Procesamiento digital de señales

Aplicaciones de la Transformada Z

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Ejercicio 1

$$h[n] = \left(\frac{1}{4}\right)^n \cos\left(\frac{\pi}{4} n\right) u[n]$$

a)

$$h[n] = \left(\frac{1}{4}\right)^n \cdot \frac{1}{2} \cdot e^{-j\frac{\pi}{4}n} \cdot u[n] + \left(\frac{1}{4}\right)^n \cdot \frac{1}{2} \cdot e^{+j\frac{\pi}{4}n} \cdot u[n]$$

$$H(z) = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{4} \cdot e^{-j\frac{\pi}{4}} \cdot z^{-1}\right)^n + \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{4} \cdot e^{+j\frac{\pi}{4}} z^{-1}\right)^n$$

$$H(z) = \frac{1}{2} \cdot \frac{1}{1 - \frac{e^{-j\frac{\pi}{4}} \cdot z^{-1}}{4}} + \frac{1}{2} \cdot \frac{1}{1 - \frac{e^{j\frac{\pi}{4}} \cdot z^{-1}}{4}} \quad \text{con } \left|\frac{1}{4} e^{j\frac{\pi}{4}} z^{-1}\right| < 1$$

$$\frac{1}{4} < |z|$$

$$H(z) = \frac{1}{2} \left(\frac{1 - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4} + 1 - \frac{e^{-j\frac{\pi}{4}} z^{-1}}{4}}{\left(1 - \frac{e^{-j\frac{\pi}{4}} z^{-1}}{4}\right)\left(1 - \frac{e^{j\frac{\pi}{4}} \cdot z^{-1}}{4}\right)} \right)$$

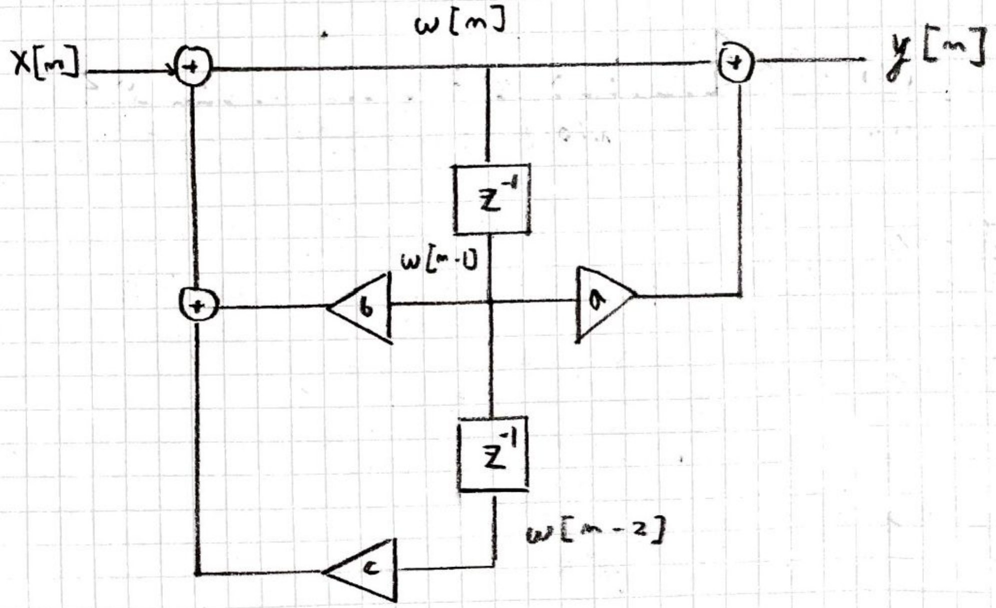
$$H(z) = \frac{\frac{1}{2} + \frac{1}{2} - \left(\frac{e^{j\frac{\pi}{4}} \cdot z^{-1} + e^{-j\frac{\pi}{4}} z^{-1}}{8} \right)}{1 - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4} - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4} + \frac{1}{16} \cdot z^{-2}}$$

$$H(z) = \frac{1 - \frac{1}{4} \cos\left(\frac{\pi}{4}\right) z^{-1}}{1 - \frac{1}{2} \cos\left(\frac{\pi}{4}\right) z^{-1} + \frac{1}{16} z^{-2}} \qquad |z| > \frac{1}{4}$$

b) $$H(z) = \frac{1 - \frac{\sqrt{2}}{8} z^{-1}}{1 - \frac{\sqrt{2}}{4} z^{-1} + \frac{z^{-2}}{16}} = \frac{Y(z)}{X(z)}$$

$$X(z) - X(z)\frac{\sqrt{2}}{8} z^{-1} = Y(z) - Y(z)\frac{\sqrt{2}}{4} z^{-1} + \frac{Y(z)}{16} z^{-2}$$

$$x[m] - \frac{\sqrt{2}}{8} x[m-1] = y[m] - \frac{\sqrt{2}}{4} y[m-1] + \frac{1}{16} y[m-2]$$



Compruebo con las transformadas

$$w[n] = x[n] + b\,w[n-1] + c\,w[n-2]$$

$$x[n] = w[n] - b\,w[n-1] - c\,w[n-2]$$

$$X(z) = W(z) - b\,W(z)\,z^{-1} - c\,W(z)\,z^{-2}$$

$$y[n] = w[n] + a\,w[n-1]$$

$$Y(z) = W(z) + a\,W(z)\,z^{-1}$$

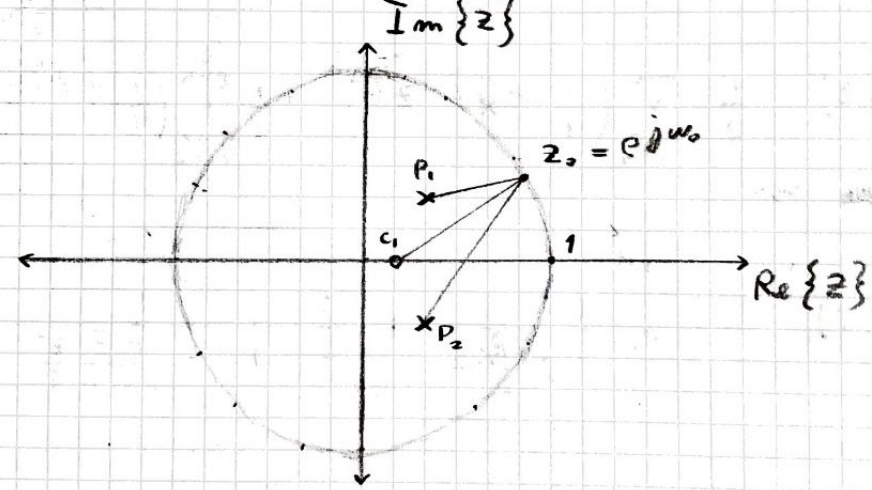
$$H(z) = \frac{Y(z)}{X(z)} = \frac{W(z) + a\,W(z)\,z^{-1}}{W(z) - b\,W(z)\,z^{-1} - c\,W(z)\,z^{-2}} = \frac{1 + a\,z^{-1}}{1 - b\,z^{-1} - c\,z^{-2}}$$

Por lo que podemos obtener los valores de a, b y c

- $a = \frac{\sqrt{2}}{8}$ • $b = \frac{\sqrt{2}}{4}$ y • $c = -\frac{1}{16}$

c) Polos $\left\{ \overset{P_2}{\frac{1}{4} e^{-j\frac{\pi}{4}}} , \overset{P_1}{\frac{1}{4} e^{j\frac{\pi}{4}}} \right\}$

Ceros $\left\{ \overset{c_1}{\frac{\sqrt{2}}{8}} \right\}$



Para $\omega_0 = 0$ $\quad |H(z)| = \dfrac{\left|1 - \frac{\sqrt{2}}{8}\right|}{\left|1 - \frac{1}{4} e^{j\frac{\pi}{4}}\right| . \left|1 - \frac{1}{4} e^{-j\frac{\pi}{4}}\right|} =$

$$\left|H(e^{j\omega_0})\right|_{\omega=0} = \frac{0{,}8232}{(0{,}8419).(0{,}8416)} = \underline{1{,}1612}$$

• Para $w_o = \frac{\pi}{4}$

$$|H(e^{jw_o})|_{w_o=\frac{\pi}{4}} = \frac{|e^{j\frac{\pi}{4}} - \frac{\sqrt{2}}{8}|}{|e^{j\frac{\pi}{4}} - \frac{1}{4}e^{j\frac{\pi}{4}}||e^{j\frac{\pi}{4}} - \frac{1}{4}e^{-j\frac{\pi}{4}}|} = \frac{0,883}{0,773} = \underline{1,1423}$$

• Para $w_o = \frac{\pi}{2}$

$$|H(e^{jw_o})|_{w_o=\frac{\pi}{2}} = \frac{|e^{j\frac{\pi}{2}} - \frac{\sqrt{2}}{8}|}{|e^{j\frac{\pi}{2}} - \frac{1}{4}e^{j\frac{\pi}{4}}||e^{j\frac{\pi}{2}} - \frac{1}{4}e^{-j\frac{\pi}{4}}|} = \frac{1,0155}{1,002} = 1,0135$$

• Para $w_o = \frac{3}{4}\pi$

$$|H(e^{jw_o})|_{w_o=\frac{3\pi}{4}} \frac{|e^{j\frac{3}{4}\pi} - \frac{\sqrt{2}}{8}|}{|e^{j\frac{3\pi}{4}} - \frac{1}{4}e^{j\frac{\pi}{4}}|.|e^{j\frac{3\pi}{4}} - \frac{1}{4}e^{-j\frac{\pi}{4}}|} = \frac{1,1319}{1,2885} = 0,8785$$

• Para $w_o = \pi$

$$|H(e^{jw_o})|_{w_o=\pi} = \frac{|e^{j\pi} - \frac{\sqrt{2}}{8}|}{|e^{j\pi} - \frac{1}{4}e^{j\frac{\pi}{4}}|.|e^{j\pi} - \frac{1}{4}e^{j\frac{\pi}{4}}|} = \frac{1,1768}{1,4161} = 0,831$$

• Para $w_0 = \frac{5}{4}\pi$

$$\left|H(e^{jw_0})\right|_{w_0=\frac{5}{4}\pi} = \frac{\left|e^{j\frac{5}{4}\pi} - \frac{\sqrt{2}}{8}\right|}{\left|e^{j\frac{5}{4}\pi} - \frac{1}{4}e^{j\frac{1}{4}}\right| \cdot \left|e^{j\frac{5}{4}\pi} - \frac{1}{4}e^{-j\frac{1}{4}}\right|} = \frac{1,1319}{1,2885} = 0,8785$$

• Para $w_0 = \frac{3}{2}\pi$

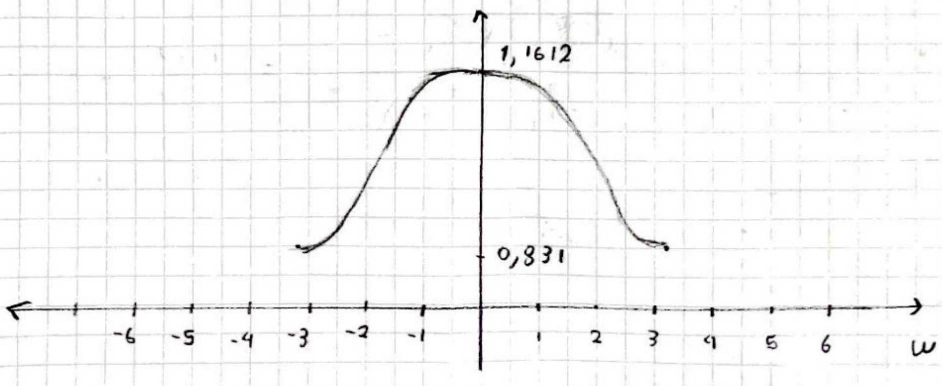
$$\left|H(e^{jw_0})\right|_{w_0=\frac{3}{2}\pi} = \frac{\left|e^{j\frac{3}{2}\pi} - \frac{\sqrt{2}}{8}\right|}{\left|e^{j\frac{3}{2}\pi} - \frac{1}{4}e^{j\frac{1}{4}\pi}\right| \left|e^{j\frac{3}{2}\pi} - \frac{1}{4}e^{-j\frac{1}{4}\pi}\right|} = \frac{1,0155}{1,002} = 1,0135$$

• Para $w_0 = \frac{7}{4}\pi$

$$\left|H(e^{jw_0})\right|_{w=\frac{7}{4}\pi} = \frac{\left|e^{j\frac{7}{4}\pi} - \frac{\sqrt{2}}{8}\right|}{\left|e^{j\frac{7}{4}\pi} - \frac{1}{4}e^{j\frac{1}{4}\pi}\right| \left|e^{j\frac{7}{4}\pi} - \frac{1}{4}e^{-j\frac{1}{4}\pi}\right|} = \frac{0,8839}{0,7731} = 1,1433$$

• Para $w_0 = 2\pi$

$$\left|H(e^{jw_0})\right|_{w=2\pi} = \frac{\left|e^{j2\pi} - \frac{\sqrt{2}}{8}\right|}{\left|e^{j2\pi} - \frac{1}{4}e^{j\frac{1}{4}\pi}\right| \left|e^{j2\pi} - \frac{1}{4}e^{-j\frac{1}{4}\pi}\right|} = \frac{0,8232}{0,7089} = 1,1612$$



d) $x[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$

$$X[z] = \frac{1}{1 - \frac{1}{4} z^{-1}} \qquad |z| > \frac{1}{4}$$

Sabiendo que $H(z) = \dfrac{Y(z)}{X(z)}$

Por lo que $Y(z) = H(z) \cdot X(z)$

$$Y(z) = \frac{1 - \frac{\sqrt{2}}{8} z^{-1}}{\left(1 - \frac{e^{-j\frac{\pi}{4}} \cdot z^{-1}}{4}\right)\left(1 - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4}\right)\left(1 - \frac{1}{4} z^{-1}\right)}$$

$$Y(z) = \frac{A_1}{\left(1 - \frac{e^{-j\frac{\pi}{4}} z^{-1}}{4}\right)} + \frac{A_2}{\left(1 - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4}\right)} + \frac{A_3}{\left(1 - \frac{1}{4} z^{-1}\right)}$$

$$A_1 = \lim_{z \to \frac{1}{4} e^{-j\frac{\pi}{4}}} \frac{1 - \frac{\sqrt{2}}{8} z^{-1}}{\left(1 - \frac{e^{j\frac{\pi}{4}} z^{-1}}{4}\right)\left(1 - \frac{1}{4} z^{-1}\right)} = 0{,}25 - 0{,}603j$$

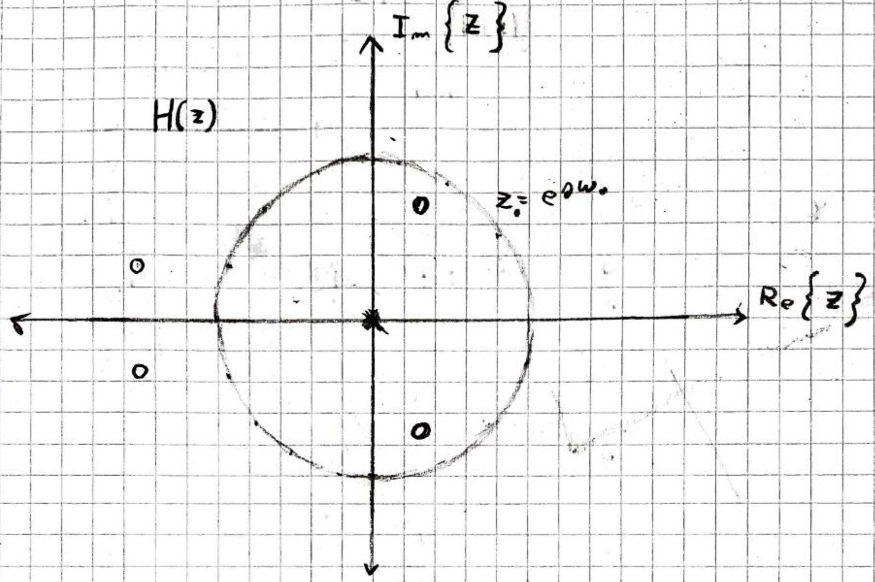
$$A_2 = \lim_{z \to \frac{1}{4}e^{j\frac{\pi}{4}}} \frac{1 - \frac{\sqrt{2}}{8} z^{-1}}{\left(1 - \frac{1}{4}e^{-j\frac{\pi}{4}} \cdot z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)} = 0{,}25 + 0{,}603j$$

$$A_3 = \lim_{z \to \frac{1}{4}} \frac{1 - \frac{\sqrt{2}}{8} z^{-1}}{\left(1 - \frac{1}{4}e^{-j\frac{\pi}{4}} z^{-1}\right)\left(1 - \frac{1}{4}e^{j\frac{\pi}{4}} z^{-1}\right)} = 0{,}5$$

$$y[n] = A_1 \cdot \left(\frac{1}{4} e^{-j\frac{\pi}{4}}\right)^n \cdot u[n] + A_2 \cdot \left(\frac{1}{4} \cdot e^{j\frac{\pi}{4}}\right)^n \cdot u[n] + A_3 \left(\frac{1}{4}\right)^n u[n]$$

Ejercicio 2

$$H(z) = (1 - 0{,}7e^{j0{,}4\pi}z^{-1})(1 - 0{,}7e^{-j0{,}4\pi}z^{-1})(1 - 1{,}5e^{j0{,}9\pi}z^{-1})(1 - 1{,}5e^{-j0{,}9\pi}z^{-1})$$



$$H(z) = 0{,}7e^{j0{,}4\pi}\left(\frac{10}{7}e^{-j0{,}4\pi} - z^{-1}\right) \cdot 0{,}7e^{-j0{,}4\pi}\left(\frac{10}{7}e^{j0{,}4\pi} - z^{-1}\right)$$

$$\cdot \frac{3}{2}e^{j0{,}9\pi} \cdot \left(\frac{2}{3}e^{-j0{,}9\pi} - z^{-1}\right) \cdot \frac{3}{2}e^{-j0{,}9\pi}\left(\frac{2}{3}e^{j0{,}9\pi} - z^{-1}\right)$$

$$H(z) = 0{,}49\left(z^{-1} - \frac{10}{7}e^{-j0{,}4\pi}\right) \cdot \left(z^{-1} - \frac{10}{7}e^{j0{,}4\pi}\right) \cdot \frac{9}{4}\left(z^{-1} - \frac{2}{3}e^{-j0{,}9\pi}\right)$$

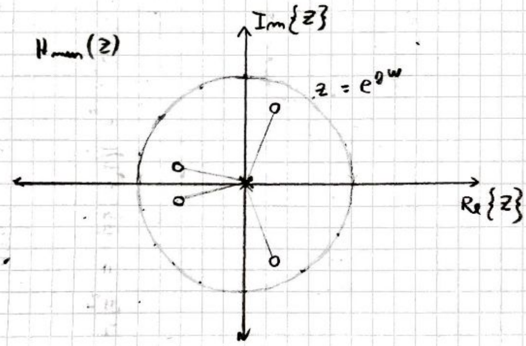
$$\left(z^{-1} - \frac{2}{3}e^{j0{,}9\pi}\right)$$

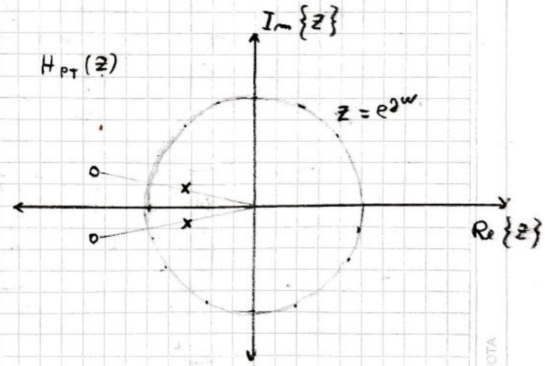
Ejercicio 2

$$H(z) = \frac{441}{400}\left(z^{-1} - \frac{10}{7}e^{-j0,4\pi}\right)\left(z^{-1} - \frac{10}{7}e^{j0,4\pi}\right)\left(z^{-1} - \frac{2}{3}e^{-j0,9\pi}\right)\left(z^{-1} - \frac{2}{3}e^{j0,9\pi}\right) \cdot \frac{\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right)}{\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right)}$$

$$H_{min}(z) = \frac{441}{400}\left(z^{-1} - \frac{10}{7}e^{-j0,4\pi}\right)\left(z^{-1} - \frac{10}{7}e^{j0,4\pi}\right)\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{+j0,9\pi}z^{-1}\right)$$

$$H_{PT}(z) = \frac{\left(z^{-1} - \frac{2}{3}e^{-j0,9\pi}\right)\left(z^{-1} - \frac{2}{3}e^{j0,9\pi}\right)}{\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right)}$$





Para que el compensador sea causal y estable

$$H_c(z) = \left[H_{min}(z)\right]^{-1}$$

$$H_c(z) = \frac{\frac{400}{441}}{\left(z^{-1} - \frac{10}{7}e^{-j0,4\pi}\right)\left(z^{-1} - \frac{10}{7}e^{j0,4\pi}\right)\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right)}$$

$$G(z) = H(z) . H_c(z) = H_{PT}(z) . \cancel{H_{min}(z)} . \cancel{H^{-1}_{min}(z)}$$

$$G(z) = \frac{\left(z^{-1} - \frac{2}{3}e^{-j0,9\pi}\right)\left(z^{-1} - \frac{2}{3}e^{j0,9\pi}\right)}{\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right)}$$

$$\left|G(e^{jw})\right| = \frac{\left|e^{-jw} - \frac{2}{3}e^{-j0,9\pi}\right|\left|e^{-jw} - \frac{2}{3}e^{j0,9\pi}\right|}{\left|1 - \frac{2}{3}e^{-j0,9\pi} . e^{-jw}\right|\left|1 - \frac{2}{3}e^{+j0,9\pi} . e^{-jw}\right|} =$$

$$= \frac{\cancel{\left|e^{-jw}\right|}^{1} . \left|1 - \frac{2}{3}e^{-j(0,9\pi - w)}\right| \cancel{\left|e^{-jw}\right|}^{1} . \left|1 - \frac{3}{3}e^{j(0,9\pi + w)}\right|}{\left|1 - \frac{2}{3}e^{-j(0,9\pi + w)}\right|\left|1 - \frac{2}{3}e^{+j(0,9\pi - w)}\right|}$$

$$= \frac{\sqrt{\left(1 - \frac{2}{3}\cos(-(0,9\pi - w))\right)^2 + \left(-\frac{2}{3}\operatorname{sen}(0,9\pi - w)\right)^2} . \sqrt{\left(1 - \frac{2}{3}\cos(0,9\pi + w)\right)^2 + \left(+\frac{2}{3}\operatorname{sen}(0,9\pi + w)\right)^2}}{\sqrt{\left(1 - \frac{2}{3}\cos(-(0,9\pi + w))\right)^2 + \left(-\frac{2}{3}\operatorname{sen}(0,9\pi + w)\right)^2} . \sqrt{\left(1 - \frac{2}{3}\cos(0,9\pi - w)\right)^2 + \left(\frac{2}{3}\operatorname{sen}(0,9\pi - w)\right)^2}}$$

como el cos es par el $\cos(a) = \cos(-a)$ y como el seno es impar $(\operatorname{sen}(a))^2 = (\operatorname{sen}(-a))^2 = (-\operatorname{sen}(a))^2$

Por lo que verifico que $|G(e^{j\omega})| = 1$ ✓

c) $H_c(z) = \dfrac{Y(z)}{X(z)}$

$$X(z) = Y(z).\left(z^{-1} - \frac{10}{7}e^{-j0,4\pi}\right)\left(z^{-1} - \frac{10}{7}e^{j0,4\pi}\right)\left(1 - \frac{2}{3}e^{-j0,9\pi}z^{-1}\right)\left(1 - \frac{2}{3}e^{j0,9\pi}z^{-1}\right).\frac{441}{400}$$

$$X(z) = Y(z)\left(z^{-2} - 0,882\,z^{-1} + 2,04\right)\left(1 + 1,268\,z^{-1} + \frac{4}{9}z^{-2}\right).\frac{441}{400}$$

$$X(z) = Y(z)\left(\frac{49}{100}z^{-4} + 1,397\,z^{-3} + z^{-2} - 0,432\,z^{-3} - 1,233\,z^{-2}\right.$$

$$\left. - 0,9724\,z^{-1} + 0,9996\,z^{-2} + 2,851\,z^{-1} + 2,249\right)$$

$$X(z) = Y(z)\,\frac{49}{100}z^{-4} + Y(z)\,\frac{193}{200}z^{-3} + Y(z)\,0,766\,z^{-2}$$

$$+\ Y(z).1,8786.\,z^{-1} + 2,249\,Y(z)$$

Utilizando propiedad de desplazamiento temporal

$$x[n] = \frac{49}{100}y[n-4] + \frac{193}{200}y[n-3] + 0,766\,y[n-2] + 1,878\,y[n-1]$$

$$+\ 2,249\,y[n]$$