Microwave Circuit Principles and Design

MSc Communication Engineering

MEng Electronic Engineering

EEEN40171/40171

2022-2023

Passive Circuits*

RF Components & Transmission Line Theory Review

Recommended Reading:

[1] David M. Pozar, “Microwave Engineering”, John Wiley, ISBN 0-471-17096-8 .

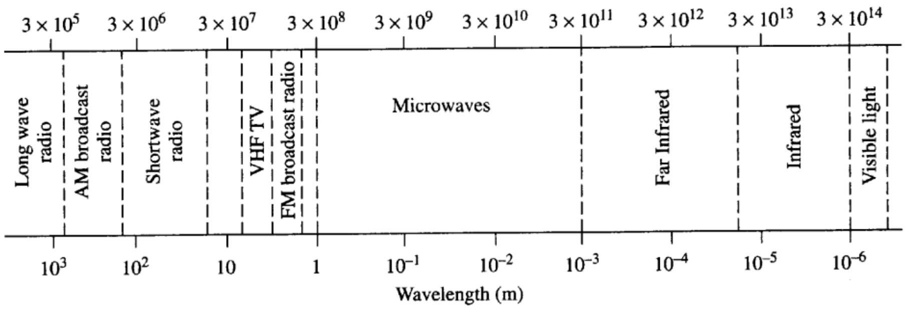
[2] R Ludwig and P Bretchko, “RF Circuit Design Theory and Applications”, Prentice-Hall, ISBN 0-13-095323-7.

*Most materials presented in this handout come from [1] and [2]

In RF and microwave frequency range, conventional circuit analysis methods applying to DC and low frequency lumped components such as resistors, capacitors and inductors, become inadequate. The circuits at such high frequency are governed by electromagnetic wave propagation. Furthermore, the parasitic effects of passive and active devices play a significant role in both circuit and system designs. The aim of this course is *to build upon fundamental RF and microwave circuit analysis and apply it to yield the rudiments of microwave passive and active circuit designs. Impact of individual component specification and its parasitic effects on overall system performance is investigated.*

The Electromagnetic Spectrum

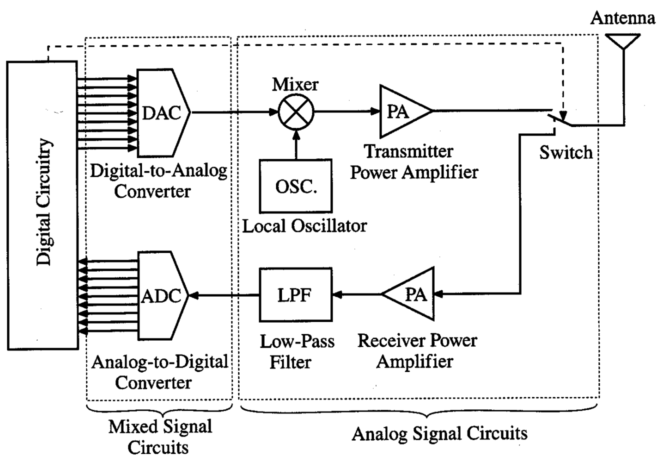
In general wavelengths of 1 mm to 1 m, or frequencies of 300 MHz to 300 GHz are regarded as RF/microwave/millimeter waves. The lower end of this spectrum could overlap with FM and higher end with lower THz.



Typical Frequencies	
AM broadcast band	535–1605 kHz
Shortwave radio	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
Microwave ovens	2.45 GHz

Approximate Band Designations	
L-band	1–2 GHz
S-band	2–4 GHz
C-band	4–8 GHz
X-band	8–12 GHz
Ku-band	12–18 GHz
K-band	18–26 GHz
Ka-band	26–40 GHz
U-band	40–60 GHz

Ref [1]



Block Diagram of a Generic RF/microwave System (Typical applications: Cellular phones and wireless local area networks) Ref [2]

Passive Circuits

1. RF Behavior of Resistors, Capacitors and Inductors [2]

1.1. Resistors:

At RF/microwave frequency, a **resistor** can no longer be presented by DC resistance. Skin effect must be taken into account. R_{RF} is inversely proportional to the skin depth, i.e.,

$R_{RF} \propto \frac{1}{2\delta} R_{DC}$, where δ is called skin depth and give by

$\delta = 1/\sqrt{\pi f \mu_0 \sigma_{cond}}$, where σ_{cond} is conductivity of the conductor, μ_0 is the abosulte permeability.

By taking into parasitic effects, an equivalent circuit model of a resistor at RF/microwave becomes

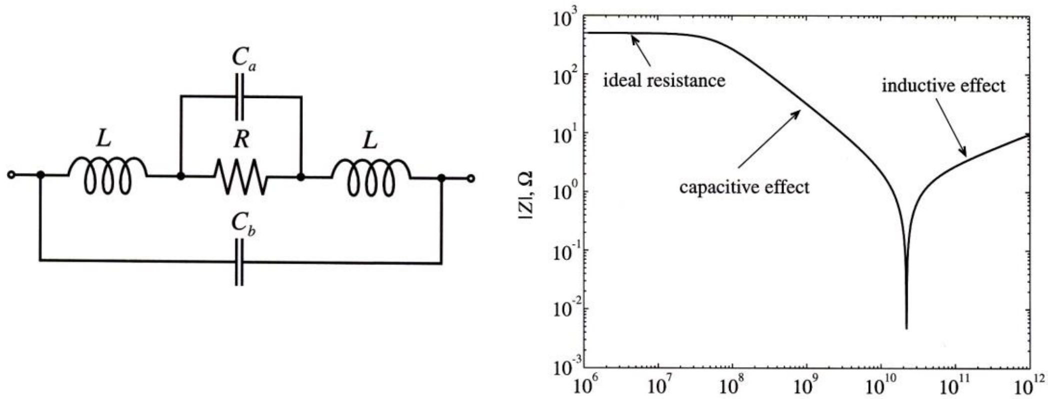


Fig. 1.1 Resistor equivalent circuit model and frequency response

L in the equivalent circuit models the leads and C_a models charge storage effects within the resistor. C_b can be neglected in our study.

1.2. Capacitors:

At RF/microwave frequency, a **capacitor** must be represented by its high frequency equivalent circuit.

L and R_S in Fig. 1.2 represent lead inductance and losses, respectively. R_e takes into account of dielectric losses and is given as

$R_e = \frac{1}{G_e}$, where G_e is the conductance of the dielectric.

$G_e = \frac{\omega C}{tan\, \Delta_S}$, where $\tan \Delta_S$ is defined as series loss tangent. It can be given as

$tan\, \Delta_S = \frac{\omega \varepsilon}{\sigma_{dielec}}$, where σ_{dielec} is the conductivity of the dielectric.

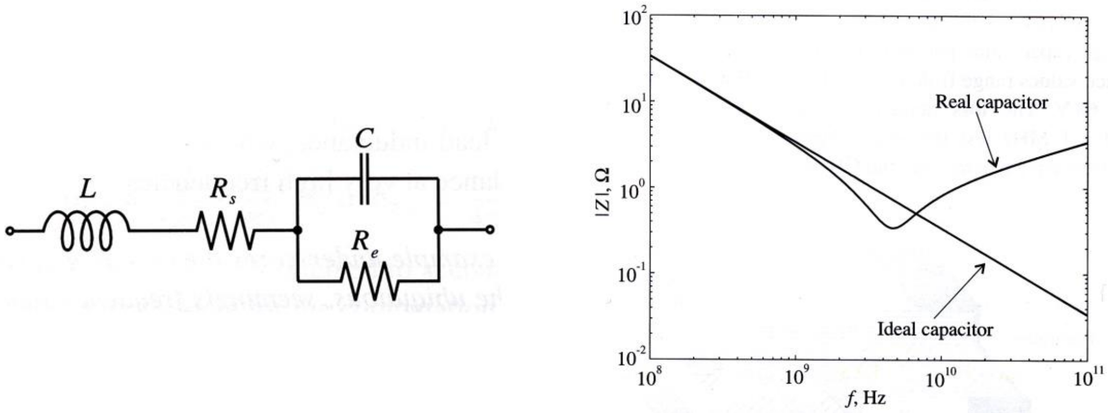


Fig. 1.2 Capacitor equivalent circuit model and frequency response

1.3. Inductors:

At RF/microwave frequency, an **inductor** equivalent circuit model can be given as below

The capacitor C_S and resistor R_S in Fig. 1.3 represent parasitic effects of distributed C_d and resistor R_d, respectively.

Assuming a coil inductor, C_S can be given as

$C_S = 4\pi\varepsilon_0 \frac{raN^2}{l}$, where *r* is the radius of the coil core, *a* the radius of the wire, *N* the number of the turns and *l* the length of the inductor.

Since the wire is normally very thin, the skin effect of the wire can be neglected, and the series resistance can be expressed by its DC resistance. That is

$R_S = \frac{2\pi r N}{\sigma_{cond}\pi a^2}$.

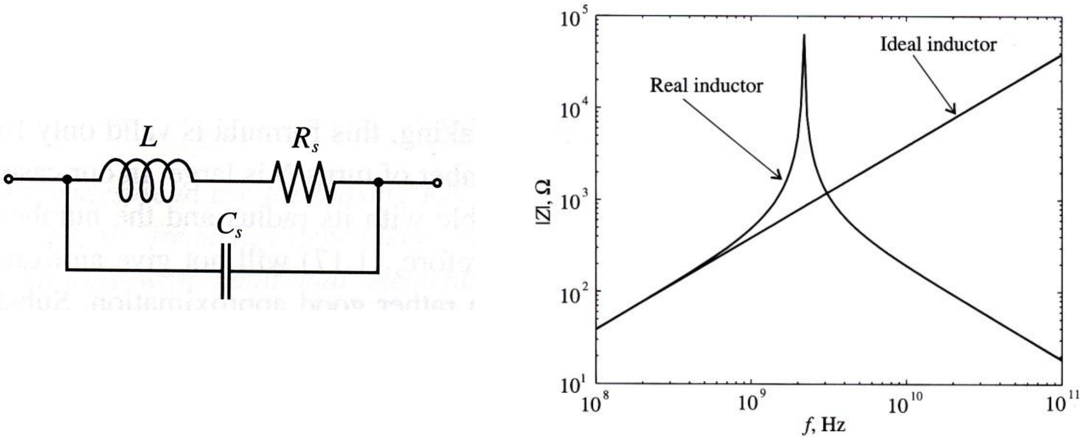
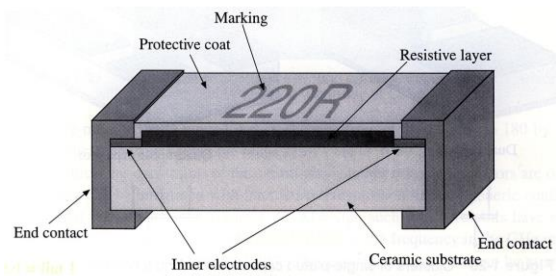
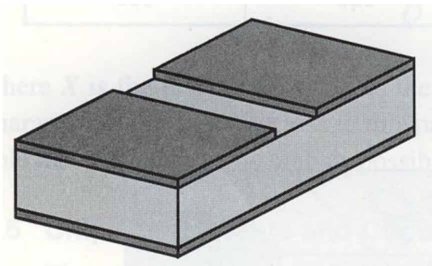


Fig. 1.3 Inductor equivalent circuit model and frequency response

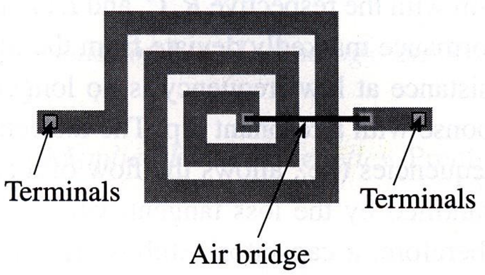
Examples of chip resistors, capacitors, and inductors



A chip resistor



A dual chip capacitor



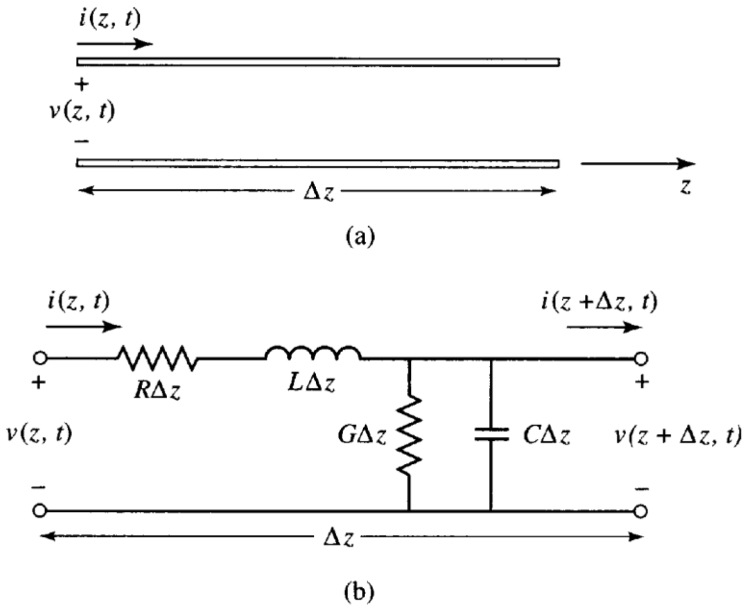
A chip inductor

Fig. 1.4 Chip resistor, capacitor, and inductor

2. **Transmission Line Theory Review** [1]

The transmission has been introduced in EEEN600121/400121. A brief review is provided below, leading to further development

2.1. Voltage and Current on a Transmission Line



R: series resistance per unit length, L: series inductance per unit length,
G: shut conductance per unit length, C: shut capacitance per unit length.

Fig. 2.1 (a) a transmission line, (b) an equivalent circuit model of the transmission line.

From Kirchoff's voltage and current laws, one has:

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \qquad (1)$$

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \qquad (2)$$

$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} - Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} = 0 \qquad (3)$$

$$-\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} - Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t} = 0 \qquad (4)$$

Let $\Delta z \to 0$

$$\frac{\partial v(z,t)}{\partial z} + Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} = 0 \qquad (5)$$

$$\frac{\partial i(z,t)}{\partial z} + Gv(z,t) + C\frac{\partial v(z,t)}{\partial t} = 0 \qquad (6)$$

2.2. Steady-State Voltage and Current on a Transmission Line

For steady state sinusoidal signal, Eq. (50 and (6) can be written as:

$$\frac{dV(z)}{dz} + (R + j\omega L)I(z) = 0 \qquad (7)$$
$$\frac{dI(z)}{dz} + (G + j\omega C)V(z) = 0 \qquad (8)$$

Taking d/dz of Eq. (7) and (8), one has:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \qquad (9)$$
$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \qquad (10)$$

$$where\ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Eq. (9) and (10) are defined as wave equations. γ is defined as **Propagation Constant**, which can also be written as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + \mathrm{j}\beta \quad (11)$$

where the real part, α is defined as **attenuation constant** and the imaginary part, β is called **phase constant**.

Solving Eq. (9) and (10), one obtains:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (12)$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad (13)$$

$e^{-\gamma z}$ represents wave propagating in the +Z direction, also called forward wave, whereas $e^{\gamma z}$ in the –Z direction, also called reflected wave. $V_0^+ e^{-\gamma z}, V_0^- e^{\gamma z}$ are the forward and reflected (backward) voltage waves, respectively. $I_0^+ e^{-\gamma z}, I_0^- e^{\gamma z}$ are the forward and reflected (backward) current waves, respectively.

Substituting Eq. (12) into Eq. (7), we have

$$I(z) = \frac{\gamma}{R + j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \qquad (14)$$

Characteristic Impedance of the line, Z_0 is define as,

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (15)$$

Substituting Z_0 into Eq. (12) and (13), one has

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (16)$$
$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad (17)$$

In time domain, these equations become

$$v(z,t) = |V_0^+| e^{-\alpha z} cos(\omega t - \beta z + \varphi_v^+) + |V_0^-| e^{\alpha z} cos(\omega t + \beta z + \varphi_v^-) \quad (18)$$
$$i(z,t) = \left|\frac{V_0^+}{Z_0}\right| e^{-\alpha z} cos(\omega t - \beta z + \varphi_i^+) - \left|\frac{V_0^-}{Z_0}\right| e^{\alpha z} cos(\omega t + \beta z + \varphi_i^-) \quad (19)$$

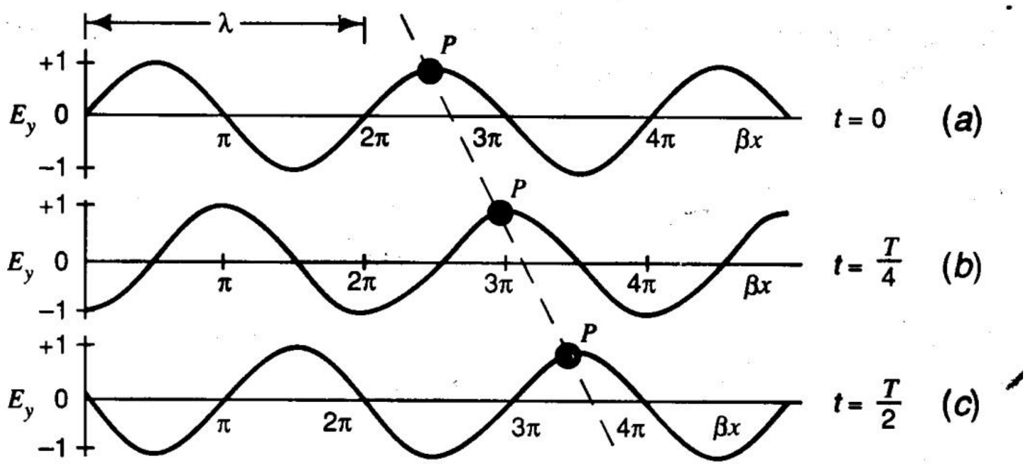


Fig. 2.2 Illustration of forwarded wave propagation.

2.3. Relations between Phase Velocity, Wavelength, Frequency and Phase Constant

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - \text{constant}}{\beta}\right) = \frac{\omega}{\beta} \qquad (20)$$
$$\lambda = v_p T = \frac{v_p}{f} = \frac{2\pi}{\beta} \qquad (21)$$

2.4. Lossless Transmission Line

A lossless line assumes that there is no conductor loss, no dielectric loss and no radiation loss. For a lossless line, R=0, G=0, the propagation constant $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ becomes

$\gamma = \sqrt{j\omega \mathrm{L} \mathrm{j}\omega C} = j\omega\sqrt{\mathrm{LC}}$, i.e.,

$$\begin{cases} \alpha = 0 \\ \beta = \omega\sqrt{\mathrm{LC}} \end{cases} \tag{22}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} \tag{23}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \tag{24}$$

For lossless case, Eq. (16) and (17) now become

$$\begin{cases} V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} & (25) \\ I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^+}{Z_0} e^{j\beta z} & (26) \end{cases}$$

2.5. Lossless Transmission Line Terminated with a Load

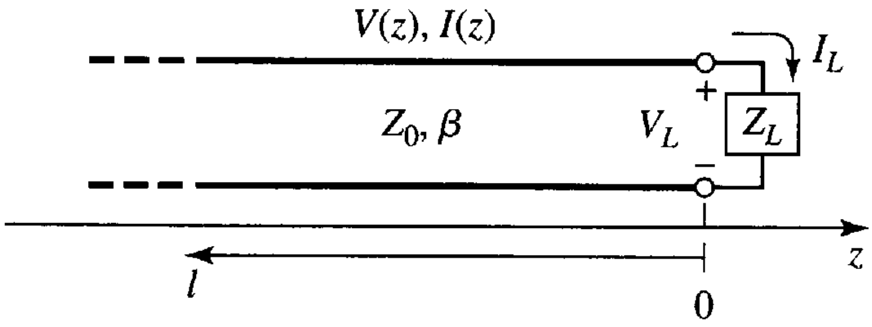


Fig. 2.3 lossless transmission terminated with a load Z_L.

(a) Voltage Reflection Coefficient

We define voltage reflection coefficient as the ratio of the reflected voltage wave over the incident voltage wave along the line, i.e.,

$$\Gamma_V(z) = \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}} = \frac{V_0^-}{V_0^+} e^{2j\beta z} \tag{27}$$

$\Gamma_V(z)$ becomes $\Gamma_V(0)$ at z=0, and $\Gamma_V(z-l)$ at z=-*l*, respectively,

$$\Gamma_V(0) = \frac{V_0^- e^{j\beta 0}}{V_0^+ e^{-j\beta 0}} = \frac{V_0^-}{V_0^+}, \tag{28}$$

$$\Gamma_V(-l) = \frac{V_0^- e^{j-\beta l}}{V_0^+ e^{-j(-)\beta l}} = \frac{V_0^-}{V_0^+} e^{-2j\beta l} = \Gamma_V(0)e^{-2j\beta l}. \tag{29}$$

At the load (z=0), from Eq (25) and (26)

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}}, \text{ ie.,} \quad \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Equation (28) becomes

$$\Gamma_V(0) = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (30)$$

Eq (25) and (26) can then be expressed as:

$$V(z) = V_0^+(e^{-j\beta z} + \Gamma_V(0)e^{j\beta z}) \qquad (31)$$

$$I(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma_V(0)e^{j\beta z}) \qquad (32)$$

(b) Power Flow along the Line and delivered to the Load

$$P(z) = \frac{1}{2}Re[V(z)I(z)^*] = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}Re\{1 - \Gamma_V^*(0)e^{-2j\beta z} + \Gamma_V(0)e^{2j\beta z} - |\Gamma_V(0)|^2\}$$

$$= \frac{1}{2}\frac{|V_0^+|^2}{Z_0} - \frac{1}{2}\frac{|V_0^+|^2}{Z_0}|\Gamma_V(0)|^2 = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma_V(0)|^2),$$

where $|\Gamma_V(0)|^2 = \frac{|V_0^-|^2(reflected\ power)}{|V_0^+|^2(incident\ power)}$.

We define power reflection coefficient $\rho(0) = |\Gamma_V(0)|^2$ as the ratio of the reflected power over the incident power at z = 0. The power reflection coefficient $\rho(z)$along the line is given as:

$$\rho(z) = \left|\Gamma_V(0)e^{2j\beta z}\right|^2 = |\Gamma_V(0)|^2 \qquad (33)$$

Power dissipated by the load is:

$$P_L = \frac{1}{2}\frac{|V_0^+|^2}{Z_0} - \frac{1}{2}\frac{|V_0^+|^2}{Z_0}|\Gamma_V(0)|^2 = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma_V(0)|^2) \qquad (34)$$

From equation (34), it becomes clear that not all of the incident power is delivered to the load. This ‘loss’ (not a real loss, but reflection) is called return loss (RL), and defined as

$$RL = -20\, log|\Gamma_V(0)| \quad \text{(dB)}. \qquad (35)$$

(c) Voltage Maximum, Minimum and Standing Wave Ratio (VSWR)

From Equations (31), we have

$$V(z) = V_0^+(e^{-j\beta z} + \Gamma_V(0)e^{j\beta z}) = V_0^+e^{-j\beta z}(1 + |\Gamma_V(0)|e^{j\theta}e^{j2\beta z})$$

$$|V(z)| = \left|V_0^+e^{-j\beta z}(1 + |\Gamma_V(0)|e^{j\theta}e^{j2\beta z})\right| = |V_0^+|\left|(1 + |\Gamma_V(0)|e^{j\theta}e^{j2\beta z})\right|$$

If $\theta + 2\beta z = 0, 2\pi, 4\pi, ..., 2n\pi$, we have

$\left|(1 + |\Gamma_V(0)|e^{j\theta}e^{j2\beta z})\right| = 1 + |\Gamma_V(0)|$, i.e., at these points the voltage magnitudes reach their maximum values. That is

$$V_{\max} = \left|V_0^+\right|(1+\left|\Gamma_V(0)\right|). \tag{36}$$

If $\theta + 2\beta z = \pi, 3\pi, ..., (2n-1)\pi$, we have

$\left|(1+\left|\Gamma_V(0)\right|e^{j\theta}e^{j2\beta z})\right| = 1-\left|\Gamma_V(0)\right|$, i.e., at these points the voltage magnitudes reach their minimum values. That is

$$V_{\min} = \left|V_0^+\right|(1-\left|\Gamma_V(0)\right|). \tag{37}$$

The voltage standing wave ratio is defined as

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+\left|\Gamma_V(0)\right|}{1-\left|\Gamma_V(0)\right|}. \tag{38}$$

The magnitude of the voltage reflection coefficient at z = 0 can be expressed as a function of VSWR as:

$$|\Gamma_V(0)| = \frac{VSWR-1}{VSWR+1}. \tag{39}$$

(d) Input Impedance along the Line

$$Z_{in} = \frac{V(z)}{I(z)} = \frac{V_0^+(e^{-j\beta z} + \Gamma_V(0)e^{j\beta z})}{\frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma_V(0)e^{j\beta z})},$$

$$= Z_0\frac{1+\Gamma_V(0)e^{2j\beta z}}{1-\Gamma_V(0)e^{2j\beta z}} = Z_0\frac{Z_L-jZ_0\,tan(\beta z)}{Z_0-jZ_0\,tan(\beta z)}. \tag{40}$$

At Z=-l,

$$Z_{in} = Z_0\frac{1+\Gamma_V(0)e^{-2j\beta l}}{1-\Gamma_V(0)e^{-2j\beta l}} = Z_0\frac{Z_L+jZ_0tan\,(\beta l)}{Z_0+jZ_Ltan\,(\beta l)}. \tag{41}$$

(e) The load is a lossless transmission line with different characteristic impedance

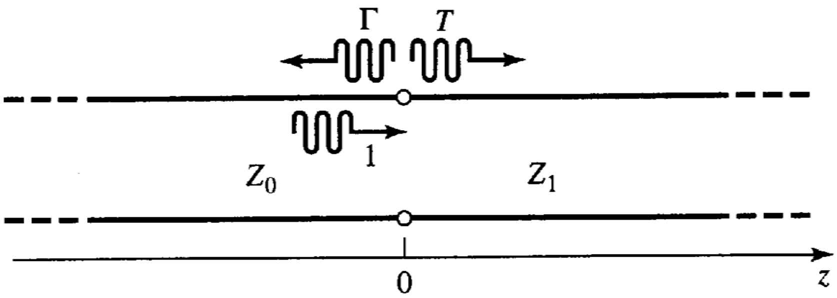


Fig. 2.4 Lossless transmission line terminated with another transmission line with characteristic impedance of Z_1.

Assuming the second line is infinitely long with the characteristic impedance of Z_1, then the reflection coefficient at z = 0 is:

$$\Gamma_V(0) = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

For z < 0

$$V(z) = V_0^+(e^{-j\beta z} + \Gamma_V(0)e^{j\beta z})$$

For z > 0

$$V(z) = V_0^+ T_V(0)e^{-j\beta z}$$

At z=0, the voltage must equal, i.e.,

$$1 + \Gamma_V(0) = T_V(0) \qquad (42)$$

$T_V(0)$ is called voltage transmission coefficient.

Insertion loss, IL, is defined as:

$$IL = -10\,log\frac{P_{Tran}}{P_{Inc}} = -10\,log\frac{P_{Inc}-P_{Ref}}{P_{Inc}} = -10\,log(1 - |\Gamma_V(0)|^2) \qquad (43)$$

(f) Transmission Line with Special Length and Terminated by a Special Load

(i) ***λ/4 open-circuited line*** ($l = \lambda/4$ and Z_L = infinite.)***:***

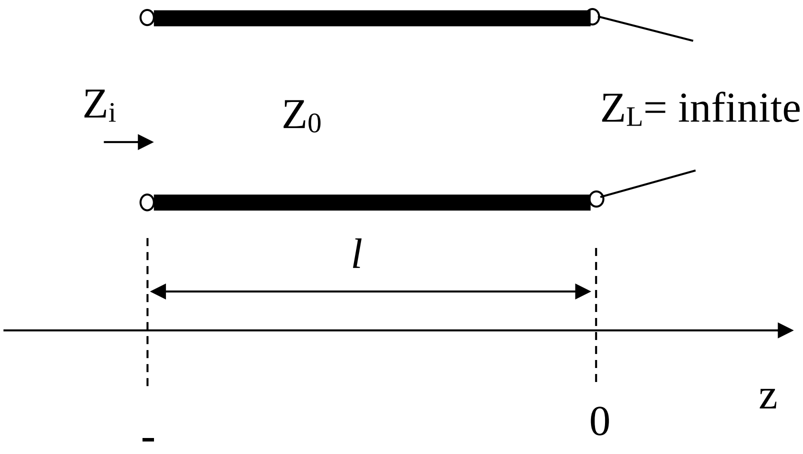


Fig. 2.5 λ/4 open-circuited transmission line.

If the line is lossless, the input impedance can be derived from Eq. (41) by setting $l = \frac{\lambda}{4}$ and Z_L = infinite. That is

$$Z_{in} = -jZ_0/\tan(\beta l) = -jZ_0/\tan(\frac{2\pi}{\lambda}\frac{\lambda}{4}) = -jZ_0/\tan(\frac{\pi}{2}) = 0. \quad (44)$$

This impedance is periodic in l, repeating for multiples of λ/2 as can be seen in Fig. 2.6.

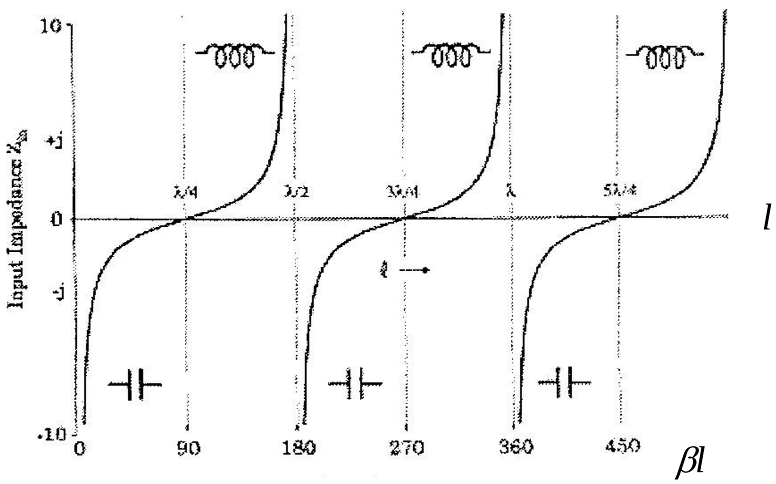


Fig. 2.6 Input impedance of the line as function of l with open-circuited load.

(ii) *λ/4 short-circuited line* ($l = \lambda/4$ and $Z_L = 0$.)**:**

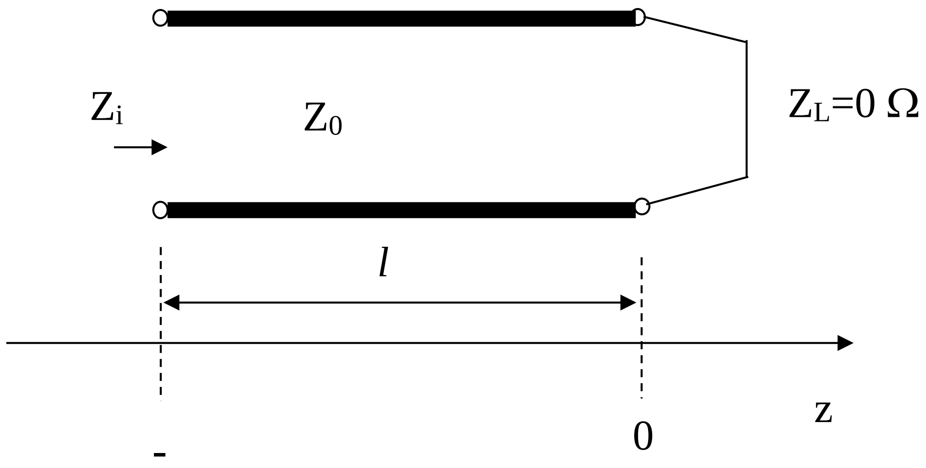


Fig. 2.7 λ/4 Short-circuited transmission line.

If the line is lossless, the input impedance can be derived from Eq. (41) by setting $l = \lambda/4$ and $Z_L = 0$. That is

$$Z_{in} = jZ_0\,tan(\beta l) = jZ_0\,tan(\frac{2\pi}{\lambda}\frac{\lambda}{4}) = jZ_0\,tan(\frac{\pi}{2}) = \infty. \qquad (45)$$

This impedance is periodic in *l*, repeating for multiples of λ/2 as can be seen in Fig. 8.

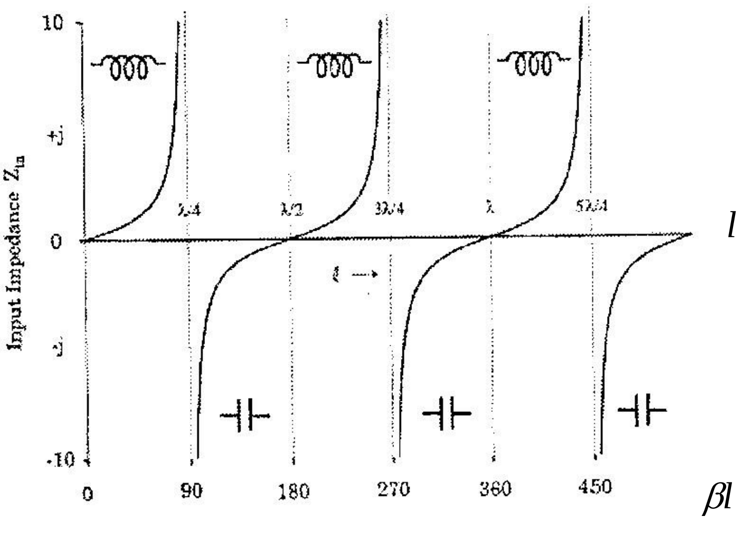


Fig. 2.8 Input impedance of the line as function of *l* with zero load

(iii) *λ/2 open-circuited line* (*l* = λ/2 and Z_L = infinite.)*:*

If the line is lossless, the input impedance can be derived from Eq. (41) by setting $l = \lambda/2$ and Z_L = infinite. That is

$$Z_{in} = -jZ_0/\tan(\beta l) = -jZ_0/\tan(\frac{2\pi}{\lambda}\frac{\lambda}{2}) = -jZ_0/\tan(\pi) = \infty. \qquad (46)$$

This impedance is periodic in *l*, repeating for multiples of λ/2, as can be seen in Fig. 2.7.

(iv) ***λ/2 short-circuited line*** ($l = \lambda/2$ and $Z_L = 0$.)**:**

If the line is lossless, the input impedance can be derived from Eq. (41) by setting $l = \lambda/2$ and $Z_L = 0$. That is

$$Z_{in} = jZ_0 \tan(\beta l) = jZ_0 \tan(\frac{2\pi}{\lambda}\frac{\lambda}{2}) = jZ_0 \tan(\pi) = 0 \qquad (47)$$

This impedance is periodic in *l*, repeating for multiples of λ/2, as can be seen in Fig. 2.8.

2.6. Lossy Transmission Line

(a) Propagation constant and Characteristic Impedance

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{\mathrm{LC}}\sqrt{1 - j(\frac{R}{\omega L} + \frac{G}{\omega C}) - \frac{RG}{\omega^2 LC}} \qquad (48)$$

For low-loss line, ωL >>R and ωC >> G, hence RG/ω^2LC → 0,

$$\gamma \approx j\omega\sqrt{\mathrm{LC}}[1 - j(\frac{R}{\omega L} + \frac{G}{\omega C})] = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{\mathrm{LC}}. \qquad (49)$$

The charateristic impedance becomes

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}} \qquad (50)$$

(b) Dispersion & Distortionless Transmission

From Eq. (48), one can see that the phase constant β can be a very complicated function of frequency if the loss is not very small, or the transmission line is very long. This results in the phase velocity varies with frequency, implying that the various frequency components of a wideband signal will travel with different phase velocities and arrive at the end of the line at different time, which causes signal distortion. This effect is called dispersion.

However, if a transmission line has R/L = G/C, one can prove that the line is distrotionless. Re-arranging Eq. (48), one has,

$$\gamma = j\omega\sqrt{\mathrm{LC}}\sqrt{1 - j(\frac{R}{\omega L} + \frac{G}{\omega C}) - \frac{RG}{\omega^2 LC}} = j\omega\sqrt{\mathrm{LC}}\sqrt{(1 - j\frac{R}{\omega L})^2} = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} \qquad (51)$$

or

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

i.e., the phase velocity is a constant value.

(c) Voltage Reflection Coefficient and Input Impedance

(i) Voltage reflection

$$\Gamma_V(z) = \Gamma_V(0)e^{2\gamma z} = \Gamma_V(0)e^{2\alpha z}e^{j2\beta z} \qquad (52)$$

At z = -l, we have (referring to Fig. 2.3 but the transmission line is lossy now),

$$\Gamma_V(l) = \Gamma_V(0)e^{-2\alpha l}e^{-j2\beta l} \qquad (53)$$

(ii) Impedance

$$Z_{in}(z) = Z_0\frac{1+\Gamma_V(0)e^{2\gamma z}}{1-\Gamma_V(0)e^{2\gamma z}} = Z_0\frac{Z_L+Z_0\,tanh(-\gamma z)}{Z_0+Z_L\,tanh(-\gamma z)} \qquad (54)$$

At z = -l, we have (referring to Fig. 2.3),

$$Z_{in}(l) = Z_0\frac{1+\Gamma_V(0)e^{-2\gamma l}}{1-\Gamma_V(0)e^{-2\gamma l}} = Z_0\frac{Z_L+Z_0\,tanh(\gamma l)}{Z_0+Z_L\,tanh(\gamma l)} \qquad (55)$$

(d) Power Flow

$$P(z) = \frac{1}{2}Re[V(z)I(z)^*] = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma_V(z)|^2)e^{-2\alpha z} \qquad (56)$$

At z = -l, we have

$$P(l) == \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma_V(l)|^2)e^{2\alpha l} \qquad (57)$$

The power delivered to the load is

$$P_L = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma_V(0)|^2).$$

The power lost to the transmission line is

$$P_{LOSS} = P(l) - P_L = \frac{|V_0^+|^2}{2Z_0}[(e^{2\alpha l} - 1) + |\Gamma_V(0)|^2(1 - e^{-2\alpha l})] \qquad (58)$$

2.7. Multiple Reflection Analysis of the Quarter Wave Transformer

The following figure shows the quarter wave transformer with reflection and transmission coefficients defined as below (the transformer is lossless).

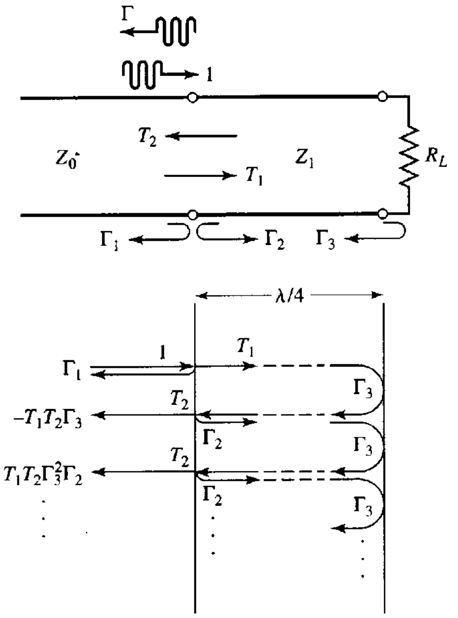


Fig. 2.9 Analysis of the quarter-wave transformer [Figure 2.18, Pozar 2012].

- Γ = overall, or total, reflection coefficient of a wave incident upon the $\lambda/4$ transformer,
- Γ_1 = partial reflection coefficient of a wave incident on a load Z_1, from the Z_0 line,
- Γ_2 = partial reflection coefficient of a wave incident on a load Z_0, from the Z_1 line,
- Γ_3 = partial reflection coefficient of a wave incident on a load R_L, from the Z_1 line,
- T_1 = partial transmission coefficient of a wave from the Z_0 line into the Z_1 line,
- T_2 = partial transmission coefficient of a wave from the Z_1 line into the Z_0 line.

These coefficients can be expressed as

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

Consider the quarter wave transformer in the time domain for a wave travelling down the Z_0 feedline towards the transformer. When the wave encounters the junction with the Z_1 line, it sees the impedance Z_1, since it has not yet reached the load R_L and therefore this load can not yet have had an effect. Part of the wave is reflected, with coefficient Γ_1 and part is transmitted through to the Z_1 line, with coefficient T_1.

The transmitted wave then travels a distance $\lambda/4$ to the load, is reflected due to the discontinuity at the load with coefficient Γ_3 and travels back $\lambda/4$ to the junction with the Z_0 transmission line. Part of this wave is transmitted and continues through (to the left) to the Z_0 line, with coefficient T_2, and part is reflected back towards the load, with coefficient Γ_2. This process creates an infinite number of reflected waves and the total reflection coefficient, Γ is the sum of all partial reflections. Each round trip along and back down the $\lambda/4$ transformer section results in a 180° phase shift and the total reflection coefficient can be expressed as

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3} = \frac{\Gamma_1 - \Gamma_3(\Gamma_1^2 + T_1 T_2)}{1+\Gamma_2\Gamma_3}.$$

The numerator of this expression can be simplified, using the coefficients defined above, as

$$\Gamma_1 - \Gamma_3(\Gamma_1^2 + T_1 T_2) = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1+Z_0)(R_L+Z_1)}.$$

If the quarter wave transformer impedance Z_1 is designed as $Z_1 = \sqrt{Z_o R_L}$, then the total reflection coefficient is zero and the Z_0 line is then matched to the load. Therefore, if the characteristic impedance and length of the matching section is selected accordingly, impedance matching is achieved due to the superposition of all of the partial reflections equating to zero.

2.8. More about Mismatches and Power Analysis

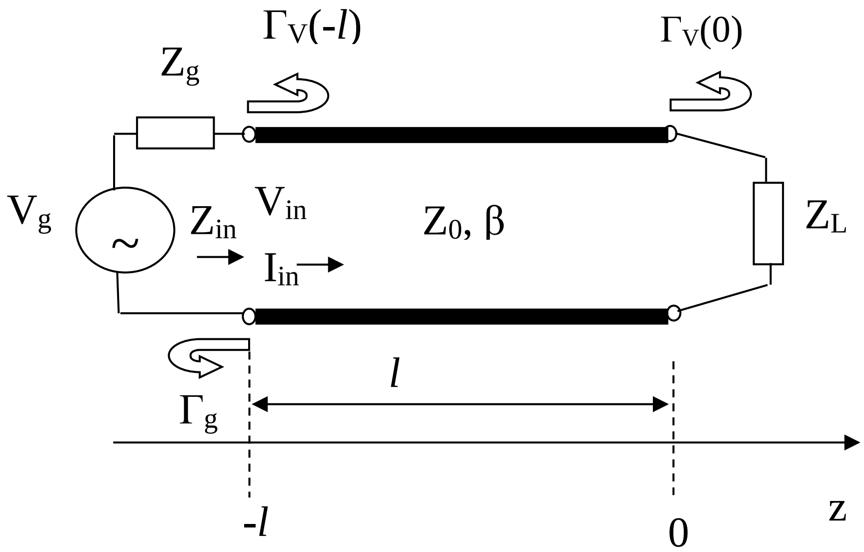


Fig. 2.10 Transmission line circuit for mismatched load and generator.

Fig. 2.10 shows the circuit diagram of a transmission line connected to an arbitrary generator. The source and load impedances are Z_g and Z_l respectively, which may be complex. The transmission line is assumed to be lossless here, with length *l* and characteristic impedance Z_0. Because both the generator and load are mismatched, multiple reflections from each discontinuity will occur on the line. The power delivered to the line is given by

$$P = \frac{1}{2} Re[V_{in} I_{in}{}^*] = \frac{1}{2}|V_{in}|^2 \, Re\{\frac{1}{Z_{in}}\} = \frac{1}{2} Re\{\frac{1}{Z_{in}}\} \left|\frac{V_g Z_{in}}{Z_{in} + Z_g}\right|^2.$$

Let $Z_{in} = R_{in} + jX_{in}$ and $Z_g = R_g + jX_g$, then we get

$$P = \frac{|V_g|^2}{2} \frac{R_{in}}{(R_{in}+R_g)^2+(X_{in}+X_g)^2}.$$

Maximum power transfer to the load will be achieved if $R_{in} = R_g$ and $X_{in} = - X_g$, i.e.

$$P_{\max} = \left|V_g\right|^2 / 8R_g \Rightarrow \text{Conjugate matching.}$$

2.9. Wave Velocities and Dispersion

Various velocities are generally encountered when discussing the propagation of electromagnetic waves, including the speed of light in a particular medium, the phase velocity and the group velocity.

- **The speed of light in a medium**
  - The speed of light in a medium is the velocity at which a plane wave would propagate in that medium, $(c/\sqrt{\mu_r \epsilon_r})$.
- **The phase velocity**
  - The phase velocity is the speed at which a constant phase point travels, $v_p = \omega/\beta$.
  - For a transverse electromagnetic (TEM) plane wave, the speed of light and the phase velocity are identical. However, for a non-TEM wave, the phase velocity may be greater or less than the speed of light.
- **Dispersion**
  - If the phase velocity of a line does not vary with frequency, the phase of a signal containing multiple frequency components will not be distorted. However, if the phase velocity varies with signal frequency, the phase relationships will change as the signal propagates along the line. The signal will be distorted and such an effect is called dispersion, i.e. some frequency components will travel ‘faster’ than others.
- **Group Velocity**
  - In a dispersive medium, there is no single phase velocity. However, if the bandwidth of the signal is relatively small, or if the dispersion is not too severe, group velocity is often used, given as

    $v_g = \left(\frac{d\beta}{d\omega}\right)^{-1}\Big|_{\omega=\omega_0}$,where ω_0 is the centre frequency of the signal.

2.10. Practical Transmission Lines

➢ Microstrip Lines

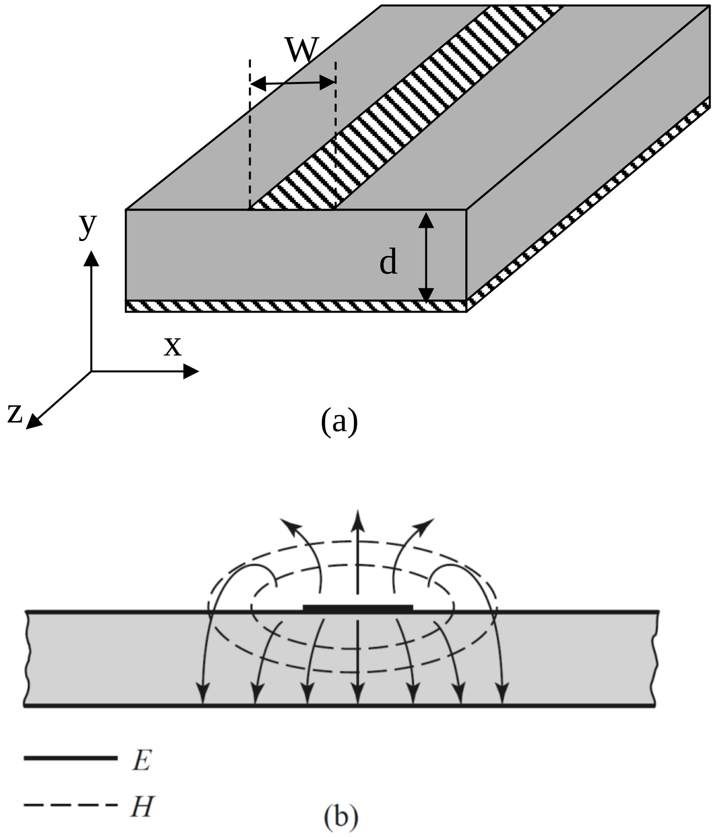


Fig. 2.11 Microstrip transmission line, (a) Geometry, (b) Electric and magnetic field lines.

The microstrip line is one of the most popular types of planar transmission lines, primarily because it can be fabricated by photolithographic processes and is easily miniaturized and integrated with both passive and active microwave devices. The geometry of a microstrip line is shown in Fig. 11 (a). A conductor of width *W* is printed on a thin, grounded dielectric substrate of thickness *d* and relative permittivity ε_r. A sketch of the electric (*E*) and magnetic (*H*) field lines is shown in Fig. 11 (b), viewed end on to the printed circuit board.

If the dielectric substrate were not present ($\varepsilon_r = 1$), we would have a two-wire line consisting of a flat strip conductor over a ground plane, embedded in a homogeneous medium (air). This would constitute a simple TEM transmission line with phase velocity ($v_p = c$) and propagation constant $\beta = k_0$. In many practical applications, a TEM mode is assumed for microstrip lines. However, it should be pointed out that due to the dielectric-air interface, a pure TEM mode cannot be supported by microstrip lines. The following expressions apply for the phase velocity, phase constant and characteristic impedance (the derivation of these equations and further insight may be obtained from [2]),

$$V_p = \frac{c}{\sqrt{\varepsilon_e}}, \;\; \beta = \beta_0\sqrt{\varepsilon_e}, \; \mathrm{Z}_0 = \frac{Z_{0\text{ in air}}}{\sqrt{\varepsilon_e}},$$

where ε_e is the effective dielectric constant of the microstrip line, which is defined as

$$\varepsilon_e = \frac{\text{Capacitance per unit length of the microstrip line with a dielectric substrate } (\varepsilon_r \neq 1)}{\text{Capacitance per unit length of the microstrip line with an air substrate } (\varepsilon_r = 1)}.$$

If the dispersion of the microstrip can be neglected, ε_e can be approximated as

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}.$$

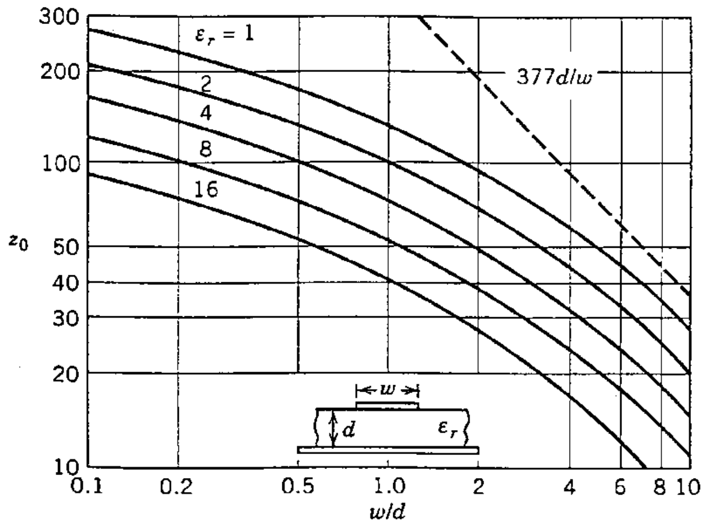
Microstrip Line Design Procedure (Old fashion)

Given ε_r, Z_0 and d, find W, or

Given ε_r and W/d, determine Z_0, or

Given ε_r, W and Z_0, determine d.

The graph shown below displays the relationship between Z_0 and W/d, for various values of ε_r. Such a graph may be used to easily select the parameters to achieve the required design specifications without calculation. Nowadays however, the design task is performed using software tools or electromagnetic simulators.



Design graph of microstrip lines.

Microstrip line can also be designed using semi-empirical equations. For those of you who are interested, please refer to Example of Microstrip line design using semi-empirical equations at the end of the this section.

➢ Coplanar Waveguide (CPW) Line

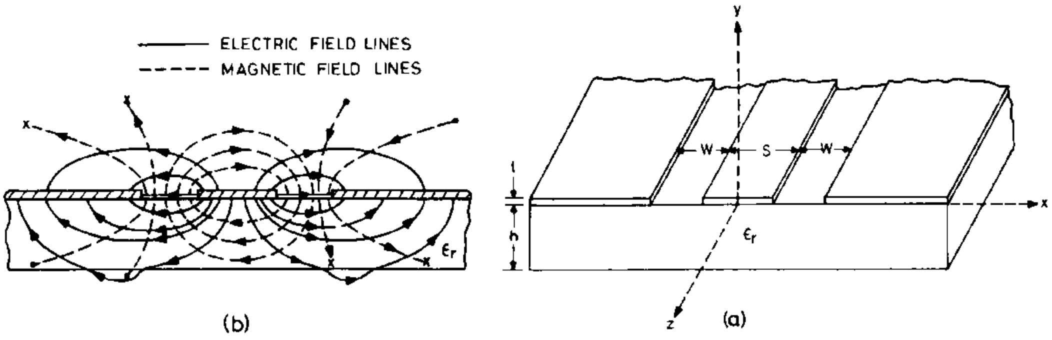


Fig. 2.12 CPW transmission line, (a) Electric and magnetic field lines, (b) Geometry.

The coplanar waveguide, shown above, is similar to the slotline and can be viewed as a slotline with a third conductor centred in the slot region. Because of the presence of this additional conductor, this type of line can support even or odd quasi-TEM modes, depending on whether the electric fields in the two slots are in the opposite direction or the same direction. Coplanar waveguides are particularly useful for fabricating active circuitry due to the presence of the centre conductor and the close proximity of the ground planes.

At low frequencies, a TEM mode is assumed to be supported by the CPW line. At higher frequencies, the longitudinal component of magnetic field in the slot due to the dielectric-air interface can no longer be neglected.

Under TEM mode assumption, we have

$$V_p = \frac{c}{\sqrt{\varepsilon_e}},\ \ \beta = \beta_0\sqrt{\varepsilon_e},\ \mathrm{Z}_0 = \frac{Z_{0\ \text{in air}}}{\sqrt{\varepsilon_e}},$$

where, ε_e is the effective dielectric constant of the CPW line.

Various design equations and models can be used to design CPW transmission lines; for further details, please consult [1].

➢ Stripline

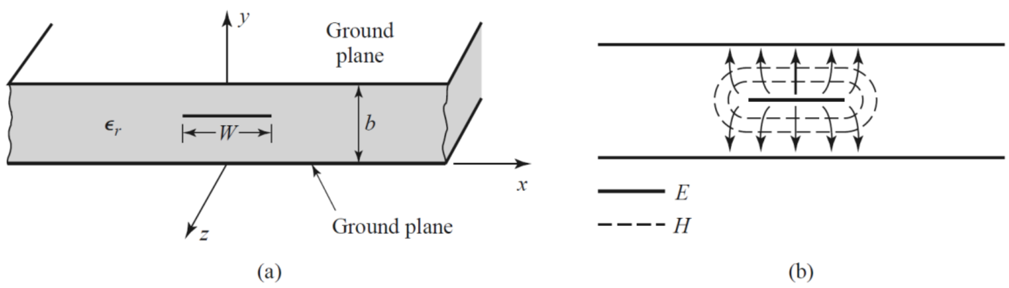


Fig. 2.13 Stripline transmission line, (a) Geometry, (b) Electric and magnetic field lines.

Stripline is another planar type of transmission line that lends itself well to microwave integrated circuitry, miniaturization and photolithographic fabrication. The geometry and field lines of stripline are shown above. A thin conducting strip of width W is centred between two wide conducting ground planes of separation b and the region between the ground planes is filled with a dielectric material. In practice, stripline is usually constructed by etching the centre conductor on a grounded dielectric substrate of thickness b/2 and then covering with another grounded substrate. Variations of the basic geometry include stripline with differing dielectric substrate thicknesses (asymmetric stripline) or different dielectric constants (inhomogeneous stripline). An air dielectric is sometimes used when it is necessary to minimize loss.

Because stripline has two conductors and a homogeneous dielectric, it supports a pure TEM mode. The phase velocity, propagation constant and characteristic impedance are given as

$$V_p = \frac{c}{\sqrt{\varepsilon_r}},\ \ \beta = \beta_0\sqrt{\varepsilon_r},\ \mathrm{Z}_0 = \frac{Z_{0\text{ in air}}}{\sqrt{\varepsilon_r}} = \frac{30\pi}{\sqrt{\varepsilon_r}}\frac{b}{W_e + 0.441b},$$

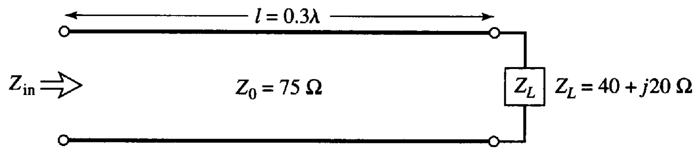
where ε_r is the relative dielectric constant of the substrate and W_e is given by

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \dfrac{W}{b} > 0.35 \\ (0.35 - W/b)^2 & \text{for } \dfrac{W}{b} < 0.35 \end{cases}.$$

Problems:

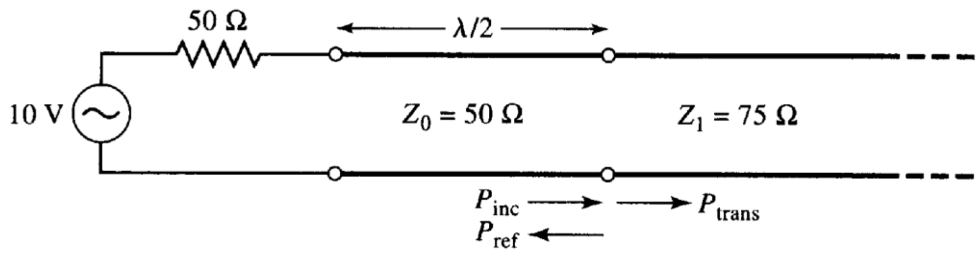
1. A transmission line has the following per unit length parameters: L=0.2 μH/m, C=300 pF/m, R=5 Ω/m, and G=0.01 S/m. Calculate the propagation constant and characteristic impedance of this line at 500 MHz. Recalculate these in the absence of loss (R=G=0).

2. A lossless transmission line of electrical length $l = 0.3\lambda$ is terminated with a complex load impedance as show below. Find the reflection coefficient at the load, the VSWR on the line, and the input impedance to the line.



3. A radio transmitter is connected to an antenna having impedance 80+j40 Ω with a 50 Ω coaxial cable. If the 50 Ω transmitter can deliver 30 W when connected to a 50 Ω load, how much power is delivered to the antenna?

4. Consider the transmission line circuit shown below. Compute the incident power, the reflected power, and the power transmitted into the infinite long 75 Ω line. Show that power conservation is satisfied.



5. A transmission line circuit as shown below has V_g = 15 V rms, Z_g = 75 Ω, Z_0 = 75 Ω, Z_L = 60 – j40 Ω, and l = 0.7λ. Compute the power delivered to the load using three different techniques.

(a) find Γ and compute

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}(1-|\Gamma|^2)$$

(b) find Z_{in} and compute

$$P_L = \left| \frac{V_g}{Z_g + Z_{in}} \right|^2 \operatorname{Re}(Z_{in}),$$

(c) find V_L and compute

$$P_L = \left| \frac{V_L}{Z_L} \right|^2 \operatorname{Re}(Z_L).$$

Discuss the rationale for each of these methods. Which of these methods can be used in the line is not lossleess?