1

Exercises

Exercise n°1

Eliminate useless productions from

$$\begin{aligned} S &\to \ a \ | \ aA \ | \ B \ | \ C \\ A &\to \ aB \ | \ \lambda \\ B &\to \ Aa \\ C &\to \ cCD \\ D &\to \ ddd \end{aligned}$$

Exercise n°2

Use the CYK algorithm to determine whether the strings $aabb$, $aabba$ and $aabbb$ are in the language generated by the grammar

$$\begin{aligned} S &\to \ AB \\ A &\to \ BB \ | \ a \\ B &\to \ AB \ | \ b \end{aligned}$$

Solution n°1

$$
\begin{aligned}
S &\to a \mid aA \mid B \mid C \\
A &\to aB \mid \lambda \\
B &\to Aa \\
C &\to cCD \\
D &\to ddd
\end{aligned}
$$

We determine all productive variable:

- S : $S \to a$
- A : $A \to \lambda$
- B : $B \to Aa$ and A is productive
- C does not rewrite to a terminal word: $a \in T^*$
- D: $B \to ddd$

Thus we remove all rules containing C:

$$
\begin{aligned}
S &\to a \mid aA \mid B \\
A &\to aB \mid \lambda \\
B &\to Aa \\
D &\to ddd
\end{aligned}
$$

We determine all reachable variable:

- S is reachable.
- From S we can reach B.
- From B we can reach A.

We remove all rules containing D:

$$
\begin{aligned}
S &\to a \mid aA \mid B \\
A &\to aB \mid \lambda \\
B &\to Aa
\end{aligned}
$$

Solution n°2

Notice that the grammar is in Chomsky Normal Form since the right-hand side of every rules consist of either two variables or a single terminal letter:

$$\begin{aligned} S &\to \; AB \\ A &\to \; BB \;\mid\; a \\ B &\to \; AB \;\mid\; b \end{aligned}$$

$$\begin{aligned} V_a &= \{A\} \\ V_b &= \{B\} \\ V_{aa} &= \{X \;\mid\; X \to V_aV_a = \{AA\}\} = \{\} \\ V_{ab} &= \{X \;\mid\; X \to V_aV_b = \{AB\}\} = \{S, B\} \\ V_{bb} &= \{X \;\mid\; X \to V_bV_b = \{BB\}\} = \{A\} \\ V_{aab} &= \{X \;\mid\; X \to V_{aa}V_b \cup V_aV_{ab} = \{AS, AB\}\} = \{S, B\} \\ V_{abb} &= \{X \;\mid\; X \to V_{ab}V_b \cup V_aV_{bb} = \{SB, BB, AA\}\} = \{A\} \\ V_{aabb} &= \{X \;\mid\; X \to V_{aa}V_{bb} \cup V_aV_{abb} \cup V_{aab}V_b = \{AA, SB, BB\}\} = \{A\} \end{aligned}$$

$$\begin{aligned} V_{ba} &= \{X \;\mid\; X \to V_bV_a = \{BA\}\} = \{\} \\ V_{bba} &= \{X \;\mid\; X \to V_bV_{ba} \cup V_{bb}V_a = \{AA\}\} = \{\} \\ V_{abba} &= \{X \;\mid\; X \to V_{ab}V_{ba} \cup V_aV_{bba} \cup V_{abb}V_a = \{AA\}\} = \{\} \\ V_{aabba} &= \{X \;\mid\; X \to V_{aabb}V_a \cup V_aV_{abba} \cup V_{aa}V_{bba} \cup V_{aab}V_{ba} = \{\}\} = \{\} \end{aligned}$$

$$\begin{aligned} V_{bbb} &= \{X \;\mid\; X \to V_bV_{bb} \cup V_{bb}V_b = \{BA, AB\}\} = \{S, B\} \\ V_{abbb} &= \{X \;\mid\; X \to V_aV_{bbb} \cup V_{ab}V_{bb} \cup V_{abb}V_b = \{AS, AB, SA, BA\}\} \\ &= \{S, B\} \\ V_{aabbb} &= \{X \;\mid\; X \to V_{aabb}V_b \cup V_aV_{abbb} \cup V_{aa}V_{bbb} \cup V_{aab}V_{bb}\} \\ &= \{AS, AB, SA, BA\}\} = \{S, B\} \end{aligned}$$

- The word *aabb* doesn't belong to the language since $S \notin V_{aabb}$
- The word *aabba* doesn't belong to the language since $S \notin V_{aabba}$
- The word *aabbb* is in the language since $S \in V_{aabbb}$

Solution n°3

$$
\begin{aligned}
S &\to AB \\
A &\to BB \mid a \\
B &\to AB \mid b
\end{aligned}
$$

$$
\begin{aligned}
V_a &= \{A\} \\
V_b &= \{B\} \\
V_{aa} &= \{X \mid X \to V_a V_a = \{AA\}\} = \{\} \\
V_{ab} &= \{X \mid X \to V_a V_b = \{AB\}\} = \{S, B\} \\
V_{bb} &= \{X \mid X \to V_b V_b = \{BB\}\} = \{A\} \\
V_{aab} &= \{X \mid X \to V_{aa} V_b \cup V_a V_{ab} = \{AS, AB\}\} = \{S, B\}
\end{aligned}
$$

Since the starting symbol S is in this set, we conclude that aab is in the language generated by the grammar:

$$
\underbrace{S}_{\text{aab}} \longrightarrow \underbrace{A}_{\text{a}}\underbrace{B}_{\text{ab}} \longrightarrow \underbrace{A}_{\text{a}}\underbrace{A}_{\text{a}}\underbrace{B}_{\text{b}} \longrightarrow aab
$$

Solution n°4

Solution n°5

$$S \rightarrow \ bSa \ | \ cSa \ | \ \lambda$$

Notice that there is no useless variable. Notice that S is erasable since $S \rightarrow \lambda$.

$\text{PreFirst}(S) = \{S, bSa, cSa, \lambda, b, c\}$
$\text{First}(S) = \{b, c, \lambda\}$ ($\lambda \in \text{First}(S)$ since S is erasable)

$\{\$\} \subseteq \text{Follow}(S)$
$\text{First}(a) = \{a\} \subseteq \text{Follow}(S)$
$\text{Follow}(S) = \{\$, a\}$

	b	a	c	\$
S	$S \rightarrow bSa$	$S \rightarrow \lambda$	$S \rightarrow cSa$	$S \rightarrow \lambda$

$$\begin{aligned}\langle S\$, bca\$\rangle &\rightarrow \langle bSa\$, bca\$\rangle \rightarrow \langle Sa\$, ca\$\rangle \rightarrow \langle cSaa\$, ca\$\rangle \rightarrow \langle Saa\$, a\$\rangle \\ &\rightarrow \langle aa\$, a\$\rangle \rightarrow \langle a\$, \$\rangle \ ✗\end{aligned}$$

$$\begin{aligned}\langle S\$, bcaa\$\rangle &\rightarrow \langle bSa\$, bcaa\$\rangle \rightarrow \langle Sa\$, caa\$\rangle \rightarrow \langle cSaa\$, caa\$\rangle \rightarrow \langle Saa\$, aa\$\rangle \\ &\rightarrow \langle a\$, a\$\rangle \rightarrow \langle \$, \$\rangle \ ✓\end{aligned}$$