UNIVERSITÉ CATHOLIQUE DE LOUVAIN

TP2 : Graphs

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3 CHAPTER 10 : GRAPHS

3.1 GRAPHS ANS GRAPHS MODELS

- **1.** Draw graph models, stating the type of graph (from Table 1) used, to represent airline routes where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with
  a) an edge between vertices representing cities that have a flight between them (in either direction).
  b) an edge between vertices representing cities for each flight that operates between them (in either direction).
  c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.
  d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.
  e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends
- **9.** The intersection graph of a collection of sets $A_1, A_2, ..., A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two

sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

a) $A_1 = \{0, 2, 4, 6, 8\}$, $A_2 = \{0, 1, 2, 3, 4\}$, $A_3 = \{1, 3, 5, 7, 9\}$, $A_4 = \{5, 6, 7, 8, 9\}$, $A_5 = \{0, 1, 8, 9\}$

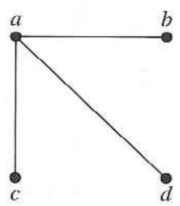
b) $A_1 = \{..., -4, -3, -2, -1, 0\}$, $A_2 = \{..., -2, -1, 0, 1, 2, ...\}$, $A_3 = \{..., -6, -4, -2, 0, 2, 4, 6, ...\}$, $A_4 = \{..., -5, -3, -1, 1, 3, 5, ...\}$, $A_5 = \{..., -6, -3, 0, 3, 6, ...\}$

c) $A_1 = \{x | x < 0\}$, $A_2 = \{x | -1 < x < 0\}$, $A_3 = \{x | 0 < x < 1\}$, $A_4 = \{x | -1 < x < 1\}$, $A_5 = \{x | x > -1\}$, $A_6 = \mathbf{R}$

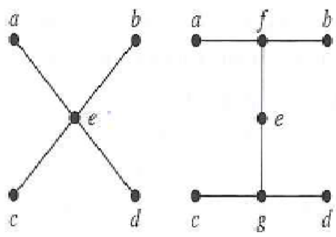
3.2 Graph Terminology and Special Types of Graphs

- **31.** A sequence $d_1, d_2, ..., d_n$ is graphic if it is the degree sequence of a simple graph. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.
  a) 3, 3, 3, 3, 2. b) 5, 4, 3, 2, 1 c) 4, 4, 3, 2, 1 d) 4, 4, 3, 3, 3 e) 3, 2, 2, 1, 0 f) 1, 1, 1, 1, 1

- **37.** Draw all subgraphs of this graph.



- **41.** Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)

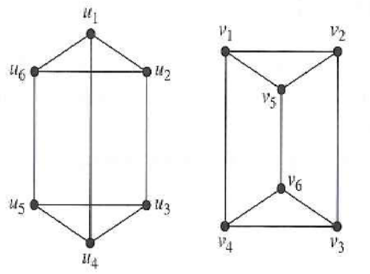


3.3 Representing Graphs and Graph Isomorphism

- **15.** Draw the graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

- **19.** What is the sum of the entries in a column of the adjacency matrix for an undirected graph? for a directed graph?
- **21.** What is the sum of the entries in a column of the incidence matrix for an undirected graph?
- **23.** Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphisme or provide a rigorous argument that none exists.

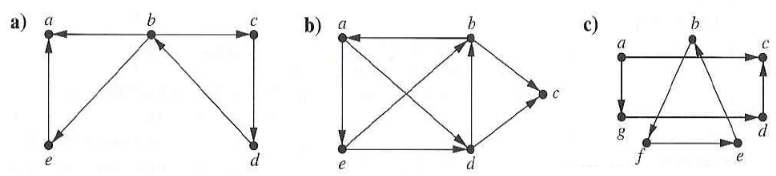


- **33.** Show that the vertices of a bipartite graph with two or more vertices can be ordered so that the adjacency matrix has the form

$$\begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{bmatrix}$$

3.4 Connectivity

- **7.** Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.



- **17.** Find the number of paths between a and c in the graph of Exercice **7.b** of length
a) 2. b) 3. c) 4. d) 5. e) 6. f) 7.
- **45.** Let $P1$ and $P2$ be two simple paths between the vertices u and v in the simple graph G that do not contain the same set of edges. Show that there is a simple circuit in G.