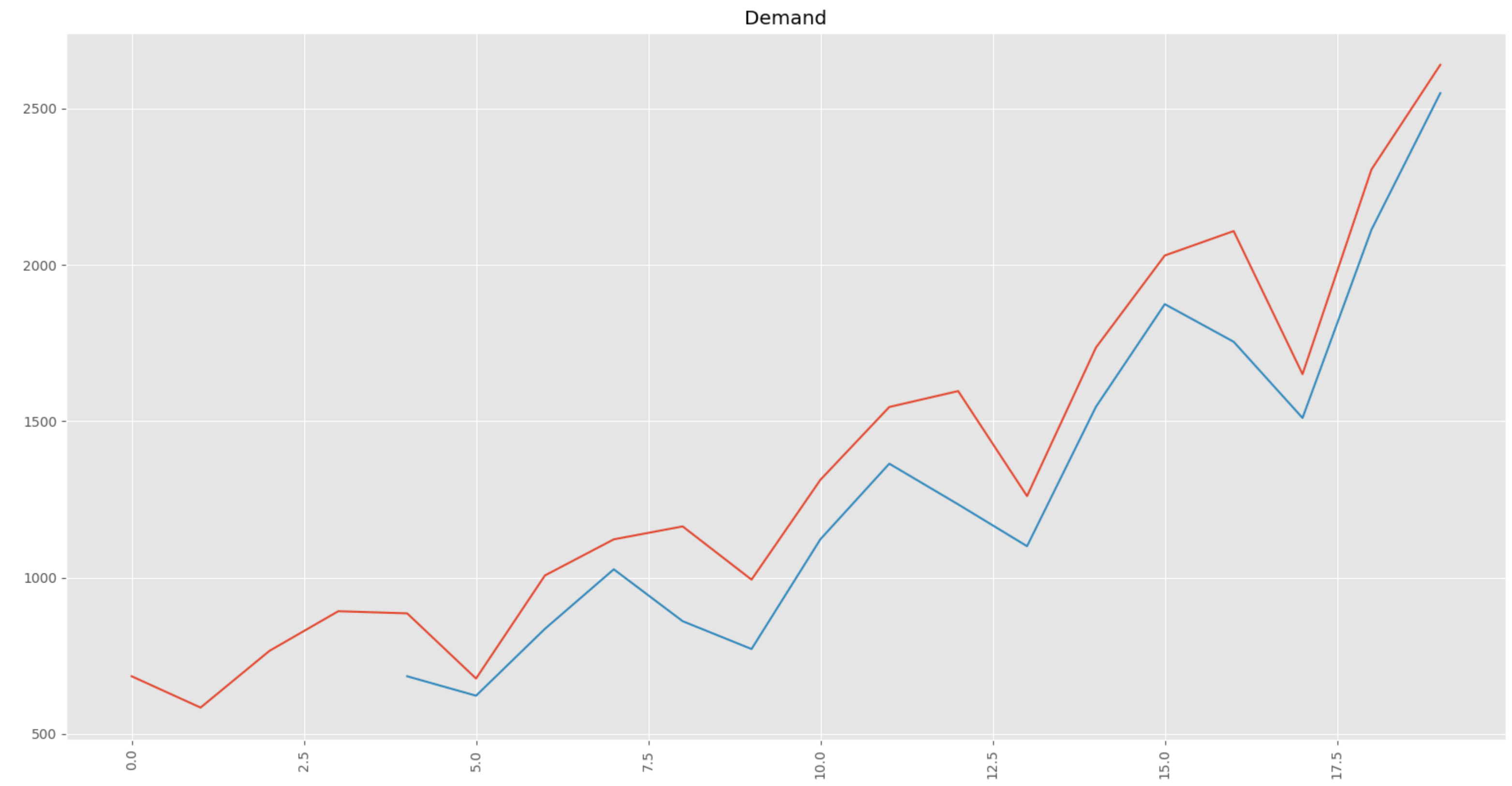
$$T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$$

$$S_t = \gamma\left(\frac{D_t}{A_{t-1} \times T_{t-1}}\right) + (1-\gamma)S_{t-m}$$

$$F_t = A_{t-1} \times T_{t-1} \times S_{t-m}$$

- $alpha$ = 0.2
- $beta$ = 0.1
- $gama$ = 0.2
- $rmse$ = 209.20

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**	**1.00**	**1.220**	
2011	1	885.40	774.52	1.01	0.990	684.20
	2	677.00	792.83	1.01	0.813	622.09
	3	1006.60	831.52	1.01	1.089	835.92
	4	1122.10	857.03	1.01	1.243	1026.25
2012	1	1163.40	929.94	1.02	1.060	860.35
	2	993.20	1003.88	1.03	0.859	771.38
	3	1312.50	1065.56	1.03	1.126	1122.46
	4	1545.30	1126.86	1.03	1.276	1363.99
2013	1	1596.20	1232.28	1.04	1.122	1233.90
	2	1260.40	1317.57	1.04	0.884	1100.18
	3	1735.20	1406.53	1.04	1.154	1545.80
	4	2029.70	1493.56	1.05	1.297	1874.13
2014	1	2107.80	1625.73	1.05	1.168	1753.88
	2	1650.30	1739.47	1.05	0.901	1510.31
	3	2304.40	1864.08	1.05	1.175	2111.77
	4	2639.40	1979.38	1.06	1.306	2548.78

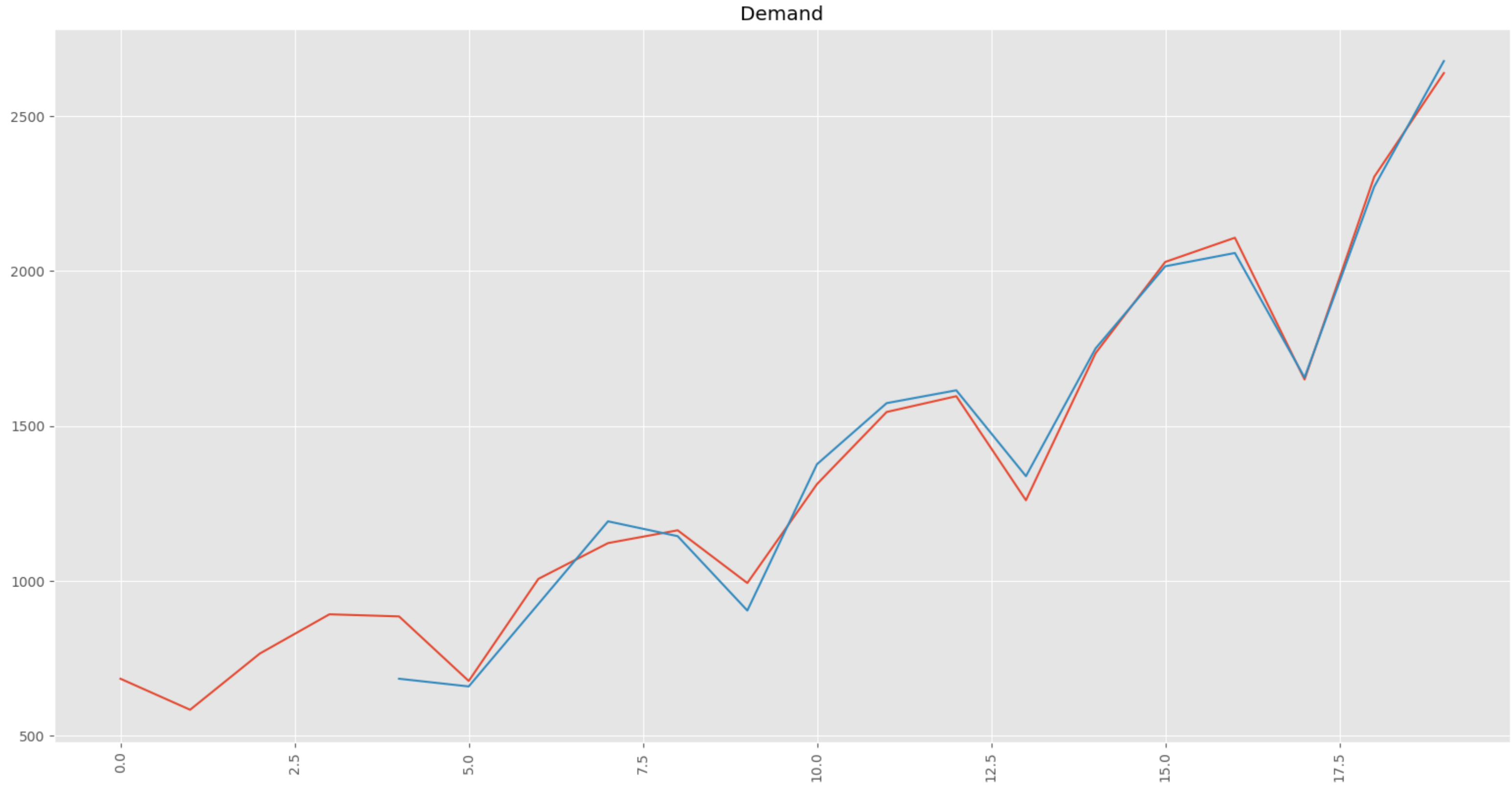


Demand
2500
2000
1500
1000
500
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Optimization

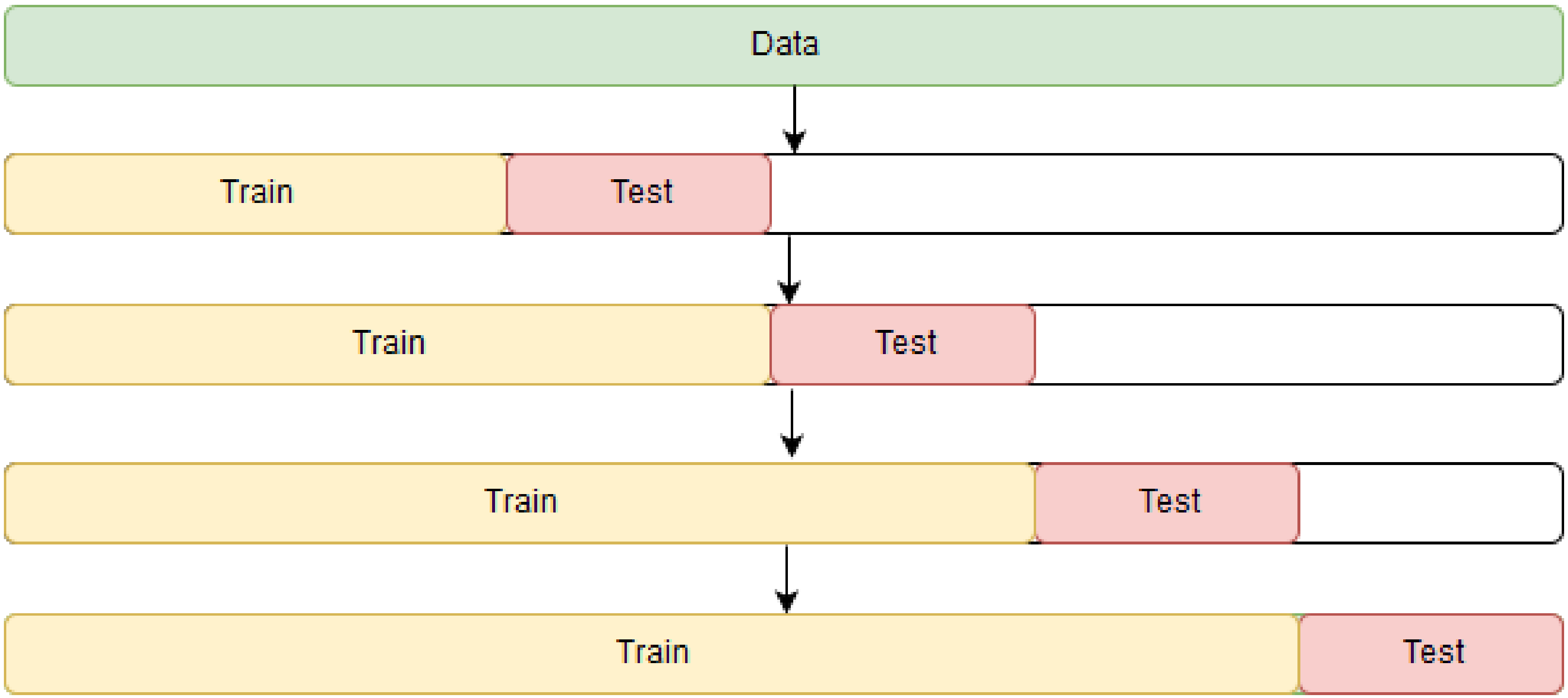
- $alpha$ = 0.22
- $beta$ = 0.90
- $gama$ = 0.63
- $rmse$ = 69.05

Year	Quarter	Demand	A	T	S	Forecast
2010	1	684.20			**0.935**	
	2	584.10			**0.798**	
	3	765.40			**1.046**	
	4	892.30	**731.50**	**1.00**	**1.220**	
2011	1	885.40	779.58	1.06	1.108	684.20
	2	677.00	830.72	1.06	0.812	659.39
	3	1006.60	901.98	1.08	1.104	925.69
	4	1122.10	964.63	1.07	1.175	1192.38
2012	1	1163.40	1036.81	1.07	1.119	1144.43
	2	993.20	1138.43	1.10	0.862	904.42
	3	1312.50	1234.37	1.09	1.071	1376.75
	4	1545.30	1334.34	1.08	1.161	1573.85
2013	1	1596.20	1439.17	1.08	1.111	1615.29
	2	1260.40	1532.48	1.07	0.830	1338.18
	3	1735.20	1630.72	1.06	1.065	1750.61
	4	2029.70	1738.31	1.07	1.166	2015.64
2014	1	2107.80	1862.62	1.07	1.128	2058.57
	2	1650.30	1993.09	1.07	0.828	1656.54
	3	2304.40	2139.56	1.07	1.075	2272.56
	4	2639.40	2288.65	1.07	1.156	2678.22



Demand
2500
2000
1500
1000
500
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Cross-validation



Data
Train
Test
Train
Test
Train
Test
Train
Test

References

- MAKRIDAKIS, S.G., WHEELWRIGHT, S.C, HYNDMAN, R.J. **Forecasting: Methods and Application**. 3rd Edition