Filtro Butterworth Implementado na Topologia Sallen Key

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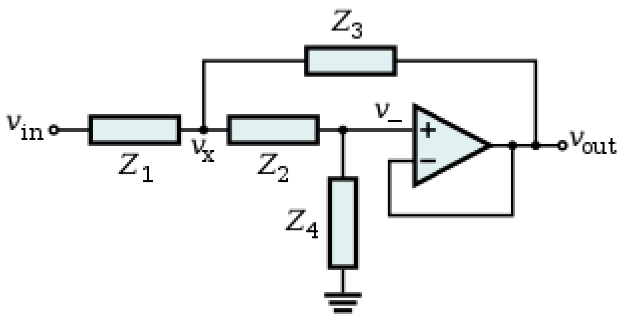
Anotações Pessoais



$$Z_1 = R_1 \quad Z_2 = R_2 \quad Z_3 = \frac{1}{sC_1} \quad Z_4 = \frac{1}{sC_2}$$

$$V_x = v_a(s) \quad Vin = v_i(s) \quad Vout = v_o(s)$$

$i_1(s)$ passa por Z_1 $i_2(s)$ passa por Z_2 e Z_4
$i_3(s)$ passa por Z_3

Figura 1: Topologia Sallen Key

$$i_1(s) = i_2(s) + i_3(s) \tag{1}$$

$$\begin{aligned} v_{(+)} &= i_2(s)\frac{1}{sC_2} \\ v_{(+)} &= v_{(-)} = v_o \\ i_2(s) &= C_2 s v_o(s) \end{aligned} \tag{2}$$

$$\begin{aligned} v_a(s) &= \left(R_2 + \frac{1}{sC_2}\right) i_2(s) \\ v_a(s) &= R_2 C_2 s v_o(s) + v_o(s) \end{aligned} \tag{3}$$

$$\begin{aligned} i_1(s) &= \frac{v_i(s) - v_a(s)}{R_1} \\ i_1(s) &= \frac{v_i(s) - R_2 C_2 s v_o(s) - v_o(s)}{R_1} \end{aligned} \tag{4}$$

$$\begin{aligned} i_3(s)\frac{1}{sC_1} &= v_a(s) - v_o(s) \\ i_3(s) &= sC_1\left[v_a(s) - v_o(s)\right] \\ i_3(s) &= R_2 C_1 C_2 s^2 v_o(s) \end{aligned} \tag{5}$$

$$\begin{aligned} & v_i(s) - R_2 C_2 s v_o(s) - v_o(s) = R_1 C_2 s v_o(s) + ... \\ & \quad + R_1 R_2 C_1 C_2 s^2 v_o(s) \\ & v_i(s) = v_o(s) + s v_o(s)\left(R_1 C_2 + R_2 C_2\right) + ... \\ & \quad + s^2 v_o(s)\left(R_1 R_2 C_1 C_2\right) \\ & H(s) = \frac{v_o(s)}{v_i(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + C_2\left(R_1 R_2\right) s + 1} \\ & H(s) = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{R_1 + R_2}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}} \end{aligned} \tag{6}$$

$$H(s) = \frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2} \tag{7}$$

$$\begin{aligned} \frac{\sqrt{2}}{2} &= \left|\frac{{\omega_0}^2}{(j\omega_c)^2 + 2\zeta\omega_0(j\omega_c) + {\omega_0}^2}\right| \\ \frac{\sqrt{2}}{2} &= \left|\frac{{\omega_0}^2}{(-{\omega_c}^2 + {\omega_0}^2) + j\left(2\zeta\omega_0\omega_c\right)}\right| \\ \frac{\sqrt{2}}{2} &= \frac{\sqrt{\left({\omega_0}^2\right)^2}}{\sqrt{\left({\omega_0}^2 - {\omega_c}^2\right)^2 + \left(2\zeta\omega_0\omega_c\right)^2}} \\ \frac{1}{2} &= \frac{{\omega_0}^4}{\left({\omega_0}^2 - {\omega_c}^2\right)^2 + \left(2\zeta\omega_0\omega_c\right)^2} \\ 2{\omega_0}^4 &= \left({\omega_0}^2 - {\omega_c}^2\right)^2 + \left(2\zeta\omega_0\omega_c\right)^2 \end{aligned} \tag{8}$$

$$Q = \frac{1}{2\zeta} \tag{9}$$

$$\begin{aligned} Q &= \frac{\sqrt{2}}{2} \\ 2\zeta &= Q^{-1} = \left(\frac{\sqrt{2}}{2}\right)^{-1} \\ 2\zeta &= \sqrt{2} \end{aligned} \tag{10}$$

$$\begin{aligned} 2{\omega_0}^4 &= \left({\omega_0}^2 - {\omega_c}^2\right)^2 + \left(\sqrt{2}\omega_0\omega_c\right)^2 \\ 2{\omega_0}^4 &= {\omega_0}^4 - 2{\omega_0}^2{\omega_c}^2 + {\omega_c}^4 + 2{\omega_0}^2{\omega_c}^2 \\ {\omega_0}^4 &= {\omega_c}^4 \\ \omega_c &= \omega_0 \end{aligned} \tag{11}$$

$$
\begin{aligned}
\omega_0^2 = \omega_c^2 &= \frac{1}{R_1 R_2 C_1 C_2} \\
\omega_0 = \omega_c &= \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&\omega_0 = 2\pi f_0 = \omega_c = 2\pi f_c \\
&2\pi f_0 = 2\pi f_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \\
&f_0 = f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&2\zeta\omega_0 = 2\zeta\left(2\pi f_c\right) = \frac{R_1 + R_2}{R_1 R_2 C_1} \\
&2\zeta = \frac{1}{2\pi f_c} \cdot \frac{R_1 + R_2}{R_1 R_2 C_1} \cdot \frac{C_2}{C_2} \\
&\frac{1}{2\zeta} = Q = \frac{2\pi f_c R_1 R_2 C_1 C_2}{C_2\left(R_1 + R_2\right)} \\
&Q = \frac{2\pi R_1 R_2 C_1 C_2}{C_2\left(R_1 + R_2\right)} \cdot \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \cdot \frac{2\pi}{2\pi} \\
&Q = \frac{{f_c}^{-2} f_c}{2\pi C_2\left(R_1 + R_2\right)} \\
&Q = \frac{1}{2\pi f_c C_2\left(R_1 + R_2\right)}
\end{aligned} \tag{14}
$$

Equações do software:

$$
\alpha = R_1 R_2 C_1 C_2 = \frac{1}{4\pi^2 {f_c}^2} \tag{15}
$$

$$
\begin{aligned}
\beta = C_2\left(R_1 + R_2\right) &= \frac{1}{2\pi f_c Q} \\
\beta = C_2\left(R_1 + R_2\right) &= \frac{1}{2\pi f_c} \cdot \frac{2}{\sqrt{2}} \\
\beta = C_2\left(R_1 + R_2\right) &= \frac{\sqrt{2}}{2\pi f_c}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\gamma &= \frac{R_1 + R_2}{R_1 R_2 C_1} = \frac{\omega_0}{Q} = \frac{2\pi f_c}{Q} \\
\gamma &= \frac{R_1 + R_2}{R_1 R_2 C_1} = 2\pi f_c \frac{2}{\sqrt{2}} \\
\gamma &= \frac{R_1 + R_2}{R_1 R_2 C_1} = 2\sqrt{2}\pi f_c
\end{aligned} \tag{17}
$$

$$
e_x = \frac{\left|x - x_0\right|}{x_0} \leqslant \delta \tag{18}
$$

$$
E = \sqrt{{e_\alpha}^2 + {e_\beta}^2 + {e_\gamma}^2} \leqslant \delta \tag{19}
$$