Developmental Science

Developmental Science 2017; 20: e12459 DOI: 10.1111/desc.12459

PAPER

Mastery of the logic of natural numbers is not the result of mastery of counting: evidence from late counters

Julian Jara-Ettinger,[1] Steve Piantadosi,[2] Elizabeth S. Spelke,[3] Roger Levy[1] and Edward Gibson[1]

1. *Department of Brain & Cognitive Sciences, Massachusetts Institute of Technology, USA*
2. *Department of Brain & Cognitive Sciences, University of Rochester, USA*
3. *Department of Psychology, Harvard University, USA*

Abstract

To master the natural number system, children must understand both the concepts that number words capture and the counting procedure by which they are applied. These two types of knowledge develop in childhood, but their connection is poorly understood. Here we explore the relationship between the mastery of counting and the mastery of exact numerical equality (one central aspect of natural number) in the Tsimane', a farming-foraging group whose children master counting at a delayed age and with higher variability than do children in industrialized societies. By taking advantage of this variation, we can better understand how counting and exact equality relate to each other, while controlling for age and education. We find that the Tsimane' come to understand exact equality at later and variable ages. This understanding correlates with their mastery of number words and counting, controlling for age and education. However, some children who have mastered counting lack an understanding of exact equality, and some children who have not mastered counting have achieved this understanding. These results suggest that understanding of counting and of natural number concepts are at least partially distinct achievements, and that both draw on inputs and resources whose distribution and availability differ across cultures.

Research highlights

- We explore the relation between children's acquisition of counting and their non-verbal understanding of exact number.
- Because, in the US, these two acquisitions develop over a limited timespan, we tested a population where children master counting at a delayed age and with greater variability: the Tsimane' from the Bolivian Amazon.
- We find that mastery of counting and the ability to understand the logic of exact numerical equality emerge together, controlling for age and education. However, these two acquisitions do not emerge in a specific order.
- These results suggest that counting and understanding the natural number system are at least partially distinct achievements, and that both draw on inputs and resources whose distribution and availability differ across cultures.

Introduction

Children's acquisition of natural numbers (the numbers we use to count) is a remarkable conceptual achievement (Carey, 2009). When learning to count, children progress through several systematic stages of knowledge (Lee & Sarnecka, 2010, 2011; Sarnecka & Lee, 2009; Wynn, 1990, 1992). By approximately age 2, children in industrialized societies learn to recite the first 10 or more number words in order (Fuson, 1988), but they do not understand their meaning, and therefore fail to produce the appropriate number of objects in response to verbal queries (Wynn, 1990, 1992; Carey, 2009). Over the following year, children sequentially learn to produce the

Address for correspondence: Julian Jara-Ettinger, Department of Brain & Cognitive Sciences, Massachusetts Institute of Technology, 77 Massachusetts Ave. Bldg. 46-4011AA, Cambridge, MA 02139, USA; e-mai: jjara@mit.edu

correct number of objects in response to ‘one’, ‘two’, ‘three’, and sometimes ‘four’. Throughout these stages, children, called subset-knowers, have limited understanding of the meaning of the other words in their count list. Subset-knowers understand that only one number word can apply to a set at any given time; that this number word continues to apply to the set as long as the set is intact; and that a new number word should be used when the set’s size changes (Brooks, Audet & Barner, 2013; Condry & Spelke, 2008; Sarnecka & Gelman, 2004). Nevertheless, subset-knowers fail to understand that sets of the same size must be associated with the same number word (Condry & Spelke, 2008; Sarnecka & Gelman, 2004).

After learning the meaning of the first three or four number words, children’s behavior undergoes a striking change (Carey, 2009; Piantadosi, Tenenbaum & Goodman 2012): they produce the appropriate exact quantity in response to any word in their count list, becoming full counters.[1] In the immediate time after they become full counters, children continue to have a fragile understanding of number words. For example, they do not understand that consecutive number words always refer to consecutive cardinal values (Davidson, Eng & Barner, 2012). Nevertheless, full counters’ understanding of number words quickly develops, and children master the logic of all number words, even those they cannot count up to (i.e. even if a full counter cannot count up to 80, they still understand that 80 refers to an exact cardinal value) (Lipton & Spelke, 2006).

Broadly, two types of accounts of the developmental relationship between counting and exact number have been proposed.[2] The first type of account proposes that humans understand the logic of exact number before they begin to learn the meaning of number words and counting (Gallistel & Gelman, 1992). That is, subset-knowers understand that collections of objects have an exact numerical size, and they understand how different manipulations affect that value (e.g. adding one item increases the set’s exact size by one, and then removing one item restores the set’s original exact size). This understanding may be available at birth, or mature before children begin to learn the meaning of number words. Under this account, children only learn that each exact number has a name (e.g. ‘one’, ‘twenty-four’) and that these names are ordered in the count list (i.e. ‘five’ is the name of the exact numerical size that is exactly one element bigger than the exact numerical size named by ‘four’). If this account is correct, then children’s failure to use number words correctly does not reflect a limit to their numerical concepts; instead, it implies that they have not learned that number words are names for exact numbers.

On this account, children’s understanding of number resembles that of adults. We, as adults, know that a bucket of sand consists of an exact number of grains. Furthermore, we understand that adding or removing a single grain of sand changes this number (even though the bucket looks the same), whereas substituting one grain for another does not. This knowledge, however, does not depend on knowledge of the exact number of grains in the bucket, on possession of any means to enumerate the grains, or on knowledge of the specific word that refers to this cardinal value. Young children may share this ability to represent and track changes to a set’s exact numerical size, but nevertheless struggle to understand how counting procedures work and what information they provide. We call these types of proposals ‘[exact numerical] concepts before counting’.

The second type of account proposes that humans do not understand the meaning and logic of exact number until they learn to enumerate sets by counting. That is, young children have a poor understanding of what a set’s exact numerical size is or how it changes under different transformations. By learning the meanings of the first number words and the workings of the counting procedure of their culture, children come to master the logic of exact number. In particular, Carey (2009) proposed that children undergo a sharp conceptual change when they decipher the meaning of their counting procedure. We call these types of proposals ‘[exact numerical] concepts through counting’.

Although there is a large literature investigating children’s understanding of number words (e.g. Sarnecka & Gelman, 2004; Sarnecka & Wright, 2013; Brooks *et al.*, 2013; Condry & Spelke, 2008), this literature does not distinguish the above accounts. Because the tasks in these papers all involve the use of number words, children may fail these tasks either because they do not understand the logic of exact number, or because they do

[1] Full counters are usually called Cardinal Principle knowers (CP-knowers). However, this term assumes that they understand the logic of the natural numbers: an assumption that the present research aims to test. Therefore, we use this more neutral terminology.

[2] The accounts we focus on here should not be confused with those of an earlier debate over the relation between the principles underlying counting and the skills needed to count effectively (Gelman & Gallistel, 1978; Briars & Siegler, 1984). With most current investigators (e.g. Leslie, Gelman & Gallistel, 2008; Carey, 2009), we assume that the cardinal principle underlying counting – the understanding that the last tag in a counting routine indicates a property of the entire set – namely its exact size – is acquired, and we focus on the open question of *what* children learn when they acquire this principle: do they only come to understand how counting works, or do they also develop an understanding of exact numerical magnitudes?

not know that number words refer to exact numerical values. Thus, children's understanding of the logic of exact number needs to be assessed without relying on number words.

A long history of research, beginning with Piaget (1968), has probed children's understanding of number, using tasks that do not require mastery of numerical language and counting. Piaget's work on number concepts sprang from a general theory of the development of quantitative concepts, whereby young children (in the preoperational stage; ages 2–7) have an undifferentiated representation of quantity that fuses number with other perceptible features of a set, such as the length of the array formed by its elements. His tasks focused on children's understanding that number remains constant when other dimensions of quantity (such as length) are transformed (see Hyde, 1970, chapter 4 for a succinct review of Piaget's views). Subsequent research revealed, however, that Piaget's theory was incorrect (Gelman, 1972; Gelman & Gallistel, 1978; Mehler & Bever 1967; Rose & Blank, 1974), and experiments now provide evidence that even human infants represent number independent of other quantities or perceptual features of arrays (e.g. Libertus, Starr & Brannon, 2014), although infants are nonetheless sensitive to other quantities (e.g. Lourenco & Longo 2010) and to relationships between number and length (de Hevia, Izard, Coubart, Spelke & Streri, 2014). These findings do not reveal, however, whether young children's numerical concepts have the full power of the natural number system: they are consistent with both the 'concepts before counting' and the 'concepts through counting' views.

The first critical experiments that bear on this question, to our knowledge, were conducted by Izard, Streri and Spelke (2014). Izard *et al.* presented 2-year-old subset-knowers with a set of five or six featurally indistinguishable finger puppets (e.g. frogs), each paired with a branch of a six-branch tree (a reference set), and then moved all the frogs into an opaque box. Then the experimenter shook the puppets in the box, surreptitiously removed one puppet from the original set of six (so that the box in both conditions now contained only five puppets), and encouraged children to put all the frogs back on the tree. After the children placed the five frogs back in their branches, they continued to search the box for a longer time if the original set consisted of six frogs. Children failed to show this pattern in a replication of the experiment using a tree with 11 branches, providing evidence that their successful discrimination of five from six puppets depended on the one-to-one correspondence of puppets to branches.

Subsequent studies therefore used the finger puppet task to probe children's understanding of exact numerical equality. Children failed to distinguish five from six puppets if they saw that one puppet was added to or removed from the box while all the puppets were hidden inside it. Critically, the same children who failed to understand these transformations with a set of five or six puppets succeeded when the set only contained two or three puppets, suggesting that children's failures on these transformations did not stem from any difficulty in perceiving, remembering, or understanding the transformations that resulted in the addition or subtraction of one object. If they did, the set's size should not have influenced their performance. Nevertheless, these transformations did not affect children's representations of the number of puppets inside the box.

Izard also tested children's reactions to two events involving both the addition and the subtraction of a single puppet. In one event, five or six puppets entered the box and then one of the puppets was removed from the box and returned to it: a transformation that preserved the identities of the individual puppets. In the other event, one puppet was removed from the box and a different, featurally indistinguishable puppet replaced it: a transformation that substituted one puppet for another. Although the same numerical transformations were presented in these two events, children responded to them differently. Children searched appropriately after the identity event but not after the substitution event, even though all the objects involved in these events were visually indistinguishable and the events themselves were highly similar in timing and appearance.

The contrast between children's understanding of the identity and substitution events is striking because they involve the same numerical transformations of addition and subtraction. If you have six cookies on a plate and take one away, and then replace it with an identical cookie, it makes no difference to the resulting exact numerical size which of the cookies you took, and which cookie you replaced it with: the resulting size will always be six. For the children in Izard's studies, however, the identities of the individual objects that were removed and added to the initial set mattered. Izard *et al.*'s findings provide evidence that young children can use the one-to-one correspondence to reproduce an exact set of objects, but that they do not represent that set's exact numerical size: a value that is transformed in a lawful way by addition or subtraction of one individual, regardless of the identity of that individual. Although young children understand that number is *conserved* over non-numerical transformations, contrary to Piaget's theory and findings, they evidently do not understand that number is *restored* after the paired numerical transformations of adding and subtracting one. The subset-knowers in these experiments failed to understand the logic of exact numerical equality.

Izard *et al.*'s results directly challenge Gallistel and Gelman's (1992) version of the 'concepts before counting' account. If 2-year-old subset-knowers do not represent a set's exact numerical size, then mastery of the logic of natural numbers cannot be reduced to learning the names for exact numbers and how to calculate them. These findings are nevertheless consistent with weaker versions of the 'concepts before counting' account. Children's understanding of the logic of exact number may still be necessary for children to master numbers and counting and simply develop sometime after the second year of life. The findings also are consistent with the 'concepts through counting' account: children may come to master the logic of exact number by mastering the logic of counting. More generally, the findings raise two questions. First, given that 2-year-old subset-knowers do not appreciate the logic of exact numerical equality, when does this understanding emerge? Second, what is the relation between children's understanding of exact equality, their learning of numerical language, and their mastery of a culture-specific counting procedure? In particular, is understanding the logic of exact equality a prerequisite for counting, a consequence of learning to count, or neither?

These questions are difficult to answer in industrialized populations, because children's numerical concepts, language mastery, and counting skill develop in parallel, over a short age span. However, the Tsimane', a native Amazonian group of farmer-foragers living in the lowlands of Bolivia (Huanca, 2008), learn to count at later ages and with a more variable timeline. While most children in industrialized societies master counting by age 4,[3] it takes Tsimane' children 2–3 times as long to learn to count[4] (Piantadosi, Jara-Ettinger & Gibson, 2014). This large variability thus enables us to evaluate the relationship between counting and exact equality while controlling for age and years of education.

To answer these questions, we assessed Tsimane' children's understanding of number words and counting (using Wynn's Give-N task), and their understanding of the logic of exact equality (using a simplified version of Izard's puppets task), over a wide age span. If children's mastery of the logic of exact equality is related to their knowledge of number words and counting, then children's performance on these two tasks should be correlated, controlling for age and education. Second, if children's mastery of the logic of exact equality is strictly tied to their understanding of counting, then there should be little evidence of subset-knowers who understand the logic of exact equality or of full counters who do not.

Experiment

Methods

We tested children both on a standard verbal task assessing children's understanding of the number words used in verbal counting (the Give-N task) and on a non-verbal set transformation task assessing children's understanding of exact equality (hereafter, the exact equality task). We additionally collected each participant's age and years of education through parental reports.

Participants

The training phase of the experiment was designed using pilot data from nine participants who were excluded from analyses. For the experiment, 63 children (mean age: 6.83 years; *SD*: 1.75 years; range: 4–11 years) were recruited from six Tsimane' communities near San Borja, Bolivia, in collaboration with the *Centro Boliviano de Investigación y de Desarrollo Socio Integral* (CBIDSI), which provided interpreters, logistical coordination, and expertise in Tsimane' culture. All analyses were performed after data collection was completed.

Procedure

First we determined each child's ability to count through a staircased version of the Give-N task (Wynn, 1992). In this task, children were asked to move N (out of 10) chips from one sheet of paper to another, where N varied between 1 and 8. All chips were returned to the first sheet of paper after each trial. We began the task by asking the child to move 4 chips from one sheet to the other. If the child did not respond to the first request (either because of shyness or because they were puzzled by the request) we discarded this first trial and asked children to move 3 chips instead. The task followed a 1-up for correct / 1-down for

[3] Data from these other countries come from a variety of studies (Negen & Sarnecka, 2009, 2012; Sarncecka & Carey, 2008; Sarnecka, Kamenskaya, Yamana, Ogura & Yudovina, 2007; Sarnecka & Lee, 2009; Slusser & Sarnecka, 2011); we thank Meghan Goldman and Barbara Sarnecka for compiling and sharing these data.

[4] This delay may be due to differences in the material circumstances, cultural experience, and linguistic experience (all of which we collectively refer to as *input*) of young Tsimane' children, relative to children in industrialized societies (Foster, Byron, Reyes-García, Huanca, Vadez *et al.*, 2005; Huanca, 2008; Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher & Gunderson 2010).

incorrect staircased procedure and ended when (1) the child correctly moved 8 chips from one sheet to the other, (2) the child's counting stage could be obtained through the standard classification rules after 8 queries (see Piantadosi *et al.*, 2014, for rules), or (3) the child wanted to stop. Because visitors to the communities are rare, and the Tsimane' are less familiar with behavioral experiments, it was often challenging to explain that we were interested only in the participant's behavior, regardless of whether they responded correctly or not. As a result, parents sometimes blurted out help (e.g. 'pick up one more!'). The experimenter noted the trials when participants received help and ran additional trials, ignoring the first two stopping rules. In some cases, a child's knower-level was not evident from their pattern of responses. In these situations, the experimenter asked the child if they would be willing to do a few additional trials. If the participant agreed, the experimenter chose the queries that would help the most in determining the child's number-knower level, based on their past responses. An undergraduate volunteer (see Acknowledgments) and the first author independently determined which number-knower level best fit each child's performance (75.00% agreement; Cohen's weighted inter-rater agreement kappa = 0.92). A third coder (second author) served as a tiebreaker for cases when the first two coders disagreed. All coders were blind to all other participant information (age, education, and performance on the non-verbal set transformation task).

The exact equality task consisted of a non-verbal assessment of children's understanding of this aspect of natural number. We began with a training phase to familiarize the participants with the displays and questions. Two drawings of children (distinguishable only by their shirt color) were placed on opposite sides of a small table. The participants were told that we would distribute paper pictures of cookies between the two children.[5] After each distribution, participants were asked whether the children had an equal quantity of cookies or different quantities of cookies (the correct answer was 'equal' in half of the trials). Participants completed four simple trials in the training phase: One cookie for each child; one cookie for one child and two cookies for the other; one cookie for one child and eight cookies for the other; and eight cookies for each child (see main task procedure for explanation of how the eight cookies were distributed). Training phase trials were presented in a random order except that the trial where eight cookies were given to each child was always the final one. When a participant responded incorrectly, we asked follow-up questions that helped the participant understand the task (e.g. 'Does one child have more than the other?', 'Can you point to which one has more?'). The interpreter then explained the task again using the current trial's cookie distribution as an example.

Figure 1 shows the exact equality task's procedure. Part 1 was identical to the last training trial. The experimenter announced that a set of (N = 16) cookies would be divided evenly between the two children. The cookies were distributed using one-to-one correspondence (i.e. taking two cookies at a time and giving one to each child) and were arranged into two 4×2 matrices. In part 2, the participant was asked to confirm that the two children had an equal quantity of cookies. If the participant responded incorrectly we asked follow-up questions similar to those in the training phase and restarted the trial. Afterwards, the experimenter rearranged the sets of cookies into piles such that the cookies overlapped with each other, making them difficult to individuate (and thus to perform one-to-one matching) and minimizing geometric cues to quantity. In part 3, the experimenter applied a simple transformation to one of the cookie piles. The interpreter described the transformation while the experimenter performed it. However, the transformation was performed so that it could be followed and understood in the absence of the linguistic description. In part 4, the experimenter asked the participant 'Do the children have an equal quantity of cookies or different quantities of cookies?' The interpreter then informed the experimenter if the child had responded 'equal' or 'different'. This procedure was repeated six times applying the following transformations in a random order: (1) Stir the cookies, (2) give one cookie (addition), (3) take one cookie (subtraction), (4) replace one cookie by another cookie of the same appearance (substitution), (5) take and return, or add and remove, one cookie (identity; the operation order in this transformation was randomized across children), and (6) take half of the cookies. Thus, half of the transformations disrupted exact equality and half did not. If children understand exact equality, they should understand which of these transformations do and do not change the set's size. Occasionally children got distracted in the middle of a trial and looked away. When this happened, the experimenter restarted the trial. Because children can succeed in the stir and the take-half transformations without understanding the logic of exact equality, we will refer to these two transformations as the control transformations, and to the rest as the primary transformations (addition, subtraction, substitution, and identity transformations).

[5] The types of objects that were given to the pictured children were the same within-trials but changed across trials for each child. The possible objects were cookies, candies, and drawings of pencils. For simplicity, we always refer to the objects as cookies.

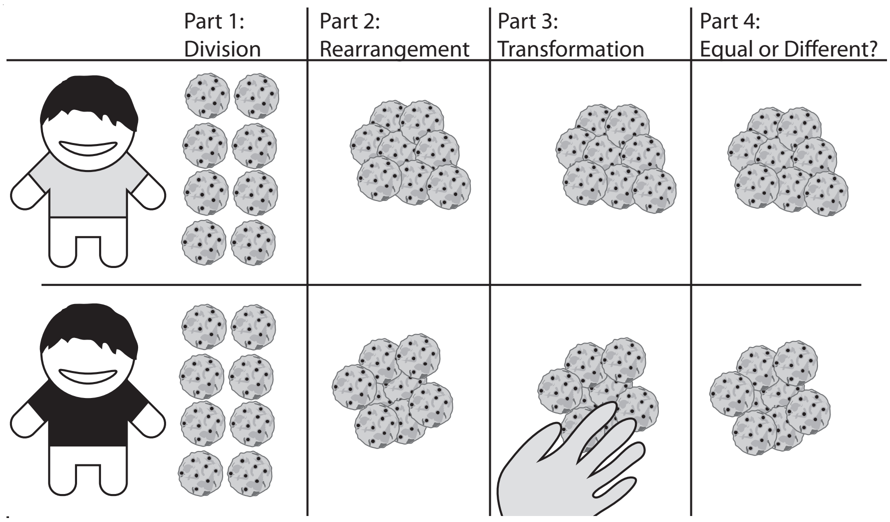

Figure 1 *General task procedure. Sixteen cutout cookie drawings were evenly distributed between two pictures of identical children with different shirts (Part 1). After the participant confirmed that the both children had the same amount of cookies the experimenter rearranged each set to remove shape cues (Part 2). Next, the experimenter performed a set transformation on one of the child's cookies (Part 3). Last, participants were asked if both children had an equal amount of cookies.*

Results

The goal of this project was to answer two questions. First, when do children master exact equality? Second, is understanding exact equality a prerequisite for counting, a consequence, or neither? We approached the first question by calculating partial correlations. If performance on the exact equality task correlates with performance on the Give-N task, controlling for age and years in school, then children likely learn to reason about exact equality as they learn number words and counting. Alternatively, if performance on the exact equality task correlates with age or years in school, controlling for performance on the Give-N task, then children likely learn to reason about exact equality independent of their knowledge of number words.[6] We approached the second question by searching for the existence of children in each of the four categories defined by crossing two states of knowledge of counting ({full counter, subset-knower} by two states of knowledge of exact equality {understands exact equality, does not understand exact equality}). Past work suggests that there are full counters who understand exact equality (Lipton & Spelke, 2006), and subset-knowers who do not understand exact equality (Izard *et al.*, 2014). Thus, our main focus is on searching for the existence of full counters who do not understand exact equality, and subset-knowers who do. Because we expect children to fall in all four quadrants simply due to noise in their performance, we use a stringent and conservative rule to categorize children who understand the logic of exact equality: perfect performance across the four primary transformations. Conversely, when searching for the existence of children who fail to understand the logic of exact equality, we analyze their errors in detail to test if they can be explained by performance errors.

Figures 2 and 3 summarize our complete data set. In Figure 2 each column is a single child and each row is a single set transformation. Children are sorted by their age in years (horizontal axis) and sub-sorted by number-knower level in each year group. Filled in squares indicate correct answers and empty squares indicate incorrect answers; children's number-knower level is color-coded. Figure 3 shows the same data as a function of children's number-knower level, their performance on the exact equality task (see below for details), and their age. To ensure that children's performance on the equality task captured their understanding of exact numerical equality, we first confirmed that children's patterns of responses did not conform to an approximate interpretation of equal (see SI text for details).

When do children understand exact equality?

To measure how understanding exact equality relates to children's number word knowledge, we computed a correlation between number-knower level (numerically

[6] The structure of the partial correlations may be more complex. However, in light of our results, discussing these alternatives is unnecessary.

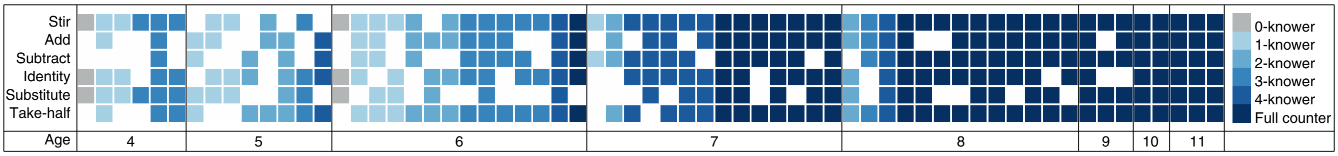

Figure 2 *Children's individual performance on each of the six transformations of exact equality task. Each column shows a child's performance on the task. Participants are sorted by age and color-coded by their number-knower stage. Each square represents success in the row's transformation.*

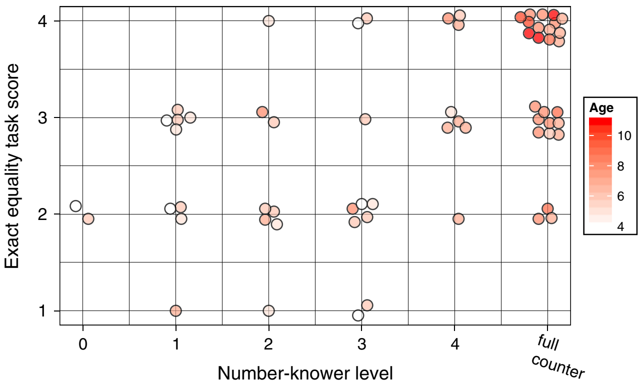

Figure 3 *Results as a function of children's number-knower level (x-axis), their performance on the exact equality (y-axis), and their age (color coded). Each point represents a child.*

coded 0–5, with 5 = full counter) and overall accuracy on the exact equality task (defined as the number of primary transformations the child reasoned about correctly).[7] In a nonparametric Kendall correlation (Johnson & Wichern, 2002) children's number-knower level significantly correlated with their performance on the exact equality task after controlling for both age and education (Tau = 0.23; 95% CI: 0.06–0.40; $p < .01$ Kendall tau rank partial correlation). This indicates that children's understanding of number words is linked to their understanding of exact equality, independent of their age or schooling. As such, this suggests that children's mastery of exact equality is linked with their understanding of, or exposure to, number words. These results, together with the findings from Izard *et al.* (2014), challenge the 'concepts before counting' account, as they show that children do not understand the logic of exact equality before they learn number words, and that this understanding does not emerge at a given age. Instead, mastery of exact equality emerges together with children's understanding of number words.

These findings, however, do not imply that age or schooling have no influence on the exact equality task. Children's performance on the exact equality task marginally correlated with their age when controlling for number word knowledge (but not schooling), and it correlated with their schooling when controlling for number word knowledge (but not age). This suggests that age, schooling, or some other factor that correlates with these two also influenced children's performance on our task (see SI text for details).

Is understanding the logic of exact equality a prerequisite for counting, a consequence of counting, or neither?

Given that understanding number words is related to understanding exact equality, we next focused on the order in which children master exact equality, on one hand, and counting on the other. To do this, we searched in our data for the existence of children in each of the four possible states regarding counting knowledge (full counters, subset-knowers) and exact equality understanding (understands exact equality, does not understand exact equality). Different concerns arise when searching for the existence of children in each quadrant. If some full counters make errors in the exact equality task, we want to ensure that they were not caused by distraction. In contrast, if some subset-knowers succeed in the exact equality task, we want to ensure that this finding cannot be explained by chance alone. Consequently, we analyzed each quadrant individually. As in the previous analysis, we used the child's aggregate performance on the four primary set-transformations (add one, take one, and the identity and substitution transformations) as an overall measure of knowledge of exact equality. Thus, perfect or ceiling performance refers to children responding the four main set-transformations correctly.

Some, but not all, full counters understand exact equality

As expected, our dataset contained children who could both count and understand exact equality. In all, 14/26 = 53.85% (95% CI: 33.37–73.41) of full counters

[7] Including the stir and take-half transformations in any of our analyses does not qualitatively change our findings.

performed at ceiling (all four main set-transformations correct) on the exact equality task. This proportion is significantly higher than expected by chance given that each participant responded to four transformations ($p < .0001$ by binomial test).

Next, we tested whether there were any full counters who did not understand the logic of exact equality. Altogether, 12/26 = 46.15% of full counters made at least one error on the exact equality task. However, all of these participants were close to ceiling, making at most two errors (9/26 = 34.62% only erred in one transformation, and 3/26 = 11.54% erred on two transformations). It is possible then that these children understand the logic of exact equality and that their errors were simply due to temporary external distractions. If this were true, then their errors should be uniformly distributed across all six transformations (all transformations were approximately matched for length, and trials were restarted if the child looked away at any point during the transformation; see Methods). However, this was not the case. Instead, full counters' errors were concentrated in the substitution transformation (7 children failing this transformation; thus mimicking patterns found in industrialized societies; see Discussion), followed by errors in the identity and the addition transformations (4 and 3 errors, respectively), and last in the take one transformation (1 error). Although the stir and take-half transformations were not included in this analysis, all of these participants also succeeded in both of the control transformations. Moreover, a linear regression fit to subset-knowers' error rate using age and education as the dependent variables predicted that full counters who did not perform at ceiling should have similar error rates to the observed ones (using their age and years in school as the predictors; see SI text for regression and prediction details). This suggests that full counters' errors were not substantially lower compared to the error rate of subset-knowers (adjusting for age and education). Altogether this suggests that full counters' errors were likely not due to the product of temporary distraction or carelessness, but because exact equality is a fragile notion even after learning how to count: their errors were concentrated on the substitution transformation ($p < .05$ by permutation test), mimicking patterns found in the US, and their error rate was similar to that of subset-knowers with similar age and schooling. Our results therefore show that children can master the counting algorithm without fully understanding exact equality. However, in contrast to studies in industrialized populations (e.g. Davidson *et al.*, 2012), our task did not use number words, thus showing that some full counters not only fail to understand how to use number words, but they also fail to understand the underlying concepts that number words capture.

Some, but not all, subset-knowers understand exact equality

Replicating findings from the US (Izard *et al.*, 2014), our dataset also contained subset-knowers who failed to understand exact equality. In all, 31/37 = 83.78% (95% CI: 67.99–93.81) of subset-knowers made at least one error in the set transformation task ($p < .0001$ by binomial test): 12.90% failed on one transformation, 48.39% failed on two transformations, and 38.71% failed on three transformations.

Last, we asked if any subset-knowers understood exact equality. Six out of the 37 subset-knowers (16.22%) performed at ceiling on the four tested transformations, compared to 6.25% expected by chance ($p < .05$).[8] Given that the number of subset-knowers performing at ceiling on the exact equality task is significantly higher than expected by chance, this finding suggests that our dataset contains at least one child who cannot count but nevertheless understands the logic of exact equality.[9] Thus, to our knowledge, our experiment is the first to provide evidence that understanding the logic of exact equality can also precede knowledge of counting. Altogether, our results suggest that there is no strict implicational relationship between the development of the logic of exact equality and mastery of counting.

Discussion

Here we explored how children's understanding of exact numerical equality relates to the mastery of counting. Our findings show that children's understanding of the

[8] The strength of evidence for this finding would drop if any of the six subset-knowers performing at ceiling were actually full counters who were misclassified by our Give-N task and subsequent coding. However, incorrect classification of a true full counter as a subset-knower is unlikely with the present Give-N procedure, which began with 4 (i.e. a quantity at the border between these categories) and proceeded for eight staircased trials. Post-hoc inspection of children who most plausibly might be misclassified suggested that there were no children who might have been misclassified as subset-knowers. If any classification was erroneous, it was only one participant classified as a full counter who was at chance (3/8 correct) on trials of $N = 5$ and above in the Give-N task, but was at ceiling in the set-transformation task. Reclassifying that child as a subset-knower would strengthen the evidence that there are some subset-knowers who understand exact equality.

[9] Importantly, however, our measure of whether a subset-knower understands exact numerical equality is conservative because it requires perfect performance on our tasks. It is therefore possible that our dataset contains more subset-knowers who understand exact equality but did not perform at ceiling due to performance errors. As such, using a less stringent measure would only strengthen our conclusions.

logic of exact equality – the understanding that collections of objects have an exact numerical size and the understanding of how set manipulations affect that value – emerges at around the same time that children master counting, independent of both age and education. These two abilities, however, do not emerge in a specific order. Instead, children can be full counters without understanding exact equality, and they can be subset-knowers who nevertheless understand exact equality. Importantly, when searching for the existence of subset-knowers who understand exact equality we used a stringent definition (perfect performance on the main set transformations), making our estimate of subset-knowers who understand exact numerical equality conservative. Conversely, when searching for the existence of full counters who do not understand exact numerical equality we analyzed their errors in detail to ensure that our conclusion is warranted that such children are present in our sample. Together, these analyses show that (1) non-verbal mastery of the logic of exact number is variable across cultures and does not emerge uniformly at a certain age; and that (2) mastery of counting is neither necessary nor sufficient for understanding the logic of exact number.

Our findings directly challenge both the 'concepts before counting' accounts and Carey's (2009) version of the 'concepts through counting' account. Mastery of the logic of exact numerical equality appears to develop gradually, but is not tightly linked with children's acquisition of counting. Together, (1) the correlation between understanding of exact equality and number-word knowledge, (2) the delay the Tsimane' show in both these acquisitions, and (3) the absence of sharp changes in children's understanding of exact equality when they master counting suggest that learning to count and learning the underlying number system that counting captures are at least partially distinct achievements, and that both draw on inputs and resources whose distribution and availability differ across cultures.

Our findings are consistent with results testing children from industrialized societies. In the US, young children have a poor understanding of the meaning of number words (Brooks *et al.*, 2013; Davidson *et al.*, 2012; Condry *et al.*, 2008). Although these studies focused on tasks with number words, our results suggest that children's failures on these tasks may be partially caused by a deeper lack of appreciation of exact number. Furthermore, our findings replicate and extend the findings of Izard *et al.* (2014). First, we found that in a similar task, but with a different culture, subset-knowers fail to represent a set's exact numerical size. Moreover, our replication matches Izard *et al.*'s at a more intricate level: we also find that substitution transformations are more challenging for children than identity transformations. This finding suggests that understanding that sets have exact numerical sizes, and appreciating how these sizes are disrupted or preserved through different transformations, develops gradually. In particular, children first learn that adding and then removing (or removing and then adding) a single individual restores the original exact size of a set, despite the fact that this event involves two numerical operations. Then children learn that adding or removing even one element from a set always changes its exact size. Finally, children learn that removing one element and then adding a different element restores the original exact size, despite the fact that this last transformation (a substitution) involves the same composition of two numerical operations as the very first transformation that they mastered (see SI text for a quantitative analysis of this development using a mixed-effects model).

Children's differing performance on the substitution and identity transformations has now been demonstrated in two laboratories using different displays and methods and conducted in different cultures. The identity and substitution transformations used in these experiments were perceptually similar, and they made similar demands on memory. Above all, these two transformations were identical in their effects on number: the combined addition and subtraction of one element from a set restores the original exact numerical size of that set, regardless of which element or elements participate in the transformation. Despite their numerical equivalence and superficial similarity, however, young children treat these two transformations differently until they have developed considerable experience with numbers and number words. These findings provide clear evidence that performance with these transformations reflects some conceptual development rather than age-dependent task constraints. This convergence is particularly striking because of the variability in the timing of Tsimane' children's learning to count: variability that allowed us to disentangle age from other factors.

Could our results be explained by performance errors, rather than the genuine presence of children who have come to understand counting but not the logic of exact equality, or the reverse? In one sense, tracking two distinct individuals (in the substitution transformation) must place more performance demands on children than does tracking a single individual (in the identity transformation). However, this explanation raises a key question: why are children in our task tracking individuals at all, if they have and use numerical concepts to reason about sets? When an older child is told that five marbles are in one box and six marbles are in another box, the child does not need to see or track any individual marbles in order to conclude that the second box has more marbles (Davidson *et al*., 2012). Similarly, if a single individual is added to or removed from a set, the change in the set's exact size does not depend on which individual participated in that transformation. If some children fail on the substitution transformation because they have difficulty tracking the individual members of that set, that failure implies that they are not using a concept of exact number to reason about this transformation. Moreover, the existence of children who understand counting, as assessed by the Give-N task, yet fail the test of exact equality, suggests that mastery of counting is not sufficient for the mastery and productive use of this key numerical concept.

Finally, it is important to note that our results do not imply that children learn non-verbal number concepts and counting independently, but only that each of these achievements *can* be attained before the other. Children may be able to reach a mature conception of exact number and counting in different ways. A child who masters counting before understanding exact number may be able to use the counting algorithm to build an understanding of exact number (e.g. by noticing that small sets of the same size produce the same output on the counting algorithm, and that adding one element to a set changes the algorithm's output by one spot on the count list). Conversely, a child who masters exact number before mastering counting may come to realize that number words refer to exact numerical values and that the counting algorithm computes these values.

Acknowledgements

We thank Ricardo Godoy and Tomas Huanca for logistical help, Dino Nate and Robertina Nate for help translating and running the task, Mika Maeda for help with coding Give-N task results, and Barbara Sarnecka, Laura Schulz, Josh Tenenbaum and two anonymous reviewers for useful comments. Research reported in this publication was supported by the Eunice Kennedy Shriver National Institute of Child Health & Human Development of the National Institutes of Health under Award Number F32HD070544 to SP. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health. This work was supported by National Science Foundation Grant 1022684 from the Research and Evaluation on Education in Science and Engineering (REESE) program to EG.

References

Briars, D., & Siegler, R.S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, **20** (4), 607–618.

Brooks, N., Audet, J., & Barner, D. (2013). Pragmatic inference, not semantic competence, guides 3-year-olds' interpretation of unknown number words. *Developmental Psychology*, **49** (6), 1066–1975.

Carey, S. (2009). *The origin of concepts*. Oxford: Oxford University Press.

Condry, K.F., & Spelke, E.S. (2008). The development of language and abstract concepts: the case of natural number. *Journal of Experimental Psychology: General*, **137** (1), 22–38.

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition*, **123** (1), 162–173.

de Hevia, M.D., Izard, V., Coubart, A., Spelke, E.S., & Streri, A. (2014). Representations of space, time, and number in neonates. *Proceedings of the National Academy of Sciences, USA*, **111** (13), 4809–4813.

Foster, Z., Byron, E., Reyes-García, V., Huanca, T., Vadez, V. *et al*. (2005). Physical growth and nutritional status of Tsimane' Amerindian children of lowland Bolivia. *American Journal of Physical Anthropology*, **126** (3), 343–351.

Fuson, K.C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Gallistel, C.R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, **44** (1), 43–74.

Gelman, R. (1972). Logical capacity of very young children: number invariance rules. *Child Development*, **43** (1), 75–90.

Gelman, R., & Gallistel, C. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gunderson, E.A., & Levine, S.C. (2011). Some types of parent number talk count more than others: relations between parents' input and children's cardinal-number knowledge. *Developmental Science*, **14** (5), 1021–1032.

Huanca, T. (2008). *Tsimane' oral tradition, landscape, and identity in tropical forest*. La Paz: South-South Exchange Programme for Research on the History of Development (SEPHIS).

Hyde, D.M.G. (1970). *Piaget and conceptual development: With a cross-cultural study of number and quantity*. New York: Holt, Rinehart & Winston.

Izard, V., Pica, P., Spelke, E.S., & Dehaene, S. (2008). Exact equality and successor function: two key concepts on the path towards understanding exact numbers. *Philosophical Psychology*, **21** (4), 491–505.

Izard, V., Streri, A., & Spelke, E.S. (2014). Toward exact number: young children use one-to-one correspondence to measure set identity but not numerical equality. *Cognitive Psychology*, **72**, 27–53.

Johnson, R.A., & Wichern, D.W. (2002). *Applied multivariate statistical analysis* (Vol. 5, No. 8). Upper Saddle River, NJ: Prentice Hall.

Lee, M.D., & Sarnecka, B.W. (2010). A model of knower-level behavior in number-concept development. *Cognitive Science*, **34**, 51–67.

Lee, M.D., & Sarnecka, B.W. (2011). Number-knower levels in young children: insights from Bayesian modeling. *Cognition*, **120**, 391–402.

Leslie, A.M., Gelman, R., & Gallistel, C.R. (2008). The generative basis of natural number concepts. *Trends in Cognitive Sciences*, **12** (6), 213–218.

Levine, S.C., Suriyakham, L.W., Rowe, M.L., Huttenlocher, J., & Gunderson, E.A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, **46** (5), 1309–1319.

Libertus, M.E., Starr, A., & Brannon, E.M. (2014). Number trumps area for 7-month-old infants. *Developmental Psychology*, **50** (1), 108–112.

Lipton, J.S., & Spelke, E.S. (2006). Preschool children master the logic of number word meanings. *Cognition*, **98** (3), B57–B66.

Lourenco, S.F., & Longo, M.R. (2010). General magnitude representation in human infants. *Psychological Science*, **21** (6), 873–881.

Mehler, J., & Bever, T.G. (1967). Cognitive capacity of very young children. *Science*, **158** (3797), 141–142.

Negen, J., & Sarnecka, B.W. (2009). Young children's number-word knowledge predicts their performance on a nonlinguistic number task. In *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 2998–3003).

Negen, J., & Sarnecka, B.W. (2012). Number-concept acquisition and general vocabulary development. *Child Development*, **83** (6), 2019–2027.

Piaget, J. (1968). Quantification, conservation, and nativism. *Science*, **162** (3857), 976–979.

Piantadosi, S.T., Jara-Ettinger, J., & Gibson, E. (2014). Children's learning of number words in an indigenous farming-foraging group. *Developmental Science*, **17** (4), 553–563.

Piantadosi, S., Tenenbaum, J., & Goodman, N. (2012). Bootstrapping in a language of thought: a formal model of numerical concept learning. *Cognition*, **123**, 199–217.

Richard, A.J., & Dean, W.W. (2002). *Applied multivariate statistical analysis.* London: Prentice Hall.

Rose, S.A., & Blank, M. (1974). The potency of context in children's cognition: an illustration through conservation. *Child Development*, **45** (2), 499–502.

Sarnecka, B.W., & Carey, S. (2008). How counting represents number: what children must learn and when they learn it. *Cognition*, **108** (3), 662–674.

Sarnecka, B.W., & Gelman, S.A. (2004). Six does not just mean a lot: preschoolers see number words as specific. *Cognition*, **92** (3), 329–352.

Sarnecka, B.W., Kamenskaya, V.G., Yamana, Y., Ogura, T., & Yudovina, Y.B. (2007). From grammatical number to exact numbers: early meanings of 'one', 'two', and 'three' in English, Russian, and Japanese. *Cognitive Psychology*, **55** (2), 136–168.

Sarnecka, B.W., & Lee, M.D. (2009). Levels of number knowledge in early childhood. *Journal of Experimental Child Psychology*, **103**, 325–337.

Sarnecka, B.W., & Wright, C.E. (2013). The idea of an exact number: children's understanding of cardinality and equinumerosity. *Cognitive Science*, **37** (8), 1493–1506.

Slusser, E.B., & Sarnecka, B.W. (2011). Find the picture of eight turtles: a link between children's counting and their knowledge of number word semantics. *Journal of Experimental Child Psychology*, **110** (1), 38–51.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, **36** (2), 155–193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, **24** (2), 220–251.

Received: 17 February 2015
Accepted: 26 April 2016

Supporting Information

Additional Supporting Information may be found online in the supporting information tab for this article:

Data S1. Supplemental material