Procesamiento digital de señales

Análisis espectral

Integrantes:

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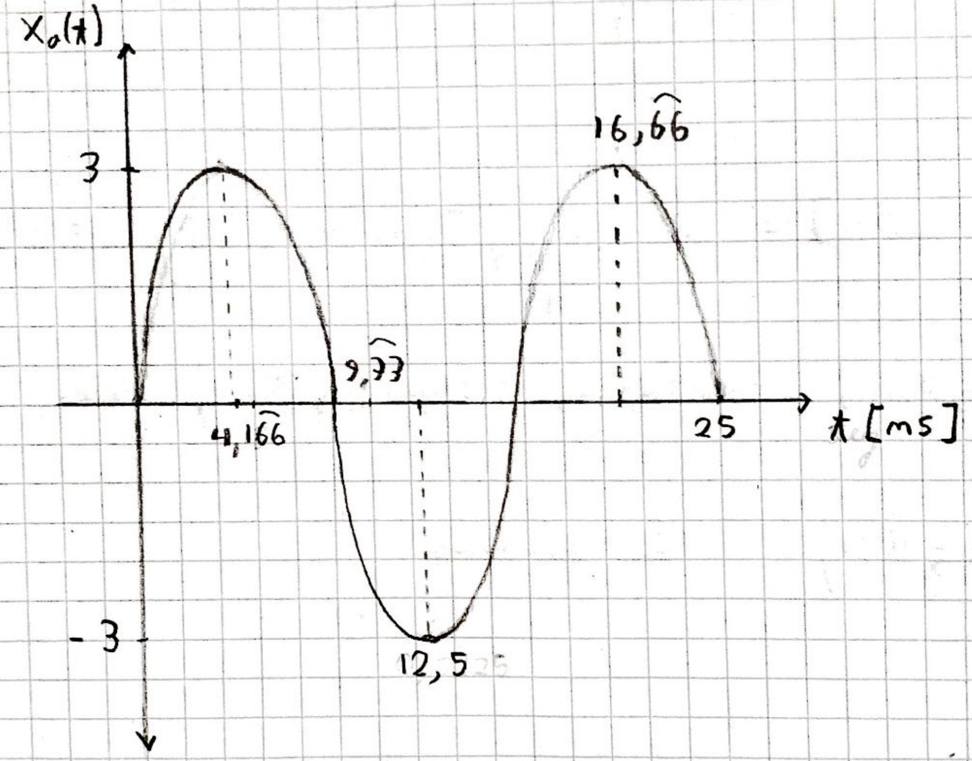
Ejercicio 1

$$x_a(t) = 3 \operatorname{sen}(120\pi t)$$

a)



b) $x[n] = x_c(t)\Big|_{t=nT}$ $\qquad T = \frac{1}{360} \frac{seg}{muestra}$

$x[n] = 3 \cdot sen\left(\frac{1}{3}\pi \cdot n\right)$

$\frac{1}{3}\pi N = 2\pi$

$\omega = \frac{1}{3}\pi \longrightarrow f = \frac{\omega}{2\pi} = \frac{1}{6}\ Hz$

$\underline{N = 6 \text{ muestras}}$

$x[n + 2\pi k] = 3 \cdot sen\left(\frac{1}{3}\pi \cdot n + 2\pi k\right) =$

$= 3\, sen\left(\frac{1}{3}\pi n\right) \cdot \underbrace{\cos(2\pi k)}_{1} + 3\cos\left(\frac{1}{3}\pi n\right) \cdot \underbrace{sen(2\pi k)}_{0}$

$= 3\, sen\left(\frac{1}{3}\pi n\right)$ $\quad$ Es periodica

i) $N = 6$

$x[0] = x[N] = 0$

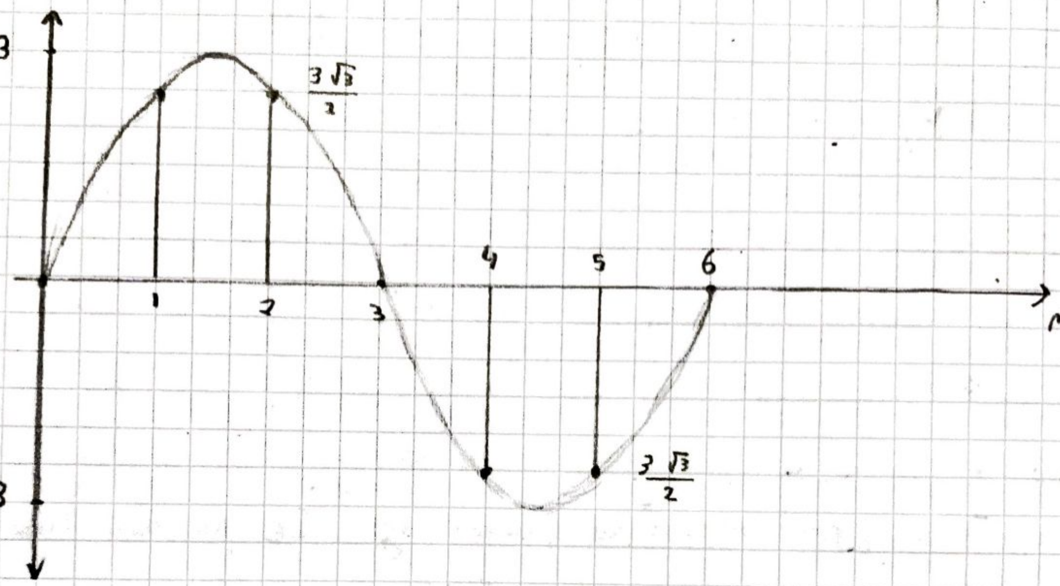
$x[1] = x[N+1] = \frac{3\sqrt{3}}{2}$

$x[2] = x[N+2] = \frac{3\sqrt{3}}{2}$

$x[3] = x[N+3] = 0$

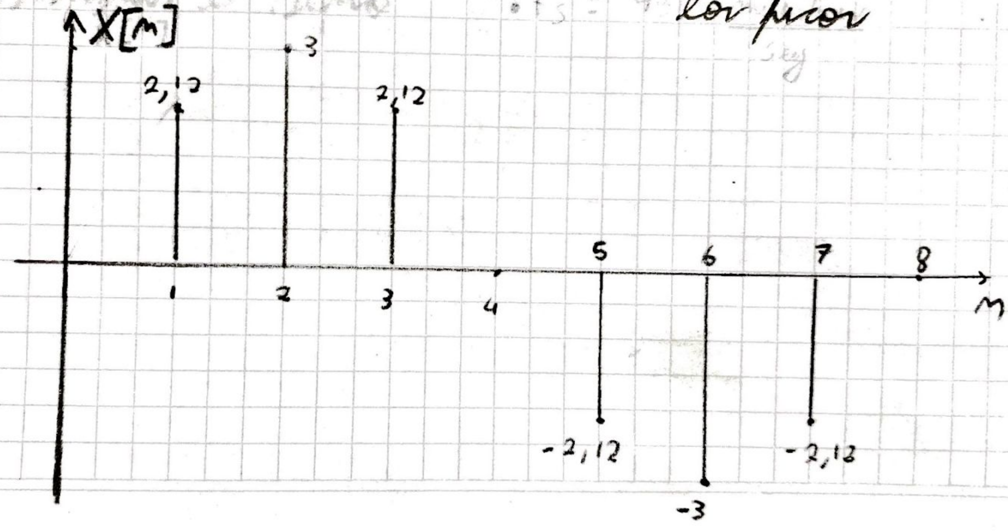
El periodo es $T = 6 \text{ muestras} \cdot \frac{1}{360} \frac{seg}{muestras} = 16{,}6\ mseg$

c)



d)

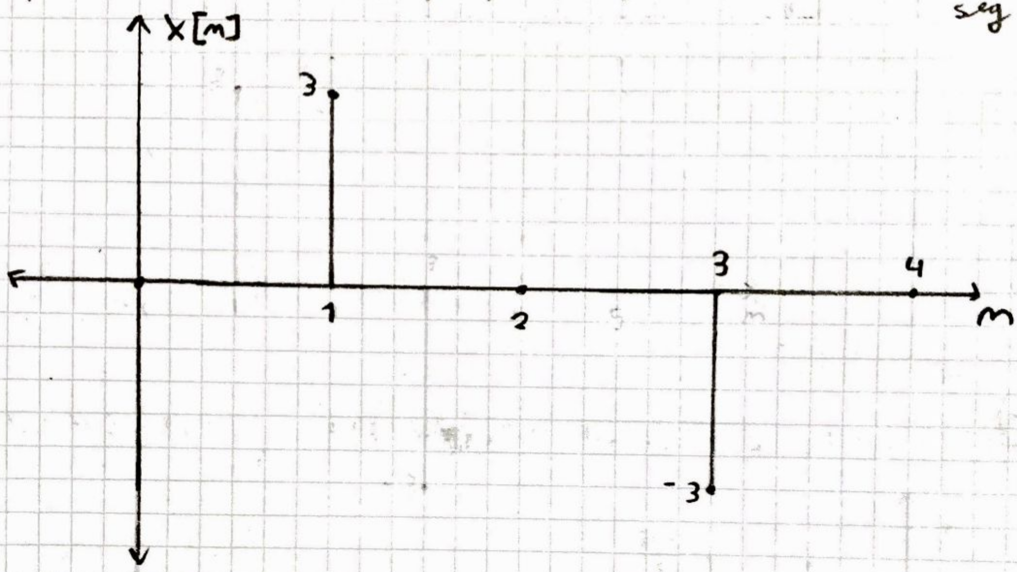
Con $F_s = 480$ muestras sobre seg. Se pueden visualizar los picos



La mínima Fs seria:

Una forma de obtener una frecuencia de muestreo en la cual la señal $x[n]$ alcance los valores pico de $x_a(t)$ es tomando 4 muestras en un periodo. De esta manera, estaremos muestreando solo en los 4 puntos de interes $(3, 0, -3, 0)$

• $F_s = 240 \frac{\text{muestras}}{\text{seg}}$



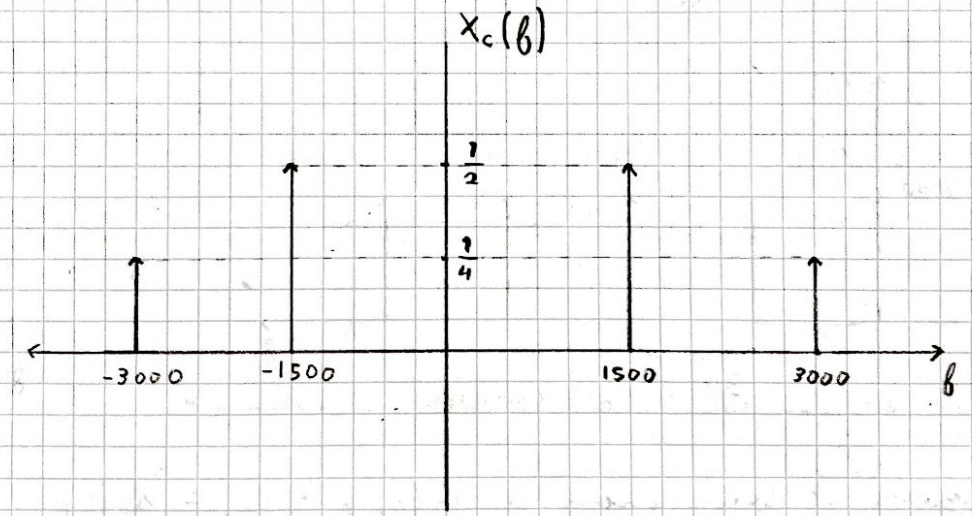
Ejercicio 2

$$x_c(t) = \cos(2\pi.\overbrace{1500}^{f_0}.t) + 0,5\cos(2\pi.\overbrace{3000}^{f_1}.t)$$

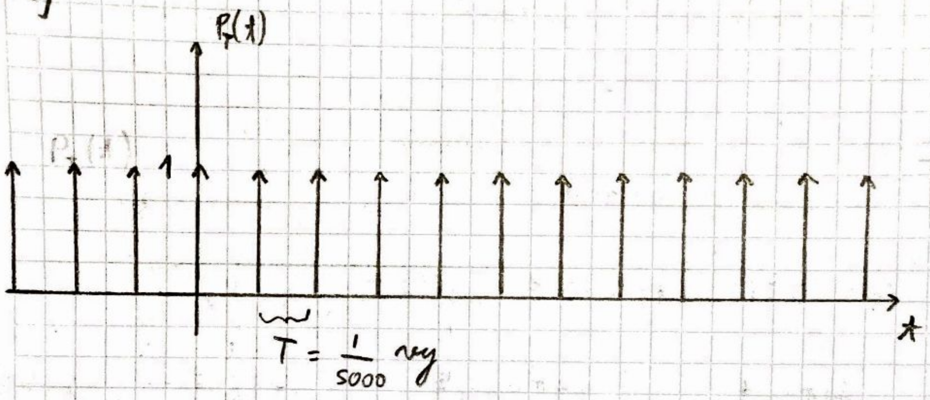
$$x_c(t) = \frac{1}{2}e^{j2\pi f_0 t} + \frac{1}{2}e^{j2\pi(-f_0)t} + \frac{1}{4}e^{j2\pi f_1 t} + \frac{1}{4}e^{j2\pi(-f_1)t}$$

$$\mathcal{F}\{x_c(t)\} = \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0) + \frac{1}{4}\delta(f - f_1) + \frac{1}{4}\delta(f + f_1)$$

$$X_c(f) = \frac{1}{2}\delta(f - 1500) + \frac{1}{2}\delta(f + 1500) + \frac{1}{4}\delta(f - 3000) + \frac{1}{4}\delta(f + 3000)$$

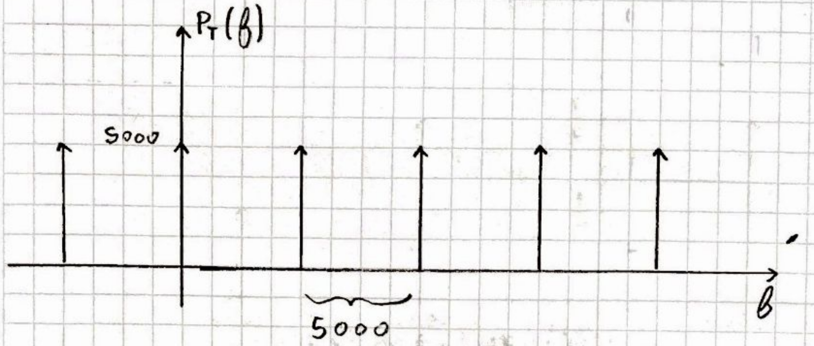


b)



$$P_T(t) = \sum_{m=-\infty}^{\infty} \delta\left(t - \frac{m}{5000}\right)$$

$$F\left\{P_T(t)\right\} = 5000 \cdot \sum_{k=-\infty}^{\infty} \delta(f - k5000)$$



Realizamos la convolucion de $P_T(f) * X_c(f) = X_s(f)$

$$X_s(f) = X_c(f) * \frac{1}{T} \sum_{-\infty}^{\infty} \delta(f - kF_s)$$

$$= \frac{1}{T} \sum_{-\infty}^{\infty} X_c(f - kF_s) = \frac{1}{T} \sum_{-\infty}^{\infty} X_c(f - kF_s)$$

$$X_s(f) = 5000 \sum_{k} X_c(f - kF_s)$$

$$X_s(f) = 5000 \cdot \sum_{k=-\infty}^{\infty} \left[\frac{1}{2} \left(\delta(f + 1500 - 5000k) + \delta(f - 1500 - 5000k) \right) + \frac{1}{4} \left(\delta(f + 3000 - 5000k) \right. \right.$$

$$\left. \left. + \delta(f - 3000 - 5000k) \right) \right]$$

$$X(e^{jw}) = X(f) \Big|_{f = \frac{w F_s}{2\pi}}$$

$$X_s(e^{jw}) = 5000 \cdot \sum_{k=-\infty}^{\infty} \frac{1}{2} \left(\delta\left(\frac{F_s}{2\pi} \left(w + \frac{3}{5}\pi - 2\pi k \right) \right) + \delta\left(\frac{F_s}{2\pi} \cdot \left(w - \frac{3}{5}\pi - 2\pi k \right) \right) \right) +$$

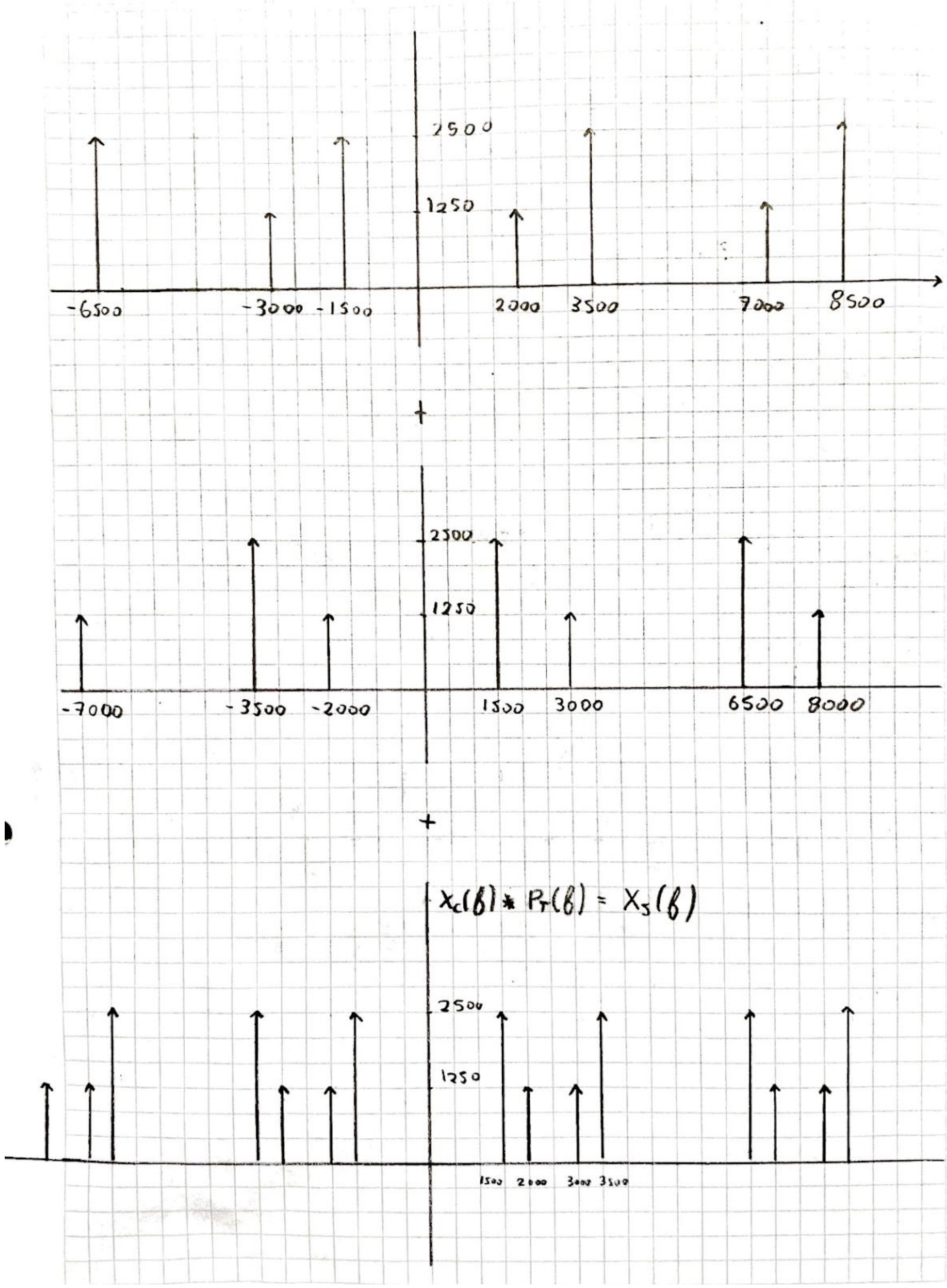
$$+ \frac{1}{4} \left(\delta\left(\frac{F_s}{2\pi} \left(w + \frac{6}{5}\pi - 2\pi k \right) \right) + \delta\left(\frac{F_s}{2\pi} \left(w - \frac{6}{5}\pi - 2\pi k \right) \right) \right)$$

Por propiedad de escalado del impulso $\delta(aw) = \frac{1}{|a|} \cdot \delta(w)$

$$X_s(e^{jw}) = \sum_{k=-\infty}^{\infty} \left[\pi \left(\delta\left(w + \frac{3}{5}\pi - 2\pi k \right) + \delta\left(w - \frac{3}{5}\pi - 2\pi k \right) \right) + \right.$$

$$\left. + \frac{1}{2} \pi \left(\delta\left(w + \frac{6}{5}\pi - 2\pi k \right) + \delta\left(w - \frac{6}{5}\pi - 2\pi k \right) \right) \right]$$

De forma gráfica sería



c)

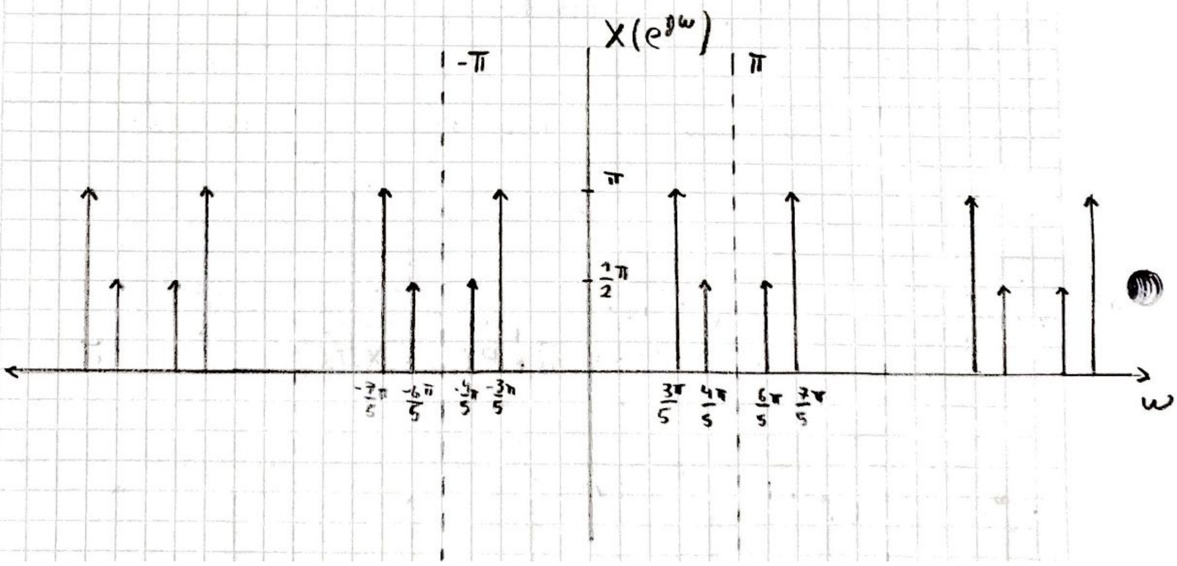
$$X(e^{j\omega}) = X(f)\Big|_{f=\frac{\omega F_s}{2\pi}}$$

$$\omega_0 = \frac{2\pi f}{F_s} = \frac{3}{5}\pi$$

$$\omega_1 = \frac{4\pi}{5}$$

$$\omega_2 = \frac{6\pi}{5}$$

$$\omega_3 = \frac{7\pi}{5}$$



d)

$$h[n] = 0{,}5\,\delta[n-10]$$

$$H(e^{jw}) = 0{,}5 \cdot e^{-j10w}$$

e) $Y(e^{jw}) = H(e^{jw}) \cdot X(e^{jw})$

Por propiedad de convolución.

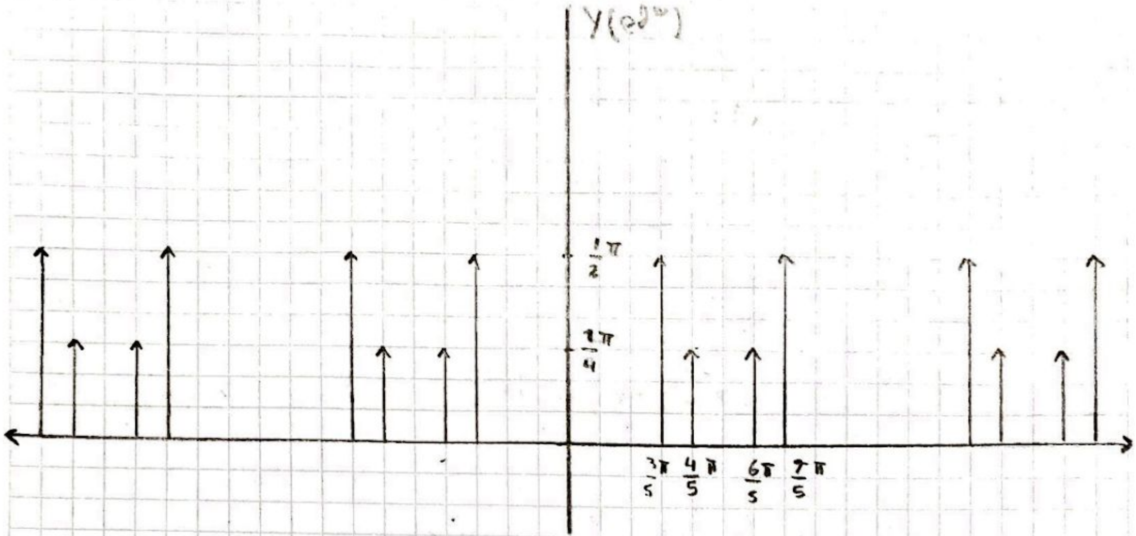
$$Y(e^{jw}) = \frac{1}{2}\sum_{-\infty}^{\infty} \pi\left(e^{j10\frac{3\pi}{5}}\,\delta\left(w+\frac{3}{5}\pi-2\pi k\right) + e^{-j10\frac{3\pi}{5}}\,\delta\left(w-\frac{3}{5}\pi-2\pi k\right)\right)$$

$$+\ \frac{1}{2}\pi\left(e^{j10\frac{6\pi}{5}}\,\delta\left(w+\frac{6}{5}\pi-2\pi k\right) + e^{-j10\frac{6\pi}{5}}\cdot\delta\left(w-\frac{6}{5}\pi-2\pi k\right)\right)$$

como $e^{j10\frac{3\pi}{5}}$, $e^{-j10\frac{3\pi}{5}}$, $e^{j10\cdot\frac{6\pi}{5}}$ y $e^{-j10\frac{6\pi}{5}}$ son reales y 1

por lo que el gráfico de $Y(e^{jw})$ no va a ser igual solo que reducido a la mitad

En este caso el retardador $h[n]$ esta retardando a los cosenos una cantidad entera de periodos, por lo que a la salida los cosenos tienen la misma fase



$$Y_s(f) = Y(e^{jw})\Big|_{w = \frac{2\pi f}{F_s}}$$

$$Y_s(f) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \pi\left(e^{j6\pi}\delta\left(\frac{2\pi}{F_s}\left(f+1500-F_s k\right)\right) + e^{-j6\pi}\delta\left(\frac{2\pi}{F_s}\left(f-1500-F_s k\right)\right)\right)$$

$$+ \frac{1}{2}\pi\left(e^{j12\pi}\cdot\delta\left(\frac{2\pi}{F_s}\left(f+3000-F_s k\right)\right) + e^{-j12\pi}\cdot\delta\left(\frac{2\pi}{F_s}\left(f-3000-F_s k\right)\right)\right)$$

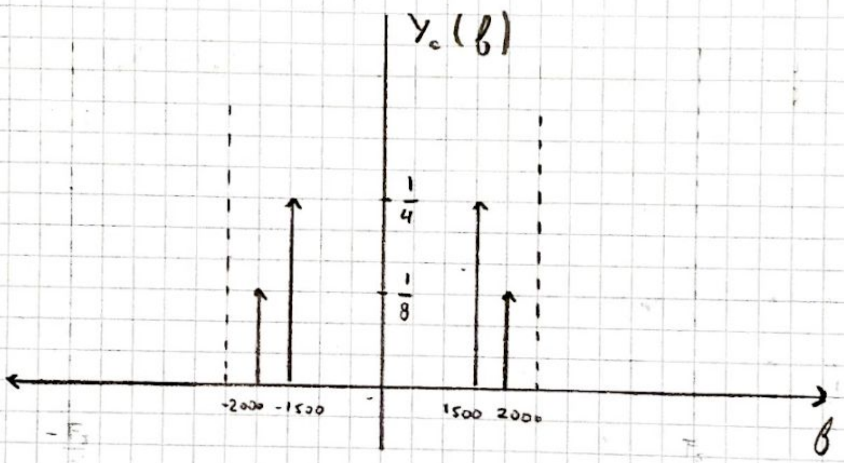
$$Y_s(f) = F_s\sum_{K=-\infty}^{\infty}\frac{1}{4}\left(e^{j6\pi}\delta\left(f+1500-F_s K\right) + e^{-j6\pi}\delta\left(f-1500-F_s k\right)\right)$$

$$+ \frac{1}{8}\left(e^{j12\pi}\delta\left(f+3000-F_s k\right) + e^{-j12\pi}\cdot\delta\left(\frac{2\pi}{F_s}\left(f-3000-F_s k\right)\right)\right)$$

$$Y_c(f) = H_r(f) \cdot Y_s(f)$$

$$H_r(f) = \begin{cases} T & |f| < \frac{F_s}{2} \\ 0 & \text{caso contrario} \end{cases}$$

$$Y_c(f) = \begin{cases} T\,Y_s(f) & |f| < \frac{F_s}{2} \\ 0 & \text{caso contrario} \end{cases}$$



$$Y_c(t) = \frac{1}{2}\cos(2\pi \cdot 1500 \cdot t) + \frac{1}{4}\cos(2\pi \cdot 2000 \cdot t)$$

En este caso, al no cumplirse el teorema de Nyquist osea que la frec de muestreo es menor al doble de la maxima frecuencia de $x_c(t)$, por lo que hay aliasing y esto me genera un corrimiento de frecuencia en el coseno $(2\pi\, 3000\, t)$.

Ejercicio 3

a)

La salida $y[n]$ desplazada n_0 muestras

$y[0-n_0] = (x[-n_0])^2$

$y[1-n_0] = (x[1-n_0])^2$

$y[2-n_0] = (x[2-n_0])^2$

$y[3-n_0] = (x[3-n_0])^2$

$y[n_0-n_0] = (x[0])^2$

La salida $y_1[n]$ ante una entrada $x_1[n] = x[n-n_0]$

$y_1[0] = (x[-n_0])^2$

$y_1[1] = (x[1-n_0])^2$

$y_1[2] = (x[2-n_0])^2$

$y_1[3] = (x[3-n_0])^2$

$y_1[n_0] = (x[0])^2$

Es invariante en el tiempo ya que $y[n-n_0] = S\{x[n$

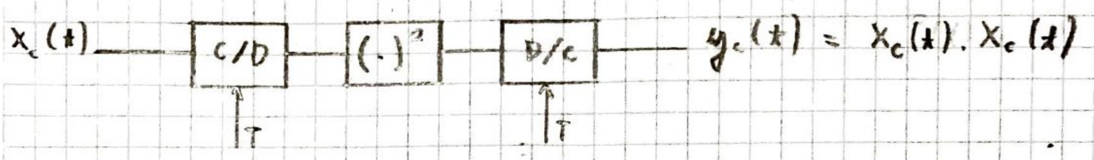
$$S\{x[n]\} = x^2[n]$$

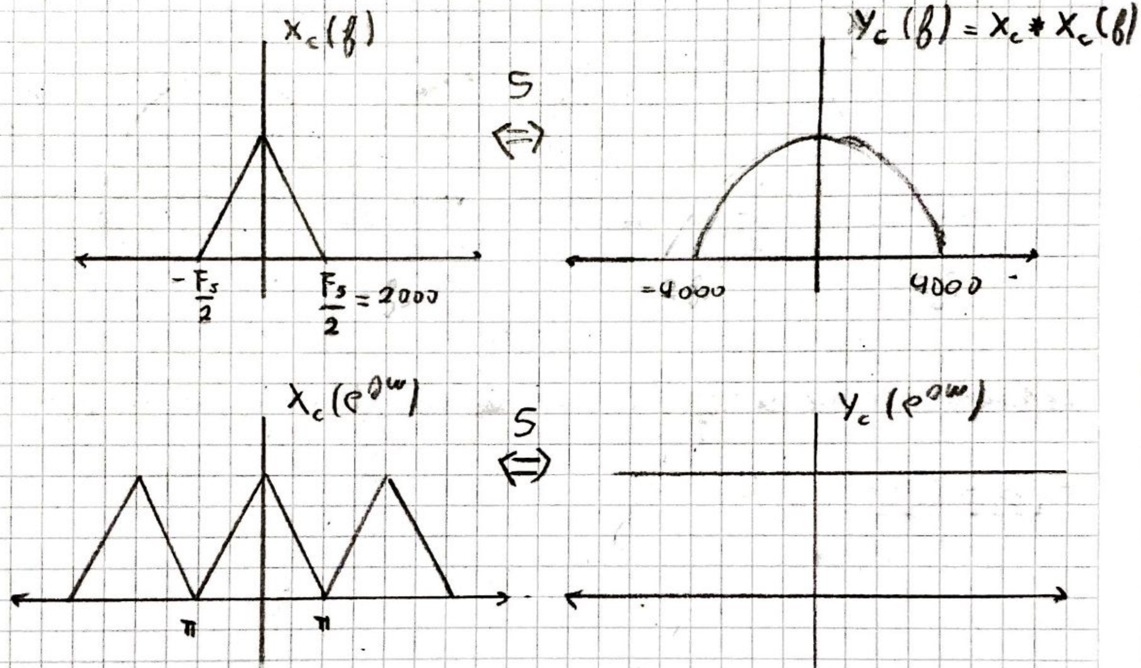
$$S\{a x_1[n] + b x_2[n]\} = a S\{x_1[n]\} + b S\{x_2[n]\}$$

$$(a x_1[n] + b x_2[n])^2 = (a x_1[n])^2 + ab\, x_1[n] \cdot x_2[n] + (b x_2[n])^2$$

<u>No</u> es un sistema lineal ya que no cumple el principio de superposicion

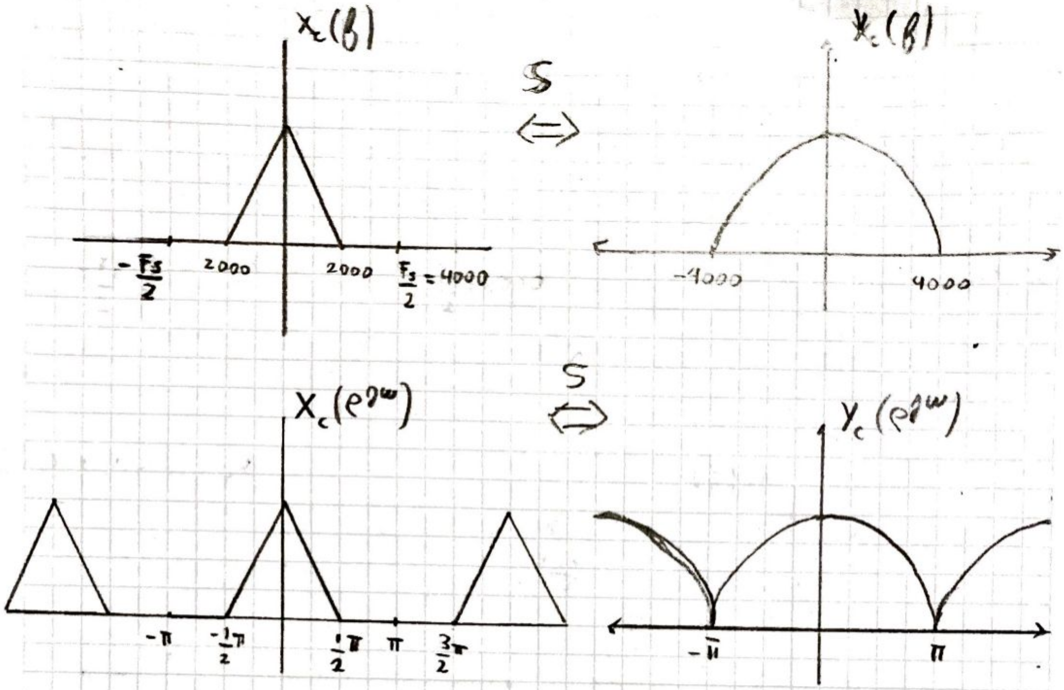
3 - b)





Podemos ver que si $\frac{Fs}{2} = 2000$ hay aliasing por lo que se deberá aumentar la F_s

Para que la convolución periódica se realice correctamente se debe tener un espacio entre cada repetición periódica del ancho de la función



Podemos ver que si $\frac{F_s}{2} = 4000$ No hay Aliasing por lo que

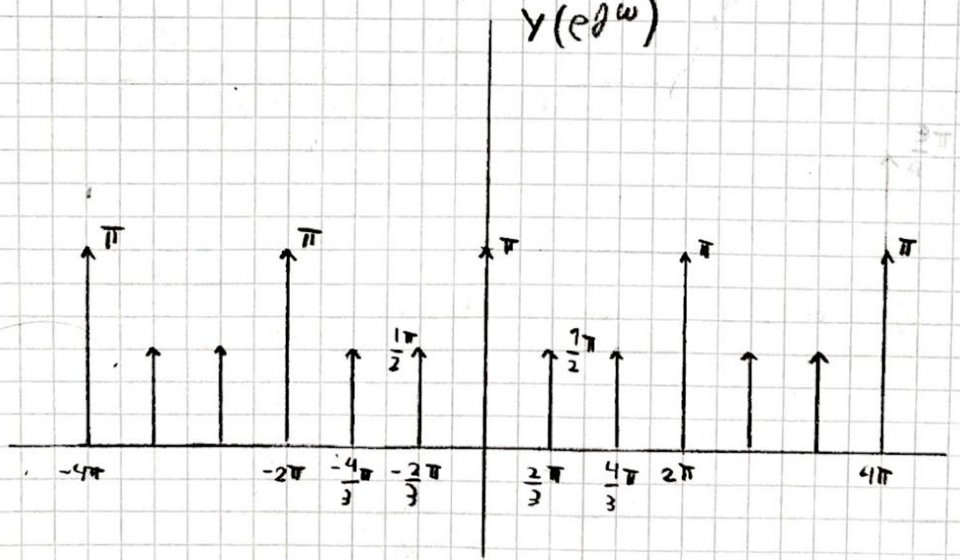
$\underline{F_s \geq 8000}$

ii)

$$Y(e^{j\omega}) = \sum_{-\infty}^{\infty} \cos^2\left(2\pi \frac{1}{6} n\right) \cdot e^{-j\omega n}$$

$$Y(e^{j\omega}) = \sum_{-\infty}^{\infty} \left(\frac{1}{2} + \frac{\cos\left(4\pi \cdot \frac{1}{6} n\right)}{2}\right) \cdot e^{-j\omega n}$$

$$Y(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \pi \cdot \delta(\omega - 2\pi k) + \sum_{k=-\infty}^{\infty}\left(\frac{\pi}{2}\, \delta\left(\omega + \frac{2}{3}\pi - 2\pi k\right) + \frac{\pi}{2}\, \delta\left(\omega - \frac{2}{3}\pi - 2\pi k\right)\right)$$

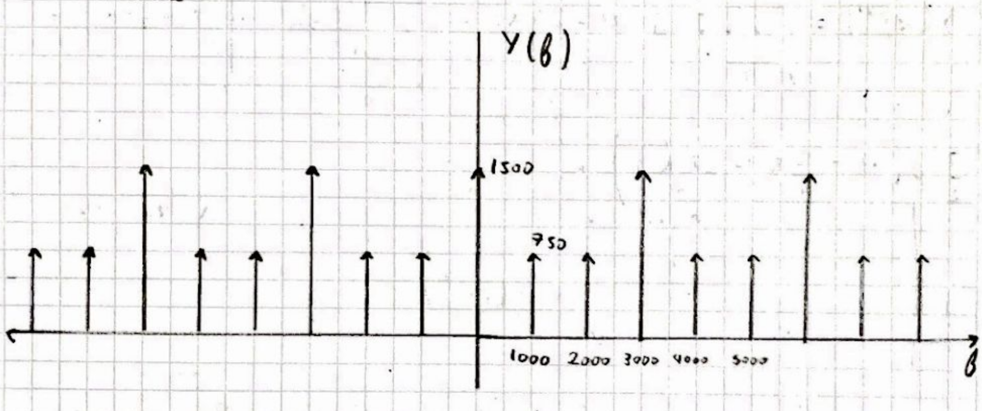


iii)

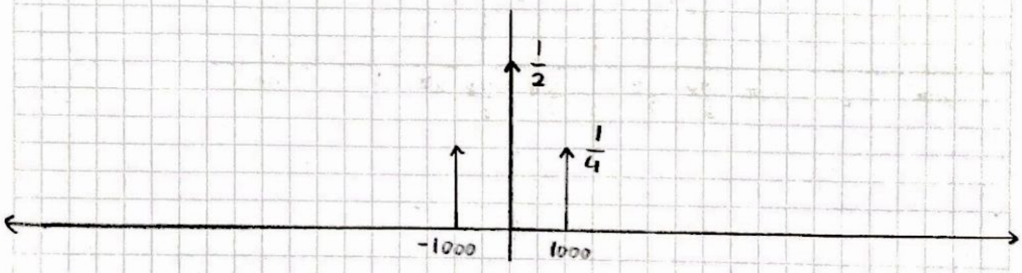
$$Y(f) = Y(e^{j\omega})\Big|_{\omega = \frac{2\pi f}{F_s}}$$

$$Y(f) = \sum_{k=-\infty}^{\infty} \pi \cdot \delta\left(\frac{2\pi}{F_s}(f - kF_s)\right) + \frac{\pi}{2}\delta\left(\frac{2\pi}{F_s}(f+1000-kF_s)\right) + \frac{\pi}{2}\delta\left(\frac{2\pi}{F_s}(f-1000-kF_s)\right)$$

$$Y(f) = \sum_{k=-\infty}^{\infty} \frac{1}{2} \cdot F_s\, \delta(f-kF_s) + \frac{1}{4}F_s\, \delta(f+1000-kF_s) + \frac{1}{4}F_s\,\delta(f-1000-kF_s)$$



$Y_c(f) = Y(f) \cdot H(f)$ sendo $H(f) = \begin{cases} T & |f| < \frac{F_s}{2} \\ 0 & cc \end{cases}$



$$y_c(t) = \frac{1}{2} + \frac{1}{2}\cos(2\pi 1000\, t)$$