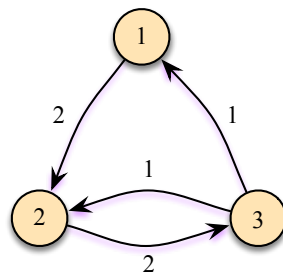

TP4 : Counting

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4 CHAPTER 2 & 6 : MATRICES AND COUNTING

4.1 JUIN 2014, QUESTION 3



Concernant la **theorie des graphes**, soit le graphe dirige et pondere (les poids sont notes a cote des arcs) ci-dessus (40 min).

1. Donnez sa matrices d'adjacence et calculez sa matrice de probabilites de transition.
2. Expliquez en detail (y compris les equations et derivations mathematiques) ce que represente le score PageRank et son interpretation en terme de marche aleatoire sur un graphe (definir la notion de teleportation et la Google Matrix a l'aide des equations, mais par contre sans la personnalisation). Justifiez bien chaque etape.
3. Calculez le score PageRank de base (sans personnalisation) de chaque noeud du graphe sans inclure de teleportation ($\alpha = 1$). Detaillez et justifiez vos calculs.
4. Que se passerait-il au niveau du score PageRank si un des noeuds devenait absorbant (reponse en une phrase) ?

2.6 MATRICES

- 5. Find a matrix \mathbf{A} such that

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \mathbf{A} = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}.$$

[Hint: Finding \mathbf{A} requires that you solve systems of linear equations.]

- 17. Let ' \mathbf{A} and \mathbf{B} be two $n \times n$ matrices. Show that

a) $(\mathbf{A} + \mathbf{B})^t = \mathbf{A}^t + \mathbf{B}^t$.

b) $(\mathbf{AB})^t = \mathbf{B}^t \mathbf{A}^t$.

- 27. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find

a) $\mathbf{A} \vee \mathbf{B}$.

b) $\mathbf{A} \wedge \mathbf{B}$.

c) $\mathbf{A} \odot \mathbf{B}$.

6.1 THE BASICS OF COUNTING

- 1. There are 18 mathematics majors and 325 computer science majors at a college.
 - a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
 - b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?
- 9. How many strings of five ASCII characters contain the character @ ("at" sign) at least once? [Note: There are 128 different ASCII characters.]
- 13. How many strings of three decimal digits
 - do not contain the same digit three times?
 - begin with an odd digit?
 - have exactly two digits that are 4s?
- 17. How many strings of eight uppercase English letters are there
 - a) if letters can be repeated?
 - b) if no letter can be repeated?

- c) that start with X, if letters can be repeated?
 - d) that start with X, if no letter can be repeated?
 - e) that start and end with X, if letters can be repeated?
 - f) that start with the letters BO (in that order), if letters can be repeated?
 - g) that start and end with the letters BO (in that order), if letters can be repeated?
 - h) that start or end with the letters BO (in that order), if letters can be repeated?
- **49.** How many diagonals does a convex polygon with n sides have? (Recall that a polygon is convex if every line segment connecting two points in the interior or boundary of the polygon lies entirely within this set and that a diagonal of a polygon is a line segment connecting two vertices that are not adjacent.)

6.2 THE PIGEONHOLE PRINCIPLE

- **3.** A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
 - a) How many socks must he take out to be sure that he has at least two socks of the same color?
 - b) How many socks must he take out to be sure that he has at least two black socks?
- **25.** Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.

6.3 PERMUTATIONS AND COMBINATIONS

- **7.** How many bit strings of length 10 contain
 - a) exactly four 1s?
 - b) at most four 1s?
 - c) at least four 1s?
 - d) an equal number of 0s and 1s?
- **13.** How many permutations of the letters ABCDEFG contain
 - a) the string BCD?
 - b) the string CFGA?
 - c) the strings BA and GF?
 - d) the strings ABC and DE?
 - e) the strings ABC and CDE?
 - f) the strings CBA and BED?

- **15.** One hundred tickets, numbered 1, 2, 3, ..., 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if
 - a) there are no restrictions?
 - b) the person holding ticket 47 wins the grand prize?
 - c) the person holding ticket 47 wins one of the prizes?
 - d) the person holding ticket 47 does not win a prize?
 - e) the people holding tickets 19 and 47 both win prizes?
 - f) the people holding tickets 19, 47 and 73 all win prizes?
 - g) the people holding tickets 19, 47, 73 and 97 all win prizes?
 - h) none of the people holding tickets 19, 47, 73 and 97 wins a prize?
 - i) the grand prize winner is a person holding ticket 19, 47, 73 or 97?
 - j) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?
- **21.** The English alphabet contains 21 consonants and 5 vowels. How many strings of six lowercase letters of the English alphabet contain
 - a) exactly one vowel?
 - b) exactly two vowels?
 - c) at least one vowel?
 - d) at least two vowels?
- **25.** How many bit strings contain exactly eight 0s and ten 1s if every 0 must be immediately followed by a 1?