Nota: Revisar 'Tapa' del Jackson.

(1) $\nabla(\vec{r}\cdot\vec{p}) = \vec{p} + \vec{r}(\nabla\cdot\vec{p}) + i(\vec{L}\times\vec{p})$

$\frac{1}{i}\vec{r}\times\cancel{(\nabla\times\vec{p})} \to \emptyset$

$\times\vec{E}$

$\nabla\times E = -\frac{\partial B}{\partial t} = \emptyset$

(2) $\int_S ds\ \vec{r}\,(\hat{n}\cdot\vec{p}(r)) = \int_S ds\ \hat{n}\left[\vec{r}\cdot\vec{p}(r)\right] = \int_S ds\ r\,\sigma_b(\vec{r})$

$$\int_V \nabla \psi \, d^3 r = \int_S \psi \, \hat{n} \, da$$ ← Teorema de la divergencia

$$\downarrow$$

$$\psi \to (\vec{r} \cdot \vec{p})$$

$$\int_V \nabla (\vec{r} \cdot \vec{p}) \, d^3 r = \int_S (\vec{r} \cdot \vec{p}) \, \hat{n} \, da$$

$$\cancel{= \int_S (\hat{n} \cdot \vec{p}) \, \hat{n} \, da}$$

$$\cancel{= \int_S (\hat{n} \cdot \vec{p}) \, \hat{n}}$$

como $\hat{n} = \dfrac{\vec{r}}{|r|} \to$

$$= \int_S (\vec{r} \cdot \vec{p}) \frac{\vec{r}}{|r|} \, da$$

$$= \int_S \left(\frac{\vec{r}}{|r|} \cdot \vec{p} \right) \vec{r} \, da$$

$$= \int_S (\hat{n} \cdot \vec{p}) \, \vec{r} \, da = \int_S \sigma_b \, \vec{r} \, da$$

Hasta aquí tenemos:

$$\int_V \nabla(\vec{r}\cdot\vec{P})\, d^3r = \int \sigma_b\, \vec{r}\, da$$

usando la ecuación 1:

$$\int_V \nabla(\vec{r}\cdot\vec{P})\, d^3r = \int_V \left[\vec{P} + \vec{r}\,(\nabla\cdot\vec{P})\right] d^3r$$

$$= \int_S \sigma_b\, \vec{r}\, da$$

despejando la integral de $\vec{P}$ sobre V:

$$\int_V \vec{P}\, d^3r = \int_S \sigma_b\, \vec{r}\, da - \int_V \vec{r}\,(\nabla\cdot\vec{P})\, d^3r$$

$$= \int_S \sigma_b\, \vec{r}\, da + \int_V \vec{r}\, \rho_b\, d^3r \quad //$$