EEEN60481 Coursework
Performance Evaluation of Wireless Communication Systems Using Monte-Carlo Simulations

September 2024

1 Performance of a Basic Wireless Communication System Using Monte-Carlo Simulations

As shown in Figure 1, let's consider a wireless communication system, where the transmitter (base station) conveys information, x, with a transmit power of P_t to the receiver (user). Let's assume that the transmitter is located r meters away from the receiver, where the horizontal distance between the transmitter and receiver is d_x meters, the height of the transmitter is h_t meters, and the height of the receiver is h_u meters.

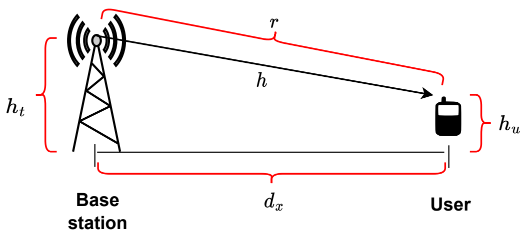

Figure 1 A basic wireless communication system.

The received signal at the receiver is expressed as

$$y = \sqrt{P_t P_L(r)} h x + n, \tag{1}$$

where h is the small-scale fading channel which is assumed to be a complex Gaussian random variable, x is the transmit signal with unit energy, and $n \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) with mean zero and variance of N_0. Furthermore, $P_L(r)$ is the free space path-loss expressed as

$$P_L(r) = G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2, \tag{2}$$

where G_t is the transmit antenna gain, G_r is the receive antenna gain, $\lambda = \frac{c}{f}$ is the wavelength, c is the speed of light and f is the operating frequency of the system.

Using (1), the received signal-to-noise ratio (SNR) is expressed as

$$\mathrm{SNR} = \frac{P_t P_L(r)|h|^2}{N_o} = \gamma_t P_L(r)|h|^2, \tag{3}$$

where $\gamma_t = \frac{P_t}{N_0}$.

Using (3), the ergodic capacity of the system is expressed as

$$C = \mathbb{E}\left[\log_2\left(1 + \mathrm{SNR}\right)\right], \tag{4}$$

where $\mathbb{E}[.]$ represents the expectation operator.

Furthermore, the probability when the SNR is below or equal to a certain threshold, γ_{th}, is termed as outage probability. Therefore, using (3), it is expressed as

$$P_{\text{out}} = \Pr(\text{SNR} \leq \gamma_{\text{th}}). \tag{5}$$

Task 1 (60 Marks): Use the Monte-Carlo simulations with 10^6 realizations to plot the **ergodic capacity** and **outage probability** of the system against $\gamma_t = [20 : 60]$ dB with the following parameters.

- $h_t = 10$ m, $h_u = 6$ m, $d_x = 80$ m, $G_t = G_r = 20$ dBi, $f = 900$ MHz, and $\gamma_{th} = 4$ dB. Furthermore, $h \sim \mathcal{CN}(\mu, 1)$, when $\mu = 0$, the channel amplitude, $|h|$, follow Rayleigh distribution, and when $\mu = \sqrt{\frac{K}{1+K}}$, $|h|$ is assumed to follow Rician distribution with a Rician factor of $K = 8$ dB.
- The distribution of $|h|$ should be appropriately chosen for the following two scenarios.
  1)- There is a line-of-sight (LOS) between the transmitter and receiver.
  2)- The direct LOS link between the transmitter and receiver has been blocked.

2 Performance of Intelligent Reflecting Surface (IRS)-assisted Wireless Communication Systems

Intelligent reflecting surface (IRS) is a new technology which significantly improves the performance of wireless communication systems. An IRS consists of a number of nearly passive reflecting elements, and it is used to form strong beams and reflect the incident signals to a desired direction by intelligently manipulating the phase of each reflecting element [1].

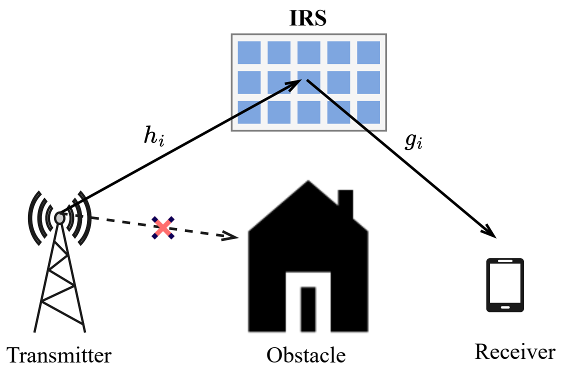

Figure 2 An IRS-assisted wireless communication system.

As shown in Figure 2, let's consider a single-input single-output (SISO) wireless communication system, where the direct link between the transmitter (T) and receiver (R) has been blocked. However, the communication is established through an IRS equipped with N reflecting elements. It is assumed that by exploiting the IRS, T transmits the signal x to R. Since the direct T-R link is not available, the transmitted signal first arrives at the IRS through the fading channels, h_i $\forall i = 1, \ldots, N$. The IRS reflecting elements then apply phase shifts, $\theta_i \in [0, 2\pi)$, to form strong beams and reflect the incident signal to R through the fading channels, g_i $\forall i = 1, \ldots, N$. The fading channel coefficients are assumed to be mutually independent and identically distributed complex Gaussian random variables, which can be expressed as $h_i \sim \mathcal{CN}(\mu_h, \sigma_h^2)$ and $g_i \sim \mathcal{CN}(\mu_g, \sigma_g^2)$, where $\{\mu_h, \mu_g\}$ and $\{\sigma_h^2, \sigma_g^2\}$ are the mean and variances, respectively. Therefore, the received signal at R is expressed as

$$y = \sqrt{P_t P_L} \sum_{i=1}^{N} h_i e^{j\theta_i} g_i x + n, \tag{6}$$

where P_t is the transmit power, x is the transmit signal with unit energy, and $n \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN). Furthermore, P_L is the large-scale path-loss expressed as [2]

$$P_L = \frac{G_t G_r}{(4\pi)^2} \left(\frac{LW}{d_h d_g} \right)^2 \cos^2(\phi), \quad (7)$$

where G_t is the transmit antenna gain, G_r is the receive antenna gain, L and W are the length and width of the IRS, d_h is the T-IRS distance, d_g is the IRS-R distance, and ϕ is the angle of incidence.

Using (6), the received SNR is expressed as

$$\text{SNR} = \gamma_t P_L \left| \sum_{i=1}^{N} h_i e^{j\theta_i} g_i \right|^2, \quad (8)$$

where $\gamma_t = \frac{P_t}{N_0}$.

2.1 IRS with Optimal Phase Shifts

Let's express the IRS fading channels in terms of their respective amplitudes and phases as $h_i = |h_i| \exp(-j\arg(h_i))$ and $g_i = |g_i| \exp(-j\arg(g_i))$, where $\{|h_i|, |g_i|\}$ and $\{\arg(h_i), \arg(g_i)\}$ are the amplitudes and phases, respectively. Moreover, it is assumed that perfect channel state information (CSI) is available at the IRS, which implies that the phases of the IRS fading channels are perfectly known at the IRS. Additionally, the IRS is assumed to have the capability to apply continuous phase shifts within the entire interval of $\theta_i \in [0, 2\pi)$. Therefore, in order to maximize the received SNR, the IRS applies phase shifts to align the phases of h_i and g_i, which leads to an optimal phase shift model, i.e. $\theta_i = \arg(h_i) + \arg(g_i)$. As a result, the received SNR given in (8) takes the following form.

$$\begin{aligned} \text{SNR} &= \gamma_t P_L \left| \sum_{i=1}^{N} |h_i||g_i| \exp\left(j \left(\underbrace{\theta_i - \arg(h_i) - \arg(g_i)}_{=0} \right) \right) \right|^2 \\ &= \gamma_t P_L \left| \sum_{i=1}^{N} |h_i||g_i| \right|^2, \end{aligned} \quad (9)$$

where $|h_i|$ and $|g_i|$ are the channel amplitudes for the T-IRS and IRS-R fading channels, respectively.

<u>For further information on IRS technology, please refer to the provided references (i.e., [1] and [2]).</u>

Task 2 (40 Marks): Use the Monte-Carlo simulations with 10^6 realizations to plot the **<u>ergodic capacity</u>** and **<u>outage probability</u>** of the system against $\gamma_t = [10 : 40]$ dB with the following parameters.

- $N = \{25, 50, 100\}$, $G_t = G_r = 10$ dBi, $L = W = 2$ m, $d_h = d_g = 25$ m, $\phi = 30^o$, and $\gamma_{th} = 10$ dB. Moreover, the IRS fading channel coefficients are assumed to be zero-mean complex Gaussian random variables , i.e. $h_i \sim \mathcal{CN}(0, 1)$ and $g_i \sim \mathcal{CN}(0, 1)$.

3 Report Structure

Each student is required to submit a report that contains

1. A cover page containing the student's full name, ID and the title of the coursework.
2. A table of content.
3. An introduction to Monte Carlo simulations.
4. Results and Discussion: All figures should be clearly visible with captions, labels and legends. Each figure must be analysed and discussed.
5. A conclusion and outcome of the coursework.
6. List of references.
7. Matlab code with comments for each line. Please note, screenshot is not permitted !

References

[1] S. Gong, X. Lu, D. T. Hoang, D. Niyato, L. Shu, D. I. Kim, and Y.-C. Liang, "Toward smart wireless communications via intelligent reflecting surfaces: A contemporary survey," *IEEE Communications Surveys & Tutorials*, vol. 22, no. 4, pp. 2283–2314, 2020.

[2] O. Ozdogan, E. Björnson, and E. G. Larsson, "Intelligent reflecting surfaces: Physics, propagation, and pathloss modeling," *IEEE Wireless Communications Letters*, vol. 9, no. 5, pp. 581–585, 2020.