1

Exercises



Exercise n°1

Eliminate useless productions from

$$\begin{split} S &\rightarrow a \mid aA \mid B \mid C \\ A &\rightarrow aB \mid \lambda \\ B &\rightarrow Aa \\ C &\rightarrow cCD \\ D &\rightarrow ddd \end{split}$$



Exercise n°2

Use the CYK algorithm to determine whether the strings aabb, aabba and aabbb are in the language generated by the grammar

$$S \to AB$$

$$A \to BB \mid a$$

$$B \to AB \mid b$$

Solution n°1

$$\begin{split} S &\rightarrow a \mid aA \mid B \mid C \\ A &\rightarrow aB \mid \lambda \\ B &\rightarrow Aa \\ C &\rightarrow cCD \\ D &\rightarrow ddd \end{split}$$

We determine all productive variable:

- $\bullet \ {\bf S}: S \to a$
- $\bullet \ {\bf A}: A \to \lambda$
- B : $B \to Aa$ and A is productive
- C does not rewrite to a terminal word: $a \in T^*$
- D: $B \rightarrow ddd$

Thus we remove all rules containing C:

We determine all reachable variable:

- S is reachable.
- From S we can reach B.
- From B we can reach A.

We remove all rules containing D:

$$S \rightarrow a \mid aA \mid B$$

$$A \rightarrow aB \mid \lambda$$

$$B \rightarrow Aa$$

Solution n°2

^^^^^^

Notice that the grammar is in Chomsky Normal Form since the right-hand side of every rules consist of either two variables or a single terminal letter:

$$S \rightarrow AB$$

$$A \rightarrow BB \mid a$$

$$B \rightarrow AB \mid b$$

$$V_{a} = \{A\}$$

$$V_{b} = \{B\}$$

$$V_{aa} = \{X \mid X \to V_{a}V_{a} = \{AA\}\} = \{\}$$

$$V_{ab} = \{X \mid X \to V_{a}V_{b} = \{AB\}\} = \{S, B\}$$

$$V_{bb} = \{X \mid X \to V_{b}V_{b} = \{BB\}\} = \{A\}$$

$$V_{aab} = \{X \mid X \to V_{aa}V_{b} \cup V_{a}V_{ab} = \{AS, AB\}\} = \{S, B\}$$

$$V_{abb} = \{X \mid X \to V_{ab}V_{b} \cup V_{a}V_{bb} = \{SB, BB, AA\}\} = \{A\}$$

$$V_{aabb} = \{X \mid X \to V_{aa}V_{bb} \cup V_{a}V_{abb} \cup V_{aab}V_{b} = \{AA, SB, BB\}\} = \{A\}$$

$$V_{ba} = \{X \mid X \to V_b V_a = \{BA\}\} = \{\}$$

$$V_{bba} = \{X \mid X \to V_b V_{ba} \cup V_{bb} V_a = \{AA\}\} = \{\}$$

$$V_{abba} = \{X \mid X \to V_{ab} V_{ba} \cup V_a V_{bba} \cup V_{abb} V_a = \{AA\}\} = \{\}$$

$$V_{aabba} = \{X \mid X \to V_{aabb} V_a \cup V_a V_{abba} \cup V_{aab} V_{ba} \cup V_{aab} V_{ba} = \{\}\} = \{\}$$

$$V_{bbb} = \{X \mid X \to V_b V_{bb} \cup V_{bb} V_b = \{BA, AB\}\} = \{S, B\}$$

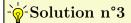
$$V_{abbb} = \{X \mid X \to V_a V_{bbb} \cup V_{ab} V_{bb} \cup V_{abb} V_b = \{AS, AB, SA, BA\}\}$$

$$= \{S, B\}$$

$$V_{aabbb} = \{X \mid X \to V_{aabb} V_b \cup V_a V_{abbb} \cup V_{aa} V_{bbb} \cup V_{aab} V_{bb}\}$$

$$= \{AS, AB, SA, BA\}\} = \{S, B\}$$

- The word aabb doesn't belong to the language since $S \notin V_{aabb}$
- The word aabba doesn't belong to the language since $S \notin V_{aabba}$
- The word *aabbb* is in the language since $S \in V_{aabbb}$



$$\begin{array}{cccc} S \rightarrow & AB \\ A \rightarrow & BB & | & a \\ B \rightarrow & AB & | & b \end{array}$$

Since the starting symbol S is in this set, we conclude that aab is in the language generated by the grammar:

$$\underbrace{S}_{aab} \longrightarrow \underbrace{A}_{a} \underbrace{B}_{ab} \longrightarrow \underbrace{A}_{a} \underbrace{A}_{a} \underbrace{B}_{b} \longrightarrow aab$$

Solution n°4

$$S \rightarrow bSa \mid cSa \mid \lambda$$

Notice that there is no useless variable. Notice that S is erasable since

 $\operatorname{PreFirst}(S) = \{S, bSa, cSa, \lambda, b, c\}$ $First(S) = \{b, c, \lambda\} \ (\lambda \in First(S) \text{ since S is erasable})$

 $First(a) = \{a\} \subseteq Follow(S)$

 $Follow(S) = \{\$, a\}$

	b	a	c	\$
S	$S \to bSa$	$S \to \lambda$	$S \to cSa$	$S \to \lambda$

 $\langle S\$, bca\$\rangle \rightarrow \langle bSa\$, bca\$\rangle \rightarrow \langle Sa\$, ca\$\rangle \rightarrow \langle cSaa\$, ca\$\rangle \rightarrow \langle Saa\$, a\$\rangle$ $ightarrow \langle aa\$, a\$
angle
ightarrow \langle a\$, \$
angle \ \mbox{\it X}$

Notice that ther $S \to \lambda$.

PreFirst(S) = {S}
First(S) = {b, c,}

{\$} \subseteq \text{Follow}(S)
First(a) = {a} \subseteq Follow(S) = {\$, ...} $\langle S\$, bca\$ \rangle \to \langle S\$, bca\$ \rangle$ $\langle S\$, bcaa\$\rangle \rightarrow \langle bSa\$, bcaa\$\rangle \rightarrow \langle Sa\$, caa\$\rangle \rightarrow \langle cSaa\$, caa\$\rangle \rightarrow \langle Saa\$, aa\$\rangle$ $\rightarrow \langle a\$, a\$ \rangle \rightarrow \langle \$, \$ \rangle$