Labs
Optimization for Machine Learning
Spring 2025

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github.com/epfml/OptML_course

Problem Set 3, March 14, 2025 (Projected Gradient Descent)

Projected Gradient Descent

Solve Exercises 23, 24 , 26 from the lecture notes.

Computing Fixed Points

Gradient descent turns up in a surprising number of situations which apriori have nothing to do with optimization. In this exercise we will see how computing the fixed point of functions can be seen as a form of gradient descent. Suppose that we have a 1-Lipschitz continuous function $g : \mathbb{R} \to \mathbb{R}$ such that we want to solve for

$$g(x) = x\,.$$

A simple strategy for finding such a fixed point is to run the following algorithm: starting from an arbitary x_0, we iteratively set

$$x_{t+1} = g(x_t)\,. \tag{1}$$

Practical exercise. We will try solve for x starting from $x_0 = 1$ in the following two equations:

$$x = \log(1 + x)\,, \text{ and} \tag{2}$$

$$x = \log(2 + x)\,. \tag{3}$$

Follow the Python notebook provided here:

colab.research.google.com/github/epfml/OptML_course/blob/master/labs/ex03/template/notebook.ipynb

What difference do you observe in the rate of convergence between the two problems? Let's understand why this occurs.

Theoretical questions.

1. We want to re-write the update (1) as a step of gradient descent. To do this, we need to find a function f such that the gradient descent update is identical to (1):

   $$x_{t+1} = x_t - \gamma f'(x_t) = g(x_t)\,.$$

   Derive such a function f.

2. Give sufficient conditions on g to ensure convergence of procedure (1). What γ would you need to pick? *Hint: We know that gradient descent on f with fixed step-size converges if f is convex and smooth. What does this mean in terms of g?*

3. What condition does g need to satisfy to ensure *linear* convergence? Are these satisfied for problems (2) and (3) in the exercise?