UNIVERSIDADE FEDERAL FLUMINENSE

Programa de Mestrado e Doutorado em Engenharia de Produção

Forecasting

Lesson: Time Series Decomposition

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Forecasting

The decomposition of time series is a statistical task that deconstructs the time series components, trend-cycle, seasonality and noise. The decomposition takes into account the additive or multiplicative nature of a time series. Many techniques can be applied to extract the components of a time series, and the simplest one involves the application of moving averages.

Forecasting

The moving average is obtained by taking the arithmetic mean of the n real values of the past demand of a time series and each average is computed by dropping the oldest observation and including the next observation. Also the moving average can be centered or not.

The moving average model assumption is that the most accurate prediction of future demand is a simple (linear) combination of past demand.

Forecasting

If $n = 1$, the forecast comes down to simply taking the value of the last actual demand (last period method or Naïve Forecast). Therefore the forecast can be calculated as:

$$F_{i+h} = \frac{\sum_i^n D_i}{n}$$

Forecasting

The weighted moving average takes n previous values of the demand for the average calculation. Each value receives different weights and usually the sum of the weights is equal to 1. Also the moving average can be centered or not.

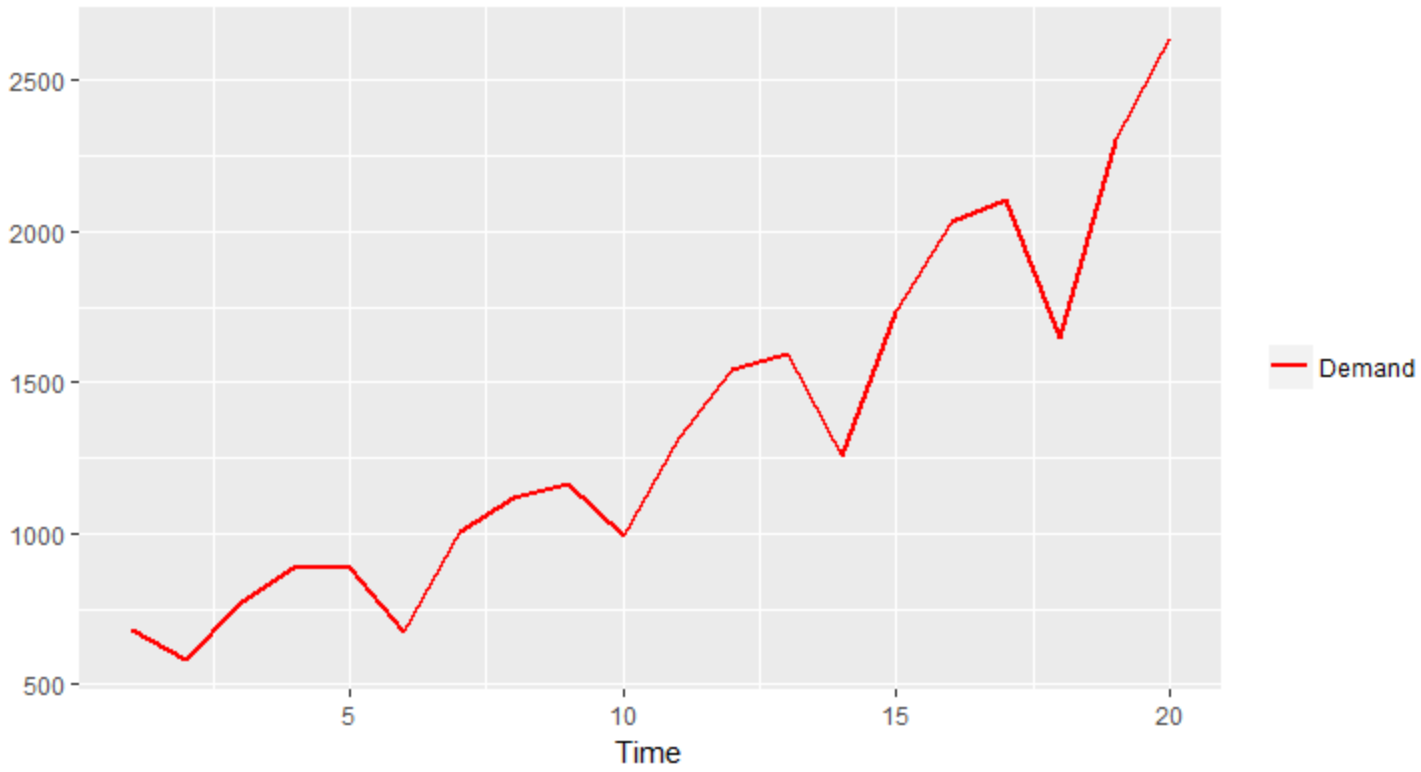
$$F_{i+h} = \sum_{i}^{n} w_i D_i$$

$$\sum_{i}^{n} w_i = 1$$

Forecasting

Period	Demand
1	684.20
2	584.10
3	765.40
4	892.30
5	885.40
6	677.00
7	1006.60
8	1122.10
9	1163.40
10	993.20
11	1312.50
12	1545.30
13	1596.20
14	1260.40
15	1735.20
16	2029.70
17	2107.80
18	1650.30
19	2304.40
20	2639.40

Forecasting

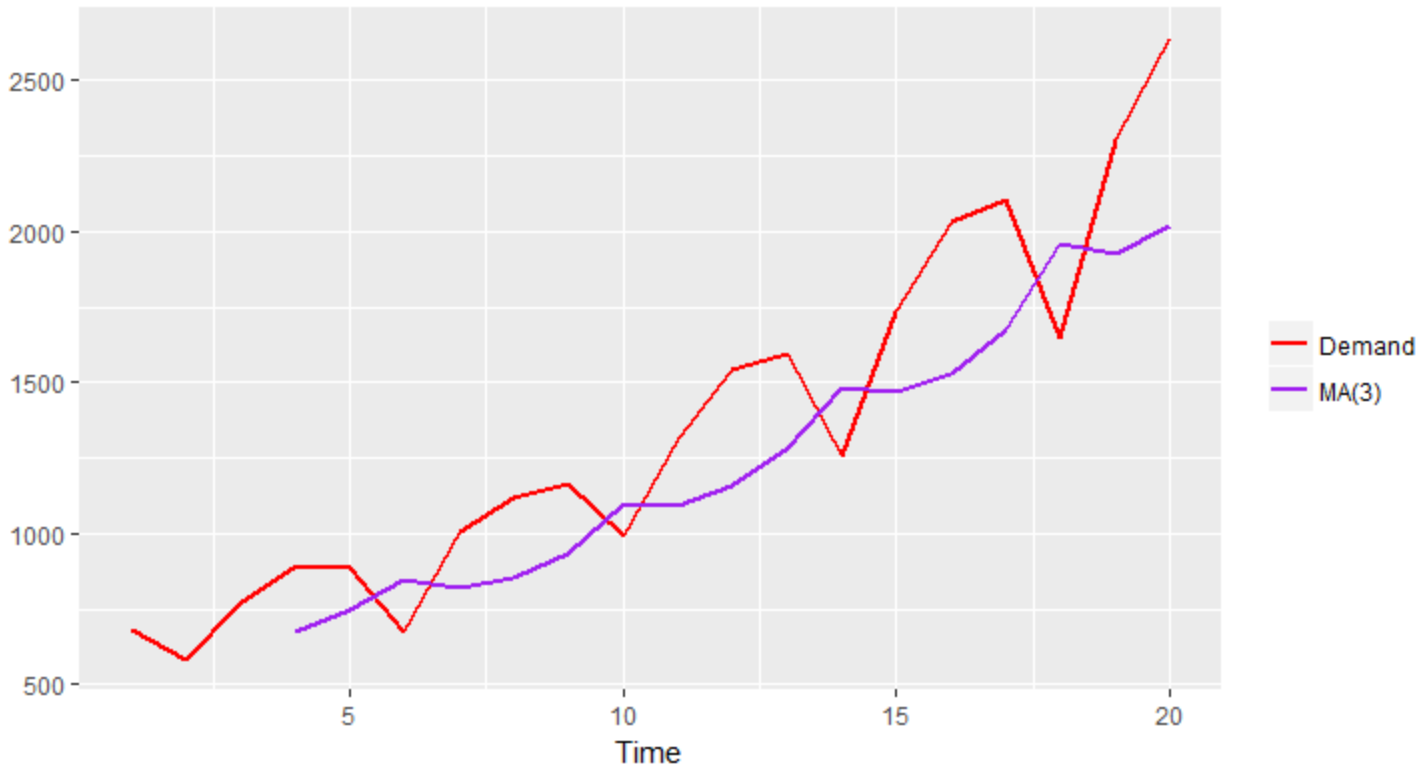


Forecasting

- MA(3)

Period	Demand	MA(3)
1	684.20	
2	584.10	
3	765.40	
4	892.30	677.90
5	885.40	747.27
6	677.00	847.70
7	1006.60	818.23
8	1122.10	856.33
9	1163.40	935.23
10	993.20	1097.37
11	1312.50	1092.90
12	1545.30	1156.37
13	1596.20	1283.67
14	1260.40	1484.67
15	1735.20	1467.30
16	2029.70	1530.60
17	2107.80	1675.10
18	1650.30	1957.57
19	2304.40	1929.27
20	2639.40	2020.83

Forecasting

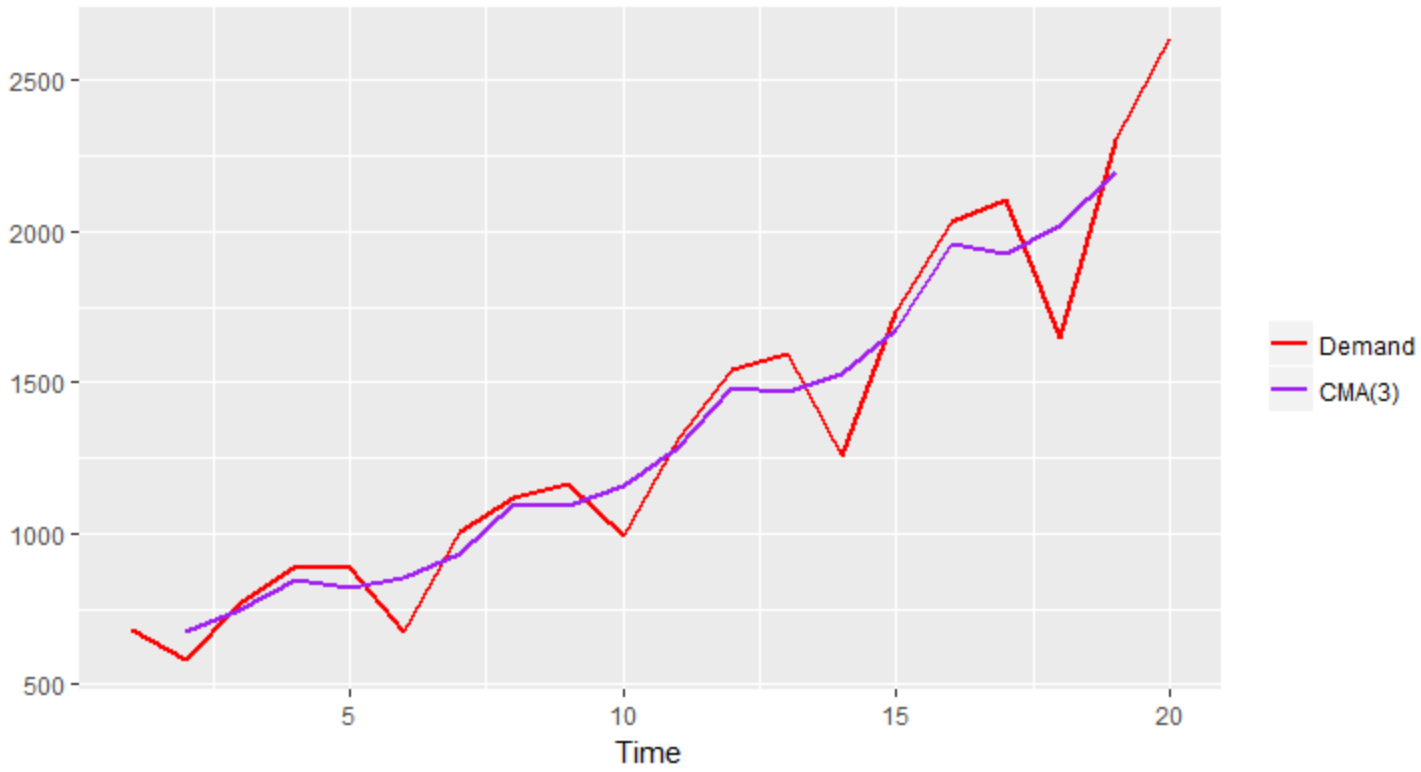


Forecasting

- CMA(3)

Period	Demand	CMA(3)
1	684.20	
2	584.10	677.90
3	765.40	747.27
4	892.30	847.70
5	885.40	818.23
6	677.00	856.33
7	1006.60	935.23
8	1122.10	1097.37
9	1163.40	1092.90
10	993.20	1156.37
11	1312.50	1283.67
12	1545.30	1484.67
13	1596.20	1467.30
14	1260.40	1530.60
15	1735.20	1675.10
16	2029.70	1957.57
17	2107.80	1929.27
18	1650.30	2020.83
19	2304.40	2198.03
20	2639.40	

Forecasting

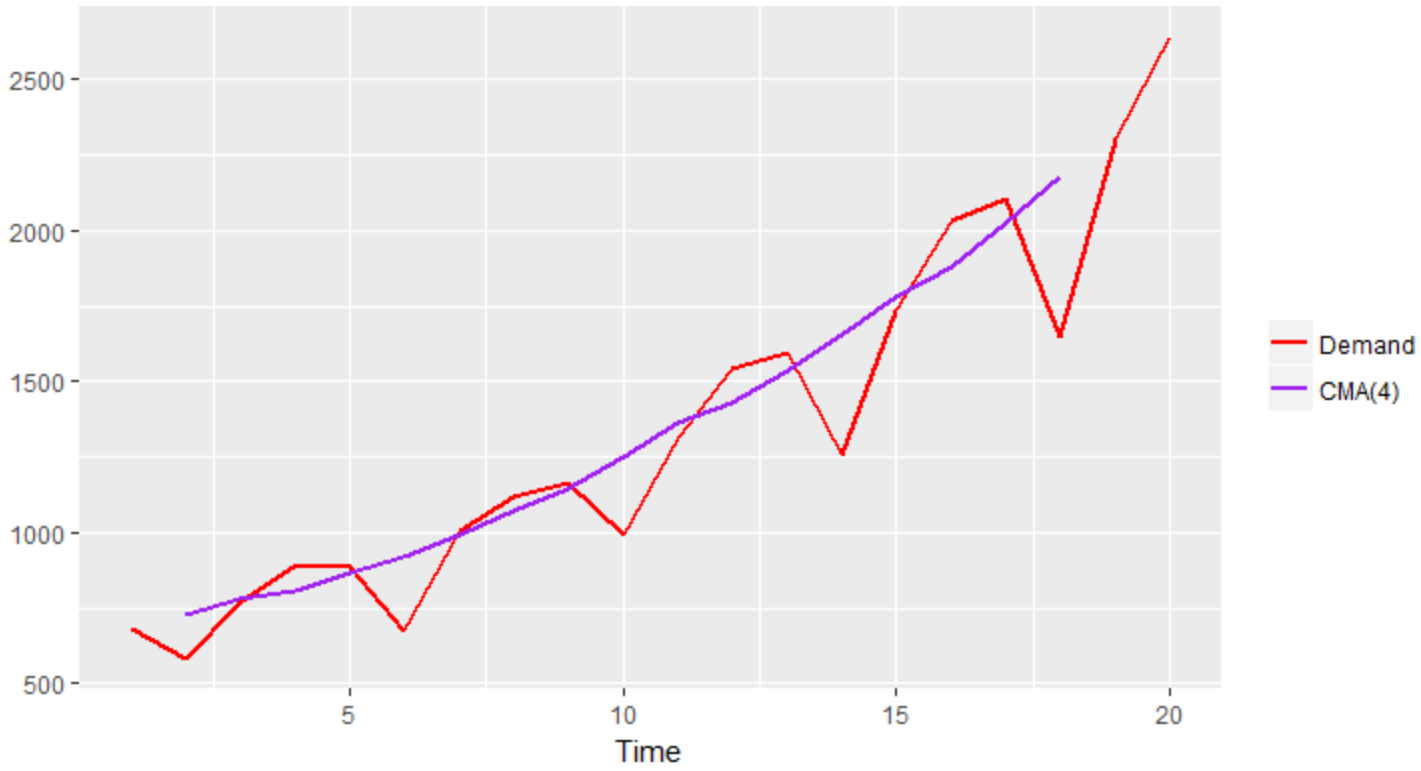


Forecasting

- CMA(4)

Period	Demand	CMA(4)
1	684.20	
2	584.10	731.50
3	765.40	781.80
4	892.30	805.03
5	885.40	865.33
6	677.00	922.78
7	1006.60	992.28
8	1122.10	1071.33
9	1163.40	1147.80
10	993.20	1253.60
11	1312.50	1361.80
12	1545.30	1428.60
13	1596.20	1534.28
14	1260.40	1655.38
15	1735.20	1783.28
16	2029.70	1880.75
17	2107.80	2023.05
18	1650.30	2175.48
19	2304.40	
20	2639.40	

Forecasting

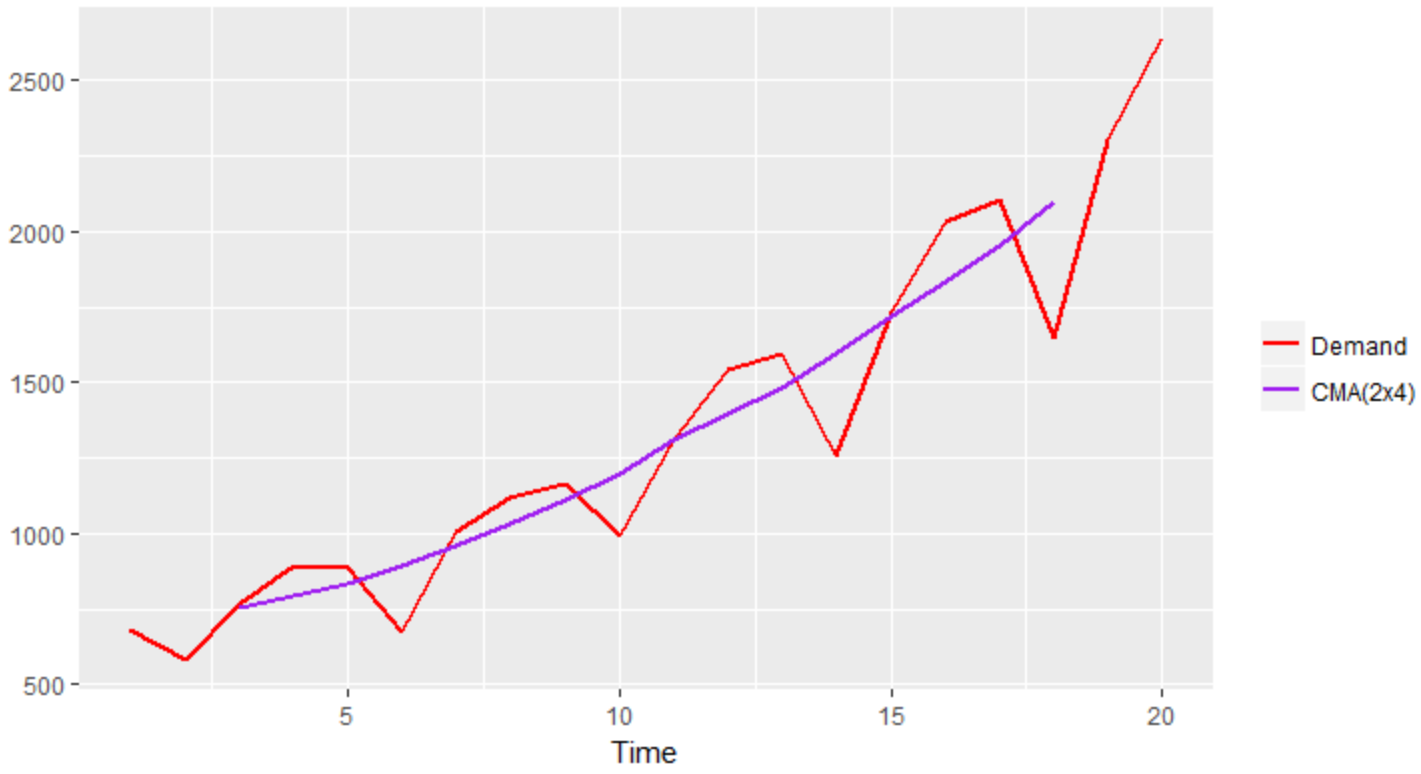


Forecasting

- CMA(2x4)

Period	Demand	CMA(4)	CMA(2x4)
1	684.20		
2	584.10	731.50	
3	765.40	781.80	756.65
4	892.30	805.03	793.41
5	885.40	865.33	835.18
6	677.00	922.78	894.05
7	1006.60	992.28	957.53
8	1122.10	1071.33	1031.80
9	1163.40	1147.80	1109.56
10	993.20	1253.60	1200.70
11	1312.50	1361.80	1307.70
12	1545.30	1428.60	1395.20
13	1596.20	1534.28	1481.44
14	1260.40	1655.38	1594.83
15	1735.20	1783.28	1719.33
16	2029.70	1880.75	1832.01
17	2107.80	2023.05	1951.90
18	1650.30	2175.48	2099.26
19	2304.40		
20	2639.40		

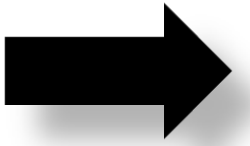
Forecasting



Forecasting

- CMA(2X4) = WMA(1/8; 1/4; 1/4; 1/4; 1/8)

Period	Demand	CMA(4)	CMA(2x4)
1	684.20		
2	584.10	731.50	
3	765.40	781.80	756.65
4	892.30	805.03	793.41
5	885.40	865.33	835.18
6	677.00	922.78	894.05
7	1006.60	992.28	957.53
8	1122.10	1071.33	1031.80
9	1163.40	1147.80	1109.56
10	993.20	1253.60	1200.70
11	1312.50	1361.80	1307.70
12	1545.30	1428.60	1395.20
13	1596.20	1534.28	1481.44
14	1260.40	1655.38	1594.83
15	1735.20	1783.28	1719.33
16	2029.70	1880.75	1832.01
17	2107.80	2023.05	1951.90
18	1650.30	2175.48	2099.26
19	2304.40		
20	2639.40		



Period	Demand	WMA
1	684.20	
2	584.10	
3	765.40	756.65
4	892.30	793.41
5	885.40	835.18
6	677.00	894.05
7	1006.60	957.53
8	1122.10	1031.80
9	1163.40	1109.56
10	993.20	1200.70
11	1312.50	1307.70
12	1545.30	1395.20
13	1596.20	1481.44
14	1260.40	1594.83
15	1735.20	1719.33
16	2029.70	1832.01
17	2107.80	1951.90
18	1650.30	2099.26
19	2304.40	
20	2639.40	

Forecasting

A MA(2x4) is equivalent to a *weighted* moving average of order 5 with weights = 1/8; 1/4; 1/4; 1/4; 1/8.

In general MA(2xk) is equivalent to a *weighted* moving average of order k+1 with weights 1/k for all observations except for the first and the last observation in the average, which have weights 1/2k.

CLASSICAL DECOMPOSITION

Forecasting

Applying a moving average to a time series captures its' trend component. And the results can be used to estimate another component, the seasonality, through the seasonal indexes.

A seasonal index is a factor that adjusts a trend value to compensate for typical seasonal fluctuation in a period of seasonality.

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	
	2	2	584.10					2	
	3	3	765.40					3	
	4	4	892.30					4	
2011	1	5	885.40						
	2	6	677.00					MSE	
	3	7	1006.60						
	4	8	1122.10						
2012	1	9	1163.40						
	2	10	993.20						
	3	11	1312.50						
	4	12	1545.30						
2013	1	13	1596.20						
	2	14	1260.40						
	3	15	1735.20						
	4	16	2029.70						
2014	1	17	2107.80						
	2	18	1650.30						
	3	19	2304.40						
	4	20	2639.40						

$$MM_4 = \frac{684.20 + 584.10 + 765.40 + 892.30}{4} = 731.50$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	
	2	2	584.10	731.50				2	
	3	3	765.40	781.80				3	
	4	4	892.30	805.03				4	
2011	1	5	885.40	865.33					
	2	6	677.00	922.78				MSE	
	3	7	1006.60	992.28					
	4	8	1122.10	1071.33					
2012	1	9	1163.40	1147.80					
	2	10	993.20	1253.60					
	3	11	1312.50	1361.80					
	4	12	1545.30	1428.60					
2013	1	13	1596.20	1534.28					
	2	14	1260.40	1655.38					
	3	15	1735.20	1783.28					
	4	16	2029.70	1880.75					
2014	1	17	2107.80	2023.05					
	2	18	1650.30	2175.48					
	3	19	2304.40						
	4	20	2639.40						

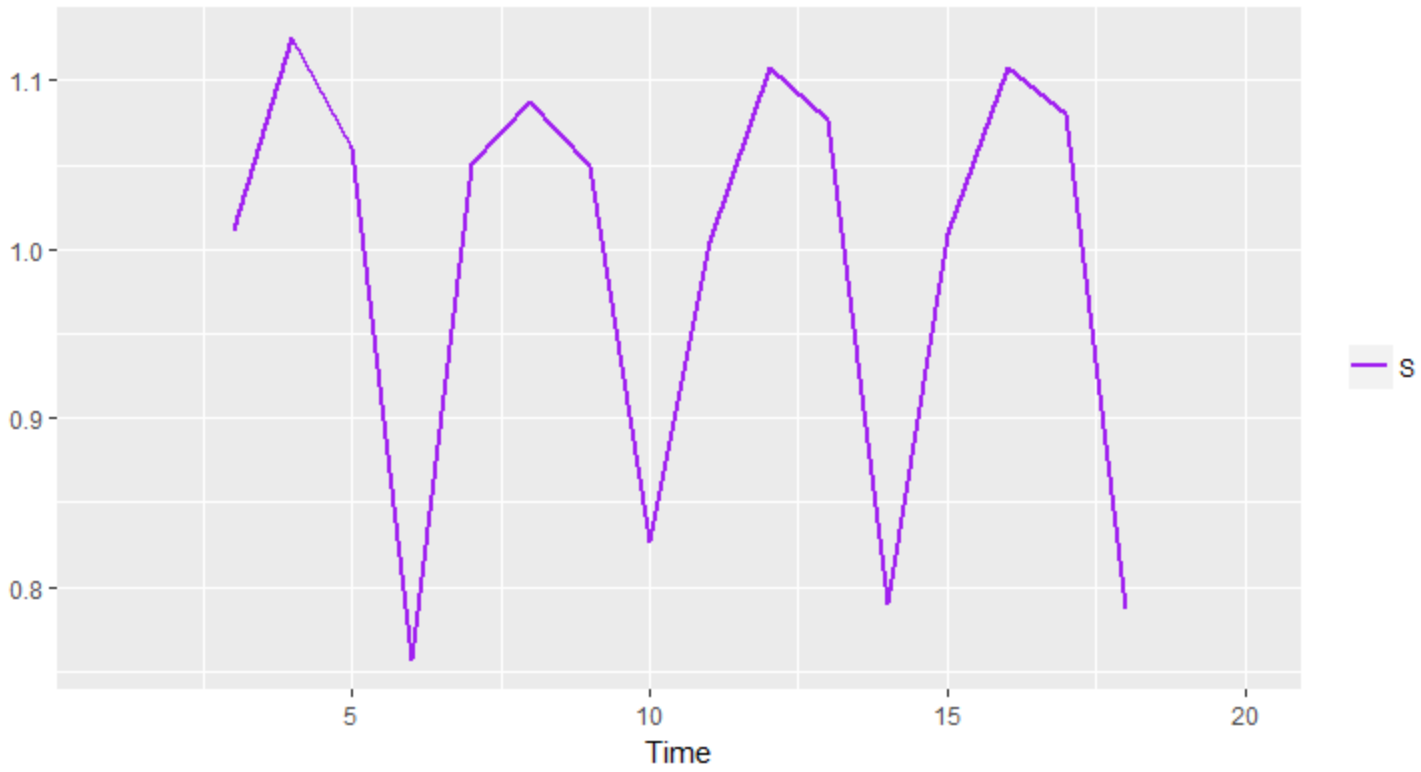
$$MM_2 = \frac{731.50 + 781.80}{2} = 756.65$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	
	2	2	584.10	731.50				2	
	3	3	765.40	781.80	756.65			3	
	4	4	892.30	805.03	793.41			4	
2011	1	5	885.40	865.33	835.18				
	2	6	677.00	922.78	894.05			**MSE**	
	3	7	1006.60	992.28	957.53				
	4	8	1122.10	1071.33	1031.80				
2012	1	9	1163.40	1147.80	1109.56				
	2	10	993.20	1253.60	1200.70				
	3	11	1312.50	1361.80	1307.70				
	4	12	1545.30	1428.60	1395.20				
2013	1	13	1596.20	1534.28	1481.44				
	2	14	1260.40	1655.38	1594.83				
	3	15	1735.20	1783.28	1719.33				
	4	16	2029.70	1880.75	1832.01				
2014	1	17	2107.80	2023.05	1951.90				
	2	18	1650.30	2175.48	2099.26				
	3	19	2304.40						
	4	20	2639.40						

$$S = \frac{765.40}{756.65} = 1.012$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	
	2	2	584.10	731.50				2	
	3	3	765.40	781.80	756.65	1.012		3	
	4	4	892.30	805.03	793.41	1.125		4	
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		**MSE**	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

Forecasting



$$SI\,(Seasonal\ Index)_1 = \frac{1.060 + 1.049 + 1.077 + 1.080}{4} = 1.066$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	
	3	3	765.40	781.80	756.65	1.012		3	
	4	4	892.30	805.03	793.41	1.125		4	
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		MSE	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

$$SI(Seasonal\ Index)_2 = \frac{0.757 + 0.827 + 0.790 + 0.786}{4} = 0.790$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012		3	
	4	4	892.30	805.03	793.41	1.125		4	
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		MSE	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

$$SI(Seasonal\ Index)_3 = \frac{1.012 + 1.051 + 1.004 + 1.009}{4} = 1.019$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012		3	1.019
	4	4	892.30	805.03	793.41	1.125		4	
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		**MSE**	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

$$SI(Seasonal\ Index)_4 = \frac{1.125 + 1.088 + 1.108 + 1.108}{4} = 1.107$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012		3	1.019
	4	4	892.30	805.03	793.41	1.125		4	1.107
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		MSE	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

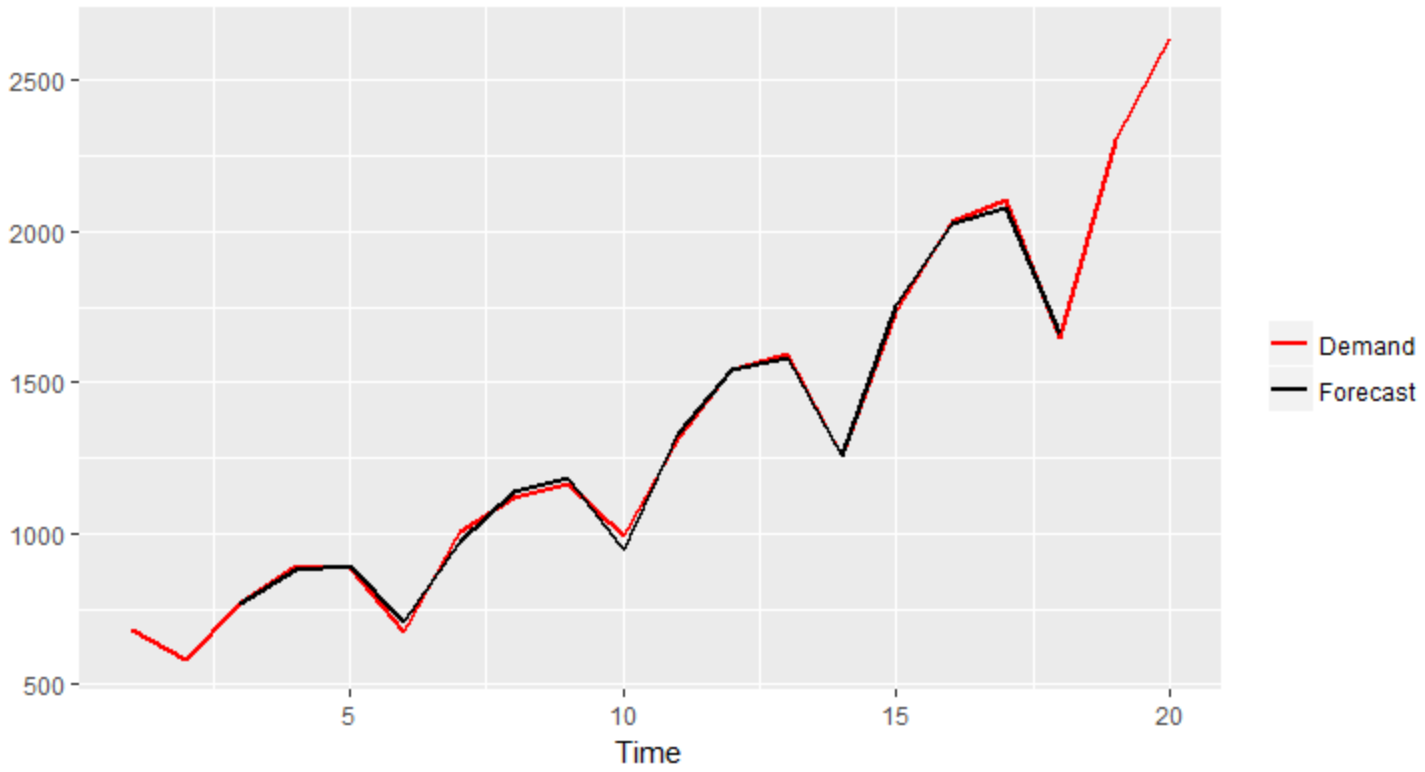
$$F(2010)_3 = 756.65 \times 1.019 = 770.97$$

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012	770.97	3	1.019
	4	4	892.30	805.03	793.41	1.125		4	1.107
2011	1	5	885.40	865.33	835.18	1.060			
	2	6	677.00	922.78	894.05	0.757		**MSE**	
	3	7	1006.60	992.28	957.53	1.051			
	4	8	1122.10	1071.33	1031.80	1.088			
2012	1	9	1163.40	1147.80	1109.56	1.049			
	2	10	993.20	1253.60	1200.70	0.827			
	3	11	1312.50	1361.80	1307.70	1.004			
	4	12	1545.30	1428.60	1395.20	1.108			
2013	1	13	1596.20	1534.28	1481.44	1.077			
	2	14	1260.40	1655.38	1594.83	0.790			
	3	15	1735.20	1783.28	1719.33	1.009			
	4	16	2029.70	1880.75	1832.01	1.108			
2014	1	17	2107.80	2023.05	1951.90	1.080			
	2	18	1650.30	2175.48	2099.26	0.786			
	3	19	2304.40						
	4	20	2639.40						

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012	770.97	3	1.019
	4	4	892.30	805.03	793.41	1.125	878.24	4	1.107
2011	1	5	885.40	865.33	835.18	1.060	890.71		
	2	6	677.00	922.78	894.05	0.757	706.49	**MSE**	
	3	7	1006.60	992.28	957.53	1.051	975.65		
	4	8	1122.10	1071.33	1031.80	1.088	1142.11		
2012	1	9	1163.40	1147.80	1109.56	1.049	1183.35		
	2	10	993.20	1253.60	1200.70	0.827	948.81		
	3	11	1312.50	1361.80	1307.70	1.004	1332.45		
	4	12	1545.30	1428.60	1395.20	1.108	1544.36		
2013	1	13	1596.20	1534.28	1481.44	1.077	1579.95		
	2	14	1260.40	1655.38	1594.83	0.790	1260.25		
	3	15	1735.20	1783.28	1719.33	1.009	1751.87		
	4	16	2029.70	1880.75	1832.01	1.108	2027.87		
2014	1	17	2107.80	2023.05	1951.90	1.080	2081.70		
	2	18	1650.30	2175.48	2099.26	0.786	1658.86		
	3	19	2304.40						
	4	20	2639.40						

Year	Quarter	Period	Demand	MA(4)	MA(2)	S	Forecast	Quarter	SI
2010	1	1	684.20					1	1.066
	2	2	584.10	731.50				2	0.790
	3	3	765.40	781.80	756.65	1.012	770.97	3	1.019
	4	4	892.30	805.03	793.41	1.125	878.24	4	1.107
2011	1	5	885.40	865.33	835.18	1.060	890.71		
	2	6	677.00	922.78	894.05	0.757	706.49	**MSE**	436.829
	3	7	1006.60	992.28	957.53	1.051	975.65		
	4	8	1122.10	1071.33	1031.80	1.088	1142.11		
2012	1	9	1163.40	1147.80	1109.56	1.049	1183.35		
	2	10	993.20	1253.60	1200.70	0.827	948.81		
	3	11	1312.50	1361.80	1307.70	1.004	1332.45		
	4	12	1545.30	1428.60	1395.20	1.108	1544.36		
2013	1	13	1596.20	1534.28	1481.44	1.077	1579.95		
	2	14	1260.40	1655.38	1594.83	0.790	1260.25		
	3	15	1735.20	1783.28	1719.33	1.009	1751.87		
	4	16	2029.70	1880.75	1832.01	1.108	2027.87		
2014	1	17	2107.80	2023.05	1951.90	1.080	2081.70		
	2	18	1650.30	2175.48	2099.26	0.786	1658.86		
	3	19	2304.40						
	4	20	2639.40						

Forecasting



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