UNIVERSIDADE FEDERAL FLUMINENSE

Programa de Mestrado e Doutorado em Engenharia de Produção

Forecasting

Lesson: Holt's Method

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Forecasting

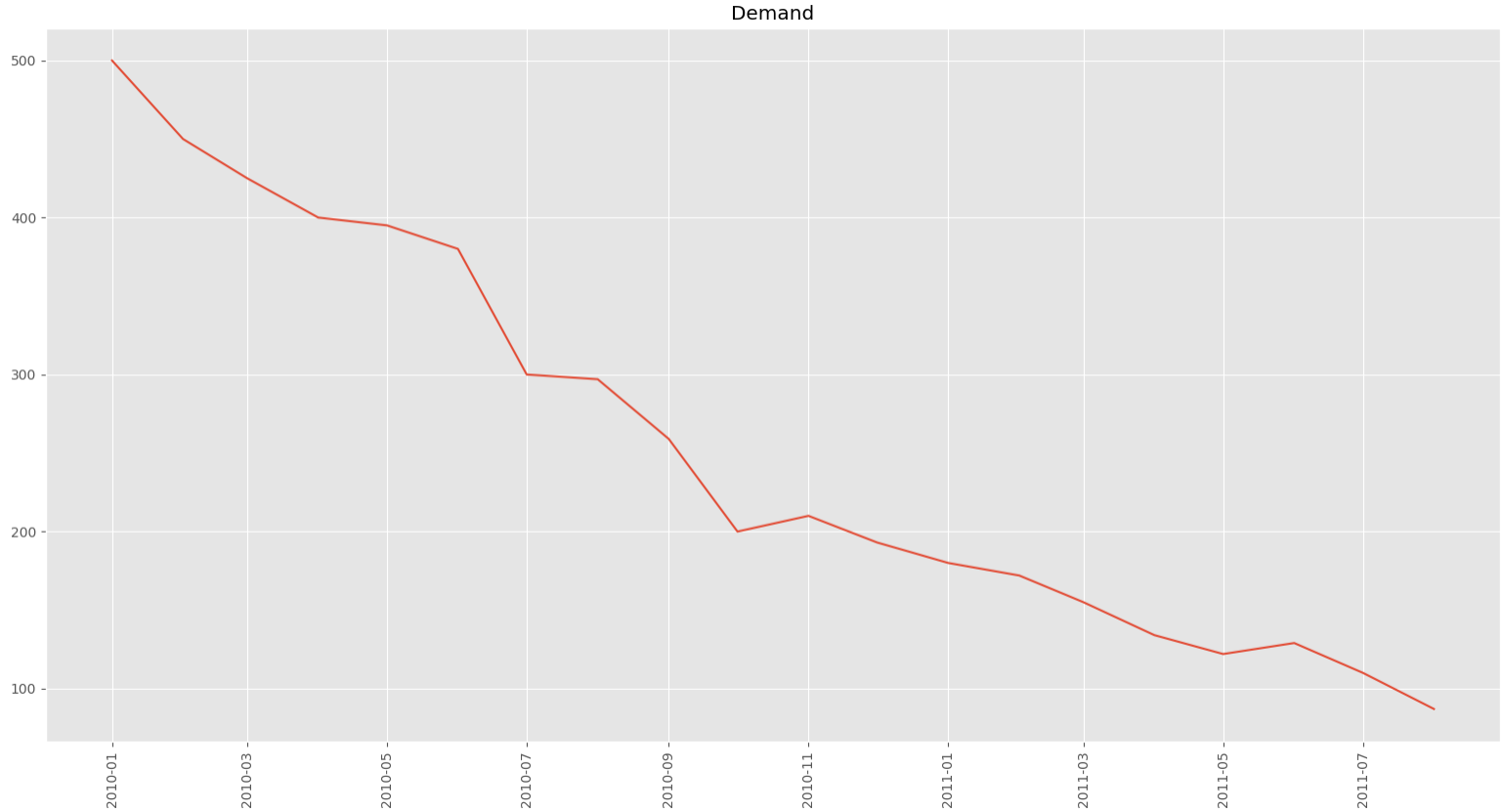
The **Holt's method** is used for the time series with multiplicative or additive trend. This model separates the data into two terms: a normal level term and a trend term.

- The normal level term is calculated based in an α variable where $0 \leq \alpha \leq 1$.

- The trend term is calculated based in a β variable where $0 \leq \beta \leq 1$.

Forecasting

Month	Demand
1	500
2	450
3	425
4	400
5	395
6	380
7	300
8	297
9	259
10	200
11	210
12	193
13	180
14	172
15	155
16	134
17	122
18	129
19	110
20	87


Demand
500
400
300
200
100
2010-01
2010-03
2010-05
2010-07
2010-09
2010-11
2011-01
2011-03
2011-05
2011-07

Seasonality / Trend	N (None)
None	(None; None)
Additive	(Additive; None)
Multiplicative	(Multiplicative; None)

Seasonality / Trend	N (None)
None	Simple Exponential Smoothing Method
Additive	Holt′s Additive Method
Multiplicative	Holt′s Multiplicative Method

Seasonality / Trend	N (None)
None	$A_t = \alpha(D_t) + (1-\alpha)A_{t-1}$ $F_t = A_{t-1}$
Additive	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} + T_{t-1})$ $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} + T_{t-1}$
Multiplicative	$A_t = \alpha(D_t) + (1-\alpha)(A_{t-1} \times T_{t-1})$ $T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1-\beta)T_{t-1}$ $F_t = A_{t-1} \times T_{t-1}$

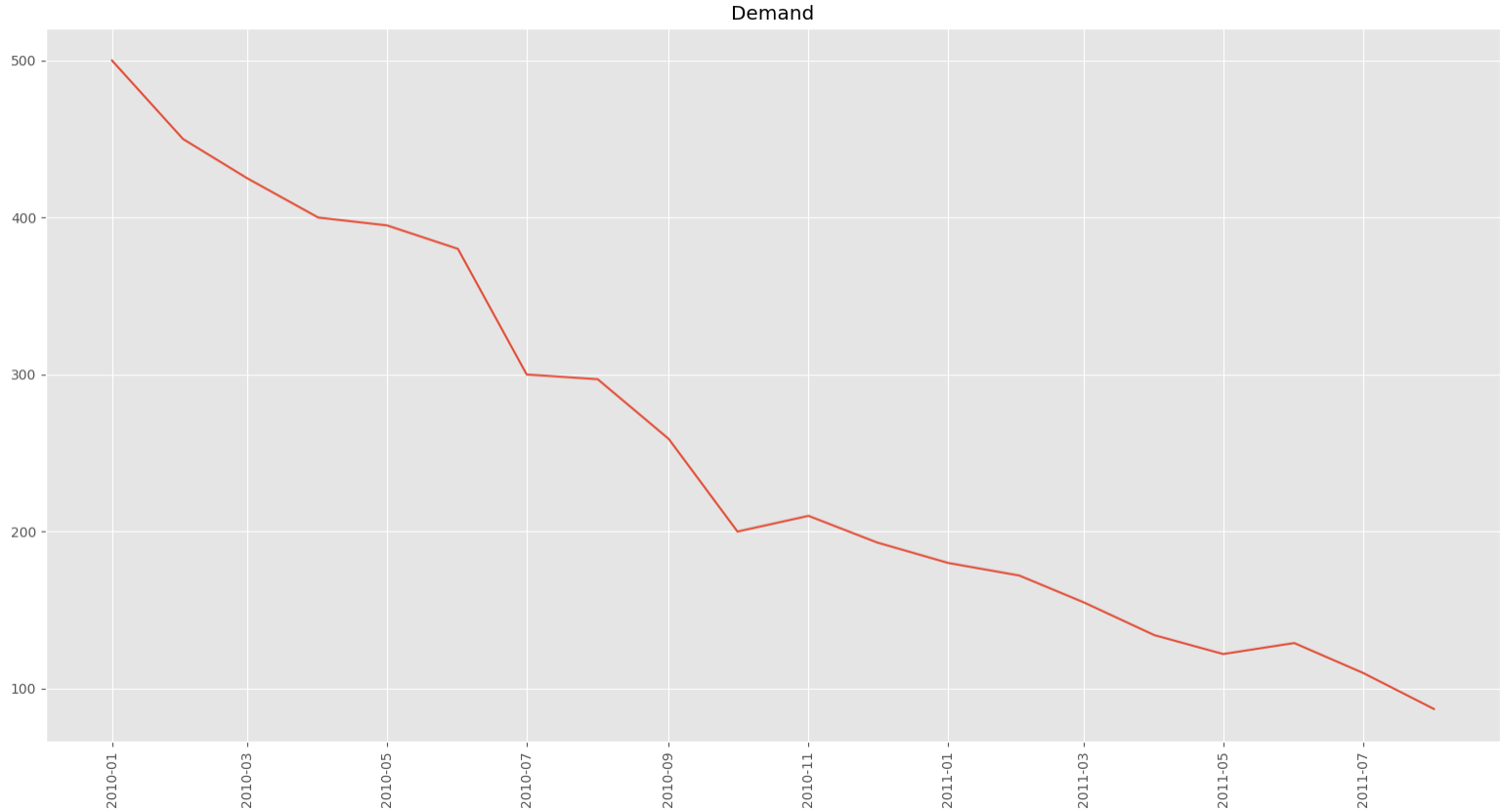
Seasonality / Trend	N (None)
None	Simple Exponential Smoothing Method
Additive	Holt's Additive Method
Multiplicative	Holt's Multiplicative Method

Forecasting

The model:

$$A_t = \alpha(D_t) + (1 - \alpha)A_{t-1}$$

$$F_t = A_{t-1}$$


Demand
500
400
300
200
100
2010-01
2010-03
2010-05
2010-07
2010-09
2010-11
2011-01
2011-03
2011-05
2011-07

Month	Demand	A	F
1	500		
2	450		
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

Month	Demand	A	F
1	500	500	
2	450		
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

Inicialization

Month	Demand	A	F
1	500	500	
2	450		
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

$$A_2 = \alpha(D_2) + (1 - \alpha)A_1$$

Month	Demand	A	F
1	500	500	
2	450	470.00	
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

$$A_2 = 0.6(450) + (0.4)500$$

Month	Demand	A	F
1	500	500	
2	450	470.00	
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

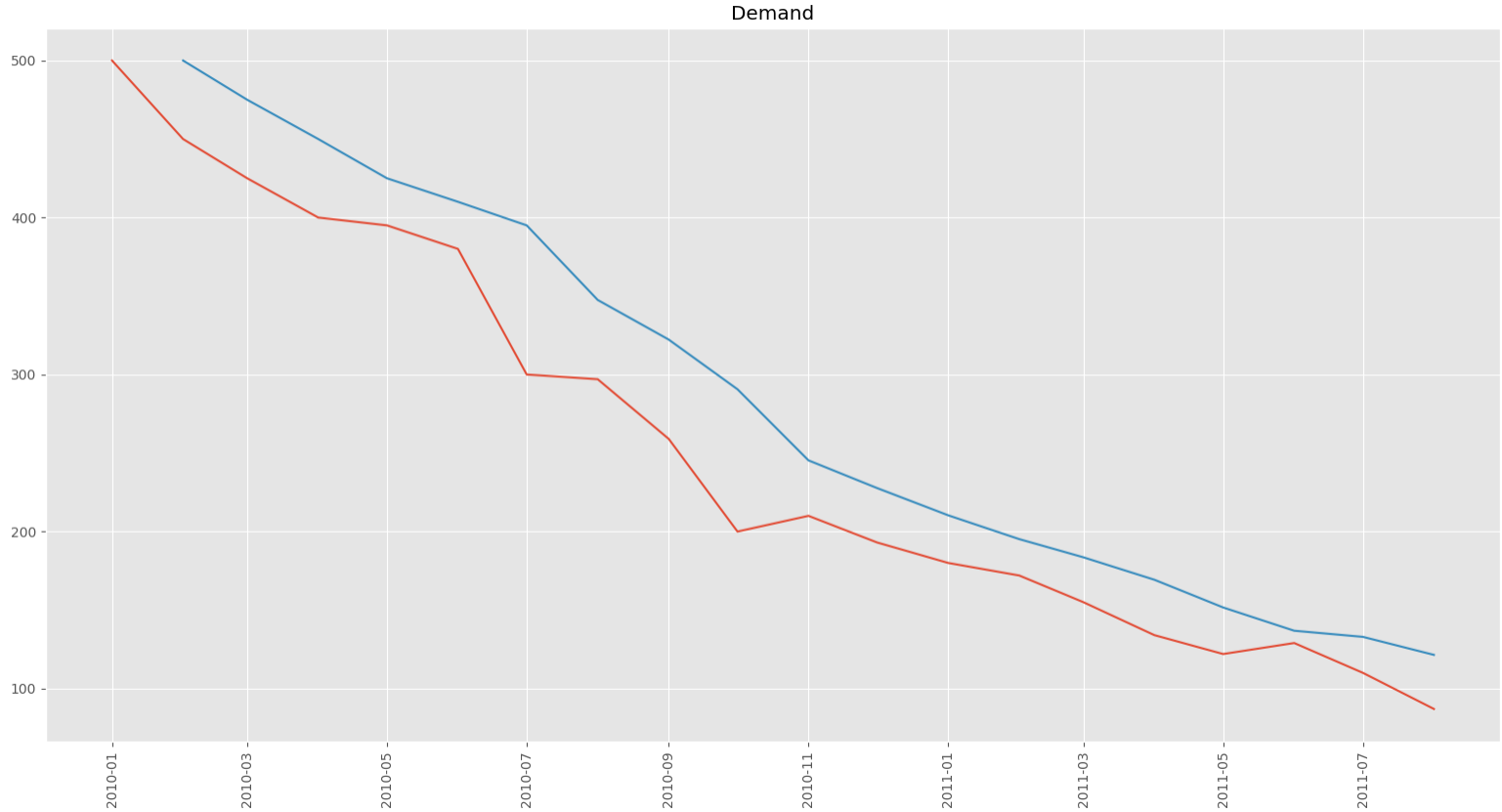
$F_2 = A_1$

Month	Demand	A	F
1	500	500	
2	450	470.00	500
3	425		
4	400		
5	395		
6	380		
7	300		
8	297		
9	259		
10	200		
11	210		
12	193		
13	180		
14	172		
15	155		
16	134		
17	122		
18	129		
19	110		
20	87		

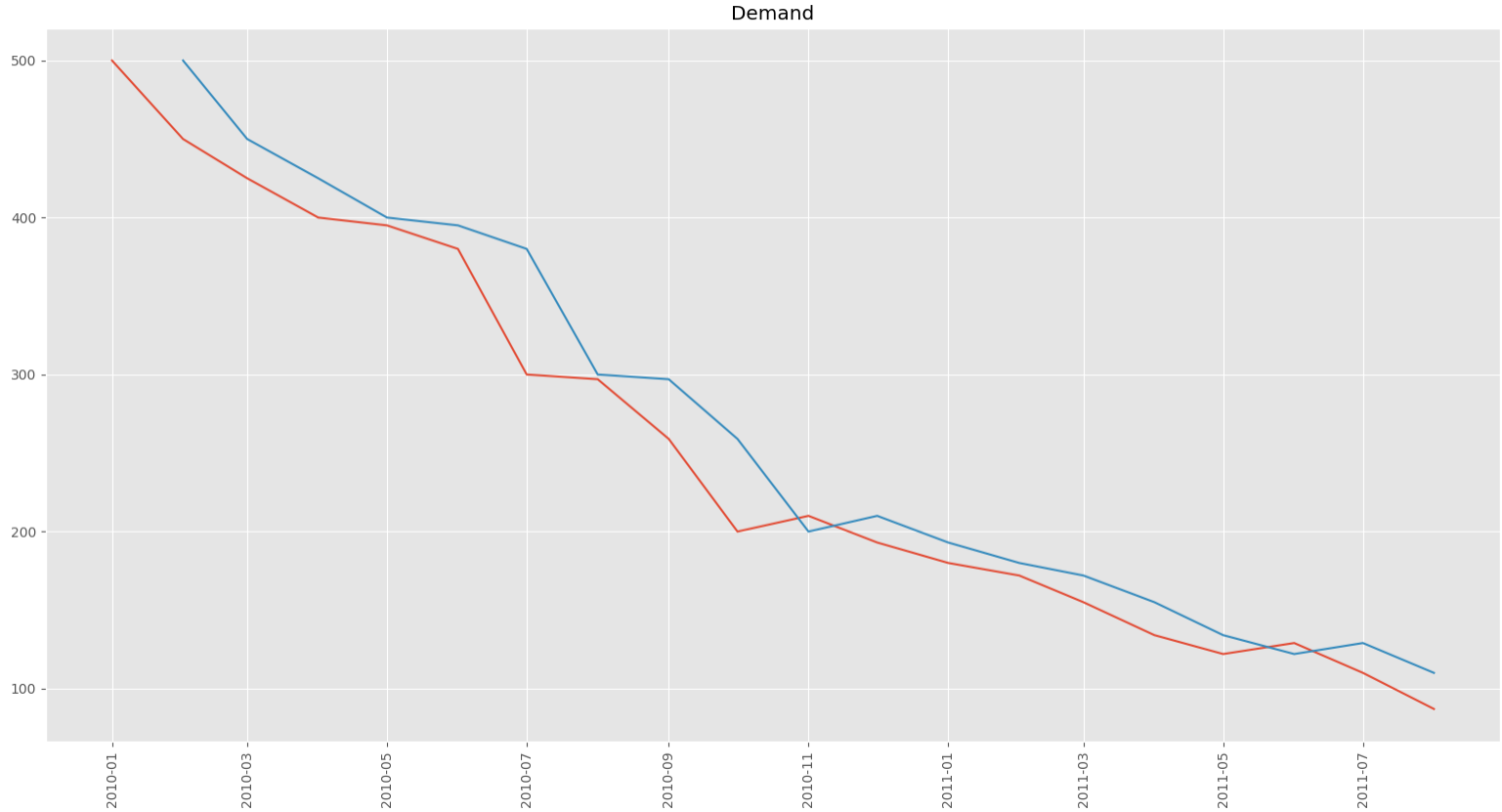
$F_2 = 500$

Month	Demand	A	F
1	500	500	
2	450	470.00	500
3	425	443.00	470.00
4	400	417.20	443.00
5	395	403.88	417.20
6	380	389.55	403.88
7	300	335.82	389.55
8	297	312.53	335.82
9	259	280.41	312.53
10	200	232.16	280.41
11	210	218.87	232.16
12	193	203.35	218.87
13	180	189.34	203.35
14	172	178.94	189.34
15	155	164.57	178.94
16	134	146.23	164.57
17	122	131.69	146.23
18	129	130.08	131.69
19	110	118.03	130.08
20	87	99.41	118.03

- $alpha$ = 0.6
- $rmse$ = 40.32



- $alpha$ = 1.0
- $rmse$ = 29.12



Trend \ Seasonality	N (None)
None	Simple Exponential Smoothing Method
Additive	Holt′s Additive Method
Multiplicative	Holt′s Multiplicative Method

Forecasting

The model:

$$A_t = \alpha(D_t) + (1 - \alpha)(A_{t-1} + T_{t-1})$$

$$T_t = \beta(A_t - A_{t-1}) + (1 - \beta)T_{t-1}$$

$$F_t = A_{t-1} + T_{t-1}$$

Month	Demand	A	T	F
1	500			
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Month	Demand	A	T	F
1	500	500	0	
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Inicialization

$$A_2 = \alpha(D_2) + (1 - \alpha)(A_1 + T_1)$$

Month	Demand	A	T	F
1	500	500	0	
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$A_2 = 0.2(450.00) + 0.8(500.00 + 0.00)$$

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00		
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$T_2 = \beta(A_2 - A_1) + (1 - \beta)T_1$$

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00		
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$T_2 = 0.1(490.00 - 500.00) + 0.9(0.00)$$

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00	-1.00	
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

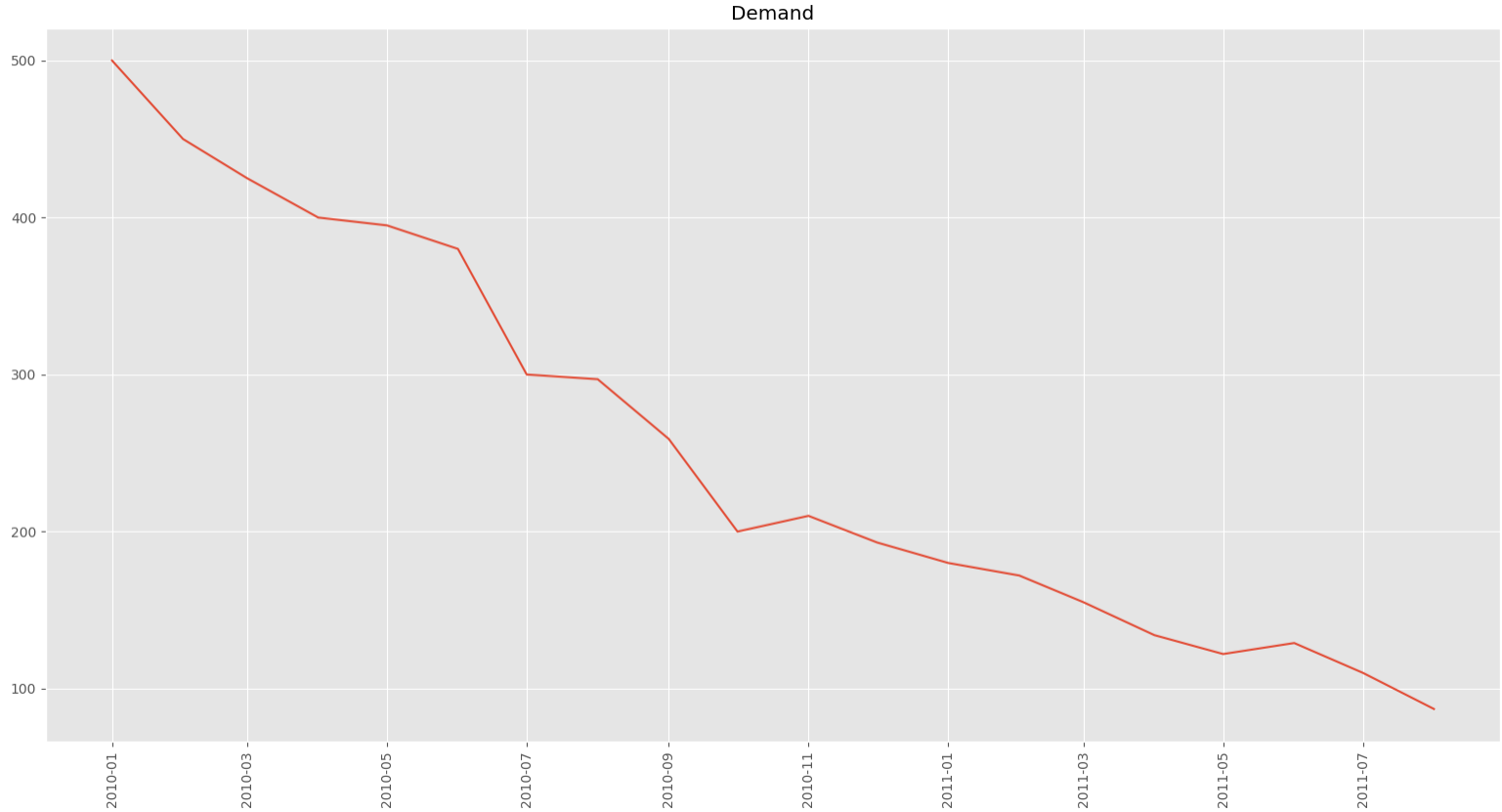
$$F_2 = A_1 + T_1$$

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00	-1.00	
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

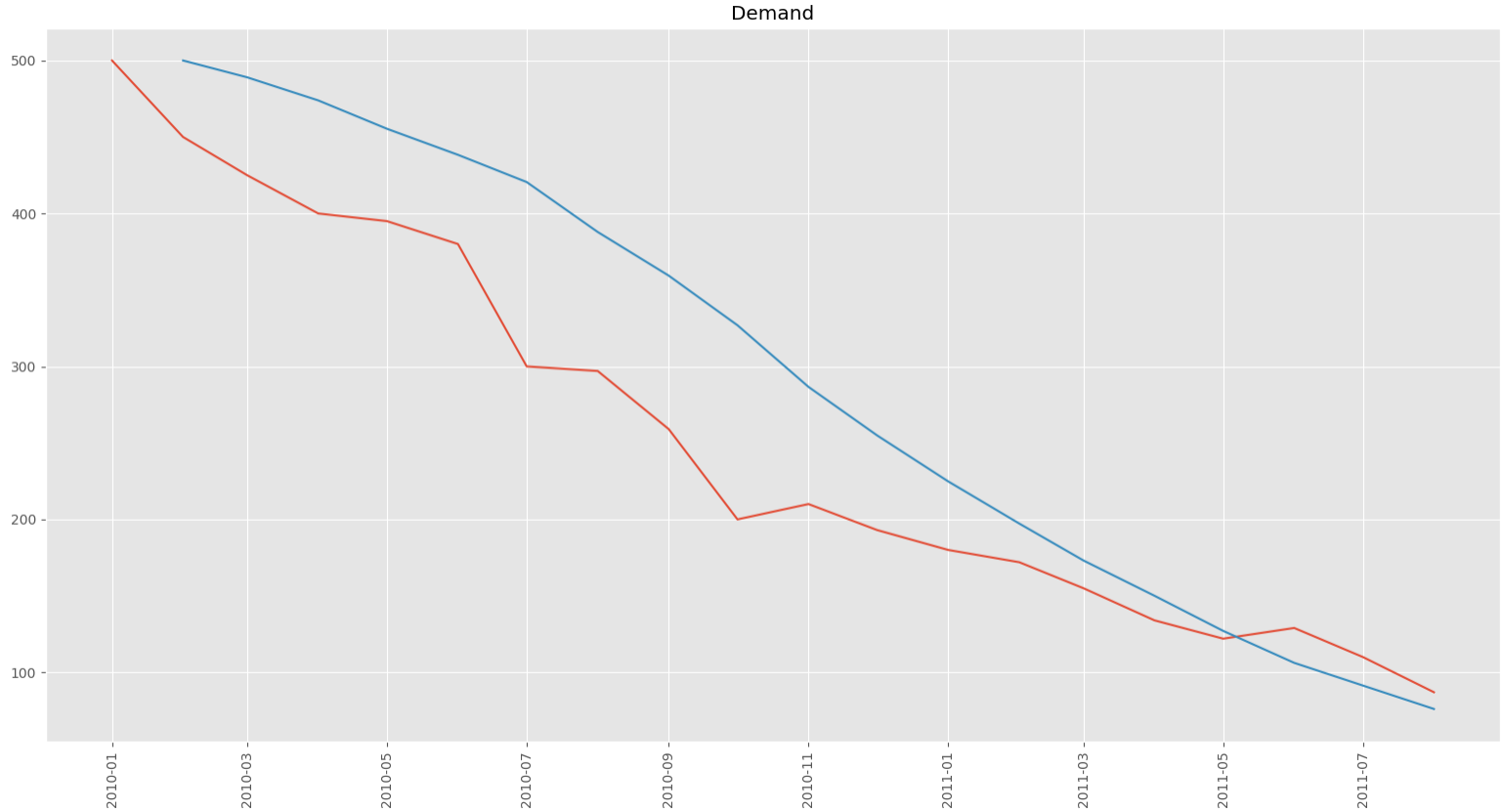
$$F_t = 500.00 + (0.00)$$

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00	-1.00	500.00
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Month	Demand	A	T	F
1	500	500	0	
2	450	490.00	-1.00	500.00
3	425	476.20	-2.28	489.00
4	400	459.14	-3.76	473.92
5	395	443.30	-4.97	455.38
6	380	426.67	-6.13	438.34
7	300	396.43	-8.54	420.54
8	297	369.71	-10.36	387.89
9	259	339.28	-12.37	359.35
10	200	301.53	-14.91	326.91
11	210	271.30	-16.44	286.62
12	193	242.49	-17.68	254.86
13	180	215.85	-18.57	224.81
14	172	192.22	-19.08	197.28
15	155	169.52	-19.44	173.14
16	134	146.86	-19.76	150.07
17	122	126.08	-19.86	127.10
18	129	110.77	-19.41	106.21
19	110	95.09	-19.04	91.36
20	87	78.24	-18.82	76.06


Demand
500
400
300
200
100
2010-01
2010-03
2010-05
2010-07
2010-09
2010-11
2011-01
2011-03
2011-05
2011-07

- $alpha$ = 0.2
- $beta$ = 0.1
- $rmse$ = 66.46



Forecasting

- What is the best value of α and β?

Forecasting

- What is the best value of α and β?
- The MS Excel Solver can find the answer.

Forecasting

- What is the best value of α and β?
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- The minimization of the MSE function can be done in order to find the optimum model.

Forecasting

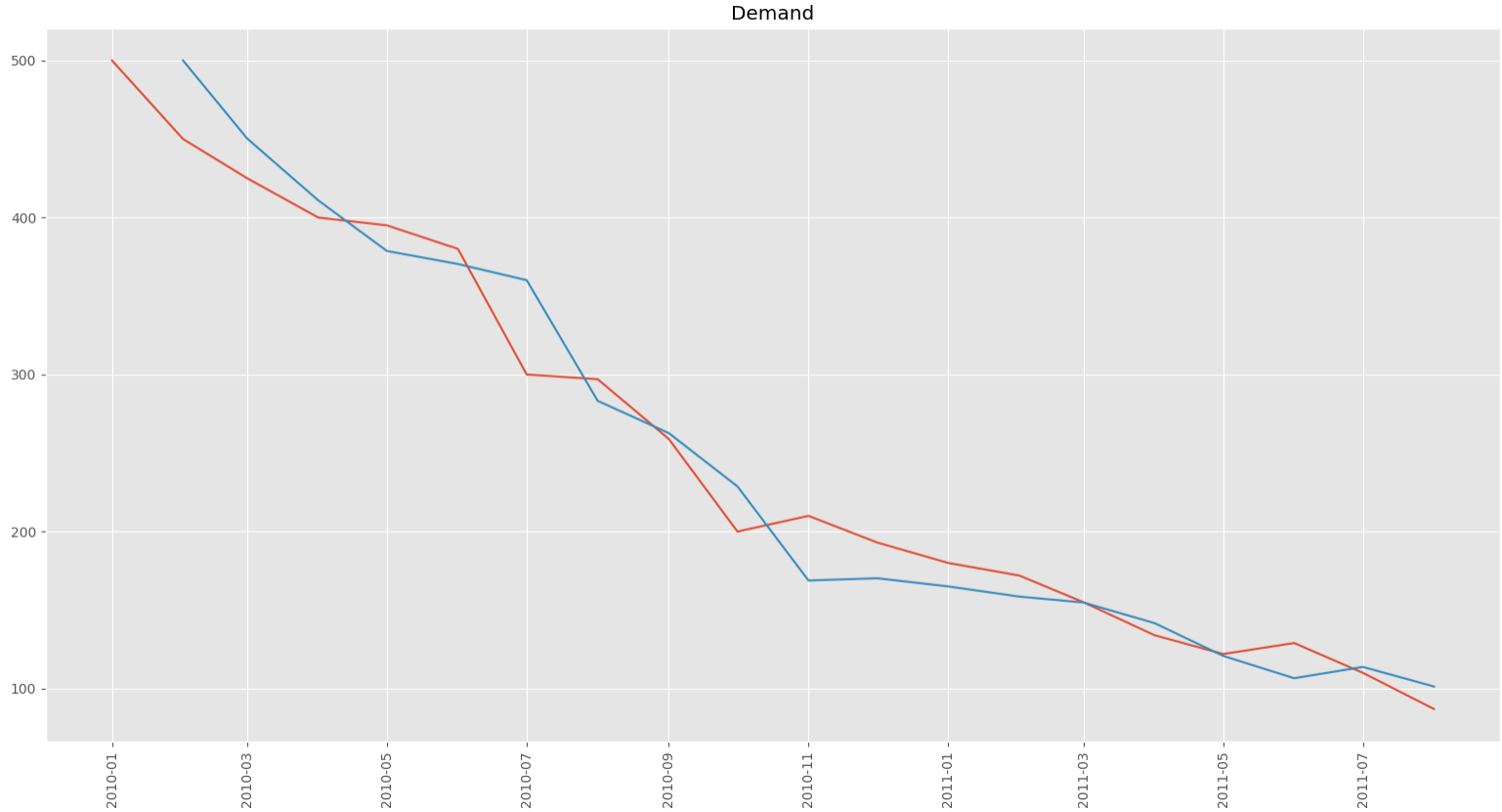
- Step 1 – Implement the Holt's Method.
- Step 2 – Choose the objective function (minimization).
- Step 3 – Choose the variable cells.
- Step 4 – Add the constraints:

$$0 \leq \alpha \leq 1; 0 \leq \beta \leq 1$$

- Step 5 – Find the solution with GRG Non Linear the method:

		beta										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
alpha	0.1	150,22	109,74	88,71	80,33	77,61	76,19	74,31	71,62	68,31	64,72	61,15
	0.2	96,50	65,70	57,44	53,91	50,15	46,17	42,54	39,49	36,95	34,80	32,92
	0.3	70,47	48,32	42,79	39,01	35,58	32,79	30,62	28,96	27,72	26,88	26,39
	0.4	55,89	38,96	34,52	31,46	29,13	37,49	26,43	25,84	25,62	25,70	25,99
	0.5	46,85	33,39	29,89	27,70	26,90	25,50	25,18	25,20	25,46	25,87	26,36
	0.6	40,89	29,98	27,30	25,86	25,12	24,88	24,98	25,30	25,75	26,26	26,81
	0.7	36,80	27,87	25,91	25,04	24,77	24,89	25,26	27,01	26,36	27,01	27,70
	0.8	33,94	26,61	25,25	24,84	24,93	25,33	25,91	26,62	27,41	28,28	29,23
	0.9	31,95	25,95	25,09	25,06	25,46	26,11	26,94	27,89	28,95	30,12	31,41
	1	30,58	25,72	25,30	25,62	26,31	27,23	28,33	29,58	30,97	32,50	34,19

- $alpha$ = 0.71
- $beta$ = 0.40
- $rmse$ = 24.77



Seasonality / Trend	N (None)
None	Simple Exponential Smoothing Method
Additive	Holt′s Additive Method
Multiplicative	Holt′s Multiplicative Method

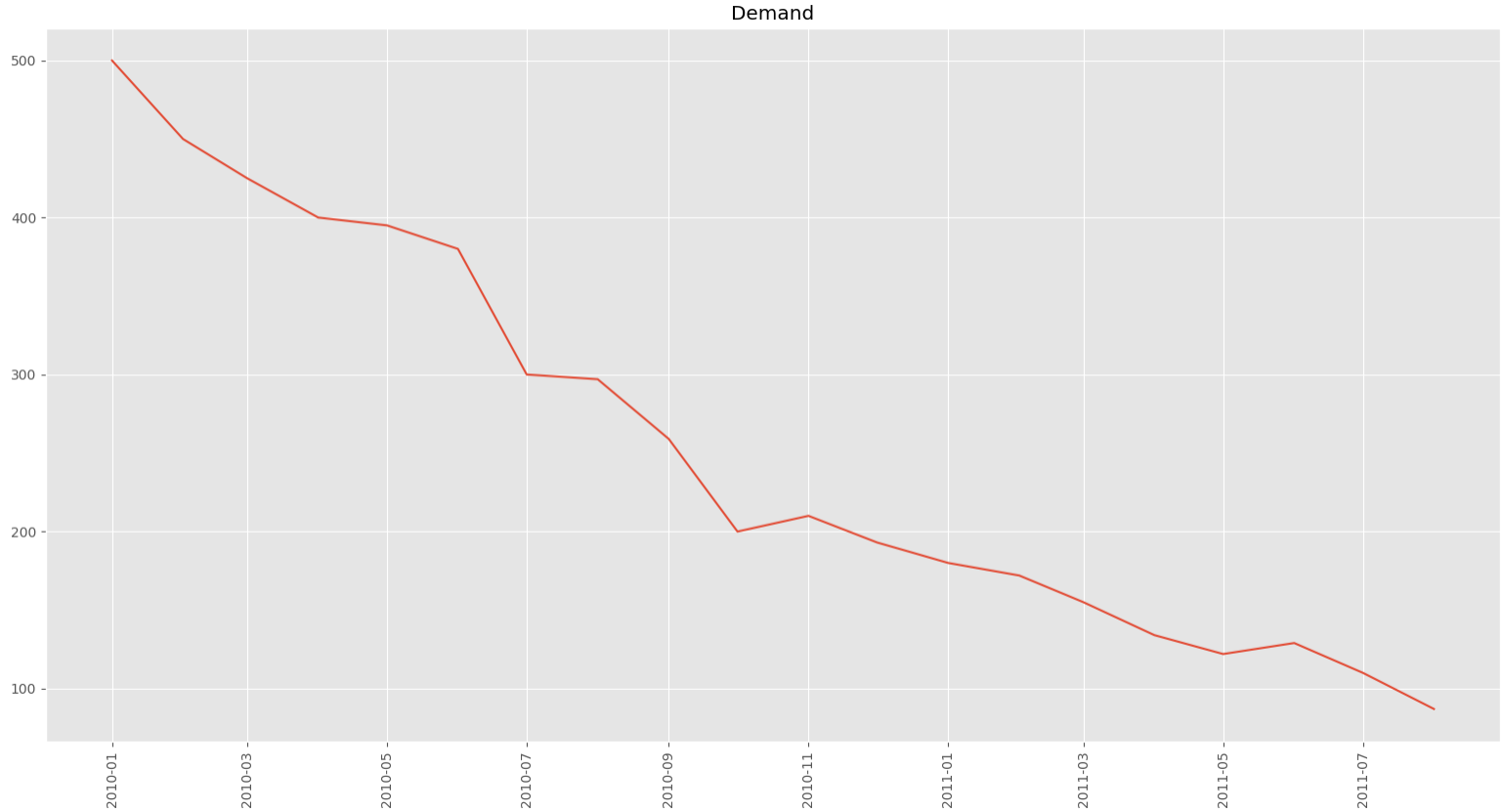
Forecasting

The model:

$$A_t = \alpha(D_t) + (1 - \alpha)(A_{t-1} \times T_{t-1})$$

$$T_t = \beta\left(\frac{A_t}{A_{t-1}}\right) + (1 - \beta)T_{t-1}$$

$$F_t = A_{t-1} \times T_{t-1}$$


Demand
500
400
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200
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2010-01
2010-03
2010-05
2010-07
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2010-11
2011-01
2011-03
2011-05
2011-07

Month	Demand	A	T	F
1	500			
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
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11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Month	Demand	A	T	F
1	500	500	1	
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Inicialization

$$A_2 = \alpha(D_2) + (1 - \alpha)(A_1 \times T_1)$$

Month	Demand	A	T	F
1	500	500	1	
2	450			
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$A_2 = 0.2(450.00) + 0.8(500.00 \times 1.00)$$

Month	Demand	A	T	F
1	500	500	1	
2	450	490.00		
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$T_2 = \beta\left(\frac{A_2}{A_1}\right) + (1-\beta)T_1$$

Month	Demand	A	T	F
1	500	500	1	
2	450	490.00		
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$T_2 = 0.1\left(\frac{490}{500}\right) + 0.9(1.00)$$

Month	Demand	A	T	F
1	500	500	1	
2	450	490.00	0.998	
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$F_2 = A_1 \times T_1$$

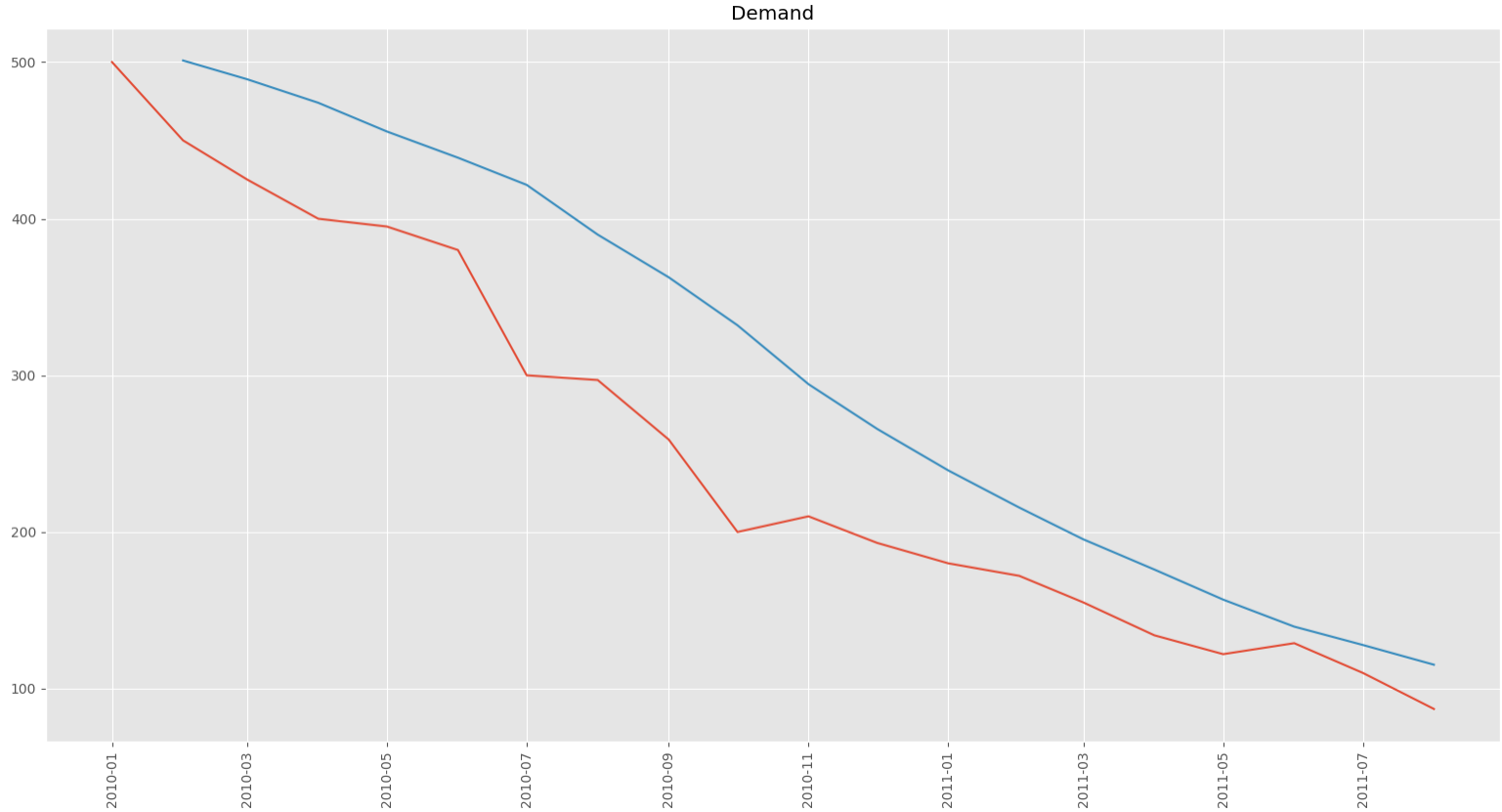
Month	Demand	A	T	F
1	500	500	1	
2	450	490.00	0.998	
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

$$F_t = 500.00 \times 1.00 = 500.00$$

Month	Demand	A	T	F
1	500	500	1	
2	450	490.00	0.998	500.00
3	425			
4	400			
5	395			
6	380			
7	300			
8	297			
9	259			
10	200			
11	210			
12	193			
13	180			
14	172			
15	155			
16	134			
17	122			
18	129			
19	110			
20	87			

Month	Demand	A	T	F
1	500	500	1	
2	450	490.00	0.998	500.00
3	425	476.22	0.995	489.02
4	400	459.22	0.992	474.02
5	395	443.54	0.990	455.67
6	380	427.15	0.987	438.94
7	300	397.27	0.981	421.59
8	297	371.27	0.977	389.84
9	259	341.87	0.971	362.59
10	200	305.57	0.963	331.97
11	210	277.49	0.958	294.36
12	193	251.22	0.953	265.78
13	180	227.44	0.948	239.30
14	172	206.86	0.944	215.57
15	155	187.22	0.940	195.27
16	134	167.60	0.936	176.00
17	122	149.85	0.931	156.81
18	129	137.46	0.930	139.58
19	110	124.28	0.927	127.85
20	87	109.61	0.923	115.26

- $alpha$ = 0.2
- $beta$ = 0.1
- $rmse$ = 71.46



Forecasting

- What is the best value of α and β?

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- What is the best value of α and β?
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- What is the best value of α and β?
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- The minimization of the MSE function can be done in order to find the optimum model.

Forecasting

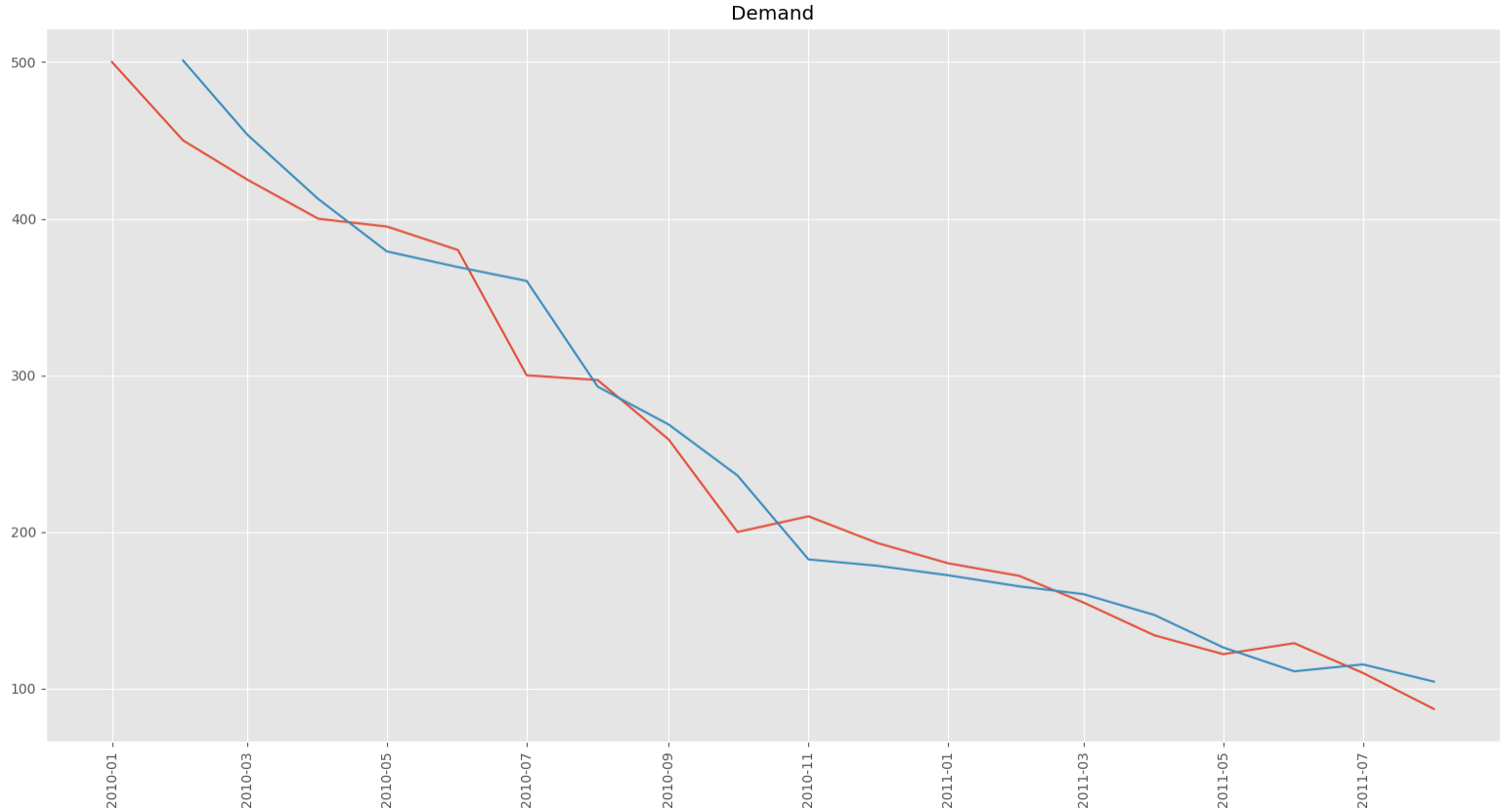
- Step 1 – Implement the Holt's Method.
- Step 2 – Choose the objective function (minimization).
- Step 3 – Choose the variable cells.
- Step 4 – Add the constraints:

$$0 \leq \alpha \leq 1; 0 \leq \beta \leq 1$$

- Step 5 – Find the solution with GRG Non Linear the method:

		beta										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
alpha	0.1	150,22	116,86	96,02	82,20	72,49	65,31	59,75	55,27	51,54	48,36	45,62
	0.2	96,50	70,50	56,95	48,71	43,08	38,93	35,74	33,24	31,27	29,70	28,44
	0.3	70,47	51,02	41,55	35,94	32,22	29,63	27,80	26,51	25,62	25,03	24,69
	0.4	55,89	40,81	33,81	29,86	27,42	25,89	24,96	24,44	24,21	24,18	24,31
	0.5	46,85	34,84	29,56	26,78	25,24	24,42	24,05	23,97	24,09	24,33	24,65
	0.6	40,89	31,16	27,15	25,22	24,30	23,94	23,91	24,10	24,40	24,79	25,22
	0.7	36,80	28,84	25,79	24,49	24,01	23,96	24,17	24,53	24,99	25,50	26,05
	0.8	33,94	27,40	25,10	24,27	24,11	24,31	24,71	25,23	25,83	26,49	27,20
	0.9	31,95	26,56	24,85	24,40	24,52	24,93	25,51	26,21	26,99	27,85	28,79
	1	30,58	26,16	24,95	24,82	25,18	25,80	26,60	27,52	28,56	29,71	30,99

- $alpha$ = 0.61
- $beta$ = 0.55
- $rmse$ = 23.89



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