THE STATIC QUARK MODEL

Outline

- Symmetry and Quantum numbers
- Hadrons : elementary or composite ?
- The eightfold way
- The discovery of the Ω^-
- The static quark model
- The mesons
- Meson quantum numbers
- Meson mixing
- The baryons
- SU(3)
- Color

Particle classification

- In the 1950s, new particles and resonances were discovered which were themselves regarded as new particles.
- An attempt was made to classify all these particles in such a way as to reveal their true nature (a similar work was done by Rydberg who found the formula to describe atomic spectra, or by Mendeleiev)
- A first symmetry found was associated with isotopic spin; particles with the same isospin are exactly the same particle for strong interactions, but e.m. interactions break the symmetry and cause a mass difference of a few% between the particles of the same multiplet.

Hadrons : "elementary" or composite?

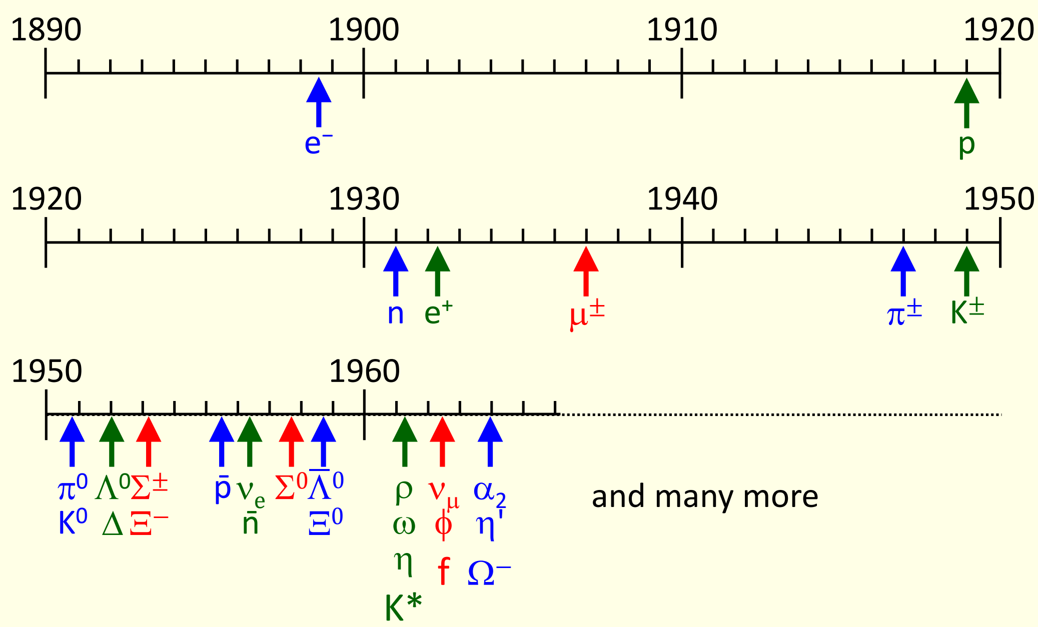
Over time the very notion of "*elementary* (???) particle" entered a deep crisis.

The existence of (too) many hadrons was seen as a contradiction with the elementary nature of the fundamental component of matter.

It was natural to interpret the hadrons as consecutive resonances of elementary components.

The main problem was then to measure the properties of the components and possibly to observe them.

[... and the leptons ? ...]

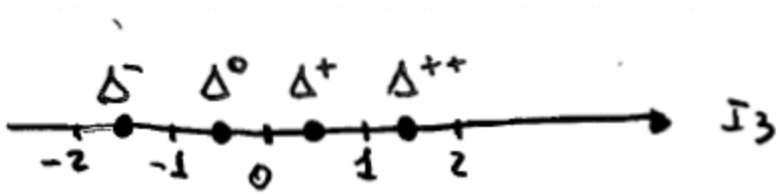


too many hadronic states: resonances ?

the figure shows the particle discoveries from 1898 to the '60s; their abundance and regularity, as a function of quantum numbers like charge and strangeness, were suggesting a possible sequence, similar to the Mendeleev table.

Particle Classification

- To extend the symmetry, an attempt was made to group several isospin multiplets into a larger group that had the same spin and parity but with different strangeness (or hypercharge).
- There are other possible a priori choices, such as same oddity but different spin and parity, but these don't work.
- The components of the isospin multiplets are represented as spaced points of a unit on the horizontal axis I_3. For example for the Δ (1232) we have:



$$\boxed{Q = I_3 + \frac{1}{2}(B+S)}$$

Hadrons : “elementary” or composite?

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 76, NO. 12 DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The ... mesons may be composite particles formed by the association of a nucleon with an anti-nucleo... extremely crude discussion of the model it appears that such a meson would have in most ...ies similar to those of the meson of the Yukawa theory.

Phys.Rev.76, 1739.

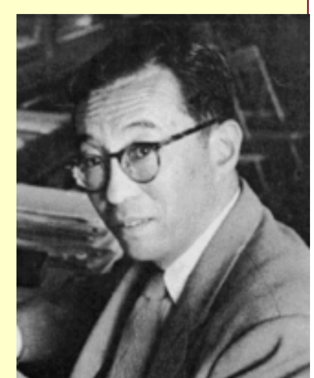
1949 : E.Fermi and C.N. Yang proposed that ALL the resonances were bound state p-n.

1956 : Sakata extended the Fermi-Yang model including the Λ, to account for strangeness : all hadronic states were then composed by (p, n, Λ) and their antiparticles.

Enrico Fermi

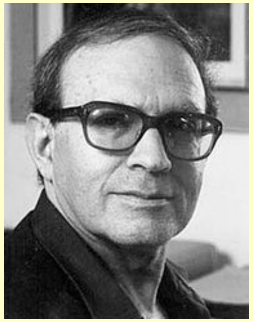
Chen-Ning Yang
(杨振宁 - 楊振寧,
Yáng Zhènníng)

Shoiki Sakata
(坂田 昌一,
Sakata Shōichi)

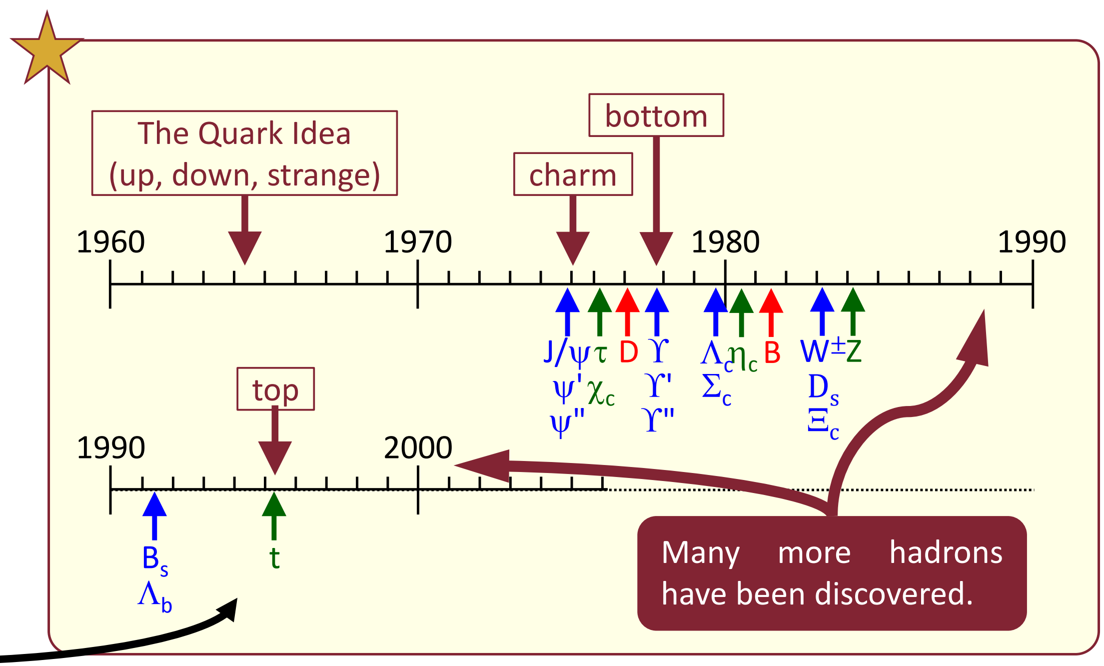


Hadrons : “elementary” or composite?

1961 : M. Gell-Mann and Y. Ne’eman (independently) proposed a new classification, the **Eightfold Way**, based on the symmetry group SU(3). The classification did **NOT** explicitly mention an **internal structure**. The name was invented by Gell-Mann and comes from the “eight commandments” of the Buddhism.



Yuval Ne'eman
(יובל נאמן)



Warning : "*t*" is a quark, not a hadron (in modern language).

Simmetries and groups (I)

Illustrative example: the rotation group

- Two successive rotations R_1 followed by R_2 are equivalent to a single rotation $R=R_2R_1$. The group is closed under multiplication.
- There is an identity element (no rotation).
- Every rotation R has an inverse R^{-1} (rotate back again).
- The product is not necessarily commutative $R_1R_2 \neq R_2R_1$, but the associative law always holds: $R_3(R_2R_1) = (R_3R_2)R_1$.
- It is a continuous group: each rotation can be labeled by a set of continuously varying parameters $(\alpha_1, \alpha_2, \alpha_3)$ which can be regarded as the components of a vector $\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \alpha_3)$ directed along the axis of rotation with magnitude given by the angle of rotation.
- The rotation group is a Lie group: every rotation can be expressed as the product of a succession of infinitesimal rotations (arbitrarily close to the identity). The group is then completely defined by the “neighborhood of the identity”.

Simmetries and groups (II)

- Rotations are a subset of the Lorentz transformations and they form a symmetry group of a physical system: Physics is invariant under rotations.
  For example suppose that under a rotation R the state of a system transforms as
  $$|\psi\rangle \xrightarrow{R} |\psi'\rangle = U|\psi\rangle$$
  Probabilities must be unchanged by R:
  $$|\langle\phi|\psi\rangle|^2 = |\langle\phi'|\psi'\rangle|^2 = |\langle\phi|U^+U|\psi\rangle|^2$$
  $U^+U = 1$, U must be a unitary operator.
  The operators *U(R)* form a group, with exactly the same structure as the original group (R_1,R_2,): they are said to form a unitary representation of the rotation group.
- The Hamiltonian is unchanged by a symmetry operation R of the system and the matrix elements are preserved

$$\langle\phi'|H|\psi'\rangle = \langle\phi|U^+HU|\psi\rangle = \langle\phi|H|\psi\rangle \qquad U^+HU = H \qquad [U,H] = 0$$

Simmetries and groups (III)

- The transformation U has no explicit time dependance and the equation of motion

$$i\frac{d}{dt}\left|\psi(t)\right\rangle = H\left|\psi(t)\right\rangle$$

is unchanged by the the symmetry operation. As a consequence the expectation value of U is a constant of the motion

$$i\frac{d}{dt}\left\langle\psi\right|U\left|\psi\right\rangle = \left\langle\psi\right|UH - HU\left|\psi\right\rangle = 0$$

Simmetries and groups (IV)

- All group properties follow from considering infinitesimal rotations in the neighborhood of the identity. Example rotation through ε around the 3-axis:

$$U = 1 - i\varepsilon J_3$$

J_3 is called the generator of rotations around the 3-axis.

$$\begin{aligned} 1 = U^+U &= (1 + i\varepsilon J_3^+)(1 - i\varepsilon J_3) \\ &= 1 + i\varepsilon(J_3^+ - J_3) + O(\varepsilon^2) \end{aligned}$$

$J_3^+ = J_3$ is therefore hermitian, and hence is an observable.

- Consider the effect of a rotation *R* on the wave function. Invariance under rotations requires:

$$\psi'(\vec{r}) = \psi(R^{-1}\vec{r}) = U\psi(\vec{r})$$

For an infinitesimal rotation ε around the 3-axis:

$$\begin{aligned} U\psi(x,y,z) &= \psi(R^{-1}\vec{r}) = \psi(x + \varepsilon y, y - \varepsilon x, z) \\ &= \psi(x,y,z) + \varepsilon\left(y\frac{\partial\psi}{\partial x} - x\frac{\partial\psi}{\partial y}\right) \\ &= \psi\left[1 - i\varepsilon(xp_y - yp_x)\right] \end{aligned}$$

$$\boxed{U\psi = \left(1 - i\varepsilon J_3\right)\psi}$$

J_3 component along the 3-axis of the angular momentum. Invariance under rotations corresponds to the conservation of angular momentum.

Simmetries and groups (V)

For a rotation through a finite angle θ:

$$U(\theta) = [U(\varepsilon)]^n = \left(1 - i\frac{\theta}{n}J_3\right)^n \xrightarrow[n\to\infty]{} e^{-i\theta J_3}$$

The commutator algebra of the generators is:

$$\left[J_m, J_n\right] = i\varepsilon_{mnl}J_l$$

The J's are said to form a Lie Algebra

ε_{ijk} = *structure constants of the group*

Nonlinear functions of the generators which commute with all the generators are called invariants or Casimir operators. For the rotation group the only Casimir operator is:

$$J^2 = J_1^2 + J_2^2 + J_3^2 \qquad [J^2, J_l] = 0 \quad l = 1,2,3$$

It follows that we can construct simultaneous eigenstates of J^2 and one of the generators, e.g. J_3:

$$J^2|j,m\rangle = j(j+1)|j,m\rangle$$

$$J_3|j,m\rangle = m|j,m\rangle$$

The group SU(2)

In the lowest-dimension nontrivial representation of the rotation group ($j=\frac{1}{2}$) the generators may be written:

$$J_k = \frac{1}{2}\sigma_k \quad k = 1,2,3 \quad \sigma_k = matrici\ di\ Pauli$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The basis for this representation is given by the eigenvectors of σ_3: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ describing a spin ½ particle of spin projections up and down.

The transformation matrices:

$$U(\theta_i) = e^{-i\theta_i \frac{\sigma_i}{2}}$$

are unitary. The set of all unitary 2×2 matrices is **U(2)** (**Unitary Group**). However U(2) is larger than the group $U(\theta_i)$, since the σ_i all have zero trace. For any hermitian traceless matrix σ it can be shown that:

$$\det e^{i\sigma} = e^{i\mathrm{Tr}(\sigma)} = 1$$

This property is preserved in matrix multiplication. The set of traceless unitary 2×2 matrices form a subgroup of U(2) called **SU(2)** (**Special Unitary**). The two-dimensional representation is the fundamental representation.

The group SU(2)

For a composite system $|j_A, j_B, m_A, m_B\rangle$ the operator $\mathbf{J}=\mathbf{J}_A+\mathbf{J}_B$ satisfies the Lie algebra and the eigenvalues $J(J+1)$ and M of J^2 and J_3 are conserved quantum numbers. The *product* of the two irreducible representations $(2j_A+1)$ and $(2j_B+1)$ may be decomposed into the sum of irreducible representations of dimension $(2J+1)$ with basis $|j_A, j_B, J, M\rangle$, where

$$|j_A - j_B| \leq J \leq |j_A + j_B| \qquad M = m_A + m_B$$

$$\left| j_A j_B J M \right\rangle = \sum_{m_A, m_B} C(m_A, m_B, J, M) \left| j_A j_B m_A m_B \right\rangle$$

Clebsch-Gordan coefficients

Example: from two two-dimensional representations (j=½) we obtain one 3-dimensional (J=1, triplet) and one 1-dimensional 1 (J=0, singolet) representation.

$$2 \otimes 2 = 3 \oplus 1$$

$$2 \otimes 2 \otimes 2 = (3 \otimes 2) \oplus (1 \otimes 2)$$

$$= \underset{\substack{\uparrow \\ \text{quadruplet} \\ \text{spin } 3/2}}{4} \oplus \underbrace{2 \oplus 2}_{\substack{\text{doublet} \\ \text{spin } 1/2}}$$

SU(2) Isospin

$$\left[I_m, I_n\right] = i\varepsilon_{mnl} I_l$$

Generators in the fundamental representation:

$$I_k = \frac{1}{2}\tau_k \qquad k = 1,2,3$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

basis

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$I = \tfrac{1}{2}, I_3 = +\tfrac{1}{2} \qquad I = \tfrac{1}{2}, I_3 = -\tfrac{1}{2}$$

Isospin for antiparticles

$$\begin{pmatrix} p' \\ n' \end{pmatrix} = e^{-i\pi\frac{\tau_2}{2}} \begin{pmatrix} p \\ n \end{pmatrix} = -i\tau_2 \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$$

$$Cp = \overline{p} \qquad Cn = \overline{n}$$

$$\begin{pmatrix} \overline{p}' \\ \overline{n}' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \overline{p} \\ \overline{n} \end{pmatrix}$$

In order for the antidoublet to transform in the same way as the doublet we must:
- Reorder the doublet
- Introduce a minus sign

$$\begin{pmatrix} -\overline{n}' \\ \overline{p}' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \overline{p} \\ \overline{n} \end{pmatrix}$$

For N $\overline{\text{N}}$

$$\begin{cases} \left| I=1, I_3=1 \right\rangle = -p\overline{n} \\ \left| I=1, I_3=0 \right\rangle = \sqrt{\tfrac{1}{2}}(p\overline{p} - n\overline{n}) \\ \left| I=1, I_3=-1 \right\rangle = n\overline{p} \end{cases}$$

$$\left| I=0, I_3=0 \right\rangle = \sqrt{\tfrac{1}{2}}(p\overline{p} + n\overline{n})$$

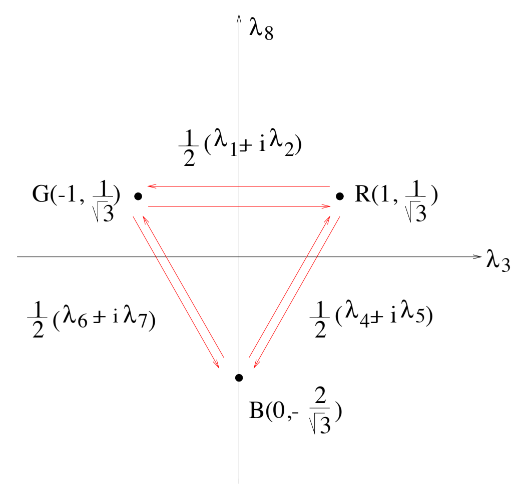
The group SU(3)

It is the group of unitary 3×3 matrices with det*U=1*. The generators may be taken to be any 3^2-1=8 linearly independent traceless hermitian 3×3 matrices. There are therefore 8 generators, of which 2 are diagonal. This is also the *maximum number of mutually commuting generators* : **Rank of the group**.
It can be shown that the rank of the group is equal to the number of Casimir operators.
The fundamental representation of SU(3) is a triplet (e.g. the three color charges of a quark). *The generators are 3×3 matrices*: *λ_i, i=1,..,8* (Gell-Mann matrices).

$$\lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

simultaneous eigenvectors of $\lambda_3$and λ_8.



Isospin and strangeness: flavour SU(3)

The introduction of a second additive quantum number S in addition to I_3 suggests to enlarge isospin symmetry to a larger group, a group of rank 2. In 1961 SU(3) was proposed. The assignment of particles to SU(3) multiplets is not straightforward due to the high mass differences between the various particles (strange and non strange).

For example, the baryonic octet group particles with mass differences up to *400 MeV*, over an average octet mass of *1100 MeV*.

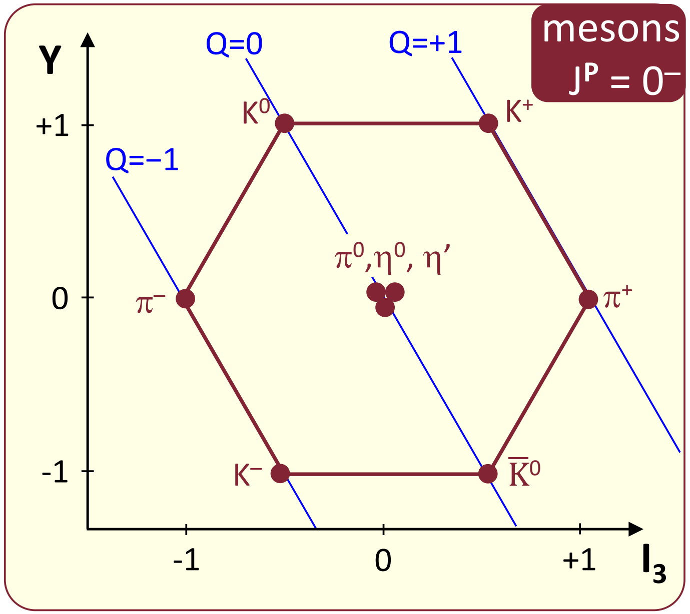
SU(3) flavor symmetry is much more approximate than SU(2) of isospin. We will see that this is due to the fact that the strange quark *s* is much heavier than *u* and *d*. SU(3) symmetry forms the basis of the quark model and it turns out to be very useful to classify hadrons and to understand some of their properties.

Color SU(3) on the other hand is an exact symmetry of fundamental origin.

The Eightfold Way: 1961-64

All hadrons (known in the '60s) are classified in the plane **($I_3 - Y$)**, (Y = strong hypercharge):

$I_3 = I_z$ = third component of isospin;
$Y = \mathscr{B} + S$ **[baryon number + strangeness]**.



The strangeness ***S***, which contributes to ***Y***, had the effect to enlarge the isospin symmetry group **SU(2)** to the larger **SU(3): Special Unitarity group**, with dimension=3.

The Gell-Mann – Nishijima formula (1956) was :

$Q = I_3 + ½(\mathscr{B}+S)$

including heavy flavors [$\mathscr{B}$:baryon, B:bottom] :

$Q = I_3 + ½(\mathscr{B}+S+C+B+T)$

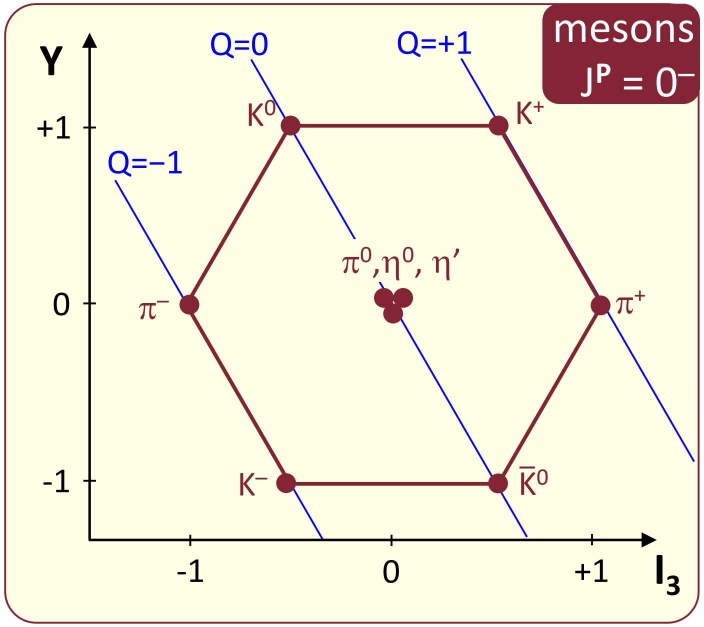
This symmetry is now called "**flavor SU(3) [$SU(3)_F$]**", to distinguish it from the "**color SU(3) [$SU(3)_C$]**", which is the exact symmetry of the strong interactions in QCD.

The Eightfold Way: SU(3)

The particles form the multiplets of **SU(3)$_F$**. Each multiplet contains particles that have the same spin and intrinsic parity. The basic multiplicity for mesons is nine ($3\times\overline{3}$), which splits in two SU(3) multiplets: (octet + singlet). For baryons there are octects + decuplets.

The gestation of SU(3) was long and difficult. It both explained the multiplets of known particles/resonances, and (more exciting) predicted new states, before they were actually discovered (*really a triumph*).

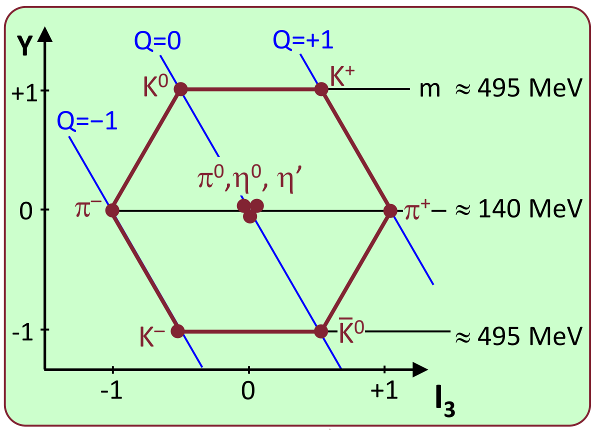
However, the mass difference p – n (or $\pi^\pm - \pi^0$) is < few MeV, while the π – K (or p – Λ) is much larger. Therefore, while the isospin symmetry **SU(2)** is almost exact, the symmetry **SU(3)$_F$**, grouping together strange and non-strange particles, is substantially violated.



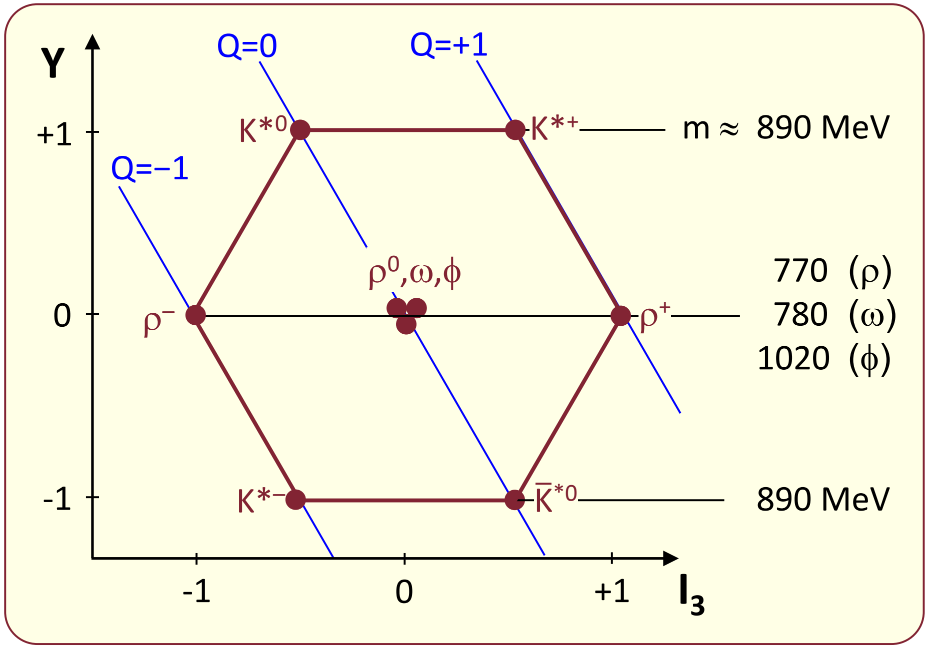
In principle, in a similar way, the discovery of heavier flavors could be interpreted with higher groups (e.g. **SU(4)$_F$** to incorporate the charm quark, and so on). However, these higher symmetries are broken even more, as demonstrated by the mass values. Therefore, **SU(6)$_F$** for all known mesons $J^P = 0^-$ is (almost) never used.

The Eightfold Way: mesons $J^P=1^-$

Another example of a multiplet: the octet of vector mesons :

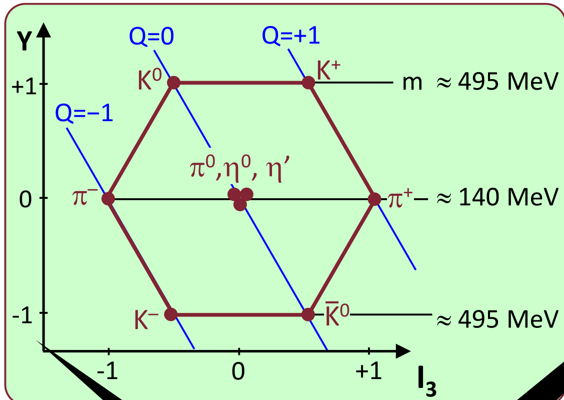


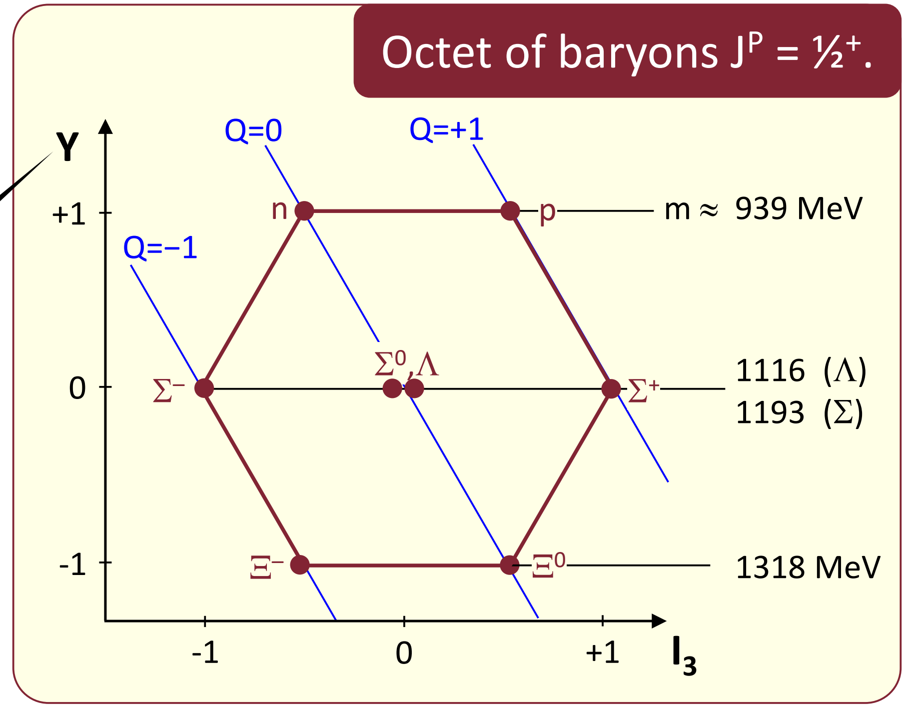
[mesons $J^P = 0^-$]



meson resonances $J^P = 1^-$ (all discovered by 1961).

The Eightfold Way: baryions $J^P=1/2^+$





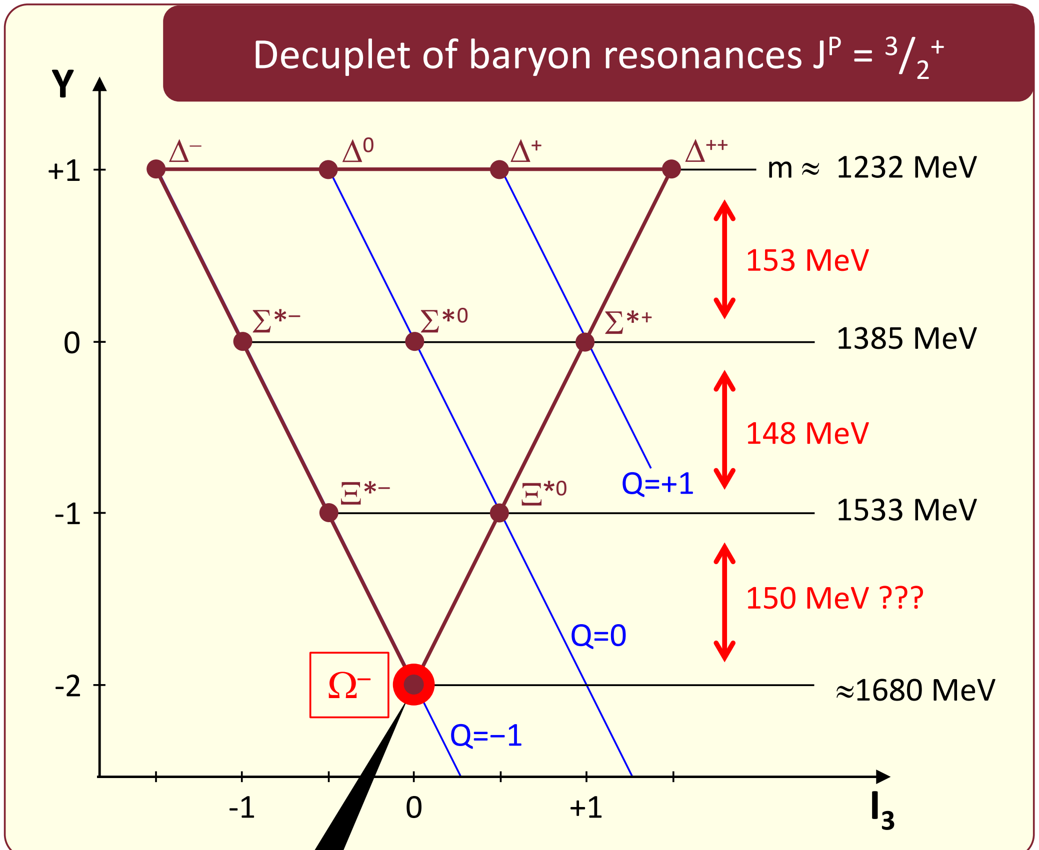
mesons: Y = S

baryons: $Y = S + \mathscr{B}$

notice the masses: for mesons, because of $\mathbb{CPT}$ ($K \leftrightarrow \bar{K}$) the masses of an octet are symmetric wrt (S=0, I_3=0), while for baryons the mass increases as –S

[*because the s-quark (S= –1) is heavier than u/d, but they did not know it*]

The Eightfold Way: baryons $J^P=3/2^+$



when the Eightfold Way was first proposed, this particle (now called Ω^-) was not known → see next slide.

The next multiplet of baryons is a decuplet $J^P = {}^3/_2{}^+$.

When the E.W. was proposed, they knew only 9 members of the multiplet, but can predict the last member:

- it is a decuplet, because of E.W.;
- the state $Y = -2$, $I_3 = 0$ ($\rightarrow Q = -1$, $S = -3$, $\mathcal{B}=1$) must exists;
- call it Ω^-;
- look the mass differences vs Y:
- mass linear in Y → $m_{\Omega^-} \approx 1680$ MeV (*NOT an E.W. requirement, but a reasonable assumption*);
- the conservation laws set the dynamics of production and decay of the Ω^-.

The discovery of the Ω^-

The particle Ω^-, predicted (★) in 1962, was discovered in 1964 by N.Samios et al., using the 80-inch hydrogen bubble chamber at Brookhaven (next slide).

The Ω^- can only decay weakly to an S = –2 final state [(1)] :

$\Omega^- \to \Xi^0 \pi^-$; $\to \Xi^- \pi^0$; $\to \Lambda^0 K^-$;

[a posteriori confirmed by the measurement $\tau_{\Omega^-} \cong 0.82 \times 10^{-10}$ s]

---

(1) Since the electromagnetic and strong interactions conserve the strangeness, the lightest (non-weak) S- and $\mathscr{B}$- conserving decay is :

$\Omega^- \to \Xi^0 K^-$ [S : –3 → –2 –1, $\mathscr{B}$: +1 → +1 +0]

which is impossible, because

$m(\Omega) \approx 1700$ MeV < $m(\Xi) + m(K) \approx 1800$ MeV.

Therefore the Ω^- must decay via strangeness-violating weak interactions : <u>the Ω^- lifetime reflects its weak (NOT strong NOR e.m.) decay</u>.

(★) ***<u>From a 1962 report:</u>***

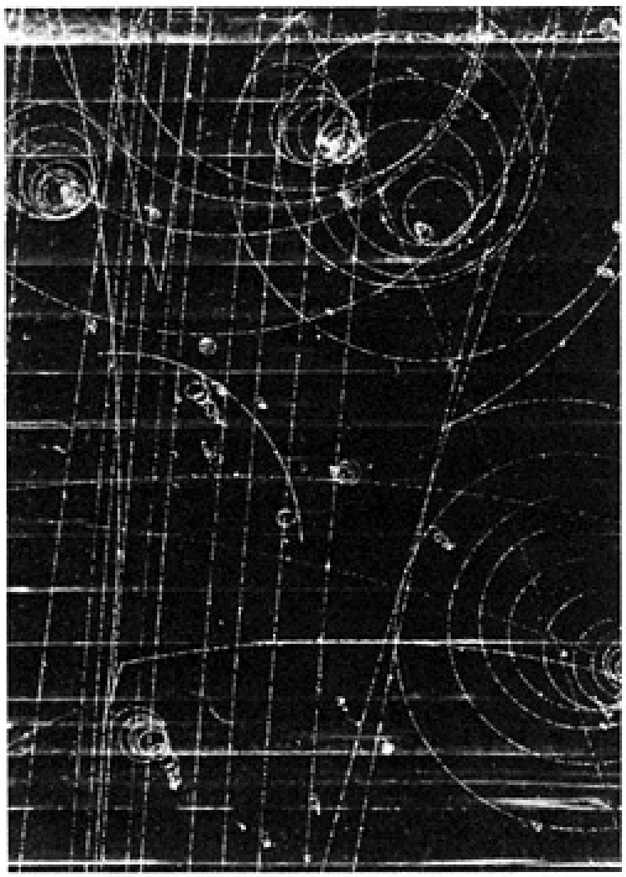
Discovery of Ξ^* resonance with mass ~1530 MeV is announced [...].

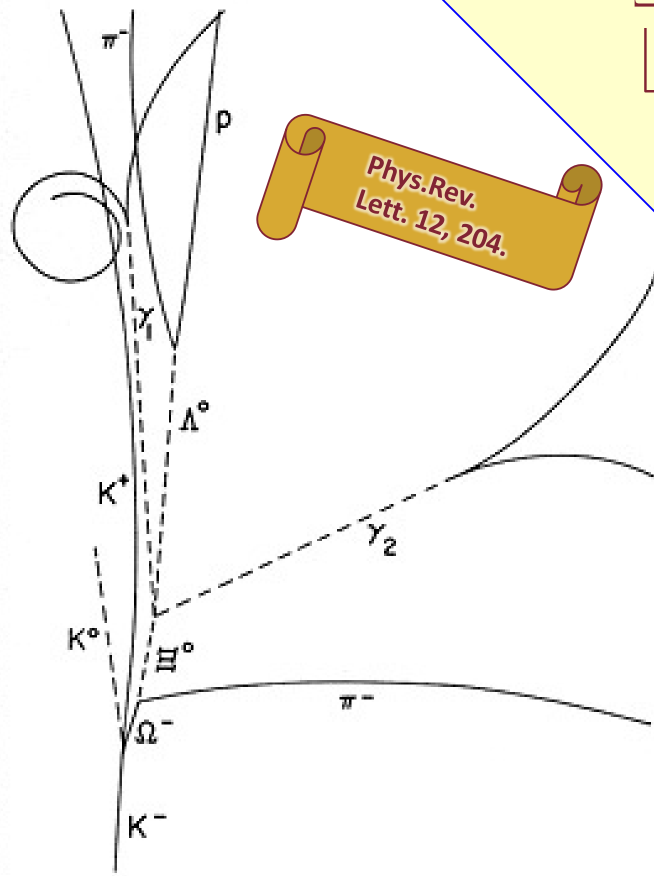
[*As a consequence,*] **Gell-Mann and Ne'eman** [...] predicted a new particle and all its properties:

- Name = Ω^- (*Omega* because this particle is the last in the decuplet);
- Mass ≈ 1680 MeV (the masses of Δ, Σ^* and Ξ^* are about equidistant ~150 MeV);
- Charge = –1;
- Spin = $^3/_2$;
- Strangeness = –3, Y = –2;
- Isospin = 0 (no charge-partners);
- Lifetime ~10^{-10} s, because of its weak decay, since strong decay is forbidden[(1)];
- Decay modes: $\Omega^- \to \Xi^0 \pi^-$ or $\Omega^- \to \Xi^- \pi^0$.

The discovery of the Ω^-: the event

the Ω^- observation required both genius and luck (e.g. compute the probability of the two γ conversions in H_2):





Phys.Rev. Lett. 12, 204.

$K^- + p \rightarrow \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)

$\quad\quad \hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ w.d.)

$\quad\quad\quad \hookrightarrow \pi^- + p$ ($\Delta S = 1$ w.d.)

$\quad\quad\quad \hookrightarrow \gamma + \gamma$ (e.m. decay)

$\quad\quad\quad\quad \hookrightarrow e^+e^-$

$\quad\quad\quad\quad \hookrightarrow e^+e^-$

Nick Samios

Brookhaven National Laboratory 80-inch hydrogen bubble chamber - 1964

The static quark model

In 1964 M. Gell-Mann and G. Zweig proposed independently that all the hadrons are composed of three constituents, that Gell-Mann called[1] quarks.

This model, enriched by both extensions (other quarks) and dynamics (electroweak interactions and QCD) is still the basis of our understanding of the elementary particles, the **Standard Model**[2].

In this chapter we consider only the static properties of the three original quarks. Sometimes, in the literature, it is referred as the *naïve quark model*.

1969 : Gell-Mann is awarded Nobel Prize *“for his contributions and discoveries concerning the classification of elementary particles and their interactions”*.

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[1] The name so whimsical was taken from the (now) famous quote "*Three quarks for Muster Mark !*", from James Joyce's novel "*Finnegans Wake*" (book 2, chapt. 4).

[2] At that time it was not clear whether the quark hypothesis was a mathematical convenience or reality. Today, as shown in the following, our understanding is clearer, but complicated: the quarks are real (to the extent that all QM particles are), but they cannot be seen as isolated single objects.

The static quark model

- They must be fermions in order to construct both fermions and bosons.
- In analogy to the idea of Fermi and Yang, mesons are $q_1 \bar{q}_2$ pairs; baryons (antibaryons) are $q_1q_2q_3$ ($\bar{q}_1 \bar{q}_3 \bar{q}_3$) states.
- In order to form nonstrange particles with charges 0,±1 at least two quarks are needed. These must form an isospin doublet (in order to have both *I=0* and *I=1*).
- In order to form strange particles a third quark is needed, to which by convention is assigned *S=-1*. *The minimal number of constituents is thus 3*.
- To properly account for baryon numbers quarks are assigned *B=1/3*.
- The simplest spin-parity assignment is $J^P=\frac{1}{2}^+$.
- From the Gell-Mann and Nishijima formula $Q = I_3 + \frac{B+S}{2}$ it follows

$$Q(u) = +\frac{2}{3} \qquad Q(d) = Q(s) = -\frac{1}{3}$$

Quarks have a **fractional electric charge.**

The static quark model

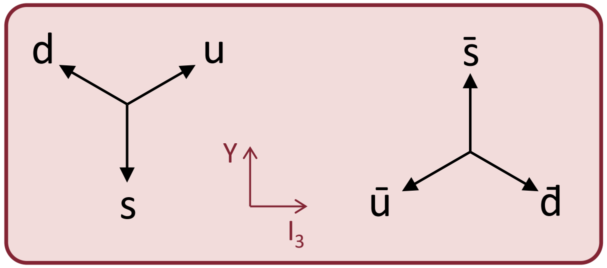
The hypothesis:

- three quarks **u**, **d**, and **s** (up, down, strange);
- quarks (q): standard Dirac fermions with spin ½ and fractional charge (±⅓e ±⅔e);
- antiquarks ($\bar{q}$): according to Dirac theory, the q-antiparticles;
- baryons: combinations qqq (e.g. uds, uud);
- antibaryons: three antiquarks (e.g $\bar{u}\bar{u}\bar{d}$);
- mesons: pairs $q\bar{q}$ (e.g $u\bar{u}$, $u\bar{d}$, $s\bar{u}$);
- "*antimesons*": a $\bar{q}q$ pair: the mesons are their own antiparticles, i.e. "anti-mesons" = mesons.

		u	d	s	c	b	t
$\mathscr{B}$	baryon	⅓	⅓	⅓			
J	spin	½	½	½			
I	isospin	½	½	0			
I_3	3rd i-spin	½	−½	0			
S	strang.	0	0	−1			
Y	$\mathscr{B}$+S	⅓	⅓	−⅔			
Q	I_3 + ½Y	⅔	−⅓	−⅓			

c, **b**, **t** not yet discovered in the '60 !!! see § 3

The quarks form a triplet, which is a basic representation of the group SU(3). Quarks may be represented in a vector shape in the plane I_3 – Y; their combinations (= hadrons) are the sums of such vectors.



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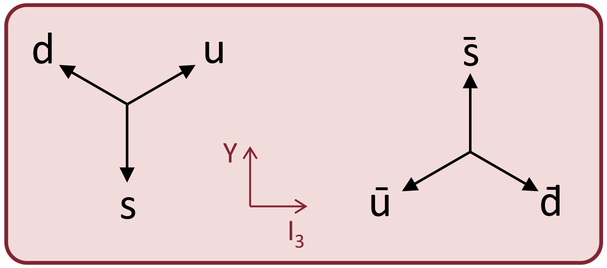
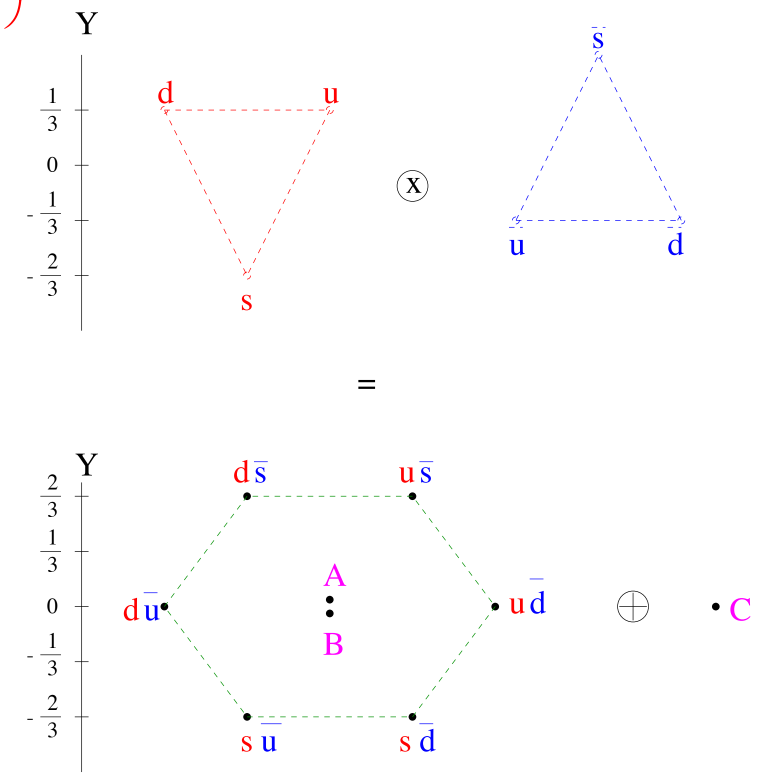


The mesons

Let us start with two quarks (*u* and *d*) $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$

$$\begin{cases} |1,1\rangle = -u\bar{d} \\ |1,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \\ |1,-1\rangle = \bar{u}d \end{cases}$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$



Adding a third quark *s* there are 9 possible combinations: 1 octet and 1 singlet (under transformations in SU(3) the 8 states transform among themselves, but they never mix with the singlet).

$$3 \otimes \bar{3} = 8 \oplus 1$$

The mesons

The singlet state, *C*, is symmetric in flavor:

$$C = \sqrt{\tfrac{1}{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

A is the neutral member of the isospin triplet:

$$\left(d\bar{u}, A, -u\bar{d}\right) \quad \Longrightarrow \quad A = \sqrt{\tfrac{1}{2}}\left(u\bar{u} - d\bar{d}\right)$$

Quarks have *spin ½*, therefore the total spin of the $q_1 \bar{q}_2$ pair can be *S=0* or *S=1*. The spin *J* of the mesons results as the combination of S and of the relative angular momentum L. The parity P of the meson is thus:

$$P = -(-1)^L = (-1)^{L+1}$$

Product of the intrinsic parities
of fermion and antifermion

The value of *C* is obtained like in positronium :

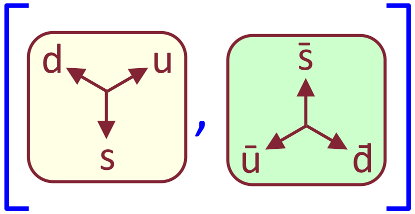
$$C = -(-1)^{S+1}(-1)^L = (-1)^{L+S}$$

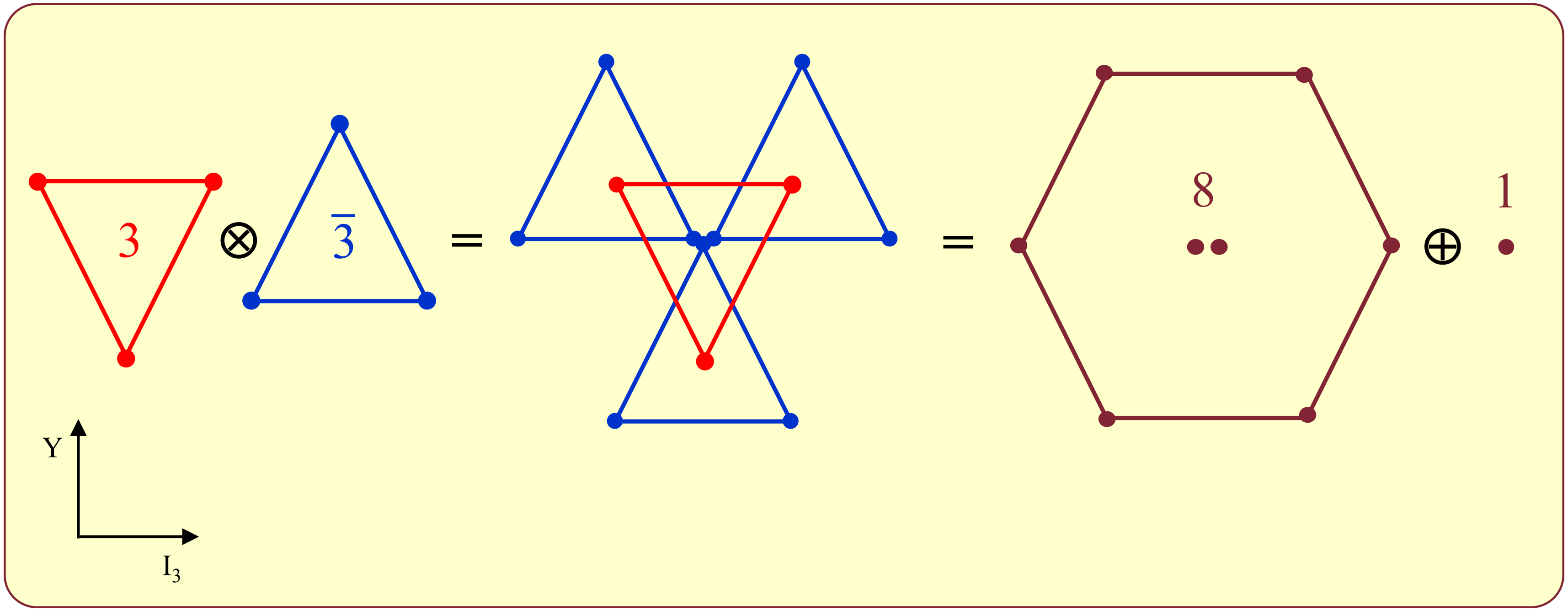
Fermion interchange

The mesons

"Build" the mesons $q\bar{q}$ with these rules :

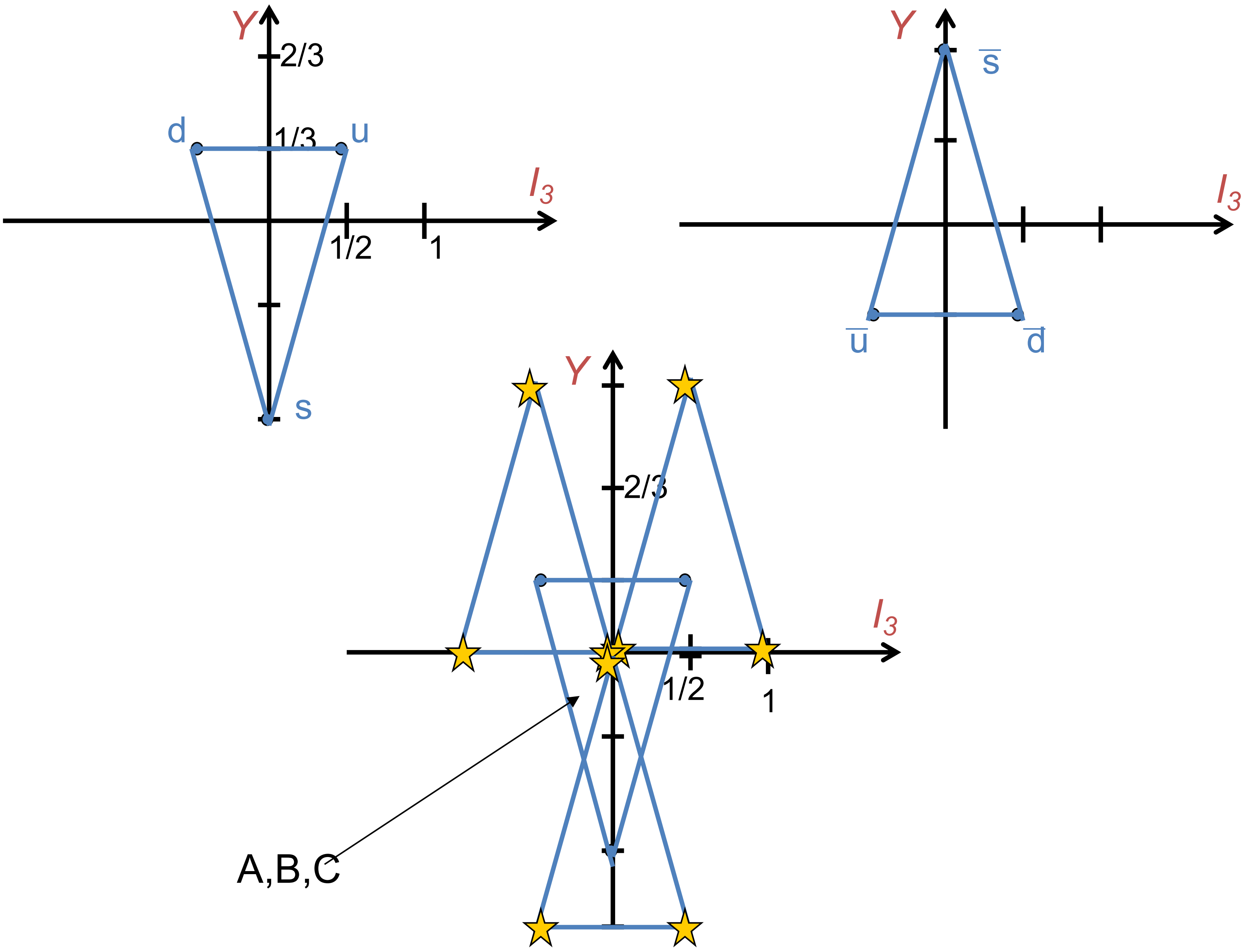
- in the space I_3 - Y, sum "vectors" (i.e. quarks and antiquarks) to produce $q\bar{q}$ pairs, i.e. mesons;
- all the combinations are allowed:



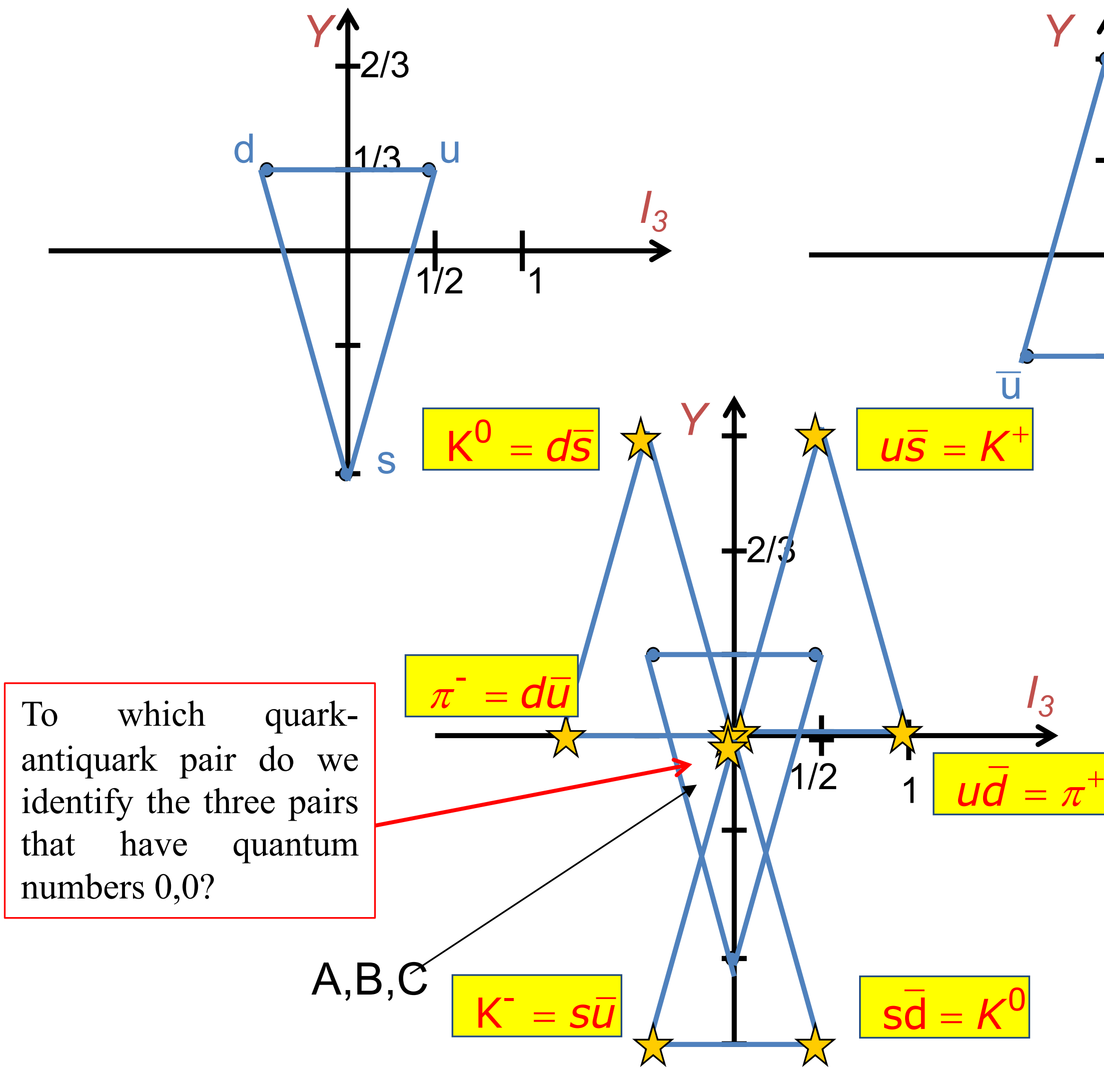


- the pseudoscalar mesons ($J^P=0^-$) are $q\bar{q}$ states in s-wave with opposite spins (⇑⇓).

Graphical construction of the meson 0^-



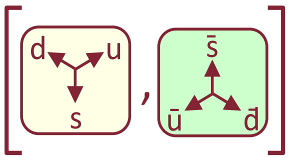
Graphical construction of the meson 0^-



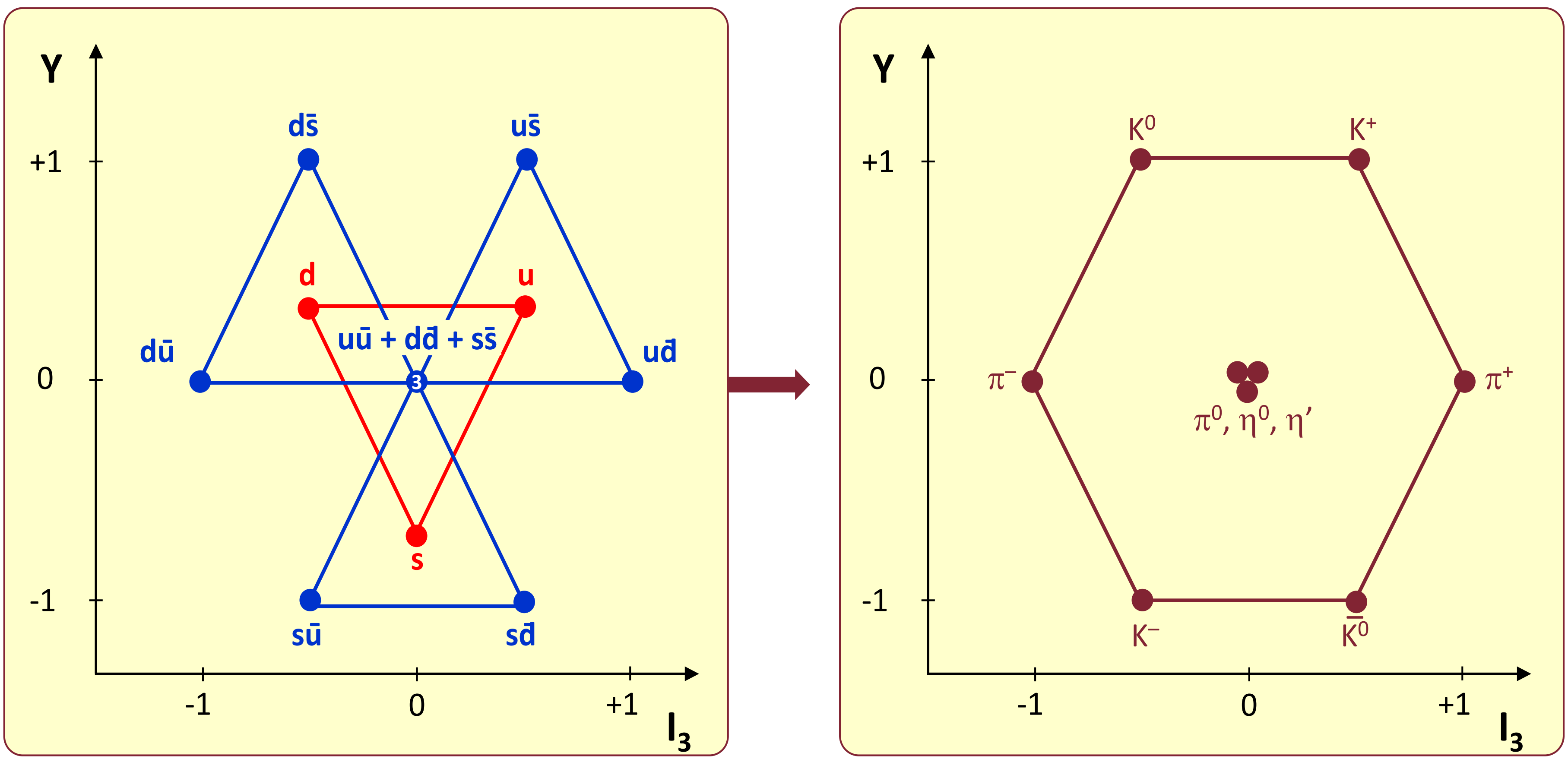
$$3 \otimes \bar{3} = 1 \oplus 8$$

The mesons: $J^{PC}=0^{-+}$

More specifically, with s-wave ($J^{PC}=0^{-+}$), we get the "pseudoscalar" nonet :



Notice that π^0, η, η' are combinations (mixing) of the three possible $q\bar{q}$ states (for the mixing parameters see later) :



Mesons 0^-

- The three states A, B, C with I_3=0 and Y=0 are orthogonal linear combinations of the states uū + dđ + sš

- Let's identify a state with $\{n, |\, I, I_3\rangle\}$ where n is the dimension of the representation.

- The SU(3) singlet must contain, because of the symmetry, all three states with the same weight (a rotation in the SU(3) space must not change the state):

$$\boxed{\eta_1 = \{1, |\, 0, 0\rangle\} = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)}$$

- One of the other two states with I_3=0 must belong to the triplet with isospin=1, therefore it can be obtained with the ladder operators, in this case with the operators of lowering or raising the charge.

Charge conjugation of the nucleons

- Let's remind the behaviour of some nucleons through the charge conjugation operation. They appear a few "minus" signs in according to the Condon-Shortley convention.

$$\begin{array}{ccccccc} I_3 & & & & & & \\ +\frac{1}{2} & |p\rangle & & |\bar{n}\rangle & |u\rangle & & |\bar{d}\rangle \\ -\frac{1}{2} & |n\rangle & & -|\bar{p}\rangle & |d\rangle & & -|\bar{u}\rangle \end{array}$$

$$\Rightarrow \begin{pmatrix} u \\ d \end{pmatrix} \text{ e } \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} \Rightarrow \begin{cases} I^- |\bar{d}\rangle = |-\bar{u}\rangle \\ I^+ |\bar{u}\rangle = |-\bar{d}\rangle \end{cases}$$

$I^{\pm}$: operator of isospin shift (raising and lowering of the charge)

- N.B. the quark s is an isospin singlet, therefore when we add it to an isospin doublet, it will not change the doublet properties:

$$\begin{pmatrix} u\bar{s} = K^+ \\ d\bar{s} = K^0 \end{pmatrix} \text{ e } \begin{pmatrix} s\bar{d} = \bar{K}^0 \\ -s\bar{u} = -K^- \end{pmatrix}$$

- Combining d with ū (or viceversa) we can have I=0 or I=1

Wave function of the π^0

- Let's apply the isospin shift operator that has the following property:

$$\mathrm{I}^{\pm} \mid \Psi(I, I_3)\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)} \mid \Psi(I, I_3 \pm 1)\rangle$$

- If we apply it to a quark we get: $\begin{cases} \mathrm{I}^{+} \mid d\ \rangle = \mid u\rangle \ ; \ \mathrm{I}^{+} \mid \bar{u}\ \rangle = \mid -\bar{d}\rangle \ ; \\ \mathrm{I}^{+} \mid u\ \rangle = \mathrm{I}^{+} \mid \bar{d}\rangle = 0 \end{cases}$

- Moreover: $\begin{cases} \mathrm{I}^{-} \mid \Psi(1,1)\rangle = I^{+} \mid \Psi(1,-1)\rangle = \sqrt{2} \mid \Psi(1,0)\rangle \\ \mathrm{I}^{+} \mid \Psi(1,0)\rangle = \sqrt{2} \mid \Psi(1,1)\rangle \ ; \ \mathrm{I}^{-} \mid \Psi(1,0)\rangle = \sqrt{2} \mid \Psi(1,-1)\rangle \\ \mathrm{I}^{+} \mid \Psi(1,1)\rangle = I^{-} \mid \Psi(1,-1)\rangle = 0 \end{cases}$

- By convention the wave function of the π^- is: $-d\bar{u}$

$$\mathrm{I}^{+} \mid \pi^{-}\ \rangle = \mathrm{I}^{+} \mid -d\bar{u}\ \rangle = \mid -\left[\left(\mathrm{I}^{+}d\right)\bar{u} + d\left(\mathrm{I}^{+}\bar{u}\right)\right]\rangle = \mid -u\bar{u} + d\bar{d}\rangle = \sqrt{2}\left(\frac{1}{\sqrt{2}} \mid -u\bar{u} + d\bar{d}\rangle\right) = \sqrt{2}\ \mid \pi^{0}\rangle$$

- The π^0 is identified with the state: $\boxed{\pi^{0} = \frac{1}{\sqrt{2}}\left(d\bar{d} - u\bar{u}\right)}$

- Indeed: $\mathrm{I}^{+} \mid \pi^{0}\ \rangle = \mathrm{I}^{+} \frac{\mid d\bar{d} - u\bar{u}\ \rangle}{\sqrt{\ }} = \frac{\mid u\bar{d} + 0 - 0 + u\bar{d}\rangle}{\sqrt{2}} = \sqrt{2}\ \mid u\bar{d}\rangle = \sqrt{2}\ \mid \pi^{+}\rangle$

Wave functions and physics states

- In order to find the singlet of the octet $\eta_8 = \{8, |0,0\rangle\}$ we need to find the quark composition that it is orthogonal to $\eta_1 = \{1, |0,0\rangle\}$ and to the π^0:

$$\eta_1 = \{1, |0,0\rangle\} = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(d\bar{d} - u\bar{u}\right)$$

$$\eta_8 = \{8, |0,0\rangle\} = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

N.B. $I^{\pm} | \eta_8\rangle = 0$

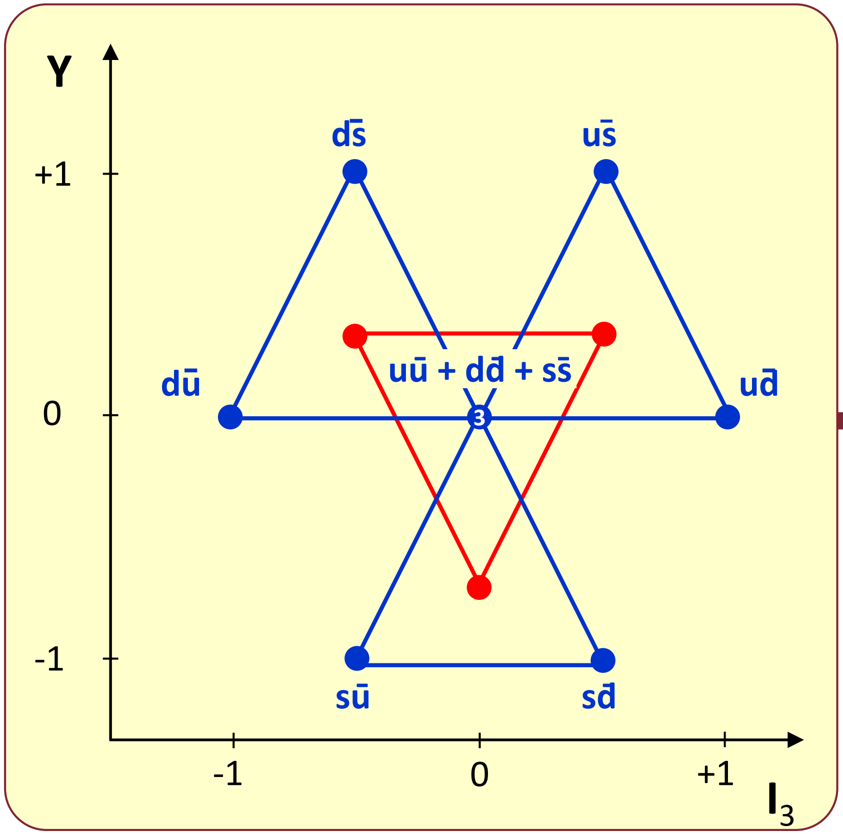
- The physical states η and η' are a linear combination of η_1 e η_8, but since the mixing angle is small (~11°), we can do the identification:

$$\eta_8 \equiv \eta \quad ; \quad m_\eta = 548 \text{ MeV}$$

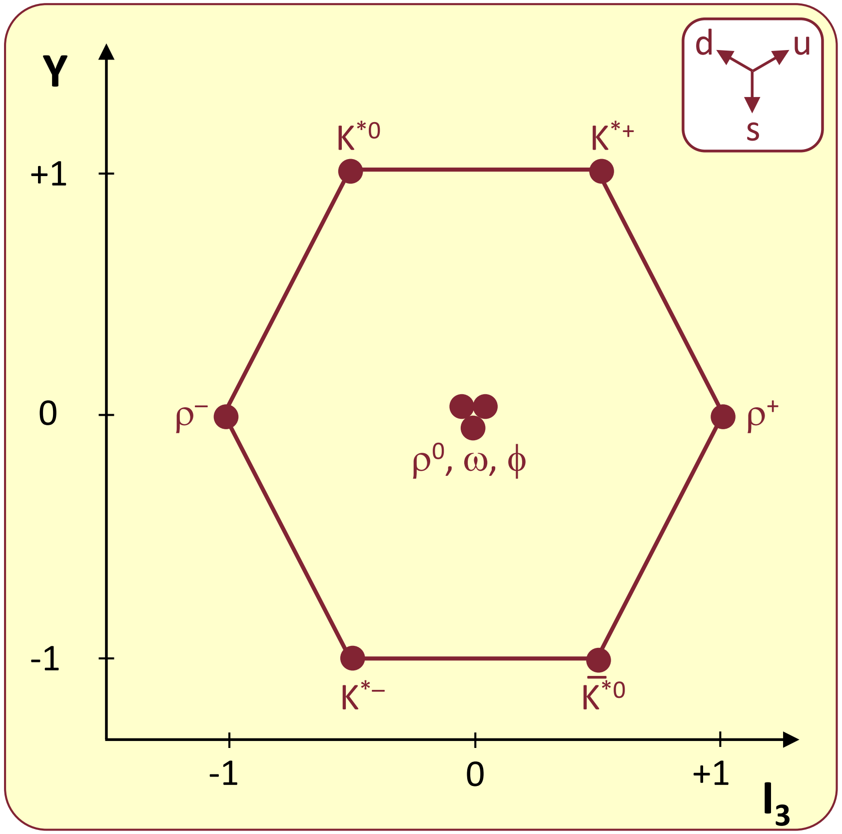
$$\eta_1 \equiv \eta' \quad ; \quad m_{\eta'} = 958 \text{ MeV}$$

The mesons: $J^{PC}=1^{--}$

If $J^{PC} = 1^{--}$ (i.e. spin ⇑⇑), the "vector" nonet :



Notice that ρ^0, ω, φ are combinations (mixing) of the three possible $q\bar{q}$ states :



Meson quantum numbers: J^{PC}

- Parity : the quarks and the antiquarks have opposite P :
  $P_{q\bar{q}} = P_1 P_2 (-1)^L = -1\,(-1)^L = (-1)^{L+1}$.

- Charge conjugation : for mesons, which are also $\mathbb{C}$ eigenstates, $\mathbb{C} = \mathbb{P}\mathbb{S}$, parity followed by spin swap (see before).

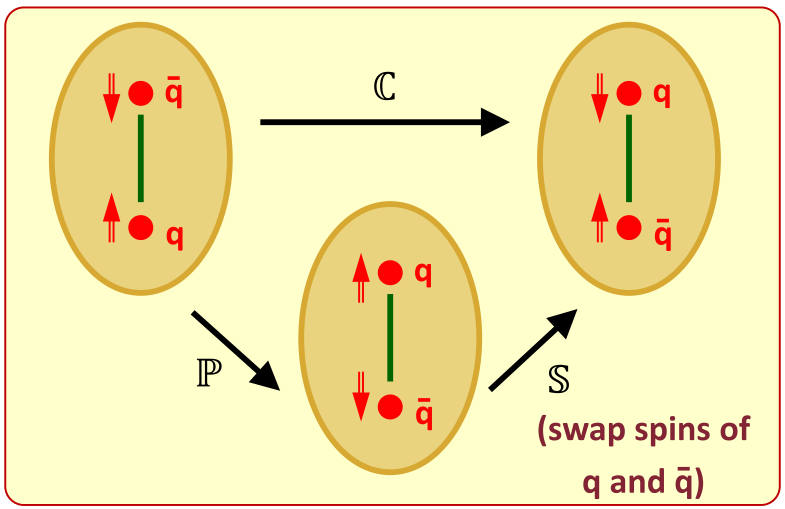
$P = (-1)^{L+1}$;

$S = (-1)^{S+1}$ (Pauli principle, [BJ, 263]);

$C = P \times S = (-1)^{L+S}$;

$G = (-1)^{L+S+I}$ (see before).

$$\underbrace{\frac{1}{\sqrt{2}}\left(\Uparrow\Downarrow - \Downarrow\Uparrow\right);}_{\substack{S\,=\,0 \\ \text{antisymmetric}}} \qquad \underbrace{\Downarrow\Downarrow;\quad \frac{1}{\sqrt{2}}\left(\Uparrow\Downarrow + \Downarrow\Uparrow\right);\quad \Uparrow\Uparrow}_{\substack{S\,=\,1 \\ \text{symmetric}}}$$



L	S	J=L⊕S	P	C	I	G
0	0	0	–	+	0	+
					1	–
	1	1	–	–	0	+
					1	–
1	0	1	+	–	0	–
					1	+
	1	0,1,2	+	+	0	–
					1	+

$J^{PC} = 0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++}, \ldots$

Meson quantum numbers: multiplets

- For the lowest state nonets, these are the quantum numbers :

L	S	J^{PC}	$^{2s+1}L_J$	I=1 state
0	0	0^{-+}	$^1S_0$	$\pi(140)$
	1	1^{--}	$^3S_1$	$\rho(770)$
1	0	1^{+-}	$^1P_1$	$b_1(1235)$
	1	0^{++}	$^3P_0$	$a_0(1450)$
		1^{++}	$^3P_1$	$a_1(1260)$
		2^{++}	$^3P_2$	$a_2(1320)$

- all these multiplets have main qn n = 1;
- as of today ~20 meson multiplets have been (partially) discovered [PDG].
- important activity from the ’50 to the ’70; still some addict;

- method (mainly bubble chambers) :
  - measure (zillions of) events; e.g. :

    $\bar{p}p \to \pi^+ \pi^+ \pi^- \pi^- \pi^0;$

  - look for "peaks" in final state combined mass, e.g. $m(\pi^+ \pi^- \pi^0)$;
  - the peaks are associated with high mass resonances, decaying via strong interactions (width $\to \Gamma \to$ strength);
  - the scattering properties (e.g. the angular distribution) and decay modes identify the other quantum numbers;
- result : an overall consistent picture;
- Great success !!!

“If I could remember the names of all these particles, I'd be a botanist.”
Enrico Fermi

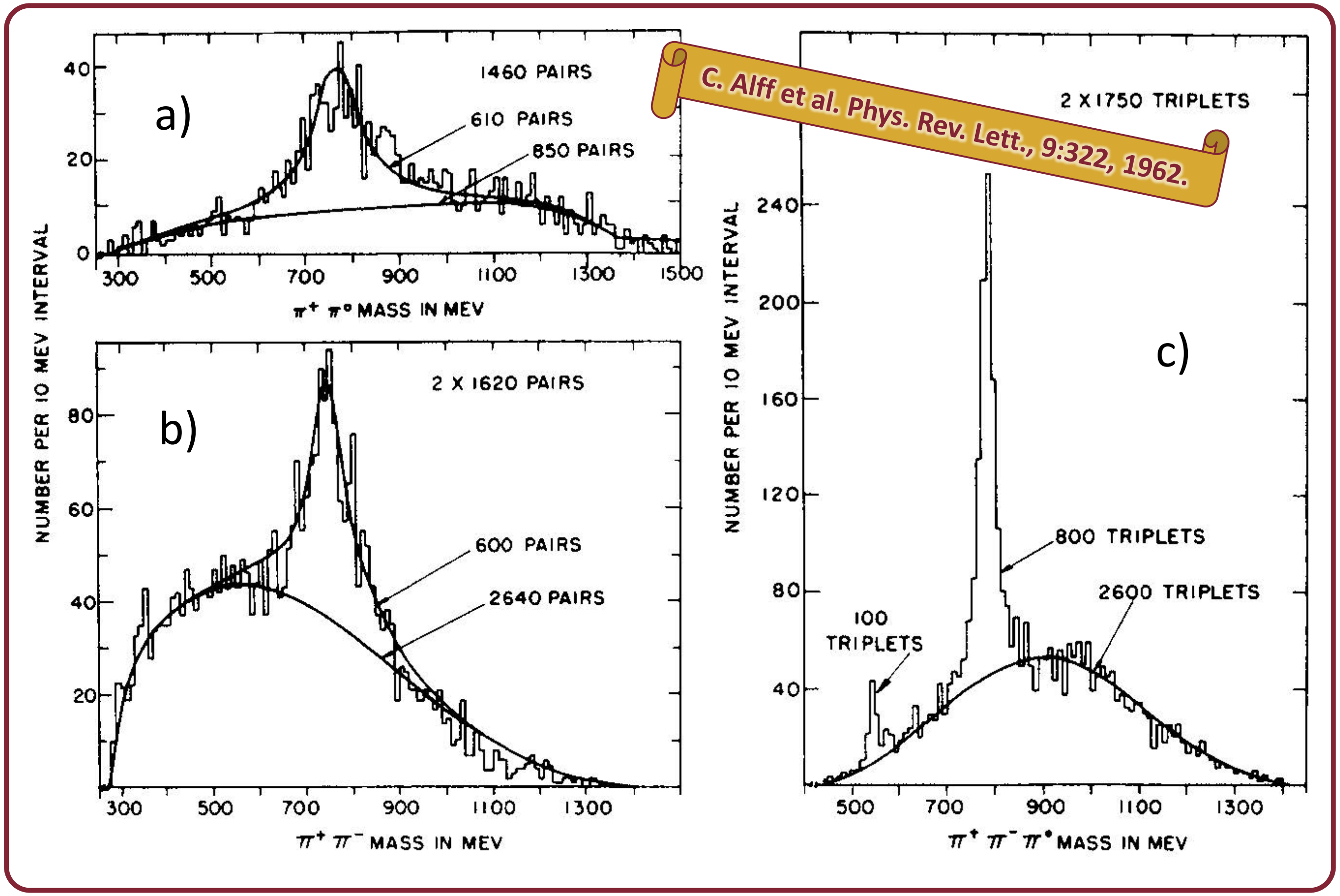
Meson quantum numbers: example

Three examples in $\pi^+ p \to X$

a) $m(\pi^+\pi^0)$ for $X = \pi^+\pi^0 p$

b) $m(\pi^+\pi^-)$ for $X = \pi^+ \pi^+ \pi^- p$

c) $m(\pi^+ \pi^-\pi^0)$ for $X = \pi^+ \pi^+ \pi^-\pi^0 p$



Q: which resonances ?

a) $\rho^+(770) \to \pi^+\pi^0$; b) $\rho^0(770) \to \pi^+\pi^-$; c) $\eta(548) \to \pi^+\pi^-\pi^0$, $\omega(782) \to \pi^+\pi^-\pi^0$.

why not the ρ^0 ?

Meson quantum numbers : $\rho^0 \not\to \pi^0\pi^0$

Problem: $\rho^0 \to \pi^0\pi^0$ is allowed ? **NO**, because of :

a) $\mathbb{C}$-parity

$C(\rho^0) = -1$; $C(\pi^0) = +1$

therefore, since the initial state is a C-eigenstate,

$-1 = (+1) \times (+1) \;\; \to$ **NO**

NB. A general rule : "a vector cannot decay into two equal (pseudo-)scalars".

But (a) and (b) do not hold for weak decays. Instead (c) is due to statistics + angular momentum conservation, and is valid for all interactions.

[(c) also forbids **Z** $\to$ **HH**]

b) Clebsch-Gordan coeff. in isospin space

$|\rho^0\rangle \;\; = |I{=}1, I_3{=}0\rangle$;

$|\pi^0\rangle \;\; = |1, 0\rangle$;

therefore the decay is

$\langle\pi^0\pi^0|\rho^0\rangle = \langle j_1 j_2 m_1 m_2 | J\ M\rangle =$
$= \langle 1\ 1\ 0\ 0\ |\ 1\ 0\ \rangle = 0$;

$\to$ **NO**.

[PDG, § 44 :

$1\otimes1$		...	1
		...	0
...	...	...	...
0	0	...	**0**

c) Spin-statistics

[Povh, problem 15-1]

- $S(\rho^0) = 1$, $S(\pi^0) = 0$ $\to L(\pi^0\pi^0) = 1$;
- ρ^0 is a boson $\to$ wave function symmetric;
- the π^0's are two equal bosons $\to$ space wave function symmetric;
- L=1 makes the wave function anti-symmetric

$\to$ **NO**.

Meson mixing

Light mesons	$q\bar{q}$	J^{PC} (1)	I	I_3	S	Q (1)	mass (MeV)	$q\bar{q}$ of I_3=0 (2)
π^+, π^0, π^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	0^{-+}	1	1, 0, -1	0	1, 0, -1	140	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
η	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	550	$\sim(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$
η'	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	960	$\sim(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{6}$
K^+, K^0 (3)	$u\bar{s}$, $d\bar{s}$	0^-	½	½,-½	+1	1, 0	495	
$\bar{K}^0, K^-$ (3)	$s\bar{d}$, $s\bar{u}$	0^-	½	½, -½	-1	0, -1	495	
ρ^+, ρ^0, ρ^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	1^{--}	1	1, 0, -1	0	1, 0, -1	770	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
ω	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	780	$\sim(u\bar{u}+d\bar{d})/\sqrt{2}$
ϕ	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	1020	$\sim s\bar{s}$
K^{*+}, K^{*0} (3)	$u\bar{s}$, $d\bar{s}$	1^-	½	½,-½	+1	1, 0	890	
$\bar{K}^{*0}, K^{*-}$ (3)	$s\bar{d}$, $s\bar{u}$	1^-	½	½, -½	-1	0, -1	890	

Notes :

(1) (L=0, $\mathscr{B}$=0) $\rightarrow$ $P = (-)^{L+1} = -$; $C = (-)^{L+S} = (-)^S$; $Q = I_3 + ½Y = I_3 + ½S$;

(2) The mesons π^0, η, η', ρ^0, ω, ϕ are mixing of $u\bar{u} \oplus d\bar{d} \oplus s\bar{s}$ (see next);

(3) States with strangeness $\neq 0$ are NOT eigenstates of C; since they have I=½, no I_3=0 exists.

Meson mixing: $J^P=1^-$

- **The vector mesons 1^- have the same quark composition of the mesons 0^-, they are in the S-state (L=0) but the two quarks (actually quark-antiquark) have the spin that are parallel (S=1).**

- There are three mesons with $I_3=0$ and Y=0; one of them belongs to the isospin triplet ρ : ρ^+, ρ^-, ρ^0.
- ρ^0 has the same wave function of the π^0 (besides a factor "-1"):

$$\rho^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

- The SU(3) singlet ϕ_1 and the isospin singlet of the octet ϕ_8 are mixed to give the mass eigenstates ϕ and ω:

$$\boxed{\begin{aligned}\omega &= \phi_1 \cos\vartheta + \phi_8 \sin\vartheta \\ \phi &= \phi_1 \sin\vartheta - \phi_8 \cos\vartheta\end{aligned}}$$

- N.B. in this case the mixing angle is $\theta \sim 35^o$

Meson mixing: $J^P=0^-, 1^-$

Mesons are bound states $q\bar{q}$. Consider only uds quarks (+ $\bar{u}\bar{d}\bar{s}$) in the nonets ($J^P = 0^-$ 1^-, the *pseudo-scalar* and *vector* nonets) :

- the states ($\pi^+=u\bar{d}$, $\pi^-=d\bar{u}$, $K^+=u\bar{s}$, $K^0=d\bar{s}$, $K^-=s\bar{u}$, $\bar{K}^0=s\bar{d}$) have no quark ambiguity;
- but ($u\bar{u}$ $d\bar{d}$ $s\bar{s}$) have the same quantum numbers and the three states ($\psi_{8,0}$ $\psi_{8,1}$ ψ_1) mix together ($\rightarrow$ 2 angles per nonet);
- the physical particles (π^0, η, η' for 0^-, ρ^0, ω, ϕ for 1^-) are linear combinations $q\bar{q}$;
- ($\psi_{8,1}$) decuples (π^0 ρ^0) ($\rightarrow$ 1 angle only);
- θ_{ps} and θ_v are computed from the mass matrices* [PDG, §15.2];
- notice: the vector mixing $\theta_v \approx 36° \approx \tan^{-1}(1/\sqrt{2})$, i.e. the ϕ meson is almost $s\bar{s}$ only [i.e. $\phi \rightarrow K\bar{K}$, see KLOE exp.];

(... continue)

$$\left.\begin{aligned} \psi_{8,1}[\mathrm{oct}, I=1] &= (u\bar{u}-d\bar{d})/\sqrt{2} \\ \psi_{8,0}[\mathrm{oct}, I=0) &= (u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6} \\ \psi_1[\mathrm{sing}] &= (u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3} \end{aligned}\right\} \begin{array}{c} \psi_{\mathrm{multi},I} \\ \text{ideal case} \end{array}$$

$$\left.\begin{aligned} \pi^0(140) \approx \psi^{ps}_{8,1} &= (u\bar{u}-d\bar{d})/\sqrt{2} \\ \eta(550) &= \psi^{ps}_{8,0}\cos\theta_{ps} - \psi^{ps}_1 \sin\theta_{ps} \\ \eta'(960) &= \psi^{ps}_{8,0}\sin\theta_{ps} + \psi^{ps}_1 \cos\theta_{ps} \end{aligned}\right\} \begin{array}{c} J^P = 0^-, \\ \theta_{\text{pseudo-scalar}} \approx -25°; \end{array}$$

$$\left.\begin{aligned} \rho^0(770) \approx \psi^{v}_{8,1} &= (u\bar{u}-d\bar{d})/\sqrt{2} \\ \phi(1020) &= \psi^{v}_{8,0}\cos\theta_{v} - \psi^{v}_1 \sin\theta_{v} \approx s\bar{s} \\ \omega(780) &= \psi^{v}_{8,0}\sin\theta_{v} + \psi^{v}_1 \cos\theta_{v} \approx \\ &\approx (u\bar{u}+d\bar{d})/\sqrt{2} \end{aligned}\right\} \begin{array}{c} J^P = 1^-, \\ \theta_{\text{vector}} \approx 36°. \end{array}$$

* in principle, both the mass spectra and the mixing angles can be computed from QCD lagrangian $\mathscr{L}_{QCD}$ ... waiting for substantial improvements in computation methods.

Meson 1^- computation of the mixing angle

- Let'assume that the Hamiltonian matrix element between two states is equal to the "mass" squared:

$$M_\omega^2 = \langle \omega | H | \omega \rangle = M_1^2 \cos^2 \vartheta + M_8^2 \sin^2 \vartheta + 2M_{18}^2 \sin\vartheta \cos\vartheta$$
$$M_\phi^2 = \langle \phi | H | \phi \rangle = M_1^2 \sin^2 \vartheta + M_8^2 \cos^2 \vartheta - 2M_{18}^2 \sin\vartheta \cos\vartheta$$

- Since ω and ϕ are mass eigenstates, they are orthogonal:

$$M_{\omega\phi}^2 = \langle \phi | H | \omega \rangle = 0 = \left(M_1^2 - M_8^2\right)\sin\vartheta\cos\vartheta + M_{18}^2\left(\sin^2\vartheta - \cos^2\vartheta\right)$$

- If we get rid of of M_{18} and M_1 from these three equations, we get:

$$\tan^2\vartheta = \frac{M_\phi^2 - M_8^2}{M_8^2 - M_\omega^2}$$

- From the mass formula of Gell-Mann – Okubo we have:

$$M_8^2 = \frac{1}{3}\left(4M_{K^*}^2 - M_\rho^2\right)$$

$M_\rho = 776$ MeV
$M_{K^*} = 892$ MeV
$M_\omega = 783$ MeV
$M_\phi = 1020$ MeV

- If we put in the formula the measured values of the masses, we get:

$$\boxed{\vartheta \approx 40^o}$$

$$\boxed{N.B.\ \sin\vartheta = \frac{1}{\sqrt{3}} \text{ if } \vartheta \approx 35^o}$$

Meson mixing: $J^P=1^-$

- If we use $\sin\vartheta = \frac{1}{\sqrt{3}}$ we have:

$$\omega = \frac{1}{\sqrt{3}}\left(\phi_8 + \sqrt{2}\phi_1\right)$$
$$\phi = \frac{1}{\sqrt{3}}\left(\phi_1 - \sqrt{2}\phi_8\right)$$

- since:

$$\phi_1 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$
$$\phi_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

- We have

$$\phi = s\bar{s} \quad ; \; \omega = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$M_\rho = 776 \text{ MeV}$$
$$M_{K^*} = 892 \text{ MeV}$$
$$M_\omega = 783 \text{ MeV}$$
$$M_\phi = 1020 \text{ MeV}$$

- In this case of "ideal mixing", that is almost true in practice, the φ is composed entirely from quark s and the ω from quarks u and d
- This implies that the mass of the ω should be similar to the one of the ρ^0 and the mass of φ should be higher, as it is observed experimentally.

Meson mixing summary

- We have the mixing in the states with I=0:
  - η,η' are linear combinations of η_1, η_8 that can mix between themselves because they have the same quantum numbers: $(I=I_3=S=0)$
  - The same thing happens to the physical states ω and ϕ that are the results of the mixing of ϕ_1 and ϕ_8

We need to introduce new parameters: the mixing angles between the states

$$|\omega\rangle = \sqrt{\frac{1}{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$|\phi\rangle = s\bar{s}$$

pure state $s\bar{s}$

$$|\eta\rangle = \sqrt{\frac{1}{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

$$|\eta'\rangle = \sqrt{\frac{1}{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

Almost exact, there is just a small mixing

Φ Meson Decay and the OZI (Okubo, Zweig, Iizuka) Rule

$$\phi \to K^+K^- \quad 49.1\% \qquad \omega \to \pi^+\pi^-\pi^0 \quad 88.8\%$$

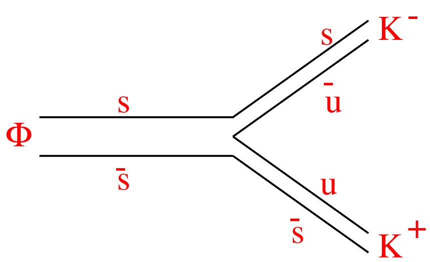
$$\to K_L^0 K_S^0 \quad 34.4\% \qquad \to \pi^0\gamma \quad 8.5\%$$

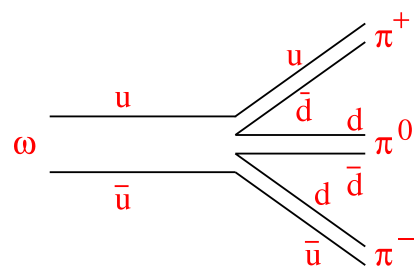
$$\to \pi^+\pi^-\pi^0 \quad 15.3\% \qquad \to \pi^+\pi^- \quad 2.2\%$$

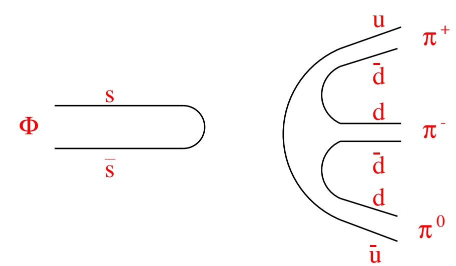
For the $\phi(1020)$ phase space would favor the 3π decay with respect to 2K:

$$Q_{3\pi} \equiv M_\phi - 2M_{\pi^\pm} - M_{\pi^0} \approx 600\ MeV$$

$$Q_{2K^0} \equiv M_\phi - 2M_{K^0} \approx 24\ MeV \qquad Q_{2K^\pm} \approx 32\ MeV$$



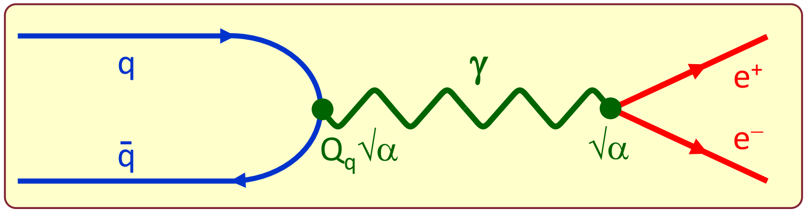




The diagram for the 3π decay is suppressed because it contains disconnected quark lines (Zweig Rule). The kinematical suppression of the 2K decays is reflected in the small total width of the ϕ: $\Gamma_\phi = (4.26 \pm 0.05)$ MeV to be compared, for example, with $\Gamma_\rho = (150.3 \pm 1.6)$ MeV.

Meson mixing: $J^P=1^-$

The decay amplitudes in the e.m. channels may be computed, up to a common factor, and compared to the experiment;



Few problems :

- the values are small*, e.g. BR($\rho^0 \to e^+e^-$) ≈ 4.7×10^{-5};
- the phase-space factor is important, especially for ϕ, which is very close to the $s\bar{s}$ threshold ($m_\phi - 2\,m_K$ = few MeV).

However, the overall picture is clear: the theory explains the data ***very well***.

$$\left.\begin{aligned}\rho^0(770) &= \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d});\\ \omega(780) &= \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d});\\ \phi(1020) &= s\bar{s};\end{aligned}\right\} \to \begin{aligned}&\mathcal{M}_{fi}(\rho^0\omega\phi \to e^+e^-)\\ &\propto \alpha \sum\nolimits_j Q_q^j;\end{aligned}$$

$$\to \begin{cases} \Gamma(\rho^0 \to e^+e^-) \propto \left[\dfrac{1}{\sqrt{2}}\left(\dfrac{2}{3}-\dfrac{-1}{3}\right)\right]^2 = \dfrac{1}{2};\\ \Gamma(\omega \to e^+e^-) \propto \left[\dfrac{1}{\sqrt{2}}\left(\dfrac{2}{3}+\dfrac{-1}{3}\right)\right]^2 = \dfrac{1}{18};\\ \Gamma(\phi \to e^+e^-) \propto \left[\dfrac{1}{3}\right]^2 = \dfrac{1}{9}; \end{cases}$$

$$\to \Gamma_\rho : \Gamma_\omega : \Gamma_\phi = \begin{cases} 9 : 1 : 2 & \text{(theo)}\\ 8.8\pm2.6 : 1 : 1.7\pm0.4 & \text{(exp).}\end{cases}$$

---

* warning: the dominant $\rho^0\omega\phi$ decay modes are strong; however, the <u>e.m.</u> decays $\rho^0\omega\phi \to e^+e^-$, with a much smaller BR, are detectable $\to \Gamma_{e.m.}$ measurable $\to$ quark charges compared.

Leptonic decay of the vector mesons

$$\rho^0 = \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\omega = \tfrac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$\phi = s\bar{s}$$

Consider the decays

$$V \to l^+l^- \quad V = \rho, \omega, \phi \quad l = e, \mu$$

The partial width is given by:

$$\Gamma(V \to l^+l^-) = \frac{16\pi\alpha^2 Q^2}{M_V^2}\left|\psi(0)\right|^2$$

$$Q^2 = \left|\sum a_i Q_i\right|^2; \quad M_V = V\ mass; \quad m_l^2 << M_V^2$$

ρ,ω,ϕ have similar masses

$$\Longrightarrow \quad \frac{\left|\psi(0)\right|^2}{M_V^2} \approx \text{costante}$$

$$\Gamma(V \to l^+l^-) \propto Q^2$$

$$\rho^0 : \left[\sqrt{\tfrac{1}{2}}\left(\tfrac{2}{3} - \left(-\tfrac{1}{3}\right)\right)\right]^2 = \tfrac{1}{2}$$

$$\omega : \left[\sqrt{\tfrac{1}{2}}\left(\tfrac{2}{3} + \left(-\tfrac{1}{3}\right)\right)\right]^2 = \tfrac{1}{18} \qquad \phi : \left(\tfrac{1}{3}\right)^2 = \tfrac{1}{9}$$

$$\Longrightarrow \quad \Gamma(\rho^0) : \Gamma(\omega) : \Gamma(\phi) = 9 : 1 : 2$$

Van Royen - Weisskopf

Experimentally

$$\frac{\Gamma(\rho^0 \to e^+e^-)}{\Gamma(\omega \to e^+e^-)} = 11.3 \qquad \frac{\Gamma(\phi \to e^+e^-)}{\Gamma(\omega \to e^+e^-)} = 2.3$$

3 Quark states: the baryons

The construction looks complicated, but in fact is quite simple :

- add the three quarks one after the other;
- count the resultant multiplicity.

In group's theory language :

$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$

i.e. a decuplet, two octects and a singlet.

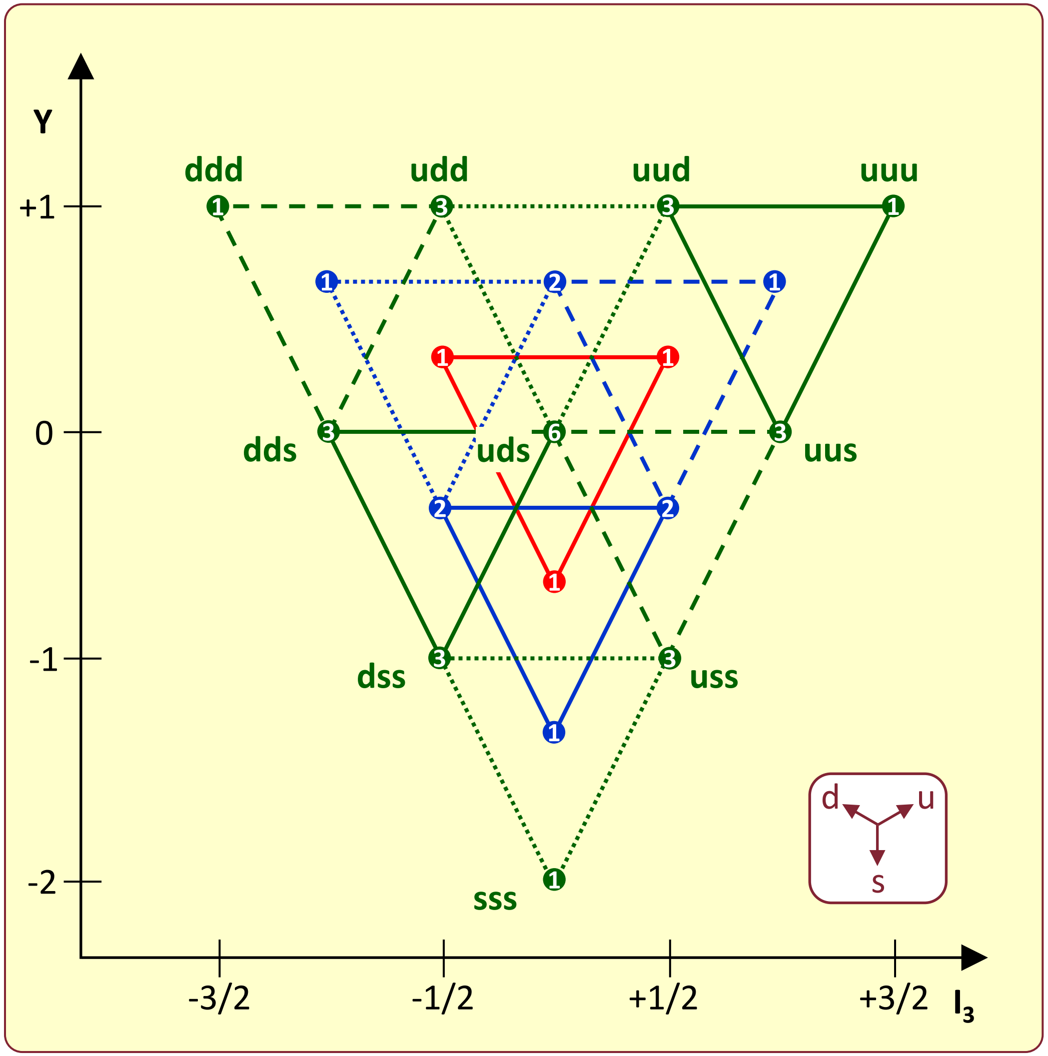
[proof. :

$3 \otimes 3 = \ 6 \oplus \bar{3};$

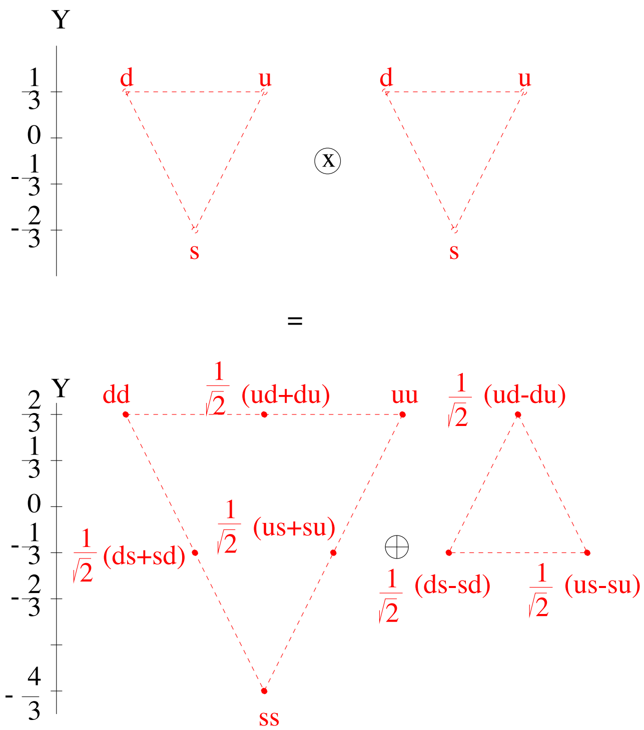
$6 \otimes 3 = 10 \oplus 8;$

$\bar{3} \otimes 3 = \ 8 \oplus 1.$ q.e.d.]

Both for 10, 8, 8' and 1 the three quarks have L = 0.

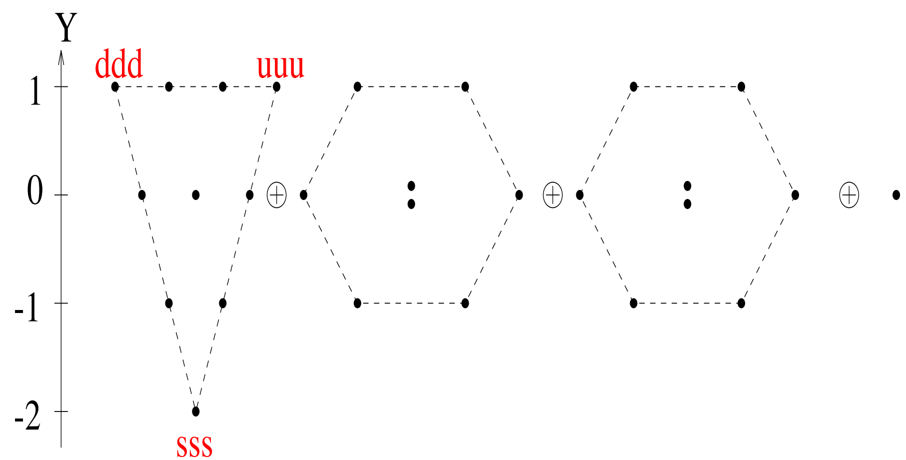


3 Quark states: the baryons



$$3 \otimes 3 = 6 \oplus \bar{3}$$

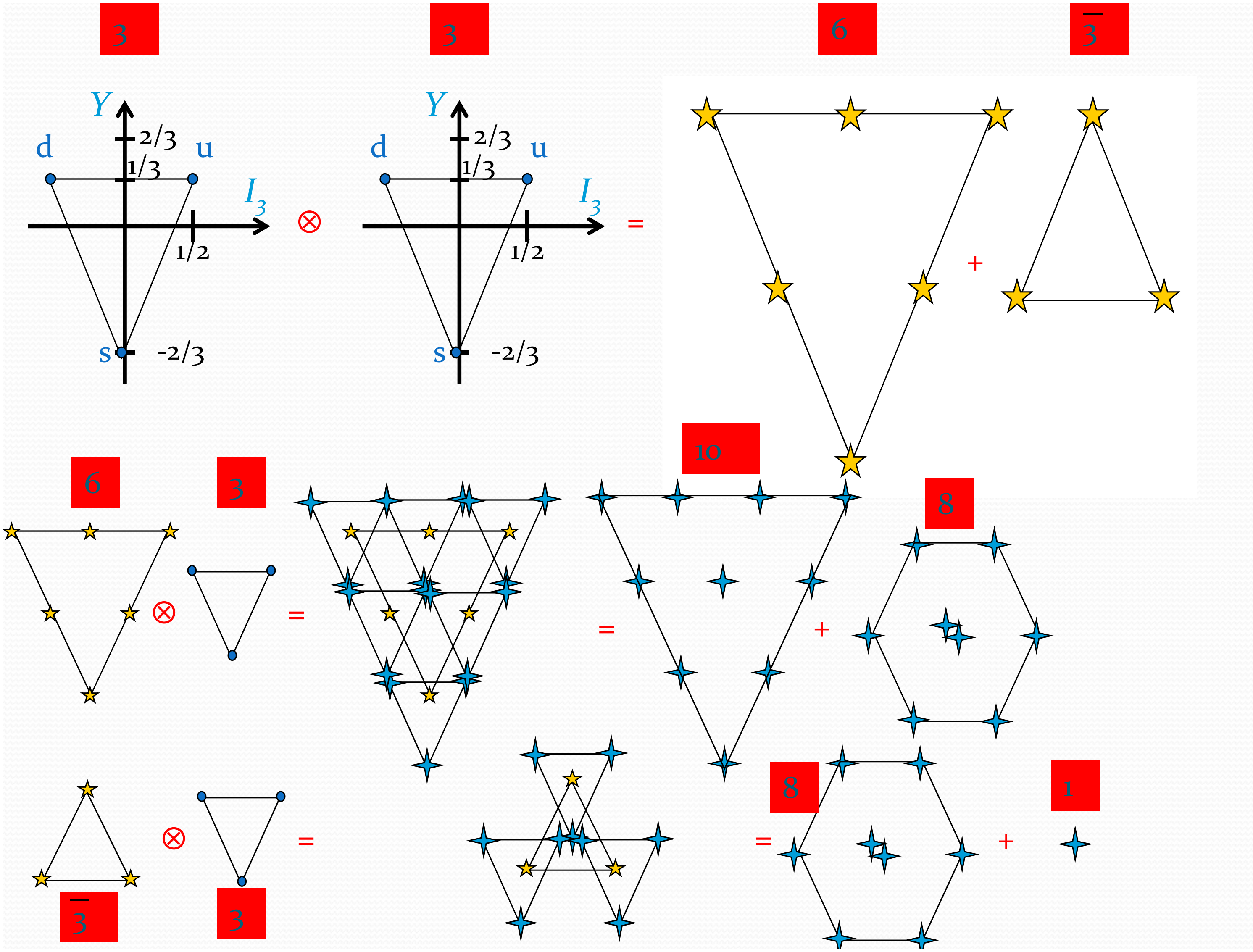
Let us now add the third quark



$$3 \otimes 3 \otimes 3 = (6 \otimes 3) \oplus (\bar{3} \otimes 3)$$

$$= 10 \oplus 8 \oplus 8 \oplus 1$$

3 Quark states: the baryons



The baryon quantum numbers

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	½⁺	½	½, -½	0	1, 0	940
Λ	uds	½⁺	0	0	-1	0	1115
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	½⁺	1	1, 0, -1	-1	1, 0, -1	1190
Ξ^0, Ξ^-	uss, dss	½⁺	½	½,-½	-2	1, 0	1320
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	uuu, uud, udd, ddd	$^3/_2{}^+$	$^3/_2$	$^3/_2$, $^1/_2$, $-^1/_2$, $-^3/_2$	0	2, 1, 0, -1	1230
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	$^3/_2{}^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ^{*0}, Ξ^{*-}	uss, dss	$^3/_2{}^+$	½	½,-½	-2	1, 0	1530
Ω^-	sss	$^3/_2{}^+$	0	0	-3	-1	1670

10
$\Uparrow\Uparrow\Uparrow$

8
$\Uparrow\Uparrow\Downarrow$

Notes :

(1) $Q = I_3 + \frac{1}{2}Y = I_3 + \frac{1}{2}(\mathscr{B} + S)$; $\mathscr{B} = 1$.

The baryons: the octet $J^P = 1/2^+$

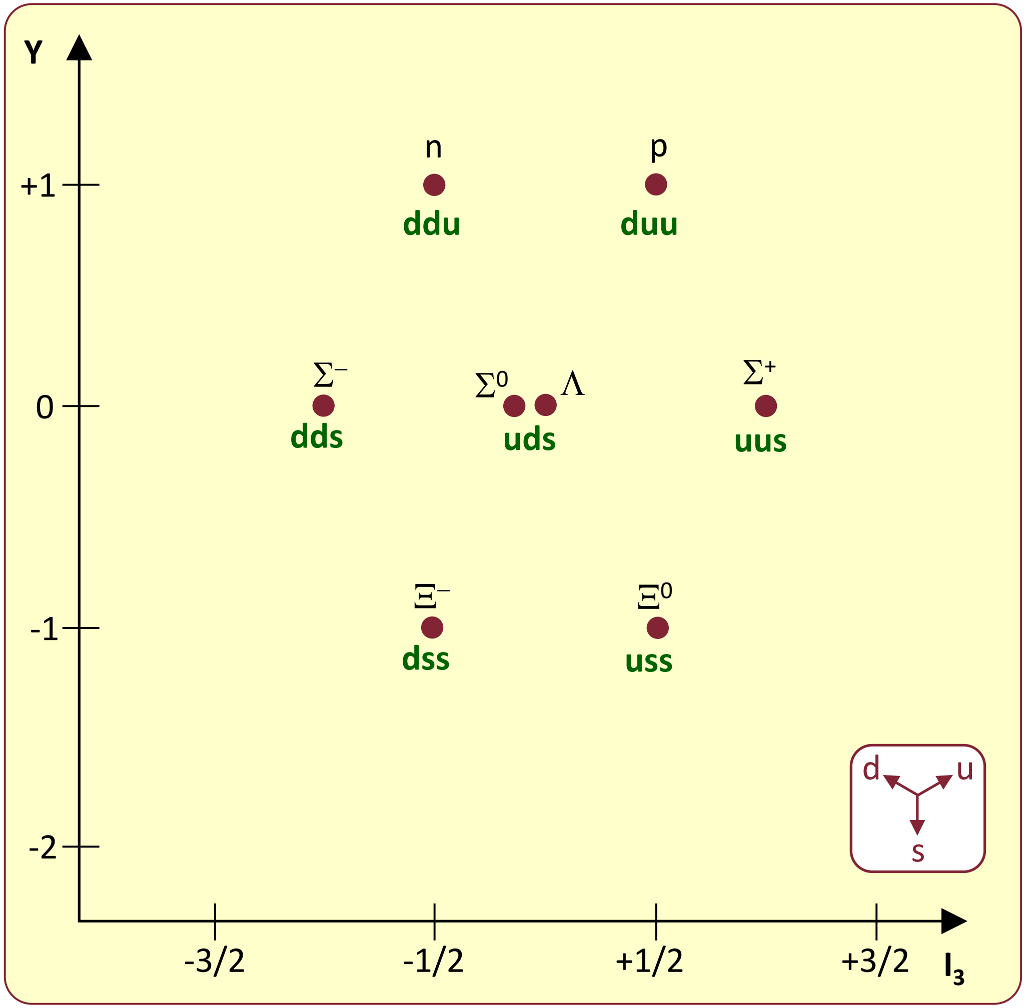
The lowest mass multiplet is an octet, which contains the familiar p and n, a triplet of S=-1 (the Σ's) a singlet S=-1 (the Λ) and a doublet of S=-2 (the Ξ's, sometimes called "cascade baryons").

The three quarks have $\ell = 0$ and spin ($\Uparrow\Uparrow\Downarrow$), i.e. a total spin of ½.

The masses are :

- ~ 940 MeV for p and n;
- ~1115 MeV for the Λ;
- ~1190 MeV for the Σ's;
- ~1320 MeV for the Ξ's;

(difference of < few MeV in the isospin multiplet, due to e-m interactions.)



The baryons: the decuplet $J^P = 3/2^+$

The decuplet is rather simple (*but there is a spin/statistics problem, see later*). The spins are aligned (⇑ ⇑ ⇑), to produce an overall $J={}^3/_2$.

The masses, at percent level, are :

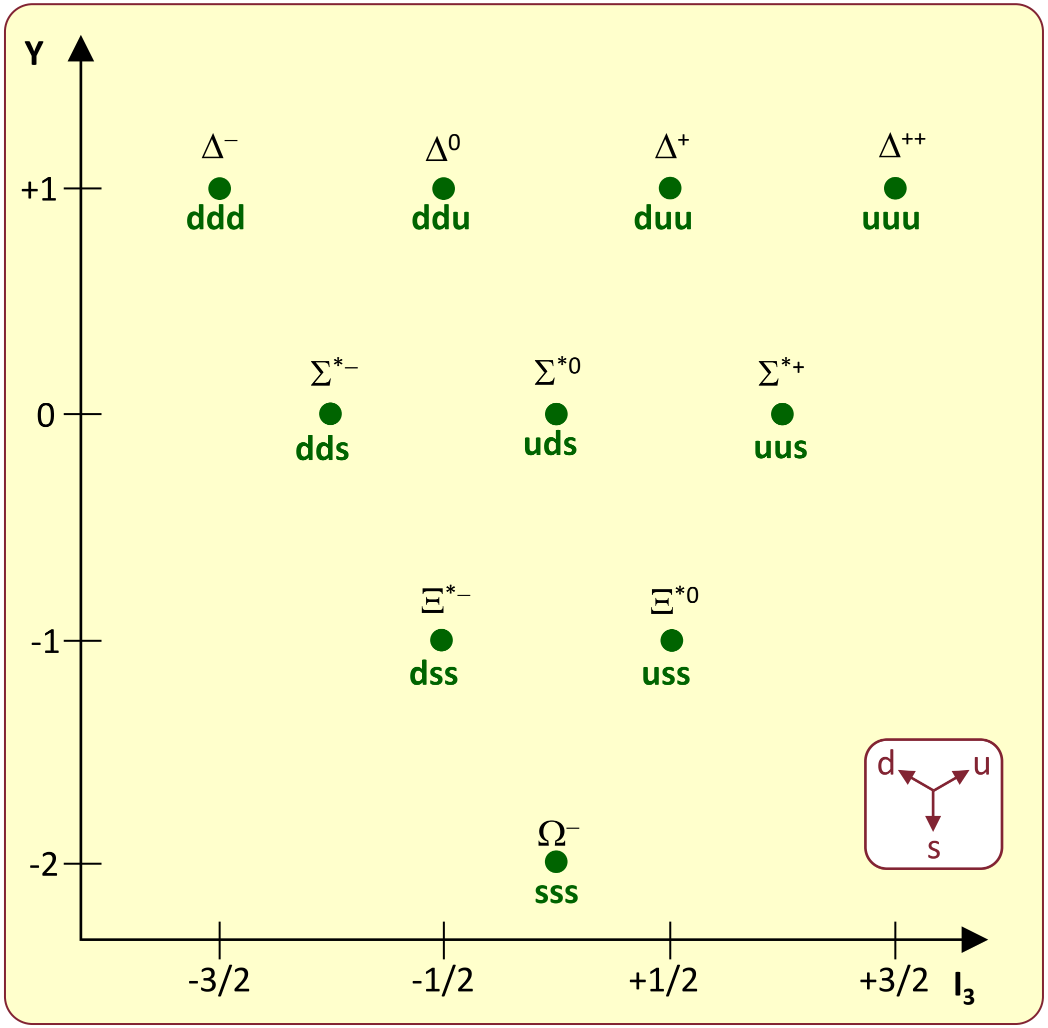
~ 1230 MeV for the Δ's;

~ 1385 MeV for the Σ^*'s,

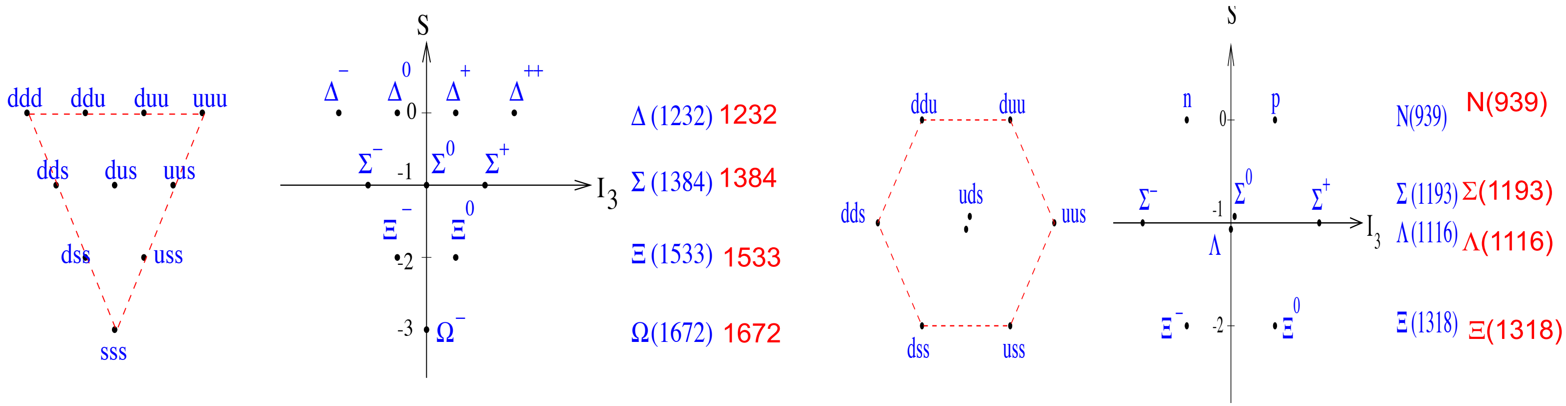
~ 1530 MeV for the Ξ^*'s

~ 1670 MeV for the Ω^-.

Notice that the mass split among multiplets is very similar, ~150 MeV (important for the Ω^- discovery, lot of speculations, no real explanation).



The baryons: the decuplet and the octet



If mass differences were solely due to the fact that the *s* quark is heavier than the *u* and *d* quarks we should have:

$$J^P = \tfrac{3}{2}^+ \quad \underbrace{\Sigma(1384) - \Delta(1232)}_{152\ MeV} = \underbrace{\Xi(1533) - \Sigma(1384)}_{149\ MeV} = \underbrace{\Omega(1672) - \Xi(1533)}_{139\ MeV}$$

$$J^P = \tfrac{1}{2}^+ \quad \Sigma(1193) = \Lambda(1116)$$

$$\underbrace{\Lambda(1116) - N(939)}_{152\ MeV} = \underbrace{\Xi(1318) - \Lambda(1116)}_{149\ MeV}$$

The order of magnitude is correct, but discrepancies are still significant.
A quantitative understanding of hadron masses must take into account the effects of the hyperfine splitting in quark interactions.

The hadron masses

If flavor SU(3) symmetry were exact all members of a given multiplet would have exactly the same mass. Yet it is not so.

$$m_\omega \approx m_\rho(u\bar{u}) = 0.78\ GeV$$

$$m_\phi(s\bar{s}) = 1.02\ GeV \qquad\qquad J^P = 1^-$$

$$m_{K^*}(s\bar{u}) = 0.89\ GeV$$

If we consider hadron masses as the sum of the masses of the constituent quarks we obtain:

$$m_u \approx m_d \approx 0.39\ GeV$$

$$m_s \approx 0.51\ GeV$$

Effective masses of quarks bound in hadrons. Constituent masses.

There are further problems:

$$m_\Delta(J = \tfrac{3}{2}) > m_N(J = \tfrac{1}{2})$$

$$m_\rho(J = 1) > m_\pi(J = 0)$$

Δ and N contain the same quarks, as do π and ρ.

The hadron masses

Since hadron masses cannot be explained solely in terms of the masses of the constituent quarks it is necessary to consider the effects of quark interaction.

In the hydrogen atom the spin-spin interaction leads to the hyperfine structure of levels.

For two pointlike fermions of magnetic moments $\vec{\mu}_i$ and $\vec{\mu}_j$ the interaction energy is $\propto \frac{\vec{\mu}_i \cdot \vec{\mu}_j}{r_{ij}^3}$

Dirac theory gives: $\vec{\mu} = \frac{e}{2m}\vec{\sigma}$

The hyperfine separation is given by:

$$\Delta E_{hf} = -\frac{2}{3}\vec{\mu}_1 \cdot \vec{\mu}_2 |\psi(0)|^2$$
$$= \frac{2\pi\alpha}{3}\frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{m_1 m_2}|\psi(0)|^2$$

It is a contact interaction: it contains the square of the wave function at zero separation and therefore it only applies to *L=0* states.

The hadron masses

For quarks the magnetic interaction associated to charge and spin is of the order of the MeV. But quarks interact through their color charges with a potential of the form:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

At small distances the term in *1/r* dominates and α_s is small enought to make the strong hyperfine splitting important:

$$\Delta E(Q\overline{Q}) = \frac{8\pi\alpha_s}{9m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2|\psi(0)|^2 \qquad \Delta E(QQ) = \frac{4\pi\alpha_s}{9m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2|\psi(0)|^2$$

In this scheme hadron masses are given by:

$$m(q_1\overline{q}_2) = m_1 + m_2 + \frac{a}{m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$m(q_1q_2q_3) = m_1 + m_2 + m_3 + \frac{a'}{2}\sum_{i>j}\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{m_im_j}$$

The hadron masses

For two quarks (or for quark-antiquark):

$$\vec{\sigma}_i = 2\vec{s}_i$$

$$\vec{s} = \vec{s}_1 + \vec{s}_2 \quad \Rightarrow \quad \vec{s}_1 \cdot \vec{s}_2 = \tfrac{1}{2}(\vec{S}^2 - \vec{s}_1^{\,2} - \vec{s}_2^{\,2})$$

Hence the eigenvalues of $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ are:

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 4\vec{s}_1 \cdot \vec{s}_2 = 2[S(S+1) - s_1(s_1+1) - s_2(s_2+1)]$$

$$= \begin{cases} +1 & S = 1 \\ -3 & S = 0 \end{cases}$$

Similarly for 3-quark systems:

$$\sum \vec{\sigma}_i \cdot \vec{\sigma}_j = 4\sum \vec{s}_i \cdot \vec{s}_j = 2[S(S+1) - 3s(s+1)]$$

$$= \begin{cases} +3 & S = \tfrac{3}{2} \\ -3 & S = \tfrac{1}{2} \end{cases}$$

$$\Rightarrow$$

$$m_\pi = m_u + m_d - \frac{3a}{m_u m_d} \qquad (S = 0)$$

$$m_{K^*} = m_u + m_s + \frac{a}{m_u m_s} \qquad (S = 1)$$

$$(\Delta E)_\Delta = +\frac{3}{m_u^2}\frac{a'}{2}$$

$$(\Delta E)_N = -\frac{3}{m_u^2}\frac{a'}{2}$$

The hadron masses

Using the experimentally measured mass values it is possible to fit the parameters m_u, m_d, m_s, a and a'. The results are:

$$m_u = m_d = 363\ MeV \qquad \frac{a'}{m_u^2} = 100\ MeV$$

$$m_s = 538\ MeV \qquad \frac{a}{m_u^2} = 160\ MeV$$

In this way the agreement with experimental data is of the order of 1 % or better.

Electromagnetic mass differences

A further contribution to hadron mass comes from the electromagnetic interaction. Let us take as an example the baryons in the octet and let us assume that the charge distributions are similar. We expect similar *electromagnetic contributions* Δm:

$$p(uud) \quad \Delta m_p = \Delta m_{\Sigma^+} \quad \Sigma^+(uus)$$

$$\Sigma^-(dds) \quad \Delta m_{\Sigma^-} = \Delta m_{\Xi^-} \quad \Xi^-(dss)$$

$$\Xi^0(uss) \quad \Delta m_{\Xi^0} = \Delta m_n \quad n(udd)$$

Let us add the bare hadron masses and sum these equations:

$$m_p + m_{\Sigma^-} + m_{\Xi^0} = m_{\Sigma^+} + m_{\Xi^-} + m_n \qquad (m_p - m_n) = (m_{\Sigma^+} - m_{\Sigma^-}) + (m_{\Xi^-} - m_{\Xi^0})$$

$$-1.3\ MeV \qquad \underbrace{-8\ MeV \qquad +6.4\ MeV}_{-1.6\ MeV}$$

Coleman-Glashow. Mass differences are associated with isospin symmetry breaking.

Electromagnetic mass differences

Electromagnetic mass differences are due to three effects:

- Difference in mass of the *u* and *d* quarks; since $m_n > m_p$ we expect $m_d > m_u$.
- Coulomb energy difference associated with the electrical energy between pairs of quarks, of the order of:

$$\frac{e^2}{R_0} \approx 2\ MeV$$

- Magnetic energy difference associated with the magnetic moment (hyperfine) interaction between quark pairs:

$$\left(\frac{e\hbar}{mc}\right)^2 \frac{1}{R_0^3} \approx 1-2\ MeV$$

Fitting the exact forms of these terms to the data it is found that:

$$m_d - m_u = 2\ MeV$$

The approximate isospin invariance can be associated with the near equality of the *u* and *d* quark masses.

Baryon Magnetic Moments

Baryon magnetic moments can be calculated as the vector sums of the moments of the constituent quarks.

For a Dirac pointlike particle of mass *m* and charge *e*:

$$\vec{\mu} = \frac{e}{2m}\vec{\sigma}$$

As and example let us calculate the magnetic moment of the proton (uud). The two u quarks are in a triplet state. Combining with a further $\left(\frac{1}{2},-\frac{1}{2}\right)$ we get:

$$\psi(\tfrac{1}{2},\tfrac{1}{2}) = \sqrt{\tfrac{2}{3}}\underbrace{\chi(1,1)\phi(\tfrac{1}{2},-\tfrac{1}{2})}_{\mu_u+\mu_u-\mu_d} - \sqrt{\tfrac{1}{3}}\underbrace{\chi(1,0)\phi(\tfrac{1}{2},\tfrac{1}{2})}_{\mu_u-\mu_u+\mu_d}$$

$$\mu_p = \tfrac{2}{3}(2\mu_u - \mu_d) + \tfrac{1}{3}\mu_d \qquad \mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

$$\boxed{\mu_p = 2.79\frac{e}{2m_p}}$$

Baryon Magnetic Moments

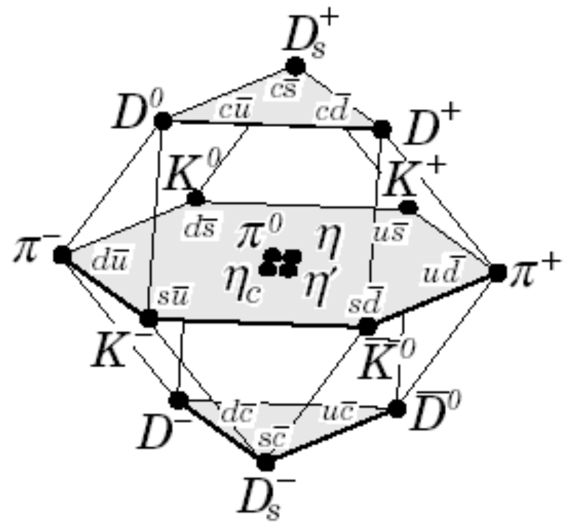
Comparison between predicted and measured magnetic moments for some baryons:

		th	$\exp$
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	2.79	2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.86	-1.913
Λ	μ_s	-0.58	-0.614
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.68	2.33
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	-1.05	-1.00
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.40	-1.25
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.47	-1.85

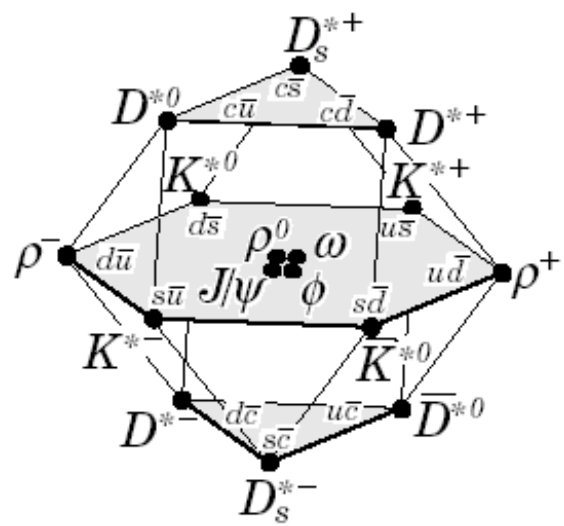
Mesons: SU(4)

SU(4): u, d, s, c $(L = 0)$

$J = 0$



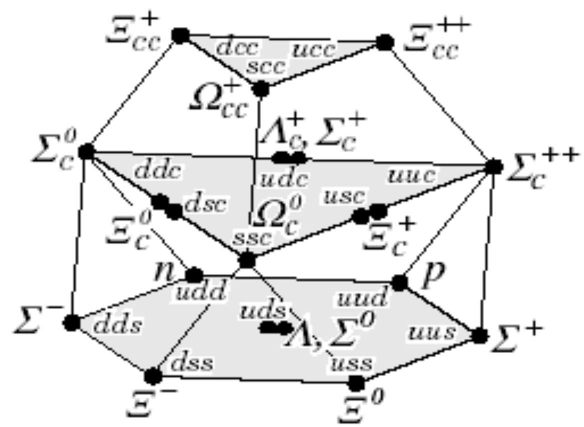
$J = 1$



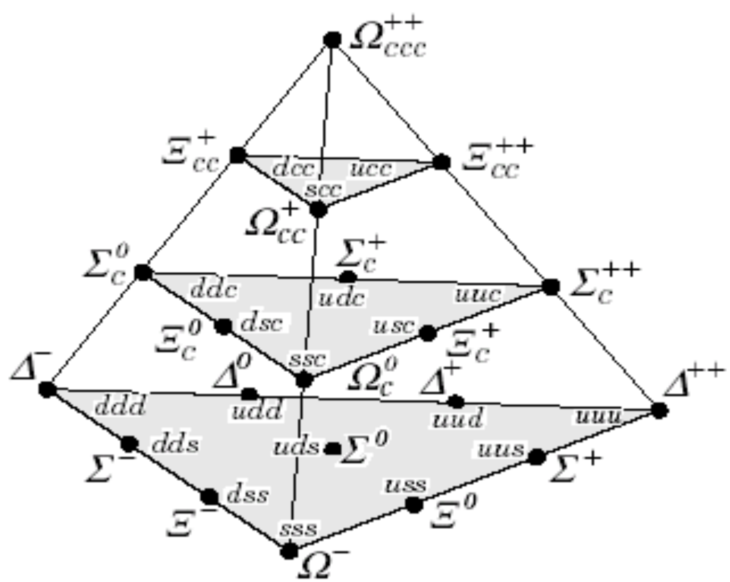
Baryons: SU(4)

SU(4): u, d, s, c $(L = 0)$

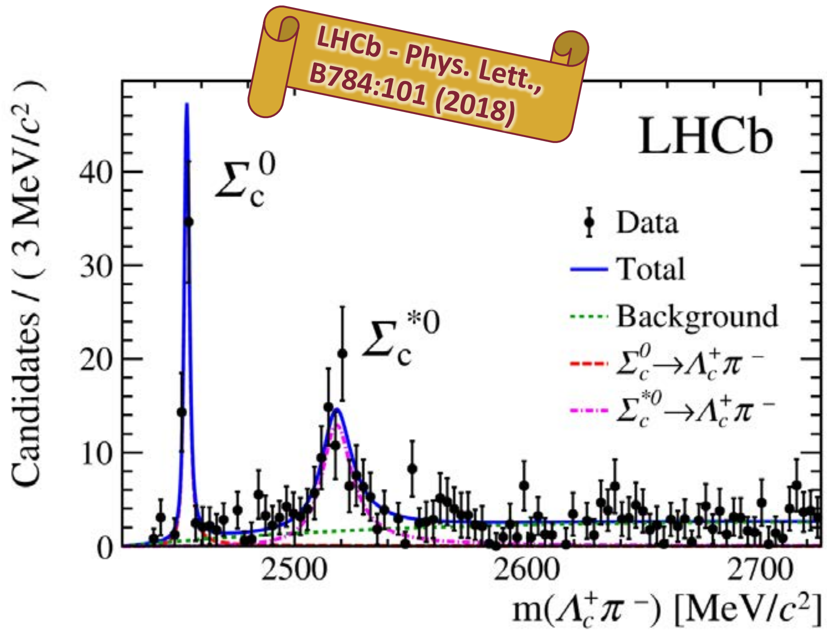
$J = \frac{1}{2}$



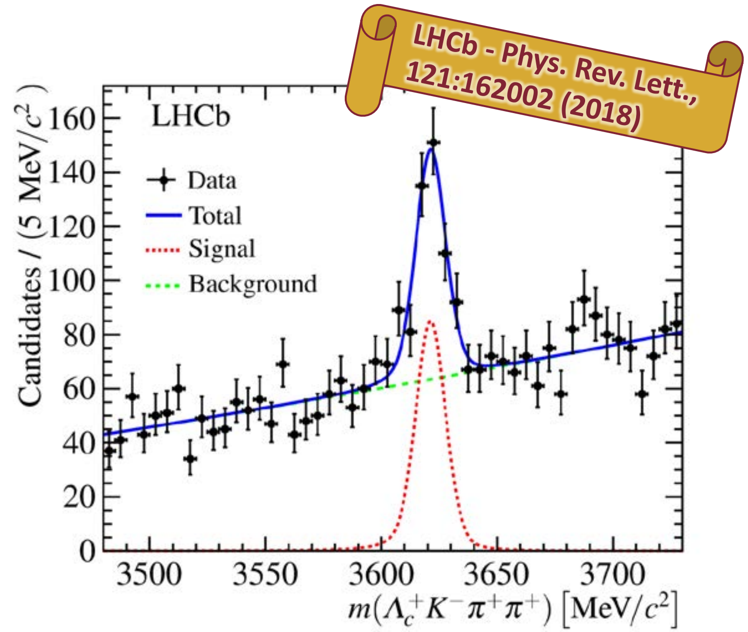
$J = \frac{3}{2}$



The baryons: newcomers!



A - $\boxed{\Sigma_c^0, \Sigma_c^{0*} \to \Lambda_c^+ \pi^-}$



B - $\boxed{\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+}$

Recently, the LHCb Collaboration at LHC has realized a nice search for baryons made with heavy quarks.

quark content:

Ξ_{cc}^{++} : ucc;
$\Sigma_c^0, \Sigma_c^{0*}$: ddc;
Λ_c^+ : udc;
K^- : $\bar{u}s$;
$\pi^\pm$: $u\bar{d}, \bar{u}d$.

Check conservation of:

✓✓ a. baryon n.
✓✓ b. charge
✓✗ c. charm
– ✗ d. strangeness

Write the Feynman diagrams of the decays

SU(3)

- For the SU(2) symmetry, the generators are the Pauli matrices. The third one is associated to the conserved quantum number I_3.
- For SU(3), the Gell-Mann matrices T_j (j=1-8) are defined (next page).
- The two diagonal ones are associated to the operators of the third component of isospin (T_3) and hypercharge (T_8).
- The eigenvectors $|u\rangle$ $|d\rangle$ $|s\rangle$ are associated with the quarks (u, d, s).

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \psi_1^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \psi_1^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix};$$

$$\sigma_2 = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \quad \psi_2^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}; \quad \psi_2^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix};$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \psi_3^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \psi_3^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix};$$

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = -i\sigma_1\sigma_2\sigma_3 = I;$$

$$\left[\sigma_i, \sigma_j\right] = 2i\sum_k \varepsilon_{ijk}\sigma_k.$$

Pauli matrices and eigenvectors

in the following, some of the properties of SU(3) in group theory: no rigorous math, only results useful for our discussions

SU(3): Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_2 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_5 = i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$\lambda_7 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}; \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Gell-Mann matrices λ_i

$$T_i = \frac{1}{2}\lambda_i$$

$$\sum_{j=1}^{8} \lambda_j^2 = \frac{16}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ diagonal.}$$

$$U = 1 + \frac{i}{2}\sum_{j=1}^{8} \varepsilon_j \lambda_j \text{ unitary matrix, } \det U = 1.$$

SU(3): eigenvectors

Definition of I_3, Y, quark eigenvectors

and related relations :

$$\hat{T}_3 = \frac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix};$$

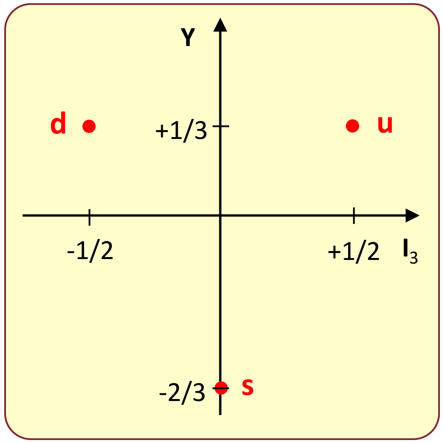
$$|u\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \qquad |d\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \qquad |s\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

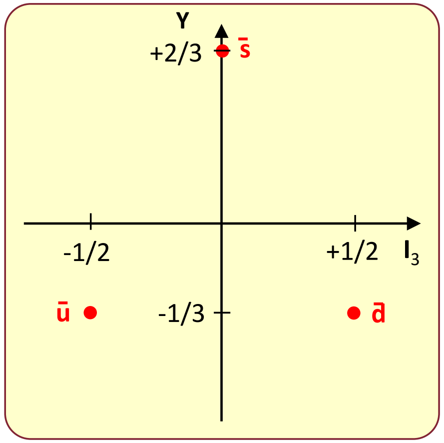
$$\hat{T}_3|u\rangle = +\frac{1}{2}|u\rangle; \qquad \hat{T}_3|d\rangle = -\frac{1}{2}|d\rangle; \qquad \hat{T}_3|s\rangle = 0;$$

$$\hat{Y}|u\rangle = +\frac{1}{3}|u\rangle; \qquad \hat{Y}|d\rangle = +\frac{1}{3}|d\rangle; \qquad \hat{Y}|s\rangle = -\frac{2}{3}|s\rangle;$$

$$\hat{T}_3|\bar{u}\rangle = -\frac{1}{2}|\bar{u}\rangle; \qquad \hat{T}_3|\bar{d}\rangle = +\frac{1}{2}|\bar{d}\rangle; \qquad \hat{T}_3|\bar{s}\rangle = 0;$$

$$\hat{Y}|\bar{u}\rangle = -\frac{1}{3}|\bar{u}\rangle; \qquad \hat{Y}|\bar{d}\rangle = -\frac{1}{3}|\bar{d}\rangle; \qquad \hat{Y}|\bar{s}\rangle = +\frac{2}{3}|\bar{s}\rangle.$$



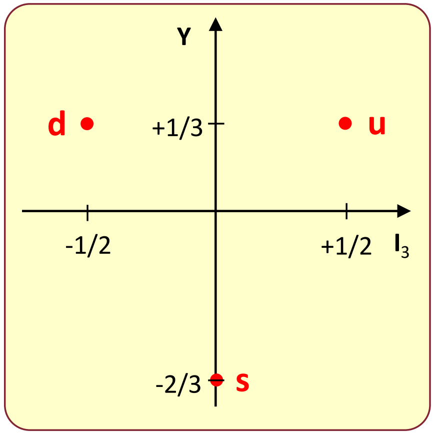


SU(3): operators

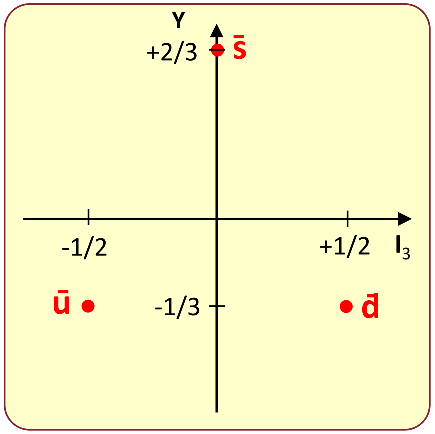
The ladder operators $T_\pm$, $U_\pm$, $V_\pm$:

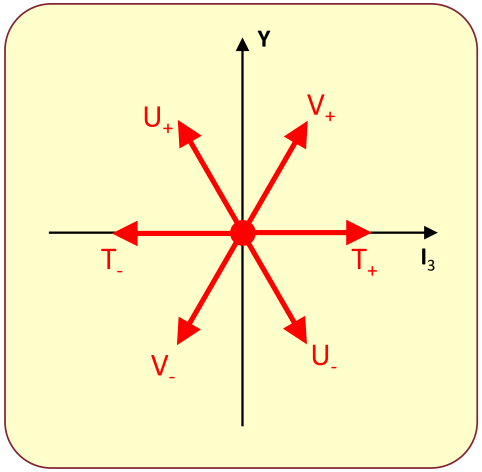
As an example, take V_+ :

$$T_\pm = T_1 \pm iT_2; \quad U_\pm = T_6 \pm iT_7; \quad V_\pm = T_4 \pm iT_5;$$



$$V_+ = T_4 + iT_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{cases} V_+\left|\bar{u}\right\rangle = -\left|\bar{s}\right\rangle; \\ V_+\left|\bar{d}\right\rangle = 0; \\ V_+\left|\bar{s}\right\rangle = 0; \end{cases}$$



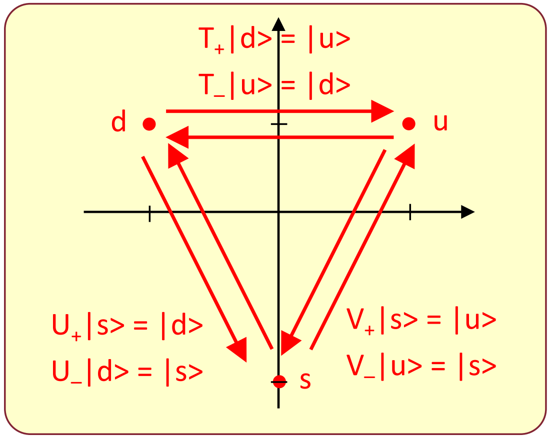


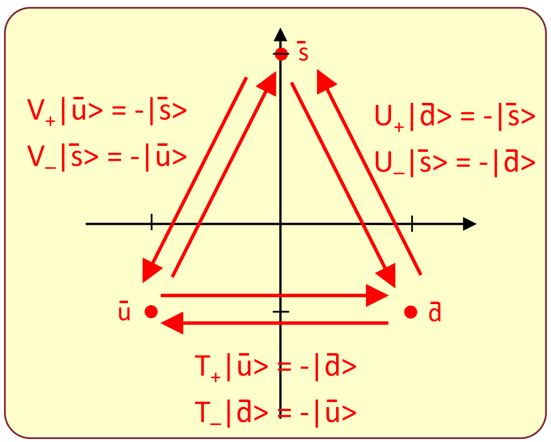
$$V_+\left|u\right\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0;$$

$$V_+\left|d\right\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0;$$

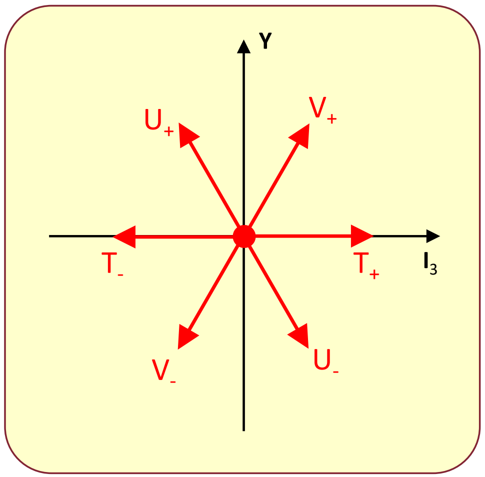
$$V_+\left|s\right\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \left|u\right\rangle.$$

SU(3): ladder operators





The ladder operators $T_\pm$, $U_\pm$, $V_\pm$.



§ QCD

Exercises

In the K^0 + proton, reaction in which the K^0 have kinetic energy of 800 MeV, explain if strange baryons can be produced. If this is not the case, explain why.

The D_s meson has charm=1, strangeness=1 and spin zero. Write its quark composition, its isotopic splin and its charge. The D^*_s meson has the same quark composition of the D_s but spin 1. The D^*_s decay in $D_s + \pi$ with a B.R. of 6%. Explain which is the ineraction responsible of this decay.

The octect of $1/2^+$ SU(3) baryons is composed by p, n,Σ^+, Σ^0, Σ^-, Ξ^-, Ξ^0, Λ. The lifetime of the Λ is $2.6 \cdot 10^{-10}$s, the one of the Σ is $7.4 \cdot 10^{-20}$s and the one of the Ξ_0 es $2.9 \cdot 10^{-10}$ s. Why he lifetime of the Σ^0 is smaller than the one of the Λ and Ξ^0.

Exercises

ρ^0 (770) and f_2^0(1270) mesons decay by strong interaction in a $\pi^+\pi^-$ pair, and have spin spin 1 and spin 2 respectively
One of the two cannot decay into two neutral pions. Identify who is the meson that cannot decay into $\pi^0\pi^0$ and explain why.

Color: a new quantum number

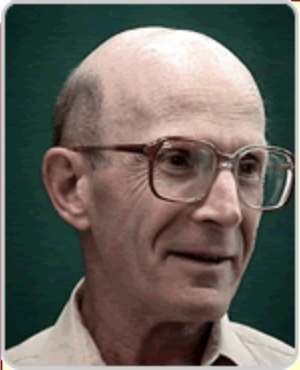
Consider the Δ^{++} resonance:

- $J^P = 3/2^+$ (measured);

$\rightarrow$ quark/spin content [*no choice*]:
$|\Delta^{++}\rangle = |\, u\Uparrow u\Uparrow u\Uparrow \rangle$

- wave function :

$\boxed{\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin}}$ NO !!!



Oskar W. Greenberg

Moo-Young Han (한무영)

Yoichiro Nambu (南部 陽一郎, *Nambu Yōichirō*)

Why "NO" ?
Consider the symmetry of $\psi(\Delta^{++})$:

- it is lightest uuu state $\rightarrow$ L = 0

$\rightarrow \psi_{space}$ symmetric;

$\rightarrow \psi_{flavor}$ and ψ_{spin} symmetric;

$\rightarrow \psi(\Delta^{++})$ = sym.$\times$sym.$\times$sym. = sym.

... but the Δ^{++} is a fermion ...NO

Anomaly : the Δ^{++} is a spin $^3/_2$ fermion and its function MUST be antisymmetric for the exchange of two quarks (Pauli principle). However, this function is the product of three symmetric functions, and therefore is symmetric $\rightarrow$ **???**.

The solution was suggested in 1964 by Greenberg, later also by Han and Nambu. They introduced a **new quantum number** for strongly interacting particles, composed by quarks : the **COLOR**.

Color: why and how

The idea:

1. quarks exist in three colors (say Red, Green and Blue, like the TV screen(*);
2. they sum like in a TV-screen : e.g. when RGB are all present, the screen is white;
3. the "anticolor" is such that, color + anticolor gives white (e.g. $\bar{R}$ = G + B);
4. anti-quarks bring ANTI-colors (see previous point);
5. Mesons and Baryons, which are made of quarks, are white and have no color: they are a "**color singlet**".

Therefore, we have to include the color in the complete wave function; e.g. for Δ^{++} :

$$\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin} \times \psi_{color}$$

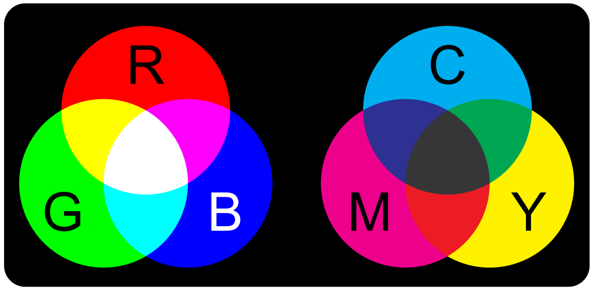
$$\psi_{color} = (1/\sqrt{6})\,(u_r^1 u_g^2 u_b^3 + u_g^1 u_b^2 u_r^3 + u_b^1 u_r^2 u_g^3 - u_g^1 u_r^2 u_b^3 - u_r^1 u_b^2 u_g^3 - u_b^1 u_g^2 u_r^3)$$

(where u_r, u_g, u_b are the color functions for u quarks of red, green, blue type).

Then ψ_{color} is antisymmetric for the exchange of two quarks and so is the global wave function.

The introduction of the color has many other experimental evidences and theoretical implications, *which we will discuss in the following*.

(*) *however, these colors are in no way similar to the real colors; therefore the names "red-green-blue" are totally irrelevant.*



Summary: Simmetries and Multiplets

1. Since the strong interactions conserve isotopic spin ("$\mathbb{I}$"), hadrons gather in I-multiplets. Within each multiplet, the states are identified by the value of I_3.

2. If no effect breaks the symmetry, the members of each multiplet would be mass-degenerate. The electromagnetic interactions, which do not respect the I-symmetry, split the mass degeneration (at few %) in I-multiplets.

3. Since the strong interactions conserve I, $\mathbb{I}$-operators must commute with the strong interactions Hamiltonian ("$\mathbb{H}_s$") and with all the operators which in turn commute with $\mathbb{H}_s$.

4. Among these operators, consider the angular momentum $\mathbb{J}$ and the parity $\mathbb{P}$. As a result, all the members of an isospin multiplet must have the same spin and the same parity.

5. $\mathbb{H}_s$ is also invariant with respect to unitary representations of SU(2). The quantum numbers which identify the components of the multiplets are as many as the number of generators, which can be diagonalized simultaneously, because are mutually commuting. This number is the *rank* of the Group. In the case of SU(2) the rank is 1 and the operator is $\mathbb{I}_3$.

6. Since $[\mathbb{I}_j , \mathbb{I}_k] = i\varepsilon_{jkm}\mathbb{I}_m$, each of the generators commutes with $\mathbb{I}^2$:

   $\mathbb{I}^2 = \mathbb{I}_1^2 + \mathbb{I}_2^2 + \mathbb{I}_3^2$.

   Therefore $\mathbb{I}^2$, obviously hermitian, can be diagonalized at the same time as $\mathbb{I}_3$.

(continue ...)

Summary: Simmetries and Multiplets

7. The eigenvalues of $\mathbb{I}$ and $\mathbb{I}_3$, can "tag" the eigenvectors and the particles.

8. This fact gives the possibility to regroup the states into multiplets with a given value of *I*.

9. We can generalize this mechanism from the isospin case to any operator : if we can prove that $\mathbb{H}$ is invariant for a given kind of transformations, then:
   a. look for an appropriate symmetry group;
   b. identify its irreducible representations and derive the possible multiplets,
   c. verify that they describe physical states which actually exist.

10. This approach suggested the idea that Baryons and Mesons are grouped in two octets, composed of multiplets of isotopic spin.

11. In reality, since the differences in mass between the members of the same multiplet are ~20%, the symmetry is "broken" (i.e. approximated).

12. Since the octets are characterized by two quantum numbers (I_3 and Y), the symmetry group has rank = 2, i.e. two of the generators commute between them.

13. We are interested in the "irreducible representations" of the group, such that we get any member of a multiplet from everyone else, using the transformations.

(… continue …)

Summary: Simmetries and Multiplets

14. The non-trivial representation (non-trivial = other than the Singlet) of lower dimension is called "Fundamental representation".

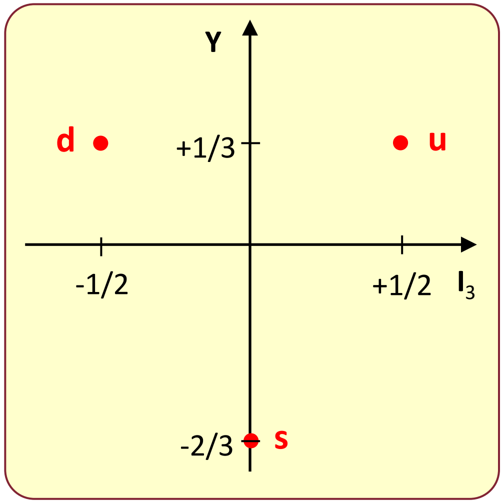
15. In SU(3) there are eight symmetry generators. Two of them are diagonal and associated to I_3 and Y.

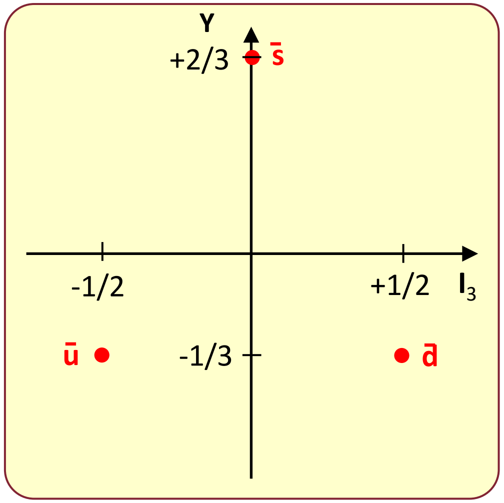
16. The fundamental representations are triplets ($\rightarrow$ quarks), from which higher multiplets ($\rightarrow$ hadrons) are derived :

    mesons: $3 \otimes \bar{3} \quad = 1 \oplus 8$;

    baryons: $3 \otimes 3 \otimes 3 \quad = 1 \oplus 8 \oplus 8 \oplus 10$.

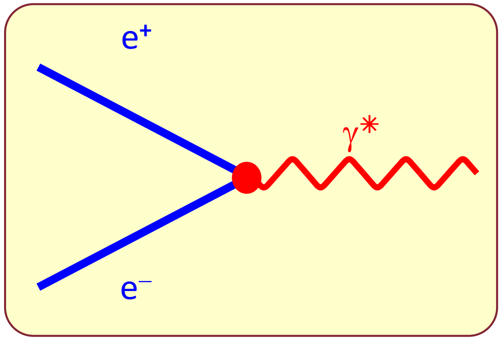
17. This purely mathematical scheme has two relevant applications:
    a. "flavour SU(3)", $SU(3)_F$ with Y_F and I_{3F} for the quarks uds – this symmetry is approximate (i.e. "broken");
    b. "color SU(3)", $SU(3)_C$ with Y_C and I_{3C} for the colors rgb; this symmetry is exact.





Collision e^+e^-: initial state

- At low energy$^{(*)}$, the main processes happen with annihilation into a virtual γ^*.
- The initial state is :
  - charge = 0;
  - lepton (+ baryon + other additive) number = 0;
  - spin = 1 ("γ^*");



- CM kinematics :
  - e^+ [E, p, 0, 0];
  - e^- [E, −p, 0, 0];
  - γ^* [2E, 0, 0, 0];
  - $m(\gamma^*) = \sqrt{s} = 2E$ [virtual photon, short lived].

Remember…Mandelstam variables

- in absence of polarization, the cross sections of a process "X" does NOT depend on the azimuth φ :

$$\frac{d\sigma_{"X"}}{d\Omega} = \frac{1}{2\pi}\frac{d\sigma_{"X"}}{d\cos\theta} = \frac{s}{4\pi}\frac{d\sigma_{"X"}}{dt}.$$

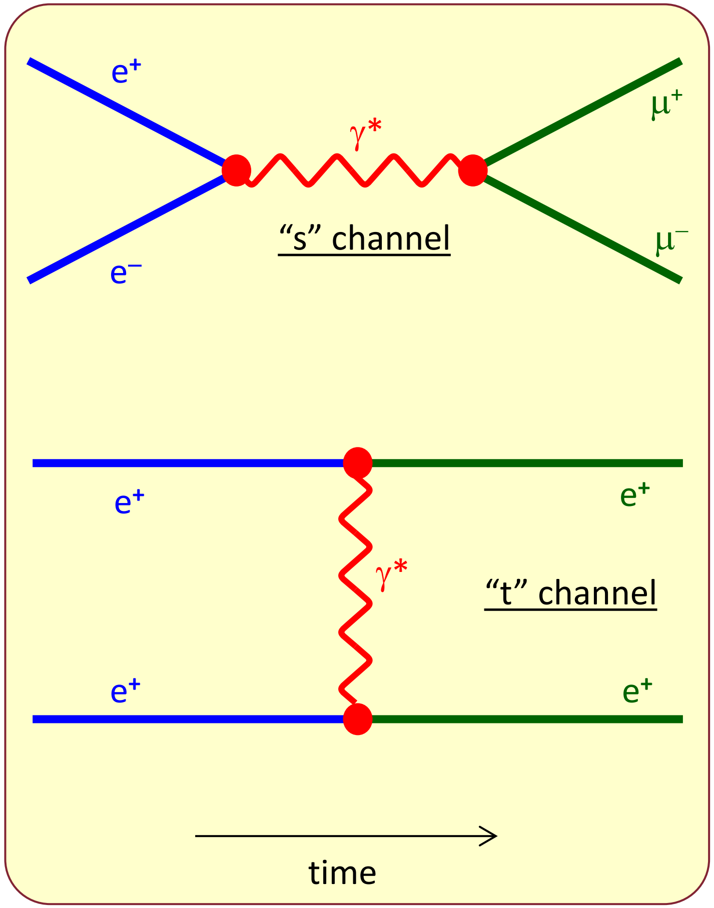
- for $m^2 \ll s$, if $\mathcal{M}_{"X"}$ is the matrix element of the process$^{(*)}$:

$$\frac{d\sigma_{"X"}}{dt} = \frac{\left|\mathcal{M}_{"X"}\right|^2}{16\pi s^2}.$$

- in lowest order QED, if $m^2 \ll s$:

$$\frac{d\sigma_{"X"}}{d\cos\theta} = \frac{\left|\mathcal{M}_{"X"}\right|^2}{32\pi s} = \frac{\alpha^2}{s} f(\cos\theta).$$

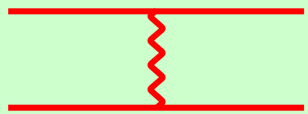
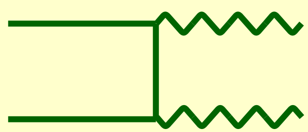
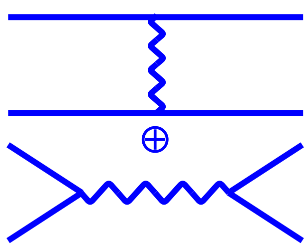
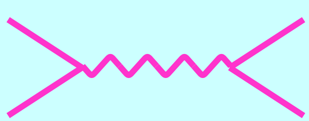
- when $\theta \to 0$, $\cos\theta \to 1$:
  - s-channel : $f(\cos\theta) \to$ constant;
  - t-channel : $f(\cos\theta) \to \infty$.



(*) also by dimensional analysis :
$[c = \hbar = 1]$, $[\sigma] = [\ell^2]$; $[t] = [s] = [\ell^{-2}]$;
therefore, in absence of any other dimensional scale,
σ [and $d\sigma/d\Omega$] = [number] × $1/s$.

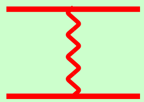
Collision e^+e^-: QED cross-section

Consider some QED processes in lowest order [$\sqrt{s} \ll m_Z$, only γ^* exchange] :

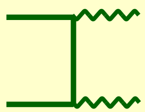
➢ $e^\pm e^\pm \to e^\pm e^\pm$		$\dfrac{d\sigma(e^\pm e^\pm \to e^\pm e^\pm)}{d\cos\theta} = \dfrac{2\pi\alpha^2}{s} \times \left(\dfrac{3+\cos^2\theta}{1-\cos^2\theta}\right)^2;$
➢ $e^+e^- \to \gamma\gamma$		$\dfrac{d\sigma(e^+e^- \to \gamma\gamma)}{d\cos\theta} = \dfrac{2\pi\alpha^2}{s} \times \dfrac{1+\cos^2\theta}{1-\cos^2\theta};$
➢ $e^+e^- \to e^+e^-$		$\dfrac{d\sigma(e^+e^- \to e^+e^-)}{d\cos\theta} = \dfrac{\pi\alpha^2}{2s} \times \left(\dfrac{3+\cos^2\theta}{1-\cos\theta}\right)^2;$
➢ $e^+e^- \to \mu^+\mu^-$		$\dfrac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\cos\theta} = \dfrac{\pi\alpha^2}{2s} \times \left(1+\cos^2\theta\right);$

Collision e⁺e⁻: QED dσ/dcosθ

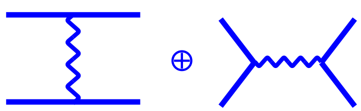
$$\frac{d\sigma(e^{\pm}e^{\pm} \to e^{\pm}e^{\pm})}{d\cos\theta} = \frac{2\pi\alpha^2}{s} \times \left(\frac{3+\cos^2\theta}{1-\cos^2\theta}\right)^2;$$



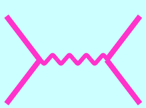
$$\frac{d\sigma(e^{+}e^{-} \to \gamma\gamma)}{d\cos\theta} = \frac{2\pi\alpha^2}{s} \times \frac{1+\cos^2\theta}{1-\cos^2\theta};$$

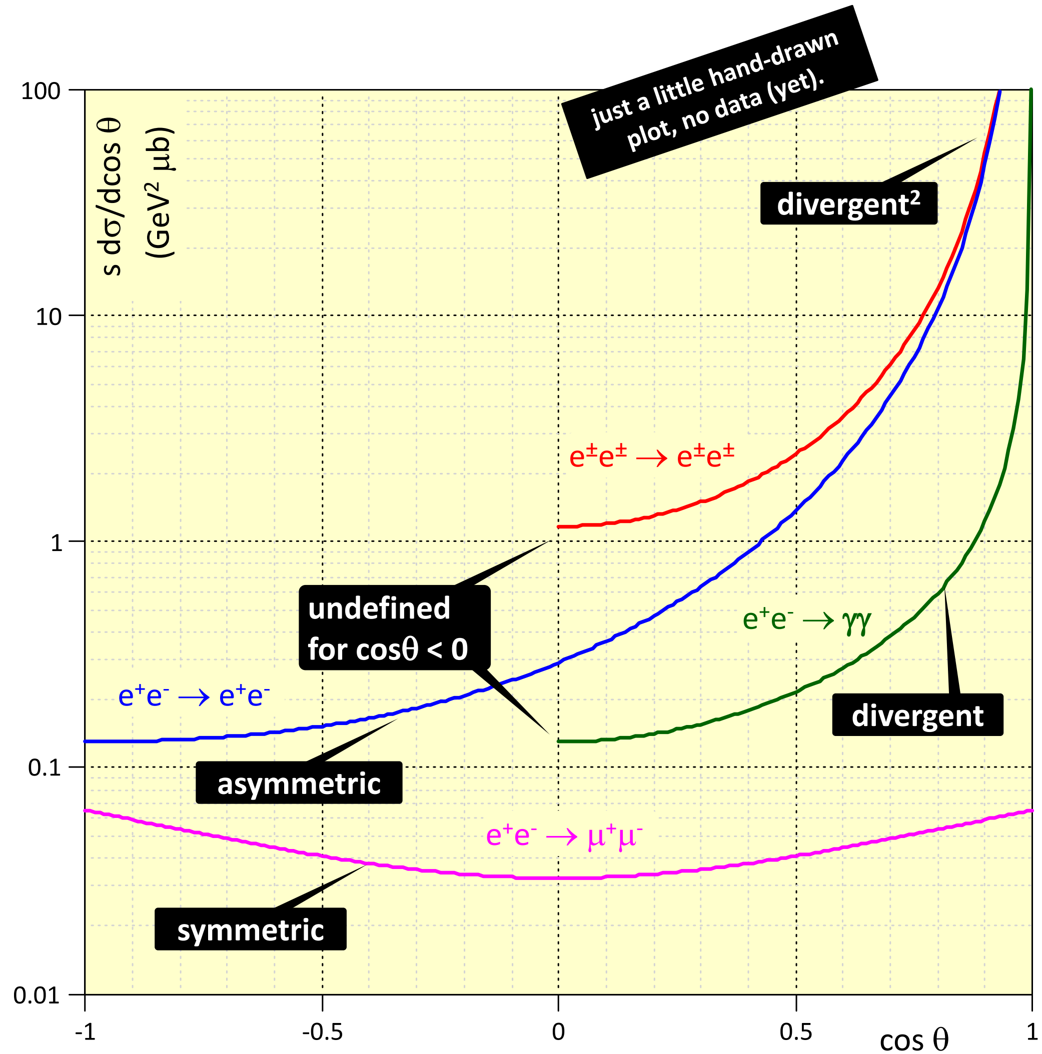


$$\frac{d\sigma(e^{+}e^{-} \to e^{+}e^{-})}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \times \left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2;$$



$$\frac{d\sigma(e^{+}e^{-} \to \mu^{+}\mu^{-})}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \times \left(1+\cos^2\theta\right);$$





Collision e^+e^-: $e^+e^- \rightarrow \mu^+\mu^-$/qqbar

- kinematics, computed in CM sys, $\sqrt{s} >> m_e, m_\mu$:

e^+ $(E, \quad p, \quad 0, \; 0);$

e^- $(E, \quad -p, \quad 0, \; 0);$

μ^+ $(E, \; p\cos\theta, \; p\sin\theta, \; 0);$

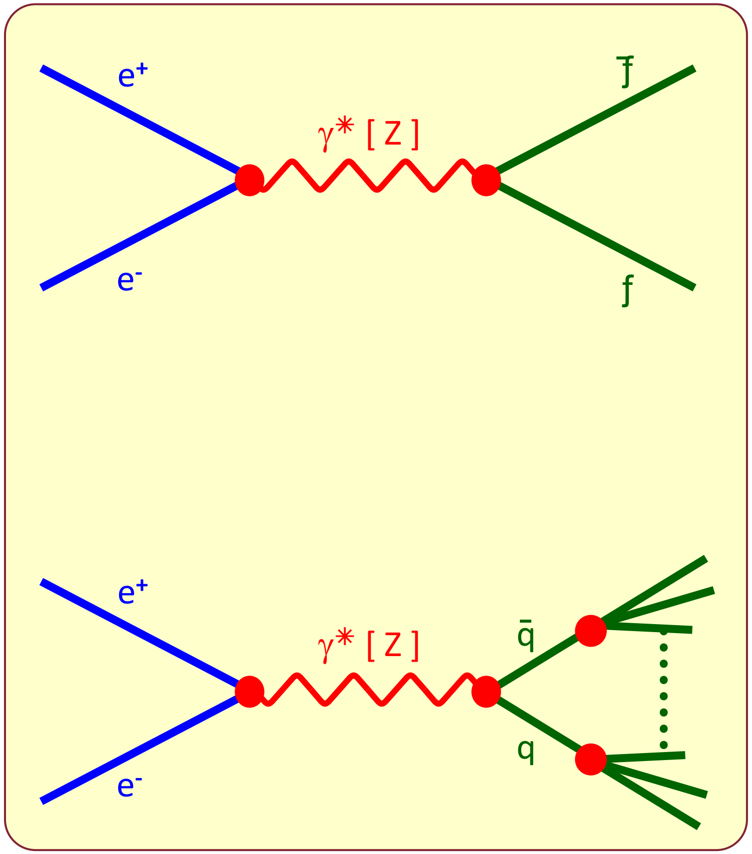
μ^- $(E, -p\cos\theta, -p\sin\theta, \; 0);$

$p \approx E = \sqrt{s}/2;$

$\vec{p}(e^+) \cdot \vec{p}(\mu^+) \approx E^2 \cos\theta \approx s \cos\theta / 4;$

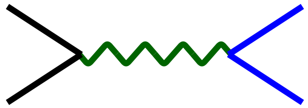
$p(e^+)\, p(\mu^+) \approx E^2 (1 - \cos\theta) = s \sin^2(\theta/2) = -t;$

- the case **$e_+e_- \rightarrow q\bar{q}$** is similar at parton level; however free (anti-)quarks do NOT exist → quarks hadronize, producing collimated jets of hadrons



Collisions e^+e^- : $\sigma(e^+e^- \to \mu^+\mu^-, q\bar{q})$

- $e^+e^- \to \mu^+\mu^-$

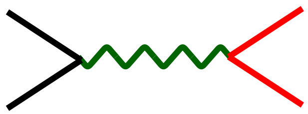


$$\sigma_{\mu\mu} = \int_{-1}^{1} d\cos\theta \left[\frac{d\sigma_{\mu\mu}}{d\cos\theta}\right] = \frac{\pi\alpha^2}{2s}\int_{-1}^{1} d\cos\theta\left(1+\cos^2\theta\right) =$$
$$= \frac{4\pi\alpha^2}{3s} = \frac{86.8\ \text{nb}}{s[\text{GeV}^2]} = \frac{21.7\ \text{nb}}{E^2_{\text{beam}}[\text{GeV}^2]}.$$

$[1+\cos^2\theta] = P_1^{\text{Legendre}}(\cos\theta)$

[spin 1 $\to$ 2 spin ½]

- $e^+e^- \to q\bar{q}$

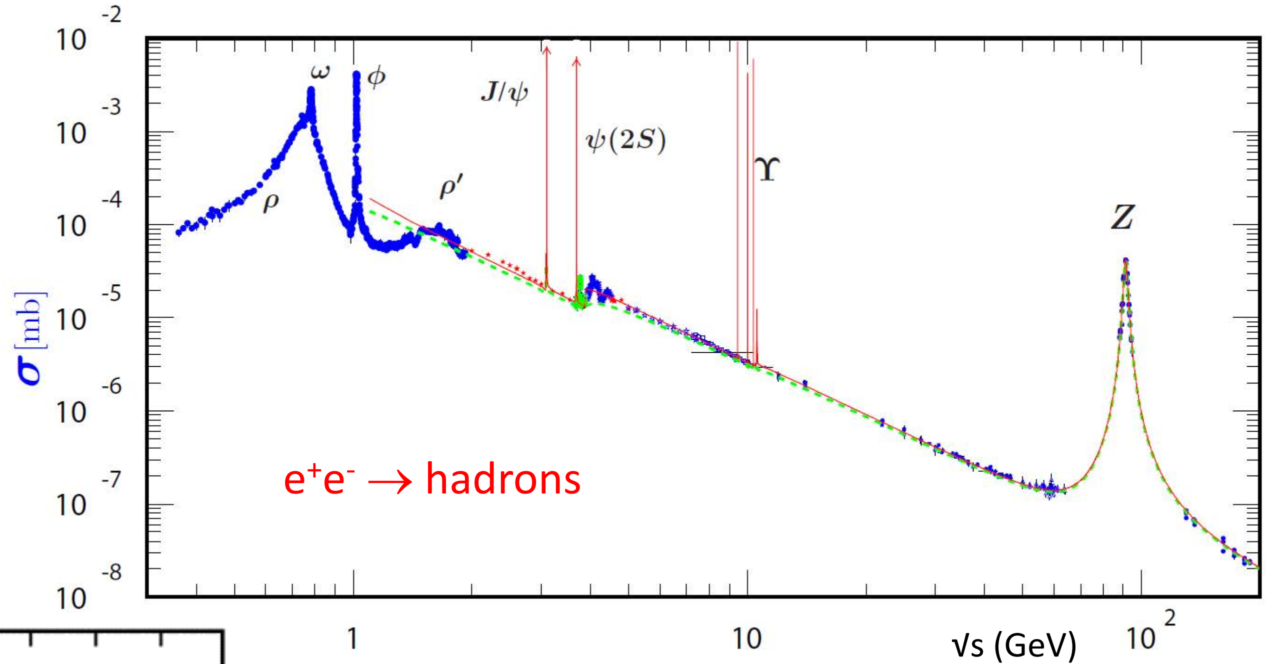


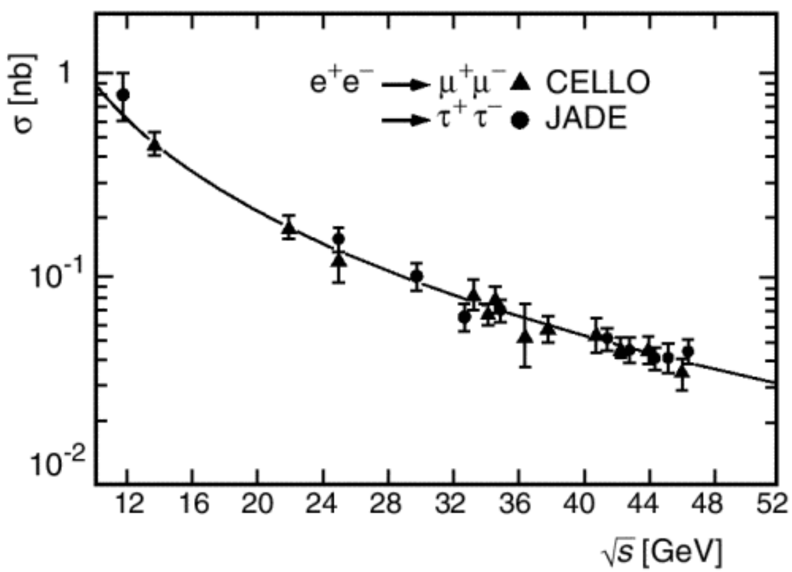
$$\frac{d\sigma_{q\bar{q}}}{d\cos\theta} = \frac{d\sigma_{\mu\mu}}{d\cos\theta}\times c_f e_f^2 = \frac{\pi\alpha^2}{2s} c_f e_f^2\left(1+\cos^2\theta\right); \quad c_f = \begin{Bmatrix} 3 & \text{quarks} \\ 1 & \text{leptons} \end{Bmatrix} \quad \text{[color]}$$

$$\sigma_{q\bar{q}} = \sigma_{\mu\mu} c_f e_f^2 = \frac{4\pi\alpha^2}{3s} c_f e_f^2; \quad e_f = \begin{Bmatrix} 1 & \text{leptons} \\ 2/3 & u\ c\ t \\ -1/3 & d\ s\ b \end{Bmatrix} \quad \text{[charge].}$$

Collisions e^+e^- : $\sigma_{large\sqrt{s}}(e^+e^- \to \mu^+\mu^-, q\bar{q})$

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s} = \\ = \frac{86.8\ \text{nb}}{s[\text{GeV}^2]} = \frac{21.7\ \text{nb}}{E^2[\text{GeV}^2]}.$$



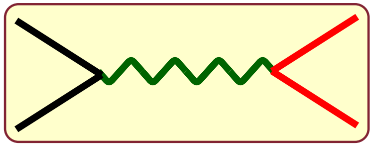


- the continuum, for $0.5 \le \sqrt{s} \le 50$ GeV, agrees well with the predicted 1/s [the line in log-log scale];
- + resonances $q\bar{q}$ [the bumps];
- for $\sqrt{s} > 50$ GeV it is dominated by the Z formation in the s-channel.

Collisions e^+e^- : $R = \sigma(q\bar{q})/\sigma(\mu^+\mu^-)$

- define the quantity, both simple conceptually and easy to measure:

$$\boxed{R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = 3\sum_{\text{quarks}} e_i^2 = R\left(\sqrt{s}\right);}$$



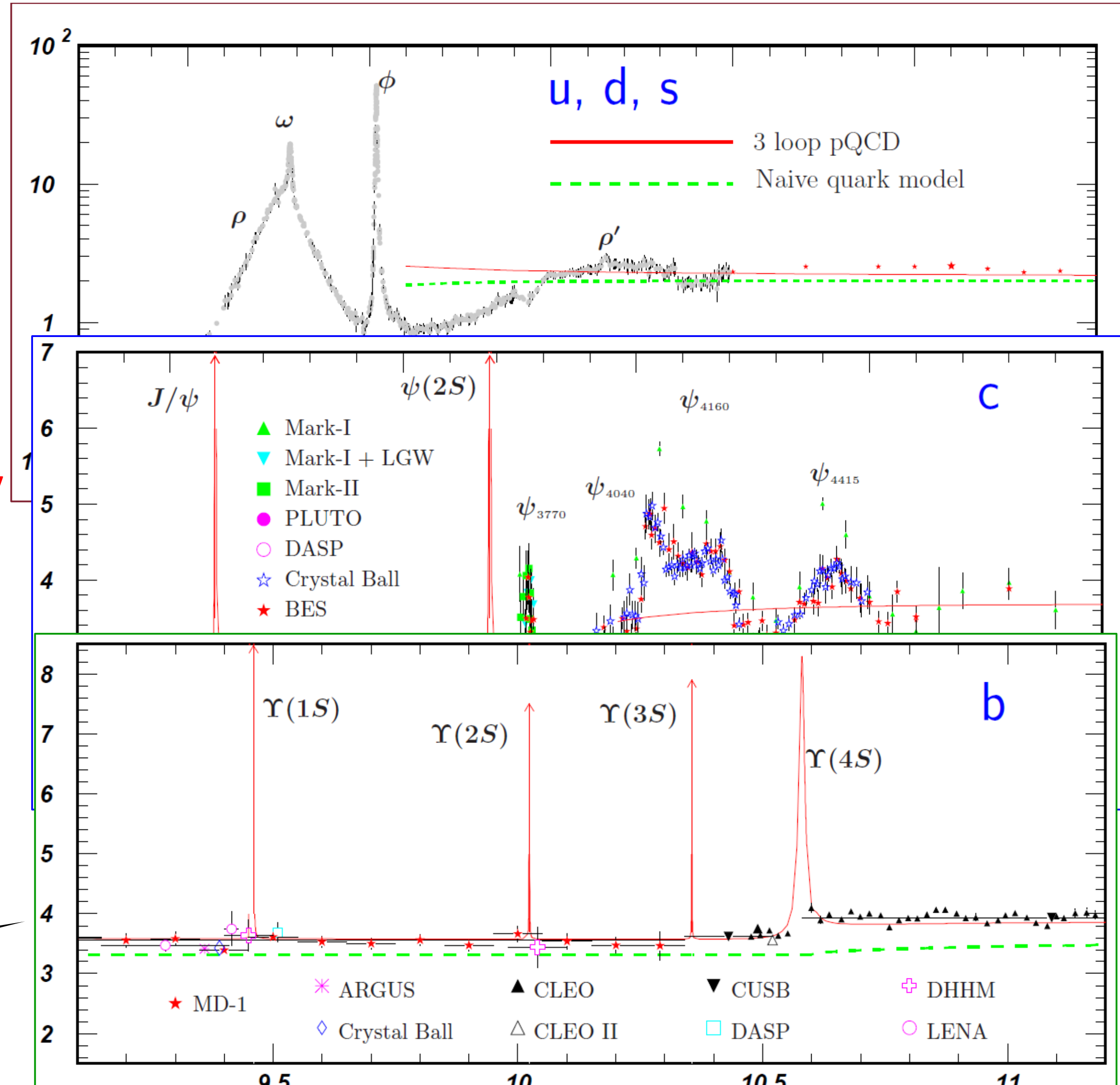
- sum over all the quarks, produced at energy $\sqrt{s}$ (i.e. $2m_q < \sqrt{s}$) :
  - $0 < \sqrt{s} < 2\,m_c$: $R = R_{uds} = 3 \times [\,(^2/_3)^2 + (^{-1}/_3)^2 + (^{-1}/_3)^2\,] = 2;$
  - $2\,m_c < \sqrt{s} < 2\,m_b$: $R = R_{udsc} = R_{uds} + 3 \times (^2/_3)^2 = 3 + {}^1/_3;$
  - $2\,m_b < \sqrt{s} < 2\,m_t$: $R = R_{udscb} = R_{udsc} + 3 \times (^{-1}/_3)^2 = 3 + {}^2/_3;$
  - ~~$2\,m_t < \sqrt{s} < \infty$: $R = R_{udscbt} = R_{udscb} + 3 \times (^2/_3)^2 = 5$~~;

Collisions e^+e^- : R vs $\sqrt{s}$ (small $\sqrt{s}$)

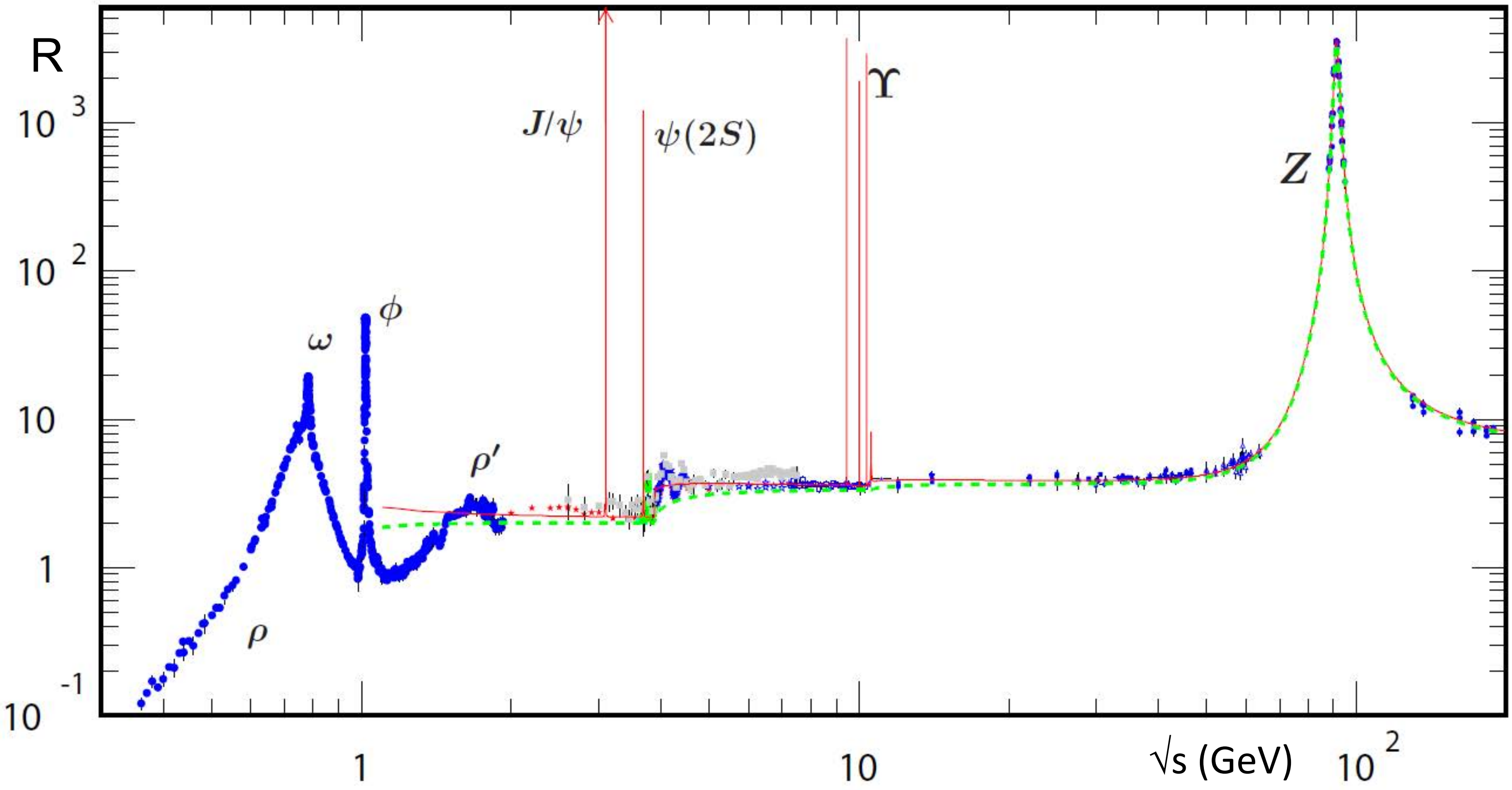
Plot R vs $\sqrt{s}$ (=2E):

- resonances $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ at 1-2 GeV (only those with $J^P=1^-$) ($\rightarrow$"vector dominance");
- step at $2m_c$ (J/ψ);
- step at $2m_b$ (Υ);
- slow increase at $\sqrt{s}$ > 50 GeV (Z, next slide);
- [lot of effort required, as demonstrated by the number of detectors and accelerators];
- strong evidence for the color (factor 3 necessary).

plots from [PDG, 588]



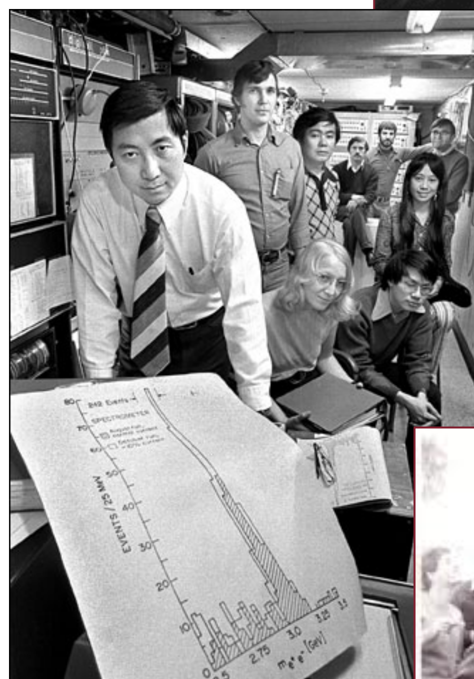
Collisions e^+e^- : R vs √s (large √s)



- The full range 200 MeV < √s < 200 GeV (3 orders of magnitude !!!).
- For √s > 50 GeV new phenomenon: electroweak interactions and the Z pole.

The November revolution

- The u,d,s quarks have not been predicted; in fact the mesons and baryons have been discovered, and later interpreted in terms of their quark content [§ 1];
- ***Some theoreticians had foreseen another quark, based on** (no $K^0 \rightarrow \mu^+\mu^-$)**, but people did not believe it.***
- In November 1974, the groups of Burton Richter (SLAC) and Samuel Ting (Brookhaven) discovered simultaneously a new state with a mass of $\approx$ 3.1 GeV and a tiny width, much smaller than their respective mass resolution.



the two experiments are quite different: we will review first the "J" and then the "ψ".

- Ting & coll. had the name "J", while Richter & coll. called it "ψ". Today's name is "J/ψ".
- We split the discussion : start with the hadronic experiment.

- The width was measured, after some time, to be 0.087 MeV, a surprisingly small value for a resonance of 3 GeV mass.

The November revolution: J

- The group of Ting at the AGS proton accelerator measured the inclusive production of e^+e^- pairs in interactions of 30 GeV protons on a plate of beryllium :

  $$p\,Be \to e^+e^-\,X.$$

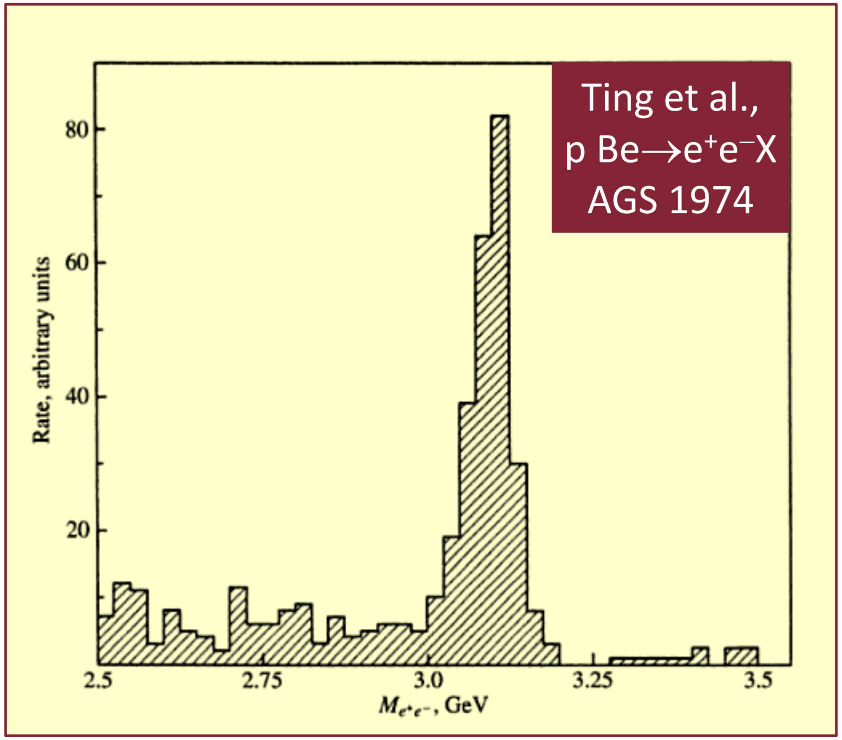
- The detector was designed to search for high mass resonances with $J^P = 1^-$ (= γ), decaying into (e^+e^-) pairs.
- They were very clever in minimizing the multiple scattering $\to$ the resolution for the invariant mass was good:

  $$\Delta m(e^+e^-) \approx 20\text{ MeV}.$$

- This resolution allowed for a much higher sensitivity wrt another previous exp. (Leon Lederman), which studied $\mu^+\mu^-$ pairs in the same range. Lederman had a "shoulder" in $d\sigma/dm(\mu^+\mu^-)$, but no conclusive evidence [next slide].

- Ting called the new particle "J", because of the e.m. current.

Measured quantum numbers of the J:
- mass ~3.1 GeV;
- width << 20 MeV (upper limit, not meas.);
- charge = 0;
- $J^P = 1^-$;
- no isospin, Γ, other decay modes ...



The November revolution: the J experiment

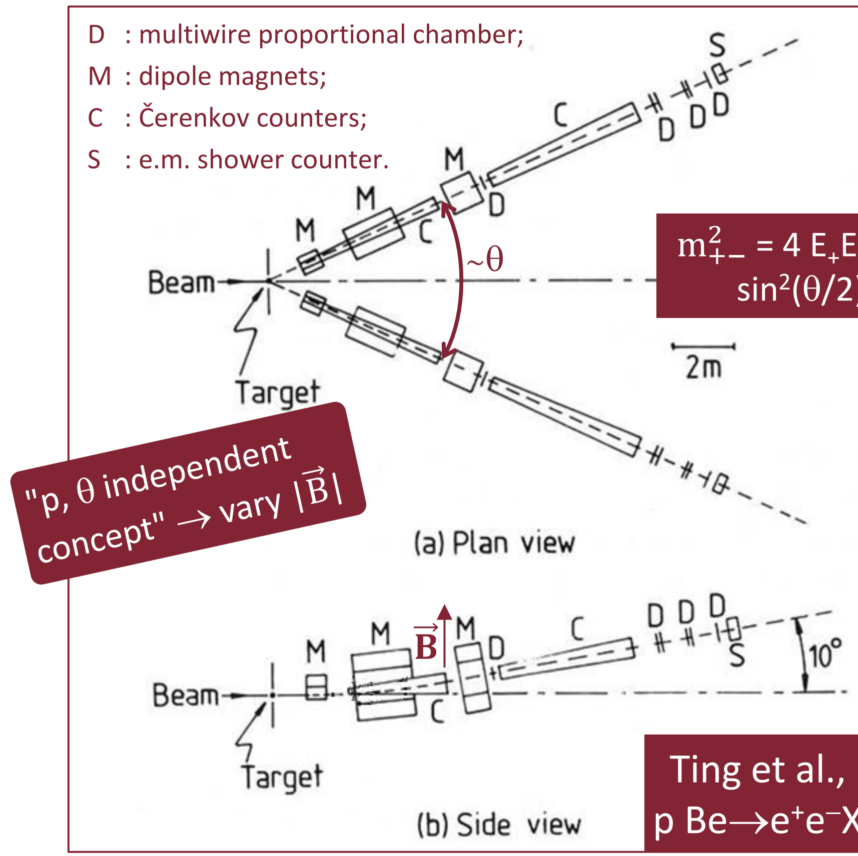
- The Ting experiment used a two arm magnetic spectrometer, to measure separately the electron and the positron.

- Ting (and also Lederman) studied the <u>Drell-Yan process</u>: hadron collisions $\to \gamma^* \to \ell^+\ell^-$ (Ting: e^+e^- / Lederman: $\mu^+\mu^-$).

- Leptonic events are rare $\to$ very intense beams (2×10^{12} ppp $^{(*)}$) $\to$ high rejection power ($\sim10^8$) to discard hadrons, that can fake <u>e^+e^-</u> or <u>$\mu^+\mu^-$</u>.

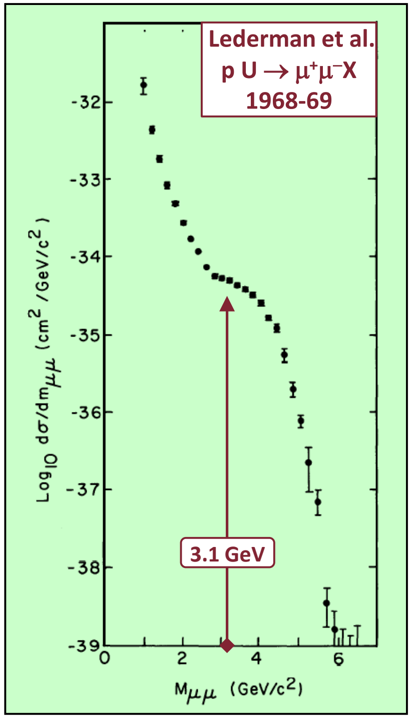
- Advantage in the <u>$\mu^+\mu^-$ case</u>: μ penetration $\to$ select leptons from hadrons with a thick absorber in a large solid angle $\to$ larger acceptance, higher counting rate.
- Disadvantage : thick absorber $\to$ multiple scattering $\to$ worst mass resolution.

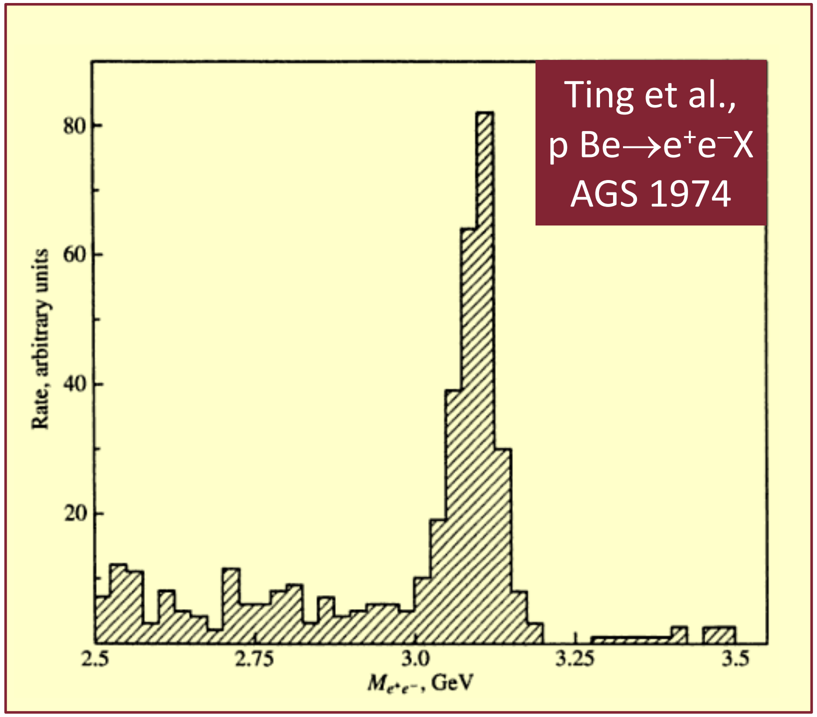
- Benefit in the <u>e^+e^- case</u>: electron identification with Čerenkov counter(s) + calorimeters $\to$ simpler setup.
  Disadvantage : small instrumented solid angle $\to$ smaller yield.

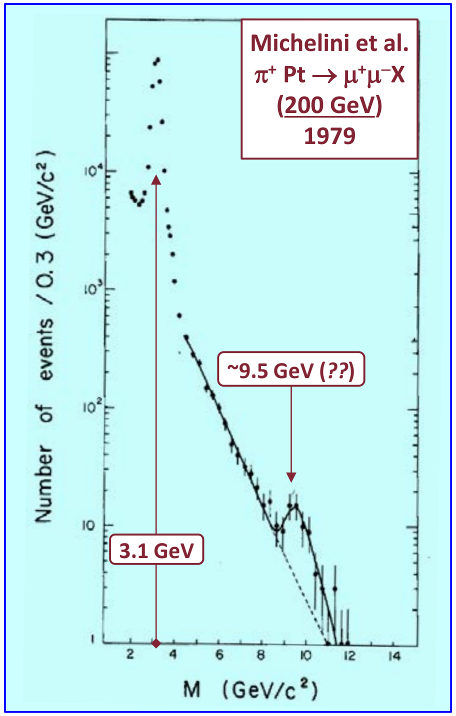


$^{(*)}$ "ppp" : "particles (or protons) per pulse", i.e. once per accelerator cycle every few seconds; it is the typical figure of merit of a beam from an accelerator.

The November revolution: the search for



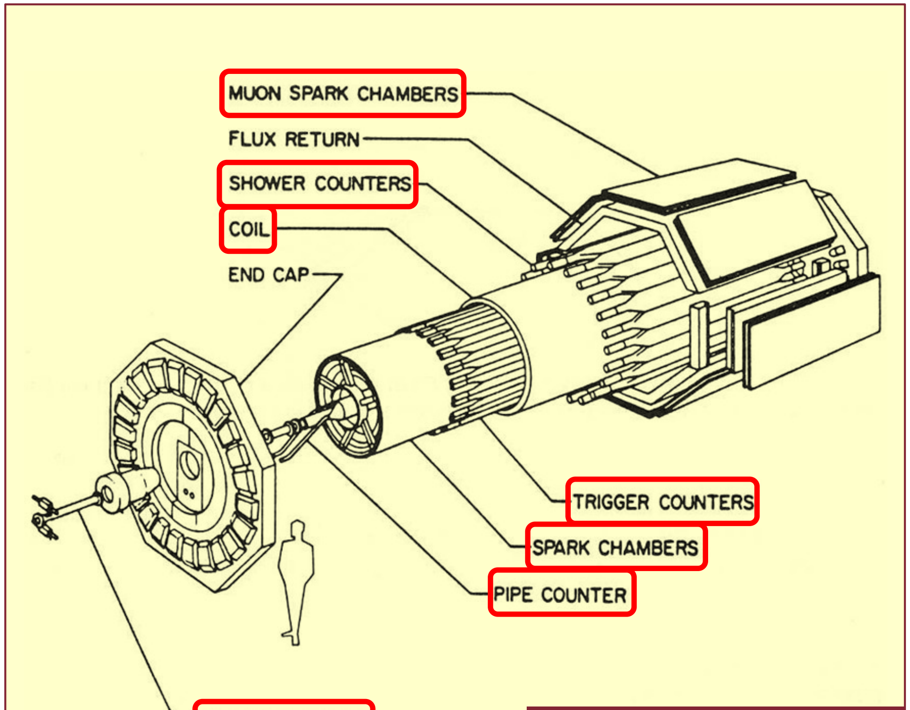




The November revolution: Mark I @ SLAC

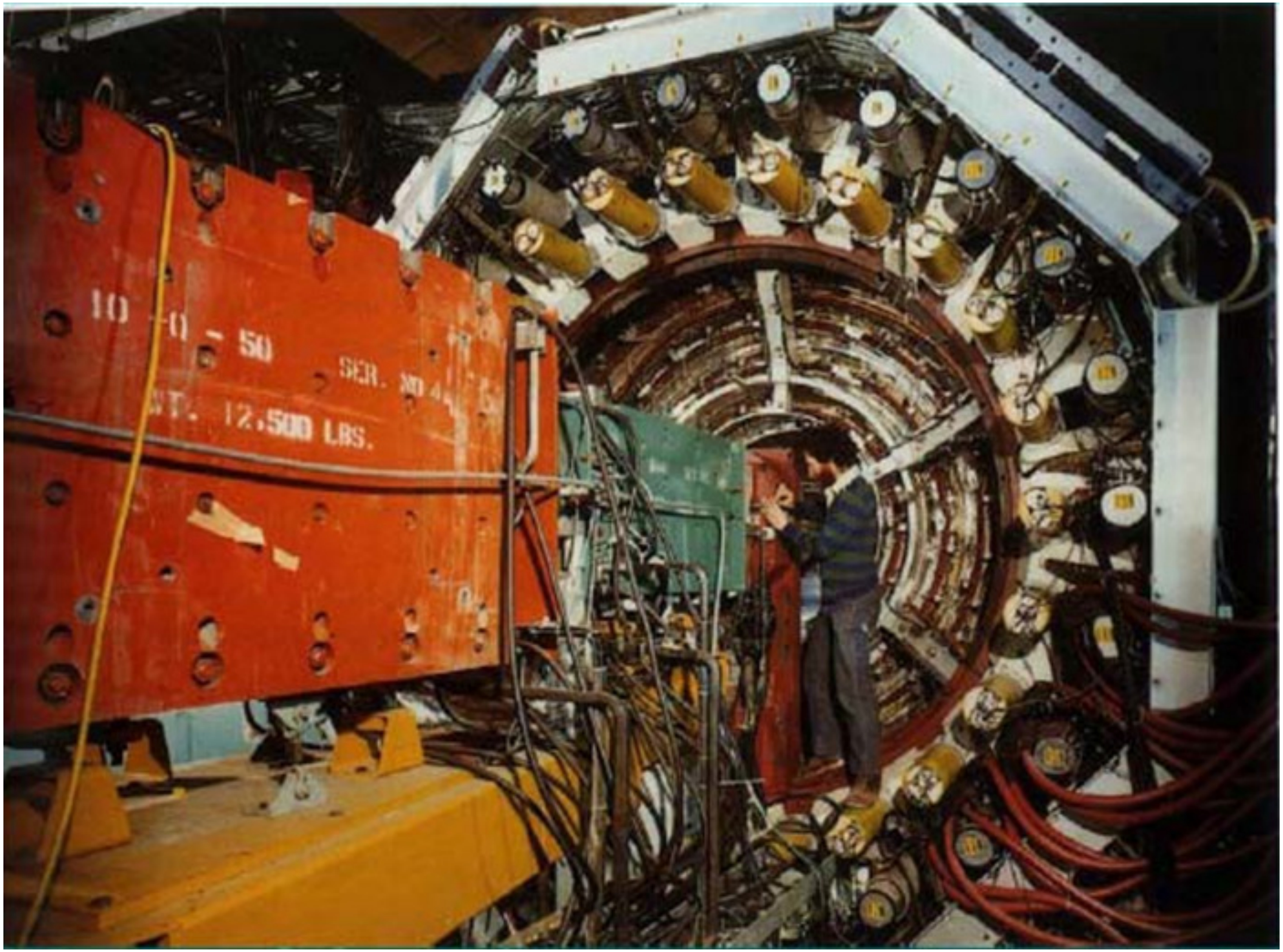
[back to 1974 : they did not know]

- Mark I at the e^+e^- collider SPEAR was studying collisions at $\sqrt{s}$ = 2.5 ÷ 7.5 GeV.
- The detector was made by a series of concentrical layers ("onion shaped").
- Starting from the beam pipe :
  - magnetostrictive spark chamber (tracking),
  - time-of-flight counters (particles' speed + trigger),
  - coil (solenoidal magnetic field, 4.6 kG),
  - electromagnetic calorimeter (energy and identification of γ's and $e^{\pm}$'s),
  - proportional chambers interlayered with iron plates (identification of $\mu^{\pm}$'s).



Mark I (SLAC –LBL)

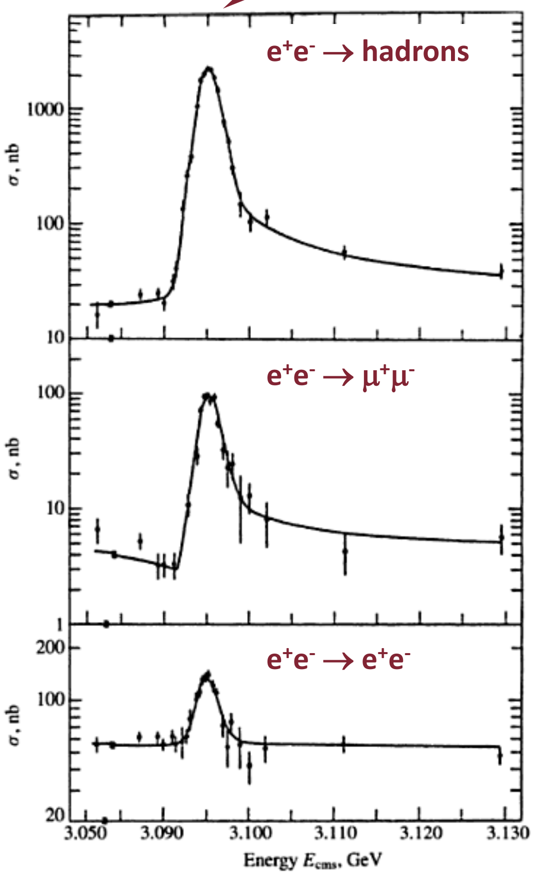
The November revolution: Mark I @ SLAC



The November revolution: ψ

- In 1974, up to the highest available energies, R = $\sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-) \approx 2$.
- Measurements at the Cambridge Electron Accelerator (CEA, Harvard) in the region of energies of SPEAR had found R ≅ 6 (a mixture of continuum and resonances). Also ADONE at LNF, which could reach an energy just sufficient, was not pushed to its max energy [*At the time the large amount of information carried by R was not completely clear*].
- At the novel Collider SPEAR, the scanning in energy was performed in steps of 200 MeV.
- The measured cross-section appeared to be a constant, NOT with expected trend $\propto 1/s$.
- When a drastic reduction in the step (200 → 2.5 MeV) increased the "resolving power", a resonance appeared, with width compatible with the beam dispersion (even compatible with a δ-Dirac).
- The particle was called "ψ".

inside Mark I acceptance and normalized to Bhabha.



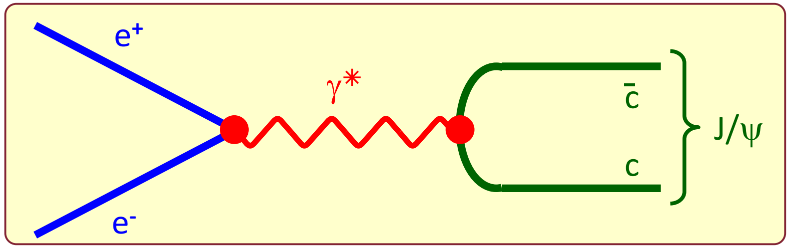
The J/ψ properties

- After some discussion, the correct interpretation emerged :
  - the resonance, now called J/ψ, is a bound state of a new quark, called **charm** (c), and its antiquark;
  - the c had been proposed in 1970 to exclude FCNC [GIM mechanism];
  - the J/ψ has $J^P = 1^-$ [*next slide*];
  - the name "charmonium" is an analogy with positronium ("onium" : bound state particle-antiparticle);
- The cross-section (Breit-Wigner) for the formation of a state ($J_R = 1$) from e^+e^- ($S_a = S_b = ½$), followed by a decay into a final state, shows that:

$$\sigma(ab \to J/\psi \to f\bar{f}, \sqrt{s}) = \frac{16\pi}{s}\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

- $\sigma(e^+e^- \to J/\psi \to f\bar{f}, \sqrt{s}) = \frac{12\pi}{s}\left[\frac{\Gamma_e}{\Gamma_{tot}}\right]\left[\frac{\Gamma_f}{\Gamma_{tot}}\right]\frac{\Gamma_{tot}^2/4}{\left(m_{J/\psi} - \sqrt{s}\right)^2+\Gamma_{tot}^2/4};$
- $\Gamma_f =$ width for the $\left(J/\psi \leftrightarrow f\bar{f}\right)$ coupling;
- $\Gamma_{tot} = \Gamma_e + \Gamma_\mu + \Gamma_{had}$ = full width of J/ψ;
- $\Gamma_f/\Gamma_{tot} = BR(J/\psi \to f\bar{f})$ [very useful].

- After 1974, many exclusive decays have been precisely measured, all confirming the above picture; the last PDG has 227 decay modes; the present most precise value of the mass and width is
  m(J/ψ) = 3097 MeV, Γ_{tot}(J/ψ) = 93 keV.



The J/ψ quantum numbers

At SPEAR they were able to measure many of the J/ψ quantum numbers :

- the resonance is asymmetric (the right shoulder is higher); therefore there is interference between J/ψ formation and the usual γ^* exchange in the s-channel; therefore the J/ψ and the γ have the same $\boxed{J^P = 1^-;}$
- from the cross section, by measuring σ_{had}, σ_μ and σ_e, they have 3 equations + a constraint (see the box, three σ_f + Γ_{tot}) for the 4 unknowns (three Γ_f + Γ_{tot}); therefore they measured everything, obtaining a $\boxed{\Gamma_{tot}}$ very small (~90 keV, a puzzling results, see next slides);
- the equality of the BR ($J/\psi \to \rho^0\pi^0$) and ($\to \rho^\pm\pi^\mp$) implies isospin $\boxed{I = 0;}$
- the J/ψ decays into an odd (3, 5) number of π, not ~~in an even (2, 4) number~~; this fact has two important consequences :

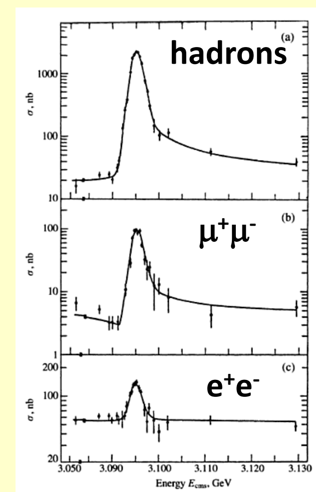
➢ the G-parity is conserved in the decay (so the $\boxed{\text{J/ψ decays via strong inter.}}$).

➢ $\boxed{\text{G-parity} = -1}$ [also $(-1)^{I+\ell+s} = -1$].

$$\sigma(e^+e^- \to J/\psi \to f\bar{f}) = \frac{3\pi}{s}\frac{\Gamma_e\Gamma_f}{\left(m_{q\bar{q}} - \sqrt{s}\right)^2 + \Gamma_{tot}^2/4} = \sigma_f(\Gamma_e, \Gamma_f, \Gamma_{tot}, \sqrt{s});$$

$$\Gamma_{tot} = \Gamma_e + \Gamma_\mu + \Gamma_{had}$$

[see previous slide].



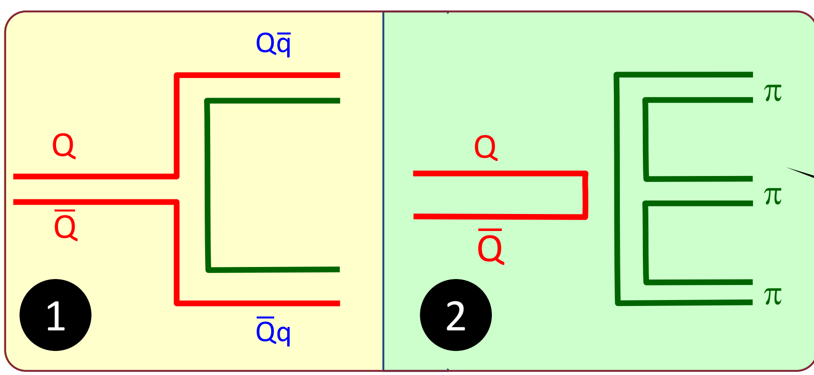
4 equations (f=e,μ,had + Γ_{tot}), 4 unknowns;
NO direct measurement of "width" required.

Charmonium the Zweig Rule (OZI)

The "Zweig rule" was set out empirically in a qualitative way before the advent of QCD :

- compare $(\phi \rightarrow 3\pi) \leftrightarrow (\phi \rightarrow KK) \leftrightarrow (\omega \rightarrow 3\pi)$;
- in the decay of a bound state of heavy quarks Q, the final states without Q's ("decays with disconnected diagrams" 2) have suppressed amplitude wrt "connected decays" 1;
- if only the decays 2 are kinematically allowed (ex. J/ψ or Υ), the total width is small and the bound state is "narrow";

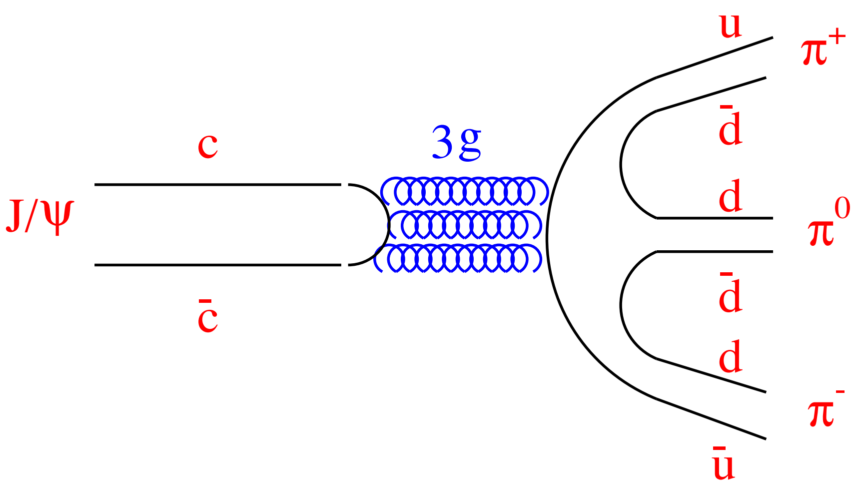
1963-1966 :
Susumu Okubo
(大久保 進
Ōkubo Susumu),
George Zweig,
Jugoro Iizuka (飯塚)

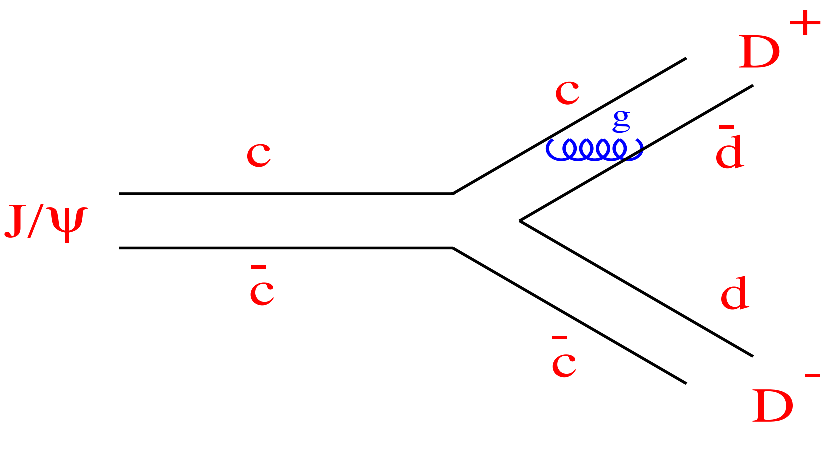


before the QCD advent, gluons were not considered.

Charmonium the Zweig Rule (OZI)

Decay rates described by diagrams with unconnected quark lines are suppressed.





OZI suppressed

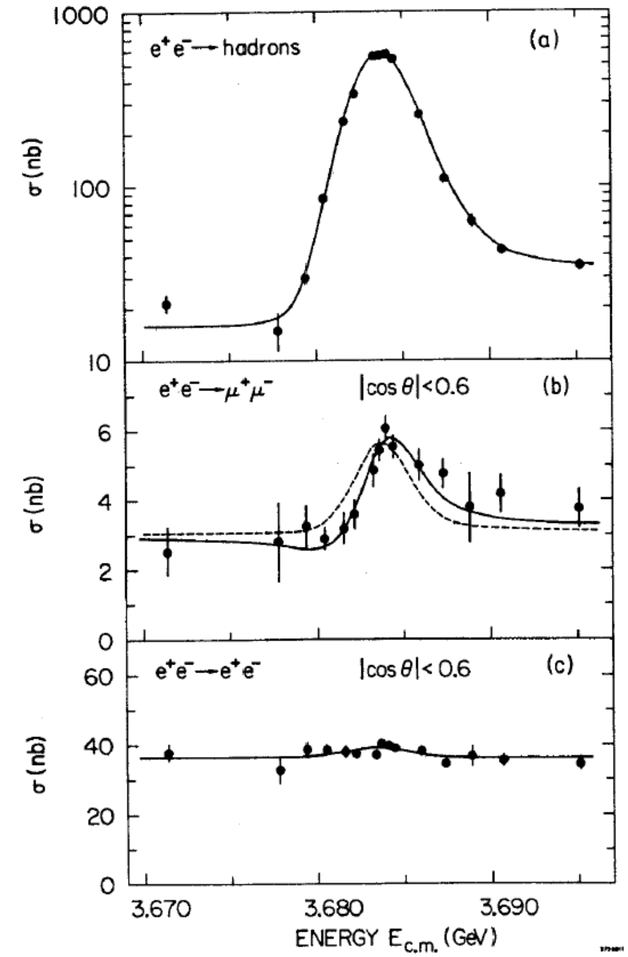
OZI allowed
forbidden by
energy conservation

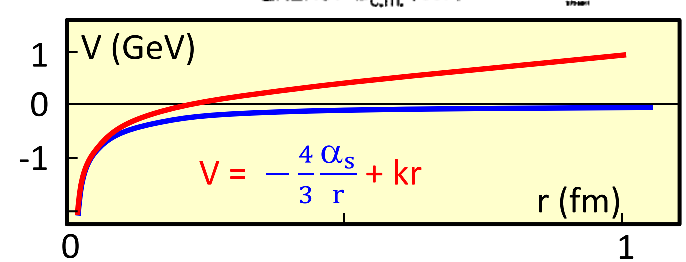
Charmonium: ψ'

- After the discovery of the J/ψ, at SPEAR they performed a systematic energy scanning with a very small step. After ten more days a second narrow resonance was found, called ψ', with the same quantum numbers of the J/ψ.
- The analysis shows that the J/ψ was the 1S state of $c\bar{c}$, while the ψ' is the 2S.
- Both particles have $J^P = 1^-$, I=0.
- The next page gives a scheme of the $c\bar{c}$ levels.
- They offer a reasonable agreement with the solution of the Schrödinger equation of a hypothetical QCD potential

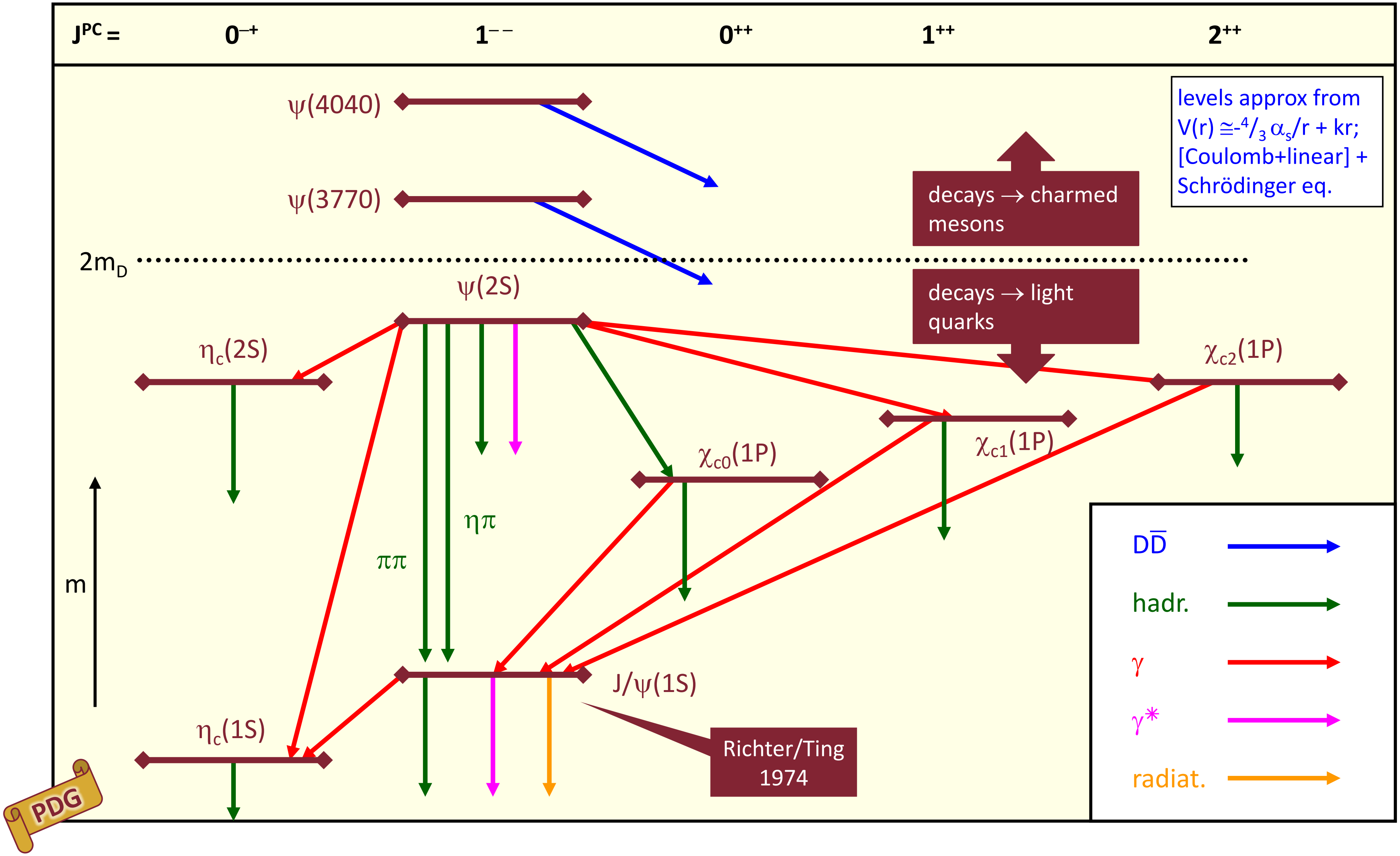
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr = \frac{A}{r} + Br.$$

- Notice that this approximation should become more realistic for heavier quarks, when the non-relativistic limit gets better.





Charmonium states



Charmonium: why it is useful?

Charmonium is a powerful tool for the understanding of the strong interaction. The high mass of the c quark ($m_c \sim 1.5$ GeV/c^2) makes it plausible to attempt a description of the dynamical properties of the ($c\bar{c}$) system in terms of non-relativistic potential models, in which the functional form of the potential is chosen to reproduce the known asymptotic properties of the strong interaction. The free parameters in these models are determined from a comparison with experimental data.

$$\beta^2 \approx 0.2 \quad \alpha_s \approx 0.3$$

Non-relativistic potential models + Relativistic corrections + PQCD

The non-relativistic potential

The functional form of the potential is chosen to reproduce the known asymptotic properties of the strong interaction.

- At small distances asymptotic freedom, the potential is coulomb-like:

$$V(r) \xrightarrow[r\to 0]{} -\frac{4}{3}\frac{\alpha_s(r)}{r}$$

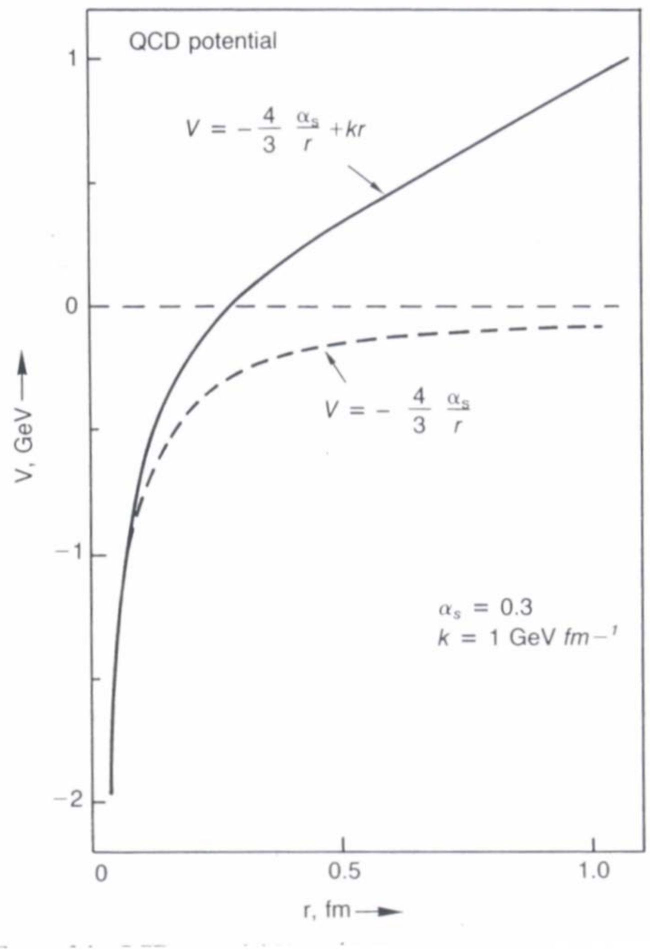
- At large distances confinement:

$$V(r) \xrightarrow[r\to\infty]{} kr$$

The non-relativistic potential

$$\alpha_s(\mu) = \frac{4\pi}{(11 - \frac{2}{3} n_f) \ln(\frac{\mu^2}{\Lambda^2})}$$

n_f = number of flavours
Λ ~ 0.2 GeV QCD scale parameter
k string constant (~ 1 GeV/fm)

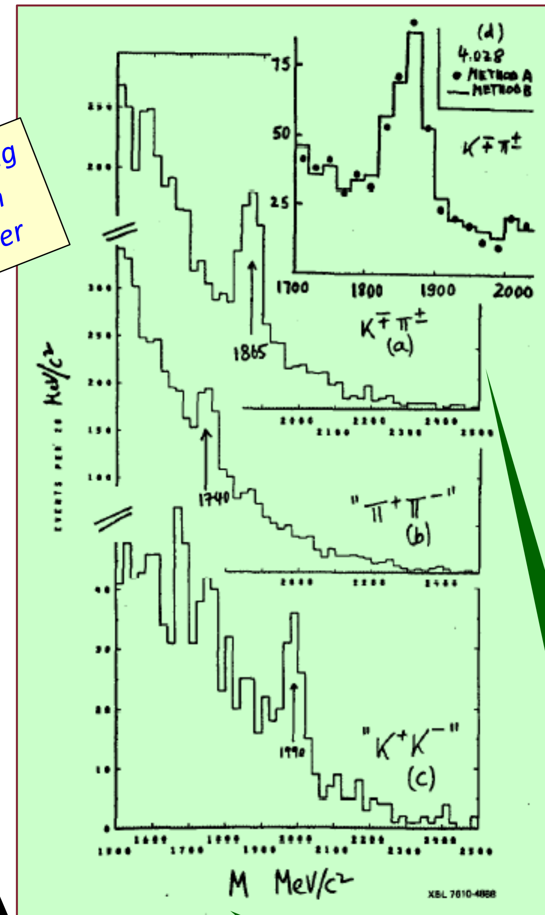


The open charm discovery

- If the J/ψ is a bound $c\bar{c}$ state, then mesons $c\bar{q}$ and $\bar{c}q$ must exist, with a mass $m_{J/\psi}/2$ + 100÷200 MeV [$3690/2 < m_D < 3770/2$ MeV].
- In 1976, the Mark I detector started the search for charmed pseudoscalar mesons, the companions of π's and K's.
- They looked at $\sqrt{s}$ = 4.02 GeV in the channels

  $e^+e^- \rightarrow D^0\,\overline{D}{}^0\,X^0; \quad \rightarrow D^+\,D^-\,X^0.$
- According to theory, D-mesons lifetimes are small, with a decay vertex not resolved (*with 1976 detectors*) wrt the e^+e^- one.
- Therefore the strategy of selection was the presence of "narrow peaks" in the combined mass of the decay products.
- A first bump at 1865 MeV with a width compatible with the experimental resolution was observed in the combined mass ($K^\pm\pi^\mp$), corresponding to the D^0 and $\overline{D}{}^0$ decay.

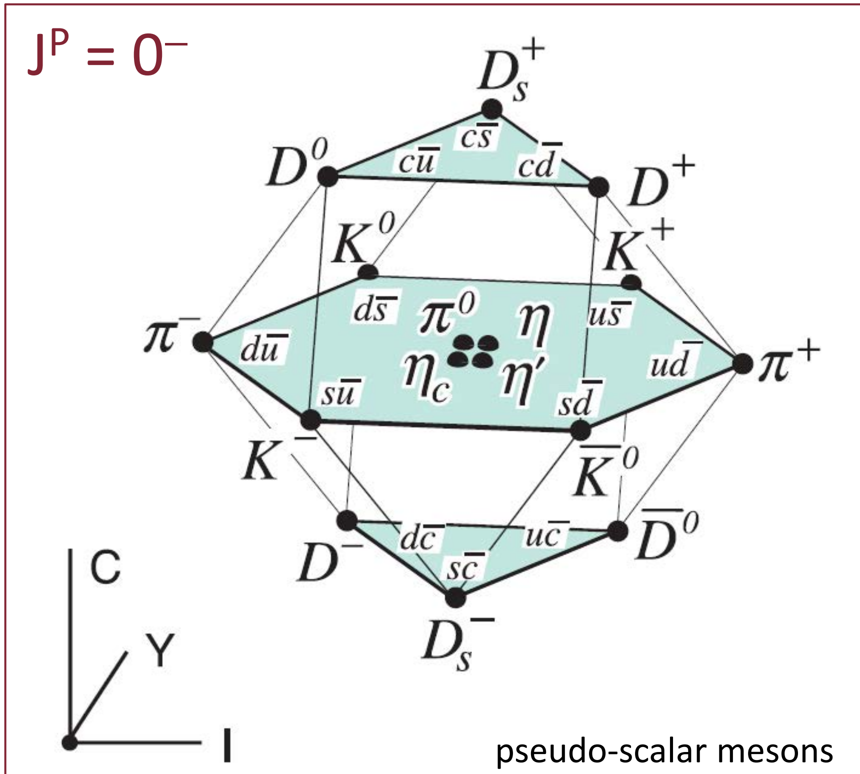
handwriting by Gerson Goldhaber



Today's technology allows for it !!!

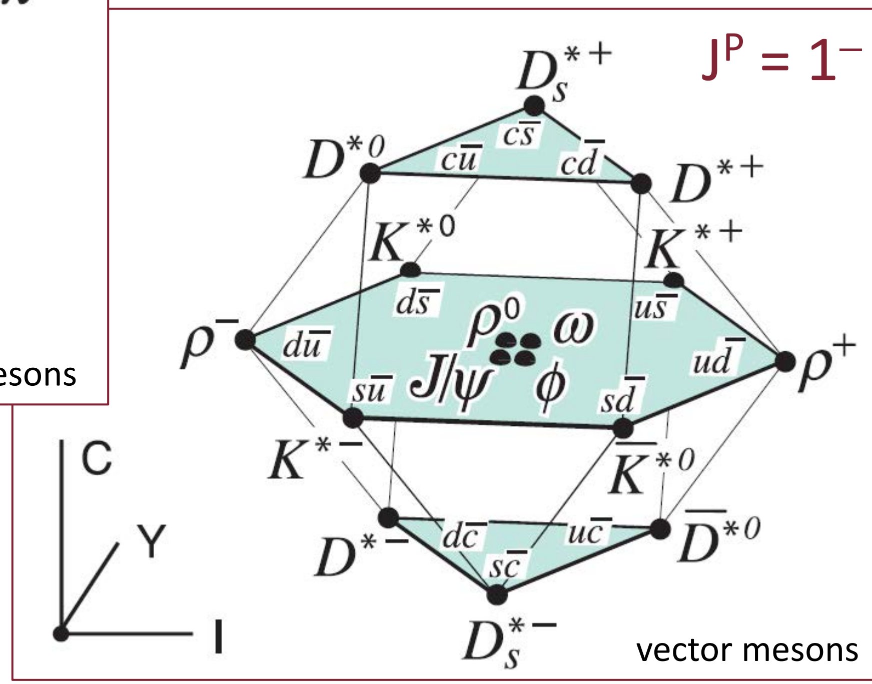
They were afraid of K/π exp. misidentification $\rightarrow$ they computed the mass twice, also with the other particle hypothesis

Open charm: meson multiplets



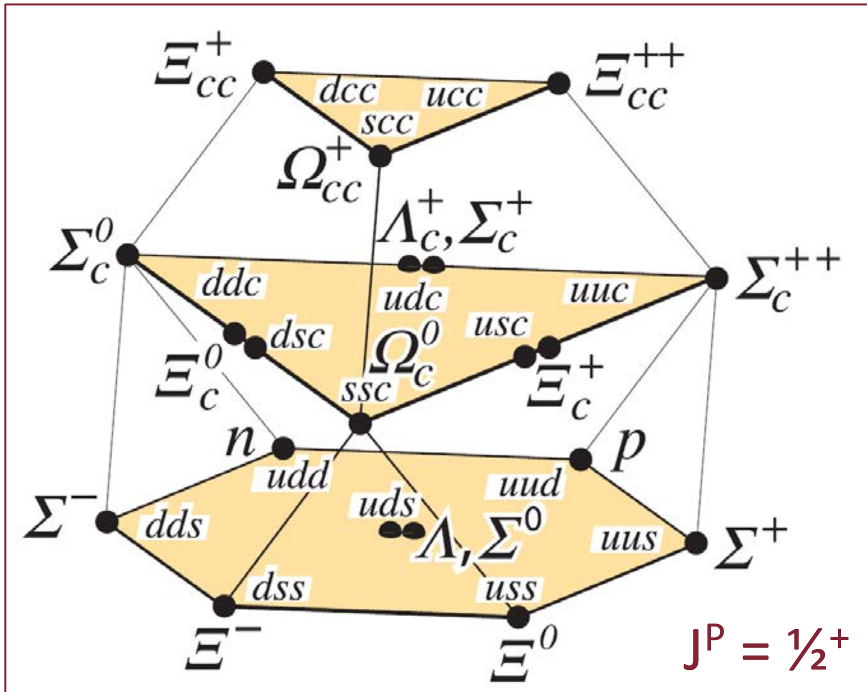
SU(3)$_{\text{flavor}}$ $\rightarrow$ SU(4)$_{\text{flavor}}$

With 4 quarks, the SU(3) nonets become multiplets in a 3-D space. However, the c quark has a large mass, so SU(4)$_{\text{flavor}}$ is much more broken that SU(3)$_{\text{flavor}}$.

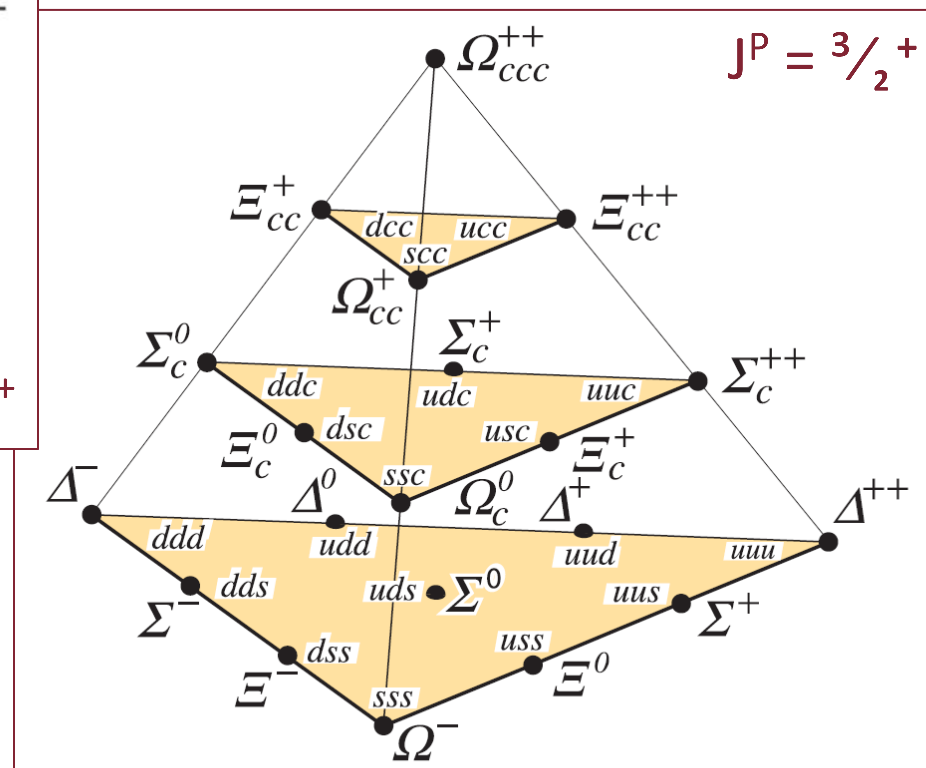


$4 \otimes \bar{4} = 15 \oplus 1.$

Open charm: meson multiplets



SU(4)$_{\text{flavor}}$ baryons



The third family: the τ lepton discovery

The analysis of Mark I data produced another beautiful discovery : the τ lepton (M. Perl won the 1995 Nobel Prize):

- the selection followed a method well known, pioneered at LNF-Frascati : the "unbalanced pairs $e^{\pm}\mu^{\mp}$" :

$$e^+e^- \to \tau^+\tau^-$$
$$\left.\begin{array}{l} \tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau \\ \tau^+ \to e^+\nu_e\bar{\nu}_\tau \end{array}\right\} \to \mu^-e^+ \ (\underline{\text{unbalanced}})$$

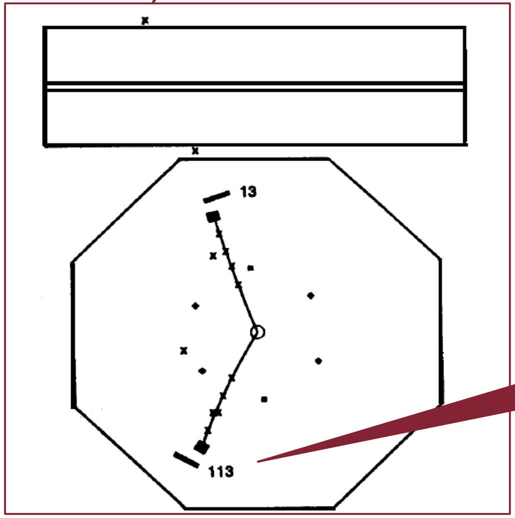
(+ CC μ^+e^-).

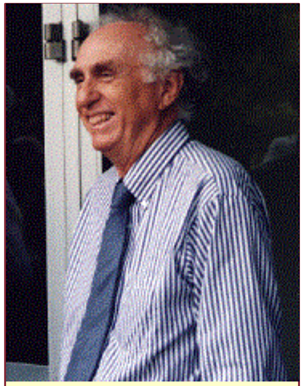
- events from this process are extremely clean and free from background [*see fig.*];
- the e^+e^- / $\mu^+\mu^-$ unbalanced pairs, which have to be present in the correct number

$$N_{unb}(e^+e^-) = N_{unb}(\mu^+\mu^-) =$$
$$= N(e^+\mu^-) = N(e^-\mu^+),$$

are only used to cross-check the sample.

In principle the τ lepton has very little to do with the c quark. However collider, detector, energy, selection and analysis are closely linked. Therefore, in experimental reviews, the τ lepton is usually treated together with the charm quark.





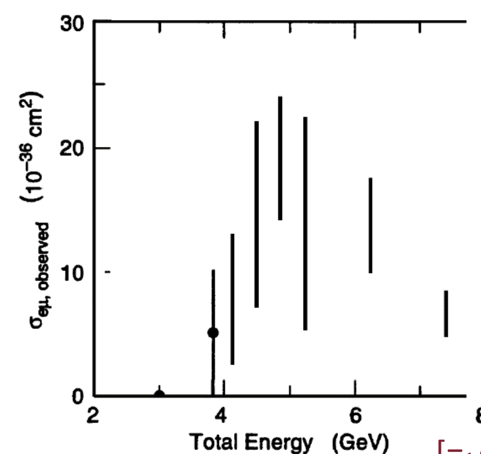
Martin Perl

The third family: the τ lepton discovery

Simple method: the yield of $e^\pm\mu^\mp$ pairs vs $\sqrt{s}$: it immediately points to the threshold $\sqrt{s} = 2m_\tau$.

- therefore : $m_\tau \approx 1780$ MeV.
  [best present value 1776.8 MeV]
- why is the $\tau^\pm$ a lepton ?
  - at the time, the evidence came from the lack of any other plausible explanation;
  - today, the evidence is solid :
    - the Z and W decays into (e μ τ) with the same BR and angular distribution;
    - the lifetime has been measured and found in agreement with predictions …
- the discovery of the τ started the hunt for the particles of a new (3rd) family, still unknown:
  - the ν_τ (possibly mixed with the others);
  - the pair of quarks q_{up} q_{down}, similar to ud (now called **t**op and **b**ottom).

$$e^+e^- \to \tau^+\tau^-$$
$$\left.\begin{array}{l} \tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau \\ \tau^+ \to e^+\nu_e\bar{\nu}_\tau \end{array}\right\} \to \mu^-e^+ \text{ (unb.)}$$

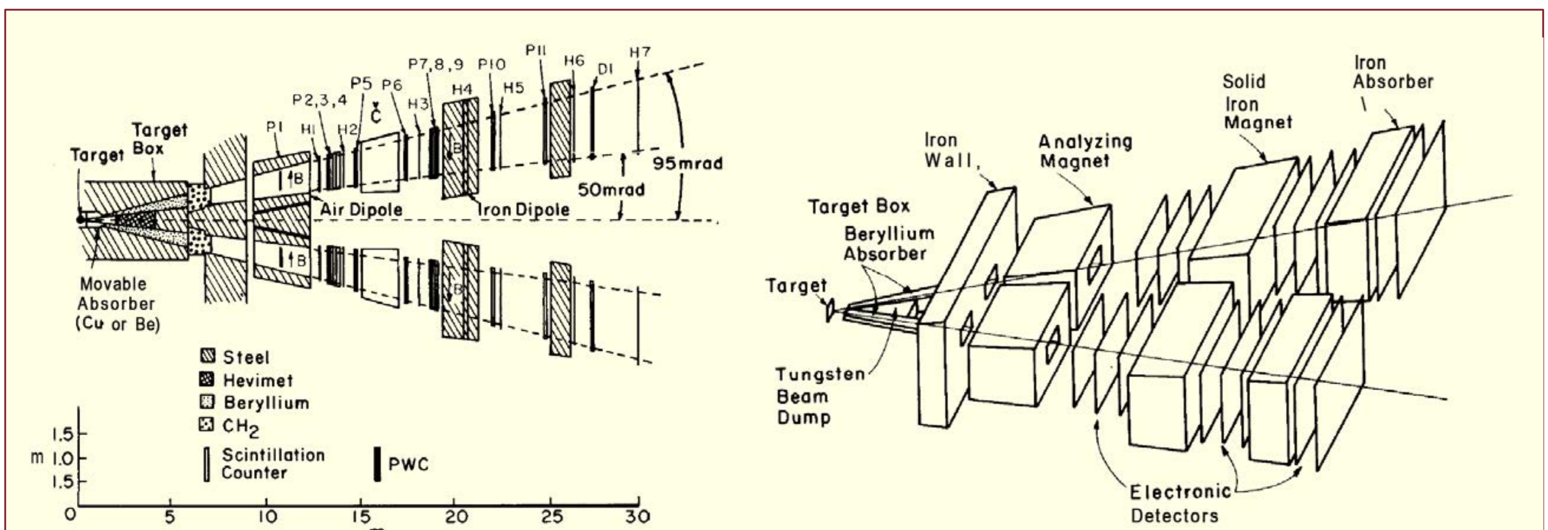


[$=\sqrt{s}$]

The third family: the b quark discovery

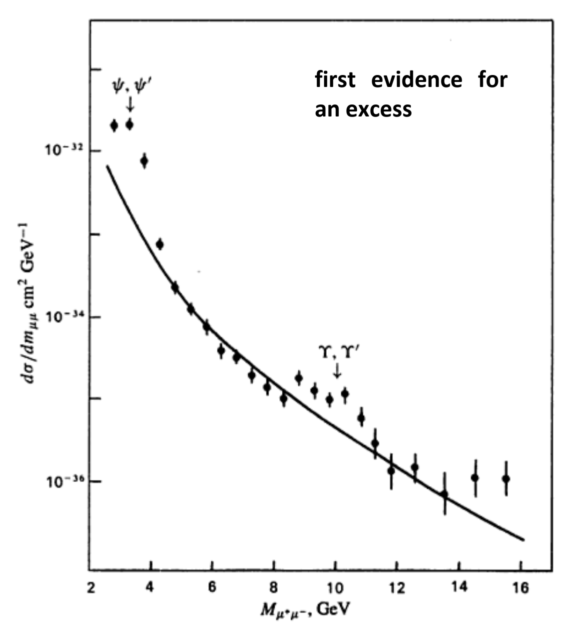
- The down quark of the 3rd family was called b (= beauty, bottom).
- In 1977 Leon Lederman and collaborators built at Fermilab a spectrometer with two arms, designed to study $\mu^+\mu^-$ pairs produced by interactions of 400 GeV protons on a copper (or platinum) target.
- The reaction under study was again the Drell-Yan process. As already pointed out, this type of events is rare, therefore requiring intense beams (in this case 10^{11} ppp) and high rejection power against charged hadrons.

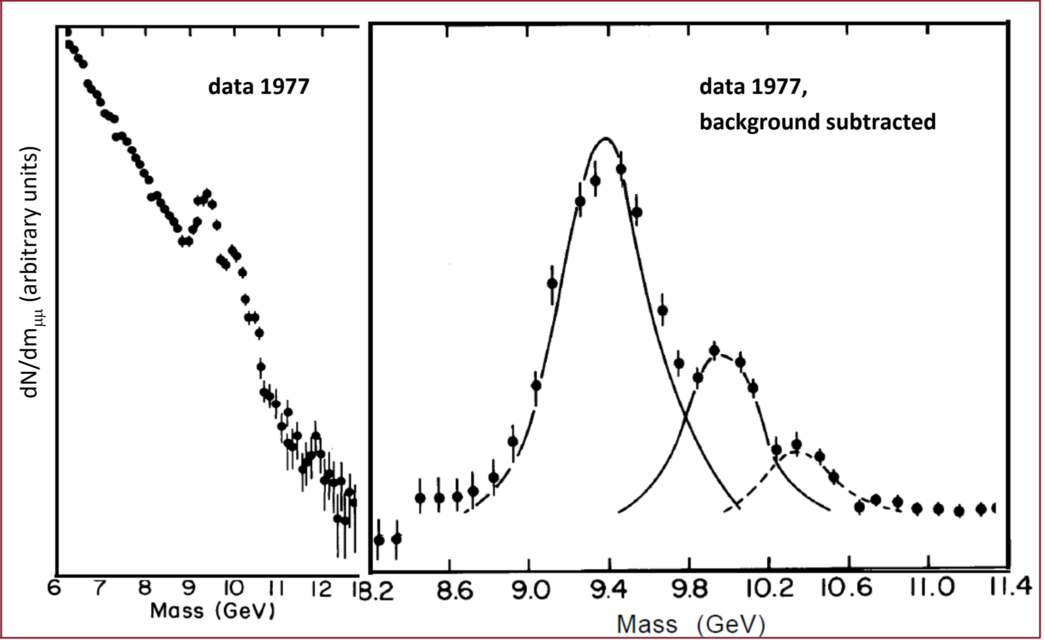
Leon Lederman



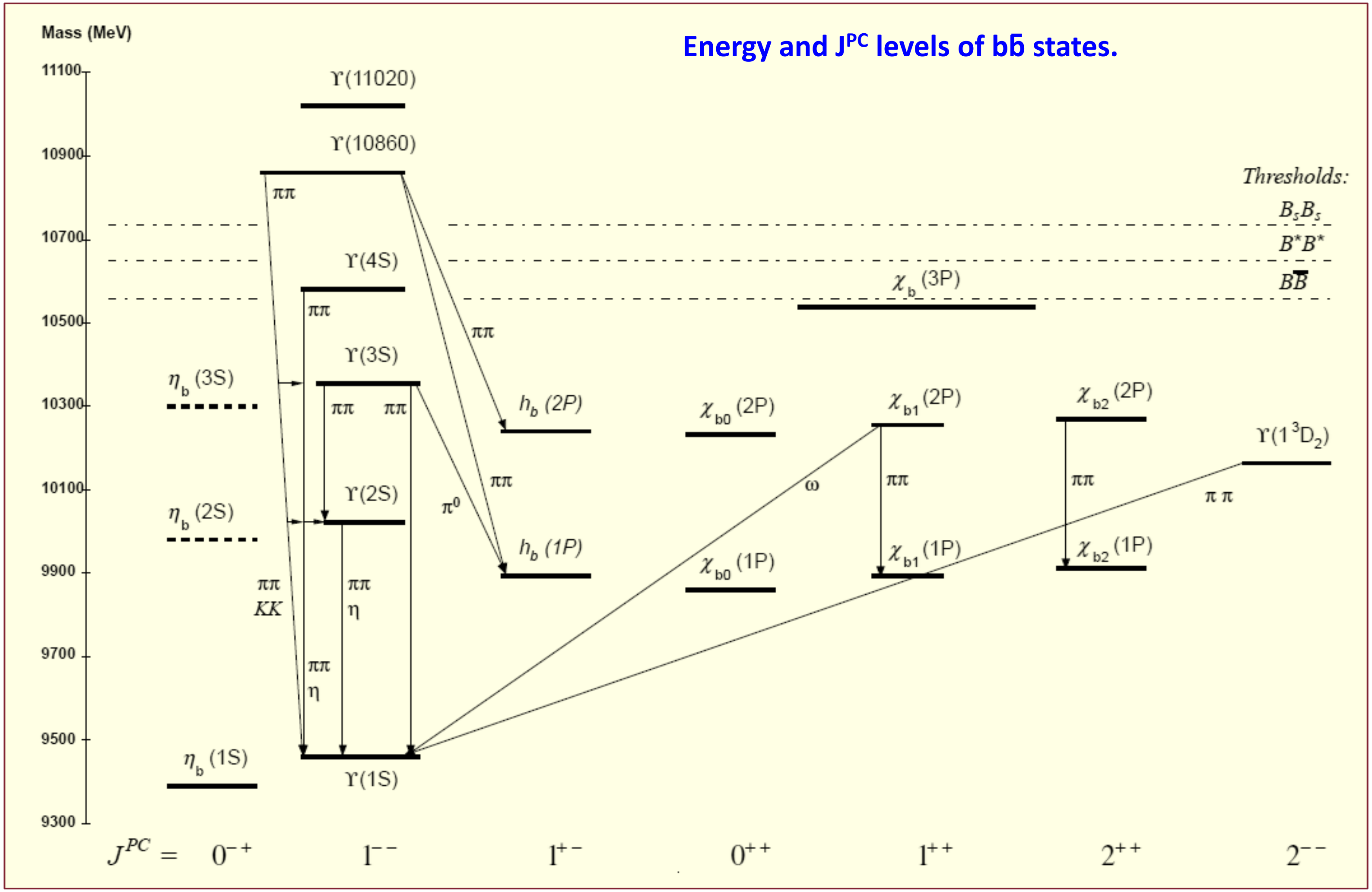
The third family: the b quark spectrum

- The usual price of the absorber technique is a loss of resolution in the muon momenta, which was $\Delta m_{\mu\mu} / m_{\mu\mu} \approx 2\%$.
- The figures show the distribution of $m_{\mu\mu}$. Between 9 and 10 GeV : there is a clearly visible excess.
- When the $\mu\mu$ continuum is subtracted, the excess appears as the superimposition of three separate states.
- The states, called $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ are bound states $b\bar{b}$.



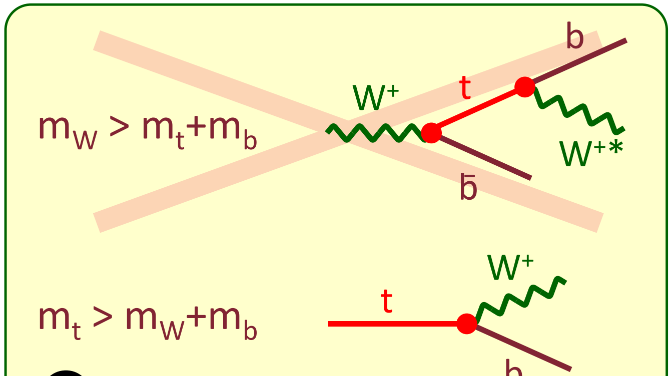


The third family: the b quark spectrum

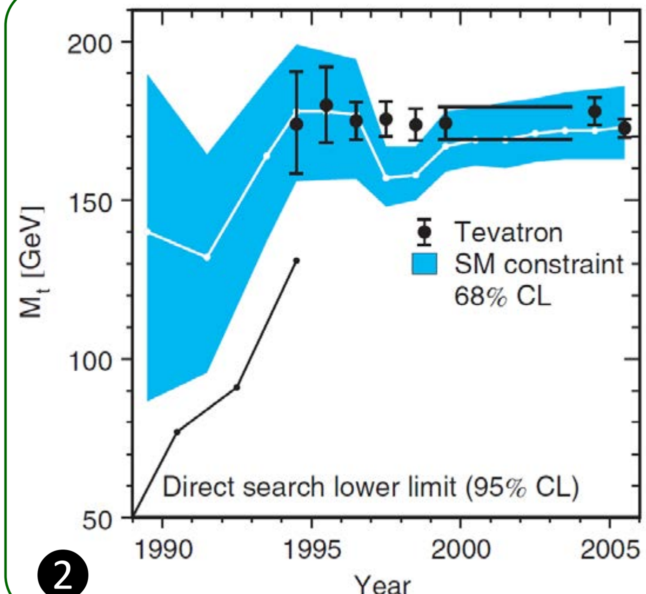


The top quark search

- The top quark was directly searched in hadron (Sp$\bar{p}$S, Fermilab) and lepton (Tristan, LEP) colliders, but was NOT found until 1990's;
- at the time the mass limit was $m_t \geq 90$ GeV;
- at $m_t \approx m_W - m_b$ ($\approx$ 75 GeV), the search changes: the "golden discovery channel" moves from ($W^+ \to t\bar{b} \to W^{+*}b\bar{b}$) to ($t \to W^+b$) [fig. ❶];
- the mass was first *computed* from the radiative corrections for m_W and m_Z [*see* § *LEP*];
- the LEP data, together with all other e.w. measurements, allowed for a prediction of $m_t \approx 175$ GeV [fig. ❷];
- in the 1990's the search was finally concluded at the Tevatron, by the CDF and D0 experiments.
- At present, we measure $m_t = 173 \pm 0.4$ GeV.

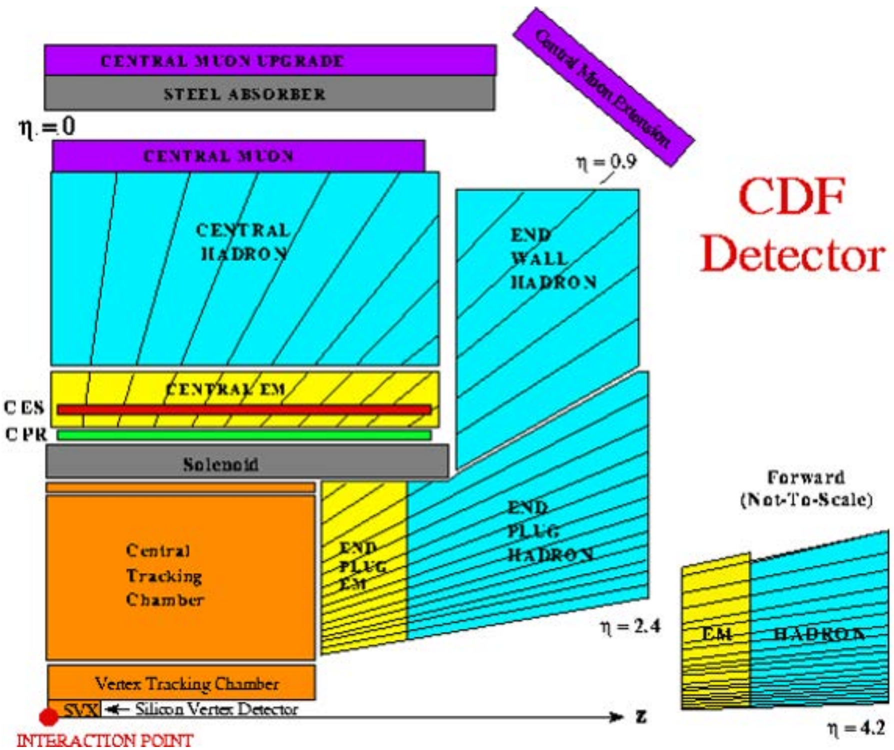


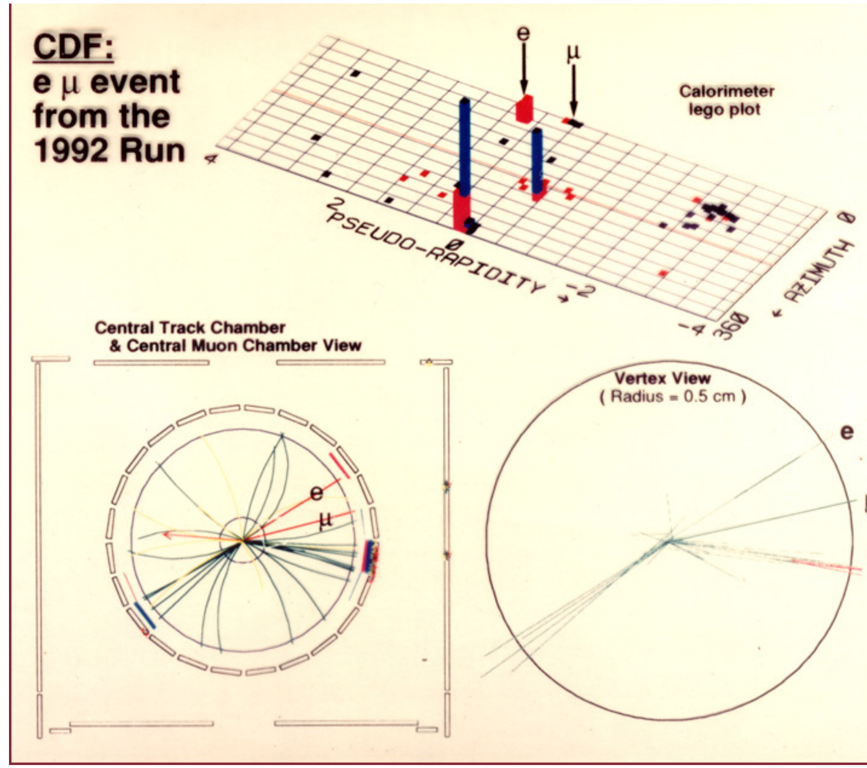
❶



❷

The top quark discovery





main tools for $t\bar{t}$ events at Tevatron (1992-4) :

- multibody final states;
- lepton id ($e^{\pm}$, $\mu^{\pm}$);
- secondary b vertices;
- mass fits.

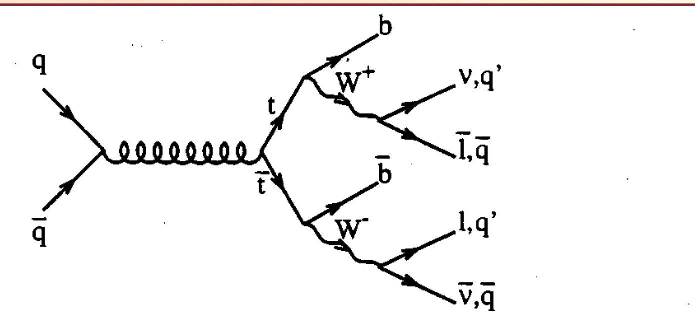


Figure 28: Tree level top quark production by $q\bar{q}$ annihilation followed by the Standard Model top quark decay chain.