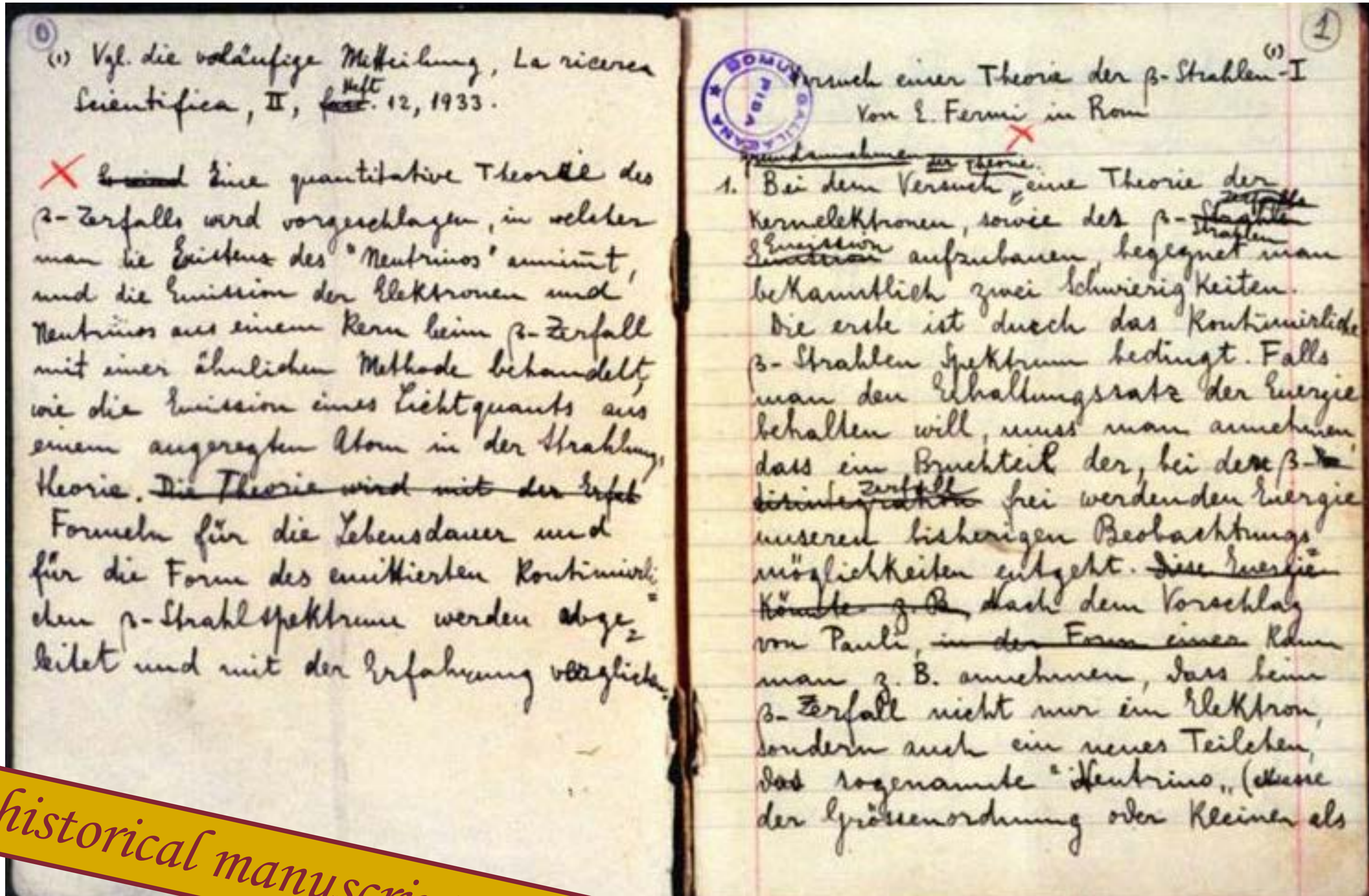
WEAK INTERACTIONS

The weak interactions: the origins



(1) Vgl. die vorläufige Mitteilung, La ricerca Scientifica, II, Heft 12, 1933.

~~Es wird~~ Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des "Neutrinos" annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. ~~Die Theorie wird mit der Erfahr~~ Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlspektrums werden abgeleitet und mit der Erfahrung verglichen.

Versuch einer Theorie der β-Strahlen. I (1)

Von E. Fermi in Rom

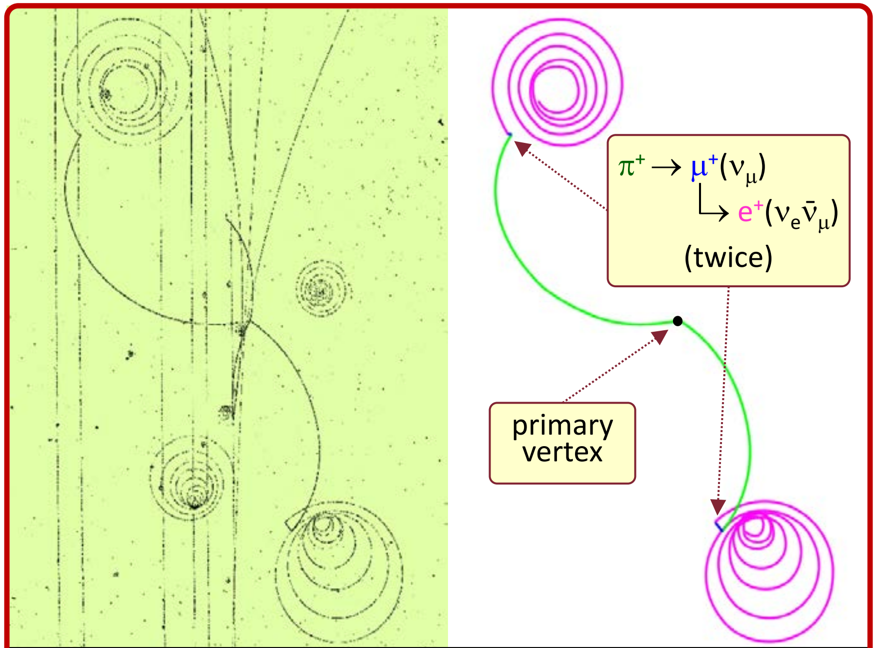
~~Grundannahmen der Theorie.~~

1. Bei dem Versuch, eine Theorie der Kernelektronen, sowie der β-Zerfalls ~~Strahlen~~ Emission aufzubauen, begegnet man bekanntlich zwei Schwierigkeiten. Die erste ist durch das kontinuierliche β-Strahlen Spektrum bedingt. Falls man den Erhaltungssatz der Energie behalten will, muss man annehmen, dass ein Bruchteil der, bei dem β-Zerfall frei werdenden Energie unseren bisherigen Beobachtungsmöglichkeiten entgeht. ~~Diese Energie könnte z. B.~~ Nach dem Vorschlag von Pauli, ~~in der Form eines~~ kann man z. B. annehmen, dass beim β-Zerfall nicht nur ein Elektron, sondern auch ein neues Teilchen, das sogenannte "Neutrino" (Masse der Grössenordnung oder kleiner als

a historical manuscript [thanks to F. Guerra]

The weak interactions: introduction

- Some rare processes, i.e. small coupling, violate the conservation laws, valid for strong and electromagnetic interactions.
- In ordinary matter the **weak interactions** (w.i.) have a negligible effect, except in cases otherwise forbidden (e.g. β decay).
- The w.i. are responsible for the fact that STABLE matter contains only u and d quarks and electrons. Other quarks and leptons are UNSTABLE because of w.i..
- Therefore, in spite of their "weakness" (small range of interaction $\approx 10^{-3}$ fm, tiny cross sections $\approx 10^{-47}$ m^2), the w.i. play a crucial role in the features of our world.
- ALL elementary particles, but ~~gluons~~ and ~~photons~~ (carriers of other interactions), are affected by w.i. : quarks and charged leptons have w.i., ν's have ONLY them.
- In the scattering processes of charged hadrons and leptons, the effects due to the strong and electromagnetic interactions "obscure" those of the w.i..
- Therefore most of our knowledge on this subject, at least until the '70s, has been obtained from the study of the decays of particles and from ν beams.



CERN 2m hydrogen bubble chamber: $K^+p \to \pi^+\pi^+X$

The weak interactions: introduction

1930 Pauli : ν existence to explain β–decay.

1933 Fermi : first theory of β–decay.

1934 Bethe and Peierls : νN and $\bar{\nu} N$ cross sections.

1936 Gamow and Teller : G.-T. transitions.

1947 Powell + Occhialini : decay $\pi^+ \to \mu^+ \to e^+$.

1956 Reines and Cowan : ν's detection from a reactor.

1956 Landè, Lederman and coll. : $K^0{}_L$.

1956 Lee and Yang : parity non-conservation.

1957 Feynman and Gell-Mann, Marshak and Sudarshan : V–A theory.

1958 Goldhaber, Grodzins and Sunyar : ν helicity.

1960 (ca) Pontecorvo and Schwarz : ν beams.

1961 Pais and Piccioni : $K_L \leftrightarrow K_S$ regeneration.

1962 First ν beam from accelerator : Lederman, Schwarz, Steinberger : ν_μ.

1963 Cabibbo theory.

1964 Cronin and Fitch : CP violation in K^0 decay.

1964 Brout, Englert, Higgs : Higgs mechanism.

1968 Weinberg–Salam model.

1968 Bjorken scaling, quark-parton model.

1970 GIM mechanism.

1972 Kobayashi, Maskawa : CKM matrix.

1973-90 ν DIS experiments : Fermilab, CERN.

1973 CERN Gargamelle : neutral currents.

1983 CERN Sp$\bar{p}$S : $W^\pm$ and Z.

1987 CERN Sp$\bar{p}$S : B^0 mixing discovery.

1989-95 CERN LEP : Z production + decay.

1997-2000 CERN LEP : W^+W^- production.

1998-2000 ν oscillations.

1999-20xx B^0 mixing detailed studies.

2012 CERN LHC : Higgs boson.

- **only major facts ≥ 1930 considered;**

The weak interactions: introduction

- Let's recall the life time of a few decays:

$\Delta^{++} \rightarrow p\pi$	$\sim 10^{-23}$ s	strong int.
$\Sigma^0 \rightarrow \Lambda\gamma$	$\sim 6 \cdot 10^{-20}$ s	1 γ, e.m. int.
$\pi^0 \rightarrow \gamma\gamma$	$\sim 10^{-16}$	2 γ, e.m. int.
$\Sigma \rightarrow n\pi$	$\sim 10^{-10}$ s	weak int.
$\pi^- \rightarrow \mu^- \nu_\mu$	$\sim 10^{-8}$ s	weak int.
$\mu^- \rightarrow e^- \nu_e \nu_\mu$	$\sim 10^{-6}$ s	weak int.
$n \rightarrow p\, e^- \nu_e$	$\sim$ 15 min	weak int.

N.B. we observe the weak interactions only when the strong and e.m. interactions are forbidden.

- We need to explain the enourmos range in the life time going from 10^{-12} s untill 15 min.
- The weak interactions are also characterized by cross-sections extremely small ($\sim 10^{-39}$ cm^2=1 fb)

$$\sigma(\nu_\mu + N \rightarrow N + \pi + \mu) = 10^{-38}\ \text{cm}^2\ (10\ \text{fb})\ \text{a}\ 1\ \text{GeV}$$

$$\sigma(\pi + N \rightarrow N + \pi) = 10^{-26}\ \text{cm}^2\ (10\ \text{mb})\ \text{a}\ 1\ \text{GeV}$$

- The weak interactios violate many conservation rules (parity, charge conjugation, strangeness, etc...)
- Because of their "weakness", the weak interactions can be observed in the "standard" matter only in the beta decay, because they do not give origin to any bound states. However they are the base fuel for the stars functioning , therefore without the weak interactions we coud not exist

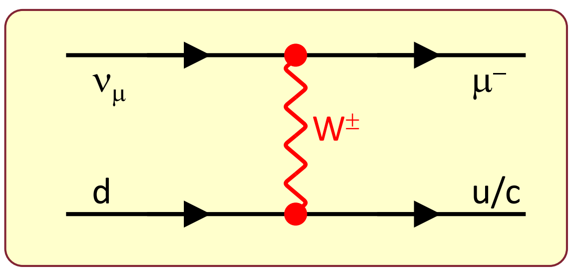
$$p + p \rightarrow d + e^+ + \nu_e$$

The weak interactions: CC and NC

In the SM, weak interactions (w.i.) are classified in two types, according to the charge of their carriers :

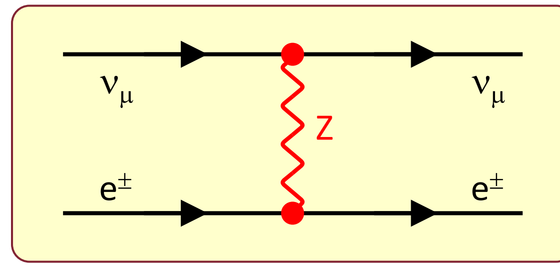
- <u>Charged currents</u> (**CC**), **W$^{\pm}$** exchange:
  - in the CC processes, the charge of quark and leptons CHANGES by ± 1; at the same time there is a variation of their IDENTITY, including FLAVOR, according to the Cabibbo theory (today Cabibbo-Kobayashi-Maskawa)

e.g.



- <u>Neutral currents</u> (**NC**), **Z** exchange:
  - in the NC case, quarks and leptons remain unchanged (no **FCNC**);
  - until 1973 no NC weak process was observed [*but another example of NC was well known, i.e. the e.m. current: γ's carry no charge !*]

e.g.

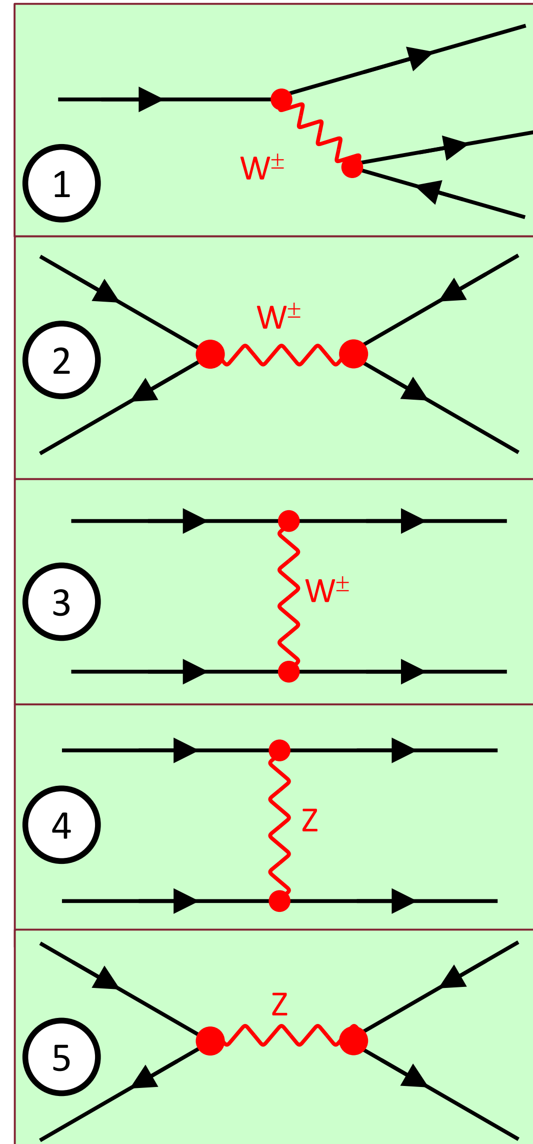


- In the 60's Glashow, Salam and Weinberg (+ many other theoreticians) developed a theory (today known as the "Standard Model", SM), that unifies the w.i. (both CC and NC) and the electromagnetism.

The SM was conceived BEFORE the discovery of NC. So the existence of NC and its carrier (the Z boson), predicted by the SM and observed at CERN in 1973 and 1983 respectively, were among the first great successes of the SM.

The weak interactions: classification

weak interactions	CC	leptonic	$\Delta S = 0$	$\mu \to e\ \nu_e\ \nu_\mu$	1
		semi-leptonic		$\pi^\pm \to \mu^\pm\ \nu_\mu$	2
				$n \to p\ e\ \nu_e$	1*
				$\nu_e\ d \to e^-\ u$	3*
				$d\bar{u} \to W^- \to\ e^-\bar{\nu}_e$	2*
			$\Delta S = \pm 1$	$K^\pm \to \mu^\pm\ \nu_\mu$	2
				$\Lambda \to p\ e\ \nu_e$	1*
		hadronic		$K^\pm \to \pi^\pm\ \pi^0$	2*
				$\Lambda \to p\ \pi^-,\ n\ \pi^0$	1*
	NC	leptonic	$\Delta S = 0$ (only)	$\nu_\mu\ e^\pm \to \nu_\mu\ e^\pm$	4
		semi-leptonic		$\nu\ N \to \nu\ N'$	4*
		hadronic		$u\ \bar{u} \to Z \to q\ \bar{q}$	5*



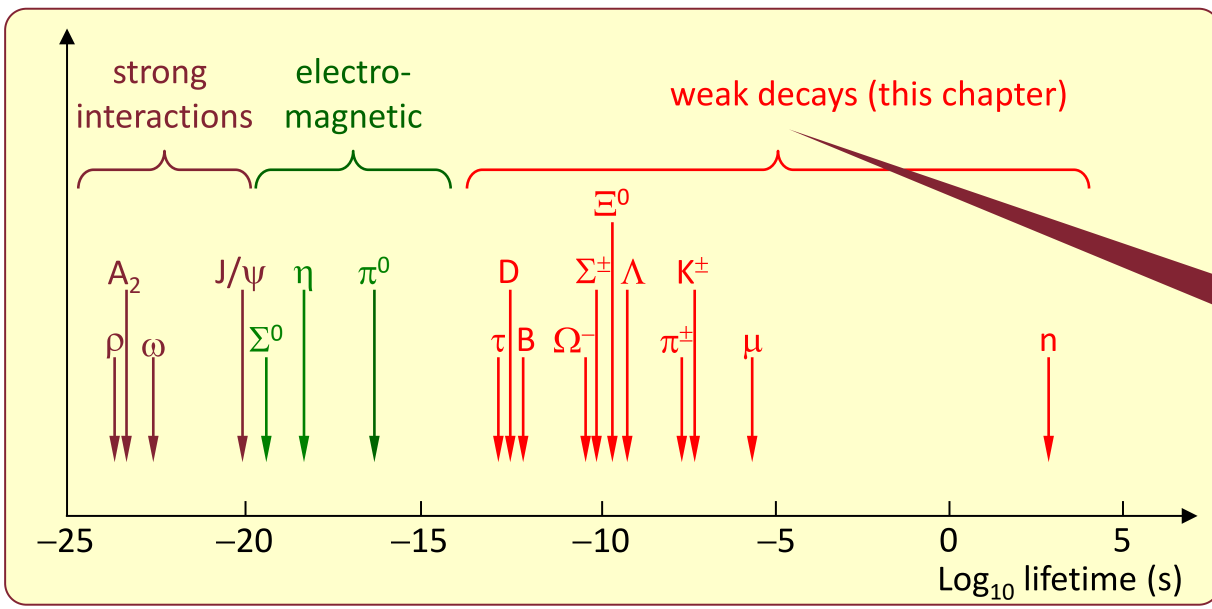
Some processes (list NOT exhaustive), classified in terms of general characteristics and Feynman diagrams.

A "*" in the last column means that the interacting hadron is composite; the diagrams shows only the interacting quark(s); the other partons (the "spectators") do not participate in the interaction, at least in 1st approximation.

In the table, ν means both ν and $\bar{\nu}$ [*only the correct one !*].

Charged Current: decays

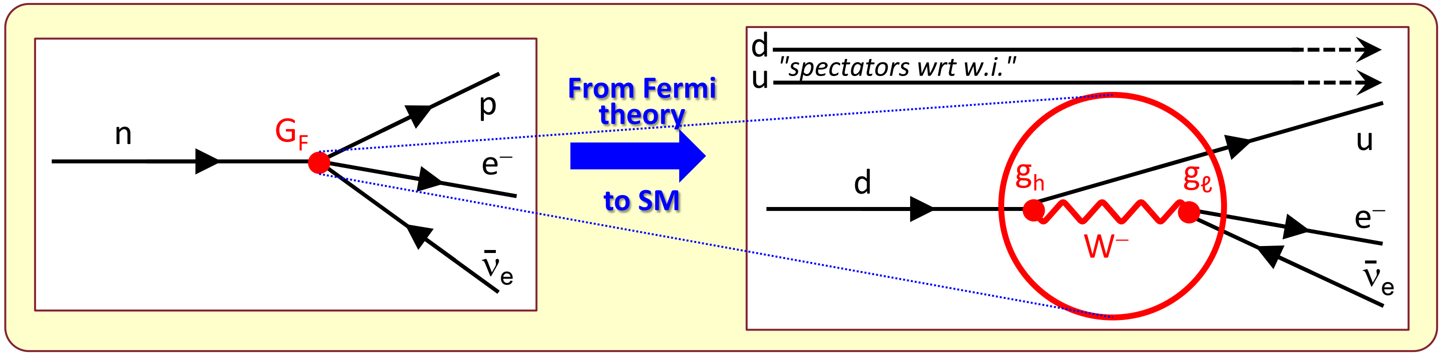
process	Lifetime (s)	comment
$\bar{\nu}_e\, p \to n\, e^+$	(none)	Neutrinos have only weak interactions (not a decay).
$n \to p\, e^-\, \bar{\nu}_e$	$\mathcal{O}(10^3)$	Long lifetime because of small mass difference (p-n).
$\pi^+ \to \mu^+\, \nu_\mu$	$\mathcal{O}(10^{-8})$	The $\pi^\pm$ is the lightest hadron, so it decays $\to$ leptons.
$\Lambda \to p\, \pi^-$	$\mathcal{O}(10^{-10})$	The decay of Λ violates strangeness conservation.



Some of the most interesting weak decays are the neutral heavy mesons of type $Q\bar{Q}$ (K^0, B^0).

Charged Current: Fermi theory

- The modern theory of the CC interactions (i.e. this part of the SM) is a successor of the Fermi theory of β decay.
- The Fermi theory describes a point-like interaction, proportional to the coupling G_F; the theory had intrinsic problems ("not renormalizable" in modern terms, i.e. cross-sections violate unitarity at high energy);
- the SM "expands" the point-like interaction, introducing a heavy charged mediator, called $W^{\pm}$.
- the SM is mathematically consistent (it is "renormalizable");
- *(more important)* it reproduces the experimental data with unprecedented accuracy.

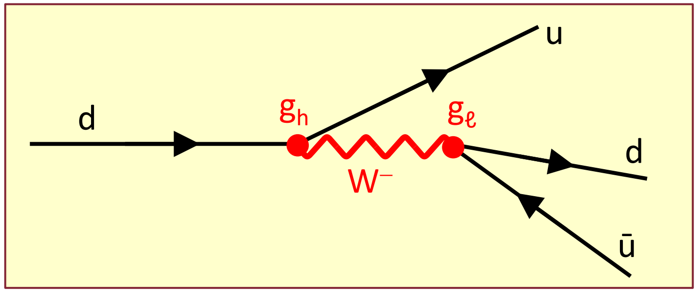


usual comment : to see a smaller scale requires higher Q^2 → higher energy

Charged Current: simple problems

Q. why is the decay $n \to p\pi^-$ (similar to $\Delta^0 \to p\pi^-$) forbidden ?

A. write the Feynman diagram



- possible ? forbidden ?

  yes, possible

- then ?

  $m(n) - m(p) \approx 1.3$ MeV

  The only possible pair ff' with $q = -1$ and baryon/lepton number = 0 is clearly $e^-\bar{\nu}_e$, since $m(e^-) + m(\bar{\nu}_e) \approx m(e^-) \approx 0.5$ MeV.

Q. why $n \to pe^-\bar{\nu}_e$ and not $p \to ne^+\nu_e$?

A. [... left to the reader]

Charged Current: coupling

A simple comparison between the couplings (g is the "charge" of the w.i. and plays a similar role as e):

- Electromagnetism :

  $\alpha \quad \propto e^2;$

  amplitude $\propto \alpha \propto e^2;$

  rate $\propto \alpha^2 \propto e^4.$

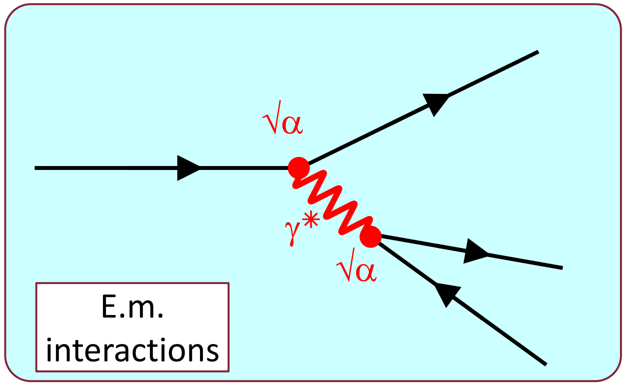
- Weak interactions :

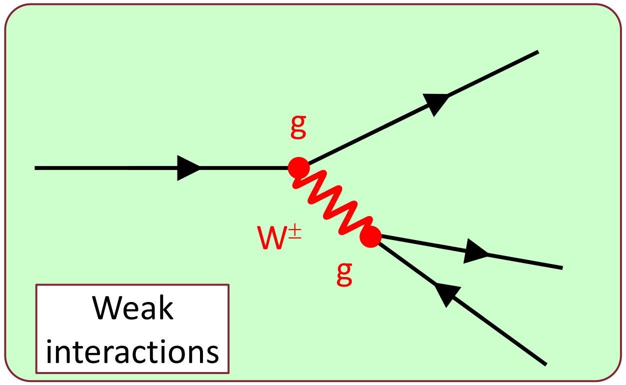
  $G_F \quad \propto g^2;$

  amplitude $\propto G_F \propto g^2;$

  rate $\propto G_F^2 \propto g^4;$

NB. unlike α, G_F is not adimensional (next slide); the <u>similarity</u> electromagnetism $\leftrightarrow$ weak interactions is hidden.





The weak interactions: beta decay

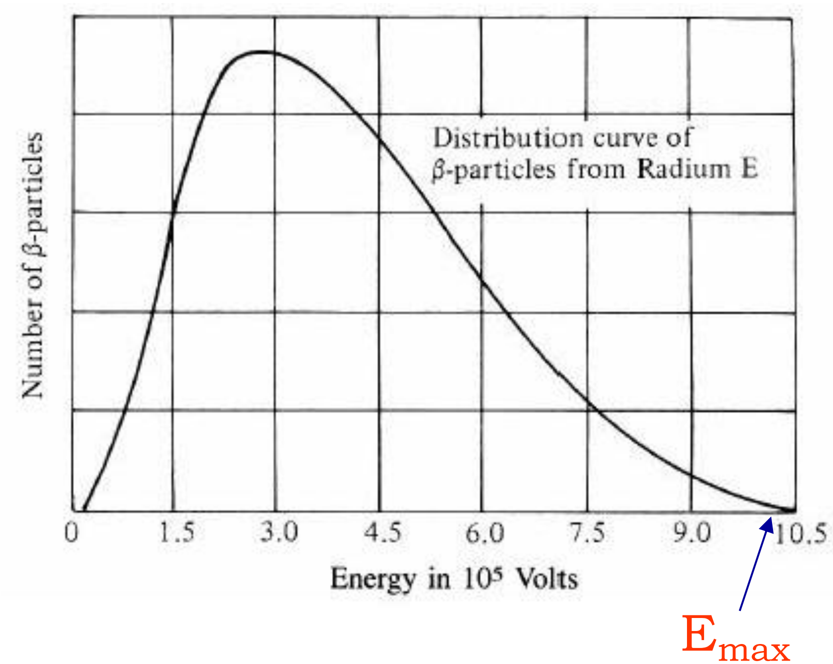
Most of our knowledge about the base principles of beta decay is based upon nuclei beta decays.

$$n \rightarrow p + e^- + \bar{\nu}_e \quad \Rightarrow \quad (A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e$$

$$p \rightarrow n + e^+ + \nu_e \quad \Rightarrow \quad (A,Z) \rightarrow (A,Z-1) + e^+ + \nu_e$$

$$e^- + p \rightarrow n + \nu_e \quad \Rightarrow \quad (A,Z) + e^- \rightarrow (A,Z-1) + \nu_e$$

- The existence of the β^+ decay was established in 1934 by Curie and Joliot.
- In 1919 Chadwick discovered that the electron in the β decay had a continuos spectrum.



- The maximum energy of the spectrum corresponds fairly well to the Q of the reaction (Q=M(A,Z)-M(A,Z+1)), while for the rest of the spectrum there is a violation of the energy conservation rule.

- Moreover there is also a violation of the momentum and angular momentum conservation rules (without the introduction of the neutrino).

The weak interactions: neutrino

- To re-establish the various conservation laws, in 1930 Pauli made the hypothesis of the existence of a very small neutral particle: the neutron (later renamed neutrino by Fermi).

December 1930: public letter sent by W. Pauli to a physics meeting in Tübingen

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the "wrong" statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin ½ and obey the exclusion principle ..... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous β-spectrum would then become understandable by the assumption that in β-decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant.

.... For the moment, however, I do not dare to publish anything on this idea ......

So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December. ....

W. Pauli

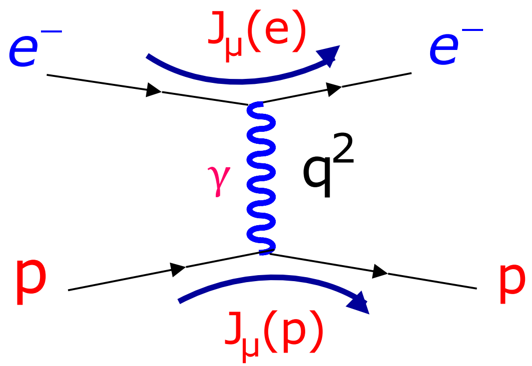
NOTES

- Pauli's neutron is a light particle ⇒ not the neutron that will be discovered by Chadwick one year later
- As everybody else at that time, Pauli believed that if radioactive nuclei emit particles, these particles must exist in the nuclei before emission

- This letter is very important for Physics … but it is also interesting from a sociological point of view ☺
- The first theory of β decay was done by Fermi in 1934.
- The neutrino was discovered by Reines e Cowan "only" in 1958
- We have three kind of neutrinos; recently have been established the flavour neutrino oscillations that imply that neutrinos have masses different from zero, although they are very small and not yet measured.

Fermi theory of β interactions

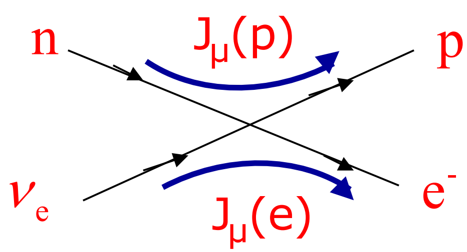
- In 1934 Fermi did the first theory of β decay; he took as a model the QED description of the electron-proton scattering:



The matrix element is proportional to:

$$\boxed{M_{fi} \approx -\frac{1}{q^2} J_\mu(e) J^\mu(p)} \qquad M_{fi} \approx \left(\bar{u}_e \sqrt{\alpha}\gamma^\mu u_e\right) \frac{g_{\mu\nu}}{q^2} \left(\bar{u}_p \sqrt{\alpha}\gamma^\nu u_p\right)$$

- Fermi made the hypothesis of a pointlike interaction like: $n+\nu \to p+e^-$ (that is like $n \to \bar{\nu}+p+e^-$)



→ There is no propagator

$$\boxed{M_{fi} \approx G\left(\bar{u}_p \gamma^\mu u_n\right)\left(\bar{u}_e \gamma_\mu u_\nu\right)}$$

vector-vector interaction

The G constant is known as Fermi's constant and it is related to the square of the "weak charge".

interaction between two (charged) currents: hadronic and leptonic currents.

$\bar{u}_p$ creates a proton (or destroys an antiproton)
u_n destroys a neutron (or creates an antineutron)
$\bar{u}_e$ creates an electron (or destroys a positron)
u_ν destroys a neutrino (or creates an antineutrino)

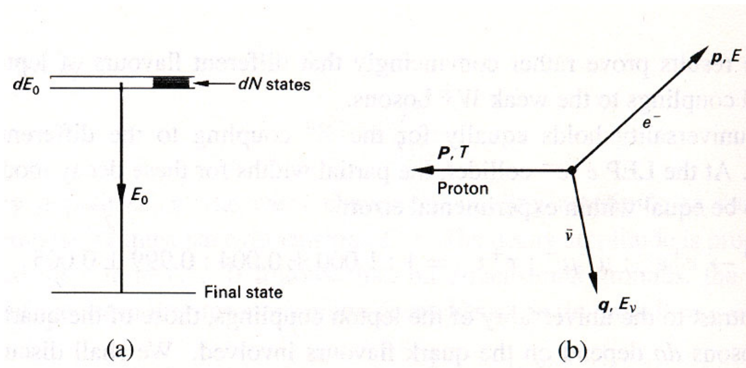
Nuclear β decay

- The transition probability (the decay rate per unit of time) can be found by using the Fermi's golden rule:

$$W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dN}{dE_0}$$

$\frac{dN}{dE_0}$: phase space

$|M|^2$ is the matrix element squared. It is computed by integrating over all angles of the final particles, by summing over the final spin states and by averaging on the initial spin states. It is a constant of order one.



Fermi decays: $J_{leptoni} = 0 \Rightarrow |M|^2 \approx 1$

Gamow-Teller decays: $J_{leptoni} = 1 \Rightarrow |M|^2 \approx 3$

- E_0 is the energy available in the final state (it is equal to the Q of the reaction). The energy spread dE_0 is present because the energy of the initial state is not precisely known due to the finite lifetime (Heisemberg's principle).

$$\vec{P} + \vec{q} + \vec{p} = 0 \qquad T + E_\nu + E = E_0$$

- In the nuclear β decays E_0 is of the order of 1 MeV. The proton kinetic energy is of the order of 10^{-3} MeV and can be neglected. The proton is there just to ensure the momentum conservation.

$q_\nu = E_0 - E_e$ The energy is shared between the electron and the neutrino

The phase space

- The number of available states for an electron with momentum between p and p+dp, confined in the volume V, within the solid angle dΩ, is:

$$dN = \frac{V d\Omega}{(2\pi)^3 \hbar^3} p^2 dp$$

- We normalize the wave function to V=1, we sum over the entire solid angle and we ignore the effect of the spin on the angular distribution. We get the following phase space for the electron and neutrino:

$$dN_e = \frac{4\pi p^2 dp}{(2\pi)^3 \hbar^3} \quad ; \quad dN_\nu = \frac{4\pi q_\nu^2 dq_\nu}{(2\pi)^3 \hbar^3}$$

- The two phase space factors are independent because there is no correlation between q and p, since it is a three bodies decay the proton will absorb the remaining momentum difference. The proton momentum is fixed (given q and p) so there is no proton phase space factor.

- The number of final states is: $dN = dN_e \cdot dN_\nu = \frac{(4\pi)^2}{(2\pi)^6 \hbar^6} p^2 q_\nu^2 dp dq_\nu$

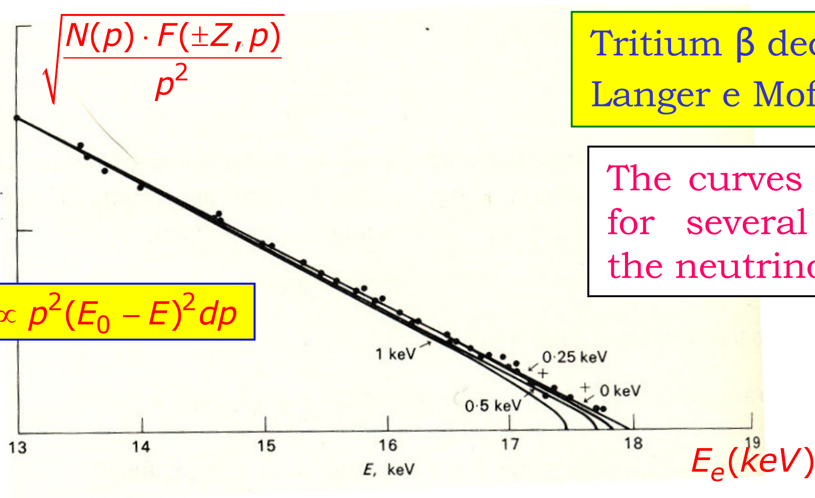
- For a given value of the electron energy E, the neutrino energy E_ν is fixed as well as its momentum:

$$q_\nu \equiv E_\nu = (E_0 - E) \quad ; \quad \Rightarrow dq_\nu = dE_0 \quad \Rightarrow \frac{dN}{dE_0} = \frac{dN}{dq_\nu} = \frac{1}{4\pi^4 \hbar^6} p^2 (E_0 - E)^2 dp$$

- Once we have integrated the transition probability W over the entire solid angle, M^2 is equal to a constant, therefore the electron energy spectrum is entirely due to the phace space form:

$$\boxed{N(p)dp \propto p^2 (E_0 - E)^2 dp}$$

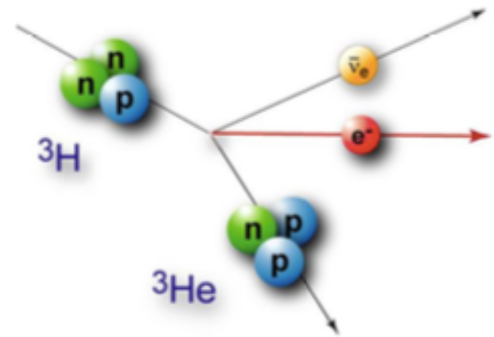
Kurie plot



$N(p)dp \propto p^2(E_0 - E)^2 dp$

Tritium β decay.
Langer e Moffatt (1952)

The curves are plotted for several values of the neutrino mass.



$E_0 = 18.6\ keV$ $\qquad T_{1/2} = 12.3$ years

- If we plot $(N(p)/p^2)^{1/2}$ versus the electron energy, we get a straight line that crosses the x-axis at $E=E_0$. This graph used to study β decay was developed by Franz N.D. Kurie.
- Experimentally we need to include a correction factor F(Z,p) to take into account the Coulomb interaction between the electron and the nucleus.
- If the neutrino has a mass, its effect would be to modify the distribution in the following way:

$$N(p)dp \propto p^2(E_0 - E)^2 \sqrt{1 - \left(\frac{m_\nu}{E_0 - E}\right)^2}\, dp$$

- The Kurie plot is modified in a way that the curve crosses the x-axis at $E=E_0-m_\nu$. This is how we try to measure the neutrino-e mass. Unfortunately in this region there are very few events and it is very difficult to perform the experiment. At the moment we have only an upper limit.

$m_{\nu_e} \leq 2.2$ eV $\qquad$ Mainz exp. ; 2000

The Sargent's rule

- The total decay rate is obtained by integrating the spectrum N(p)dp. It can be done analitically; however in the cases where the electron is relativistic we can use the approximation p≈E and we get a very simple formula:

$$\boxed{N \propto \int_0^{E_0} E^2 (E_0 - E)^2 dE \propto E_0^5}$$

- The decay rate is proportional to the fifth power of the energy available in the process. This is the **Sargent's rule.**

- If we consider all the numerical factors in the process, we get: $W = \dfrac{G^2 |M|^2 E_0^5}{60\pi^3 (\hbar c)^6 \hbar}$ (per $E_0 >> m_e$)

- The Fermi's constant G can be found, as we will see later, from the life time measurements of a few β decays (and with some theoretical speculations, see Cabibbo's angle) or in a more precise way from the muon life time.

- From the PDG we get: $\boxed{\dfrac{G}{(\hbar c)^3} = 1.16637(1) \cdot 10^{-5}\ \text{GeV}^{-2}}$

- Replacing the numerical values in the formula we get: $\dfrac{1}{\tau} = W = \dfrac{1.11}{10^4} |M|^2 E_0^5\ \text{s}^{-1}$ (E_0 in MeV)

- For instance, if E_0≈100 MeV like in the muon decay and M^2=1, we get: $\tau_\mu = \dfrac{1}{W} \approx 10^{-6}$ s (τ_μ=2.2 μs)

N.B. it is the E_0^5 dependence that explains the huge range in the life time of the decays mediated by the weak interactions.

The nuclear β decay

- The nuclear β decays can be discriminated as allowed transitions and forbidden transitions.
- The allowed ones are the most common and are characterized by the fact that the electron and neutrino emitted DOES NOT carry any spatial angular momentum, that is they are in the S-state (L=0). This is justified by the fact that the two leptons have energies of the order a 1 MeV.
- The transitions with L=1 are called first forbidden, the ones with L=2 second forbidden and so on. They have a lifetime considerably longer that the allowed transitions.
- Since e and ν have spin ½ , the nucleus spin change can be either 0 or 1. The transitions with ΔJ=0 are called Fermi transitions while the ones with ΔJ=1 are called Gamow-Teller transitions.

transitions	Δ J nucleus	Leptonic state	
Fermi	$\Delta J=0$	singlet	$\uparrow\downarrow$
Gamow-Teller	$\Delta J=1$ $\Delta J_z=0,\pm 1$	triplet	$\uparrow\uparrow$ $\uparrow\downarrow$ $\downarrow\downarrow$

- Since e-ν have L=0, there is no change in the spatial angular momentum of the nucleus, therefore its parity will not change. The nuclues undergoes a spin flip for the G.T. transitions.

Fermi: $0^+ \to 0^+, \Delta\vec{J} = 0$	**G.-T. :** $1^+ \to 0^+, \Delta\vec{J} = 1$	**Mixed:** $\frac{1}{2}^+ \to \frac{1}{2}^+, \Delta\vec{J} = 0,1$
$^{10}C \to {}^{10}B^* e^- \bar{\nu}_e$ $^{14}O \to {}^{14}N^* e^+ \nu_e$	$^{12}B \to {}^{12}C e^- \bar{\nu}_e$	$n \to p e^- \bar{\nu}_e$

The Fermi constant

- The decay rate can be written in a different way with respect to the Sargent rule. We write explicitly the proton mass m, then we include the phase space factors and the Coulombian correction F(±Z,p) in a dimensionless function $f(\pm Z, E_0/m_e)$ that can be computed analitically.

$$\frac{1}{\tau} = W = \frac{(mc^2)^5}{2\pi^3\hbar(\hbar c)^6} G^2 |M|^2 f(\pm Z, E_0)$$

$$G^2 |M|^2 = \frac{\text{constant}}{f \cdot \tau} \qquad (\text{constant} = \frac{2\pi^3}{m^5})$$

decadimento	*transizione*	τ (s)	W (E_0 (MeV))	p_e^{max}	$f\ \tau$	$g^2\ \lvert M_{if}\rvert^2$ ($MeV^2 \cdot fm^6$)
$n \to p\ e^-\ \bar\nu$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	890	1.29	1.18	$1.61\ 10^3$	$4.25\ 10^{-8}$
${}^3_1H \to {}^3_2He\ e^-\ \bar\nu$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	$5.60\ 10^8$	0.53	0.14	$1.63\ 10^3$	$4.20\ 10^{-8}$
${}^{14}_8O \to {}^{14}_7N^*\ e^+\ \nu$	$0^+ \to 0^+$	102	2.32	2.26	$4.51\ 10^3$	$1.52\ 10^{-8}$
${}^{34}_{17}Cl \to {}^{34}_{16}S\ e^+\ \nu$	$0^+ \to 0^+$	2.21	4.97	4.94	$4.54\ 10^3$	$1.51\ 10^{-8}$
${}^6_2He \to {}^6_3Li\ e^-\ \bar\nu$	$0^+ \to 1^+$	1.15	4.02	3.99	$1.17\ 10^3$	$5.85\ 10^{-8}$
${}^{13}_5B \to {}^{13}_6C\ e^-\ \bar\nu$	$\frac{3}{2}^- \to \frac{1}{2}^-$	$2.51\ 10^{-3}$	13.4	13.4	$1.11\ 10^3$	$6.17\ 10^{-8}$

- In spite of the big variations of lifetime due to the strong dependency of the function f from p_e^{max}, the product G^2M^2 is about the same in all decays.
- However we observe a small difference due to the type of nuclear transition: Fermi, Gamow-Teller or mixed transitions.
- If we consider a pure Fermi transition, we get: $$\frac{G}{(\hbar c)^3} = 1.140(2) \cdot 10^{-5}\ \text{GeV}^{-2}$$

that is slightly different from the one quoted by the PDG taken from the muon decay. We will see later the reason of such a discrepancy (Cabibbo's angle).

Charged Current: effect of m_W on coupling

- The e.m. coupling constant α is proportional to the square of the electric charge e :

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}.$$

- In a similar way, the intensity of the CC is G_F (Fermi constant), proportional to the square of the "weak charge" g.

- The matrix elements of the transitions are proportional to the square of the "weak charge" g and to the propagator :

$$\mathcal{M}_{fi} \propto g \frac{1}{Q^2 + m_W^2} g \xrightarrow{Q^2 << m_W^2} \frac{g^2}{m_W^2} \equiv G_F.$$

- The difference respect to the e.m. case is the mass of the carrier: while the γ is massless, the CC carrier is the $W^\pm$, a massive particle of spin 1. Therefore the range of CC turns out to be small ($1/m_W$).

- Unlike the case of the massless photon, for small Q^2 the propagator term "stays constant".

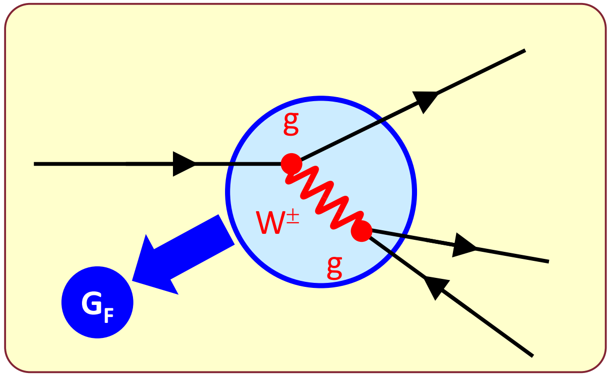
- Therefore the Fermi constant G_F has dimensions :

$$[G_F] = [m_W^{-2}] = [m^{-2}] = [\ell^2],$$

- and a small value, due to m_W :

$$\frac{G_F}{(\hbar c)^3} = O\left(10^{-5}\,\mathrm{GeV}^{-2}\right) = O\left[(10^{-3}\,\mathrm{fm})^2\right].$$

- This effect obscures the similarity of the e.m. and weak charges ($e \leftrightarrow g$), which are indeed of the same order.



Charged Current: G_F

- the most precise value of the Fermi constant G_F is measured by considering the muon decay $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$:
  - low energy process ($\sqrt{Q^2} \approx m_\mu << m_W$);
  - approximated by a four-fermion point-like process, determined by the Fermi constant ($\approx g^2/m_W^2$);
  - only leptons $\rightarrow$ free from hadronic interactions which affect other processes, e.g. the nuclear β decays.
- if $m_e \approx 0$, m_μ is the only scale of the decay $\rightarrow$ dimensional analysis:

$$\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = 1/\tau_\mu \propto G_F^2\, m_\mu^5,$$

- while the correct computation gives :

$$\Gamma\left(\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\varepsilon),$$

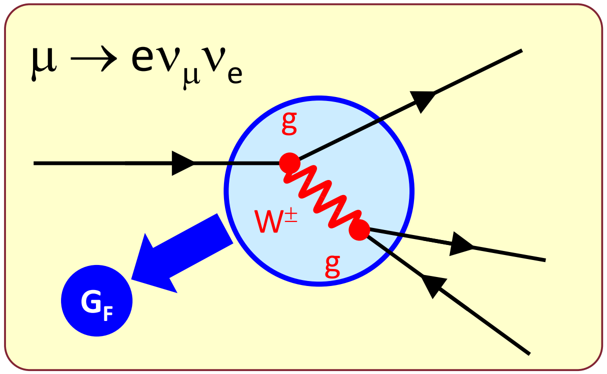
where ε is small and depends on the radiative corrections and on the electron mass.

- the mass of the muon and its average lifetime were measured with great precision:

$$m_\mu = (105.658389 \pm 0.000034) \text{ MeV};$$
$$\tau_\mu = (2.197035 \pm 0.000040) \times 10^{-6} \text{ s}.$$

- then the value of the Fermi constant is

$$G_F = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}.$$



lepton universality : (τ→e) ↔ (τ→μ)

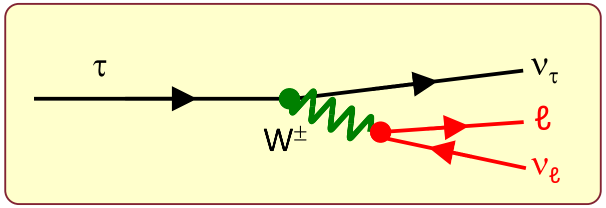
Q. Is the weak CC the same for all leptons and quarks ? Do they share the same coupling constant G_F for all the processes ?

- the **CC universality** has received extensive tests.
- [*absolutely true for leptons, some further refinement* – CKM – *for quarks*]
- The **e–μ universality** is measured by analyzing the leptonic decays of the $\tau^\pm$ (ℓ^- is the appropriate lepton, e^- / μ^-) :

$$\Gamma\left(\tau^- \to \ell^- \overline{\nu}_\ell \nu_\tau\right) \equiv \Gamma^\tau_\ell = \frac{g_\tau^2 g_\ell^2}{m_W^2 m_W^2} m_\tau^5 \rho_\ell;$$

[where ρ_ℓ is the phase space factor]

$$BR\left(\tau^- \to \ell^- \overline{\nu}_\ell \nu_\tau\right) \equiv BR^\tau_\ell = \frac{\Gamma^\tau_\ell}{\Gamma^\tau_{tot}};$$



- it follows that :

$$\frac{\Gamma^\tau_\mu}{\Gamma^\tau_e} = \frac{BR^\tau_\mu}{BR^\tau_e} = \frac{g_\mu^2 \rho_\mu}{g_e^2 \rho_e} \to$$

$$\left.\frac{BR^\tau_\mu}{BR^\tau_e}\right|_{meas.} = \frac{(17.36 \pm .05)\%}{(17.84 \pm .05)\%} = 0.974 \pm .004,$$

and, taking into account the values of ρ_μ and ρ_e :

$$\boxed{g_\mu / g_e\big|_{meas.} = 1.001 \pm .002.}$$

!!!

lepton universality : ($\mu \to e$) $\leftrightarrow$ ($\tau \to e$)

The measurement of the **μ–τ universality** is similar [$BR_x = \Gamma_x / \Gamma_{tot} = \tau\ \Gamma_x$] :

$BR(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \approx 100\%$ (experimentally);

$$\frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{\tau_\tau}{\tau_\mu} \frac{BR(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)};$$

the prediction is :

$$\frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{g_e^2}{g_e^2} \frac{g_\mu^2}{g_\tau^2} \frac{m_\mu^5}{m_\tau^5} \frac{\rho_\mu}{\rho_\tau} = \frac{g_\mu^2}{g_\tau^2} \frac{m_\mu^5}{m_\tau^5} \frac{\rho_\mu}{\rho_\tau},$$

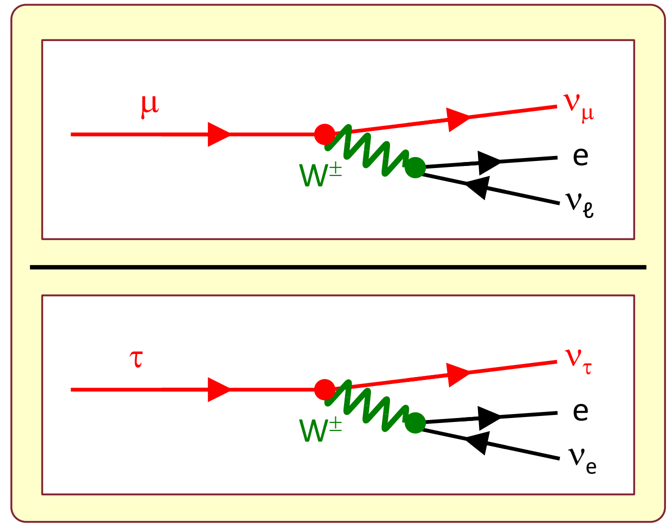
$$\to \frac{g_\mu^2}{g_\tau^2} = \frac{\tau_\tau}{\tau_\mu} \frac{1}{BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{m_\tau^5 \rho_\tau}{m_\mu^5 \rho_\mu},$$

- from the measured values of m_μ, m_τ, τ_μ, τ_τ,

and $BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$, we finally get :

$$\boxed{\left.\frac{g_\mu}{g_\tau}\right|_{meas.} = 1.001 \pm .003.}$$

!!!



"τ_τ" ?

lepton universality : τ decays

More ambitious test: extend universality to **τ hadronic decays** :

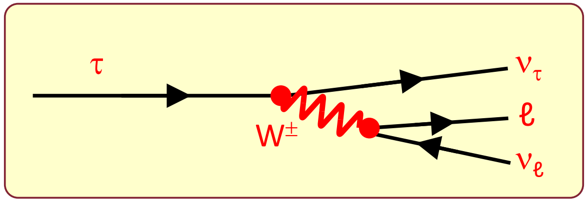
- consider again the leptonic decays of the τ lepton: mainly the following three decay modes :

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau; \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau; \quad \tau^- \to \bar{u} d \nu_\tau.$$

- from the BR_i ratio, expect (3 for color) :

$$\Gamma^{\text{meas.}}_{\tau\to e} \approx \Gamma^{\text{meas.}}_{\tau\to\mu} \approx \Gamma^{\text{meas.}}_{\tau\to\bar{u}d}/3,$$

in excellent agreement with universality and presence of color in the hadronic sector [*it is the first time we see the color appear in the weak interactions sector*].



Another test is the **τ lifetime** :

$$\Gamma_{\tau\to\mu} \approx \frac{\Gamma^{\text{tot}}_\tau}{5} = \frac{m_\tau^5}{m_\mu^5}\Gamma_{\mu\to e} = \frac{m_\tau^5}{m_\mu^5}\frac{1}{\tau_\mu};$$

$$\tau_\tau = 1/\Gamma^{\text{tot}}_\tau \approx \frac{\tau_\mu m_\mu^5}{5 m_\tau^5} \approx \boxed{3.1\times 10^{-13}\ \text{s};}\ \textbf{!}$$

experimentally it is found :

$$\tau^{\text{exp}}_\tau = \boxed{(2.956 \pm .031)\times 10^{-13}\ \text{s}.}\ \textbf{!}$$

- Many other experimental tests [... *but I suppose that you are convinced*].
- At least for CC weak interactions (but also in e.m., and in NC, as in the Z decay) all three leptons have exactly the same interactions.
- The only differences are due to their different mass.
- *Isidor Isaac Rabi said in the 30's about the muon: "who ordered that ?".*

lepton universality : Z decays

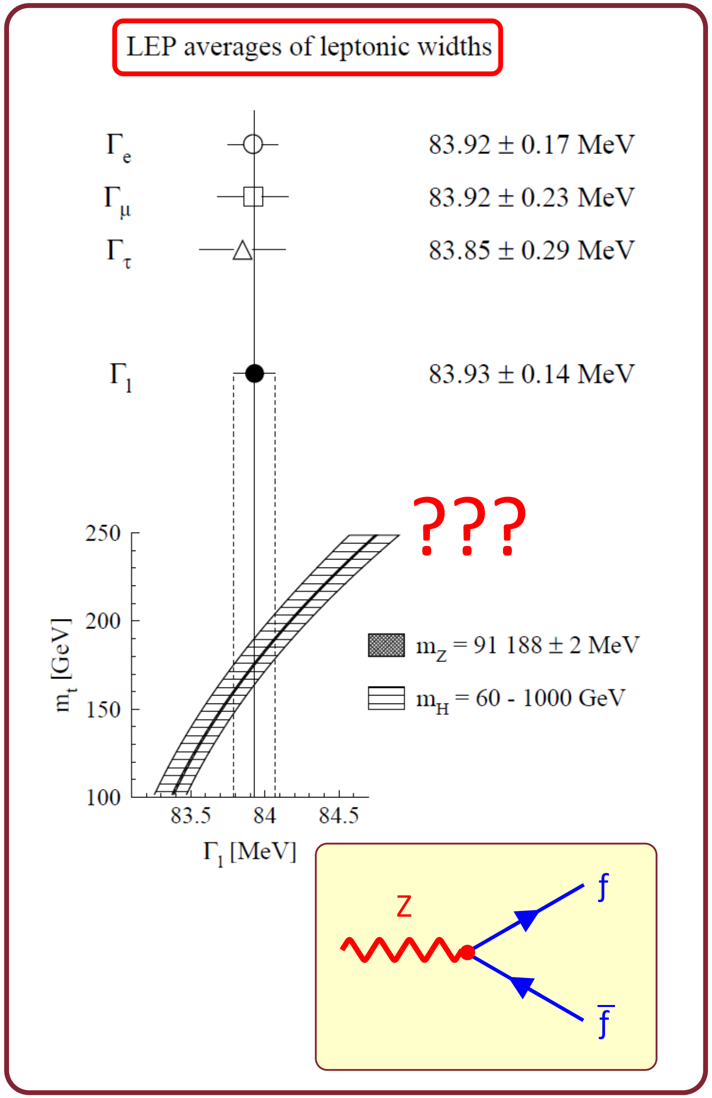
- A similar test on lepton universality has been performed at LEP, in the decay of the Z (a NC process).
- The experiments have measured the decay of the Z into fermion-antifermion pairs.
- They [*well, WE*] have found :

$Z \rightarrow \; e^+e^- \; : \quad \mu^+\mu^- \quad : \quad \tau^+\tau^-$

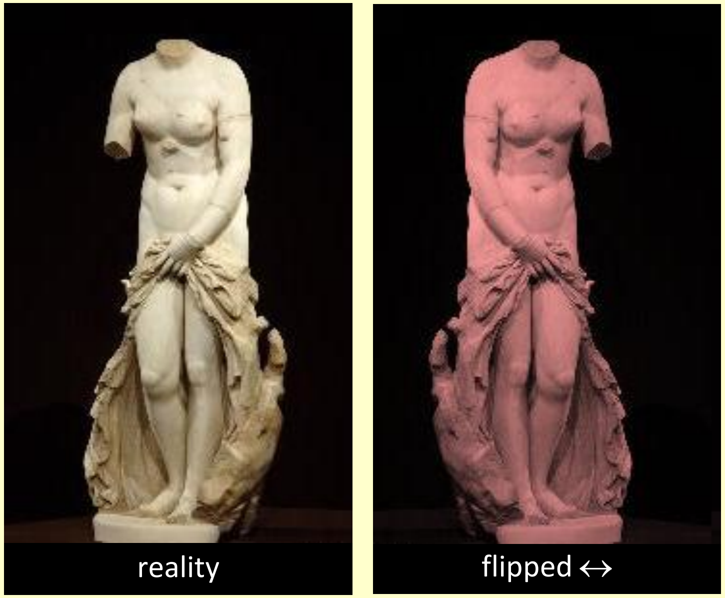
$1. \quad : 1.000 \pm .004 \; : .999 \pm .005.$

- Similar – more qualitative – tests can be carried with angular distributions, higher orders, …
- The total amount of information is impressive and essentially no margin is left to any alternative theory.

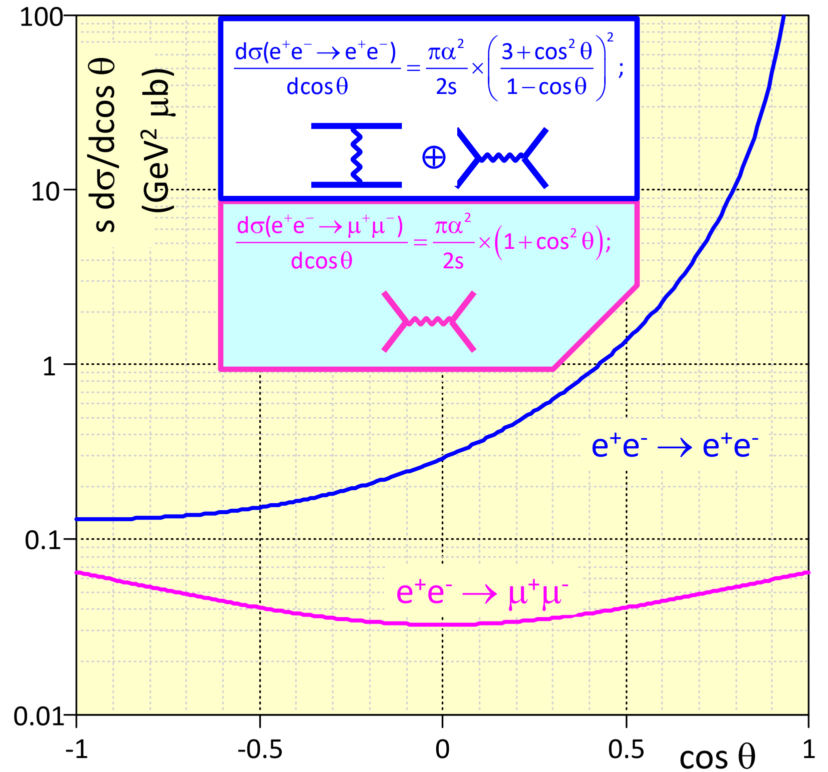
warning – in these pages we mix measurements of different ages, e.g. μ-decay in the '50s, τ-decay in the '80s, Z-decay in the '90s.



Parity violation: meaning



Venus Landolina (Syracuse, Italy)

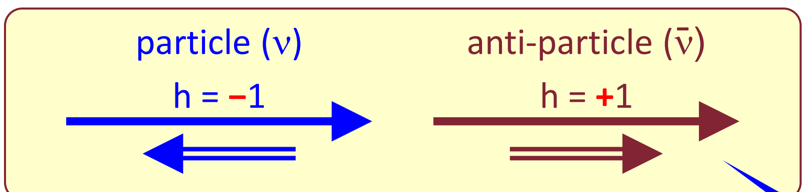


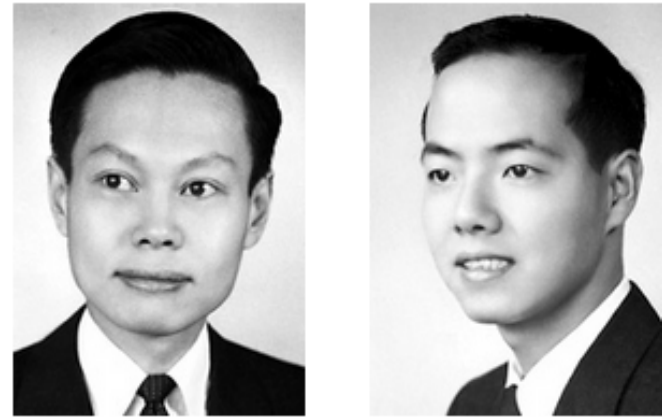
- Look at these two pictures (an ancient sculpture and a modern cross-section);
- one is human-made, the other a law of nature;
- both contain a **symmetry** (left-right legs, forward-backward $\mu^+\mu^-$) and an **asymmetry** (the broken arm, e^+e^-);
- are they examples of ***parity violation*** ?
- Obviously **NO** [*if for no other reason, because p.v. was discovered in the '50s, not in the IV century B.C.*];
- figure out a reasonable explanation
- [*consider flipping the pictures; does it help ?*].

Parity violation: history

- The effect was proposed in 1956 by two young theoreticians in a classical paper and immediately verified in a famous experiment (Mme Wu) [*FNSN 1*] and in the $\pi^{\pm}$- and $\mu^{\pm}$-decays by Lederman and coll.

- The historical reason was a review of weak interaction processes and the explanation of the "θ-τ puzzle", i.e. the K^0 decay into 2π or 3π systems.





Nobel Prize 1957
Tsung-Dao Lee (*Lǐ Zhèngdào*, 李政道)

Chen-Ning Franklin Yang (Yáng Zhènníng, 杨振宁 or 楊振寧)

for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles.

- ν only h=–1;
- $\bar{\nu}$ only h=+1;

→ **PARITY VIOLATION**

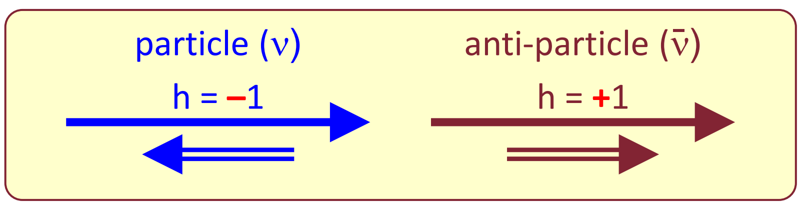
→ vectors & co.

Parity violation: mechanism

- The two authors found that parity conservation in weak decays was NOT really supported by measurements.

[then experiment, and then a new theory]

- The CC current is "V – A", which is an acronym for the factor $\gamma_\mu(1 - \gamma_5)$ in the current; it shows that the CC have a "preference" for <u>left-handed particles</u> and <u>right-handed anti-particles</u>.



- These effects clearly violates the parity : the parity operator $\mathbb{P}$ flips the helicity:

$$\mathbb{P}\ |\nu, h = -1 > = |\nu, h = +1 >$$

$\rightarrow$ it changes ν's with a –ve helicity into ν's with +ve helicity, which DO NOT EXIST (or do not interact).

- Few comments :
  - V or A alone would NOT violate the parity. The violation is produced by the simultaneous presence of the two, technically by their interference.
  - The conservation is restored, applying also $\mathbb{C}$, the charge conjugation:

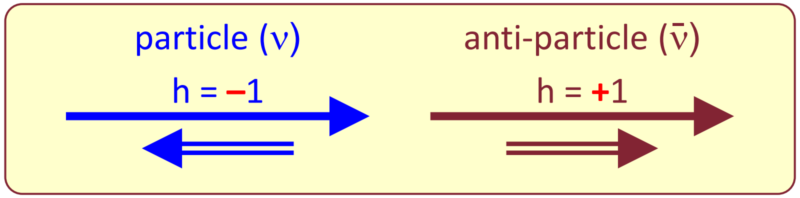
$$\mathbb{C}\mathbb{P}|\nu, h{=}-1> = \mathbb{C}|\nu, h{=}+1> = |\bar{\nu}, h{=}+1>,$$

i.e. $\nu_{h=-1} \rightarrow \bar{\nu}_{h=+1}$, which <u>does exist</u>. Therefore, "<u>$\mathbb{C}\mathbb{P}$ is not violated</u>" [*not by ν's in these experiments, at least*].

> *the above discussion holds only if* m_ν = 0 *(NOT TRUE), or* $m_\nu << E_\nu$ *(ultra-relativistic approximation -* ***<u>u.r.a.</u>****); the u.r.a. for* ν*'s is used in this chapter.*

Parity violation: ν helicity

- For massless ν's or in the u.r.a. approximation(*), V–A implies :



- Therefore in the "forbidden" amplitudes, there is a factor [$\propto$ (1 − β)] for massive particles, which vanishes when β → 1.

- If we assume a factor (1 ± β) for the production of (h = ∓ 1) particles (the opposite for anti-particles), we get :

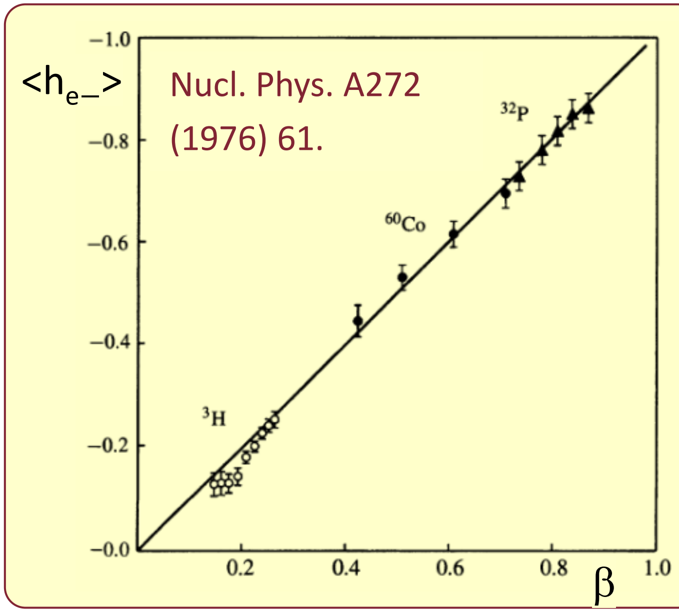
$\langle h \rangle_{part}$ = ½ [(1 + β) (-1) + (1 - β)(+1)]= − β;

$\langle h \rangle_{\overline{part}}$ = ½ [(1 + β) (+1)+ (1 - β)(-1)] = + β;

i.e., when produced in CC interactions, particles in average have –ve helicity, while anti-particles have +ve helicity.

- The effect is maximal for ν's ($\beta_\nu \approx 1$), which also have no other interactions.

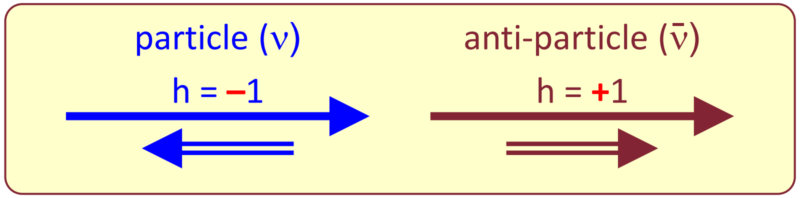
- For e^-, it is also well confirmed by data in β decays [YN1, 570] :



(*) *If $m_\nu > 0 \rightarrow \beta_\nu < 1$; a L-transformation can reverse the sign of the momentum, and hence the ν helicity, so the following argument is NOT L-invariant for massive particles [previous slide].*

Parity violation: ν helicity

- For massless ν's or in the u.r.a. approximation(*), V–A implies :



- Therefore in the "forbidden" amplitudes, there is a factor [$\propto (1 - \beta)$] for massive particles, which vanishes when $\beta \to 1$.

- If we assume a factor $(1 \pm \beta)$ for the production of ($h = \mp 1$) particles (the opposite for anti-particles), we get :

  $\langle h \rangle_{part} = ½ [(1 + \beta) (-1) + (1 - \beta)(+1)]= - \beta;$

  $\langle h \rangle_{\overline{part}} = ½ [(1 + \beta) (+1)+ (1 - \beta)(-1)] = + \beta;$

  i.e., when produced in CC interactions, particles in average have –ve helicity, while anti-particles have +ve helicity.

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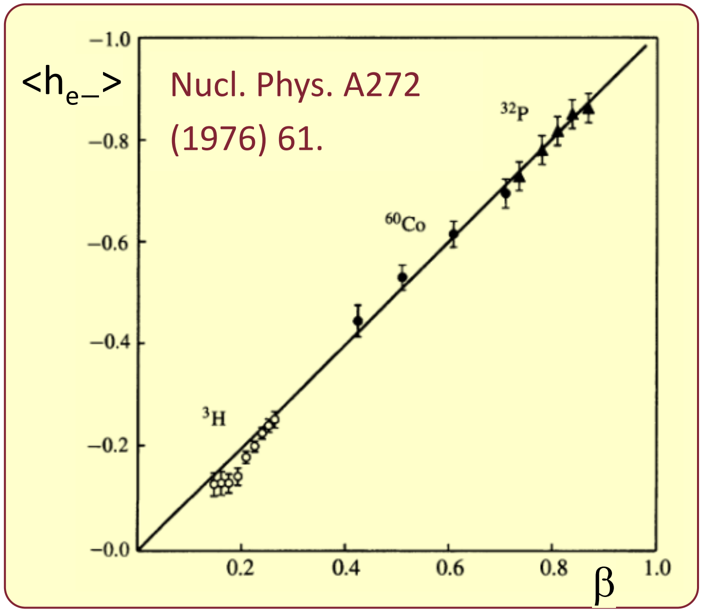


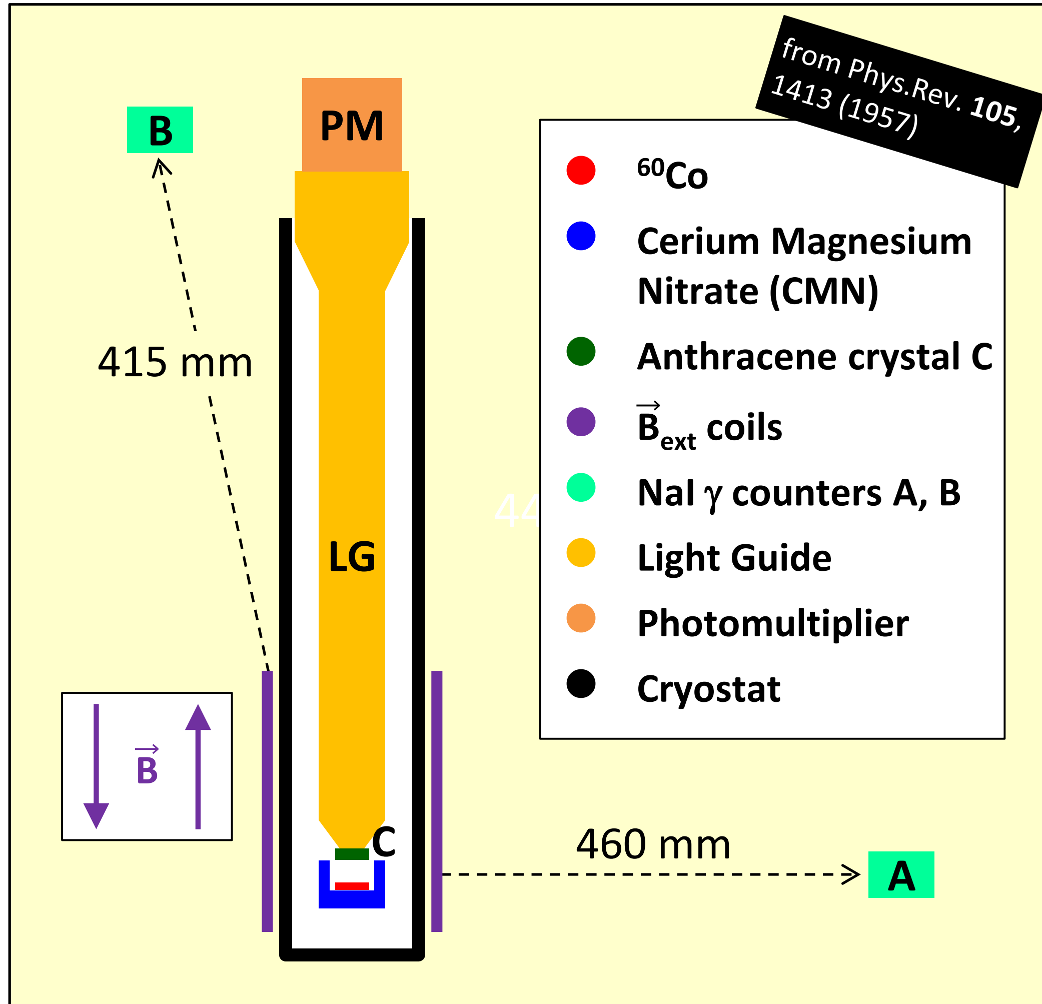
(*) *If $m_\nu > 0 \to \beta_\nu < 1$; a L-transformation can reverse the sign of the momentum, and hence the ν helicity, so the following argument is NOT L-invariant for massive particles [previous slide].*

Parity violation: Wu experiment -1

Chien-Shiung Wu
吳健雄 1912 – 1997

The "Madam Wu" experiment (1957) discovered the parity violation in ^{60}Co decay.

A difficult elegant application of state-of-the-art technologies in nuclear physics and cryogenics.



a very important part of the 3rd year course – repeated here just for completeness

Parity violation: Wu experiment -2

Technicalities:

Align the nuclear spins with an external $\vec{B}$:

- at a given value of T, $E_T = k_B T$ (k_B : Boltzmann constant);
- the magnetic field $E_B = \vec{\mu} \cdot \vec{B}$;
- good alignment if $E_B \geq E_T$ (e.g. $T \approx 10^{-2}$ K, $B \approx 20$ T [*see box*]);

such a large $|\vec{B}|$?

- use external $|\vec{B}_{ext}|$ of few $\times 10^{-2}$ T;
- it polarizes the electrons in the CMN;
- since (μ_e / μ_N = m_N / $m_e \approx 2{,}000$) $\rightarrow$ it produces a strong $|\vec{B}|$ of few T; ☺☺☺

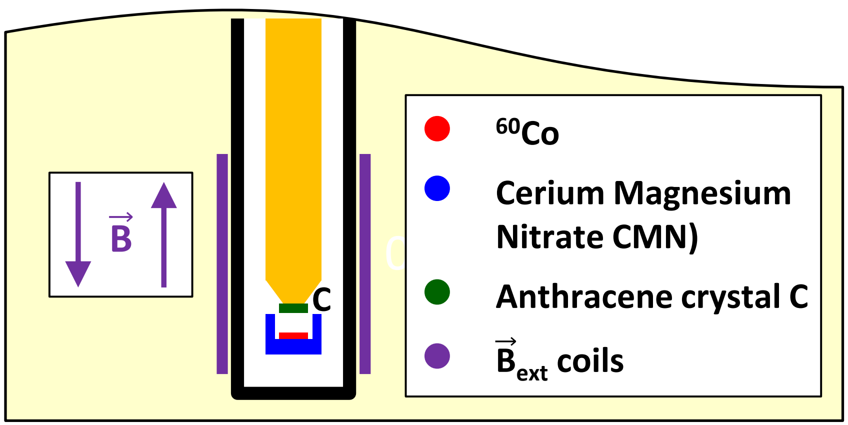
k_B = 8.62×10^{-5} eV / K;
μ_N = 3.15×10^{-8} eV / T;
T = 10^{-2} K $\rightarrow E_T \approx 8 \times 10^{-7}$ eV;
B = 20 T $\rightarrow E_B \approx 6 \times 10^{-7}$ eV. ☺☺☺

such a small T ?

- everything in a cryostat;
- produce $T \approx 10^{-2}$ K using **adiabatic depolarization**;

how to operate ?

- switch the field off ($\rightarrow$ "t_0");
- start counting as a function of time;
- the polarization goes away in few minutes and the effect disappears.



Parity violation: Wu experiment -3

a) ${}^{60}_{27}\text{Co}(J^P{=}5^+)^{\Uparrow} \rightarrow {}^{60}_{28}\text{Ni}^{**}(J^P{=}4^+)^{\Uparrow}$ $\boxed{e^-}$ $\bar{\nu}_e$;

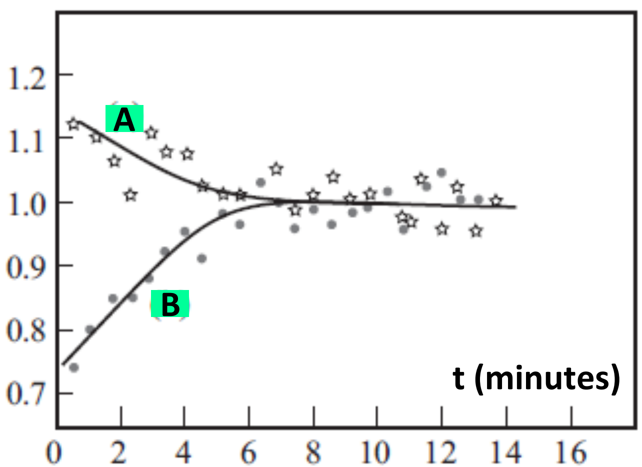
b) ${}^{60}_{28}\text{Ni}^{**}(J^P{=}4^+)^{\Uparrow} \rightarrow {}^{60}_{28}\text{Ni}^{*}(J^P{=}2^+)^{\Uparrow}$ $\boxed{\gamma_1^{[1.173\text{ MeV}]}}$;

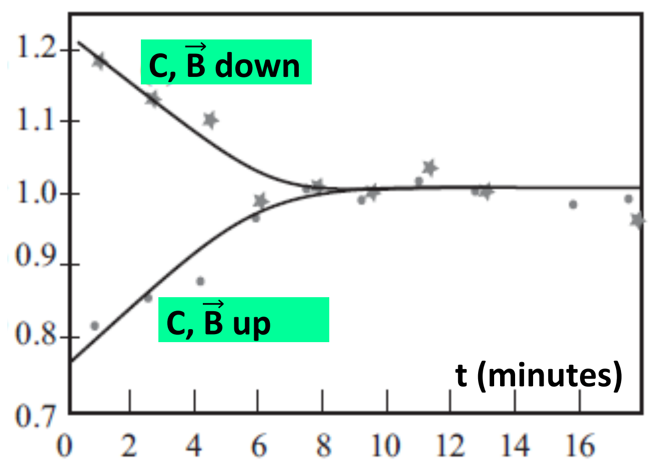
c) ${}^{60}_{28}\text{Ni}^{*}(J^P{=}2^+)^{\Uparrow} \rightarrow {}^{60}_{28}\text{Ni}(J^P{=}0^+)^{\Uparrow}$ $\boxed{\gamma_2^{[1.332\text{ MeV}]}}$;

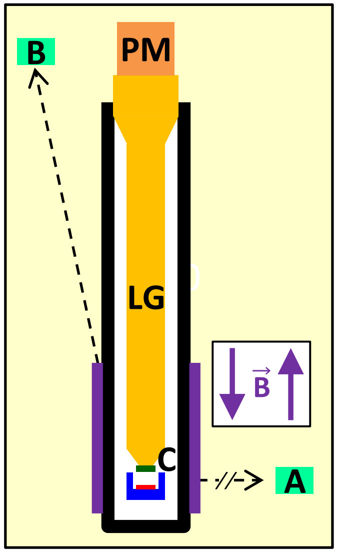
- the chain decay [*box above*];
- decay (a) is **weak** [*interesting*];
- decays (b), (c) are e.m. $\rightarrow \mathbb{P}$ conserved;
- both (a) (b) (c) conserve angular mom.;
- in A : see $\gamma_{1,2}$ if $\perp$ to $\vec{B}$;
- in B : see $\gamma_{1,2}$ if // to $\vec{B}$ [or anti-// to $\vec{B}$];
- in C : see e^- if // to $\vec{B}$ [or anti-// to $\vec{B}$].

Plots (=normalized counts in ABC, for $\pm\vec{B}$) :

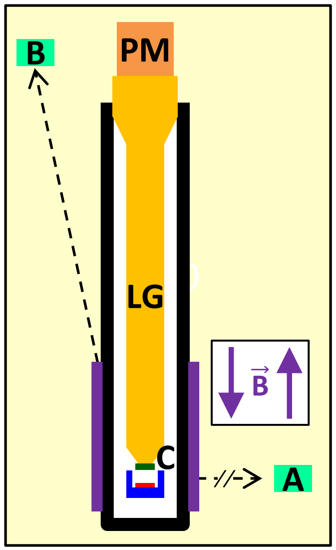
- asymmetries at $t{=}t_0$, then go away;
- A > B because of polarization $\mathcal{P}$ $\rightarrow$ measure $\mathcal{P}$, to be used later;
- A and B do not depend on $\vec{B}$ direction [e.m. conserves $\mathbb{P}$];
- C does depend on $\vec{B}$ direction, with a rate equal to $\mathcal{P} \rightarrow$ **$\mathbb{P}$ is violated**.

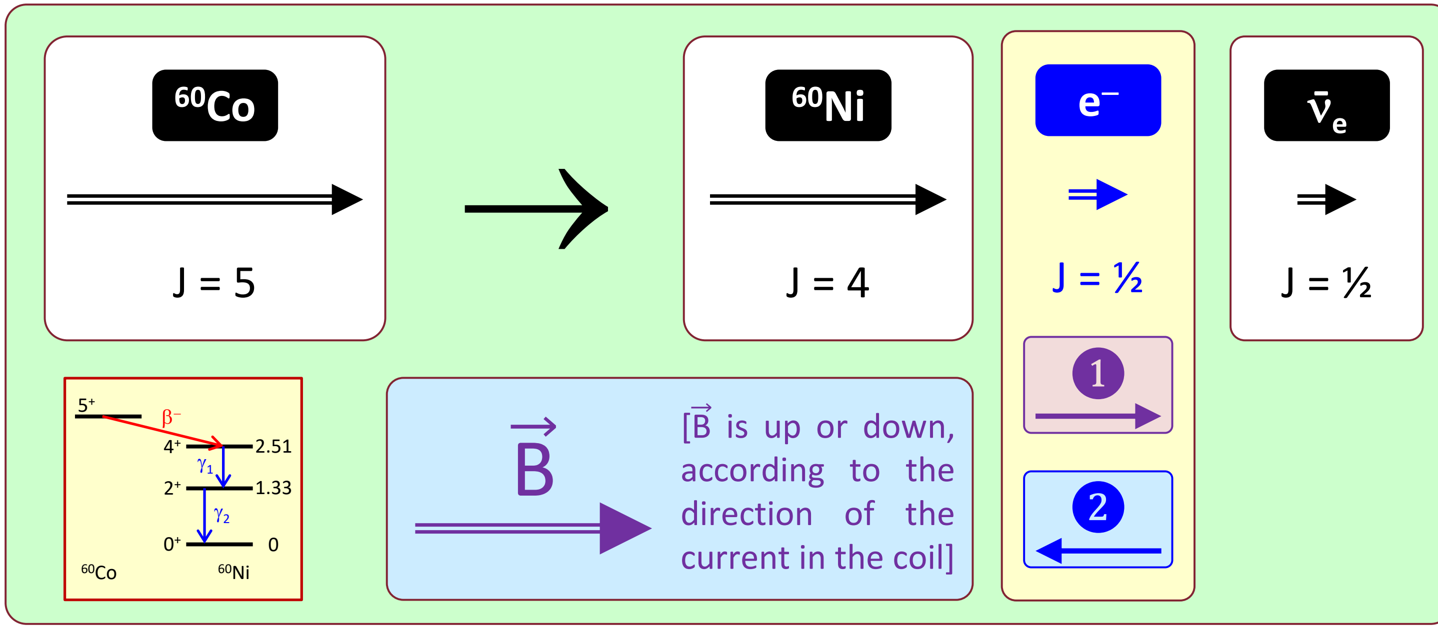






Parity violation: Wu experiment -4





reinterpret the exp. with V – A theory:

- J conservation + $\mathcal{P}$olarization → force spin direction of e⁻;
- case 1:
  - ➢ h_e = +1 → forbidden (∝ 1 – β_e);
- case 2:
  - ➢ h_e = –1 → allowed;

- conclusion:
  - ➢ direction opposite to $\vec{B}$ preferred;
  - ➢ electron rate W depends on cos θ, the angle $\vec{B}$ - $\vec{v}_e$:

$$\boxed{W(\cos\theta) \propto 1 - \mathcal{P}\,\beta_e \cos\theta.}$$

≈ 0.6 (computed)

≈ 0.65 (from counters A,B)

Parity violation: Feynman view

[… *I*]magine that we were talking to a Martian, or someone very far away, by telephone. We are not allowed to send him any actual samples to inspect; for instance, if we could send light, we could send him right-hand circularly polarized light. […] But we cannot give him anything, we can only talk to him.

[*Feynman explains how to communicate: math, classical physics, chemistry, biology are simple*]

[…] "Now put the heart on the left side." He says, "Duhhh - the left side?" […] We can tell a Martian where to put the heart: we say, "Listen, build yourself a magnet, [… *repeat the mme Wu exp …;*] then the direction in which the current goes through the coils is the direction that goes in on what we call the right.

[… *However,*] does the right-handed matter behave the same way as the right-handed antimatter? Or does the right-handed matter behave the same as the left-handed antimatter? Beta-decay experiments, using positron decay instead of electron decay, indicate that this is the interconnection: matter to the "right" works the same way as antimatter to the "left."

[… *We then*] make a new rule, which says that matter to the right is symmetrical with antimatter to the left.

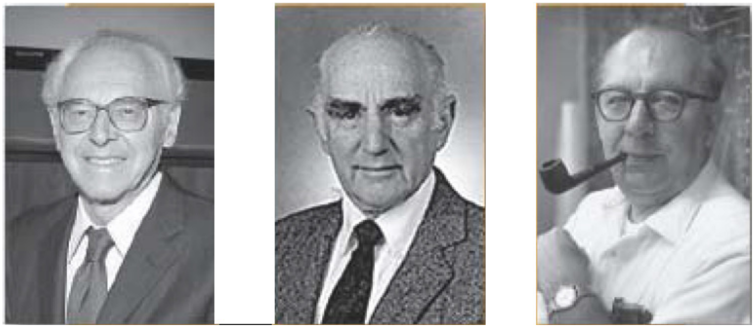
So if our Martian is made of antimatter and we give him instructions to make this "right" handed model like us, it will, of course, come out the other way around. What would happen when, after much conversation back and forth, we each have taught the other to make space ships and we meet halfway in empty space? […] Well, if he puts out his left hand, watch out!

From Feynman Lectures on Physics, 1, 52: "Symmetry in Physical Laws".

Quite amusing and great physics :

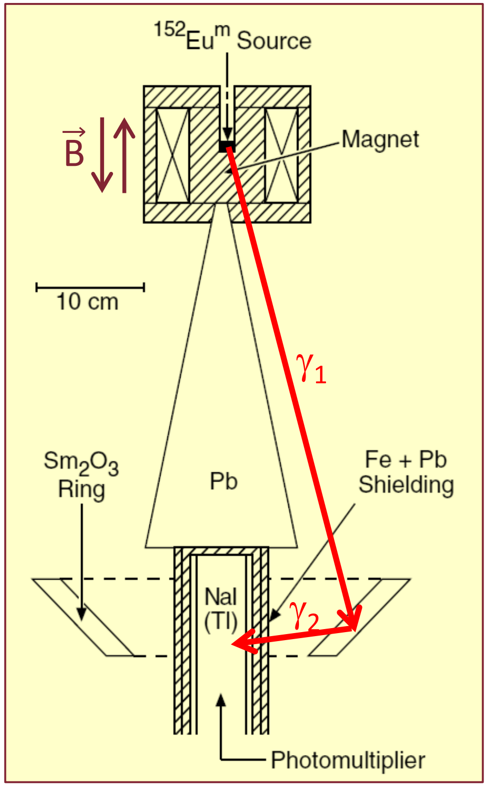
- the symmetry he is talking about is "ℂℙ" and NOT simply "ℙ" or "ℂ" !!!
- but ℂℙ is also violated [see § K^0].

Parity violation: the ν_e helicity



In 1958, Goldhaber, Grodzins and Sunyar measured the **helicity of the electron neutrino** ν_e with an ingenious experiment.

- A crucial confirmation of the V−A theory; pure V or A had been ruled out, but V+A was still in agreement with data.
- Metastable Europium (Eu) decays via K-capture → excited Samarium (Sm*) + $\boldsymbol{\nu_e}$, whose helicity is the result of the exp.;
- the Sm* decays again into more stable Samarium (Sm), emitting a γ [γ_1 in fig.].
- For such a γ the transmission in matter depends on the e^- spins; therefore a large B-field is applied to polarize the iron.
- The γ's are used to excite again another Sm; only γ's from the previous chain may do it; another γ is produced [γ_2 in fig.].
- The resultant γ's are detected.



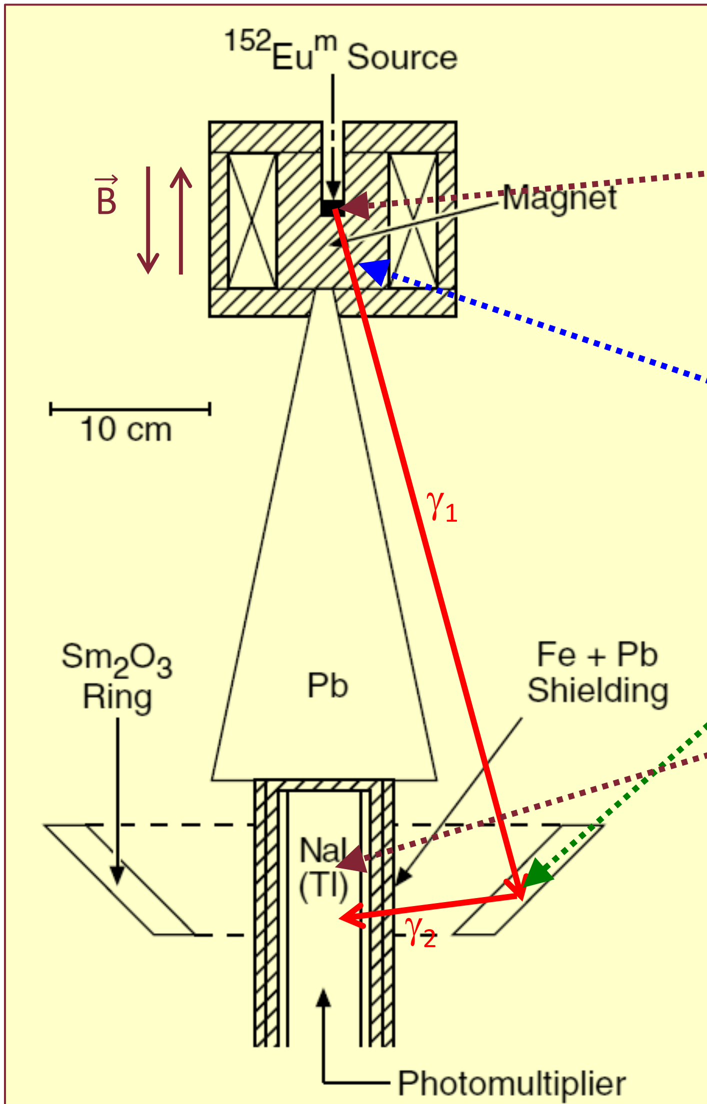
- Final result :

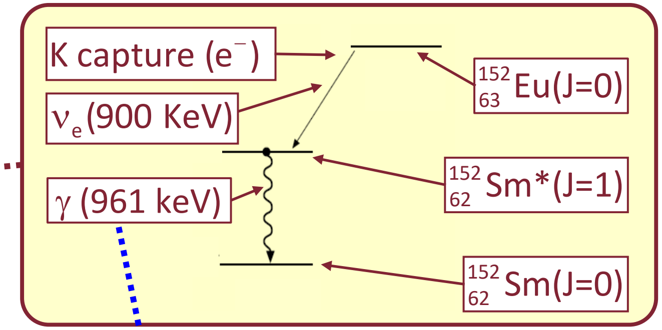
$$\boxed{h(\nu_e) = -1.0 \pm 0.3}$$

consistent with V−A only.

[the experiment is ingenuous and complex: it is discussed step by step.]

Parity violation: summary of the experiment





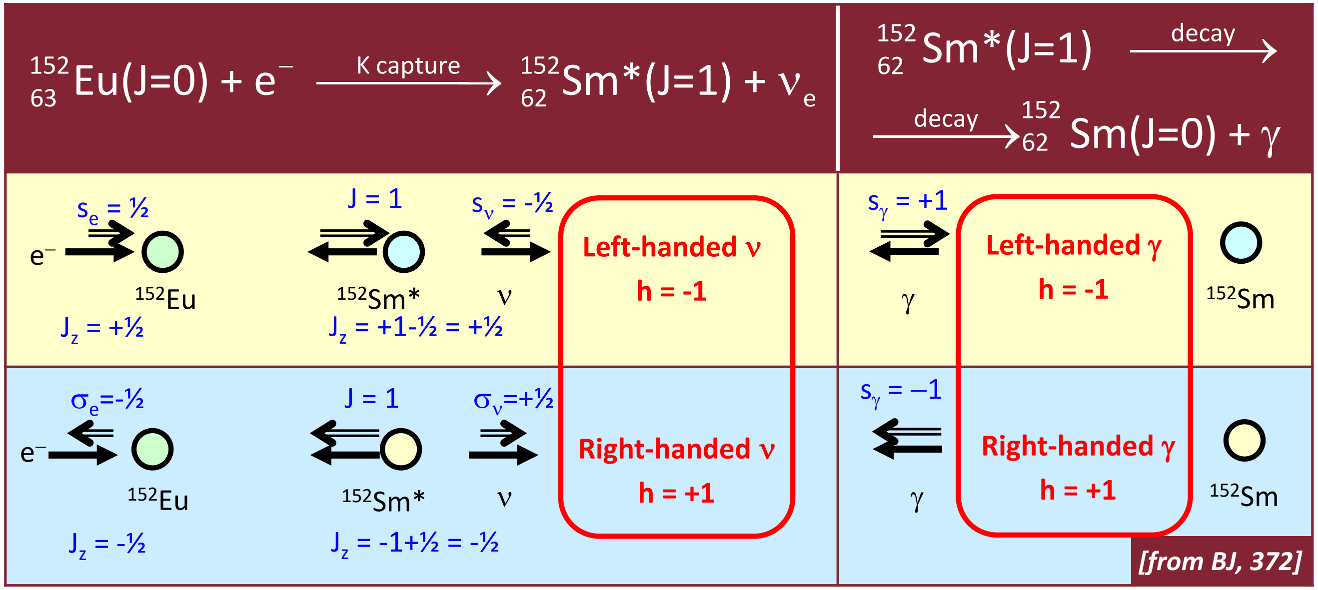
Compton effect does depend on the γ_1-spin wrt $\vec{B}$ (NB γ_1 in the figure escapes Compton effect).

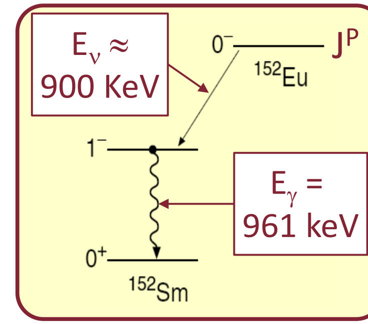
$\gamma_1 + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma_2$.

γ_2 detection via photomultiplier.

The experiment detects the number of γ_2 when $\vec{B}$ is (anti-)parallel to γ_1. The asymmetry depends on the (ν_e-helicity $\rightarrow$) γ_1-spin.

Parity violation: Europium → Samarium → γ





- ν_e monochromatic, $E_\nu \approx 900$ keV;
- Sm* lifetime = ~10^{-14} s, short enough to neglect all other interactions;
- Sm* excitation energy = 961 KeV (≈ E_ν);
- *only* for γ in the direction of Sm* recoil, angular momentum conservation implies Sm* helicity = ν_e helicity = γ helicity = ±1 [*see box with 2 alternative hypotheses*].

- Therefore, the method is:
  - [cannot measure directly the ν_e spin]
  - select and measure the γ's emitted anti-parallel to the ν_e's, i.e. in the same direction of the (^{152}Sm*);
  - measure their spin;
  - reconstruct the **ν_e helicity**.

Parity violation: the resonant scattering

- For γ of 961 keV, the dominant interaction with matter is the Compton effect; the Compton cross section is spin-dependent: the transmission is larger when the γ and e^- spin are parallel.

- Therefore, a strong and reversible $\vec{B}$ (saturated iron) selects the polarized γ's, producing an asimmetry between the two $\vec{B}$ orientations.

- Need also to select only the γ's polarized according to the ν_e spin, i.e. produced opposite to the ν_e's $\rightarrow$ use the method of *resonant scattering* in the Sm_2O_3 ring:

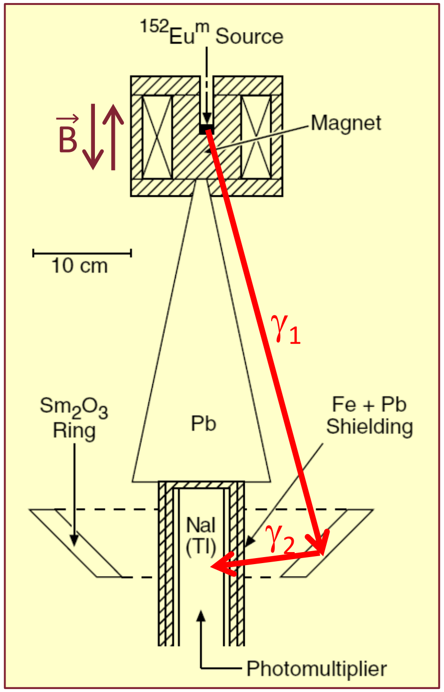
$$\gamma_1 + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma_2.$$

- [kinematics (next slide) : a nucleus at rest, excited by an energy E_0, decays with a γ emission; the γ energy in the lab. is reduced by a factor $E_0/(2M)$].

- In general, γ_1 energy is degraded and NOT sufficient for Sm excitation (i.e. to produce γ_2).

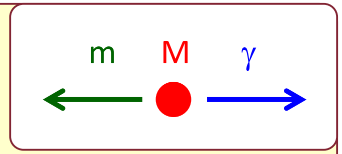
- But, if γ_1 is anti-parallel to ν_e, the Sm* recoils against ν_e. The resultant Doppler effect in the correct direction provides γ_1 of the necessary amount of extra energy ($E_\nu \approx E_\gamma$).

- In conclusion, only the γ's anti-parallel to ν_e's are detected, but those γ's carry the information about ν_e helicity.



Parity violation: kinematics

<u>Kinematics</u>



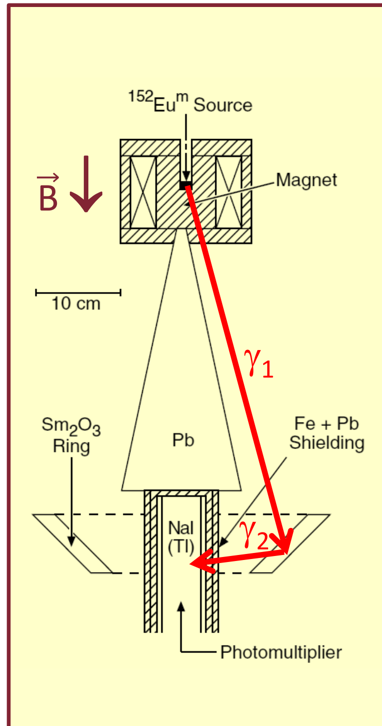
$$M \rightarrow m\,\gamma; \qquad E_0 = M - m;$$

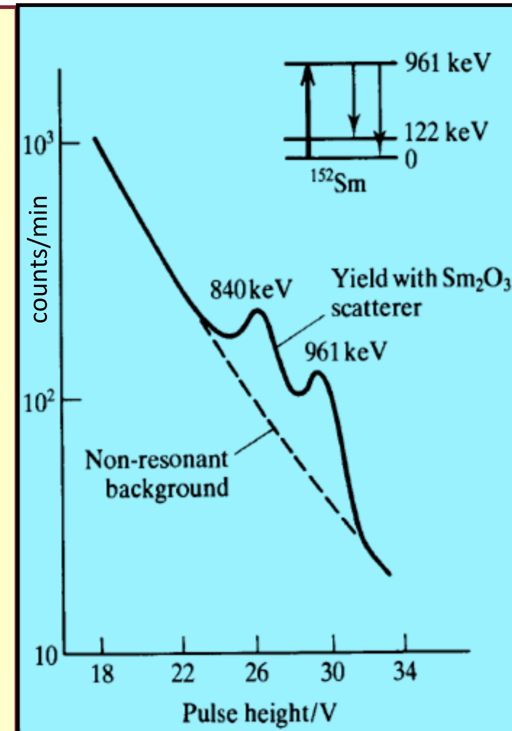
$$\text{M sys.} \begin{cases} M = [M, & 0, & 0,0]; \\ \gamma = [E_\gamma, & E_\gamma, & 0,0]; \\ m = [M - E_\gamma, & -E_\gamma, & 0,0]; \end{cases}$$

$$m^2 = (M - E_\gamma)^2 - E_\gamma^2 = M^2 + \cancel{E_\gamma^2} - 2ME_\gamma - \cancel{E_\gamma^2};$$

$$E_\gamma = \frac{M^2 - m^2}{2M} = \frac{M+m}{2M} E_0 =$$

$$= \frac{M + M - E_0}{2M} E_0 = E_0 \left(1 - \frac{E_0}{2M}\right).$$

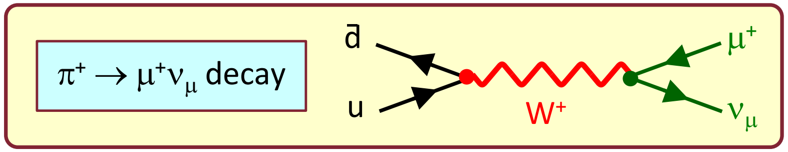




$h_\gamma = -1 \rightarrow \boxed{h_\nu = -1}$

→ if the excited nucleus (M) is <u>at rest</u>, the energy of the γ in the lab. is smaller than the excitation energy E_0; therefore it is insufficient to excite another nucleus at rest; for this to happen, the excited nucleus has to move in the right direction with the <u>appropriate energy</u>.

Weak decays : $\pi^{\pm}$



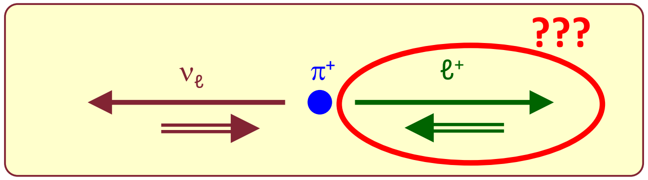
- The $\pi^{\pm}$ is the lightest hadron; therefore it may only decay through semileptonic CC weak processes, like (consider only π^+, for π^-, apply $\mathbb{C}$) :

$$\pi^+ \to \mu^+ \nu_\mu; \quad \pi^+ \to e^+ \nu_e.$$

- In reality, it almost decays only into μ's: the electron decay is suppressed by a factor ≈ 8,000, NOT understandable, also because the ($\pi \to e$) decay is favored by space phase.
- The reason is the **helicity**:

ℓ =lepton, i.e. e/μ

  - in the π^+ reference frame, the momenta of the ℓ^+ and the ν_ℓ must be opposite;
  - since the π^+ has spin 0, the spins of the ℓ^+ and the ν must also be opposite;
  - therefore the two particles must have the same helicity;
  - since the ν (a ~massless particle) must have negative helicity, the ℓ^+ (a non-massless antiparticle) is also forced to have negative helicity;
  - therefore the transition is suppressed by a factor $(1 - \beta_\ell)$;
  - the e^+ is ultrarelativistic ($p_e \approx m_\pi / 2 >> m_e$), while the μ^+ has small β [*compute it !!!*];
  - therefore the decay $\pi \to e$ is strongly suppressed respect to $\pi \to \mu$.



Kinematics (next slide) :

- $p_\ell = [(m_\pi^2 - m_\ell^2) / (2\ m_\pi)]$;
- $\beta_e = (1 - 2.6 \times 10^{-5})$;
- $\beta_\mu = 0.38$.

Weak decays : kinematics

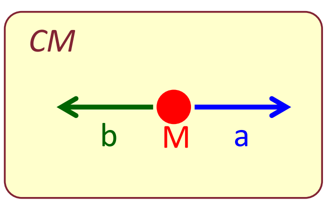
<u>SOLUTION</u> : (more general)

Decay M $\rightarrow$ a b. Compute p = $|\vec{p}_a|$ = $|\vec{p}_b|$

in the CM system, i.e. the system of M:

$$\text{CM}\begin{cases}(M, & 0, & 0,0)\\ (\sqrt{m_a^2+p^2}, & p, & 0,0);\\ (\sqrt{m_b^2+p^2}, & -p, & 0,0)\end{cases}$$

$$p^2 = \frac{\left[M^2-(m_a-m_b)^2\right]\left[M^2-(m_a+m_b)^2\right]}{4M^2}.$$



energy conservation : $M=\sqrt{m_a^2+p^2}+\sqrt{m_b^2+p^2}$;

$2\sqrt{m_a^2+p^2}\sqrt{m_b^2+p^2}=M^2-m_a^2-m_b^2-2p^2$;

$4\left[m_a^2m_b^2+p^2\left(m_a^2+m_b^2\right)+\cancel{p}^4\right]=\left(M^2-m_a^2-m_b^2\right)^2+4\cancel{p}^4-4p^2\left(M^2-m_a^2-m_b^2\right)$;

$4p^2\left[\left(\cancel{m_a^2}+\cancel{m_b^2}\right)+\left(M^2-\cancel{m_a^2}-\cancel{m_b^2}\right)\right]=-4m_a^2m_b^2+\left(M^2-m_a^2-m_b^2\right)^2$;

$4p^2M^2=\left[\left(M^2-m_a^2-m_b^2\right)+2m_am_b\right]\left[\left(M^2-m_a^2-m_b^2\right)-2m_am_b\right]=$(see above)

a) $m_a=m_b=m$; e.g. $K^0\rightarrow\pi^0\pi^0$;

$$p^2=\frac{M^2-4m^2}{4}=\frac{(M+2m)(M-2m)}{4};$$

b) $m_a=m_b=0$; e.g. $\pi^0\rightarrow\gamma\gamma$, $H\rightarrow\gamma\gamma$;

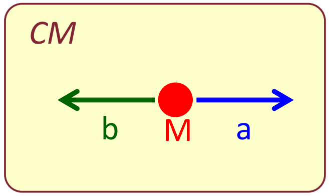
$$p^2=\frac{M^2}{4};\quad p=\frac{M}{2};$$

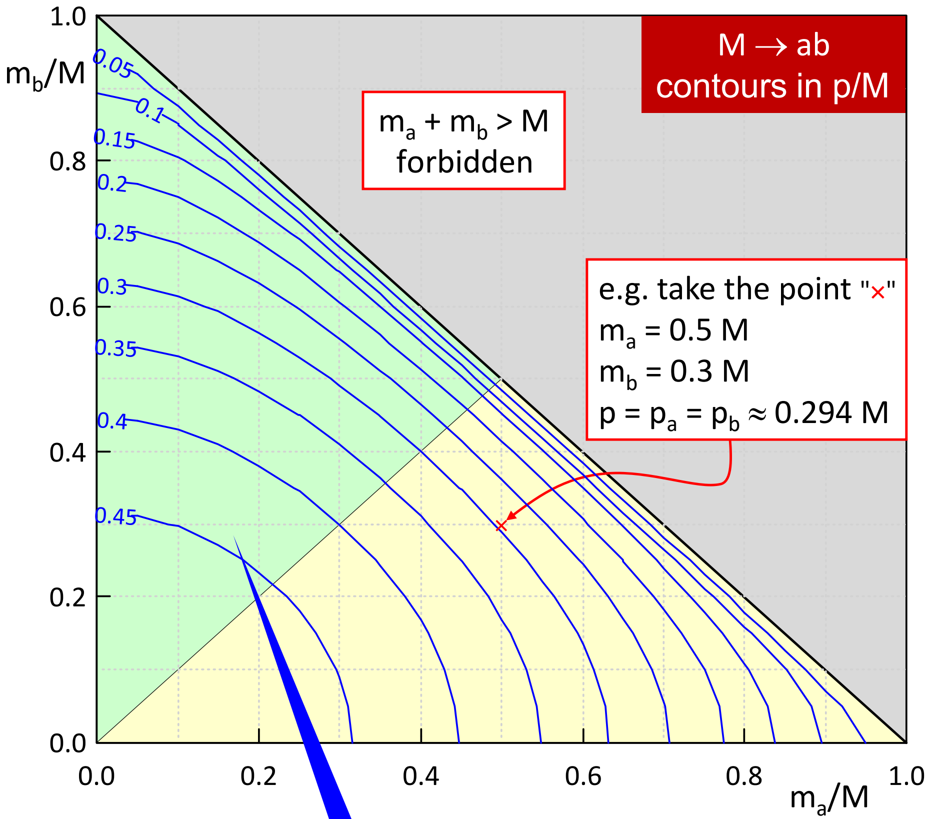
c) $m_a=m$; $m_b=0$; e.g. $\pi^+\rightarrow\mu^+\nu_\mu$, $W^*\rightarrow W\gamma$;

$$p=\frac{M^2-m^2}{2M}=\frac{M}{2}\left[1-\left(\frac{m}{M}\right)^2\right].$$

Weak decays : contour plot

same info as in previous slide, only "easier" to see

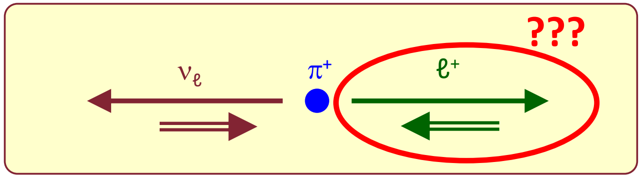




the plot is only here to show you how easy it is to produce an apparently sophisticated and professional plot.

symmetric for m_a vs m_b, plot only $m_a > m_b$.

Weak decays : $\pi^\pm \to (e^\pm / \mu^\pm)$



<u>Problem: compute the factor in the $\pi^\pm$ decay between μ and e.</u>

Assume for the decay $\pi \to \ell$ [$\ell = \mu$ or e] :

p	= decay product momentum;
ρ_ℓ	$= dN/dE_{tot}$ = phase space factor;
dN	$= Vp^2dpd\Omega/(2\pi)^3$;
$(1-\beta_\ell)$	= helicity suppression;
BR_ℓ	$= \Gamma_\ell / \Gamma_{tot} \propto \rho_\ell \times (1-\beta_\ell)$.

only factors different for μ/e (ℓ-universality)

In this case the decay is isotropic. Then :

$\rho_\ell \quad \propto p^2dp/dE_{tot}$;

4-momentum conservation [use previous slide and keep only terms ℓ-dependent]:

$$p_\ell = p_\nu = E_\nu \equiv p; \qquad E_{tot} = m_\pi; \qquad E_\ell = m_\pi - E_\nu = m_\pi - p;$$

$$p = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} = \frac{E_{tot}}{2} - \frac{m_\ell^2}{2E_{tot}}; \quad \frac{dp}{dE_{tot}} = \frac{1}{2} + \frac{m_\ell^2}{2m_\pi^2} = \frac{m_\pi^2 + m_\ell^2}{2m_\pi^2};$$

$$\rho_\ell \propto \left(\frac{m_\pi^2 - m_\ell^2}{2m_\pi}\right)^2 \frac{m_\pi^2 + m_\ell^2}{2m_\pi^2} = \frac{\left(m_\pi^2 + m_\ell^2\right)\left(m_\pi^2 - m_\ell^2\right)^2}{\boxed{8m_\pi^4}};$$

irrelevant

$\rho_e > \rho_\mu$

$$1 - \beta_\ell = 1 - \frac{p_\ell}{E_\ell} = 1 - \frac{p}{m_\pi - p} = \frac{m_\pi - 2p}{m_\pi - p} =$$

$$= \frac{m_\pi - 2(m_\pi^2 - m_\ell^2)/(2m_\pi)}{m_\pi - (m_\pi^2 - m_\ell^2)/(2m_\pi)} = \frac{2m_\ell^2}{m_\pi^2 + m_\ell^2};$$

$$BR_\ell \propto \left(m_\pi^2 + m_\ell^2\right)\left(m_\pi^2 - m_\ell^2\right)^2 \frac{\cancel{2}m_\ell^2}{m_\pi^2 + m_\ell^2} =$$

$$\propto m_\ell^2\left(m_\pi^2 - m_\ell^2\right)^2.$$

$1-\beta_e <<< 1-\beta_\mu$

For electrons, $m_e << m_\pi$, so :

$$\frac{BR\left(\pi^+ \to e^+\nu_e\right)}{BR\left(\pi^+ \to \mu^+\nu_\mu\right)} = \left(\frac{m_e}{m_\mu}\,\frac{m_\pi^2 - \cancel{m_e^2}}{m_\pi^2 - m_\mu^2}\right)^2 \approx 1.28 \times 10^{-4}.$$

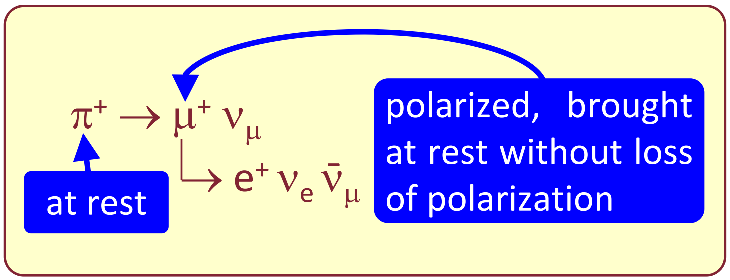
Experimentally, it is measured

$$\frac{BR\left(\pi^+ \to e^+\nu_e\right)}{BR\left(\pi^+ \to \mu^+\nu_\mu\right)} = 1.23 \times 10^{-4}.$$

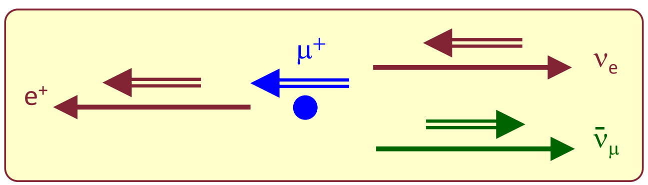
i.e. $N(\pi \to \mu) \approx$ 8,000 $N(\pi \to e)$

Weak decays : $\pi^{\pm} \rightarrow \mu^{\pm}$

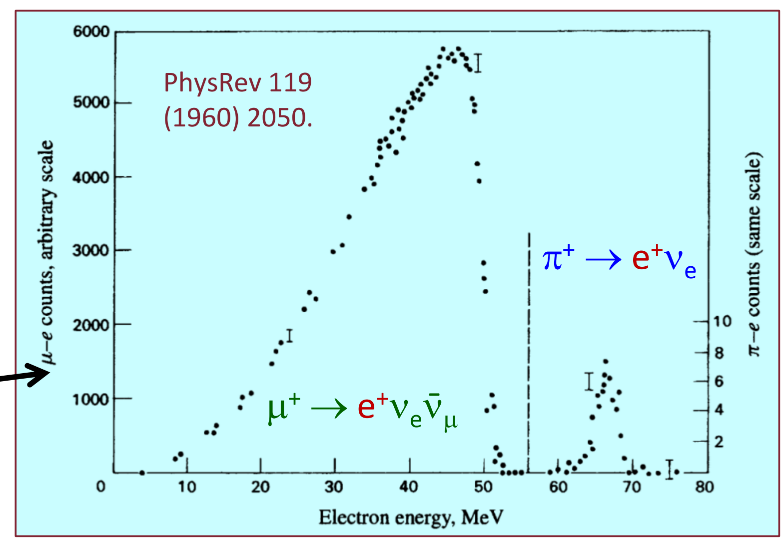
- Consider a famous experiment (Anderson et al., 1960) :



- In the μ^+ ref. frame (=LAB), this configuration is clearly preferred :

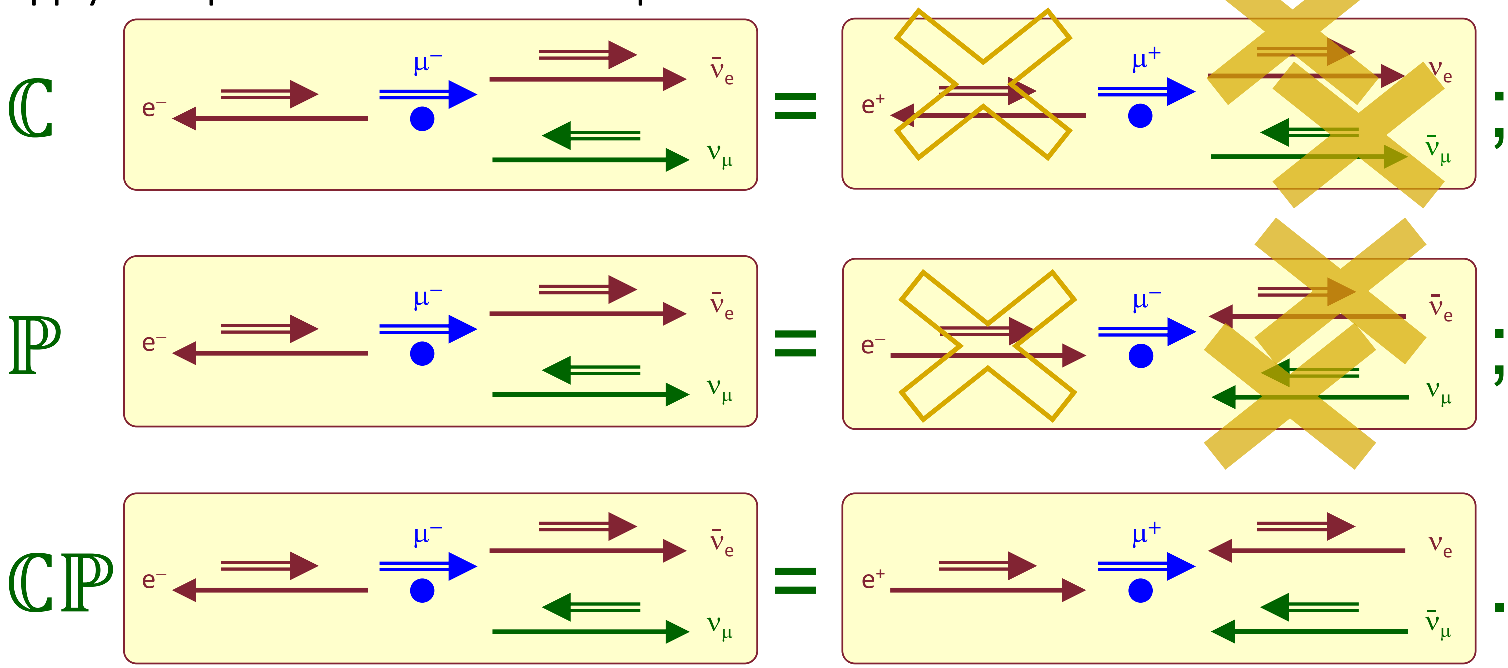


- In this angular configuration, both space and angular momentum are conserved, the particles are left- and the anti-particles right-handed.
- From the figure :
  - ➢ few e^+ directly from π^+ decay, shown in the right part ($\int\mu$ / $\int e \approx$ 8,000);
  - ➢ the electron energy is the only measurable variable;
  - ➢ kinematical considerations show that it is correlated with the angular variables, and that the value $E_e \approx m_\mu$ / 2 is possible only for parallel ν's.
  - ➢ the distribution clearly shows the parity violation in muon decay.



Weak decays: C, P in μ decays

Apply the operators ℂ and ℙ to the previous cases :



- [the "×" shows the forbidden – not existent – particles]
- both ℂ and ℙ alone transforms the decay into non-existent processes (we say "**both ℂ and ℙ separately are not conserved in this process**");
- instead, the application of ℂℙ turns a μ^- decay (**which does exist**) into a μ^+ decay (**which also exists**) → "ℂℙ is conserved in this process".

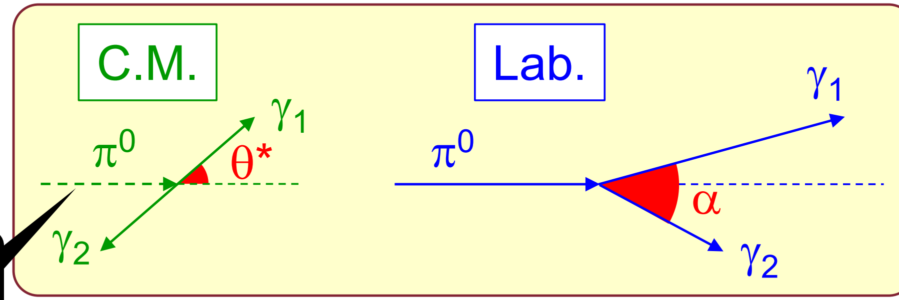
decay $\pi^0 \to \gamma\gamma$: L-transf.

L-transf
$$\begin{cases} E = \gamma(E^* + \beta p^*_\ell); \\ p_\ell = \gamma(p^*_\ell + \beta E^*); \\ p_T = p^*_T; \end{cases}$$

NB: L-transf. CM → Lab.

$$m \equiv m_{\pi^0}; \quad \beta \equiv \frac{p_{\pi^0}}{E_{\pi^0}}; \quad \gamma \equiv \frac{E_{\pi^0}}{m_{\pi^0}}.$$

In CM, π^0 at rest.



- the $\pi^0 \to \gamma\gamma$ decay is an e.m. process; it is here just for convenience;
- see also FNSN1, § Cinematica, 22-26.

$$\begin{cases} & \text{C.M.} & \text{Lab.} \\ \pi^0 & m\{1,0,0,0\} & m\{\gamma,\beta\gamma,0,0\} \\ \gamma_1 & \frac{m}{2}\{1,\cos\theta^*,\sin\theta^*,0\} & \frac{m}{2}\{\gamma(1+\beta\cos\theta^*),\gamma(\cos\theta^*+\beta),\sin\theta^*,0\} \\ \gamma_2 & \frac{m}{2}\{1,-\cos\theta^*,-\sin\theta^*,0\} & \frac{m}{2}\{\gamma(1-\beta\cos\theta^*),\gamma(-\cos\theta^*+\beta),-\sin\theta^*,0\} \end{cases}$$

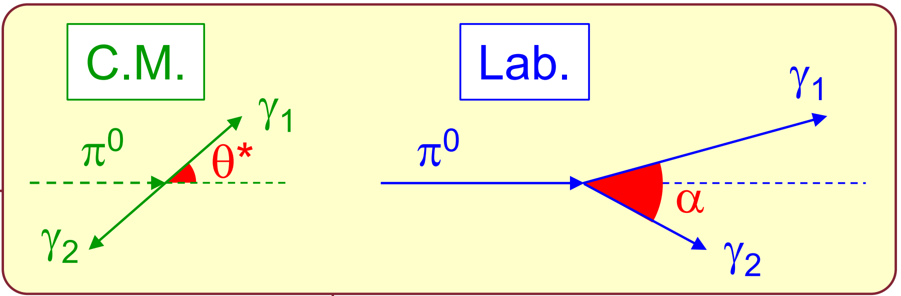
$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2} = \frac{\vec{p}_1^{\,Lab}\cdot\vec{p}_2^{\,Lab}}{E_1^{Lab}E_2^{Lab}} = \frac{\gamma^2\left(\beta^2-\cos^2\theta^*\right) - \sin^2\theta^*\left[\gamma^2(1-\beta^2)\right]}{\gamma^2\left(1-\beta^2\cos^2\theta^*\right)} = \frac{\beta^2\left(1+\sin^2\theta^*\right)-1}{1-\beta^2\cos^2\theta^*};$$

[...] = 1

for a γ : $|\vec{p}| = E$

$$\sin^2\frac{\alpha}{2} = -\frac{1}{2}\left(\frac{\beta^2\left(1+\sin^2\theta^*\right)-1}{1-\beta^2\cos^2\theta^*} - \frac{1-\beta^2\cos^2\theta^*}{1-\beta^2\cos^2\theta^*}\right) = \frac{\boxed{\beta^2+\beta^2-2}}{\boxed{-2}\left(1-\beta^2\cos^2\theta^*\right)} = \frac{1}{\boxed{\gamma^2}\left(1-\beta^2\cos^2\theta^*\right)}.$$

decay $\pi^0 \to \gamma\gamma$: angle α



$$\sin^2\frac{\alpha}{2} = \frac{1}{\gamma^2\left(1-\beta^2\cos^2\theta^*\right)};$$

$$\xrightarrow[\theta^*=90^\circ,\ \cos\theta^*=0]{} \quad [\mapsto\updownarrow] \quad \sin^2\frac{\alpha}{2}\bigg|_{min} = \frac{1}{\gamma^2} = \left(\frac{m_{\pi^0}}{E_{\pi^0}}\right)^2 \quad \to \alpha_{min} \cong \frac{2m_{\pi^0}}{E_{\pi^0}};$$

$$\xrightarrow[\theta^*=0^\circ,\ \cos\theta^*=1]{} \quad [\mapsto\leftrightarrow] \quad \sin^2\frac{\alpha}{2}\bigg|_{max} = \frac{1}{\gamma^2\left(1-\beta^2\right)} = 1 \quad \to \alpha_{max} = 180^\circ;$$

	$f(\theta^*)$	$\alpha\vert_{min}\,[\cos\theta^*=0]$	$\alpha\vert_{max}\,[\cos\theta^*=1]$
π^0	$m\{\gamma,\beta\gamma,0;1\}$	$m\{\gamma,\beta\gamma,0;1\}$	$m\{\gamma,\beta\gamma,0;1\}$
γ_1	$\frac{m}{2}\{\gamma(1+\beta\cos\theta^*),\gamma(\cos\theta^*+\beta),\sin\theta^*;0\}$	$\frac{m}{2}\{\gamma,\beta\gamma,1;0\}$	$\frac{m}{2}\{\gamma(1+\beta),\gamma(1+\beta),0;0\}$
γ_2	$\frac{m}{2}\{\gamma(1-\beta\cos\theta^*),\gamma(-\cos\theta^*+\beta),-\sin\theta^*;0\}$	$\frac{m}{2}\{\gamma,\beta\gamma,-1;0\}$	$\frac{m}{2}\{\gamma(1-\beta),\gamma(-1+\beta),0;0\}$

decay $\pi^0 \rightarrow \gamma\gamma$: P(α)

$\text{spin}(\pi^0) = 0 \rightarrow \mathcal{P}(\cos\theta^*) = \text{flat} = 1/2.$

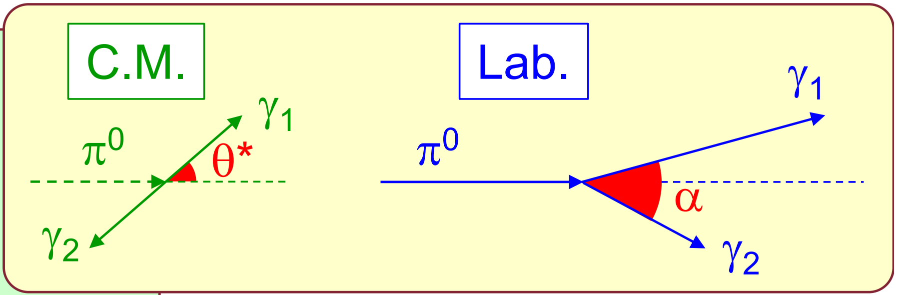
Therefore :

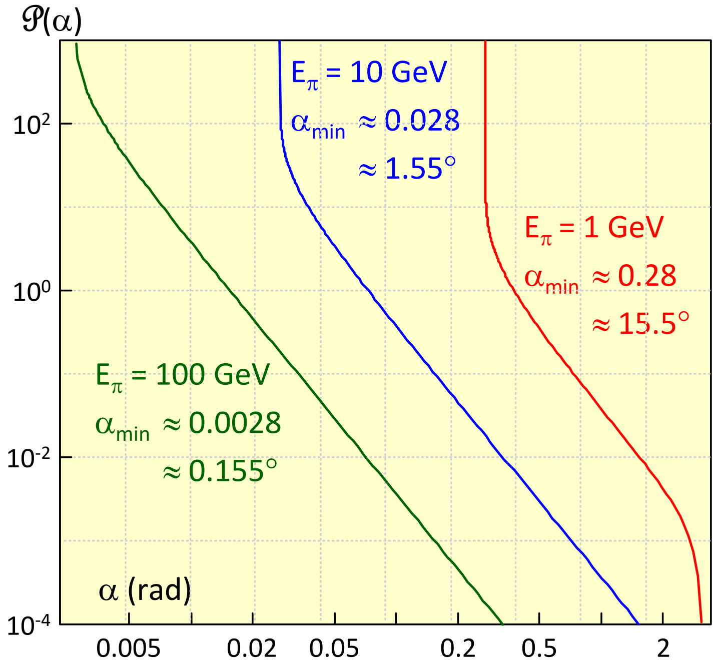
$$E_\gamma^{1,2} = \frac{m\gamma}{2}(1 \pm \beta\cos\theta^*) \rightarrow \frac{dE_\gamma^{1,2}}{d\cos\theta^*} = \pm\frac{m\beta\gamma}{2} \rightarrow$$

$$\boxed{\mathcal{P}\left(E_\gamma^{1,2}\right) = \mathcal{P}(\cos\theta^*) \Big/ \left|\frac{dE_\gamma^{1,2}}{d\cos\theta^*}\right| = \frac{1}{2}\frac{2}{m\beta\gamma} = \frac{1}{m\beta\gamma} = \frac{1}{p_{\pi^0}} \quad \text{flat in } \left[\frac{m\gamma}{2}(1-\beta), \frac{m\gamma}{2}(1+\beta)\right].}$$

$$\boxed{\mathcal{P}(\alpha) = \frac{1}{4\beta\gamma}\frac{\cos(\alpha/2)}{\sin^2(\alpha/2)\sqrt{\gamma^2\sin^2(\alpha/2) - 1}}}$$

[no proof, $\rightarrow$ FNSN1, §cinematica, 26].



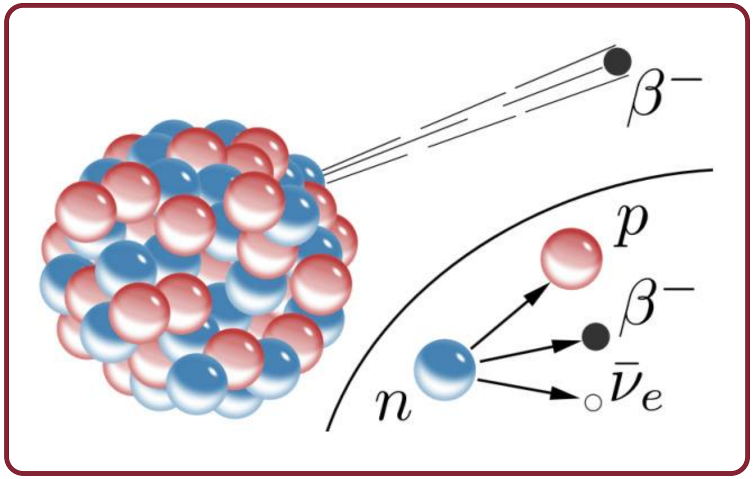


nota bene –
mutatis mutandis, similar kinematics also for $H \rightarrow \gamma\gamma$ [$\text{spin}(\pi^0)$ = spin(H) = 0].

β decay introduction

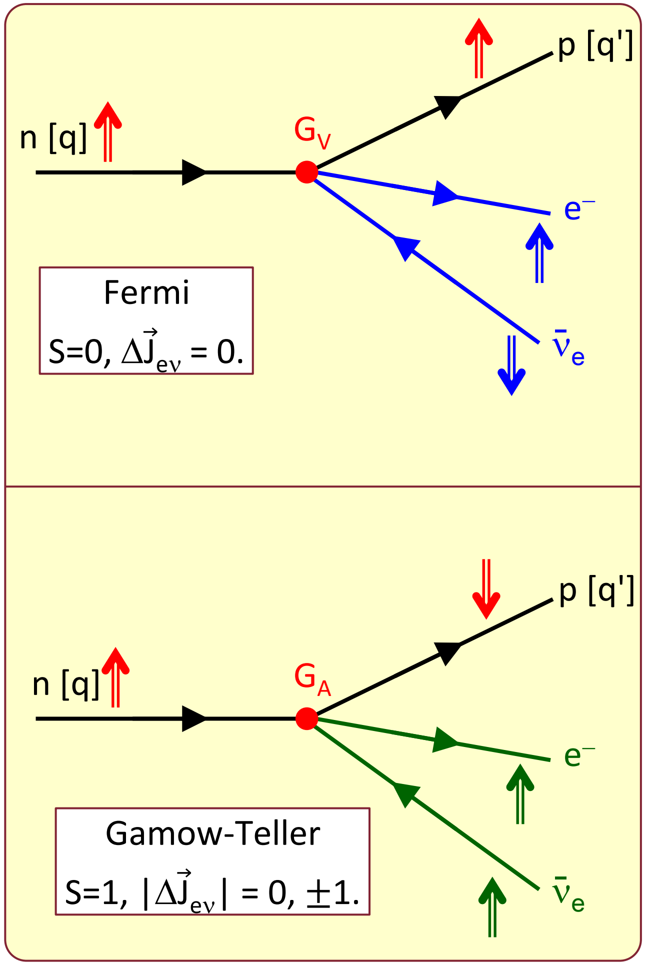
- For **point-like fermions**, CC is "V – A", both for leptons and quarks [*the only difference for hadrons being the CKM "rotation", see later*];
- however, nucleons and hyperons (p, n, Λ, Σ, Ξ, Ω) are **bound states of non-free quarks**;
- for low Q^2 processes, the "spectator model" (in this case the free quark decay) is an **unrealistic** approximation;
- **strong interaction corrections** are important → modify V – A dynamics;
- the standard approach, due to Fermi, is to produce a parameterization, based on the vector properties of the current (S-P-V-A-T, see) and then compute ↔ measure the coefficients;
- pros : quantitative theory, which reproduces the experiments well;
- cons : lack of deep understanding of the parameters.

the simple and successful approach, used for point-like decays, is not valid here, because of strong interaction corrections; those are (possibly understood, but) non-perturbative and impossible to master with present-day math; same as chemistry ↔ electromagnetism.



β decay: Fermi/Gamow-Teller

- In Fermi theory, CC currents were classified according to the properties of the transition operator.
- In neutron β-decay, the e-ν pair may be created as a spin singlet (S=0) or triplet (S=1). In case of NO orbital angular momentum, there are two possibilities to conserve the total angular momentum :
  - Fermi transitions [F], S=0, $\Delta J_{e\nu}=0$: the direction of the spin of the nucleon remains unchanged; in modern language, [*it can be shown that*] the interaction takes place with vector coupling G_V;
  - Gamow-Teller transitions [G-T], S=1, $\Delta J_{e\nu}= 0, \pm 1$: the direction of the spin of the nucleon is turned upside down (it "flips"); […] the transition happens with axial-vector coupling G_A.
- In principle, F and G-T processes are completely different : there is no a-priori reason why the coupling should be similar or even related.



β decay: S, P, V, A, T

- Study the **neutron β decay**; assume :
  - p and n are spin-½ fermions;
  - $e^{\pm}$ and ν are spin-½ fermions, but only ν's with "– helicity" exist [interact].

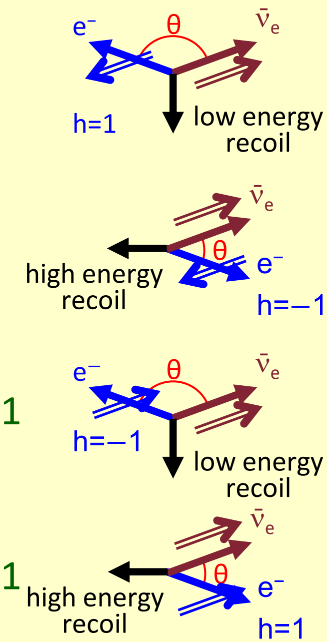
- Then, the most general matrix element for the four-body interaction is

$$\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \sum_j C_j \left[\bar{u}_p O_j u_n \right] \left[\bar{u}_e O_j \left(\frac{1-\gamma_5}{2} \right) u_\nu \right],$$

  - G_F : the overall coupling;
  - $\bar{u}_{p,n,e,\nu}$ ($u_{p,n,e,\nu}$) : creation (destruction) operators for p, n, e, ν;
  - $(1-\gamma_5)/2$: projector of –ve ν helicity;
  - C_j : sum coefficients (adimensional free parameters, *possibly of order 1*);
  - O_j : current operators with given vector properties : **S** = scalar, **P** = pseudo-scalar, **V** = vector, **A** = axial-vector, **T** = tensor.

- For β-decay, the pseudo-scalar term is irrelevant : P can only be built from the proton velocity v_p in the neutron rest frame, which are depressed by v_p/c;

- For the other four terms, the angular distributions are [*BJ 399, YN1 561*] (1, ⅓ for singlet and triplet, β=electron velocity) :



- **S** ΔJ=0 1–βcos θ
- **V** ΔJ=0 1+βcos θ
- **A** |ΔJ|=1 1–⅓βcos θ
- **T** |ΔJ|=1 1+⅓βcos θ

β decay: V, A

- From comparison with data, some terms can be excluded:
  - (S and V) are Fermi transitions : they cannot be both present, due to the lack of observed interference between them;
  - (A and T) are G-T transitions : same argument holds;
  - the angular distributions of the electrons are only consistent with V for F and A for G-T.

- So the matrix element becomes :

$$\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \left[\bar{u}_p \gamma^\mu \left(C_V + C_A \gamma_5 \right) u_n \right] \left[\bar{u}_e \gamma^\mu \left(\frac{1-\gamma_5}{2} \right) u_\nu \right],$$

- the value of C_V can be measured by comparing (<u>composite</u>) hadrons with (<u>free, pure V–A</u>) leptons; it turns out

$$C_V \approx 1.$$

- The value of $(C_A)^2$ can be measured from the relative strength of F and G-T, by comparing neutron β–decay with a pure Fermi ($^{14}O \to {}^{14}N\ e^+\nu$); for β decay:

$$|C_A| \cong 1.267.$$

- The sign of C_A could be measured from the polarization of the protons (a very difficult measurement); in practice from the interference between F and G-T in polarized neutrons decays :

$$C_A \cong -1.267.$$

Fermi did not know about parity violation, and would have written different matrix elements for his ("Fermi") transitions.

However, the final result for leptons and free quarks is very similar to his original proposal, but the factor $(1-\gamma_5)/2$ *:*

$$\mathcal{M}_{fi} = \frac{G}{\sqrt{2}} \left[\bar{u}_p \gamma^\mu \left(\frac{1-\gamma_5}{2} \right) u_n \right] \left[\bar{u}_e \gamma^\mu \left(\frac{1-\gamma_5}{2} \right) u_\nu \right].$$

β decay: CVC, PCAC

Focus on the hadron current $\propto [C_V + C_A\gamma_5]$:

- for leptons $C_A = -C_V$, i.e. "V–A" [much simpler, because leptons are free];
- for quarks, when no spectators are present, as in $\pi^\pm$ decays, similar picture (but CKM corrections);
- for composite hadrons, the picture works when their partons (quarks) interact as "quasi-free" particles;
- e.g. the "spectator approximation" works well in ν DIS and in hadron colliders, where the CC looks "V–A" as well;
- however, **at low Q^2** hadrons behave as coherent particles and not as parton containers → "V–A" is **not** valid.

$$\boxed{\mathcal{M}_{fi} \propto \left[\bar{u}_p\gamma^\mu\left(1+\frac{C_A}{C_V}\gamma_5\right)u_n\right]\left[\bar{u}_e\gamma^\mu(1-\gamma_5)u_\nu\right]}$$

- In low Q^2 processes, [*it can be shown that*] the vector part of the hadronic current stays constant (**CVC**, conserved vector current), while the axial part is broken (**PCAC**(*), "*partially conserved axial current*").
- In baryon β-decays, it is measured :
  - $n \to p\, e\, \bar{\nu}_e$, $C_A/C_V = -1.267$
  - $\Lambda \to p\, \pi^-,\ n\, \pi^0$ $= -0.718$
  - $\Sigma^- \to n\, e\, \bar{\nu}_e$ $= +0.340$
  - $\Xi^- \to \Lambda e^- \bar{\nu}_e$ $= -0.25$
  - [high Q^2 (free quarks) $= -1$].

____

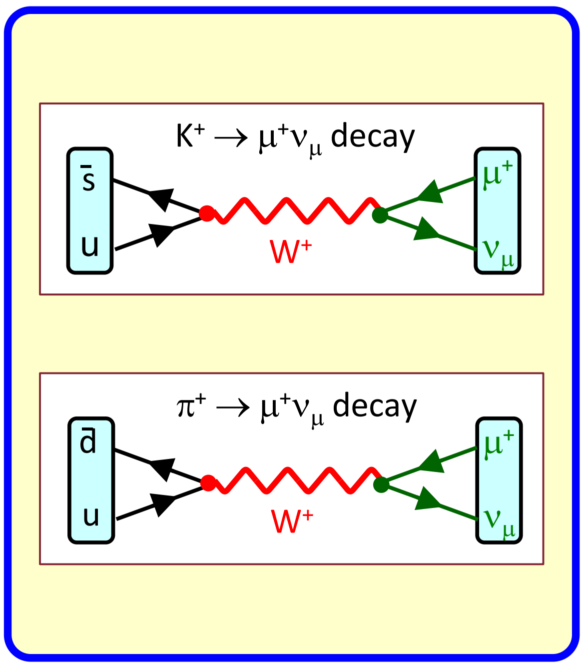
(*) *at the time, they preferred to say "partially conserved" instead of "badly broken"; it now seems that the acronym "PCAC" is slowly disappearing from the texts : you are kindly requested to forget the term "PCAC" forever.*

Quark decays: the puzzle

- For high mass quarks and at high Q^2, the structure "V−A" seems restored: quarks behave as free, point-like particles, exactly like the leptons [*Coll.Phys.*] .

- However, with more accurate data, some discrepancies appear, not due to strong interactions (see boxes).
- An apparent violation of CC universality ? A mistake ?

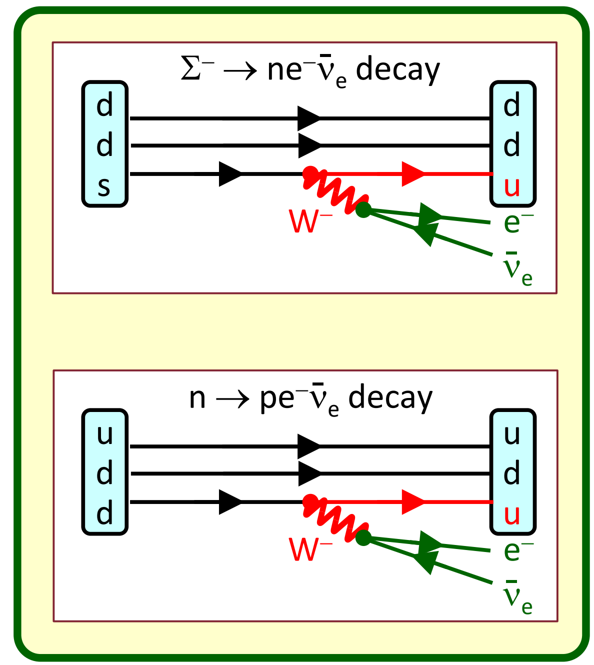
(*continue…*)



It is measured :

$$\frac{G_F^{2'}\left[\begin{matrix} K^+ \to \mu^+\nu_\mu, \\ \Delta S = 1 \end{matrix}\right]}{G_F^{2''}\left[\begin{matrix} \pi^+ \to \mu^+\nu_\mu, \\ \Delta S = 0 \end{matrix}\right]} \approx 0.05;$$

$$\frac{\Gamma\left[\begin{matrix} \Sigma^- \to n e^- \bar{\nu}_e, \\ \Delta S = 1 \end{matrix}\right]}{\Gamma\left[\begin{matrix} n \to p e^- \bar{\nu}_e, \\ \Delta S = 0 \end{matrix}\right]} \approx 0.05.$$



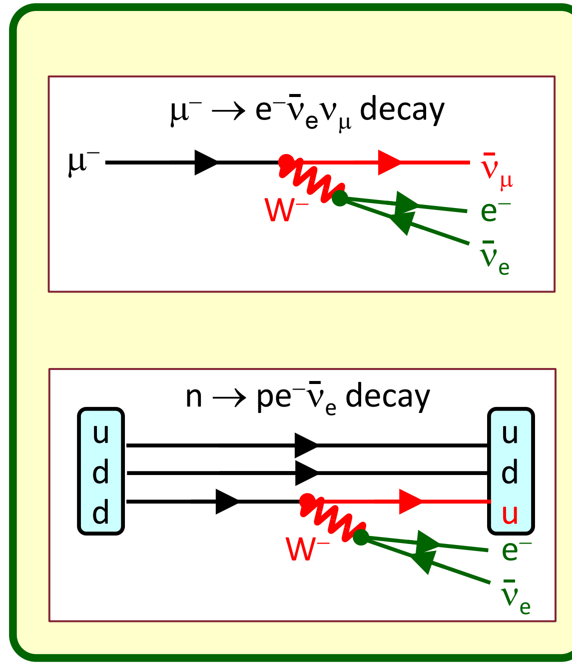
Quark decays: Cabibbo theory

(*... continue ...*)

Even tiny, but well measured effects seem to contradict the universality; "G_F" is slightly larger for leptons :

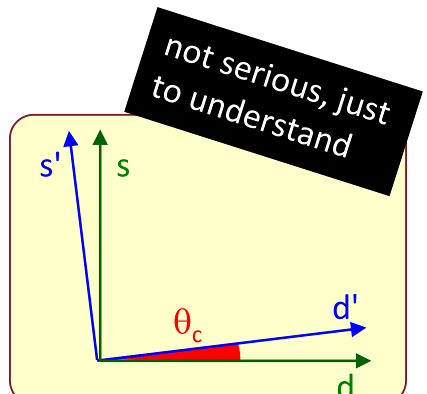
$$G_F\left[\mu^- \to e^- \bar{\nu}_e \nu_\mu\right] \approx 1.166 \times 10^{-5}\ \text{GeV}^{-2};$$

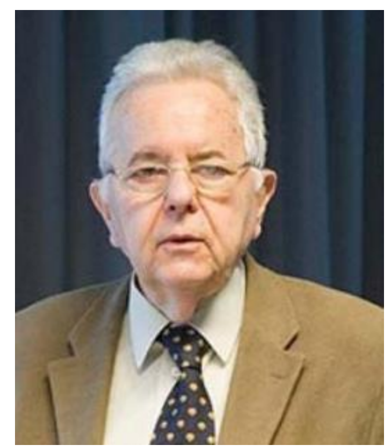
$$G_F\left[\begin{array}{r} n \to p e^- \bar{\nu}_e, \\ \text{i.e. } d \to u e^- \bar{\nu}_e \end{array}\right] \approx 1.136 \times 10^{-5}\ \text{GeV}^{-2}.$$



In 1963 N. Cabibbo [*at the time much younger than in the image*], invented a theory to explain the effect : the "Cabibbo angle" θ_c :

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}.$$





Quark decays: Cabibbo rotation

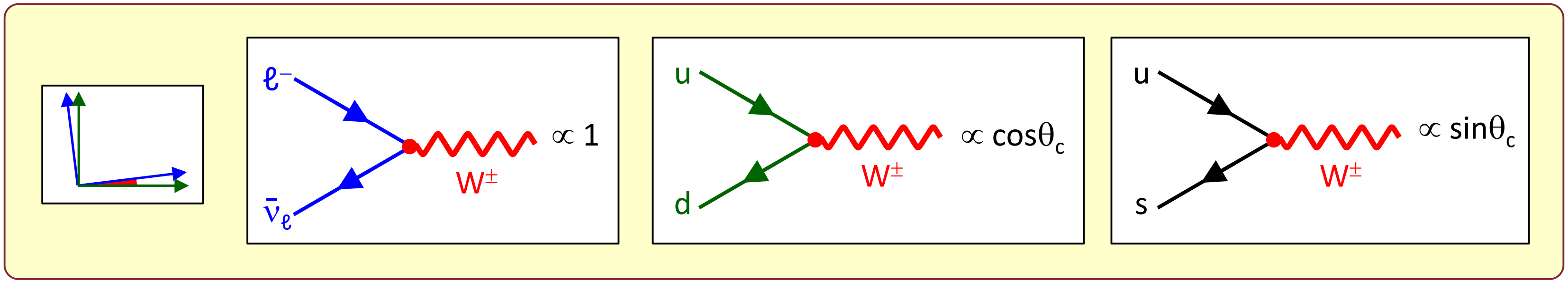
The idea was the following :

- the hadrons are built up with quarks **u d s** (**c b t** not yet discovered);
- however, in the CC processes, the quarks (d s) – same quantum numbers but S – mix together (= "rotate" by an angle θ_c), in such a way that the CC processes see "rotated" quarks (d' s') :

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}.$$

- therefore, respect to the strength of the leptonic processes (no mix), the ud coupling is decreased by $\cos\theta_c$ and the us coupling by $\sin\theta_c$, since the real process in ud', not ud or us.
- therefore the processes with $\Delta S = 0$ happen $\propto \cos^2\theta_c$ and those with $\Delta S = 1$ $\propto \sin^2\theta_c$;
- even processes $\propto \sin^4\theta_c$ may happen (e.g. in the charm sector, see §3), when two "Cabibbo suppressed" couplings are present in the same process;
- all the anomalies come back under control if

$$\sin^2\theta_c \approx .03, \cos^2\theta_c \approx .97.$$



Quark decays: GIM mechanism

In this context the GIM mechanism was invented to explain the absence of FCNC:

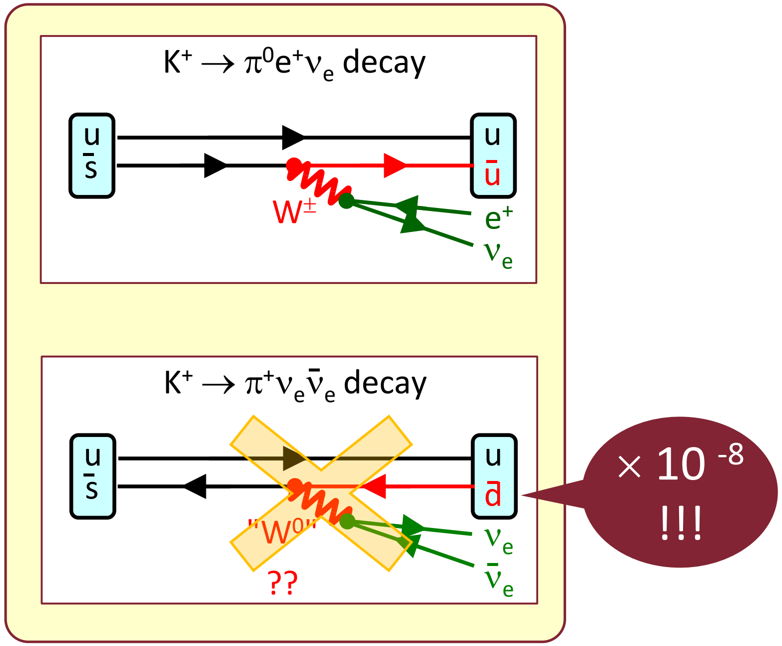
- data, at the time not understandable :

$$\left.\begin{aligned} BR\left(K^0 \to \mu^+\mu^-\right) &= 7\times10^{-9} \\ BR\left(K^+ \to \mu^+\nu_\mu\right) &= 0.64 \end{aligned}\right\} \left[\begin{matrix}\text{already} \\ \text{mentioned}\end{matrix}\right];$$

$$\left.\begin{aligned} BR\left(K^+ \to \pi^+\nu\bar{\nu}\right) &= \left(1.5^{+1.3}_{-0.9}\right)\times10^{-10} \\ BR\left(K^+ \to \pi^0 e^+\nu_e\right) &= \left(4.98\pm0.07\right)\times10^{-2} \end{aligned}\right\}.$$

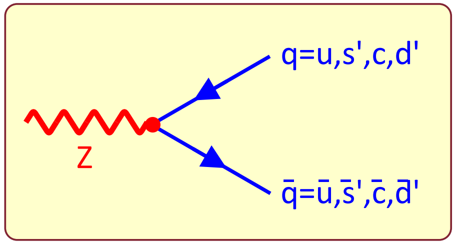
  i.e. a factor ~10^{-8} between NC and CC decays;

- if the Z, carrier of NC, see the same quark mixture as the $W^\pm$ in CC, then the NC decay would be suppressed only by a factor 5%;
- the idea was to introduce a fourth quark, called c (charm), with charge ⅔, as the u quark; this solves the FCNC problem;
- the c quark was discovered in 1974 [*see* § *3*].



Quark decays: no FCNC

In the GIM mechanism, NC contain four hadronic terms, coupled with the Z.



Assume Cabibbo theory and sum all terms:

$$u\bar{u} + d'\bar{d}' + c\bar{c} + s'\bar{s}' =$$
$$= u\bar{u} + (d\cos\theta_c + s\sin\theta_c)(\bar{d}\cos\theta_c + \bar{s}\sin\theta_c) +$$
$$+ c\bar{c} + (s\cos\theta_c - d\sin\theta_c)(\bar{s}\cos\theta_c - \bar{d}\sin\theta_c) =$$
$$= u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s} + \text{"0"}. \qquad (!!!)$$

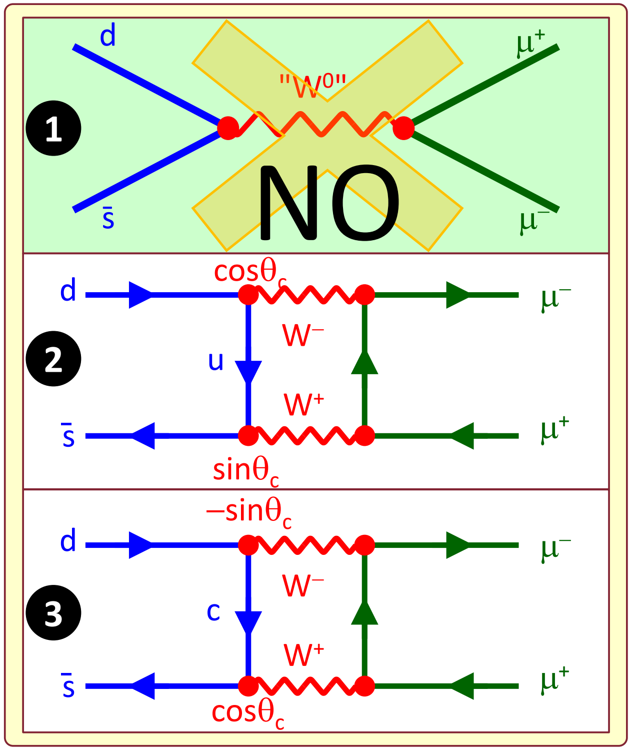
the "non-diagonal" terms, which induce FCNC, **disappear**.

Why $(K^0 \to \mu^+\mu^-)$ is small, but NOT = 0 ?

Look at the "box diagrams" ❷;

- technically a 2^{nd} order ($\propto g^4 \sin\theta_c \cos\theta_c$) CC;
- same final state as a 1^{st} order FCNC ❶;
- incompatible with data (BR too large);
- cured by the diagram ❸ with a c quark, whose contribution cancels the first in the limit $m_c \to m_u$.

The cancellation depends on m_c. The data on $(K^0 \to \mu^+\mu^-)$ put limits on m_c between 1 and 3 GeV [$J/\psi \to 2m_c \approx 3.1$ GeV, *see*].



Quark decays: third family

In 1973, Kobayashi and Maskawa extended the Cabibbo scheme to a new generation of quarks : the new mixing matrix (analogous to the Euler matrix in ordinary space) is a three-dimension unitary matrix, with three real parameters ("Euler angles") and one imaginary phase :

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \updownarrow W^{\pm}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The matrix is known as **CKM** (*Cabibbo-Kobayashi-Maskawa*) matrix.

K-M observed that the $\mathbb{CP}$ violation, already discovered, is automatically generated by the matrix, when the imaginary phase is non-zero.

In addition to the $\mathbb{CP}$-violation, the nine elements of the CKM matrix govern the flavor changes in CC processes.

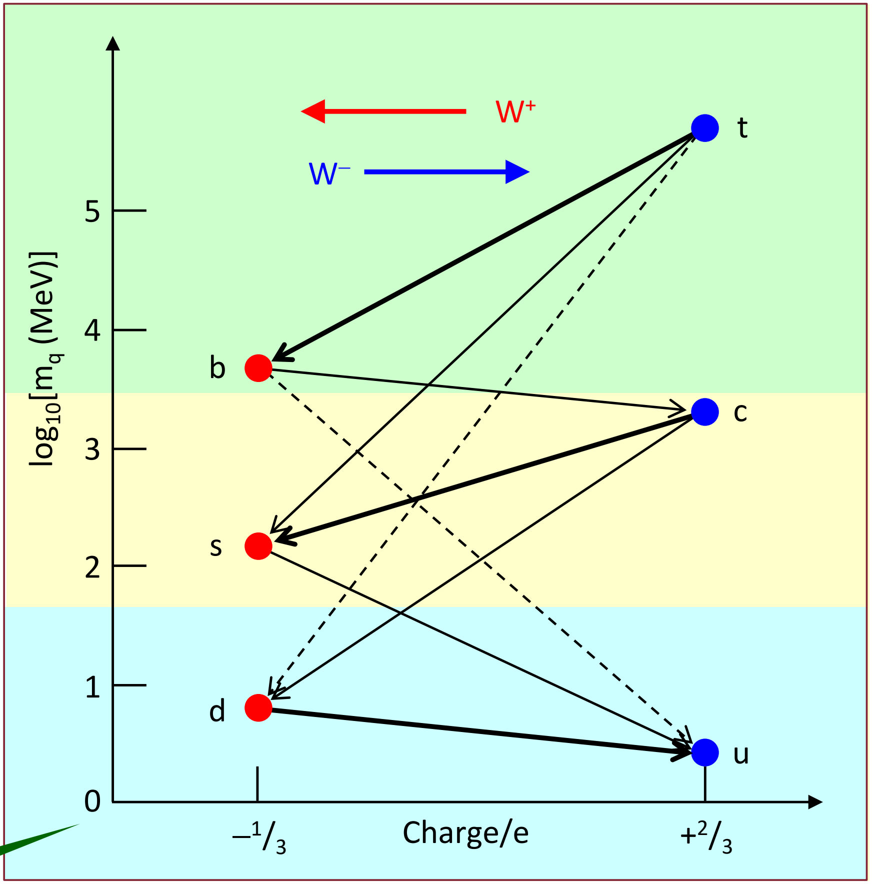
The measurement of the elements and the check of the unitarity relations is an important subject of physics studies : e.g. if some element is too small, this could be an indication of term(s) missing in the sum, i.e. the presence of a next generation of quarks.

[*A discussion of the CKM matrix in* §*5.*]

Makoto Kobayashi Toshihide Maskawa

Summary: charged current decays

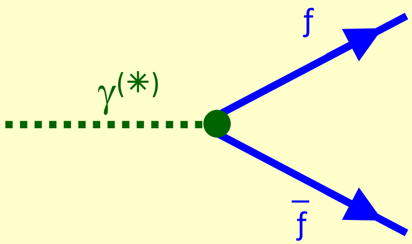
- The quark flavor changes only as a consequence of a weak CC interaction (*).
- Each type of quark can convert into each other with charge ±1, emitting or absorbing a W boson.
- The coupling is modulated by the strength of the mixing (the width of the line in fig.); in the SM it is described by the V_{CKM} matrix [§5].

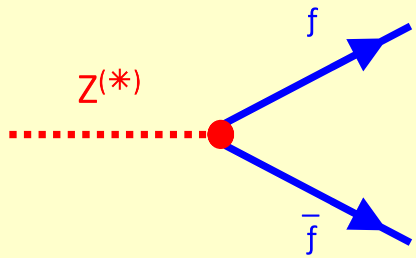


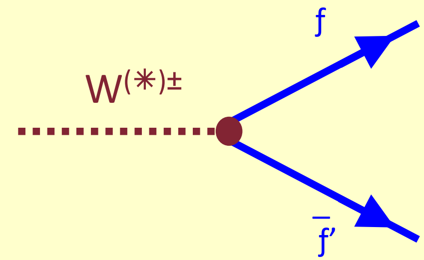
(*) since FCNC do NOT [seem to] exist, NC processes – with Z mediators – do NOT play any role in flavor decays.

+ the equivalent table for q̄'s.

Summary: EM, NC, CC







photon (γ)
(electromagnetism)

$$\mathscr{L}_F = -eJ^\mu_{e.m.} A_\mu;$$
$$J^\mu_{e.m.} = Q_f \bar{\Psi}_f \gamma^\mu \Psi.$$

[V]

neutral IVB (Z)
(neutral current)

$$\mathscr{L}_F = \frac{-e}{\sin\theta_W \cos\theta_W} J^\mu_{nc} Z_\mu;$$
$$J^\mu_{nc} = \bar{\Psi}_f \gamma^\mu \frac{g^f_V - g^f_A \gamma^5}{2} \Psi_f.$$

[combination g^f_V V + g^f_A A]

charged IVB ($W^\pm$)
(charged current)

$$\mathscr{L}_F = \frac{-e}{\sqrt{2}\sin\theta_W} J^\mu_{cc} \tau^\pm W^\pm_\mu;$$
$$J^\mu_{cc} = \bar{\Psi}_f \gamma^\mu \frac{1-\gamma^5}{2} \Psi_f.$$

[V - A]

Vectors & Co

vector properties of physical quantities :

- a 4-vector $\vec{\mathbf{v}}$ is the well-known quantity, which transforms canonically under a L-transformation $\mathbb{L}$ (both boosts and rotations), and Parity $\mathbb{P}$ in space :
  - ➢ space-time, 4-momentum, electric field, …
- an axial vector $\vec{\mathbf{a}}$ transforms like a vector under $\mathbb{L}$, but gains an additional sign flip under $\mathbb{P}$:
  - ➢ cross-products $\vec{v} \times \vec{v}'$, magnetic field, angular momentum, spin, …
- a scalar **s** is invariant both under $\mathbb{L}$ and $\mathbb{P}$:
  - ➢ [4-]dot-products $\vec{v} \cdot \vec{v}'$ or $\vec{a} \cdot \vec{a}'$, module of a vector, mass, charge, …
- a pseudoscalar **p** is invariant under $\mathbb{L}$, but changes its sign under $\mathbb{P}$:
  - ➢ a triple product $\vec{v} \cdot \vec{v}' \times \vec{v}''$;
  - ➢ a scalar product $\vec{a} \cdot \vec{v}$ between a vector and an axial vector, e.g. the helicity(*);
- a tensor **t** is a quantity which also transforms canonically under $\mathbb{L}$ and $\mathbb{P}$, with ≥ 2 dimensions :
  - ➢ the electro-magnetic tensor $F^{\mu\nu}$.

____

(*) the helicity h is the projection of the spin $\vec{s}$ along the momentum $\vec{p}$:

$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}| \cdot |\vec{p}|}.$$

Q. : this "parity violation" does NOT happen. Why ?

Helicity vs Chirality

Two different concepts:

- h for a particle is defined from its spin and momentum[1];
- χ is a spinor property[2], related to the eigenstates of γ_5.

➢ The χ operator γ_5 does NOT commute with the mass term of the free Hamiltonian, so χ is NOT conserved for a massive particle;

➢ a massive particle with definite spin and momentum has a definite h, but is a mixture of the two eigenstates of χ;

➢ for a massless particle (or in the u.r.a. approximation) χ is conserved and its value reduces to h;

➢ this approximation is generally valid in this chapter, so the slides do not stress the difference $h \leftrightarrow \chi$.

---

[1] $h = \vec{s}\cdot\vec{p}/(|\vec{s}|\ |\vec{p}|)$; sometimes $h = \vec{s}\cdot\vec{p}/|\vec{p}|$; however, the different definition does not affect the difference $h \leftrightarrow \chi$.

[2] define the projectors:

$$\psi_R = \tfrac{1}{2}(1+\gamma_5)\psi; \qquad \psi_L = \tfrac{1}{2}(1-\gamma_5)\psi;$$

$$\gamma_5\psi_R = +\psi_R; \qquad \gamma_5\psi_L = -\psi_L;$$

$\psi_{R,L}$: eigenstates of χ with eigenvalues ± 1.

References:

[Povh, 10.5], [Bettini, 7.4], [YN1, 4.3.5]