Procesamiento digital de señales

Sistemas discretos

Integrantes:

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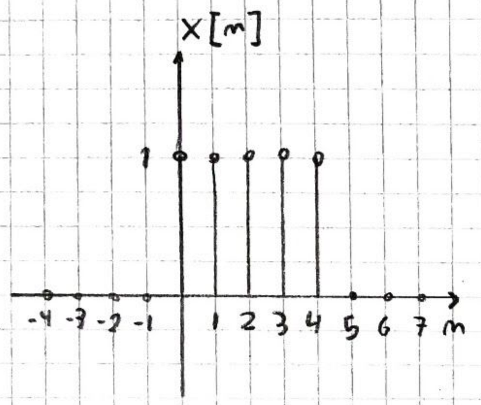
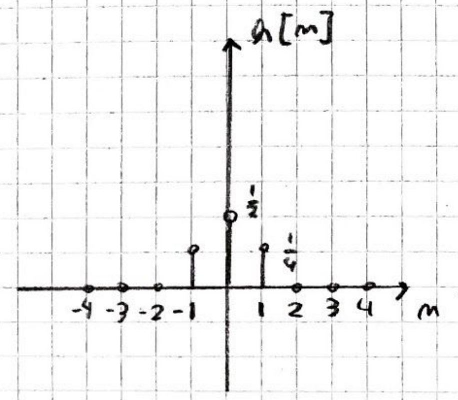
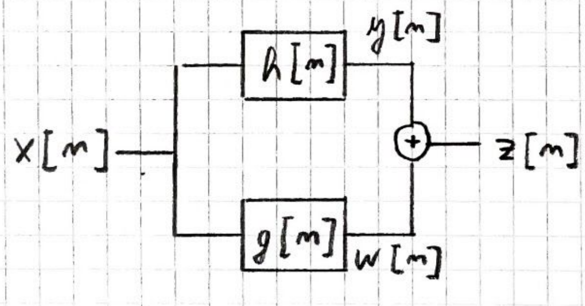
1)

$$x[n] = u[n] - u[n-5]$$

$$h[n] = \frac{1}{4}\delta[n+1] + \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1]$$







a) Como en este caso $h[n]$ solo presenta 3 terminos es mas sencillo efectuar la convolucion termino a termino

$$y[n] = x[n] * \left(\frac{1}{4}\delta[n+1] + \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1]\right)$$

$$= \frac{1}{4}x[n+1] + \frac{1}{2}x[n] + \frac{1}{4}x[n-1]$$

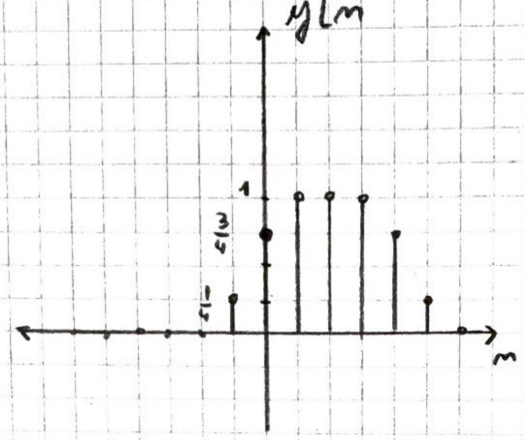
- $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$

- $x[n+1] = \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

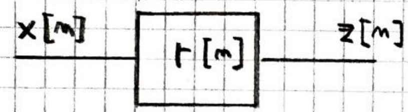
- $x[n-1] = \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$

$$y[n] = \frac{1}{4}\delta[n+1] + \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3] + \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{2}\delta[n-2] + \frac{1}{2}\delta[n-3] + \frac{1}{2}\delta[n-4] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3] + \frac{1}{4}\delta[n-4] + \frac{1}{4}\delta[n-5]$$

$$y[n] = \frac{1}{4}\delta[n+1] + \frac{3}{4}\delta[n] + \delta[n-1] + \delta[n-2]$$
$$+ \delta[n-3] + \frac{3}{4}\delta[n-4] + \frac{1}{4}\delta[n-5]$$



b)

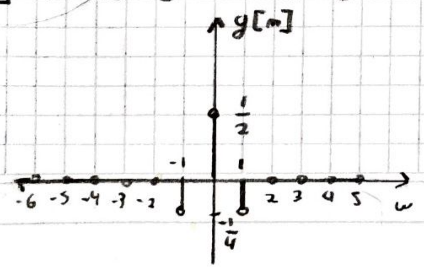


$$r[n] = h[n] + g[n]$$

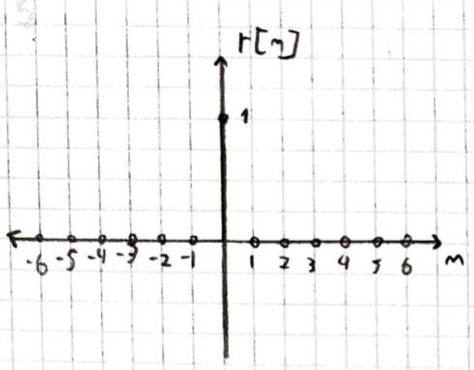
Para que $z[n] = x[n]$, $r[n]$ debe ser un impulso en cero ya que convolucionado con $x[n]$ dará la misma señal $x[n]$.

$$r[n] = \delta[n]$$

$$g[n] = r[n] - h[n] = -\frac{1}{4}\delta[n+1] + \frac{1}{2}\delta[n] - \frac{1}{4}\delta[n-1]$$



c)



d) Como $h[n]$, $g[n]$ y $r[n]$ son de respuestas impulsos entonces son invariantes en el tiempo, por lo tanto para que sean causales se debe cumplir $h[n]=0 \quad n<0$

$h[n]$: No causal $\quad h[n] \neq 0 \quad n<0$

$g[n]$: No causal $\quad g[n] \neq 0 \quad n<0$

$r[n]$: Causal $\quad r[n] = 0 \quad n<0$

Ejercicio 2

$$y[n] = a\,y[n-1] + b\,x[n-1], \quad y[-1] = 0$$

$$y[0] = a\,y[-1] + b\,x[-1] = b\,x[-1]$$

$$y[1] = a\,y[0] + b\,x[0] = ab\,x[-1] + b\,x[0]$$

$$y[2] = a\,y[1] + b\,x[1] = a^2 b\,x[-1] + ab\,x[0] + b\,x[1]$$

a)

$h[0] = b\,\cancel{\delta[-1]} = 0$

$h[1] = ab\,\cancel{\delta[-1]} + b\,\delta[0] = b$

$h[2] = a^2 b\,\cancel{\delta[-1]} + ab\,\delta[0] + \cancel{b\,x[1]} = ab$

$h[3] = a^3 b\,\cancel{\delta[-1]} + a^2 b\,\delta[0] + \cancel{ab\,x[1]} + b\,x[0] = a^2 b$

$h[n] = b \cdot a^{n-1} \cdot u[n-1] = \frac{b}{a} \cdot a^n\, u[n] - \frac{b}{a}\, a^n\, \delta[n]$

con $u[n] = \begin{cases} 1 & \text{si } n \geq 0 \\ 0 & \text{si } n < 0 \end{cases}$ y $\delta[n] = \begin{cases} 1 & \text{si } n = 0 \\ 0 & \text{si } n \neq 0 \end{cases}$

b) $|h[n]| \leq \underbrace{\sum_{n=0}^{\infty} \left|\frac{b}{a}\right| |a|^n}$ $+ \left|\frac{b}{a}\right| |a|^n$

Como es una serie geométrica de razón a, si $|a|<1$ la sumatoria converge a $\frac{|b/a|}{1-|a|}$

Por lo tanto si $h[n] = \sum \frac{b}{a} a^n\, u[n] - \frac{b}{a}\, a^n\, \delta[n]$

$|h[n]| \leq \frac{|b/a|}{1-|a|} B_x + \frac{b}{a} |a|^n B_x < \infty$

El sistema va a ser estable para $|a|<1$ y $b<\infty$

c)

$$H(e^{j\omega}) = \sum_{n} h[n]\, e^{-j\omega n} = \sum_{n} \frac{b}{a} a^{n} \cdot e^{-j\omega n} - \sum_{n} \frac{b}{a} a^{n} \cdot \delta[n] \cdot e^{-j\omega n}$$

$$= \frac{b}{a} \sum_{n=0}^{\infty} \left(a \cdot e^{-j\omega}\right)^{n} - \frac{b}{a}$$

Esto es una serie geometrica de razon $a e^{-j\omega}$, entonces la sumatoria converge si solo si $|a e^{-j\omega}| < 1 \rightarrow |a| < 1$

y converge a $\frac{b}{a} \cdot \frac{1}{1 - a e^{-j\omega}}$

Por lo tanto $H(e^{j\omega}) = \frac{b}{a}\left(\frac{1}{1 - a e^{-j\omega}} - 1\right)$

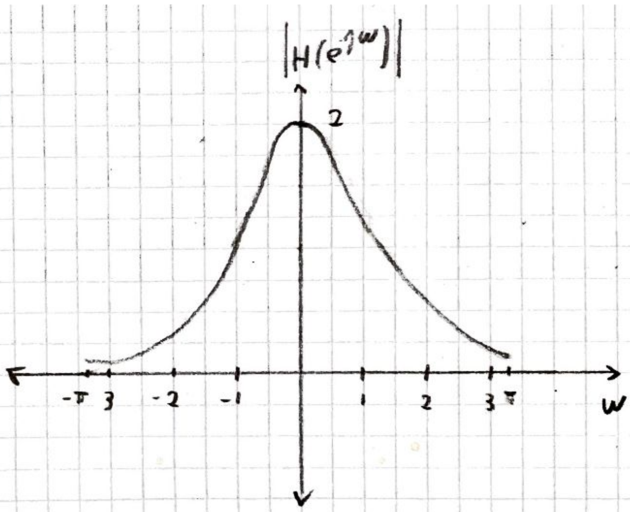
$$= \frac{b}{a} \cdot \frac{1 - 1 + a e^{-j\omega}}{1 - a e^{-j\omega}} = \frac{b e^{-j\omega}}{1 - a e^{-j\omega}}$$

$$|H(e^{j\omega})| = \left| \frac{b e^{-j\omega}}{1 - a\cos(\omega) - j a \operatorname{sen}(\omega)} \right| = \frac{|b e^{-j\omega}|}{\sqrt{(1 - a\cos(\omega))^{2} + (a \operatorname{sen}(\omega))^{2}}} =$$

$$= \frac{b}{\sqrt{1 - 2a\cos(\omega) + a^{2}}}$$

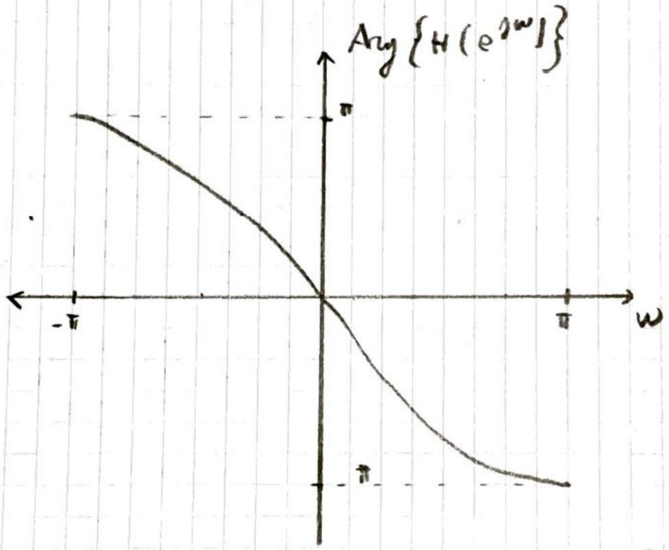
$$\operatorname{Arg}\{H(e^{j\omega})\} = -\arctan\left(\frac{a \operatorname{sen}\omega}{1 - a\cos(\omega)}\right) - \omega$$

d)



$$\left|H(e^{j\omega})\right|\Big|_{\omega=0} = \frac{b}{1-a} = 2$$

$$\left|H(e^{j\omega})\right|\Big|_{\omega=\pi} = \frac{b}{1+a} = \frac{2}{3}$$



e) Tomando $a = 0,5$ y $b = 1$ el sistema actua como un pasa bajos, filtrando las altas frecuencias y amplificando o dejando pasar las bajas, en cambio tomando valores negativos de "a" el sistema se comporta como un pasa altos. El valor de b solo influye en la amplitud y el offset de la respuesta en frecuencia

3)

$$y[m] = x[m] + x[m-1]$$

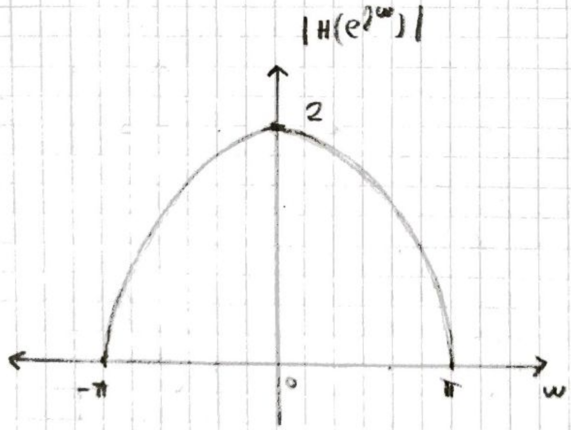
a) $h[m] = \delta[m] + \delta[m-1]$

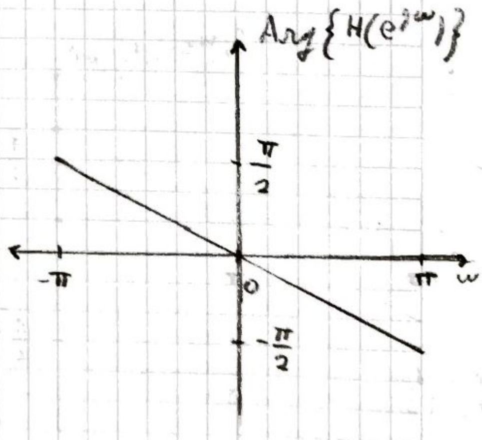
b) $H(e^{j\omega}) = \sum_{m} h[m] \cdot e^{-j\omega m} = \sum_{m} (\delta[m] + \delta[m-1]) e^{-j\omega m}$

$= e^{-j\omega 0} + e^{-j\omega 1} = 1 + e^{-j\omega} = e^{-j\frac{\omega}{2}} \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right)$

$= e^{-j\frac{\omega}{2}} \, 2\cos\left(\frac{\omega}{2}\right)$

c) $|H(e^{j\omega})| = \left|2\cos\left(\frac{\omega}{2}\right)\right|$ y $\mathrm{Arg}\{H(e^{j\omega})\} = \frac{\omega}{2}$





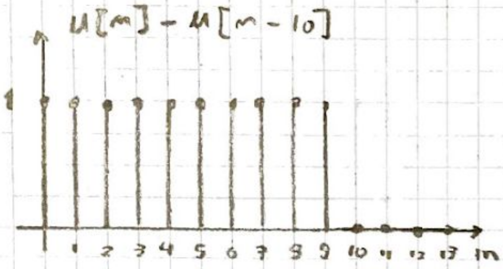
FECHA

d-i) $y[n] = \cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi(n-1)}{4}\right)$

$$y[n] = \cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{4}\right) \cdot \cos\left(\overset{\sqrt{2}/2}{\frac{\pi}{4}}\right) + \operatorname{sen}\left(\frac{\pi n}{4}\right) \cdot \operatorname{sen}\left(\overset{\sqrt{2}/2}{\frac{\pi}{4}}\right)$$

$$y[n] = \frac{1+\sqrt{2}}{2} \cos\left(\frac{\pi n}{4}\right) + \frac{\sqrt{2}}{2} \operatorname{sen}\left(\frac{\pi n}{4}\right)$$

ii) $y[n] = u[n] - u[n-10] + u[n-1] - u[n-11]$



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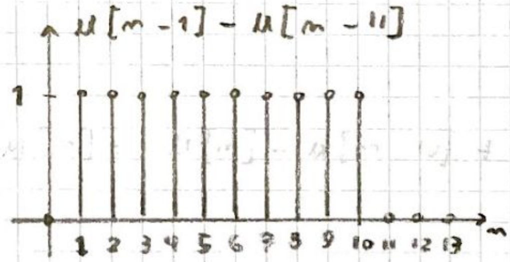

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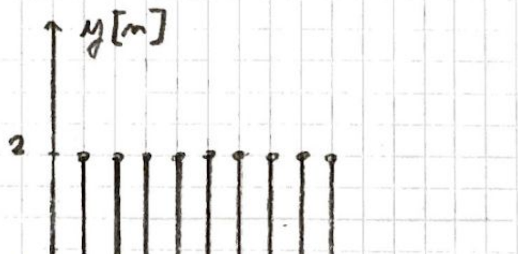

NOTA