# Microwave Circuit Principles and Design

MSc Communication Engineering

MEng Electronic Engineering

EEEN60183/40183

# Passive Circuits*

## Power Dividers & Directional Couplers

**Recommended Reading:**

David M. Pozar, "Microwave Engineering", John Wiley, ISBN 0-471-17096-8

F.A. Benson, T.M. Benson, "Fields, Waves and Transmission Lines", Chapman and Hall, ISBN 0-412-36370-4

Ed. by I. D. Robertson, S. Lucyszyn, "RFIC and MMIC Design and Technology", IEE Circuits, Devices and Systems, ISBN 0-85296-786-1

## 3. Power Dividers and Directional Couplers [1, 2]

Power dividers and directional couplers are passive microwave components used for power division or power combining (Figure 3.1). In power division, an input signal is divided into two (or more) output signals of lesser power, while a power combiner accepts two or more input signals and combines them at an output port. The coupler or divider may have three ports, four ports, or more and may be assumed (ideally) lossless. Three-port networks take the form of T-junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids. Power dividers usually provide in-phase output signals with an equal power division ratio (3 dB) but unequal power division ratios are also possible. Directional couplers can be designed for arbitrary power division, while hybrid junctions usually have equal power division. Hybrid junctions have either a 90° or a 180° phase shift between the output ports.

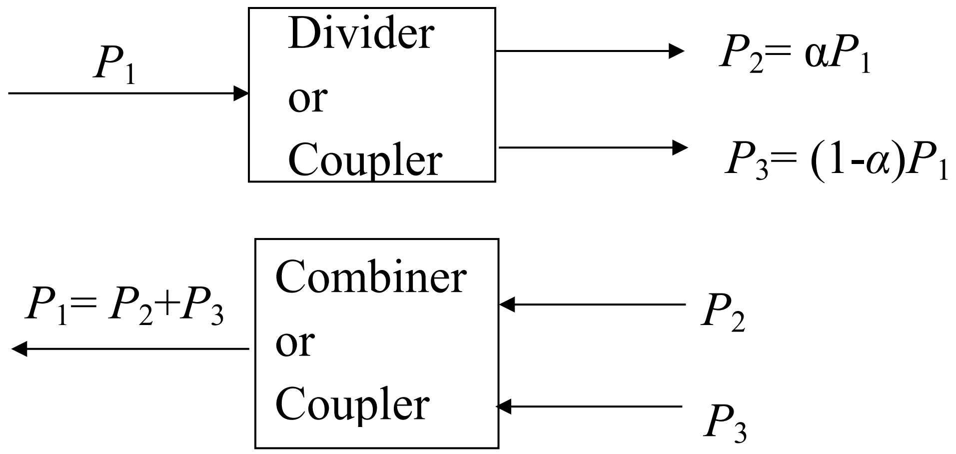



**Figure 3.1: Power divider/combiner/coupler.**

The simplest type of power divider is a T-junction, which is a three-port network with one input and two outputs (Figure 3.2). The scattering matrix of an arbitrary three-port network has nine independent elements.

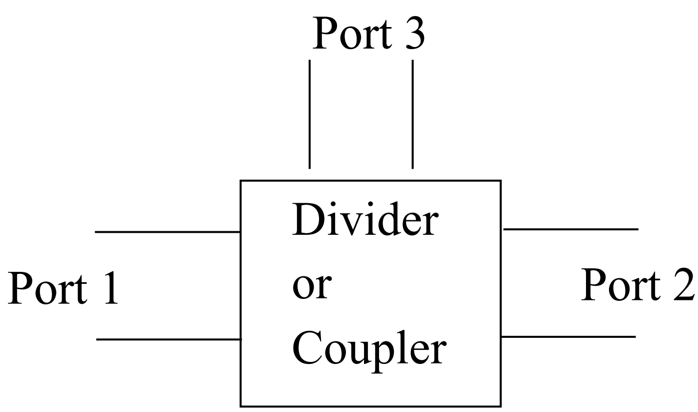



**Figure 3.2: Three-port divider/coupler.**

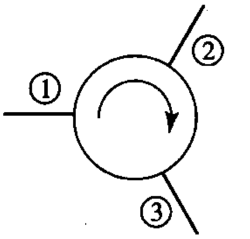
The scattering parameters may be written as,

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}.$$

If the device is passive and contains no anisotropic materials, then it must be reciprocal and its scattering matrix will be symmetric ($S_{ij} = S_{ji}$). Usually, to avoid power loss, we would like to have a junction that is lossless and matched at all ports. We can easily show however, that it is impossible to construct such a three-port lossless reciprocal network that is matched at all ports.

If the three-port network is nonreciprocal, the conditions of lossless and input matching at all ports can be satisfied. Such devices are called ***circulators***. These generally rely on an anisotropic material, such as ferrite, to achieve nonreciprocal behaviour, as depicted in Figure 3.3.

$$[S]=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$




$$[S]=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

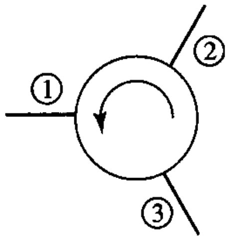



**Figure 3.3: Circulator schematic symbols and *S* matrices.**

A lossless and reciprocal three-port network is realisable if only two of its ports are matched. If the three-port network is lossy, it can be reciprocal and matched at all ports. In addition, a lossy three-port can be made to have isolation between its output ports.

### 3.1 The T-Junction Power Divider

The T-junction power divider is a simple three-port network that can be used for power division or power combining and it can be implemented in virtually any type of transmission line medium. Figure 3.4 shows commonly used T-junctions. The junctions shown here are, in the absence of transmission line loss, lossless junctions.

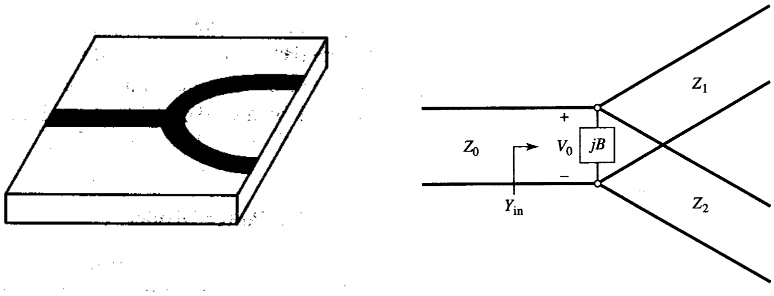



**Figure 3.4: Transmission line T-junction representations.**

The lossless T-junctions can all be modelled as a junction of three transmission lines. In general, there may be fringing fields and higher order modes associated with the discontinuity at such a junction, leading to stored energy that can be accounted for by a lumped susceptance, $B$. In order for the divider to be matched to the input line of characteristic impedance $Z_0$, we must have,

$$\boldsymbol{Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2}} \, (B = 0 \text{ for ideal case}).$$

In practice, if $B$ is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

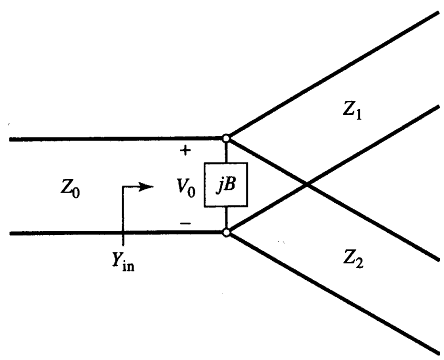
The output line impedances, $Z_1$ and $Z_2$, can be selected to provide various power division ratios. Thus, for a 50 Ω input line, a 3 dB (equal split) power divider can be made by using two 100 Ω output lines. If necessary, quarter-wave transformers can be used to bring the output line impedances back to the desired levels. If the output lines are matched, then the input line will be matched. There will be no isolation between the two output ports, however, and there will be a mismatch looking into the output ports.

**Example: T-Junction Power Divider**

A lossless T-junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

**Solution**

Consider that voltage at the junction is $V_0$, as shown in the figure below.




The input power to the matched divider and the output powers are given by,

$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0},$$

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in},$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in}.$$

The characteristic impedances may be found by equating the above equations,

$$Z_1 = 3Z_0 = 150\ \Omega,$$

$$Z_2 = \frac{3Z_0}{2} = 75\Omega.$$

The input impedance to the junction is,

$$Z_{in} = 75 \| 150 = 50\ \Omega.$$

This shows the input is matched to the 50 Ω source. Looking into the 150 Ω output line, we see an impedance of 50 || 75 = 30 Ω, while at the 75 Ω output line we see an impedance of 50 || 150 = 37.5 Ω. The reflection coefficients seen looking into these ports are,

$$\Gamma_1 = \frac{30-150}{30+150} = -0.666,$$

$$\Gamma_2 = \frac{37.5-75}{37.5+75} = -0.333.$$

**3.1.1 Resistive Divider**

If a three-port divider contains lossy components, it can be made to be matched at all ports, although the two output ports may not be isolated. The circuit for such a divider is illustrated in Figure 4.5, using lumped-element resistors. An equal-split (-3 dB) divider is shown but unequal power division ratios are also possible.

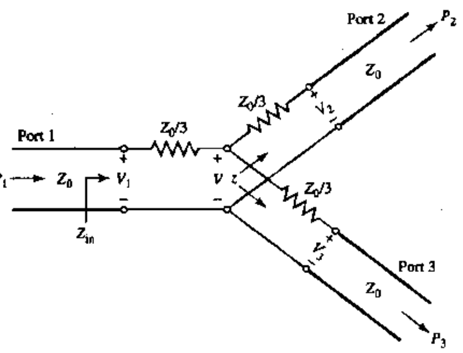



**Figure 4.5: Resistive divider schematic.**

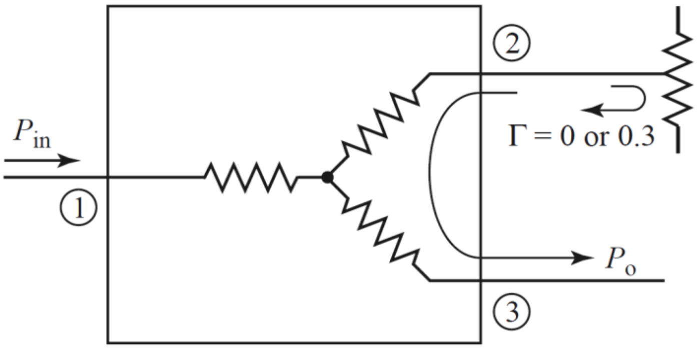
$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix},$$

$$P_2 = P_3 = \frac{1}{4} P_{in},$$

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0}.$$

The input impedance of the resistive divider is equal to the characteristic impedance and the two output ports cannot be isolated.

**Example: Three Port Resistive Divider**
Design a three-port resistive divider for an equal power split and a 100 Ω system impedance. If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load and then to a load having a mismatch of $\Gamma = 0.3$. See the figure below.




**Solution**

The system impedance is $Z_0 = 100$ Ω, while the value of $R = 33.3$ Ω. The scattering parameters may be written as follows,

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Now considering the following two cases:

**Case A) Ports 2 and 3 are matched to 100 Ω, i.e.:**

$$V_2^+ + V_3^+ = 0.$$

Considering if:

$$V_1^+ = 1,$$

$$V_3^- = \frac{1}{2}\left[V_1^+ + V_2^+\right] = \frac{1}{2}.$$

Thus,

$$V_3 = V_3^+ + V_3^- = \frac{1}{2},$$

$$P_3 = \frac{V_3^2}{Z_0} = \frac{0.25}{Z_0}.$$

**Case B) Port 3 is matched, $\Gamma$ = 0.3 at port 2, i.e.:**

$$V_3^+ = 0.$$

Considering if:

$$V_1^+ = 1,$$

$$V_2^- = \frac{1}{2}\left[V_1^+ + V_3^+\right] = \frac{1}{2},$$

$$V_2^+ = \Gamma V_2^- = (0.3)\left(\frac{1}{2}\right) = 0.15,$$

$$V_3^- = \frac{1}{2}\left[V_1^+ + V_2^+\right] = \frac{1}{2}(1.15) = 0.575\,\mathrm{V},$$

$$V_3 = V_3^+ + V_3^- = 0.575\ \mathrm{V},$$

$$P_3 = \frac{V_3^2}{Z_0} = \frac{0.331}{Z_0}.$$

Now the ratio between the two cases may be found as follows,

$$\frac{P_3(\text{port 2 mismatched})}{P_3(\text{port 2 matched})}(\mathrm{dB}) = 10\log\left(\frac{0.331}{0.25}\right) = 1.2\ \mathrm{dB}.$$

### 3.1.2 The Wilkinson Power Divider

The lossless T-junction divider suffers from the disadvantage of not being matched at all ports and it does not have isolation between output ports. The resistive divider can be matched at all ports but even though it is not lossless, isolation is still not achieved. A lossy three-port network can be made to have all ports matched, with isolation between output ports. The Wilkinson power divider, shown in Figure 3.6, is such a network, with the useful property of appearing lossless when the output ports are matched; that is, only reflected power from the output ports is dissipated.

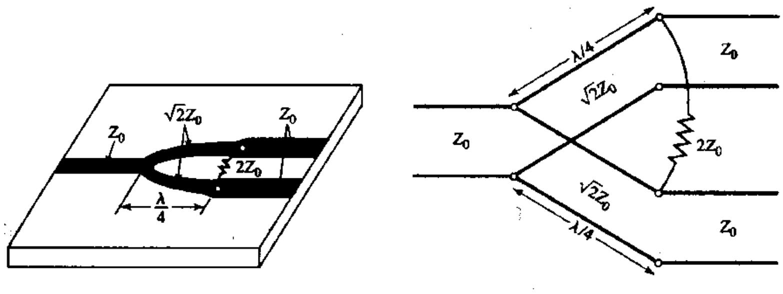



**Figure 3.6: Wilkinson power divider layout and schematic.**

**Even-Odd Mode Analysis**

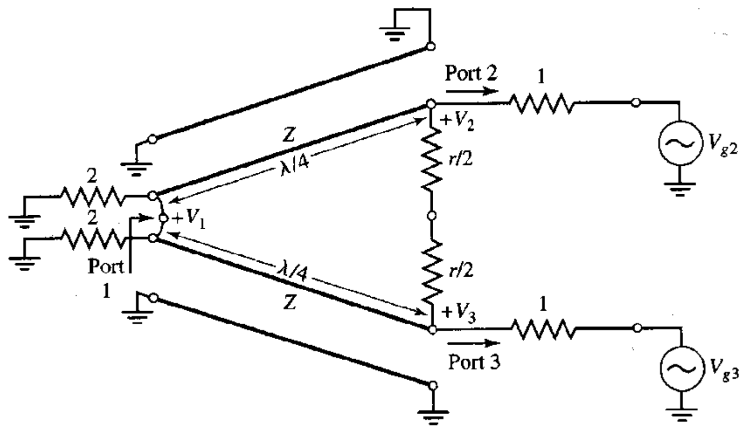



**Figure 3.7: Wilkinson power divider schematic for even-odd mode analysis.**

With reference to Figure 3.7 & Figure 3.8, $z$, $r$, 1 and 2 are all normalised to the characteristic impedance $Z_0$ of the transmission line, i.e., $z_{g2} = Z_{g2}/Z_0 = 1$, $z_{g3} = Z_{g3}/Z_0 = 1$, $z_1 = Z_1/Z_0 = 1$.

Even mode: $V_{g2} = V_{g3}$. For the sake of simplicity, assuming $V_{g2} = V_{g3}$= 2 V.

Odd mode: $V_{g2} = -V_{g3}$. For the sake of simplicity, assuming $V_{g2} = -V_{g3}$= 2 V.

The even mode input impedance looking into Port 2 is:

$$Z^{e}_{in,port2} = z^2 / 2.$$

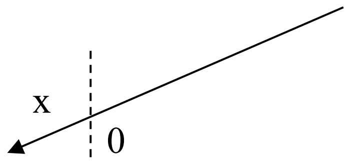
If we select

$$z = \sqrt{2},$$

then port 2 will be matched for even mode excitation and

$$V_2^e = 1\text{V}.$$

The voltage along the transmission line with $z$ characteristic impedance is given by,




$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x}).$$

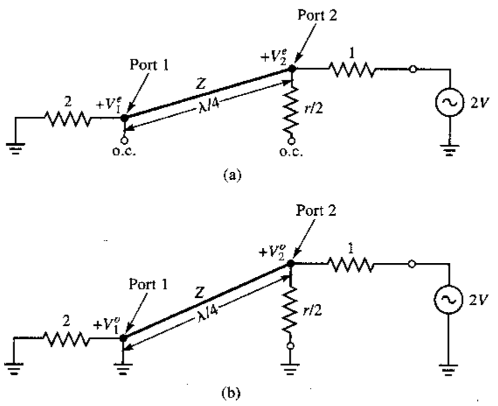



**Figure 3.8: Wilkinson power divider schematic, (a) Even mode, (b) Odd mode.**

Find $V_1^e$ :

At $x = 0$,

$V(0) = V_1^e = V^+(1 + \Gamma),$

and at $x = -\lambda/4$,

$V(-\lambda/4) = V_2^e = V^+(j - j\Gamma) = 1\,\text{V}.$

So,

$V^+ = V_2^e / (j - j\Gamma),$

$$V_1^e = V_2^e \frac{1+\Gamma}{j-j\Gamma} = 1\,\mathrm{V}\frac{1+\Gamma}{j-j\Gamma}.$$

Where,

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}.$$

The odd-mode input impedance looking into Port 2 is,

$$Z_{in,port2}^o = r/2.$$

If we select

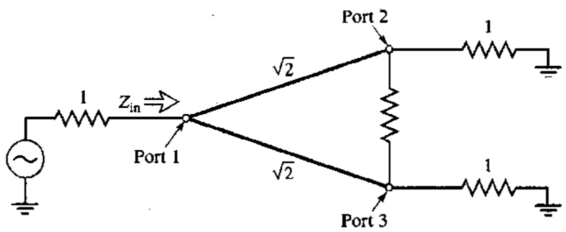
$r = 2$, then port 2 will be matched for odd mode excitation and

$$V_2^o = 1\,\mathrm{V}.$$

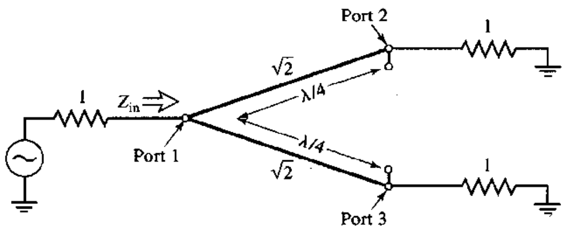
From (b), we can see that

$$V_1^o = 0.$$

The input impedance at port 1, as shown below:



(a)



(b)

$$z_{in} = 2 // 2 = 1.$$

$S_{11} = 0$   (because $z_{in} = 1$),

$S_{22} = S_{33} = 0$   (because Ports 2 and 3 are matched for even and odd modes),

$S_{23} = S_{32} = 0$   (because $V_3^e + V_3^o = 0$).

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{V_1^e}{V_2^e + V_2^o} = \frac{1V\dfrac{1+\Gamma}{j - j\Gamma}}{1V + 1V} = \frac{1+\Gamma}{2(j - j\Gamma)} = \frac{1}{2j}\frac{1+\dfrac{2-\sqrt{2}}{2+\sqrt{2}}}{1-\dfrac{2-\sqrt{2}}{2+\sqrt{2}}} = -j/\sqrt{2}.$$

*S* parameters for the equal-split (3 dB) case (Figure 3.9),

$$[S] = \begin{bmatrix} 0 & -j\dfrac{1}{\sqrt{2}} & -j\dfrac{1}{\sqrt{2}} \\ -j\dfrac{1}{\sqrt{2}} & 0 & 0 \\ -j\dfrac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}, \quad P_2 = P_3 = \frac{1}{2}P_1.$$

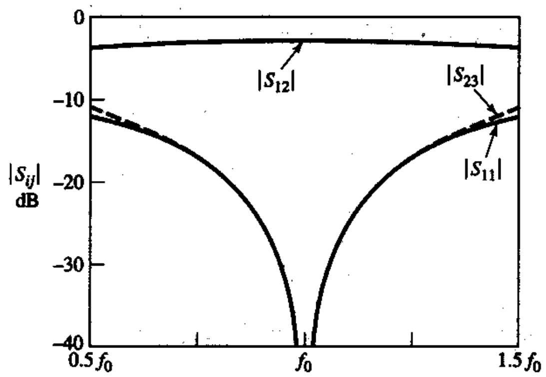
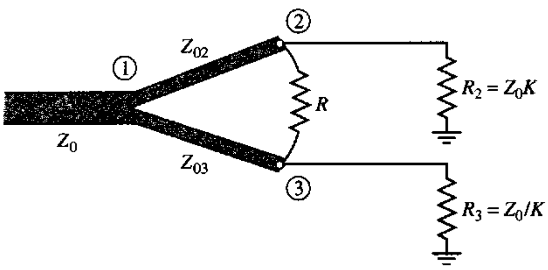



**Figure 3.9: Wilkinson power divider *S*-parameters.**

The Wilkinson power divider is lossless when the outputs are matched; only reflected power from ports 2 and 3 is dissipated in the resistor. Ports 2 and 3 are isolated. For the unequal-split case, we define the power ratio between ports 2 and 3 as $K_2 = P_3/P_2$, the design equations are:




$$Z_{03} = Z_0\sqrt{(1+K^2)/K^3},$$
$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)},$$
$$R = Z_0(K + 1/K).$$

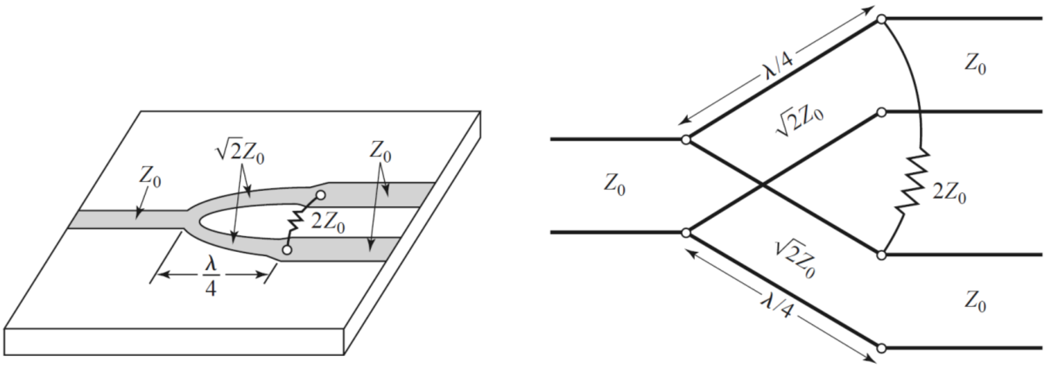
The Wilkinson divider can be made as an $N$-way divider or combiner.

**Example: Equal Split Wilkinson Power Divider**

Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency $f_0$, and plot the return loss ($S_{11}$), insertion loss ($S_{21} = S_{31}$), and isolation ($S_{23} = S_{32}$) versus frequency from 0.5 $f_0$ to 1.5 $f_0$.

**Solution**

Consider the equal split Wilkinson power divider and the equivalent transmission line circuit shown in the figure below:



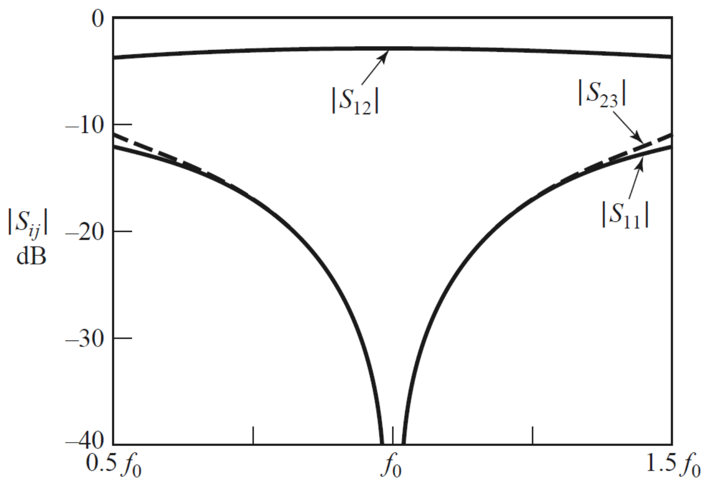

The quarter wavelength transmission lines in the divider should have a characteristic impedance of (indicated from the equivalent transmission line circuit)

$$Z = \sqrt{2}Z_0 = 70.7\ \Omega.$$

The shunt resistor has a value of

$$R = 2Z_0 = 100\,\Omega.$$

The transmission lines are $\lambda/4$ long at the frequency $f_0$. Scattering parameter magnitudes have been plotted in the figure below.



### 3.2 Directional Couplers

The scattering matrix of a four-port network has the following form:

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}.$$

A directional coupler is a reciprocal, lossless and matched four-port network (Figure 3.10). There are two types of directional coupler (Figure 3.11):

1. A symmetric coupler;
2. An antisymmetric coupler.

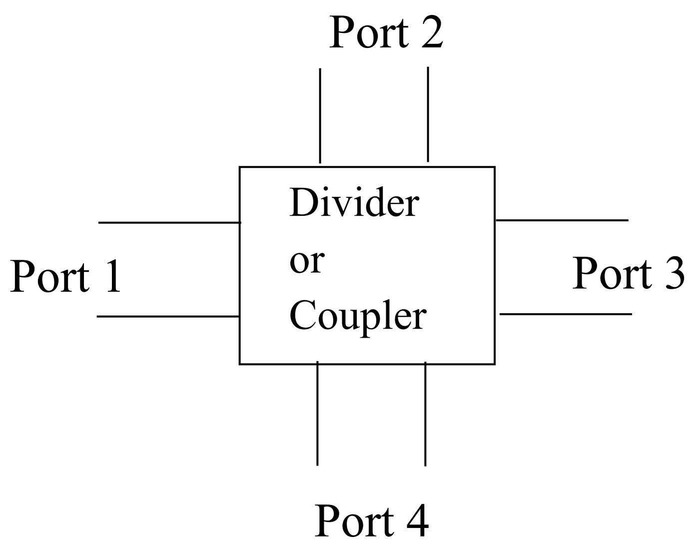



**Figure 3.10: Four-port divider or coupler network.**

$$[\mathrm{S}]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \rightarrow \textit{Symmetric,}$$

$$[\mathrm{S}]=\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \rightarrow \textit{Antisymmetric.}$$

For both cases, $\alpha^2+\beta^2=1$.

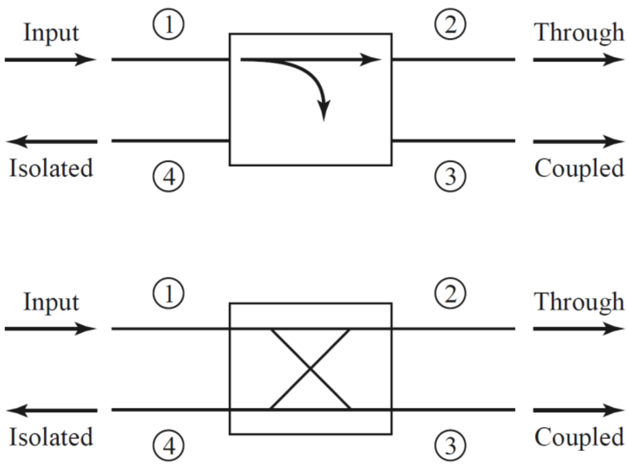



**Figure 3.11: Directional coupler symbols.**

- **Coupling factor**, indicating the fraction of the input power that is coupled to the output port:

$$C = 10\log\frac{P_1}{P_3} \text{ dB.}$$

- **Directivity**, being a measure of the coupler's ability to isolate forward and backward waves:

$$D = 10\log\frac{P_3}{P_4} \text{ dB.}$$

- **Isolation:**

$$I = 10\log\frac{P_1}{P_4} \text{ dB.}$$

***I*, *D* and *C*** are related as:

$$I = D + C \text{ dB.}$$

Hybrid couplers are special cases of directional couplers, where the coupling factor is 3 dB, which implies that $\alpha = \beta = 1/\sqrt{2}$. There are two types of hybrids. The quadrature hybrid has a 90◦ phase shift between ports 2 and 3 ($\theta = \varphi = \pi/2$) when fed at port 1 and is an example of a symmetric coupler. Its scattering matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix} \rightarrow \textit{Hybrid.}$$

The directivity of a directional coupler is a measure of the coupler's ability to separate forward and reverse wave components and applications of directional couplers often require high (35 dB or greater) directivity. Poor directivity will limit the accuracy of a reflectometer and can cause variations in the coupled power level from a coupler when there is even a small mismatch on the through line.

### 3.2.1 The quadrature 90° Hybrid Branch Line Coupler

Quadrature hybrids are 3 dB directional couplers with a 90° phase difference in the outputs of the through and coupled arms. This type of hybrid is often made in microstrip line or stripline form and is also known as a branch-line hybrid. Other 3 dB couplers, such as coupled line couplers or Lange couplers, can also be used as quadrature couplers. With all ports matched, power entering port 1 is evenly divided between ports 2 and 3, with a 90° phase shift between these outputs. No power is coupled to port 4 (the isolated port). The scattering matrix has the following form:

$$[S]=\frac{-1}{\sqrt{2}}\begin{bmatrix}0 & j & 1 & 0\\ j & 0 & 0 & 1\\ 1 & 0 & 0 & j\\ 0 & 1 & j & 0\end{bmatrix}.$$

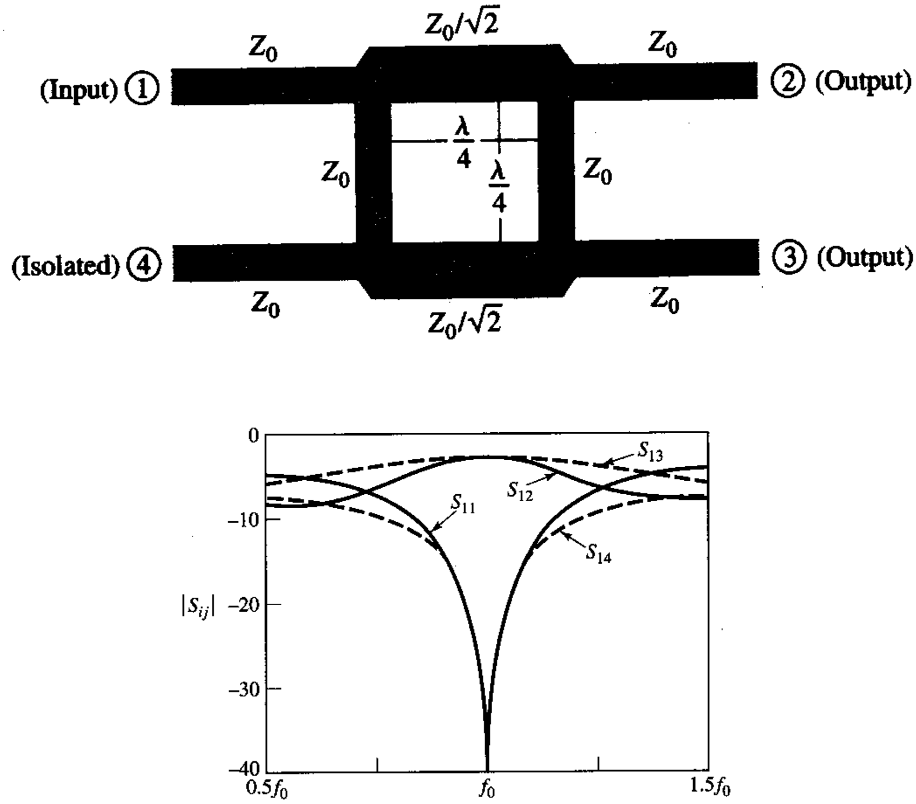



**Figure 3.12: Quadrature 90° hybrid coupler layout and *S*-parameters.**

Typical performance of the quadrature coupler is shown in Figure 3.12. 3 dB power division at ports 2 and 3 and high isolation and return loss at ports 4 and 1, respectively, at the design frequency $f_0$ is shown. All of these quantities, however, degrade rapidly as the frequency departs from $f_0$.

Due to the quarter-wave length requirement, the bandwidth of a branch-line hybrid is limited to 10-20 %. However, as with multi-section matching transformers and multi-hole directional couplers, the bandwidth of a branch-line hybrid can be increased to a decade or more by using multiple sections in cascade. In addition, the basic design can be modified for unequal power division and/or different characteristic impedances at the output ports. The discontinuities at the junction may require the shunt arms to be 10˚ – 20˚ longer.

**Example: Quadrature Hybrid Coupler**
Design a 50 Ω branch-line quadrature hybrid junction and plot the scattering parameter magnitudes from 0.5 $f_0$ to 1.5 $f_0$, where $f_0$ is the centre design frequency.

**Solution**

The lines of the quadrature hybrid are $\lambda/4$ at the design frequency, $f_0$ and the branch line impedances may be written as

$$\frac{Z_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.4\,\Omega.$$

### 4.2.2 Coupled Line Directional Couplers

When two unshielded transmission lines are in close proximity (Figure 3.13), power can be coupled from one line to the other, due to the interaction of the electromagnetic fields. Such lines are referred to as coupled transmission lines and they usually consist of three conductors in close proximity, although more conductors can be used. Coupled transmission lines are sometimes assumed to operate in the TEM mode, which is rigorously valid for coaxial line and stripline structures but only approximately valid for microstrip line, coplanar waveguide or slotline structures. Coupled transmission lines can support two distinct propagating modes and this feature can be used to implement a variety of practical directional couplers, hybrids and filters.

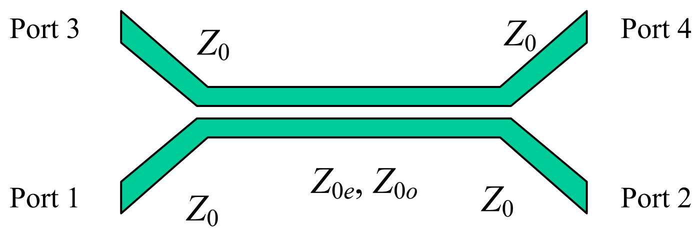



**Figure 3.13: Coupled transmission lines.**

**Coupled Line Theory**

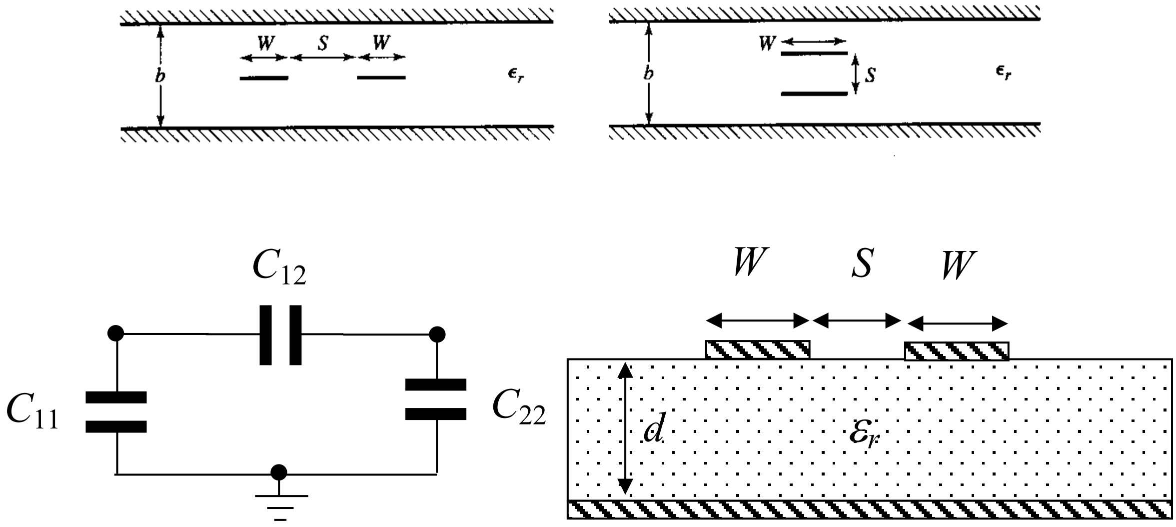



**Figure 3.14: Coupled transmission line layouts and equivalent circuit.**

If TEM mode propagation is assumed, the electrical characteristics of the coupled lines can be completely determined from the effective capacitances between the lines and the velocity of the propagation on the line.

In the equivalent circuit of Figure 3.14, $C_{12}$ is the capacitance between the two strip conductors in the absence of the ground conductor. $C_{11}$, $C_{22}$ are the capacitance between one strip conductor and ground in the absence of the other strip conductor.

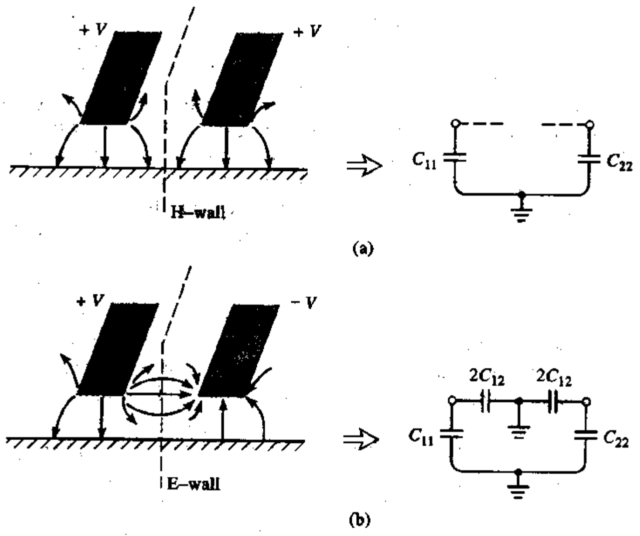
**Even and Odd Mode Analysis**

- **Even Mode:**
  The currents in the strip are equal in magnitude and direction. Assuming the two strips are identical in size and location, the capacitance and characteristic impedance of each line are,

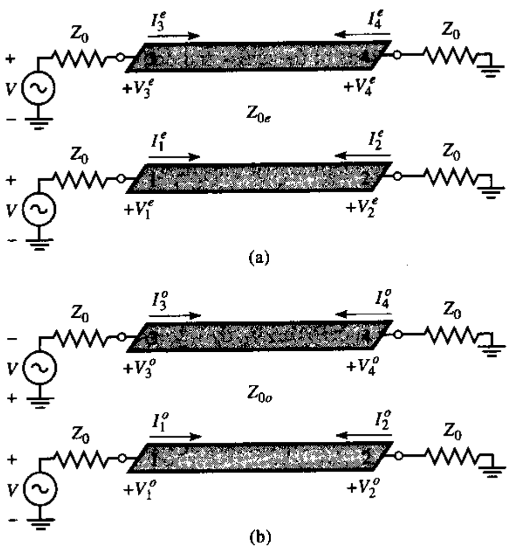
$$C_e = C_{11} = C_{22}, Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{v_e C_e}.$$

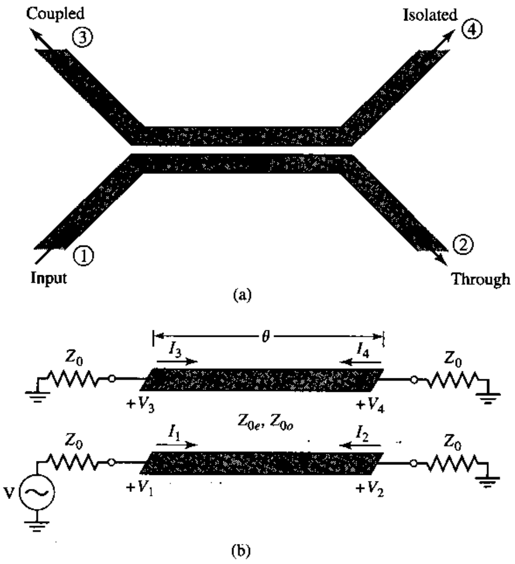
- **Odd Mode:**
  The currents in the strip are equal in magnitude but in opposite directions. There is a voltage null existing between the two strips. The capacitance between either strip and ground and the characteristic impedance of each line, respectively are:

$$C_0 = C_{11} + 2C_{12}, Z_{0o} = \sqrt{\frac{L}{C_o}} = \frac{1}{v_o C_o}.$$




## Using the Even-Odd Mode Technique to Analyse the Coupled Line Coupler




The even and odd mode input impedances are:

$$Z_{in}^{e} = \frac{V_1^e}{I_1^e} = Z_{0e}\frac{Z_0 + jZ_{0e}\tan(\beta l)}{Z_{0e} + jZ_0\tan(\beta l)}, \qquad Z_{in}^{o} = \frac{V_1^o}{I_1^o} = Z_{0o}\frac{Z_0 + jZ_{0o}\tan(\beta l)}{Z_{0o} + jZ_0\tan(\beta l)},$$

$$V_1^e = \frac{Z_{in}^e}{Z_{in}^e + Z_o}V, \qquad V_1^o = \frac{Z_{in}^o}{Z_{in}^o + Z_o}V,$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_o}, \qquad I_1^o = \frac{V}{Z_{in}^o + Z_o},$$

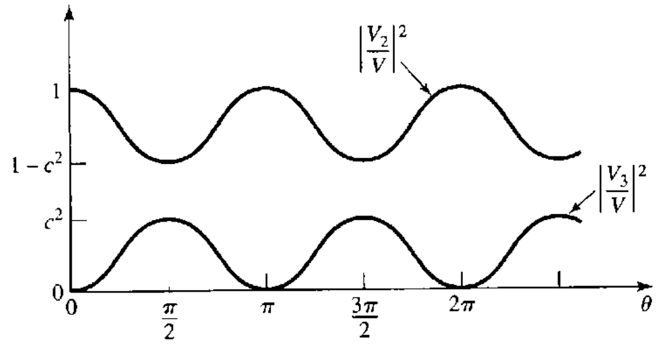
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = Z_0 + \frac{2(Z_{in}^e Z_{in}^o - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}.$$

If $Z_0 = (Z_0^e Z_0^o)^{1/2}$, $Z_{in}^e$ and $Z_{in}^o$ become:

$$Z_{in}^{e} = Z_{0e}\frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan(\beta l)}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan(\beta l)}, \quad Z_{in}^{o} = Z_{0o}\frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan(\beta l)}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan(\beta l)},$$

and $Z_{in}^{e} Z_{in}^{o} - Z_0^2$ is equal to zero. Hence $Z_{in} = Z_0$ and $V_1 = V_1^e + V_1^o = V$.

$$\text{If } l = \lambda/4, \text{i.e.}, \beta l = \pi/2, \frac{V_3}{V_1} = C, \frac{V_2}{V_1} = -j\sqrt{1-C^2}, \frac{V_4}{V_1} = 0.$$




There is a 90 phase shift between the two output voltages. The coupled voltage is in phase with the input voltage but the through voltage is 90° lagging. As long as $Z_o = (Z_{oe}Z_{oo})^{1/2}$, the coupler will be matched at the input and will have perfect isolation at any frequency.

The design equations for the coupled line directional coupler are given below:

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}},$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}}.$$

### 3.2.3 The Lange Coupler

Generally, the coupling in a coupled line coupler is too loose to achieve coupling factors of 3 or 6 dB. One way to increase the coupling between edge-coupled lines is to use several lines parallel to each other, so that the fringing fields at both edges of a line contribute to the coupling. One of the most practical implementations of this idea is the Lange coupler, shown in Figure 4.15, where four parallel coupled lines are used with interconnections to provide tight coupling.

This coupler can easily achieve 3 dB coupling ratios, with an octave or more bandwidth. The design tends to compensate for unequal even-mode and odd-mode phase velocities, which also improves the bandwidth. There is a 90° phase difference between the output lines (ports 2 and 3), so the Lange coupler is a type of quadrature hybrid.

The main disadvantage of the Lange coupler is probably practical, as the lines are very narrow and close together and the required bonding wires across the lines increase complexity. This type of coupled line geometry is also referred to as interdigitated; such structures can also be used for filter circuits.

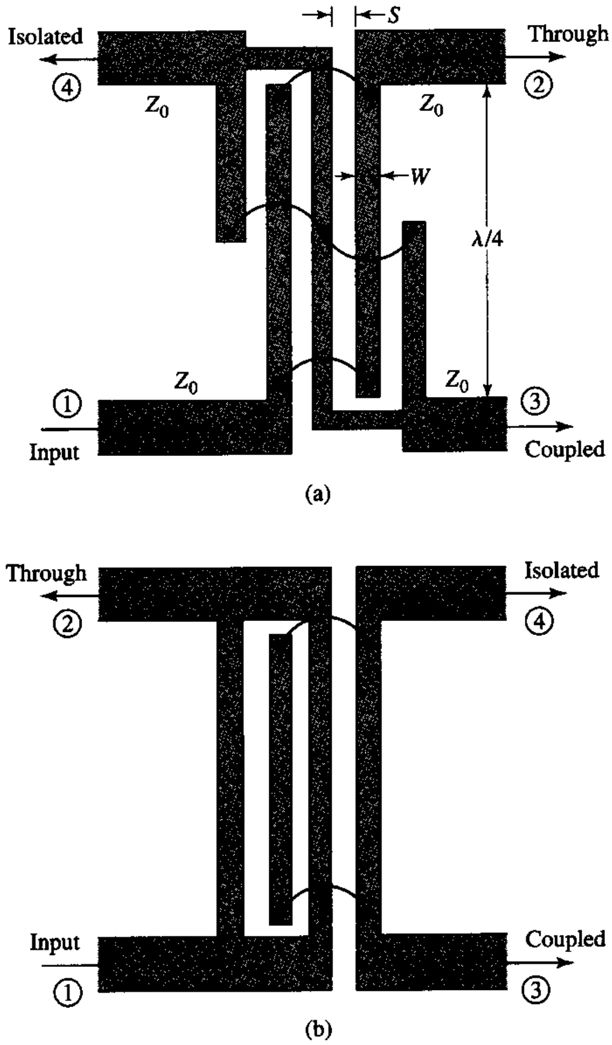



**Figure 4.15: Lange coupler layout.**

The necessary even and odd mode impedances in terms of a desired characteristic impedance and coupling coefficient for the design of the Lange coupler are given below:

$$Z_{oe} = Z_o \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}},$$

$$Z_{oo} = Z_o \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}}.$$

### 3.2.4 The 180° Hybrid

The 180° hybrid junction is a four-port network with a 180° phase shift between the two output ports. It can also be operated so that the outputs are in phase. With reference to the 180° hybrid shown in Figure 4.16, a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3 and port 4 will be isolated. If the input is applied to port 4, it will be equally split into two components with a 180° phase difference at ports 2 and 3 and port 1 will be isolated.

When operated as a combiner, with input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4. Hence, ports 1 and 4 are referred to as the sum and difference ports, respectively.

The 180° hybrid can be fabricated in several forms. The ring hybrid, or rat-race, shown in Figure 4.16, can easily be constructed in planar (microstrip or stripline) form, although waveguide versions are also possible. Another type of planar 180° hybrid uses tapered matching sections and coupled lines; this is known as the tapered coupled line coupler.

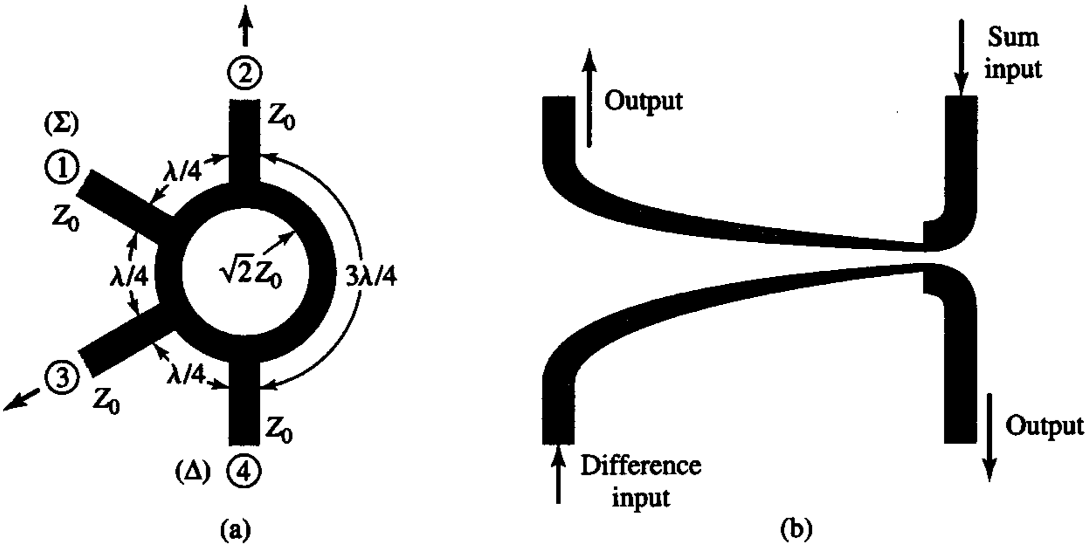



**Figure 4.16: (a) Ring hybrid, or rat-race coupler, (b) Tapered coupled line coupler.**

The scattering matrix for the ideal 3 dB 180° hybrid thus has the following form:

$$[S] = -j\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}.$$

The scattering parameter magnitudes versus frequency for the ring hybrid are shown in Figure 4.17.

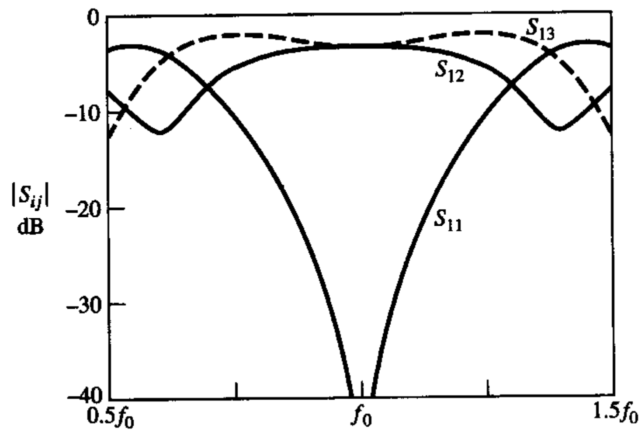



**Figure 4.17: Ring hybrid *S*-parameter magnitudes.**

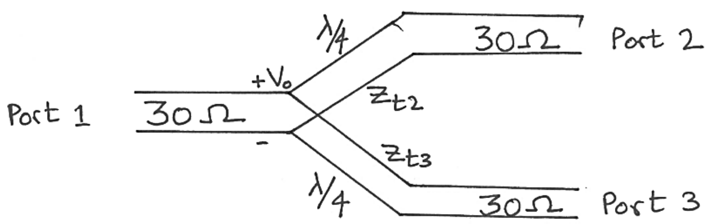
## 3.3 Additional Examples

**Example 1: Lossless T-junction Divider**

Design a lossless T-junction divider with a 30 Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30 Ω. Determine the magnitude of the scattering parameters for this circuit, using a 30 Ω characteristic impedance.

**Solution**

Consider the following figure, showing a lossless T-junction divider with ports 1-3 labelled accordingly. Quarter wave matching transformers have been shown that will convert the impedances of the output lines to 30 Ω at port 1.




The power at the ports can be given as:

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1},$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{3}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{3}{4Z_0}\right),$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{1}{4Z_0}\right).$$

Equating the above equations, we can find the characteristic impedances as follows:

$$Z_2 = \frac{4}{3}Z_0 = 40\Omega,$$

$$Z_3 = 4Z_0 = 120\Omega.$$

The impedances of the $\lambda/4$ matching transformers have been labelled as $Z_{t2}$ and $Z_{t3}$ in the figure and can be found as:

$$Z_{t2} = \sqrt{30\times 40} = 34.6\Omega,$$

$$Z_{t3} = \sqrt{30\times 120} = 60.0\Omega.$$

The $S$-parameters can be found by considering the phase reference at the 30 Ω ports as follows:

$$S_{11} = \frac{30-30}{30+30} = 0,$$

$$S_{22} = \frac{30||120-40}{30||120+40} = \frac{24-40}{24+40} = -0.25,$$

$$S_{33} = \frac{30||40-120}{30||40+120} = \frac{17.1-120}{17.1+120} = -0.75,$$

$$S_{21} = S_{12} = \sqrt{\frac{P_2}{P_1}}e^{-j\theta} = \sqrt{\frac{3}{4}} < -90^0 = 0.866 < -90^0,$$

$$S_{31} = S_{13} = \sqrt{\frac{P_3}{P_1}}e^{-j\theta} = \sqrt{\frac{1}{4}} < -90^0 = 0.5 < -90^0.$$

Since the divider network is lossless, we have the following condition:

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1.$$

Thus,

$$S_{23} = S_{32} = \sqrt{1-(0.25)^2-(0.866)^2}e^{-2j\theta} = 0.433 < -180^0.$$

The complete scattering parameters may be written as:

$$[S]=\begin{bmatrix} 0 & 0.866e^{-j\theta} & 0.5e^{-j\theta} \\ 0.866e^{-j\theta} & -0.25 & 0.433e^{-j\theta} \\ 0.5e^{-j\theta} & 0.433e^{-j\theta} & -0.75 \end{bmatrix}.$$

**Example 2: Unequal Split Wilkinson Power Divider**

Design a Wilkinson power divider with a power division ratio of $P_3/P_2$ = 1/3 and a source impedance of 50 Ω.

**Solution**

This is an unequal power division case and the following design equations may be used:

$$Z_{03} = Z_0\sqrt{(1+K^2)/K^3},$$
$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)},$$
$$R = Z_0(K+1/K).$$

Now,

$$K^2 = \frac{P_3}{P_2} = 1/3,$$
$$\Rightarrow K = 0.577.$$

Now,

$$Z_{03} = Z_0\sqrt{(1+(0.577)^2)/(0.577)^3} = 131.7\,\Omega,$$

$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)} = 50\sqrt{(0.577)(1+0.577^2)} = 43.9\,\Omega.$$

Thus,

$$R = Z_0(K+1/K) = 50(0.577 + 1/0.577) = 115.5\,\Omega.$$

The output impedances are:

$$R_2 = Z_0 K = 50\times 0.577 = 28.9\,\Omega,$$

$$R_3 = Z_0 / K = 50/0.577 = 28.9\,\Omega.$$

[1] Reinhold, Ludwig and Pavel Bretchko, 'RF Cirucit Design – Theory and Applications, 2000
[2] David M. Pozar, "Microwave Engineering", John Wiley, ISBN 0-471-17096-8