UNIVERSITÉ CATHOLIQUE DE LOUVAIN

TP1 : Logic and Proofs

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1 CHAPTER 1 : LOGIC AND PROOFS

1.1 PROPOSITIONAL LOGIC

- **7.** Let p and q be the propositions
  - p : It is below freezing.
  - q : It is snowing.

  Write these propositions using p and q and logical connectives (including negations).

  a) It is below freezing and snowing.

  b) It is below freezing but not snowing.

  c) It is not below freezing and it is not snowing.

  d) It is either snowing or below freezing (or both).

  e) If it is below freezing, it is also snowing.

  f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

  g) That it is below freezing is necessary and sufficient for it to be snowing.

- **23.** Construct a truth table for each of these compound propositions.

  a) $p \rightarrow (\neg q \vee r)$

  b) $\neg p \rightarrow (q \rightarrow r)$

  c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

  d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

e) $(p \rightarrow q) \vee (\neg q \rightarrow r)$

f) $(\neg p \rightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

- **29.** Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

  a) 101 1110, 010 0001

  b) 1111 0000, 1010 1010

  c) 00 0111 0001, 10 0100 1000

  d) 11 1111 1111, 00 0000 0000

1.2 Application of Propositional Logic

- **3.** You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of *e*: "You are eligible to be President of the U.S.A.," *a*: "You are at least 35 years old," *b*: "You were born in the U.S.A," *p*: "At the time of your birth, both of your parents where citizens," and *r*: "You have lived at least 14 years in the U.S.A."

- **13.** This exercise relates to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, *A* and *B*. Determine, if possible, what *A* and *B* are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions? *A* says "At least one of us is a knave" and *B* says nothing.

- **19.** A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

1.3 Propositional Equivalences

- **5.** Show that each of these conditional statements is a tautology by using truth tables.

  a) $(p \wedge q) \rightarrow p$

  b) $p \rightarrow (p \vee q)$

  c) $\neg p \rightarrow (p \rightarrow q)$

  d) $(p \wedge q) \rightarrow (p \rightarrow q)$

  e) $\neg(p \rightarrow q) \rightarrow p$

  f) $\neg(p \rightarrow q) \rightarrow \neg q$

- **13.** Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

- **37.** Determine whether each of these compound propositions is satisfiable.
  a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
  b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
  c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

1.4 Predicates and Quantifiers

- **7.** Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of P(x), $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
  a) There is a student at your school who can speak Russian and who knows $C++$.
  b) There is a student at your school who can speak Russian but who doesn't know $C++$.
  c) Every student at your school either can speak Russian or knows $C++$.
  d) No student at your school can speak Russian or knows $C++$.

- **21.** Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")
  a) Some old dogs can learn new tricks.
  b) No rabbit knows calculus.
  c) Every bird can fly.
  d) There is no dog that can talk.
  e) There is no one in this class who knows French and Russian.

- **33.** What are the truth values of these statements?
  a) $\exists! x P(x) \to \exists x P(x)$
  b) $\forall x P(x) \to \exists! x P(x)$
  c) $\exists! x \neg P(x) \to \neg \forall x P(x)$

1.5 Nested Quantifiers

- **7.** Use quantifiers and predicates with more than one variable to express these statements.
  a) Every computer science student need a course in discrete mathematics.
  b) There is a student in this class who owns a personal computer.
  c) Every student in this class has taken at least one computer science course.

d) There is a student in this class who has taken at least one course in computer science.

e) Every student in this class has been in every building on campus.

f) There is a student in this class who has been in every room on campus.

g) Every student in this class has been in at least one room of every building on campus.

- **13.** Determine the thruth value of each of hese statements if the domain for all variables consists of all integers.

  a) $\forall n \exists m (n^2 < m)$

  b) $\exists n \forall m (n < m^2)$

  c) $\forall n \exists m (n + m = 0)$

  d) $\exists n \forall m (nm = m)$

  e) $\exists n \exists m (n^2 + m^2 = 5)$

  f) $\exists n \exists m (n^2 + m^2 = 6)$

  g) $\exists n \exists m (n + m = 4 \wedge n - m = 1)$

  h) $\exists n \exists m (n + m = 4 \wedge n - m = 2)$

  i) $\forall n \forall m \exists p (p = (m + n)/2)$

- **17.** Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

  a) $\neg \forall x \forall y P(x, y)$

  b) $\neg \forall y \exists x P(x, y)$

  c) $\neg \forall y \forall x (P(x, y) \wedge Q(x, y))$

  d) $\neg (\exists x \exists y \neg P(x, y) \vee \forall x \forall y Q(x, y))$

  e) $\neg \forall x (\exists y \forall z P(x, y, z) \vee \exists z \forall y P(x, y, z))$

1.6 Rules of Inference

- **5.** What rules of inference are used in this famous argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

- **11.** For each of these arguments determine whether the argument is correct or incorrect and explain why.

  a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

  b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

d) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

- **19.** Use rules of inference to show that if $\forall x(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

1.7 Normal Forms

Obtain disjunctive normal form, conjonctive normal form, principal disjunctive normal form, and principal conjunctive normal form for the following expression

- **1.** $Q \wedge (P \vee \neg Q)$
- **3.** $(Q \to P) \wedge (\neg P \wedge Q)$