Procesamiento digital de señales

Transformada de fourier

Integrantes:

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Ejercicio 1:

$$x(t) = \begin{cases} A\cos(2\pi f_0 t) & \text{si } |t| < \frac{\varepsilon}{2} \\ 0 & \text{caso contrario} \end{cases}$$

Para lograr esta funcion necesito multiplicar un $\cos(x)$ por un pulso.

$$\cdot\ A\cos(2\pi f_0 t) = \frac{A}{2}\cdot\left(e^{j2\pi f_0 t} + e^{j2\pi(-f_0)t}\right) =$$

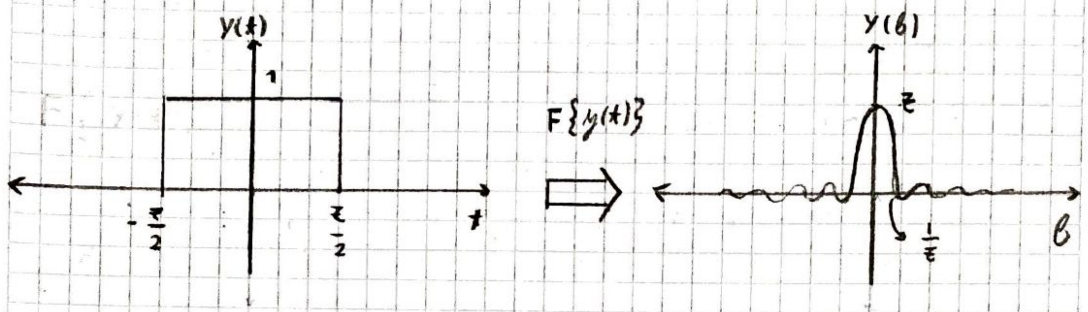
$$= \frac{A}{2}\cdot e^{j2\pi f_0 t} + \frac{A}{2} e^{j2\pi(-f_0)t}$$

$$y(t) \begin{cases} 1 & \text{si } |t| < \frac{\varepsilon}{2} \\ 0 & \text{caso contrario} \end{cases}$$ Pulso rectangular

$$x(t) = y(t)\cdot A\cos(2\pi f_0 t) = \frac{A}{2}\cdot\left(y(t)e^{j2\pi f_0 t} + y(t)e^{-j2\pi f_0 t}\right)$$

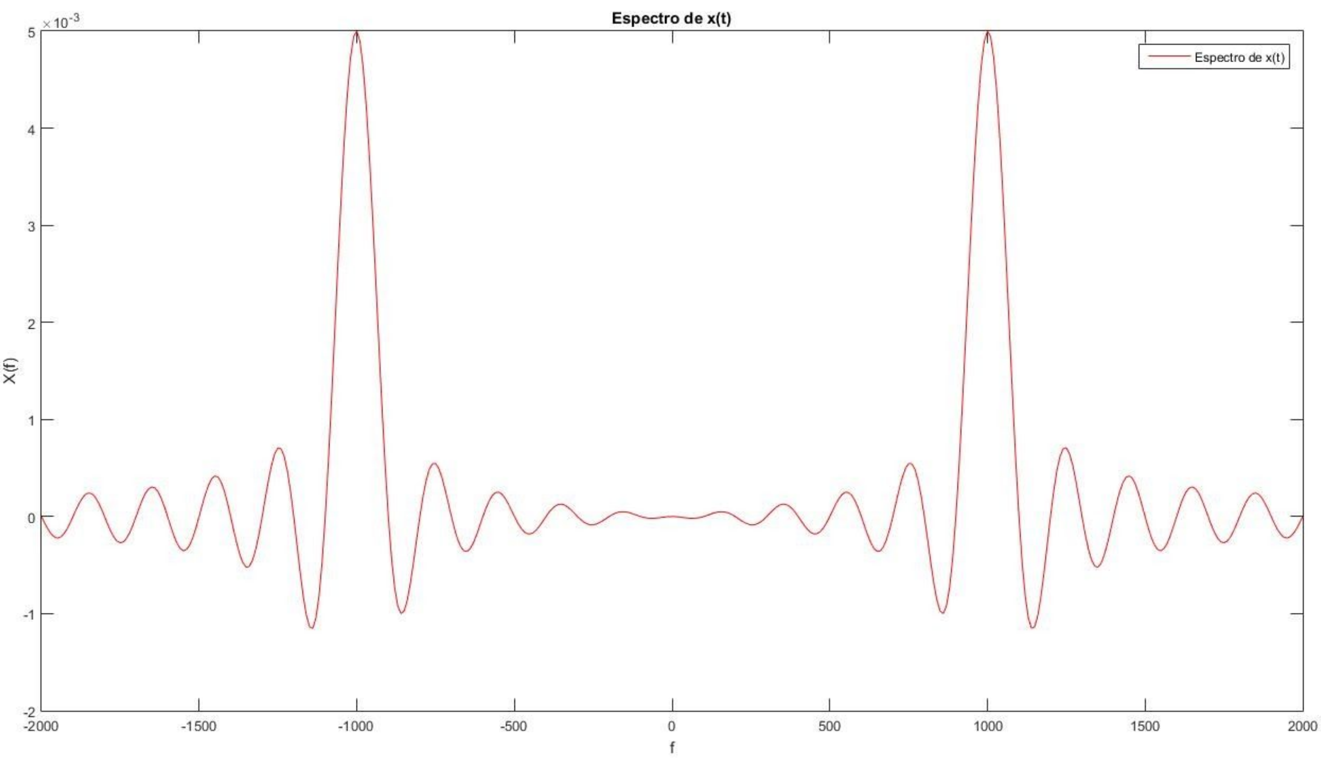
Utilizando propiedad de linealidad

$$F\{x(t)\} = \frac{A}{2}\left[Y(f-f_0) + Y(f+f_0)\right]$$



$$F\{x(t)\} = \frac{A}{2}\left[\tau\,\mathrm{sinc}\left[\tau(f-f_0)\right] + \tau\,\mathrm{sinc}\left[\tau(f+f_0)\right]\right]$$

Espectro



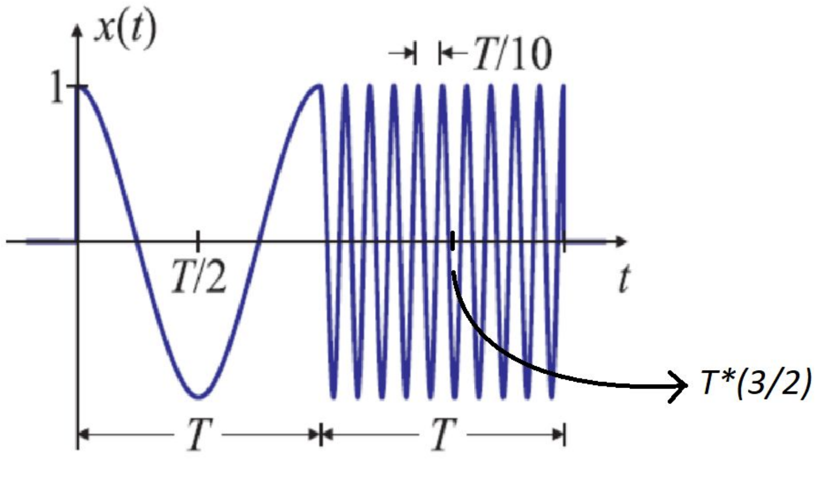
Ejercicio 2

$$y(t) = \begin{cases} 1 & -\frac{T_0}{2} < t < \frac{T_0}{2} \\ 0 & \text{caso contrario} \end{cases}$$

$$u(t) = y\left(t - \frac{T_0}{2}\right) . \cos\left(2\pi \frac{t}{T_0}\right) = \frac{1}{2} y_1\left(t - \frac{T_0}{2}\right)\left[e^{j 2\pi \frac{1}{T_0} . t} + e^{j \pi \left(\frac{-1}{T_0}\right) t}\right]$$

$$p(t) = y\left(t - \frac{3T_0}{2}\right) . \cos\left(2\pi \frac{10 t}{T_0}\right) = \frac{1}{2} y_2\left(t - \frac{3T_0}{2}\right)\left[e^{j 2\pi \frac{10}{T_0} . t} + e^{j 2\pi \left(\frac{-10}{T_0}\right) t}\right]$$

$$x(t) = u(t) + p(t)$$



Aplicando propiedad de linealidad y desplazamiento en tiempo

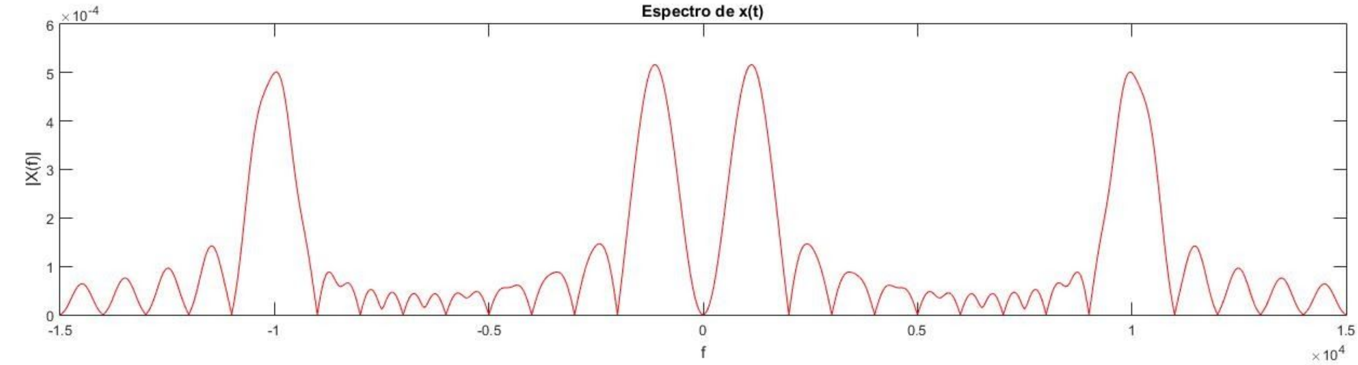
$$F\{x(t)\} = F\{u(t) + p(t)\} = F\{u(t)\} + F\{p(t)\}$$

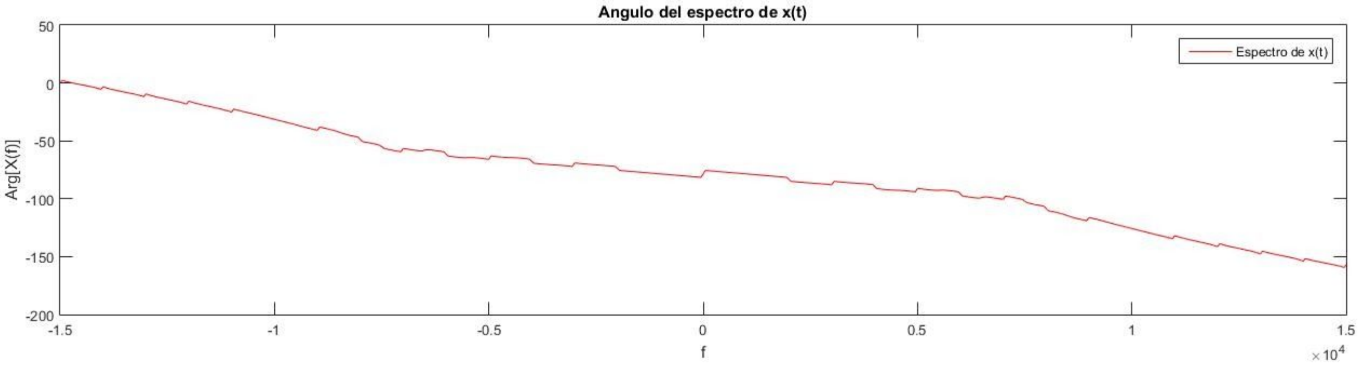
$$F\{x(t)\} = \frac{1}{2}\left[e^{-j2\pi f\frac{T_0}{2}}\left(Y\left(f-\frac{1}{T_0}\right)+Y\left(f+\frac{1}{T_0}\right)\right)+e^{-j2\pi f\frac{3}{2}T_0}\left(Y\left(f-\frac{10}{T_0}\right)+Y\left(f+\frac{10}{T_0}\right)\right)\right]$$

$$= \frac{1}{2}\cdot\left[e^{-j2\pi f\frac{T_0}{2}}T_0\left(\text{sinc}\left[T_0\left(f-\frac{1}{T_0}\right)\right] + \text{sinc}\left[T_0\left(f+\frac{1}{T_0}\right)\right]\right) +\right.$$

$$\left. + e^{-j2\pi f\frac{3}{2}T_0}\cdot T_0\cdot\left(\text{sinc}\left[T_0\left(f-\frac{10}{T_0}\right)\right] + \text{sinc}\left[T_0\left(f+\frac{10}{T_0}\right)\right]\right)\right]$$

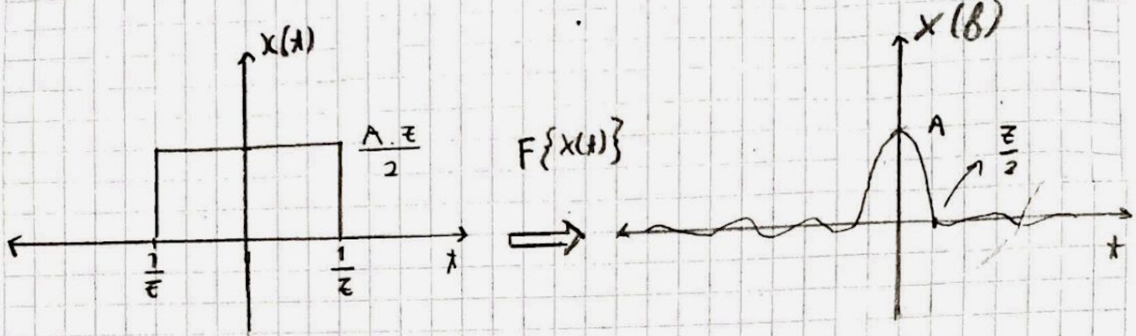
Espectro y ángulo



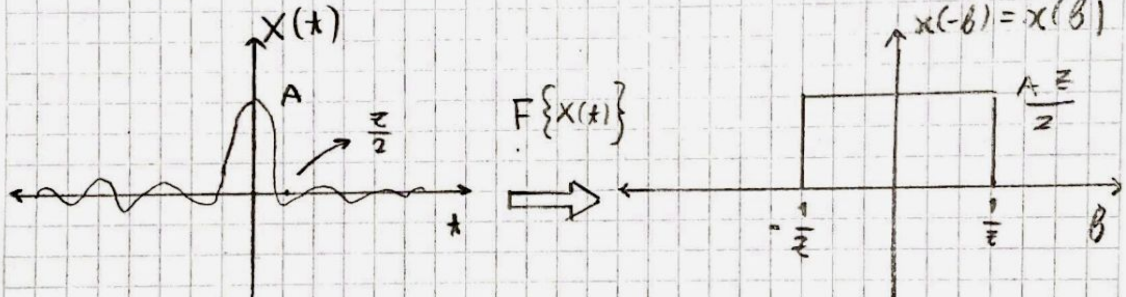


Ejercicio 3

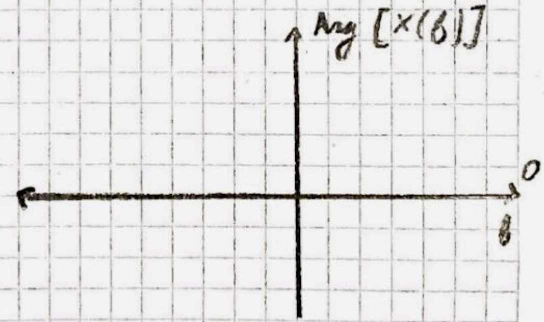
a) La transformada de Fourier de un pulso rectangular es un sinc.



Por propiedad de dualidad se puede asumir que la transformada de un sinc es un pulso rectangular



$$X(f) = \begin{cases} \frac{A\tau}{2} & \frac{-1}{\tau} < f < \frac{1}{\tau} \\ 0 & \text{caso contrario} \end{cases}$$



El argumento es cero ya que X(f) es real puro

NOTA

b) $\tilde{y}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk2\pi f_0 t}$

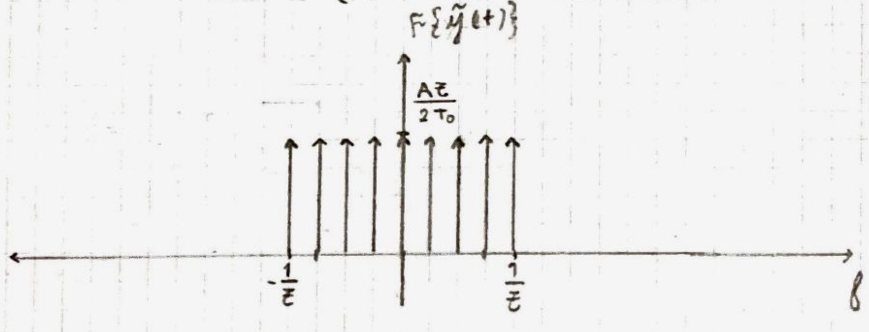
$F\{\tilde{y}(t)\} = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \left(c_k e^{jk2\pi f_0 t}\right) \cdot e^{-jk2\pi f t} \, dt$

$F\{\tilde{y}(t)\} = \sum_{k=-\infty}^{\infty} c_k \underbrace{\int_{-\infty}^{\infty} e^{-jk2\pi (f-f_0) t} \, dt}_{\text{Delta de Dirac}}$

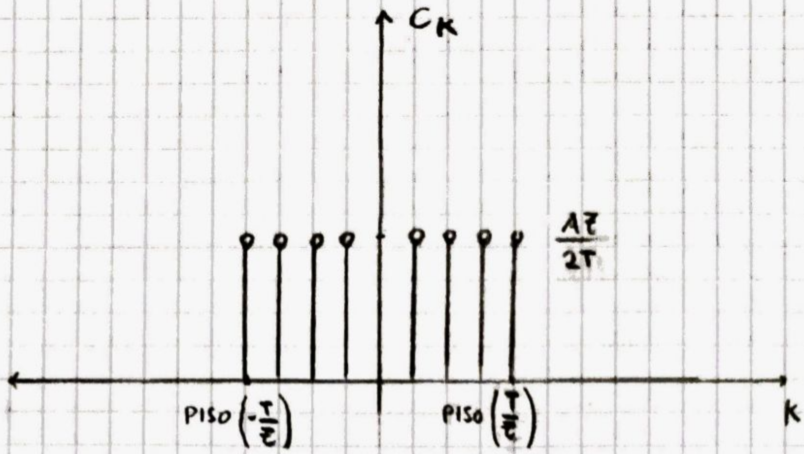
constante en este caso ya que no depende de k

$F\{\tilde{y}(t)\} = \sum_{k=-\infty}^{\infty} c_k \, \delta(f - kf_0) = \sum_{k=-\infty}^{\infty} c_k \, \delta\left(f - \frac{k}{T_0}\right)$

$c_k = \frac{1}{T_0} \cdot X(f)\Big|_{f=\frac{k}{T_0}} = \begin{cases} \frac{A\tau}{2T_0} & -\frac{T_0}{\tau} < k < \frac{T_0}{\tau} \\ 0 & \text{caso contrario} \end{cases}$



c)



Siendo "PISO" el valor entero por debajo mas cercano, ya que K solo puede tomar valores enteros.