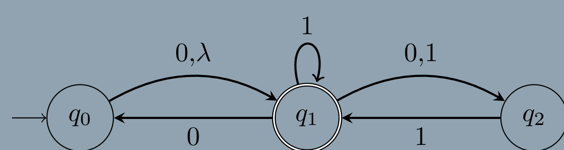
1 Exercises

Exercise n°1

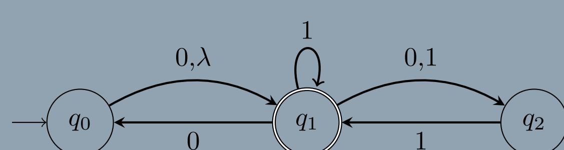
Consider the following NFA:



Which of the words 00, 01001, 10010, 000 and 0000 does the automaton accept?

Exercise n°2

Convert the following NFA into an equivalent DFA:



Exercise n°3

Is it true that for every NFA $M = (Q, \Sigma, \delta, S, F)$ the complement of L(M) is equal to the set

$$\{w \in \Sigma^* \ \mid \ (q_0, w) \vdash^* (q, \lambda) \ \text{ for some } \ q \in (Q \setminus F)\}$$

Exercise n°4

Show that L^R is regular if L is regular

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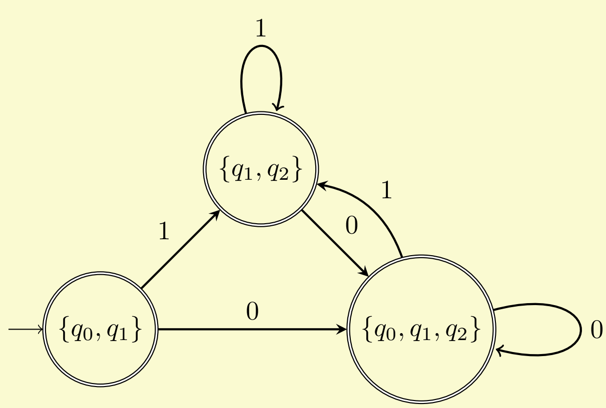
Solutions

Solution n°1

Only 01001 and 000 are accepted:

- $(q_0, 01001) \vdash (q_1, 1001) \vdash (q_1, 001) \vdash (q_0, 01) \vdash (q_1, 1) \vdash (q_1, \lambda)$
- $(q_0, 000) \vdash (q_1, 00) \vdash (q_0, 0) \vdash (q_1, \lambda)$

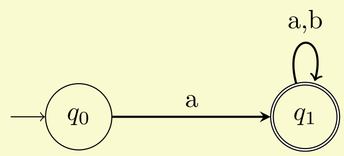
Solution n°2



Solution n°3

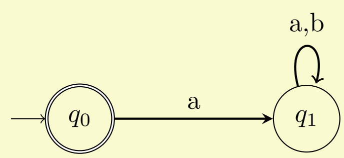
This NFA M accepts the language

$$L = \{a, ab, ab, \dots\} = \{a^{n+1}b^m \ \mid \ n \geq 0, m \geq 0\}:$$



Then L(M)=L. Now we want to construct:

$$L(\overline{M}) = \{w \in \Sigma^* \ \mid \ (q_0, w) \vdash^* (q, \lambda) \ \text{ for some } \ q \in (Q \setminus F)\}$$



This NFA accepts the language $L(\overline{M}) = \{\lambda\}$.

However

$$\overline{L} = \{\lambda, b, ba, bb, bab, \dots\}.$$

It proves that the statement is false, the language accepted by the complement of a NFA is not equal to the complement of the language:

$$\overline{L} = \overline{L(M)} \neq L(\overline{M}).$$

Solution n°4

Regular languages closed under complement proof:

Let L be any regular language.

$$\text{L is regular} \iff \exists \ \text{DFA M} = (Q, \Sigma, \delta, q_0, F) \ \text{ with } \ L(M) = L$$

Define the DFA $\overline{M} = (Q, \Sigma, \delta, q_0, Q \setminus F)$.

Notice that the set of accepting states and non-accepting ones of M are inverted in $\overline{M}$. Thus:

$$\begin{aligned} w \in L &\iff M \ \text{ accepts } \ w \\ &\iff (q_0, w) \vdash^* (q, \lambda) \ \text{ with } \ q \in F \end{aligned}$$

The transitions are the same but $q \notin Q \setminus F$, therefore:

$$w \in L \Rightarrow \overline{M} \ \text{ does not accepts w} \tag{i}$$

Similary,

$$w \notin L \Rightarrow \overline{M} \ \text{ accepts } \ w \tag{ii}$$

$$(i) \wedge (ii) \Rightarrow L(\overline{M}) = \overline{L}$$

We have found a DFA for the complement language so $\overline{L}$ is regular.