UNIVERSITÉ CATHOLIQUE DE LOUVAIN

TP5 : Counting

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5 CHAPTER 6 & 8 : COUNTING AND ADVANCED COUNTING TECHNIQUES

8.1 APPLICATIONS OF RECURRENCE RELATIONS

- **5.**
  a) Find a recurrence relation for the number of bits strings of length n that contain a pair of consecutive 0s.
  b) What are the initial conditions?
  c) How many bit strings of length seven contain two consecutive 0s?

- **13.** Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.
  a) Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.
  b) What are the initial conditions?
  c) How many different messages can be sent in 10 microseconds using these two signals?

8.2 SOLVING LINEAR RECURRENCE RELATIONS

- **2.** Solve these recurrence relations together with the initial conditions given.

a) $a_n = 2a_{n-1}$ for $n \geq 1, a_0 = 3$

b) $a_n = a_{n-1}$ for $n \geq 1, a_0 = 2$

c) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2, a_0 = 1, a_1 = 0$

d) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 6, a_1 = 8$

e) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 0, a_1 = 1$

f) $a_n = 4a_{n-2}$ for $n \geq 2, a_0 = 0, a_1 = 4$

g) $a_n = a_{n-2}/4$ for $n \geq 2, a_0 = 1, a_1 = 0$

- **11.** Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a0 = 7, a1 = -4$, and $a2 = 8$.

- **19.** Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

  a) Show that $a_n = -2^{n+1}$ is a solution of this recurrence relation.

  b) Use Theorem 5 to find all solutions of this recurrence relation.

  c) Find the solution with $a_0 = 1$.

- **21.** What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if

  a) $F(n) = n^3$?

  b) $F(n) = (-2)^n$?

  c) $F(n) = n2^n$?

  d) $F(n) = n^2 4^n$?

  e) $F(n) = (n^2 - 2)(-2)^n$?

  f) $F(n) = n^4 2^n$?

  g) $F(n) = 2$?

6.4 Binomial Coefficients and Identities

- **1.** Find the expansion of $(x + y)^4$

  a) using combinatorial reasoning, as in Example 1.

  b) using the binomial theorem.

- **13.** Prove that if n and k are integers with $1 \leq k \leq n$, then $k\binom{n}{k} = n\binom{n-1}{k-1}$

  a) using a combinatorial proof. [Hint: Show that the two sides of the identity count the number ofways to select a subset with k elements from a set with n elements and then an element of this subset.]

  b) using an algebraic proof based on the formula for $\binom{n}{k}$ given in Theorem 2 in Section 6.3.

- **15.** Show that if n and k are positive integers, then $\binom{n+1}{k} = (n+1)\binom{n}{k-1}/k$ Use this identity to construct an inductive definition of the binomial coefficients.

- **3.** How many ways are there to assign three jobs to five employees if each employee can be given more than one job?
- **13.** Suppose that a large family has 14 children, including two sets of identical triplets, three sets of identical twins, and two individual children. How many ways are there to seat these children in a row of chairs if the identical triplets or twins cannot be distinguished from one another?
- **23.** How many different bit strings can be transmitted if the string must begin with a 1 bit, must include three additional 1 bits (so that a total of four 1 bits is sent), must include a total of twelve 0 bits, and must have at least two 0 bits following each 1 bit?