Algorithmique et structures de données - NF16

Arbres binaires - Tas

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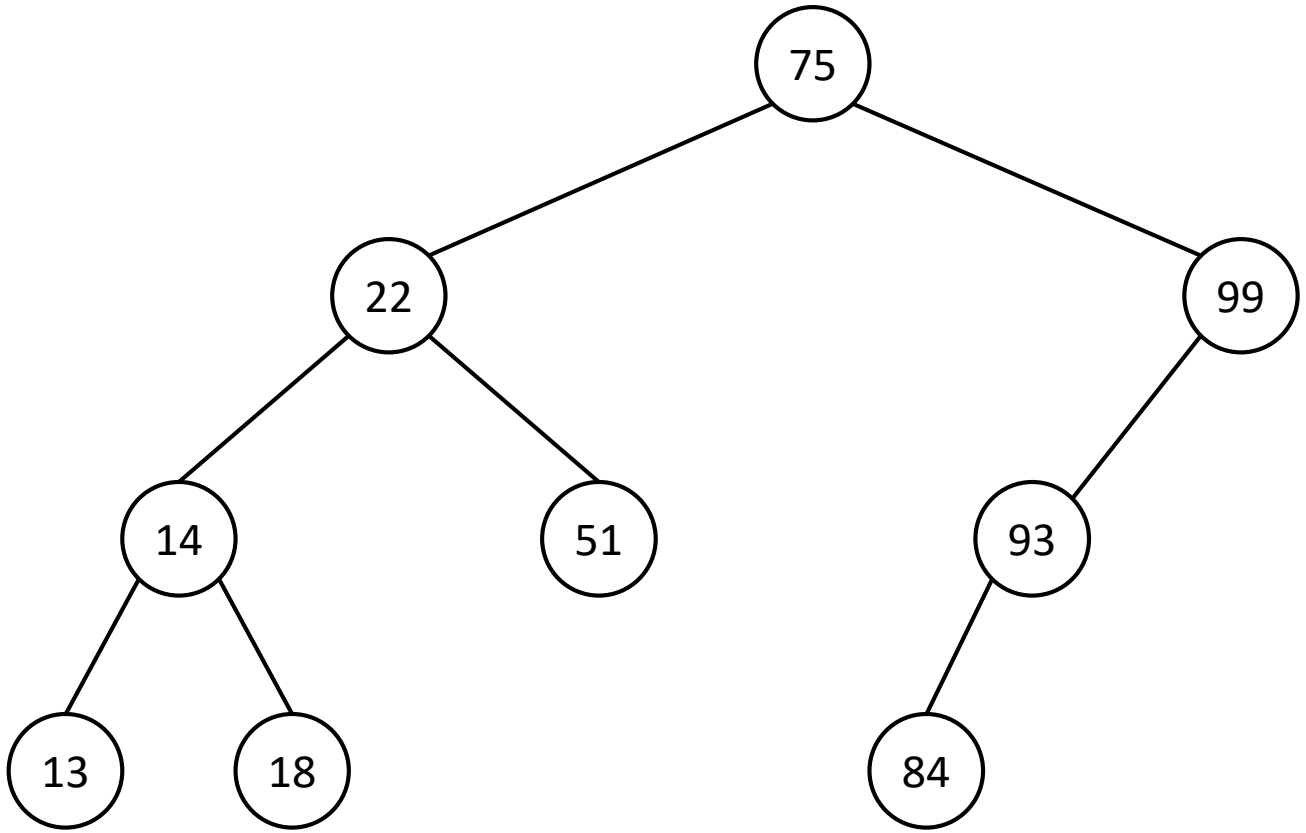
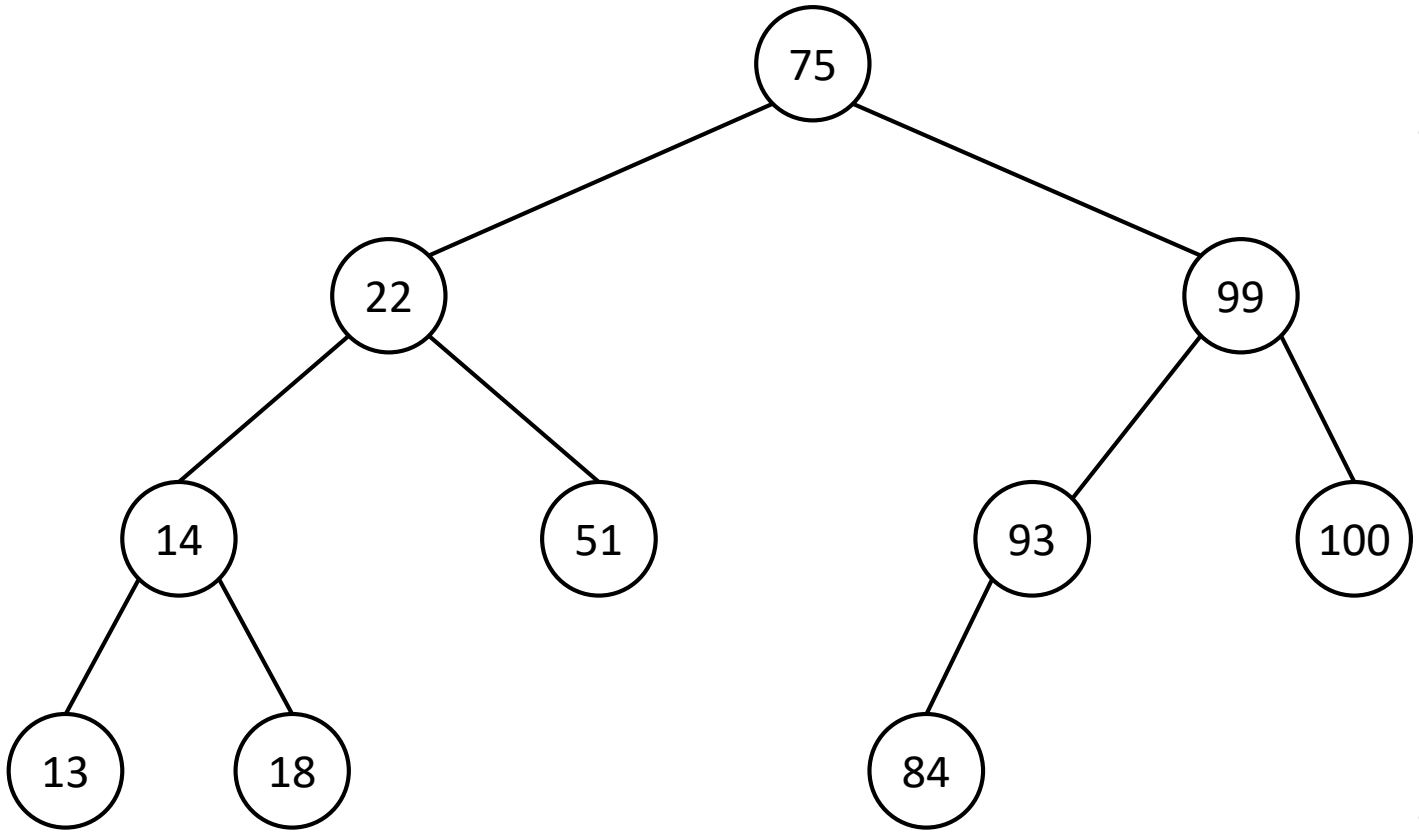
Les arbres de recherche binaires



- Un nœud ne peut avoir que deux fils au maximum.
- Les clés sont ordonnées de la façon suivante:
  - Chaque noeud du sous-arbre gauche a une clé inférieure (ou égale) à celle du noeud considéré.
  - Chaque noeud du sous-arbre droit a une clé supérieure (ou égale) à celle du noeud considéré.

Les AVLs

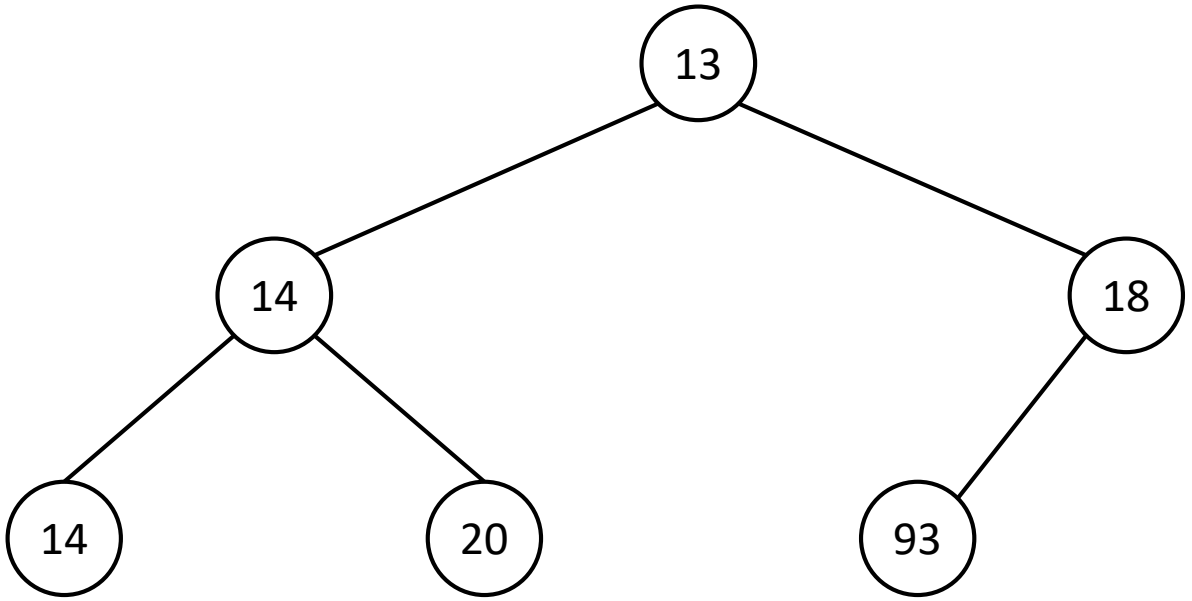


Recherche ➔ $O(log_2(n))$

- Un nœud ne peut avoir que deux fils au maximum.
- Les clés sont ordonnées de la façon suivante:
  - Chaque noeud du sous-arbre gauche a une clé inférieure (ou égale) à celle du noeud considéré.
  - Chaque noeud du sous-arbre droit a une clé supérieure (ou égale) à celle du noeud considéré.
- Pour tout nœud N :

$$|h\big(gauche(N)\big) - h\big(droit(N)\big)| \leq 1$$

Les tas

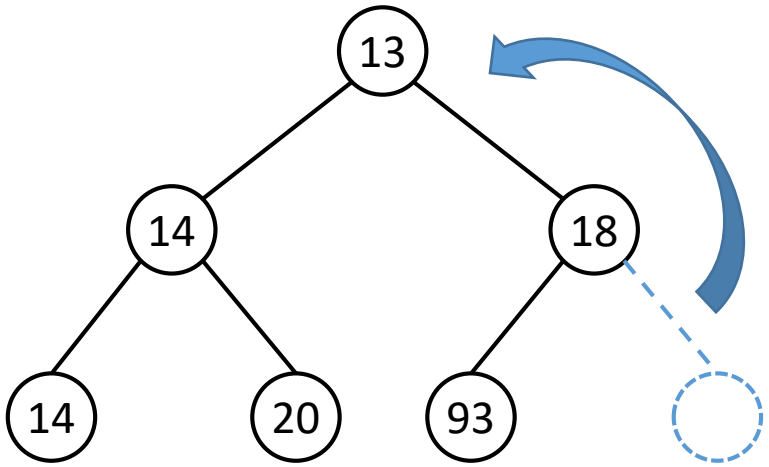


Tas Min

- Un nœud ne peut avoir que deux fils au maximum.
- Tous les niveaux sont totalement remplis, sauf le dernier ou les nœuds sont regroupés le plus à gauche possible (arbre parfait).
- $\begin{cases} \text{Tas Min} : \text{Val(Pere)} \leq \text{Val(Fils)} \\ \text{Tas Max} : \text{Val(Pere)} \geq \text{Val(Fils)} \end{cases}$

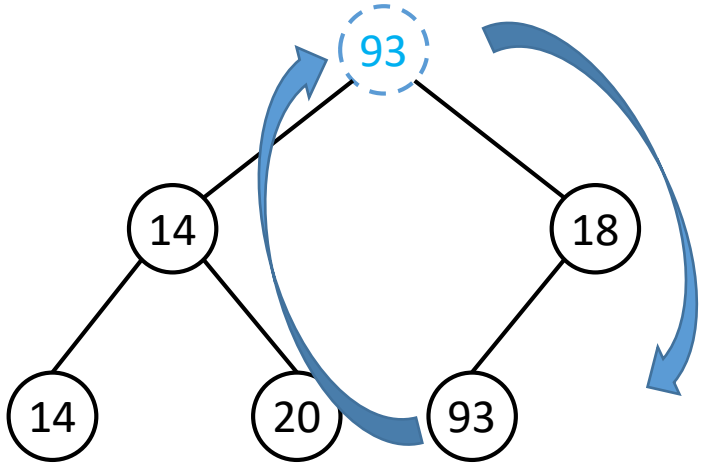
Les tas

Insertion



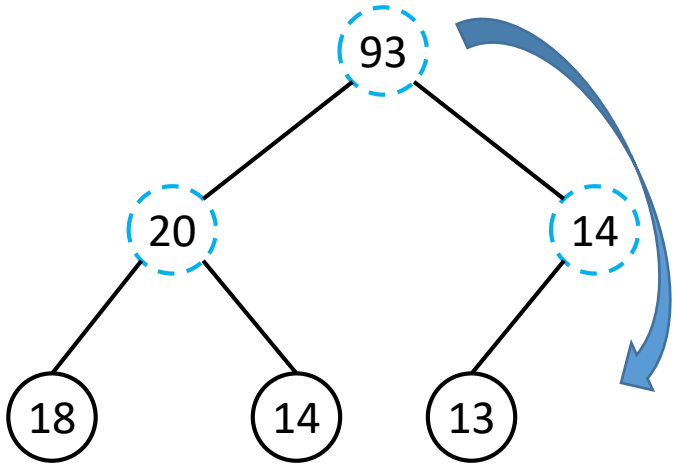
- Insérer à la fin
- Remonter

Suppression (Racine)



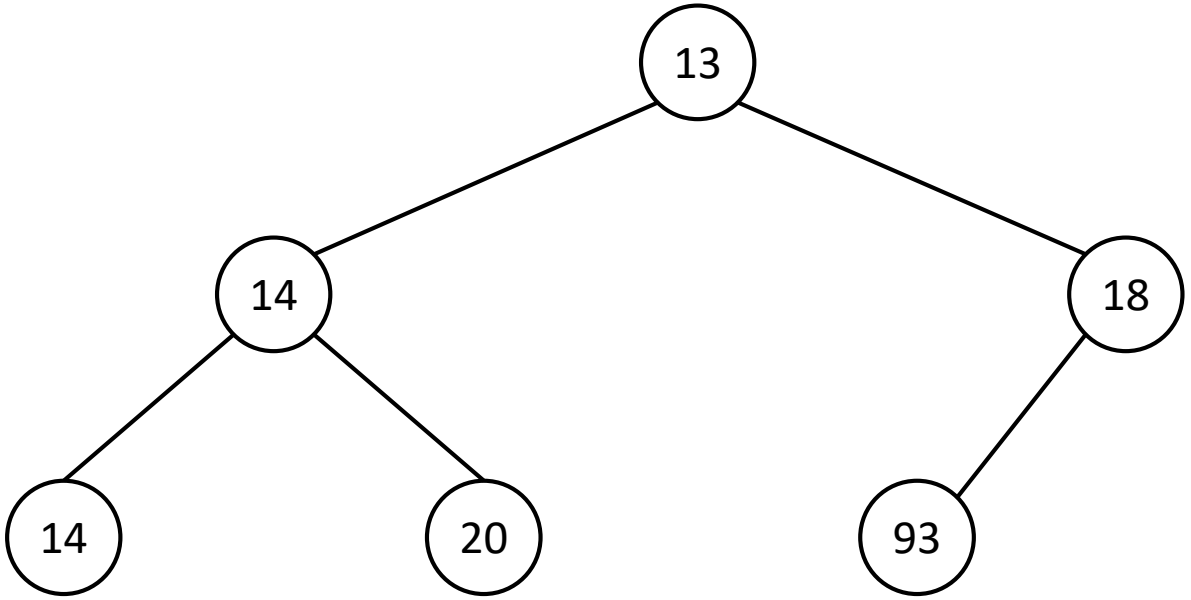
- Remplacer la racine par le dernier élément
- Entasser

Construction



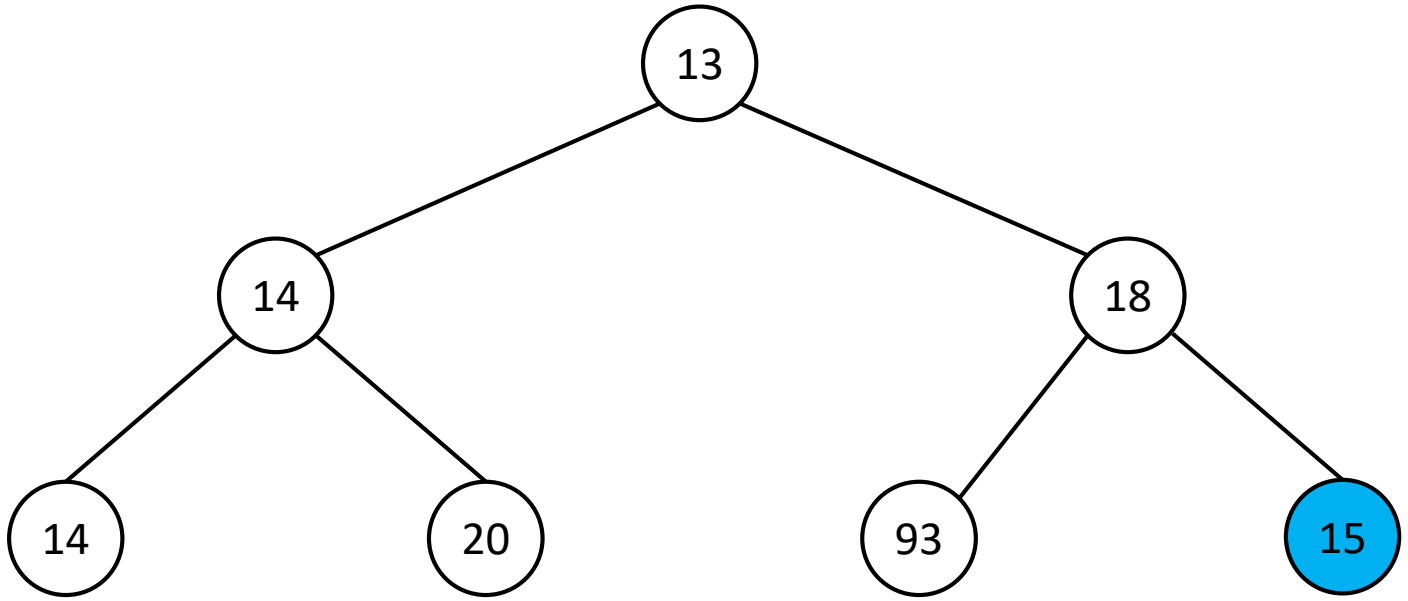
- Entasser tous les éléments sauf les feuilles

Les tas (Insertion)



Insérer 15

Les tas (Insertion)



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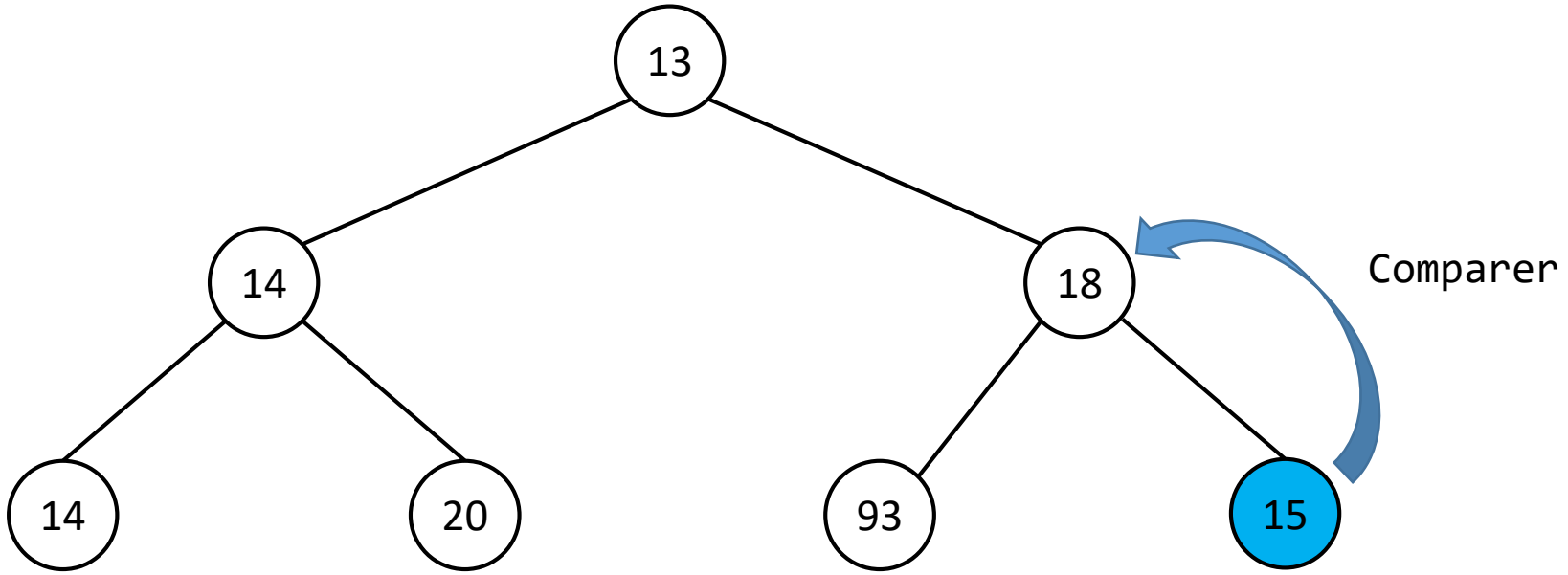

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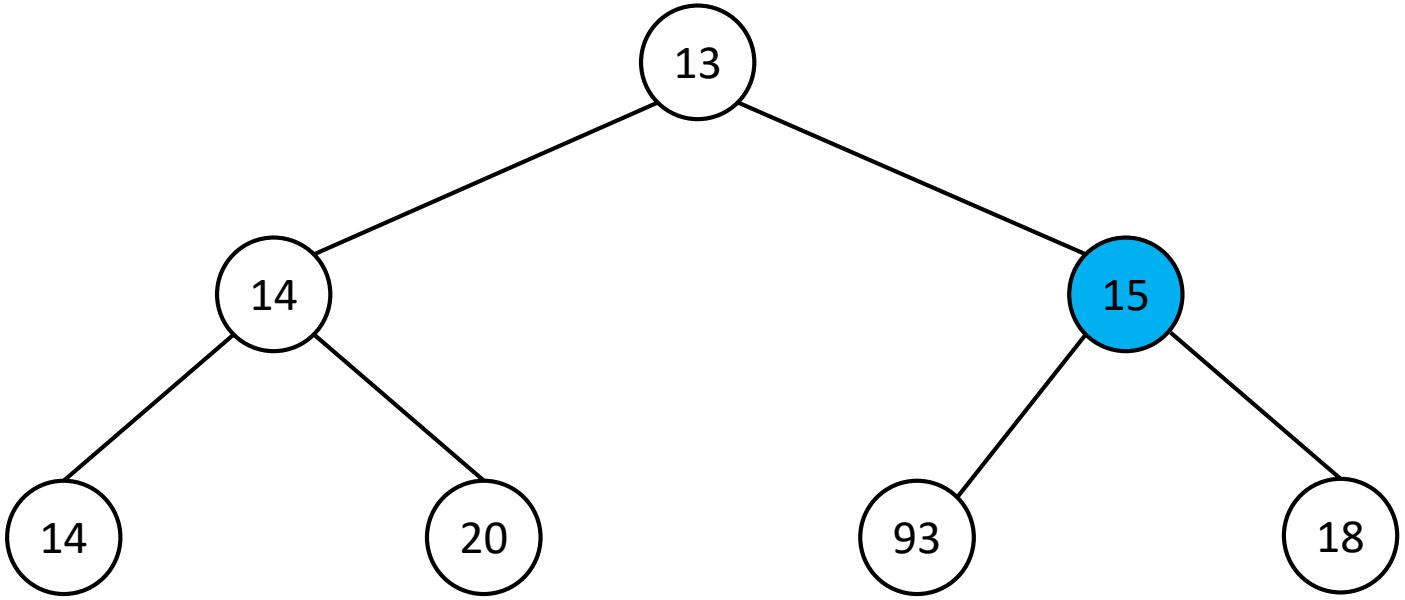

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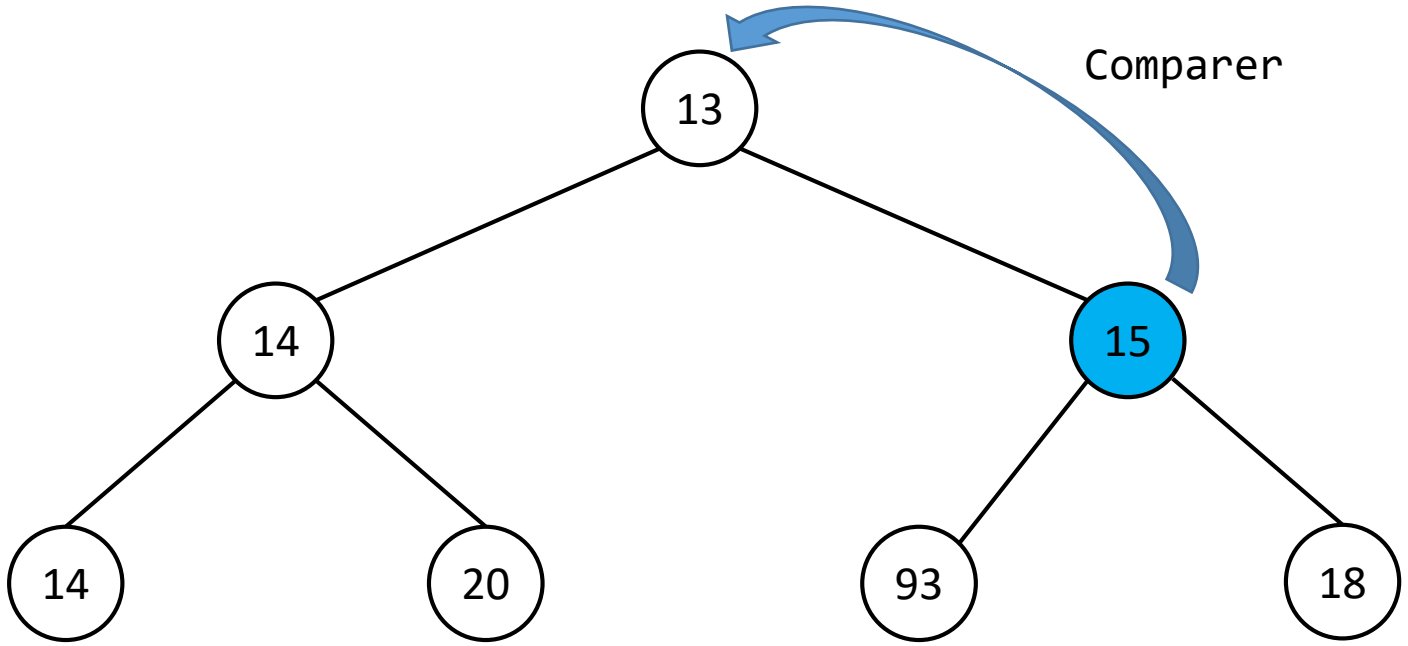

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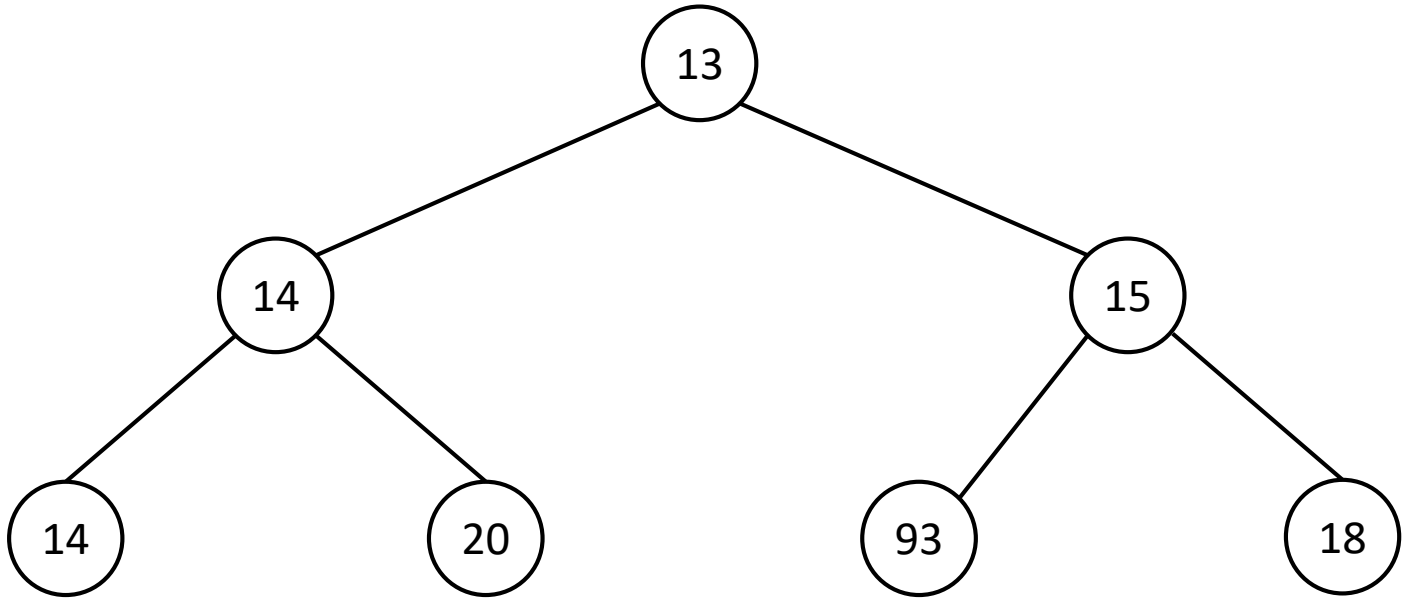

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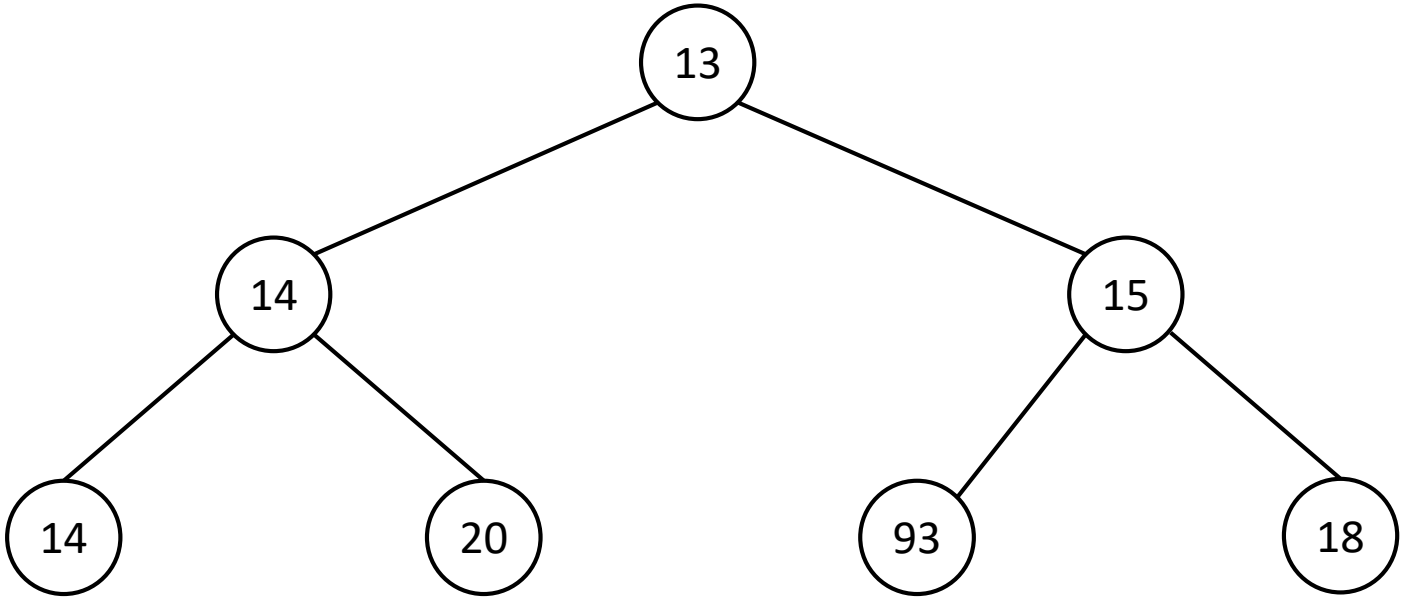

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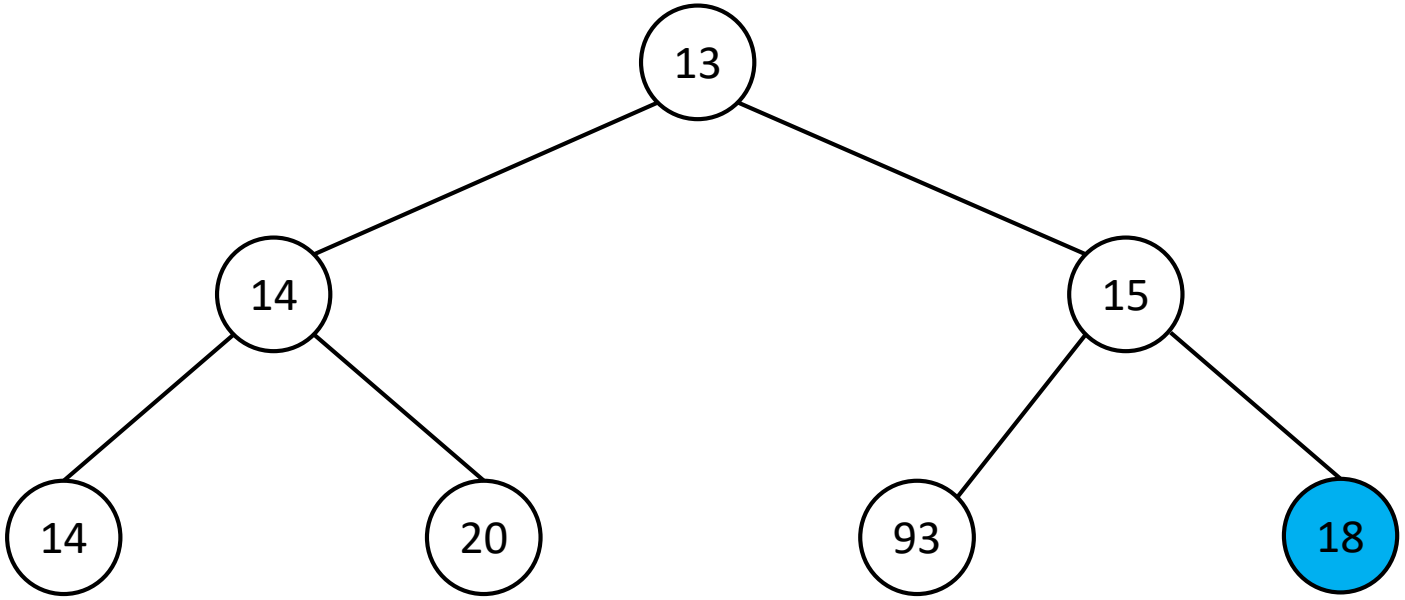

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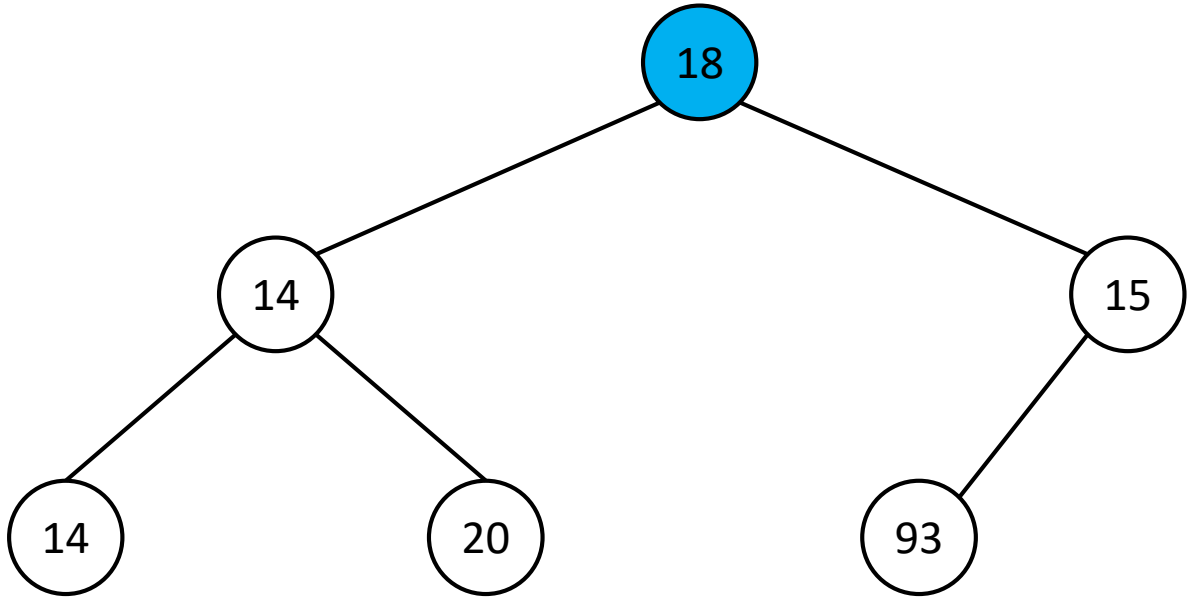
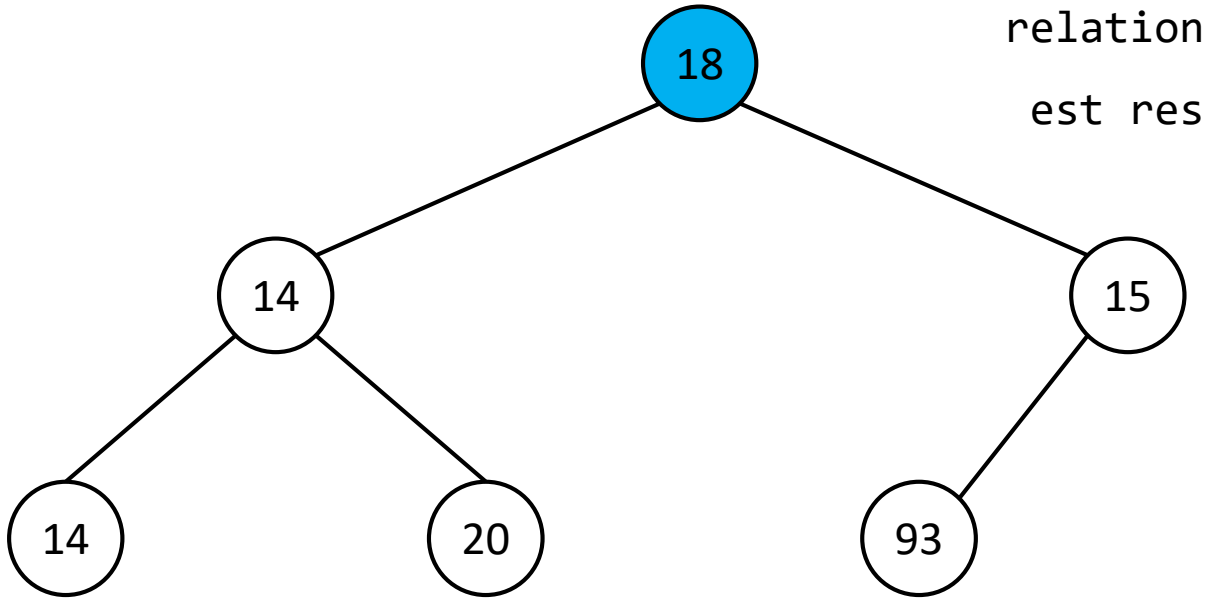


Les tas (Suppression racine)

Est-ce que la relation d'ordre est respectée?



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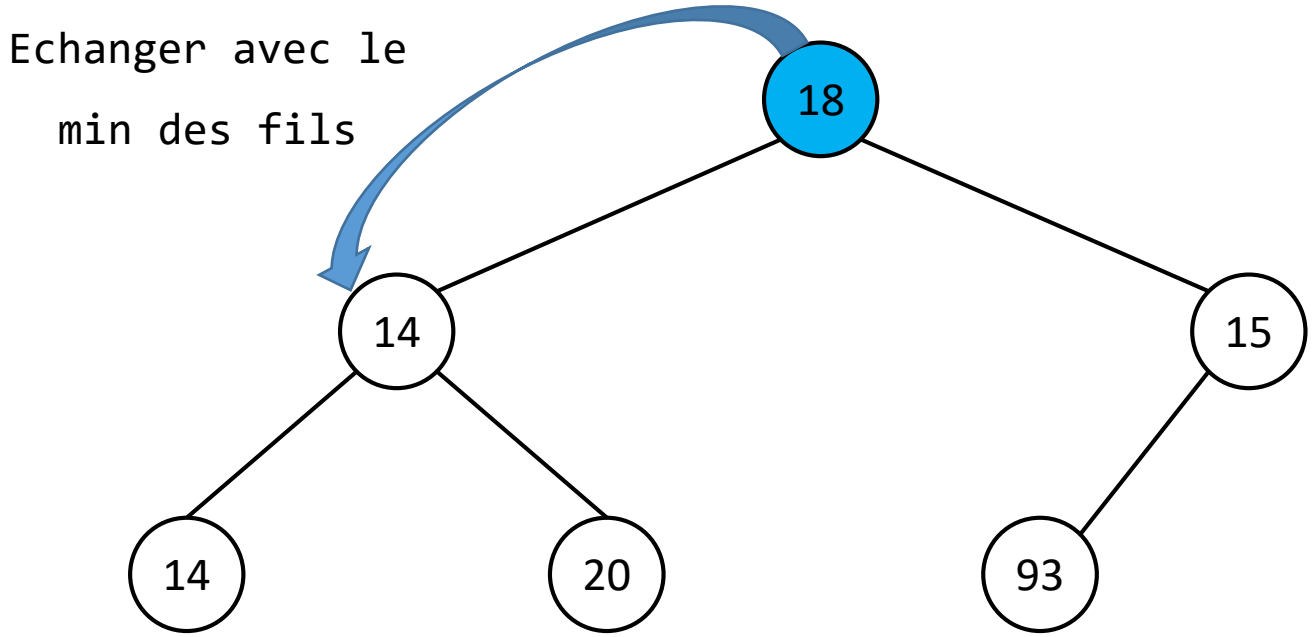

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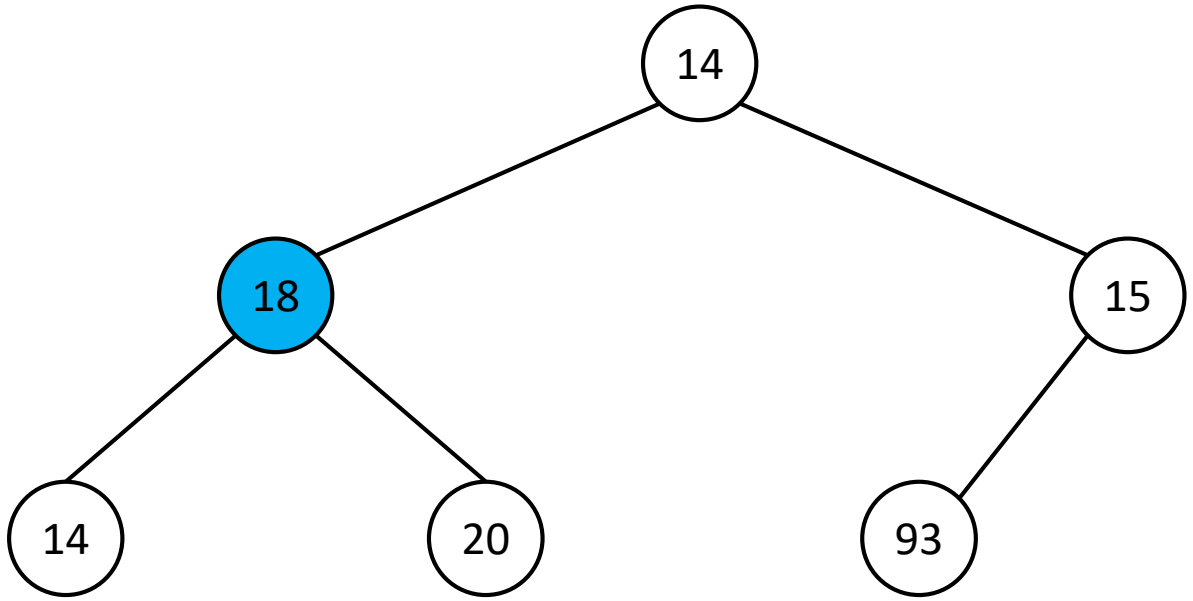

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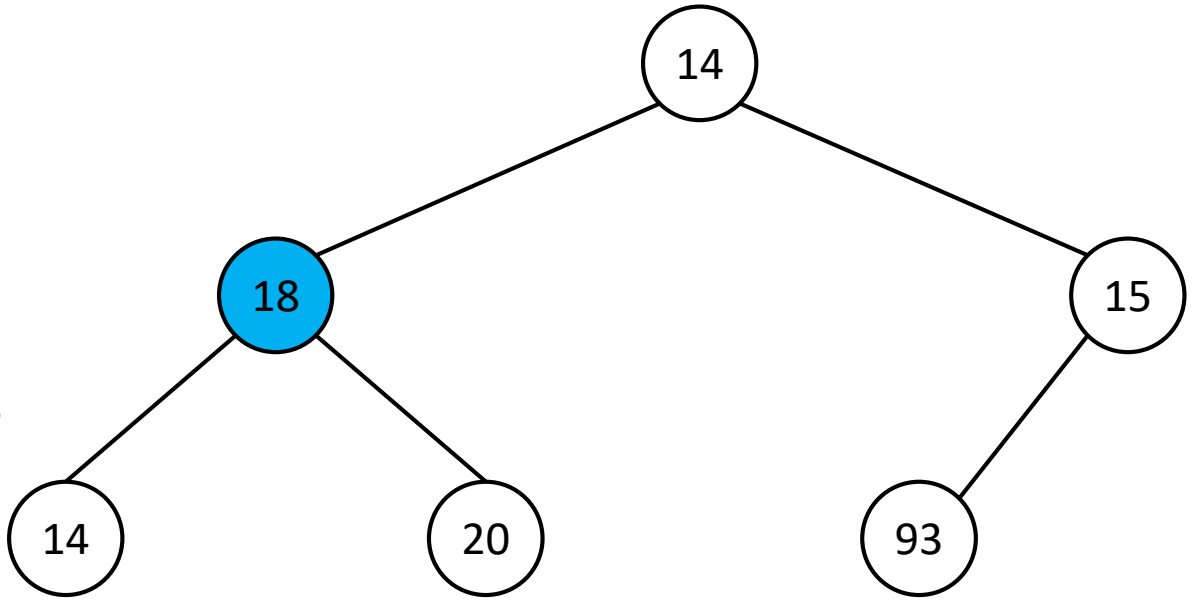
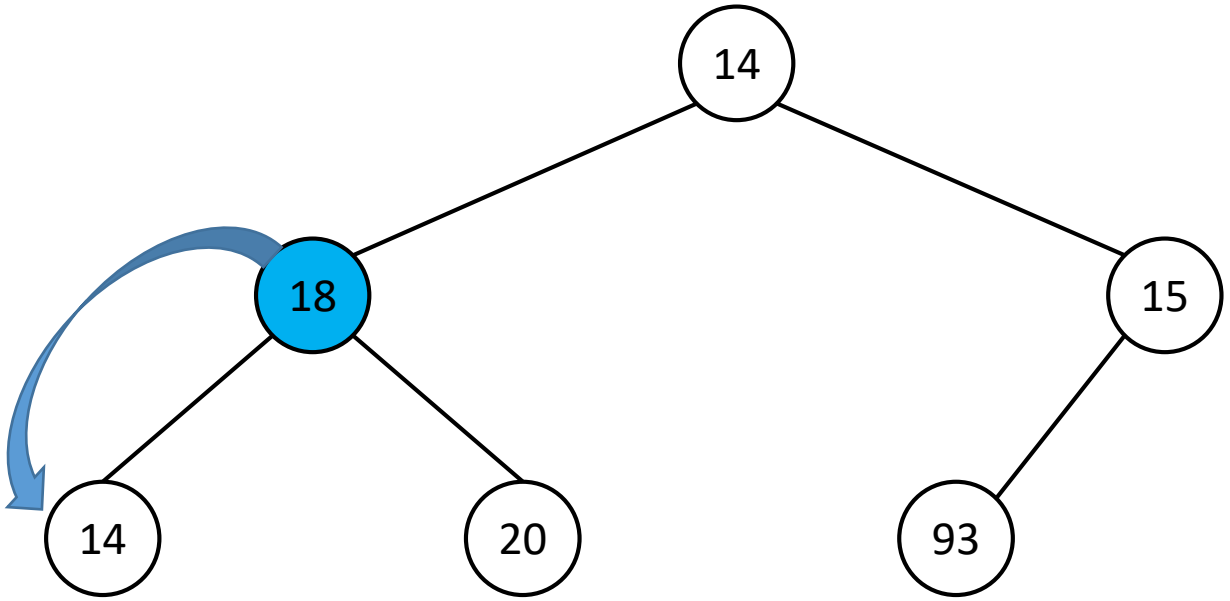


Les tas (Suppression racine)

Echanger avec le min des fils



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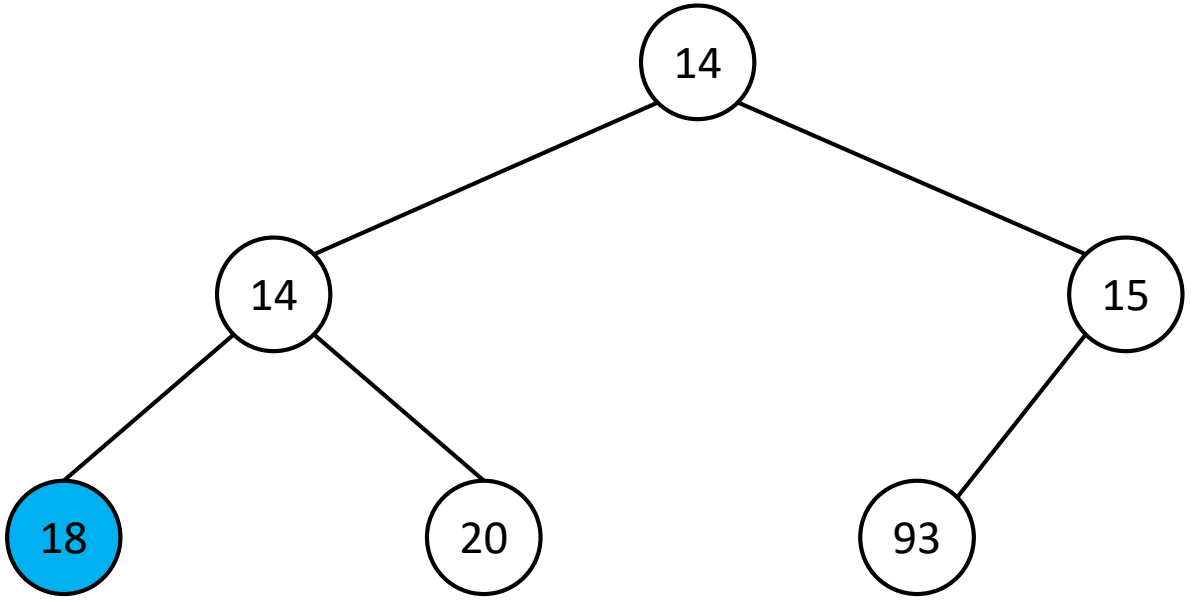
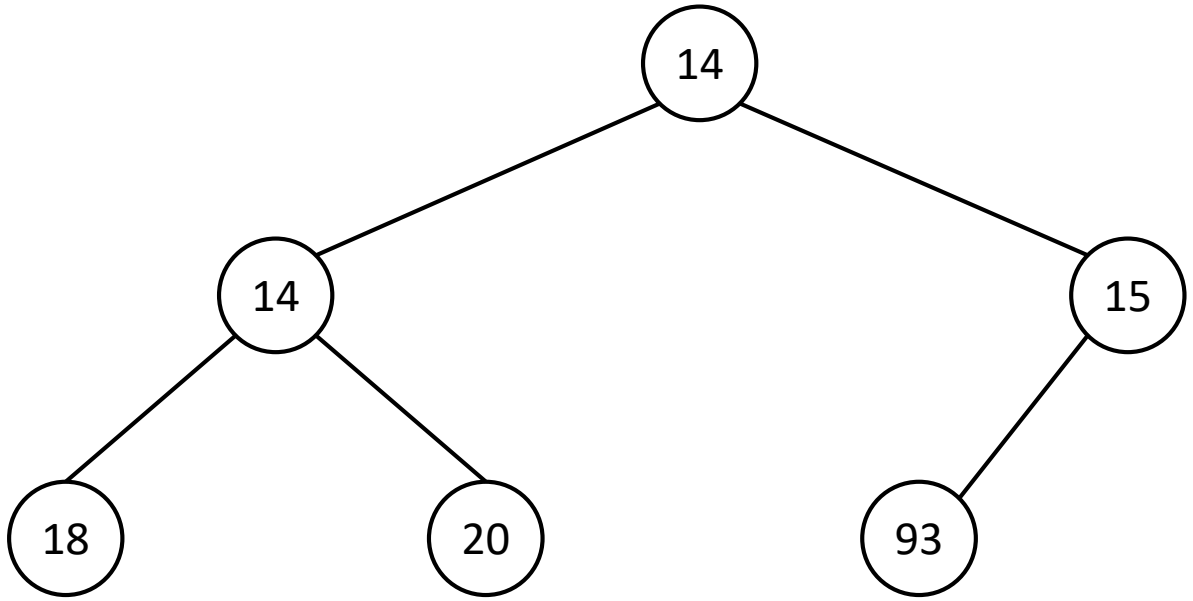
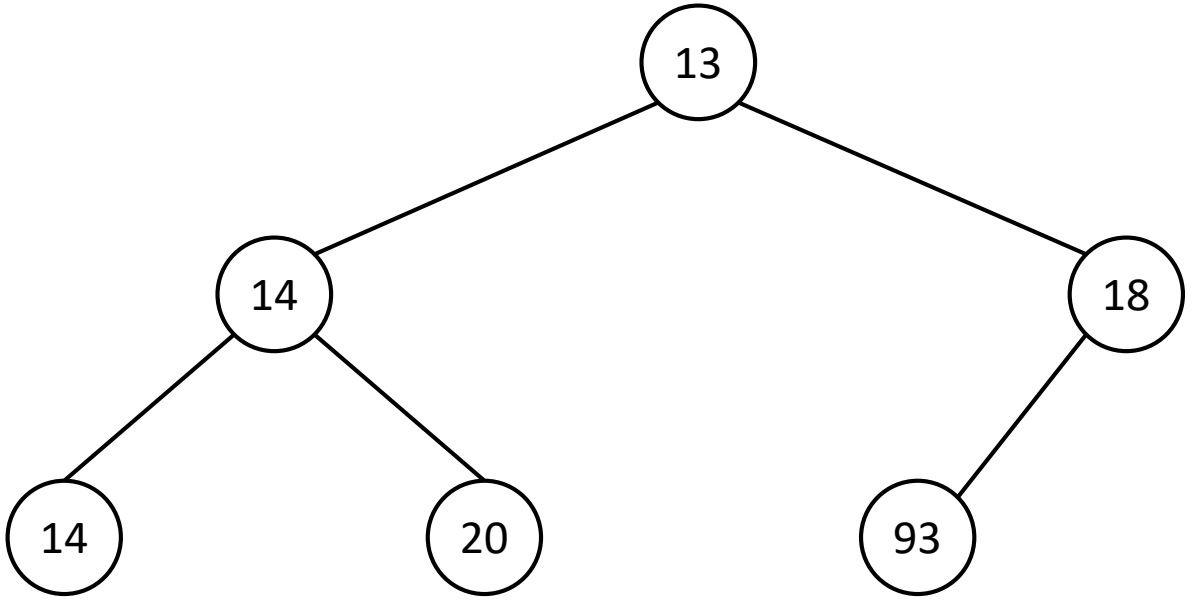


Les tas (Suppression racine)



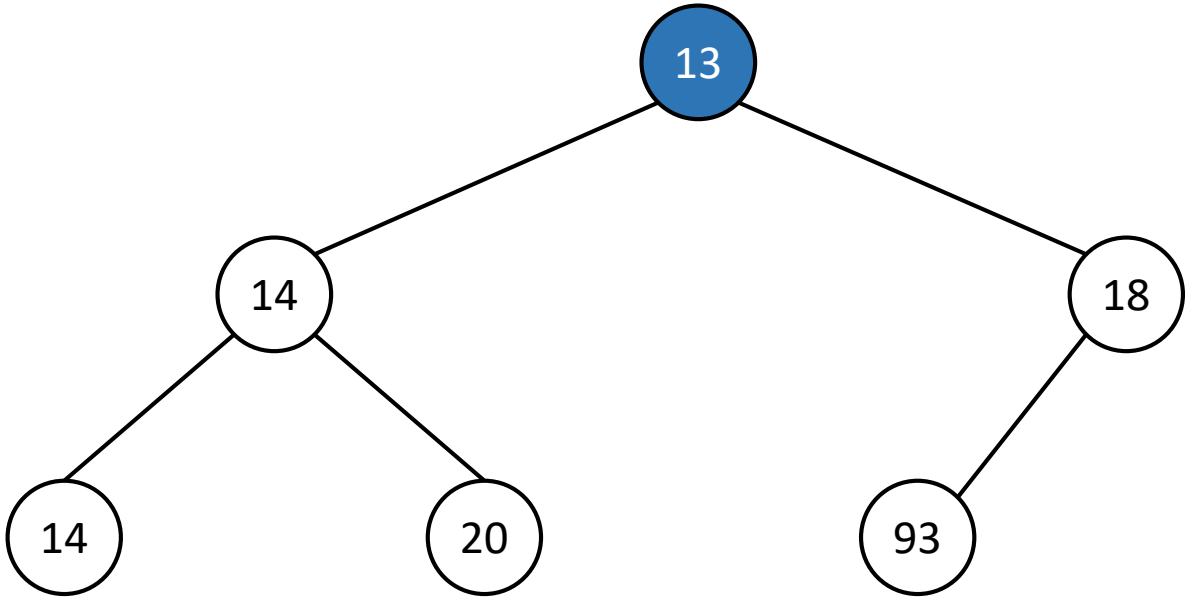
Les tas (Représentation par un tableau)

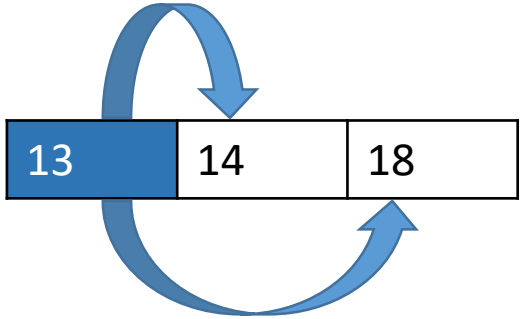


13

Tas Min

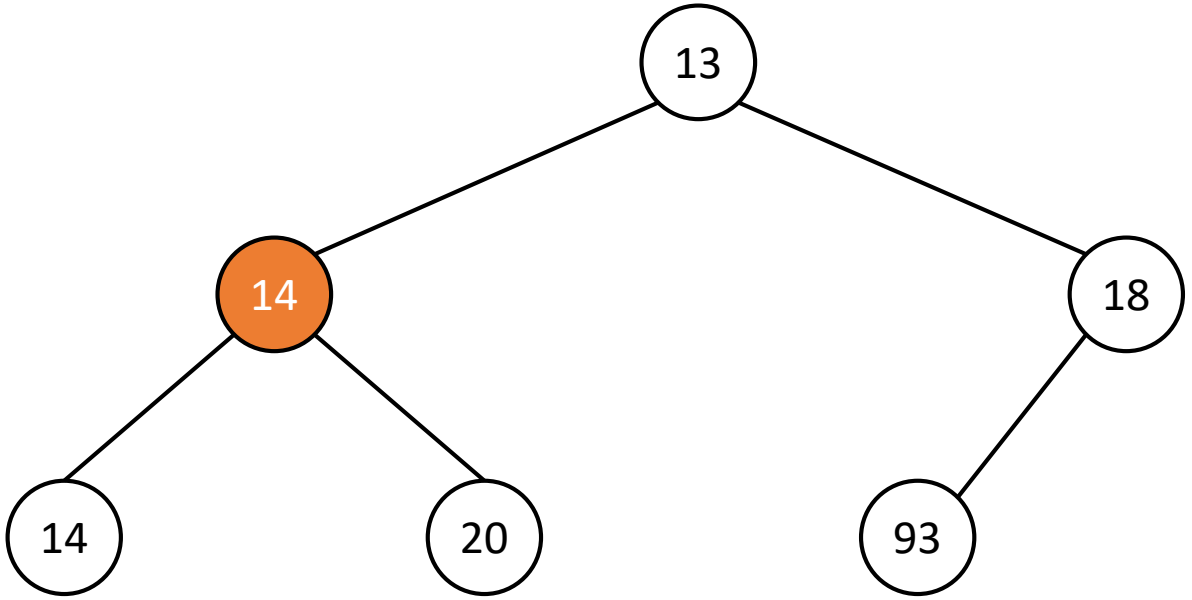
Les tas (Représentation par un tableau)



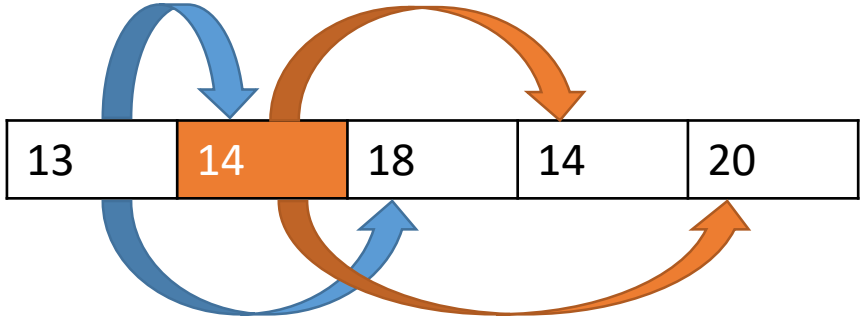


Tas Min

Les tas (Représentation par un tableau)



Tas Min



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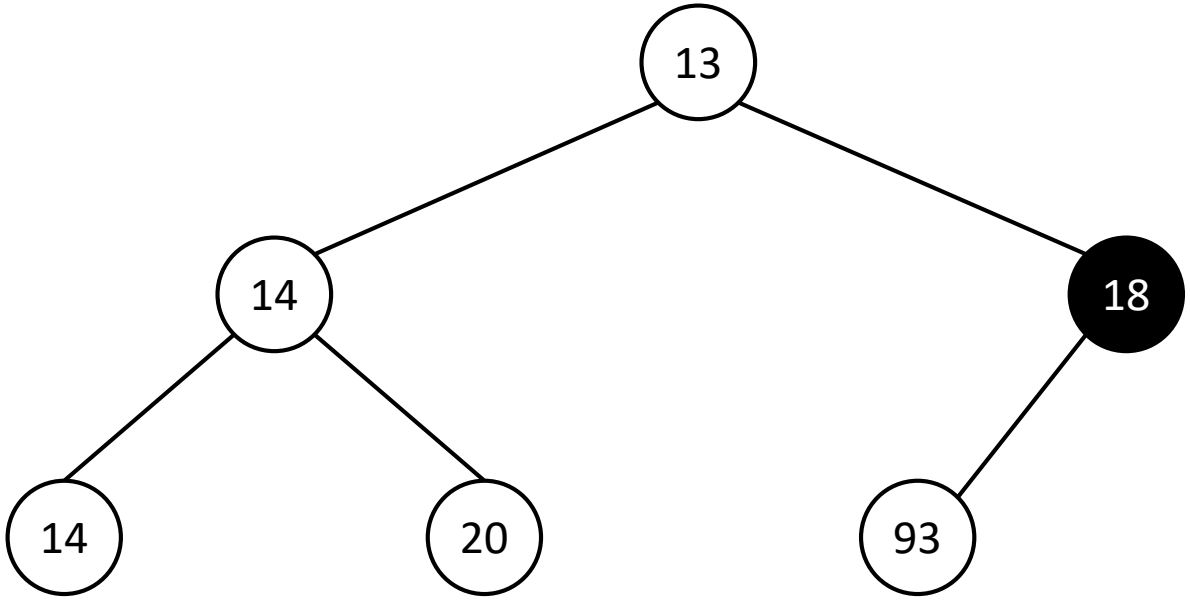
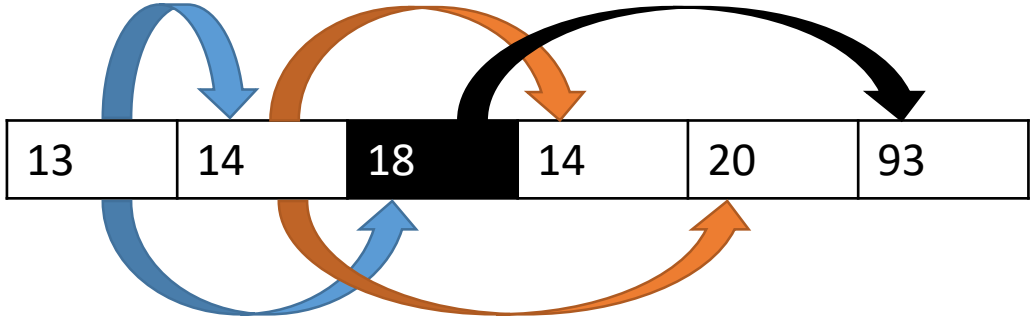


Tas Min



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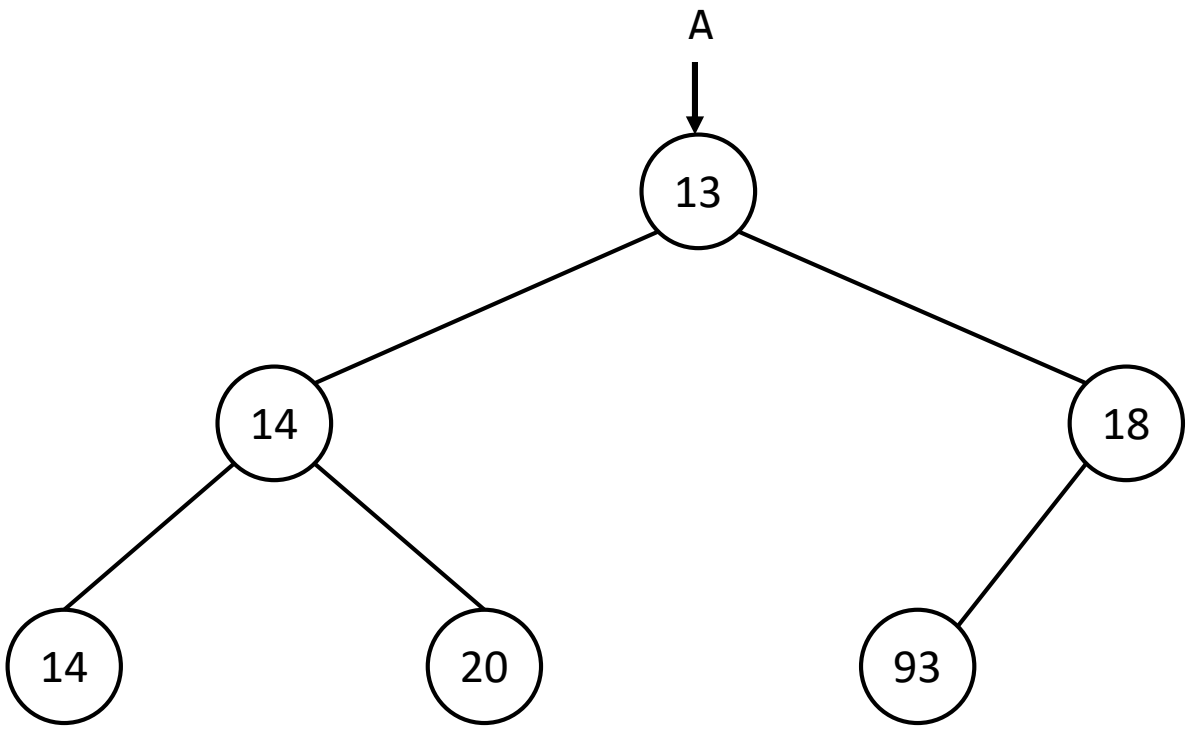
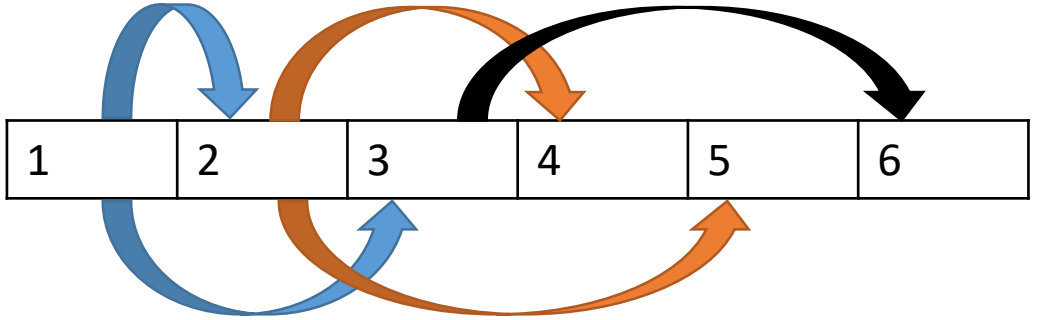


Tas Min



Racine(A): Case 1
Pere(i): Case ⌊i/2⌋
Gauche(i): Case 2i
Droit(i): Case 2i + 1

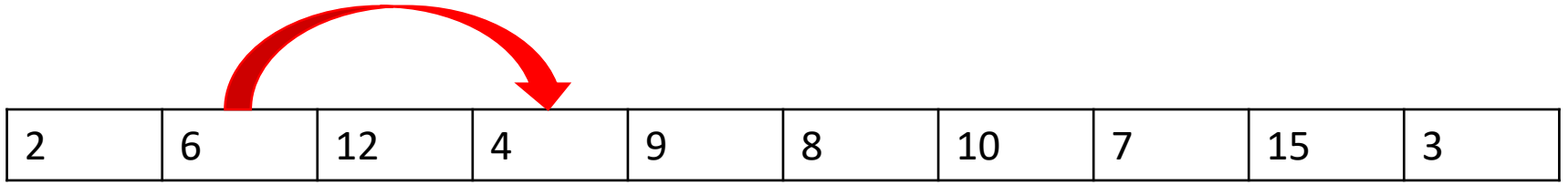
Question 1

- **Liste L1:** L1[i] ≥ L1[⌊i/2⌋] ∀i>1 donc cette liste respecte les propriétés d'un tas.

1	3	1	6	9	7	12	8	7	9	10	10

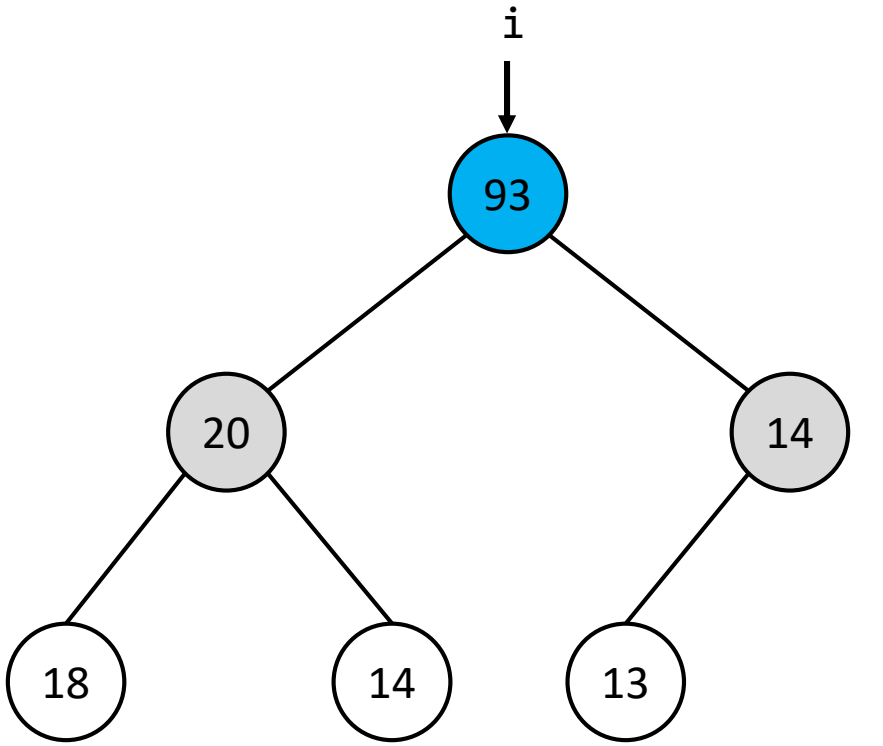
Question 1

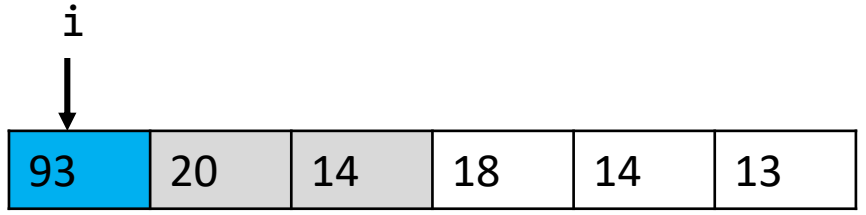
- **Liste L2:** L2[4] < L2[2] donc cette liste ne respecte pas les propriétés d'un tas.



Question 2

```
Entasser (A: Tableau, i: noeud)
```





Question 2

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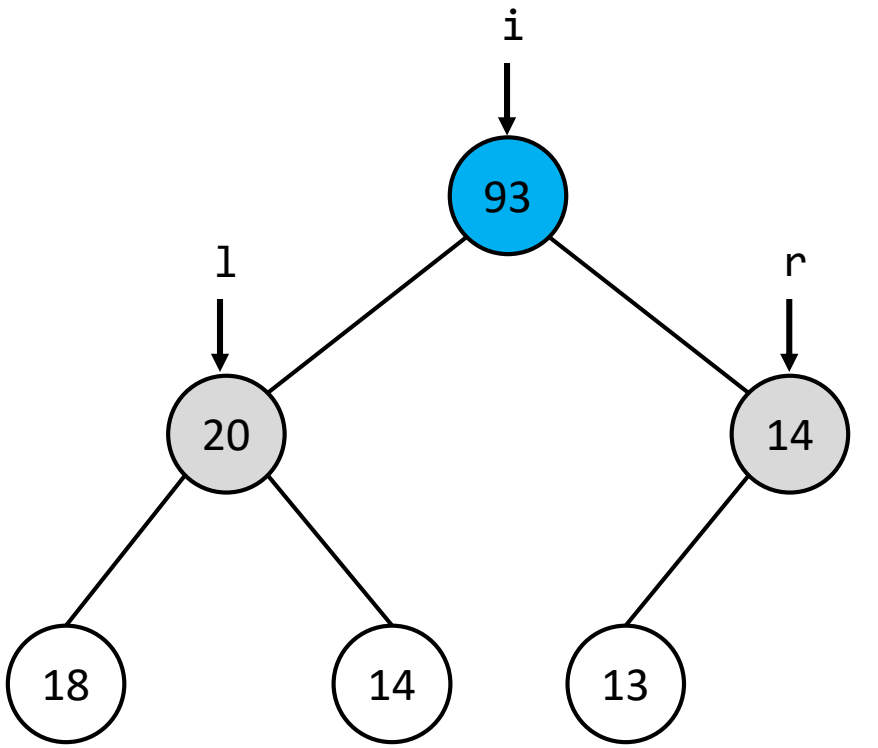
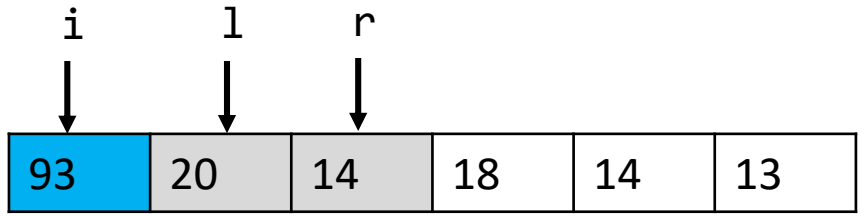
```





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    // Recuperer le min entre i son fils gauche
    Si l <= Taille[A] et A[l] < A[i]
        min ← l
    Sinon
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```

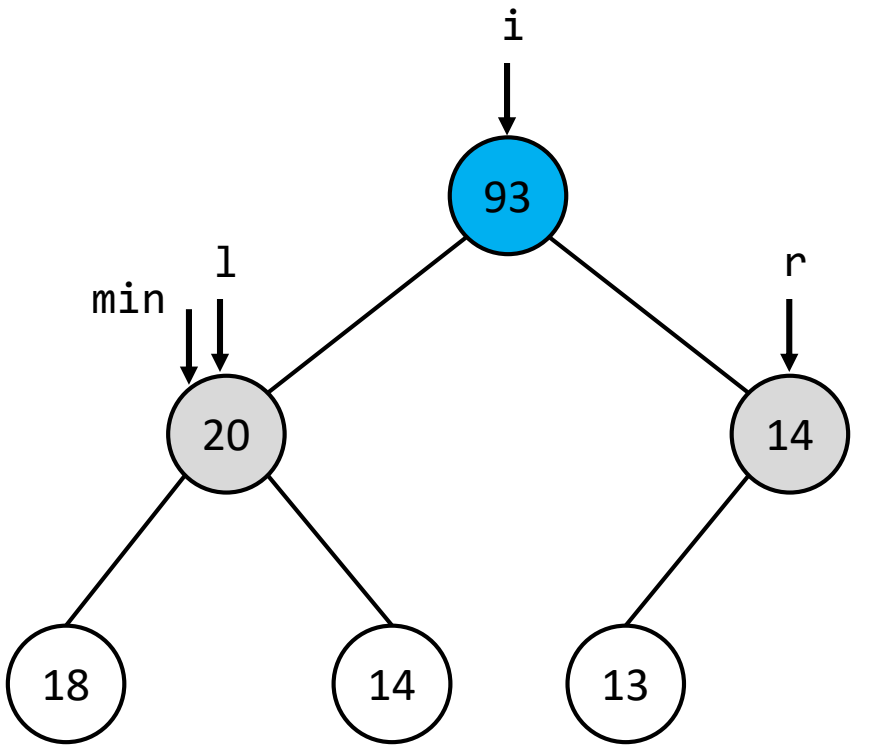

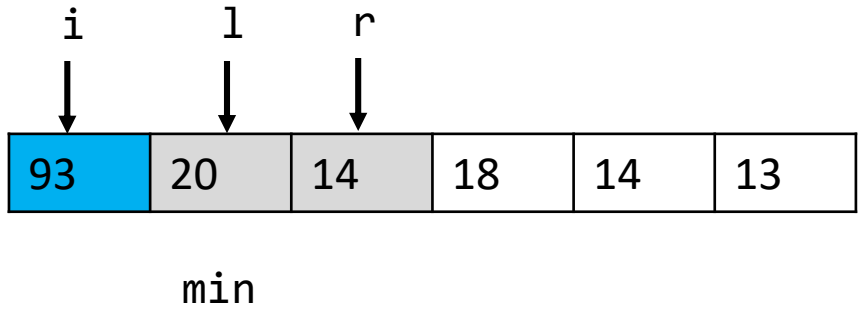

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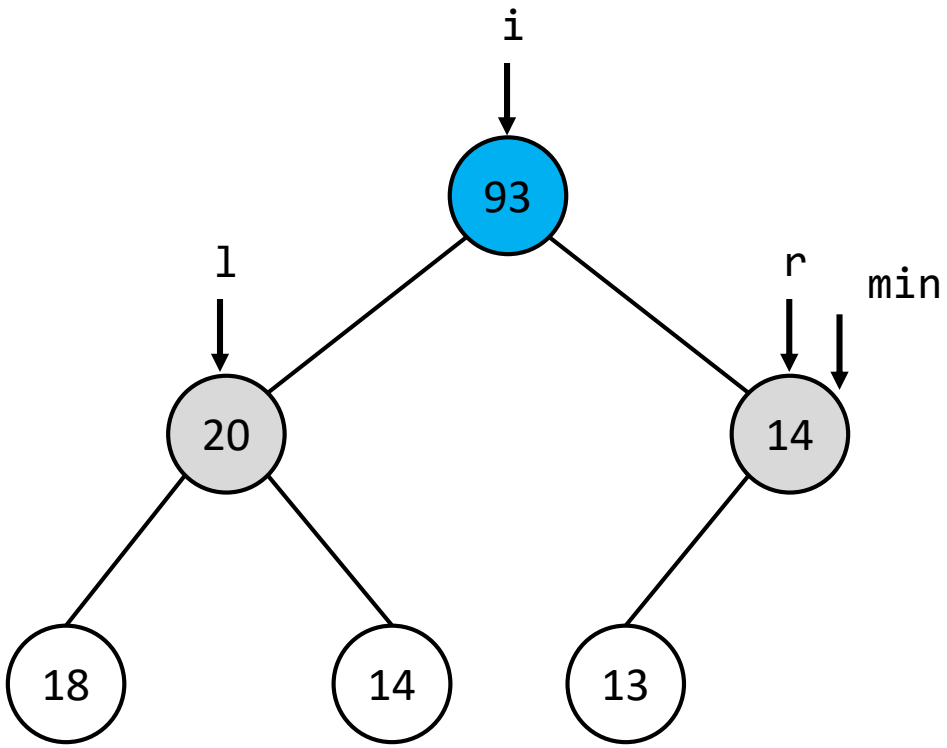

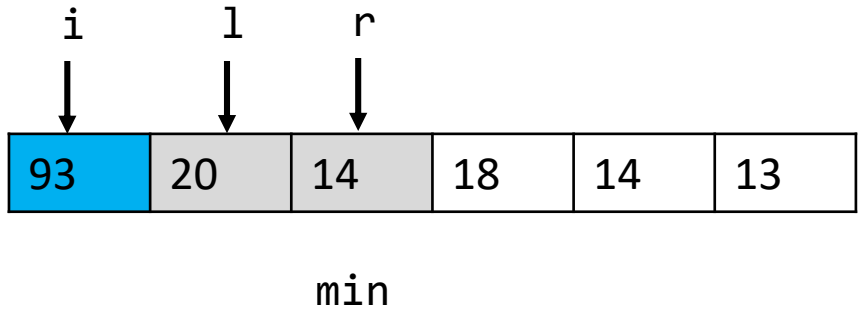

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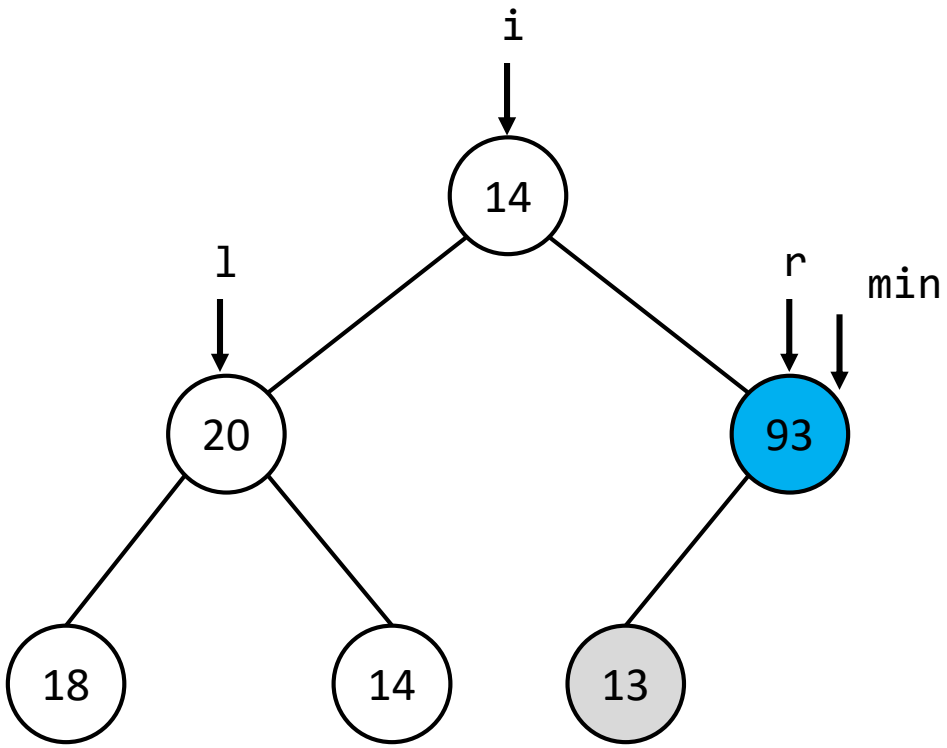
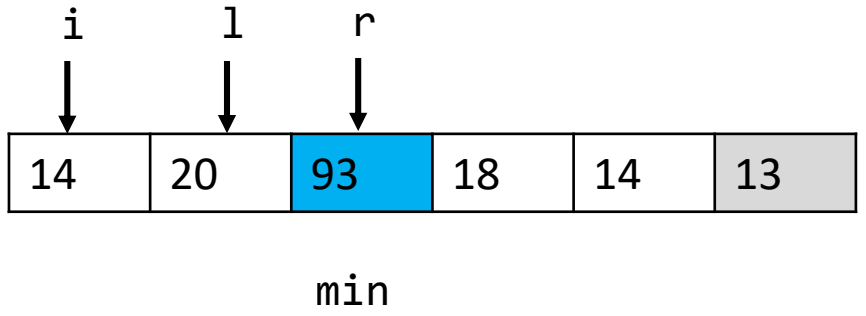
    // Si i a un fils inférieur à lui
    Si min != i
        Echanger (A, i, min)
```





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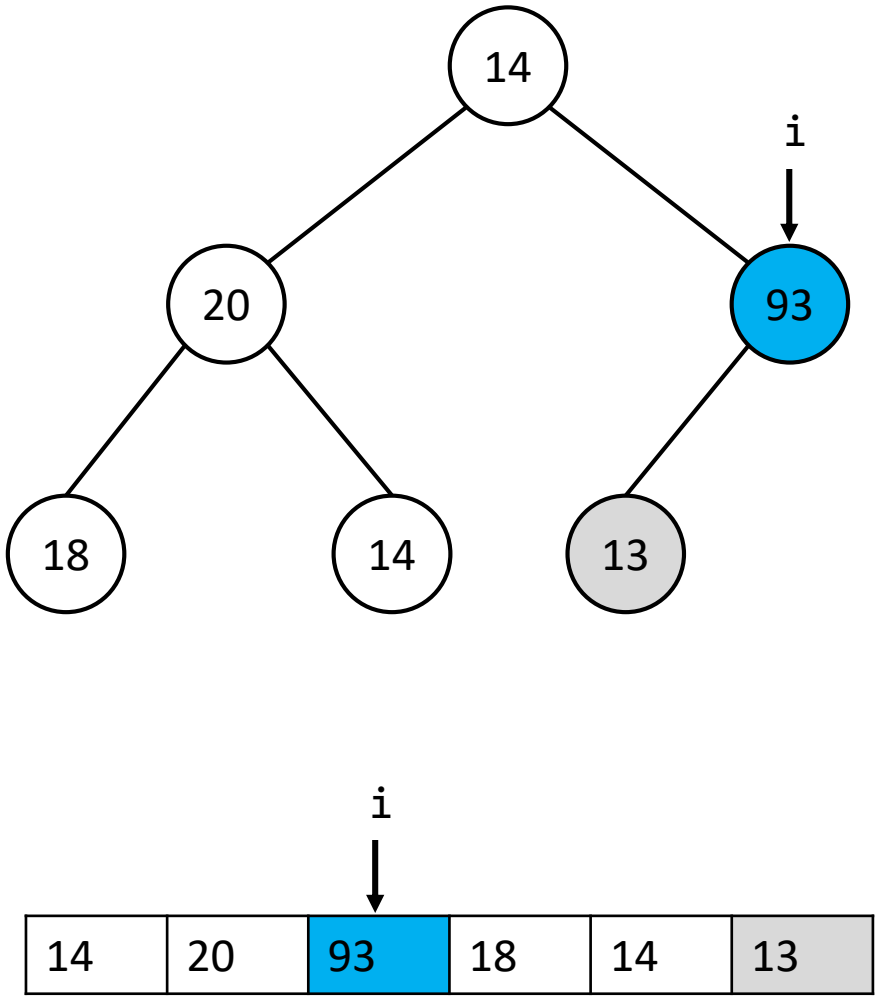

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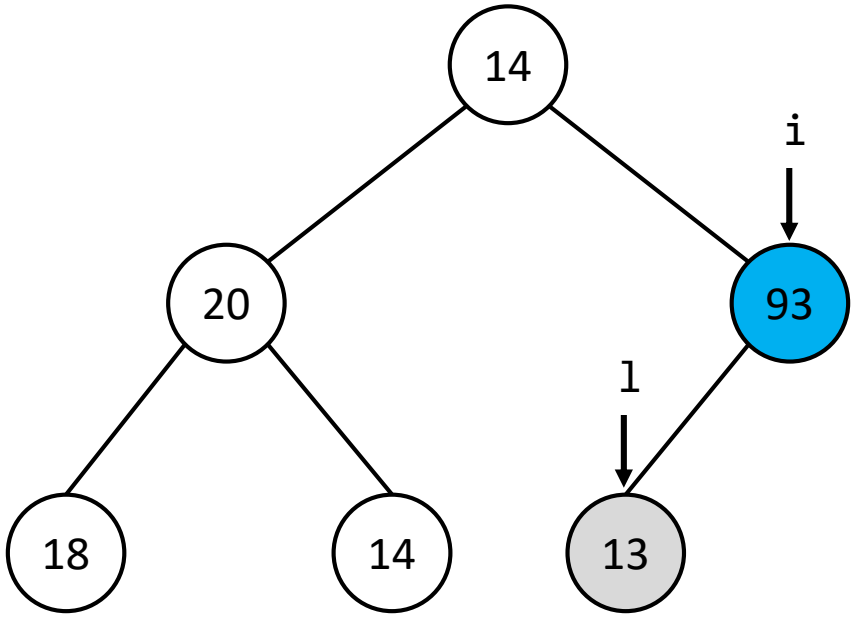

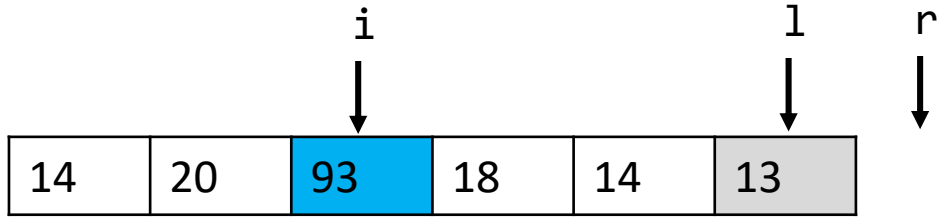

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```
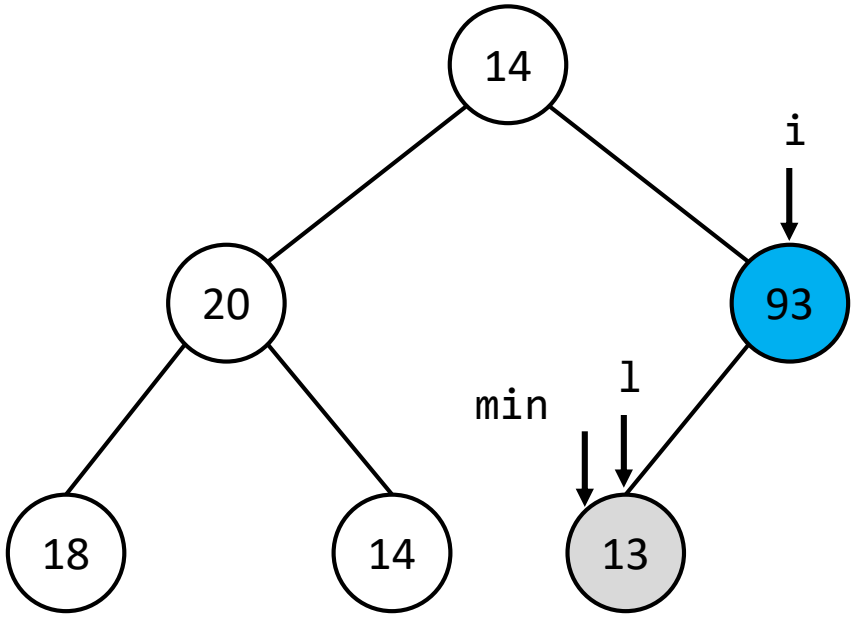
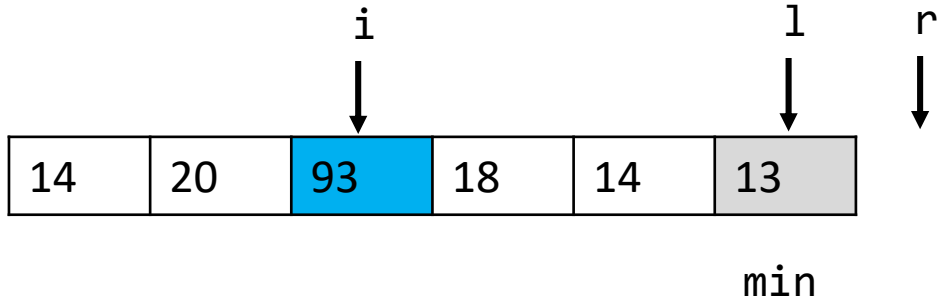
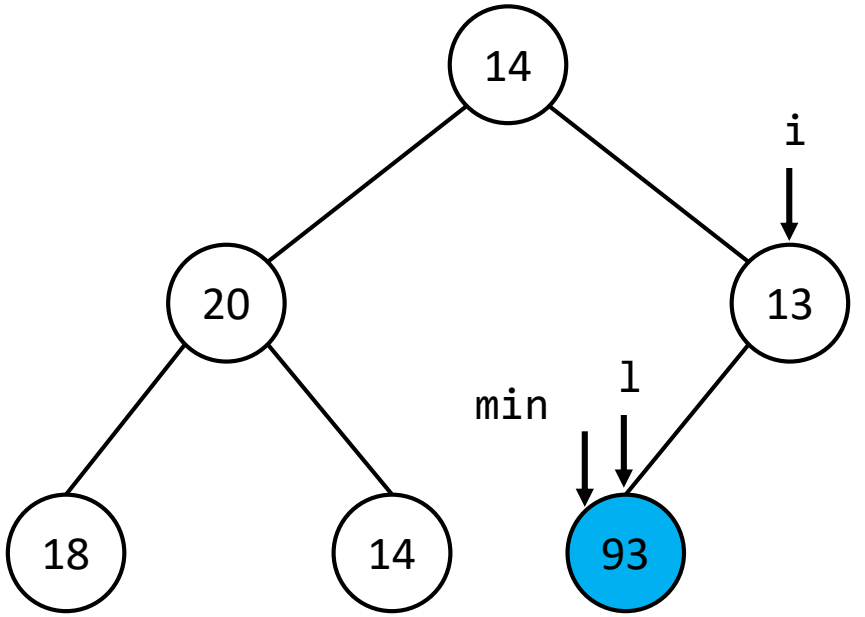
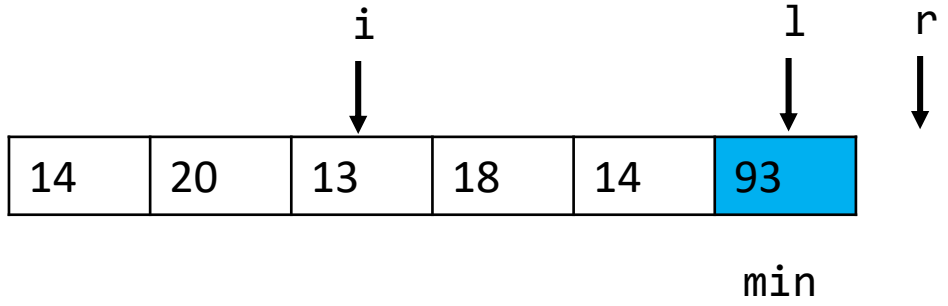





Question 2

```
Entasser (A: Tableau, i: noeud)
    l ← Gauche[i]
    r ← Droit[i]

    // Recuperer le min entre i son fils gauche
    Si l <= Taille[A] et A[l] < A[i]
        min ← l
    Sinon
        min ← i

    // Recuperer le min entre i et ses deux fils
    Si r <= Taille[A] et A[r] < A[min]
        min ← r

    // Si i a un fils inférieur à lui
    Si min != i
        Echanger (A, i, min)
        Entasser (A, min)
```





Question 2

```
Entasser (A: Tableau, i: noeud)
    l ← Gauche[i]
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    Si r <= Taille[A] et A[r] < A[min]
        min ← r

    // Si i a un fils inférieur à lui
    Si min != i
        Echanger (A, i, min)
        Entasser (A, min)
```

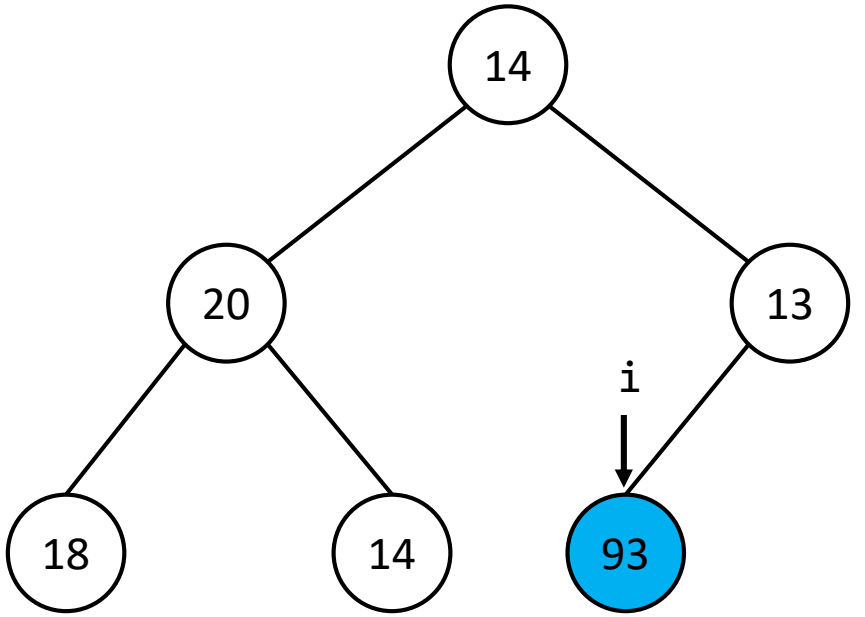

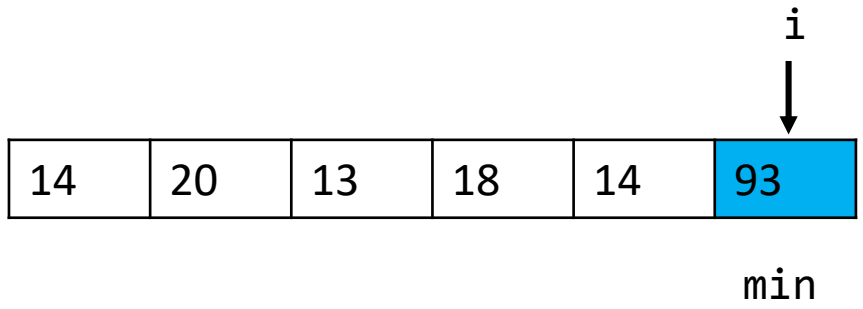

Question 2

```
Entasser (A: Tableau, i: noeud)
    l ← Gauche[i]
    r ← Droit[i]

    // Recuperer le min entre i son fils gauche
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    // Recuperer le min entre i et ses deux fils
    Si r <= Taille[A] et A[r] < A[min]
        min ← r

    // Si i a un fils inférieur à lui
    Si min != i
        Echanger (A, i, min)
        Entasser (A, min)
```

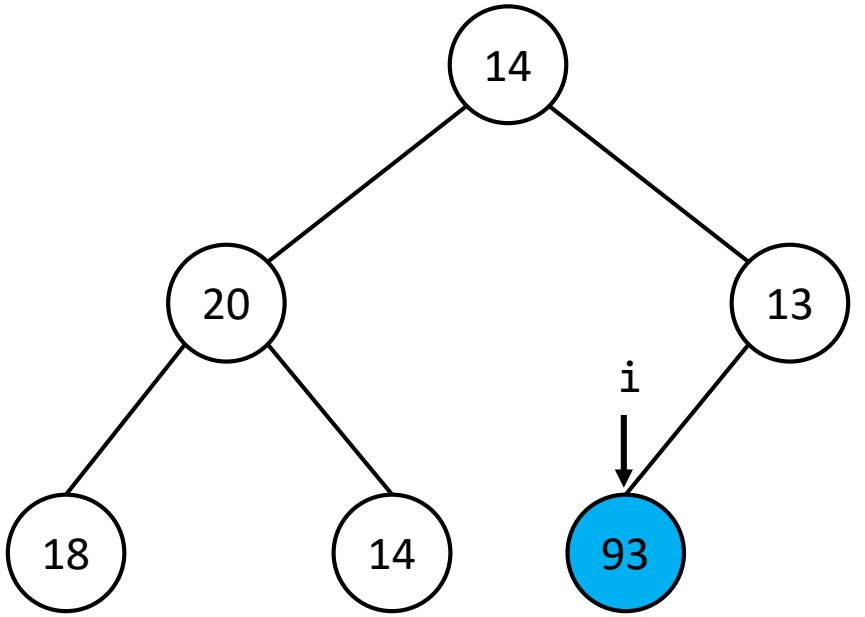

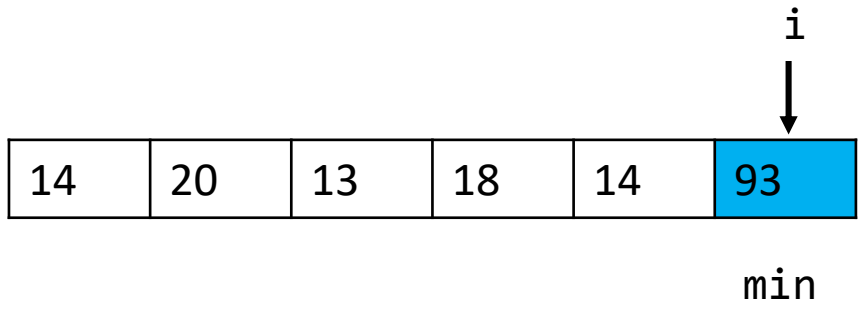

Question 2

```
Entasser (A: Tableau, i: noeud)
    l ← Gauche[i]
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    // Recuperer le min entre i son fils gauche
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    // Recuperer le min entre i et ses deux fils
    Si r <= Taille[A] et A[r] < A[min]
        min ← r

    // Si i a un fils inférieur à lui
    Si min != i
        Echanger (A, i, min)
        Entasser (A, min)
```
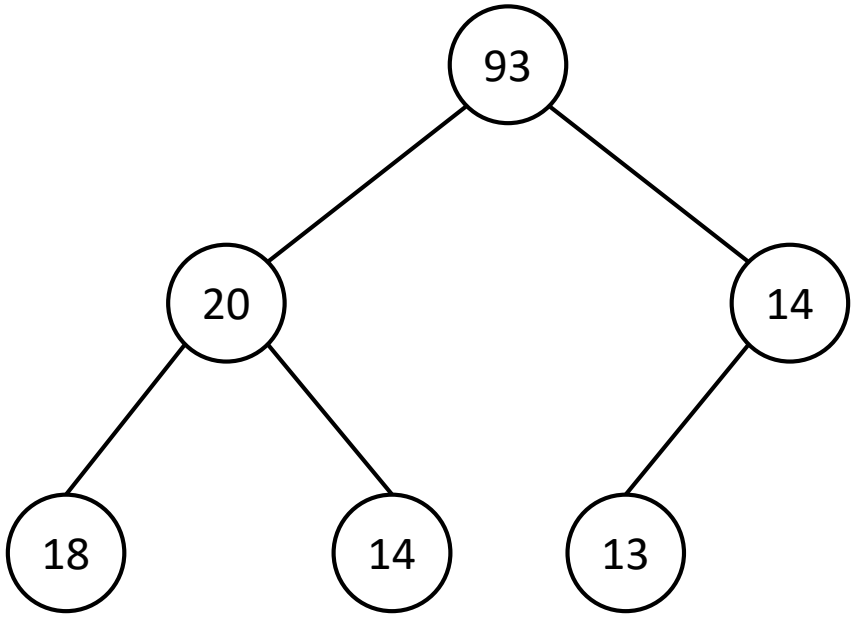

Complexité: O(h)

Question 2

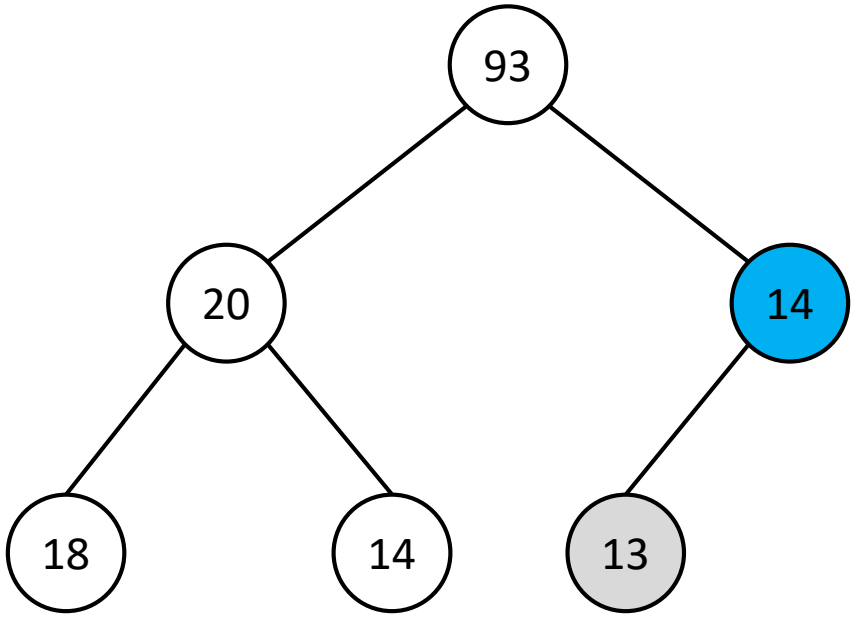
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

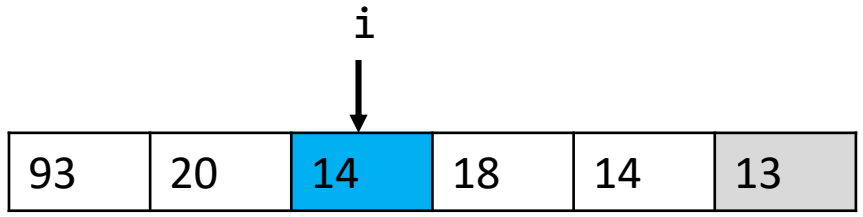


93	20	14	18	14	13

Question 2

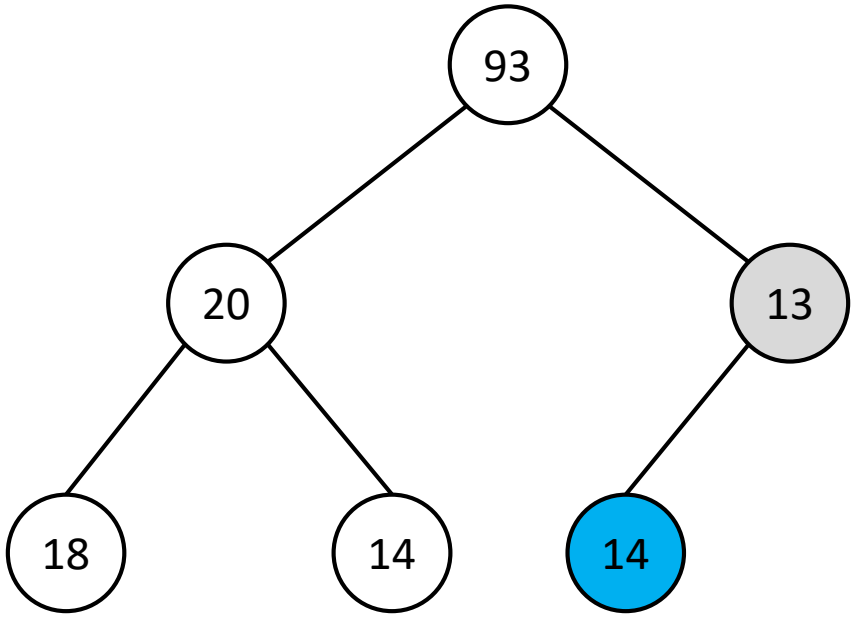
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

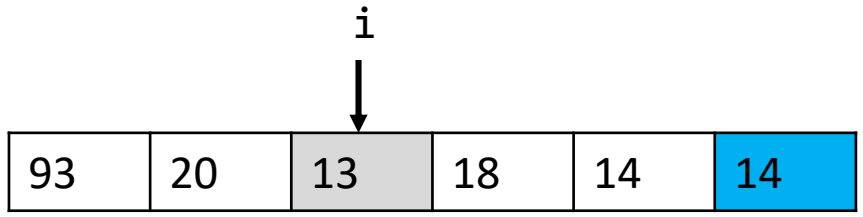




Question 2

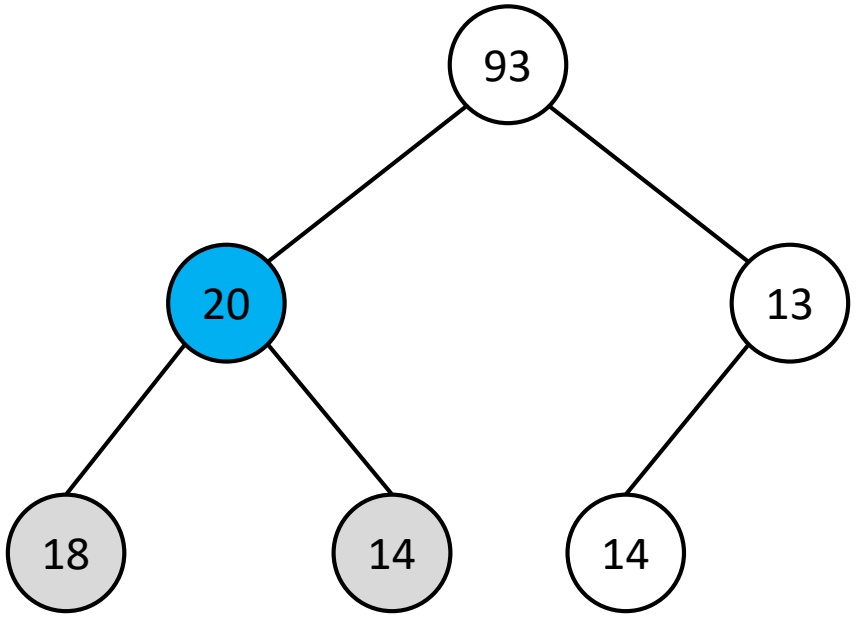
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

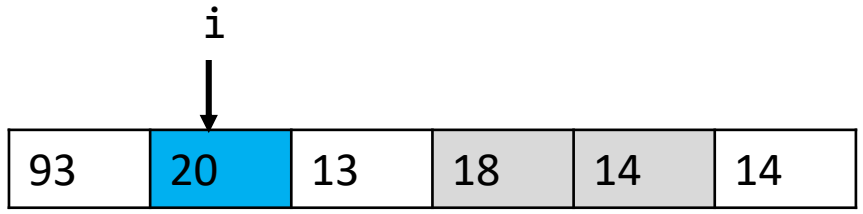




Question 2

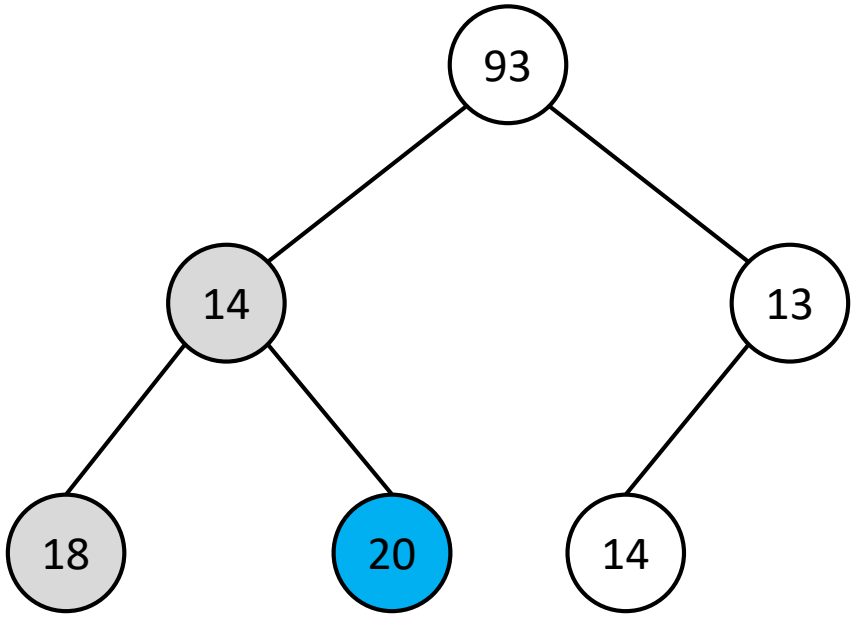
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

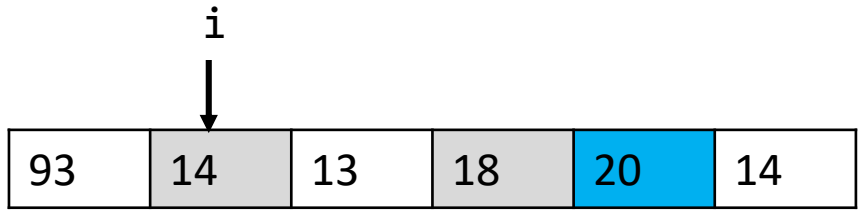




Question 2

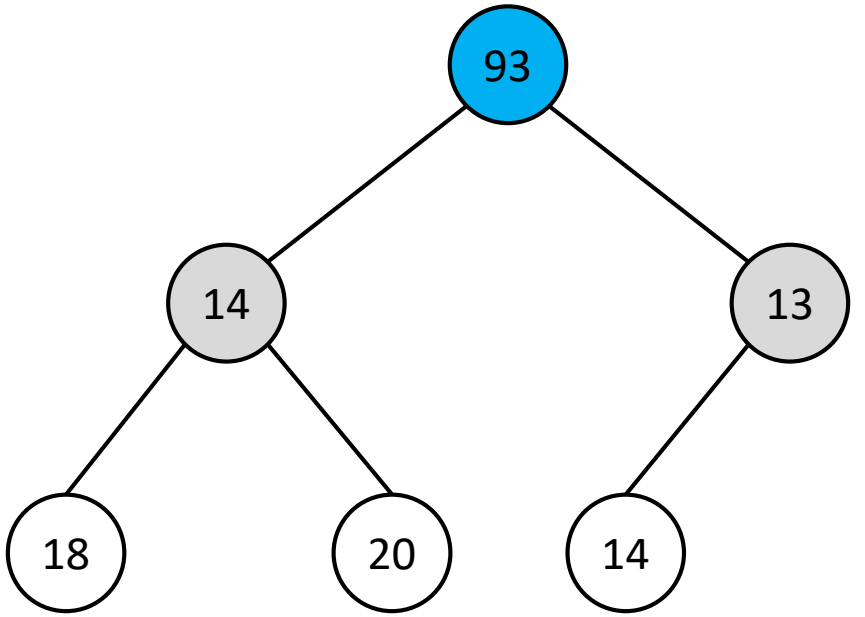
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

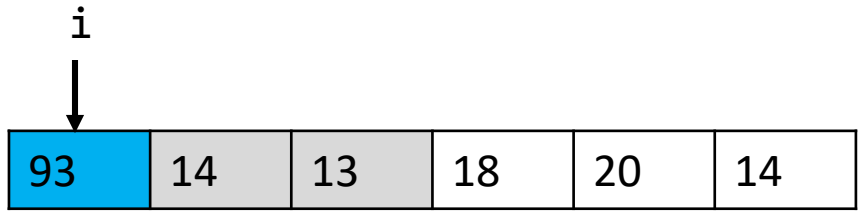




Question 2

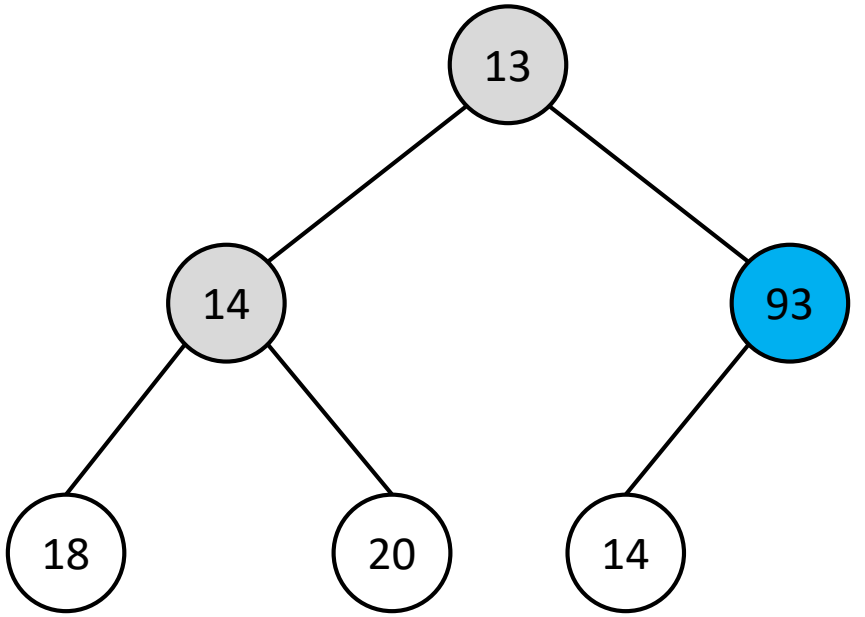
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

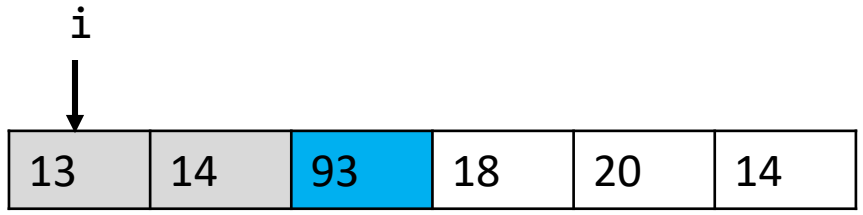




Question 2

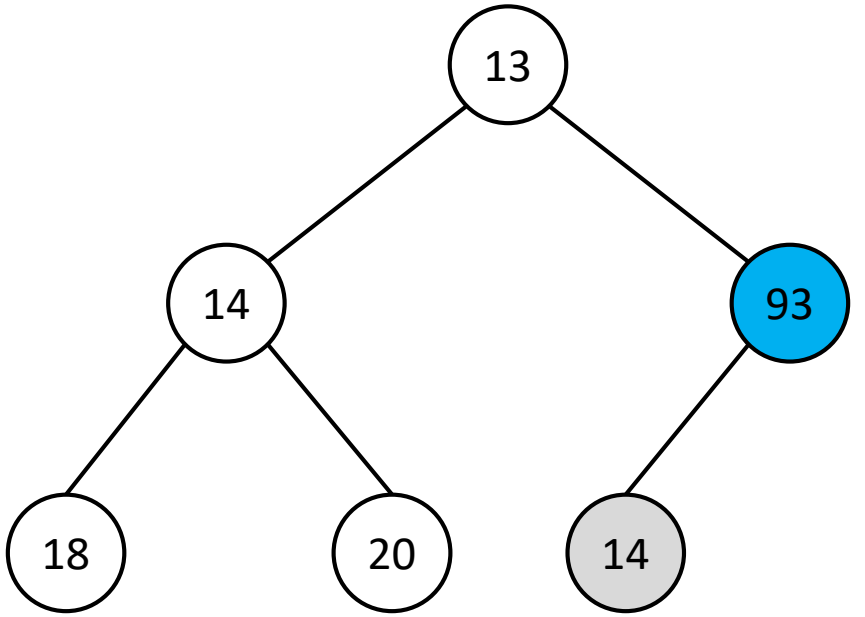
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

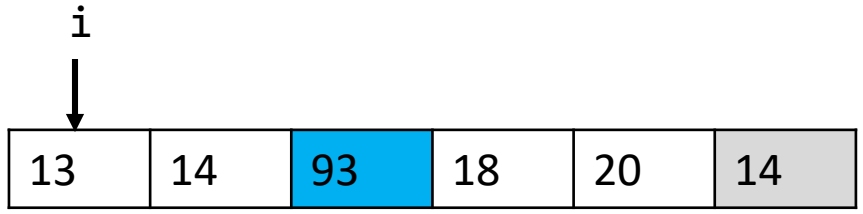




Question 2

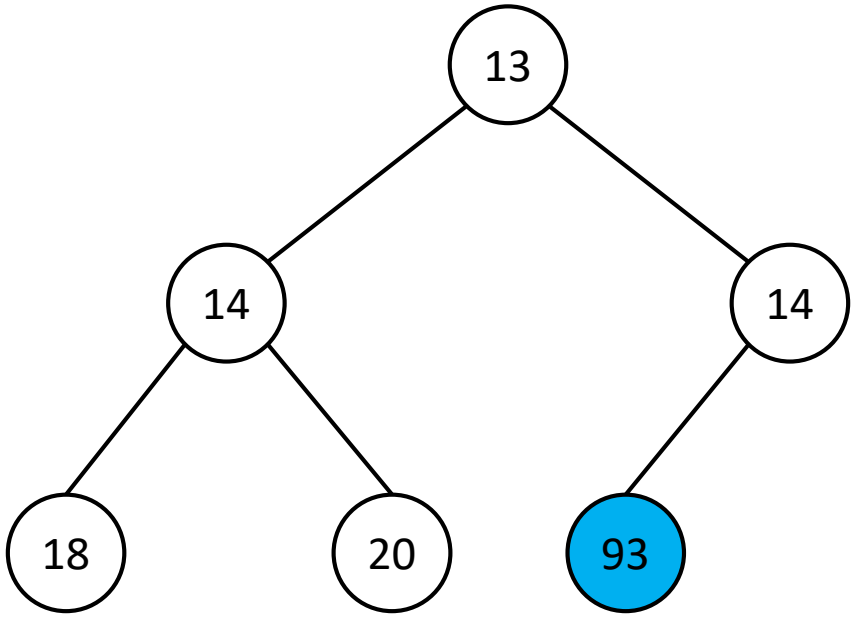
```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

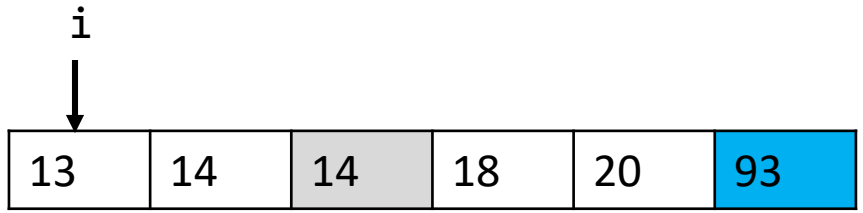




Question 2

```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```





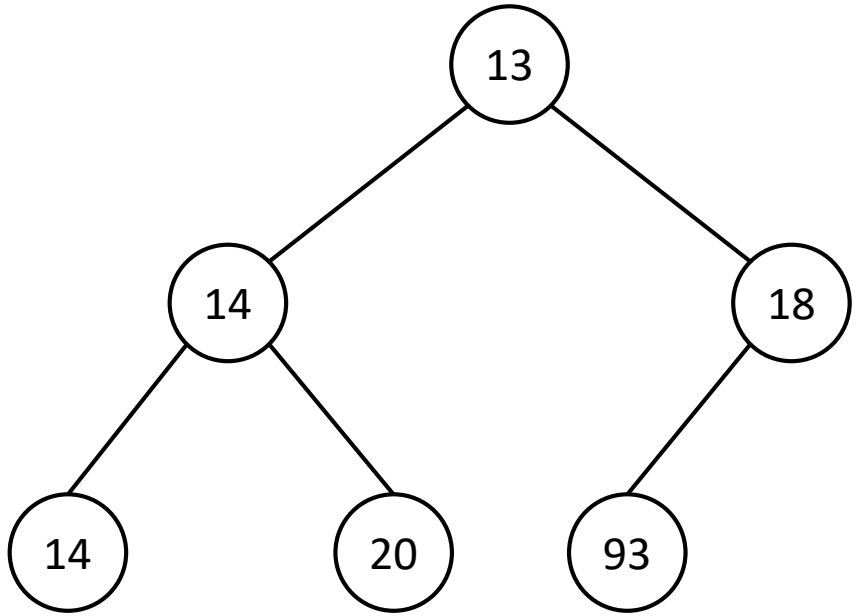
Question 2

```
construire_tas(A: Tableau)
    Pour i allant de Taille[A]/2 à 1 par pas de -1
        Entasser (A, i)
```

Complexité: O(n)

Question 3

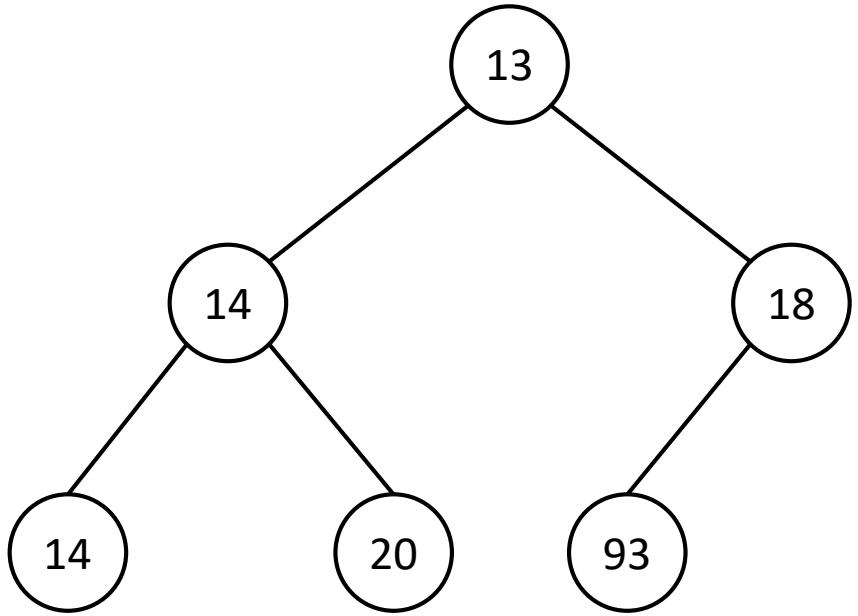
```
inserer(A: Tas , cle: valeur)
```



13	14	18	14	20	93

Question 3

```
inserer(A: Tas , cle: valeur)
```

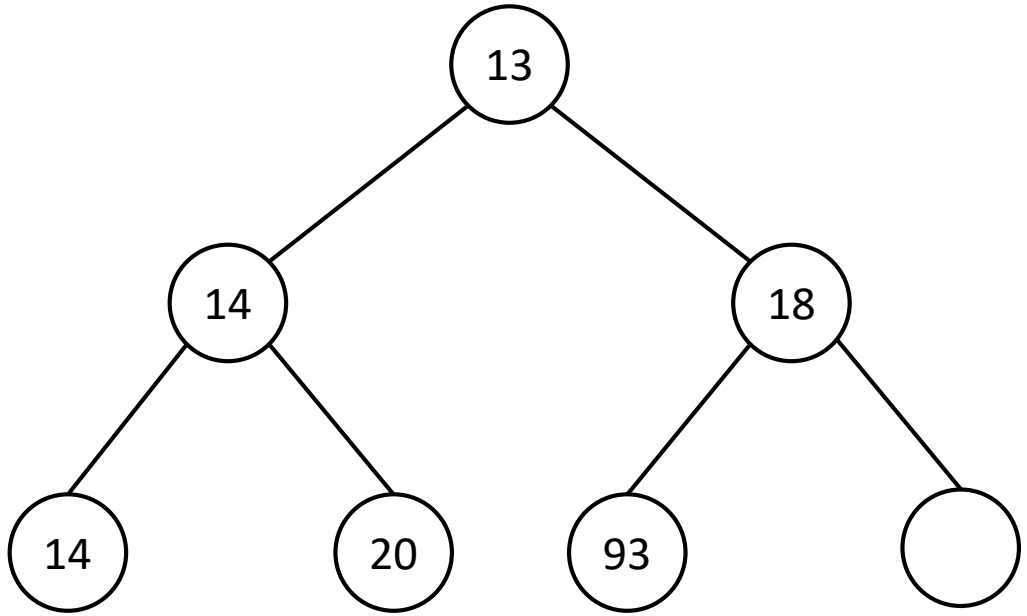


Insérer 15

13	14	18	14	20	93

Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
```

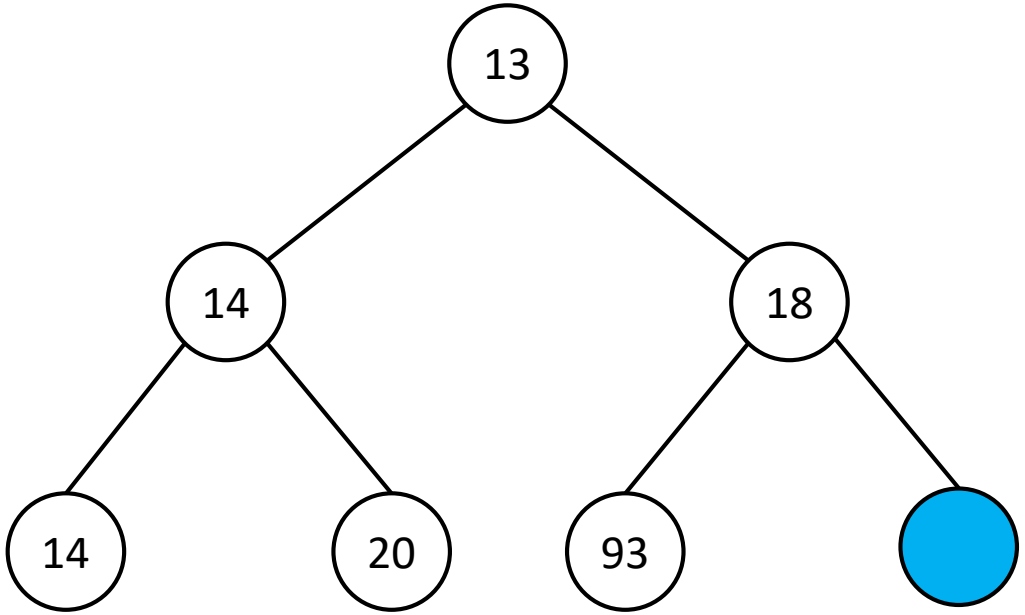


Insérer 15

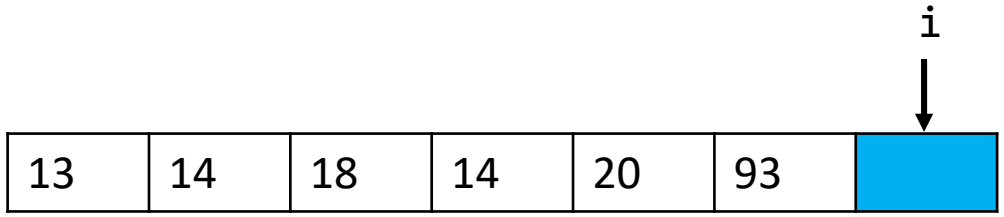
13	14	18	14	20	93	

Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
```

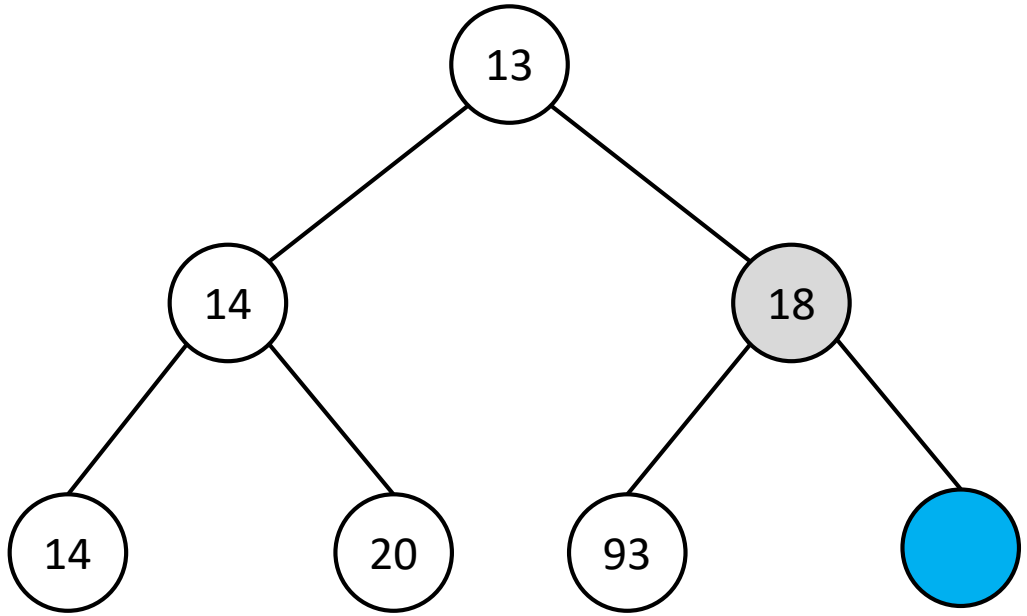


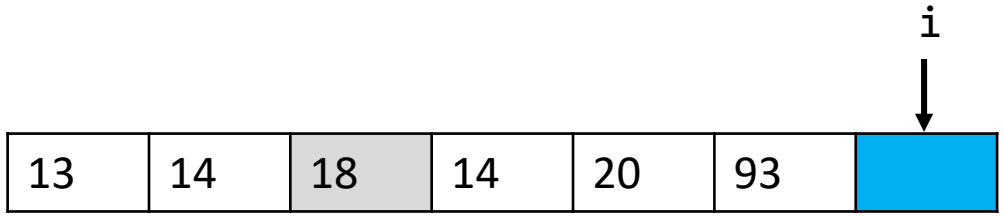
Insérer 15



Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
```

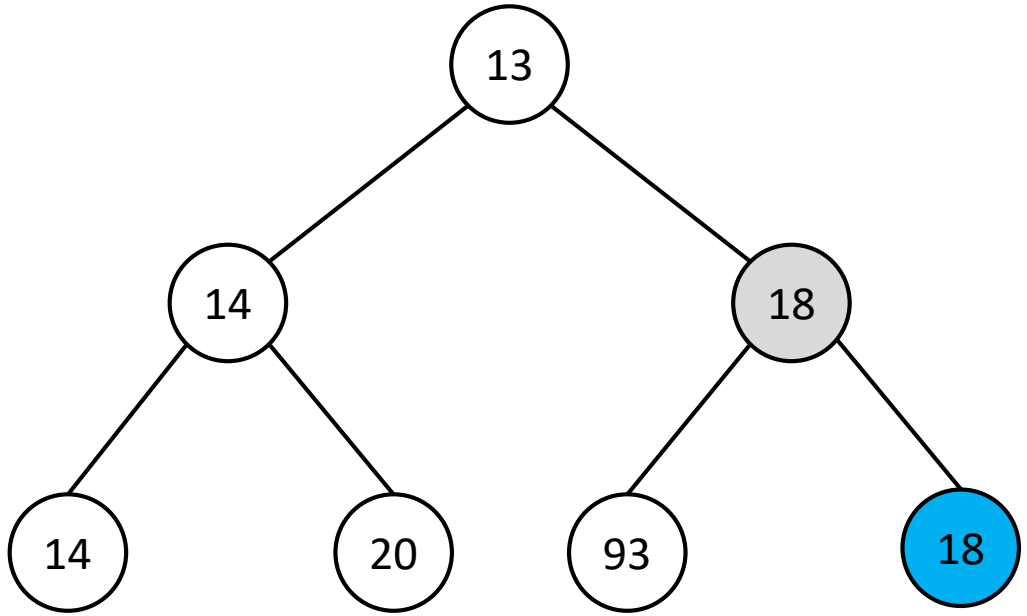




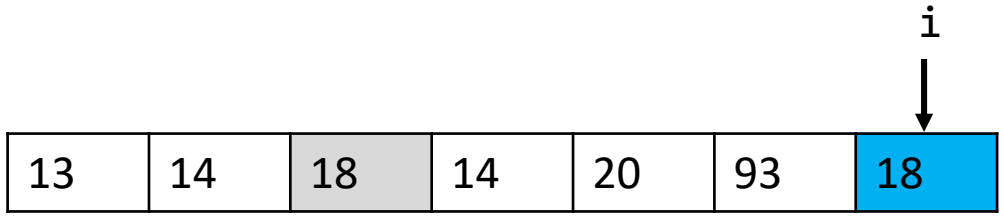
Insérer 15

Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
        A[i] ← A[Pere[i]]
```

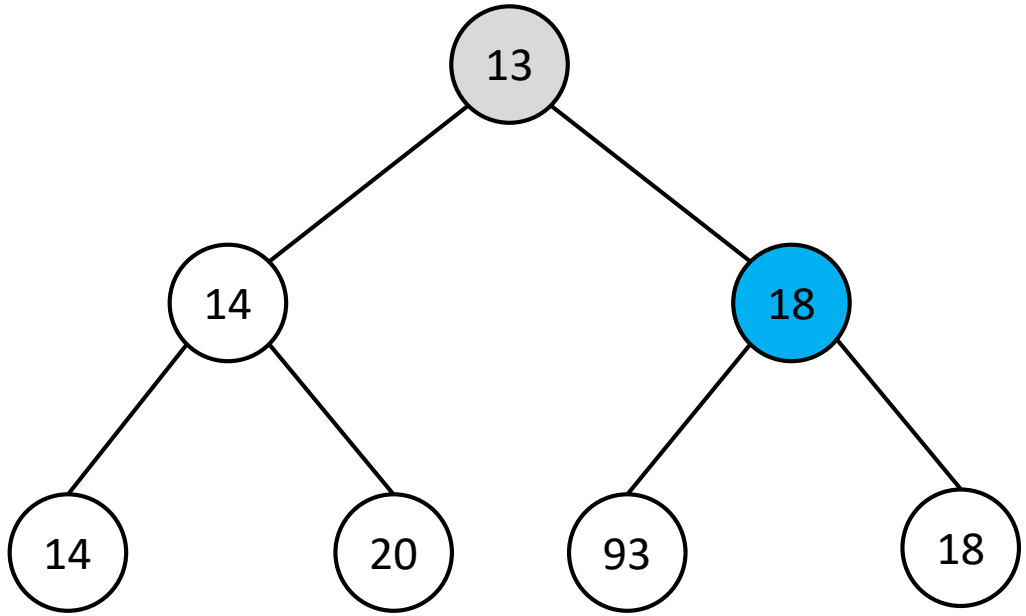


Insérer 15

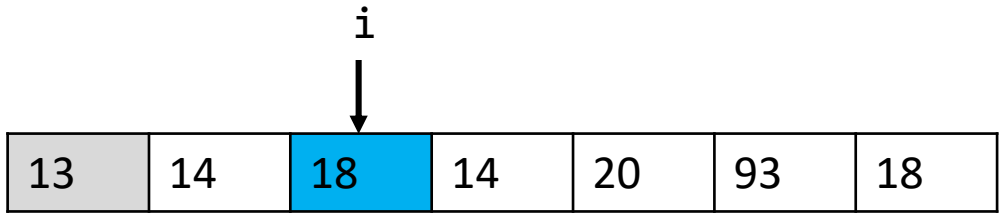


Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
        A[i] ← A[Pere[i]]
        i ← Pere[i]
```

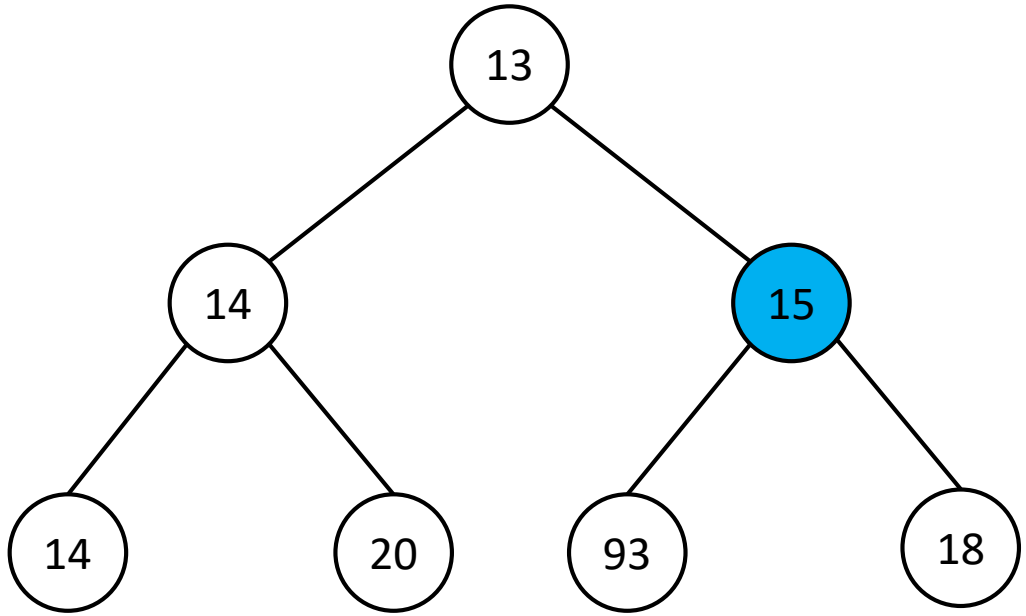


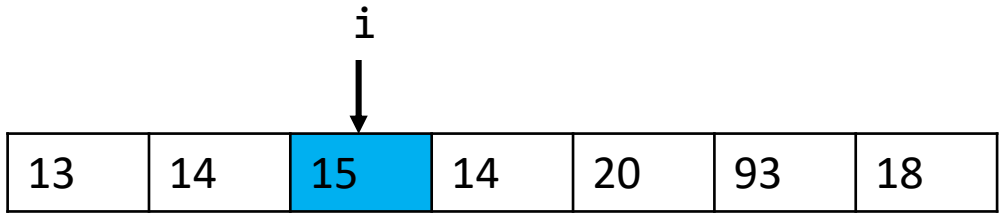
Insérer 15



Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
        A[i] ← A[Pere[i]]
        i ← Pere[i]
    A[i] ← cle
```

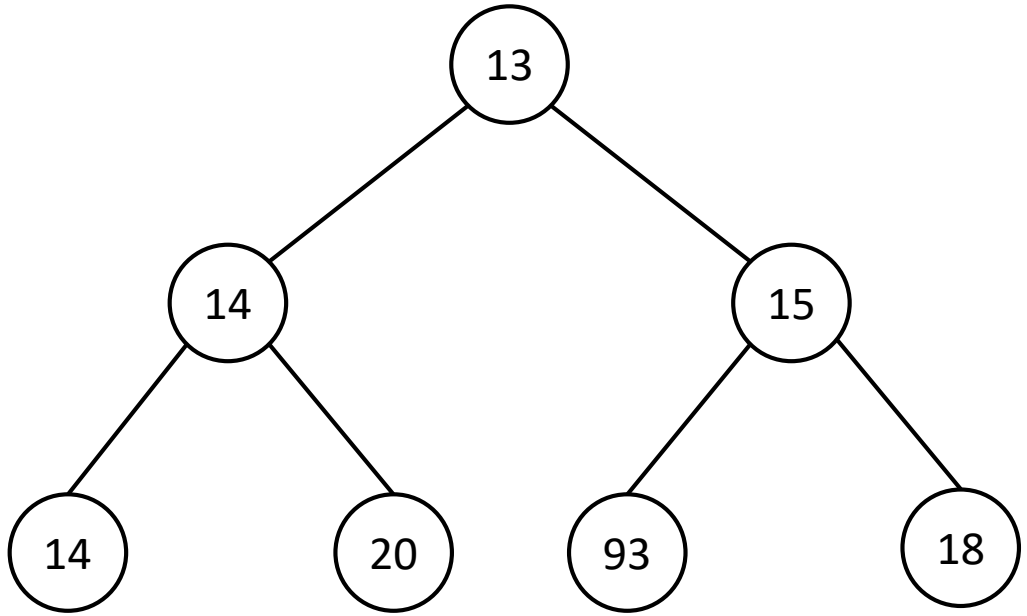




Insérer 15

Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
        A[i] ← A[Pere[i]]
        i ← Pere[i]
    A[i] ← cle
```



Insérer 15

13	14	15	14	20	93	18

Question 3

```
inserer(A: Tas , cle: valeur)
    Taille[A] ← Taille[A]+1
    i ← Taille[A]
    TantQue i > 1 et A[Pere[i]]> cle
        A[i] ← A[Pere[i]]
        i ← Pere[i]
    A[i] ← cle
```

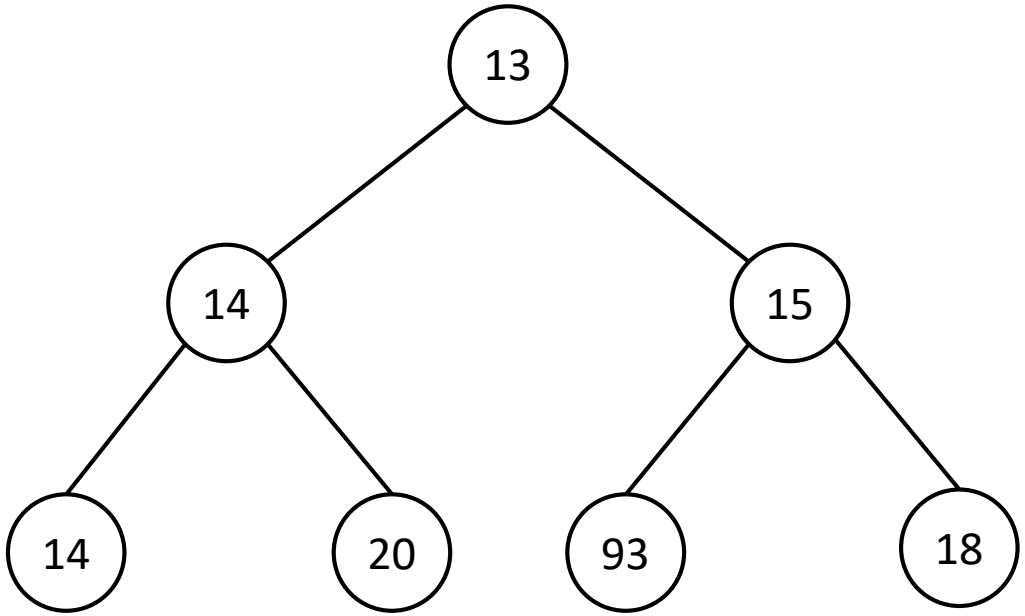
Complexité: O(h)

Question 4

- **Plus petit élément :** à la racine du tas.
- **Plus grand élément :** au niveau d’une feuille.

Question 5

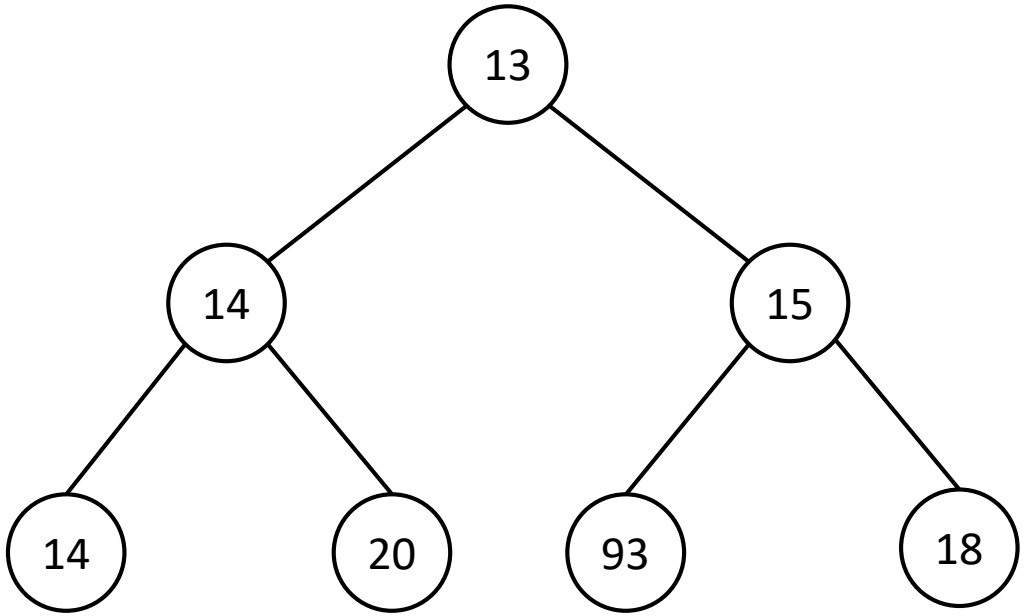
```
supprimer_min(A:Tas)
```



13	14	15	14	20	93	18

Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
```

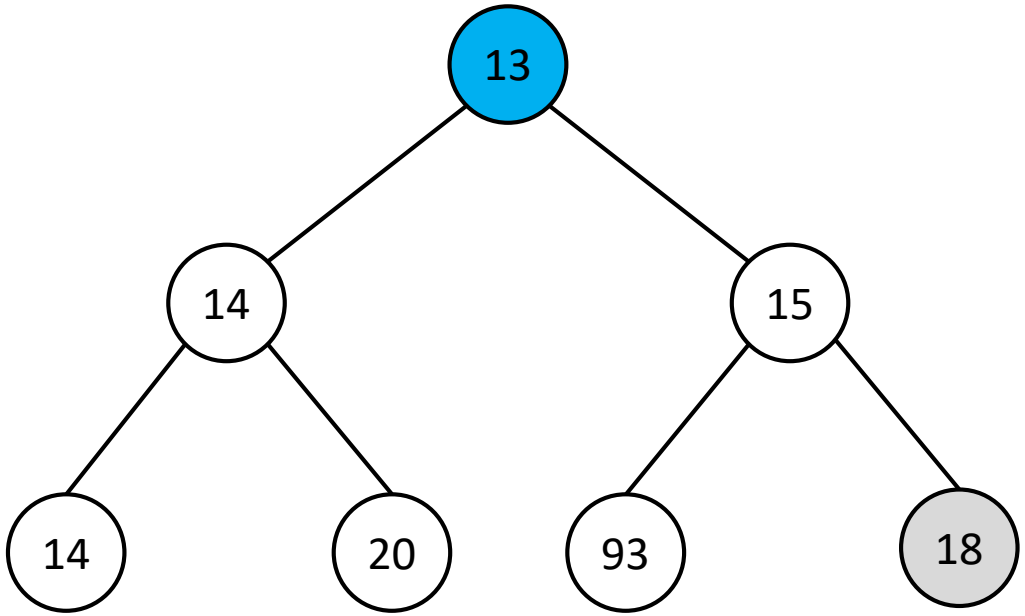


min ← 13

13	14	15	14	20	93	18

Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
    A[1] ← A[Taille[A]]
```

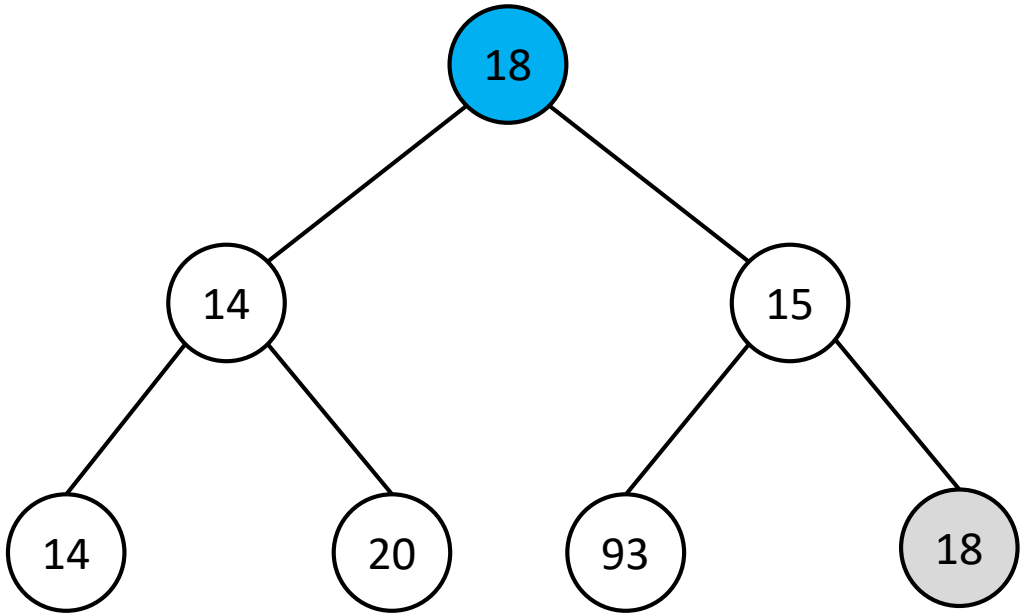


min ← 13

13	14	15	14	20	93	18

Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
    A[1] ← A[Taille[A]]
```

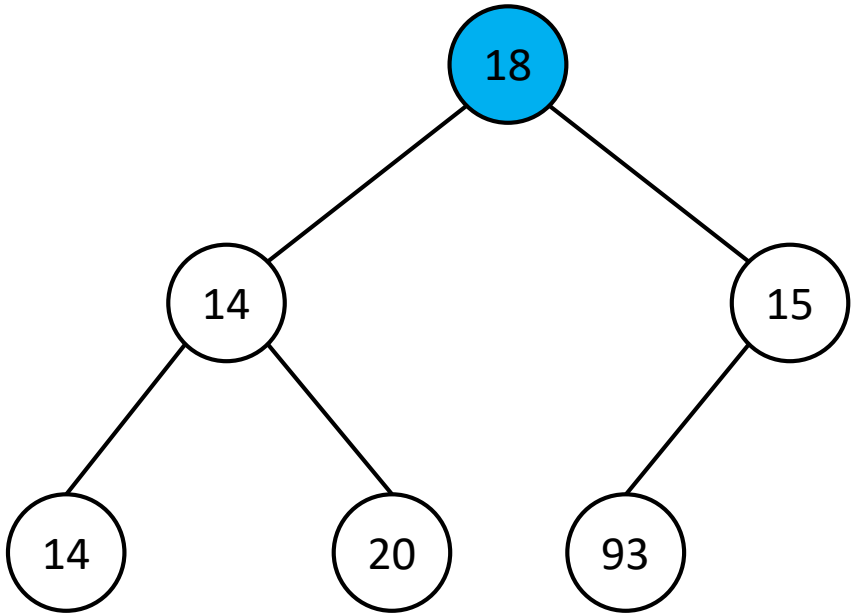


min ← 13

18	14	15	14	20	93	18

Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
    A[1] ← A[Taille[A]]
    Taille[A] ← Taille[A]-1
```

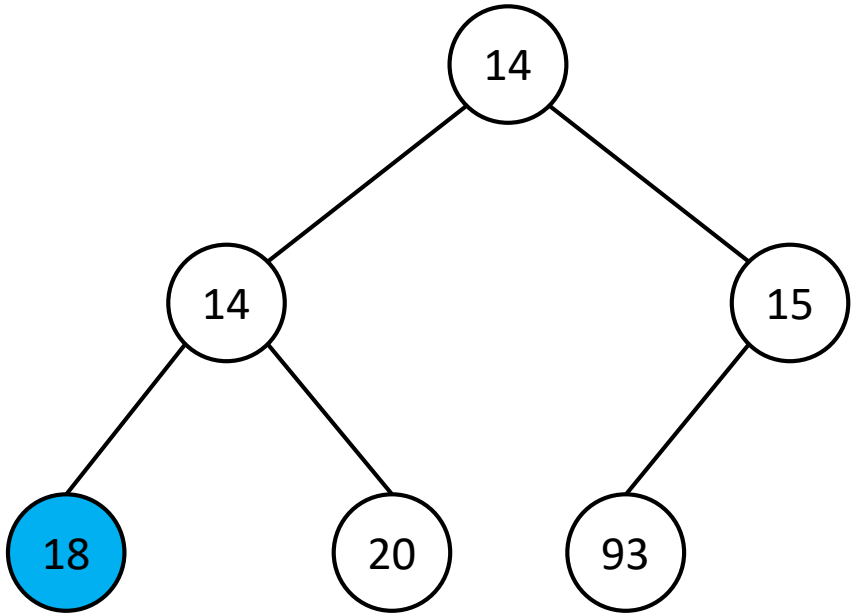


min ← 13

18	14	15	14	20	93

Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
    A[1] ← A[Taille[A]]
    Taille[A] ← Taille[A]-1
    Entasser (A, 1)
```



min ← 13

14	14	15	18	20	93

Question 5

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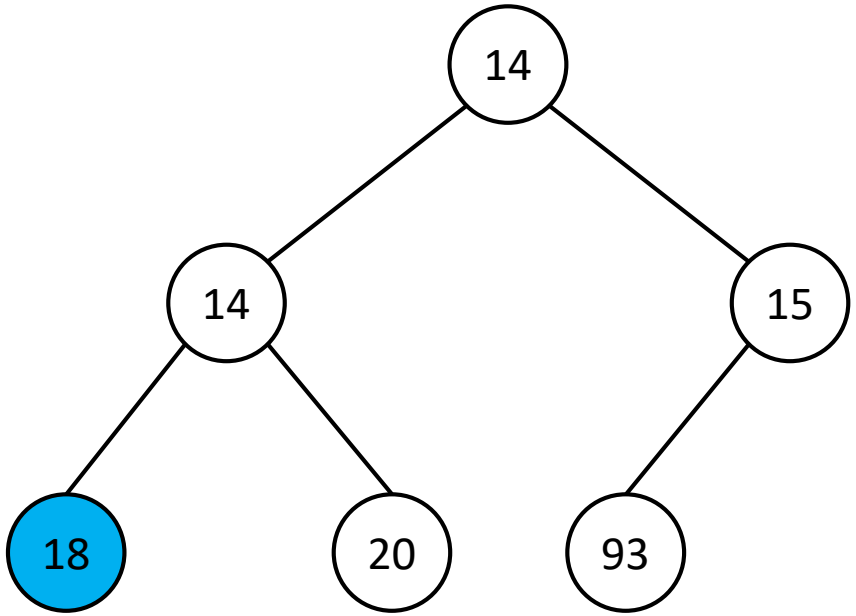
    min ← A[1]
    A[1] ← A[Taille[A]]
    Taille[A] ← Taille[A]-1
    Entasser (A, 1)
    retourner min
```



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    Entasser (A, 1)
    retourner min
```

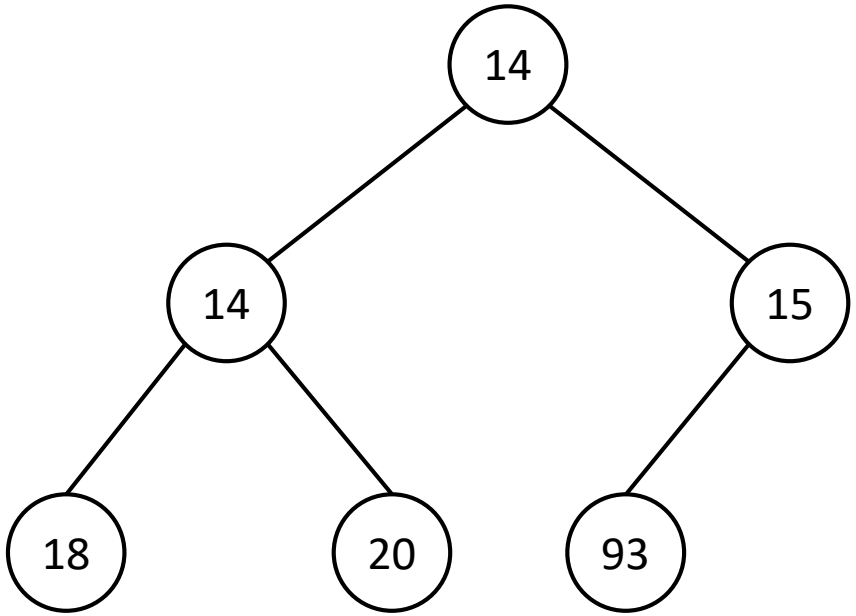

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Question 5

```
supprimer_min(A:Tas)
    min ← A[1]
    A[1] ← A[Taille[A]]
    Taille[A] ← Taille[A]-1
    Entasser (A, 1)
    retourner min
```

Complexité: O(h)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 7, 12, 8, 7, 9, 10, 10)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 7, 12, 8, 7, 9, 10, 10, 2)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 7, 12, 8, 7, 9, 10, 10, 2)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10, 7)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10, 7)

Question 6

Insérer 2 dans L1 : (1, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10, 7)

Question 6

Supprimer le min dans L1 : (1, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10, 7)

Question 6

Supprimer le min dans L1 : (1, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10, 7)

Question 6

Supprimer le min dans L1 : (7, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (7, 3, 1, 6, 9, 2, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (1, 3, 7, 6, 9, 2, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (1, 3, 7, 6, 9, 2, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (1, 3, 2, 6, 9, 7, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (1, 3, 2, 6, 9, 7, 12, 8, 7, 9, 10, 10)

Question 6

Supprimer le min dans L1 : (1, 3, 2, 6, 9, 7, 12, 8, 7, 9, 10, 10)

Question 7

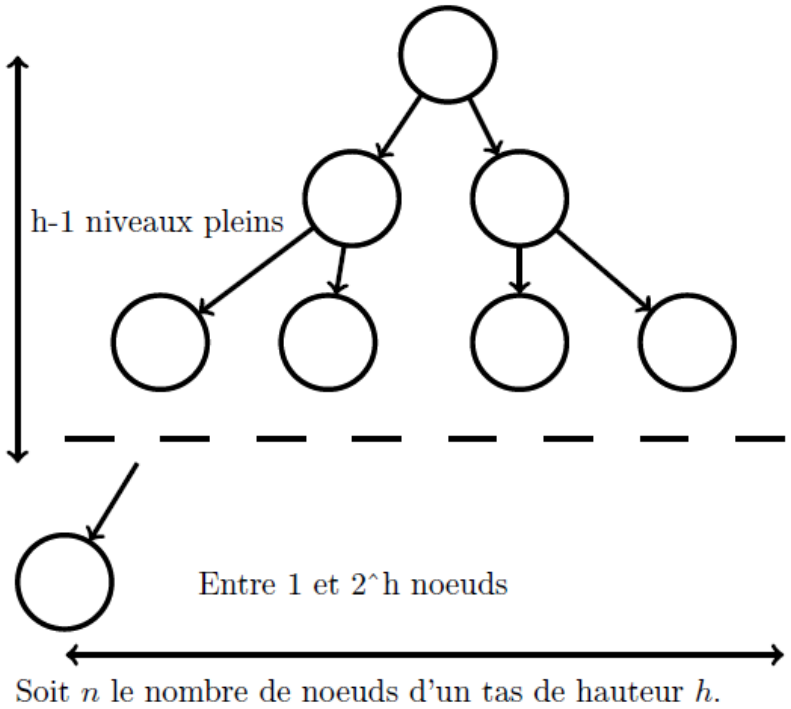
- Une liste triée est toujours un tas.

  ➔ Dans une liste triée : $A[i] \geq A[\lfloor i/2 \rfloor] \; \forall \; i > 1$

- Un tas n'est pas toujours une liste triée.

  ➔ Contre-exemple : L1.

Question 8



1

2

2^2

…

2^{h-1}

entre 1 et 2^h

Nombre de nœuds :

$$\sum_{i=0}^{h-1} 2^i + 1 \leq n \leq \sum_{i=0}^{h} 2^i$$

Question 8

$$\sum_{i=0}^{h-1} 2^i + 1 \leq \mathrm{n} \leq \sum_{i=0}^{h} 2^i$$

$$(2^h - 1) + 1 \leq \mathrm{n} \leq 2^{h+1} - 1$$

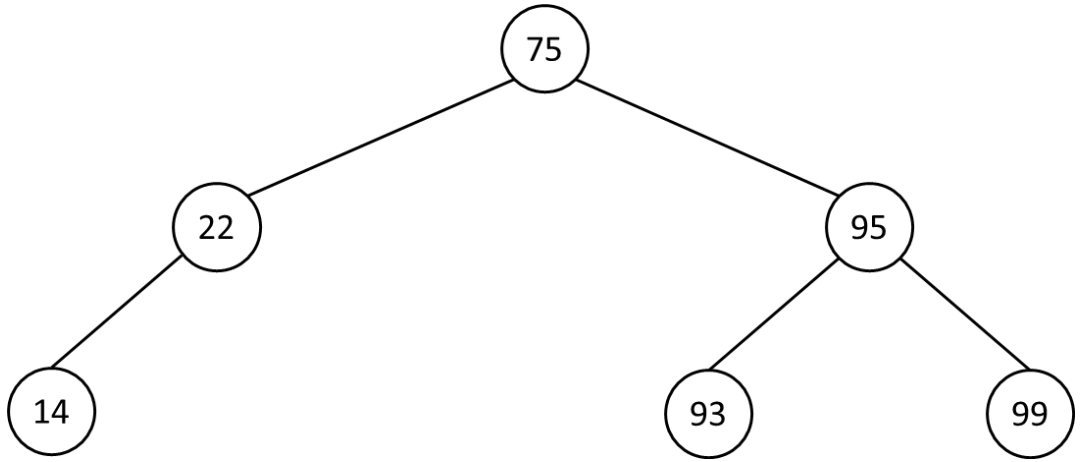
$$2^h \leq \mathrm{n} < 2^{h+1}$$

$$\mathrm{h} \leq log_2(\mathrm{n}) < \mathrm{h} + 1$$

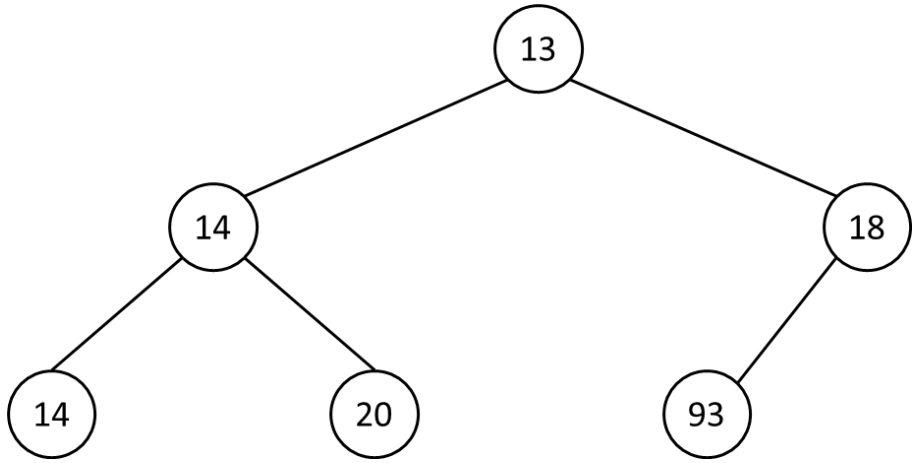
$$h = \lfloor log_2(\mathrm{n}) \rfloor$$

AVL vs TAS

AVL



TAS



Operation	AVL	TAS
Recherche	$\boldsymbol{O(log_2(n))}$	$O(n)$
Insertion	$O(log_2(n))$	$O(log_2(n))$
Suppression	$O(log_2(n))$	$O(log_2(n))$
Recherche MIN/MAX	$O(log_2(n))$	$\boldsymbol{O(1)}$
Insertion (Moyenne)	$O(log_2(n))$	$\boldsymbol{O(1)}$
Création	$O(n.log_2(n))$	$\boldsymbol{O(n)}$