I Comments

Note that I use "good components", that is at some point in the computation I multiply (from the middle-direction of the operator) by γ^+, and by anticommuting find usually a term that is proportional to γ^+ and a term that drops. To my understanding i can do this since the full operator will contain a Gamma struncture inside that will contain this γ^+. This should only affect loop diagrams containing gamma structure (quark propagators).

II gluon diagrams

Here i present the Diagrams for twist 2 with the intermediate computation performed in the Mathematica files. I will write the expressions for the diagrams, then isolte the results like it is done in Mathematica. The variables are $z1$=zn , $z2$=σn, $z3 = \tau n$ (integrate over) Note that the propagator symbol here usually is only the "one over distance squared" term, like in Mathematica, no factor of pi, 4 or Gamma included.

.i Diagram A

$$\int_{z_2}^{z_1} \mathrm{d}z_3 F^A_{\mu+}(z_1)\mathrm{i}gB^B_+(z3)T^B_{AC}F^D_{\nu N}T^D_{CE}\left(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\right)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.1}$$

The common "prefactor" is

$$\int_{z2}^{z1} \mathrm{d}z3 \int \mathrm{d}^d x A^{A'}_\iota(x)(-\mathrm{i}^2)\frac{\Gamma^2\left(\frac{d}{2}-1\right)}{(4\pi^2)^2} \tag{2.2}$$

The two contrations are

$$\partial_{z1^\mu}\partial_{x^\alpha}\Delta_{M\beta}(z_1-x)\Delta_{+\gamma}(z_3-x) \tag{2.3}$$

$$+\,\partial_{z1^\mu}\Delta_{M\gamma}(z_1-x)\partial_{x^\alpha}\Delta_{+\beta}(z_3-x) \tag{2.4}$$

The color factor is: all structure function structure, f and T with the δ-s coming from propagators computed The color factor is(the notation +1-1 indicates the first contraction is + the second is -, it does NOT mean the color factor is zero!):

$$iT^D_{A'E}C_A(+1-1) \tag{2.5}$$

Plug this into mathematica. Output: (be aware that now z_i is only z, σ and τ without n) prefactor

$$\frac{g^2}{8\pi^2\epsilon}F^D_{\nu+}T^D_{A'E} \tag{2.6}$$

integral term

$$\int_0^1 \mathrm{d}u \int_{z_3-z_2}^{0} \mathrm{d}z_3 u\partial_+ F_{\mu+}(z_1 n + \overline{u} z_3 n) \tag{2.7}$$

$$= \int_0^1 \mathrm{d}u \frac{u}{\overline{u}} \left(F^{A'}_{\mu+}(z_1 n) - F^{A'}_{\mu+}(z_2 n + u(z_1 - z_2) n) \right) \tag{2.8}$$

.ii Diagram A'

$$\int_{-\infty}^{z_2} \mathrm{d}z_3 F^A_{\mu+}(z_1) F^D_{\nu N}(z_2) T^D_{AC} \mathrm{i}g B^B_+(z3) T^B_{CE} \Big(- \mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.9}$$

The common "prefactor" is

$$\int_{z2}^{z1} \mathrm{d}z3 \int \mathrm{d}^d x A^{A'}_\iota(x)(-\mathrm{i}^2) \frac{\Gamma^2\left(\frac{d}{2} - 1\right)}{(4\pi^2)^2} \tag{2.10}$$

The two contrations are

$$\partial_{z1^\mu} \partial_{x^\alpha} \Delta_{M\beta}(z_1 - x) \Delta_{+\gamma}(z_3 - x) \tag{2.11}$$

$$+ \partial_{z1^\mu} \Delta_{M\gamma}(z_1 - x) \partial_{x^\alpha} \Delta_{+\beta}(z_3 - x) \tag{2.12}$$

The color factor is:

$$i T^D_{A'E} \frac{C_A}{2}(+1 - 1) \tag{2.13}$$

Plug this into mathematica. Output: (be aware that now z_i is only z, σ and τ without n) prefactor

$$\frac{g^2}{16\pi^2 \epsilon} F^D_{\nu+} T^D_{A'E} \tag{2.14}$$

integral term

$$\int_0^1 \mathrm{d}u \int_{-\infty}^{z_3-z_2} \mathrm{d}z_3 u\partial_+ F_{\mu+}(z_1 n + \overline{u} z_3 n) \tag{2.15}$$

$$= \left(1 - \ln\left(\frac{\mathrm{i}\hat{p}_+}{\delta}\right)\right) F^{A'}_{\mu+}(z_1 n) + \int_0^1 \mathrm{d}u \frac{u}{\overline{u}} \left(F^{A'}_{\mu+}(z_1 n + \overline{u}(z_2 - z_1) n) - F^{A'}_{\mu+}(z_1 n) \right) \tag{2.16}$$

.iii Diagram B

$$\int_{z_2}^{z_1} \mathrm{d}z_3 F^A_{\mu+}(z_1) \mathrm{i}g B^B_+(z3) T^B_{AC} F^D_{\nu N}(z_2) T^D_{CE} \Big(- \mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.17}$$

The common "prefactor" is

$$\int_{z2}^{z1} \mathrm{d}z3 \int \mathrm{d}^d x A^{A'}_\iota(x)(-\mathrm{i}^2) \frac{\Gamma^2\left(\frac{d}{2} - 1\right)}{(4\pi^2)^2} \tag{2.18}$$

The two contrations are

$$\partial_{z2^\mu}\partial_{x^\alpha}\Delta_{M\beta}(z_2-x)\Delta_{+\gamma}(z_3-x) \tag{2.19}$$

$$+\,\partial_{z2^\mu}\Delta_{M\gamma}(z_2-x)\partial_{x^\alpha}\Delta_{+\beta}(z_3-x) \tag{2.20}$$

The color factor is:

$$iT^{A'}_{AE}\frac{C_A}{2}(+1-1) \tag{2.21}$$

Plug this into mathematica. Output: (be aware that now z_i is only z, σ and τ without n) prefactor

$$\frac{g^2}{16\pi^2\epsilon}F^A_{\mu+}T^{A'}_{AE} \tag{2.22}$$

integral term

$$\int_0^1 \mathrm{d}u\int_{z_1-z_2}^0 \mathrm{d}z_3 u\partial_+ F_{\nu+}(z_2n+\overline{u}z_3n) \tag{2.23}$$

$$=\int_0^1 \mathrm{d}u\frac{u}{\overline{u}}\Big(F^{A'}_{\nu+}(z_2n+\overline{u}(z_1-z_2)n)-F^{A'}_{\nu+}(z_2n)\Big) \tag{2.24}$$

.iv Diagram B'

$$\int_{-\infty}^{z_2}\mathrm{d}z_3F^A_{\mu+}(z_1)F^D_{\nu N}(z_2)T^D_{AC}\mathrm{i}gB^B_+(z3)T^B_{CE}\Bigg(-\mathrm{i}g\int\mathrm{d}^dxA^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\Bigg)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.25}$$

The common "prefactor" is

$$\int_{z2}^{z1}\mathrm{d}z3\int\mathrm{d}^dxA^{A'}_\iota(x)(-\mathrm{i}^2)\frac{\Gamma^2\left(\frac{d}{2}-1\right)}{(4\pi^2)^2} \tag{2.26}$$

The two contrations are

$$\partial_{z_2^\mu}\partial_{x^\alpha}\Delta_{M\beta}(z_2-x)\Delta_{+\gamma}(z_3-x) \tag{2.27}$$

$$+\,\partial_{z_A2^\mu}\Delta_{M\gamma}(z_2-x)\partial_{x^\alpha}\Delta_{+\beta}(z_3-x) \tag{2.28}$$

The color factor is:

$$iT^{A'}_{AE}\frac{C_A}{2}(+1-1) \tag{2.29}$$

Plug this into mathematica. Output: (be aware that now z_i is only z, σ and τ without n) prefactor

$$\frac{g^2}{16\pi^2\epsilon}F^A_{\mu+}T^{A'}_{AE} \tag{2.30}$$

integral term

$$\int_0^1 \mathrm{d}u \int_{-\infty}^0 \mathrm{d}z_3 u \partial_+ F_{\nu+}(z_2 n + \overline{u} z_3 n) \tag{2.31}$$

$$= \left(1 - \ln\left(\frac{\mathrm{i}\hat{p}_+}{\delta}\right)\right) F^{A'}_{\nu+}(z_2 n) \tag{2.32}$$

III quark Twist 2

A Diagram D

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \overline{q}^A(z_1) \mathrm{i}g B^Z_+ T^Z_{AC} F^D_{\mu+}(z_2) T^D_{CE}(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota \partial_\alpha B^{B'}_\beta D^{C'}_\gamma) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{3.1}$$

common prefactor:

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \int \mathrm{d}^d x \overline{q}(z_1) A^{A'}_\iota(x)(-\mathrm{i}^2) \frac{\Gamma^2(\frac{d}{2}-1)}{(4\pi^2)^2} \tag{3.2}$$

Two contractions:

$$\partial_{x^\alpha} \partial_{z_1^\mu} \Delta_{M\beta}(z_2 - x) \Delta_{\gamma+}(z_3 - x) \tag{3.3}$$

$$+ \partial_{z_1^\mu} \Delta_{M\gamma}(z_2 - x) \partial_{x^\alpha} \Delta_{\beta+}(z_3 - x) \tag{3.4}$$

Color factor

$$-\mathrm{i}\frac{C_F}{2} T^{A'}_{AD} \tag{3.5}$$

Output: prefactor:

$$-\frac{C_F g^2}{16\pi^2 \epsilon} \overline{q}(z_1 n)^A T^{A'}_{AD} \tag{3.6}$$

integral term

$$\int_0^1 \mathrm{d}u \int_0^{z_1 - z_2} \mathrm{d}z_3 u \partial_+ F^{A'}_{\mu+} \tag{3.7}$$

$$= \int_0^1 \mathrm{d}u \frac{u}{\overline{u}} \left(F^{A'}_{\mu+}(z_1 n + u(z_2 - z_1)n) - F^{A'}_{\mu+}(z_2 n)\right) \tag{3.8}$$

B Diagram H

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \overline{q}^A(z_1) F^D_{\mu+}(z_2) T^D_{AC} \mathrm{i}g B^Z_+ T^Z_{CE}(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota \partial_\alpha B^{B'}_\beta D^{C'}_\gamma) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{3.9}$$

common prefactor:

$$\int_{-\infty}^{z_2} \mathrm{d}z_3 \int \mathrm{d}^d x \overline{q}(z_1) A_\iota^{A'}(x)(-\mathrm{i}^2) \frac{\Gamma^2(\frac{d}{2}-1)}{(4\pi^2)^2} \tag{3.10}$$

Two contractions:

$$\partial_{x^\alpha} \partial_{z_1^\mu} \Delta_{M\beta}(z_2 - x) \Delta_{\gamma+}(z_3 - x) \tag{3.11}$$

$$+ \partial_{z_1^\mu} \Delta_{M\gamma}(z_2 - x) \partial_{x^\alpha} \Delta_{\beta+}(z_3 - x) \tag{3.12}$$

Color factor

$$+\mathrm{i}\frac{C_F}{2} T_{AD}^{A'} \tag{3.13}$$

Output: prefactor:

$$\frac{C_F g^2}{16\pi^2 \epsilon} \overline{q}(z_1 n)^A T_{AD}^{A'} \tag{3.14}$$

integral term

$$\int_0^1 \mathrm{d}u \int_{-\infty}^0 \mathrm{d}z_3 u \partial_+ F_{\mu+}^{A'} \tag{3.15}$$

$$= \left(1 - \ln\left(\frac{\mathrm{i}\hat{p}_+(z_2)}{\delta}\right)\right) F_{\mu+}^{A'}(z_2) \tag{3.16}$$

C Diagram E

Note that in the quark propagator there is hidden a factor if i!

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \overline{q}^A(z_1) \mathrm{i}g B_+^B T_{AC}^B F_{\mu+}^D(z_2) T_{CE}^D (+\mathrm{i}g \int \mathrm{d}^d x \overline{q}^X \gamma^\nu B_\nu^Z T_{XY}^Z q^Y) \tag{3.17}$$

common prefactor:

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \int \mathrm{d}^d x \mathrm{i}^3 g^2 \frac{\Gamma(\frac{d}{2})\Gamma(\frac{d}{2}-1)}{8\pi^4} F_{\mu+}^D \tag{3.18}$$

One contraction:

$$\blacktriangle(z_1 - x) \Delta_{\nu+}(z_3 - x) \tag{3.19}$$

Color factor

$$\frac{C_F}{T}{}^{D}_{\ XE} \tag{3.20}$$

Output: prefactor:

$$-\frac{C_F g^2}{8\pi^2 \epsilon} F_{\mu+}^D(z_2 n) T_{ZE}^D \tag{3.21}$$

integral term

$$\int_0^1 \mathrm{d}u \int_0^{z_1-z_2} \mathrm{d}z_3 u\partial_+ \overline{q}^Z(z_1+\overline{u}z_3) \tag{3.22}$$

$$= \int_0^1 \mathrm{d}u \frac{u}{\overline{u}} \left(\overline{q}^Z((z_2+u(z_1-z_2))n) - \overline{q}^Z(z_1 n)\right) \tag{3.23}$$

D Diagram G

Note that in the quark propagator there is hidden a factor if i!

$$\int_{-\infty}^{z_2} \mathrm{d}z_3 \overline{q}^A(z_1) F^D_{\mu+}(z_2) T^D_{AC} \mathrm{i}g B^B_+ T^B_{CE}(+\mathrm{i}g \int \mathrm{d}^d x \overline{q}^X \gamma^\nu B^Z_\nu T^Z_{XY} q^Y) \tag{3.24}$$

common prefactor:

$$\int_{z_2}^{z_1} \mathrm{d}z_3 \int \mathrm{d}^d x \mathrm{i}^3 g^2 \frac{\Gamma(\frac{d}{2})\Gamma(\frac{d}{2}-1)}{8\pi^4} F^D_{\mu+} \tag{3.25}$$

One contraction:

$$\blacktriangle(z_1-x)\Delta_{\nu+}(z_3-x) \tag{3.26}$$

Color factor

$$\frac{C_F}{2} T^D_{XE} \tag{3.27}$$

Output: prefactor:

$$-\frac{C_F g^2}{16\pi^2\epsilon} F^D_{\mu+}(z_2 n) T^D_{ZE} \tag{3.28}$$

integral term

$$\int_0^1 \mathrm{d}u \int_{-\infty}^{z_2-z_1} \mathrm{d}z_3 u\partial_+ \overline{q}^Z(z_1+\overline{u}z_3) \tag{3.29}$$

$$= \int_0^1 \mathrm{d}\frac{u}{\overline{u}} \left(\overline{q}^Z((z_2+u(z_1-z_2))n) - \overline{q}^Z(z_1 n)\right) \tag{3.30}$$

$$+ \left(1 - \ln\left(\frac{\mathrm{i}\hat{p}_+(z_1)}{\delta}\right)\right) \overline{q}^Z(z_1 n) \tag{3.31}$$

E Diagram F

$$\overline{q}^A(z_1) f^{DXY} B^X_\mu(z_2) B^Y_+(z_2) T^D_{AC}(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota \partial_\alpha B^{B'}_\beta D^{C'}_\gamma) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{3.32}$$

common prefactor:

$$\int \mathrm{d}^d x - \mathrm{i}g^2 \frac{\Gamma^2(\frac{d}{2}-1)}{16\pi^4} \overline{q}^A(z_1) \tag{3.33}$$

contractions:

$$\partial_{x^\alpha}\Delta_{\mu\beta}(z_2-x)\Delta_{\gamma+}(z_2-x) \tag{3.34}$$

$$+\Delta_{\mu\gamma}(z_2-x)\partial_{x^\alpha}\Delta_{\beta+}(z_2-x) \tag{3.35}$$

Color factor

$$C_F T^{A'}_{AC} \tag{3.36}$$

Output: prefactor:

$$-\frac{C_F g^2}{4\pi^2\epsilon}\overline{q}^A(z_1)T^{A'}_{AE} \tag{3.37}$$

integral term(here the integral over u is trivial and gives factor $\frac{1}{2}$)

$$F^{A'}_{\mu+}(z_2 n) \tag{3.38}$$

F Diagram C

$$\overline{q}^A(z_1)gf^{DLK}B^L_\mu(z_2)B^K_+(z_2)T^D_{AE}(+\mathrm{i}g\int \mathrm{d}^d x\overline{q}^X\gamma^\nu B^Z_\nu T^Z_{XY}q^Y) \tag{3.39}$$

common prefactor:

$$\int \mathrm{d}^d x\mathrm{i}g^2\frac{\Gamma(\frac{d}{2}-1)\Gamma(\frac{d}{2})}{8\pi^4}\overline{q}^X(x) \tag{3.40}$$

contractions:(i could drop the second one from the start)

$$\blacktriangle(z_1-x)\Delta_{\nu+}(z_2-x)A^L_\mu(z_2)\gamma^\nu \tag{3.41}$$

$$+\blacktriangle(z_1-x)\Delta_{\nu\mu}(z_2-x)A^L_+(z_2)\gamma^\nu \tag{3.42}$$

Color factor

$$\frac{C_F}{2}T^L_{XE}(+1-1) \tag{3.43}$$

Output: prefactor:

$$-\frac{C_F g^2}{16\pi^2\epsilon} \tag{3.44}$$

integral term

$$\int_0^1 \mathrm{d}u u \overline{q}^X (uz_1 n + \overline{u} z_2 n) \overleftarrow{\partial_+} A^L_\mu (z_2 n) T^L_{XE} \tag{3.45}$$

$$= \int_0^1 \mathrm{d}u (z_1 - z_2)^{-1} \frac{\partial}{\partial u} \overline{q}(z^u_{2\to 1} n) A_\mu (z_2 n) \tag{3.46}$$

$$= (z_1 - z_2)^{-1} \left(- \int_0^1 \mathrm{d}\overline{q}(z^u_{2\to 1} n) + \overline{q}(z_1 n) \right) A_\mu (z_2 n) \tag{3.47}$$

$$= (z_1 - z_2)^{-1} \int_{-\infty}^{z_2} \mathrm{d}\beta \left(\int_0^1 \mathrm{d}\overline{q}(z^u_{2\to 1} n) - \overline{q}(z_1 n) \right) F_{\mu +}(\beta n) \tag{3.48}$$

G Diagram K

$$\overline{q}^A (z_1) F^D_{\mu +}(z_2) T^D_{AE} (+\mathrm{i} g \int \mathrm{d}^d x \overline{q}^X \gamma^\nu B^Z_\nu T^Z_{XY} q^Y) \tag{3.49}$$

common prefactor:

$$\int \mathrm{d}^d x \mathrm{i} g \frac{\Gamma(\frac{d}{2} - 1)\Gamma(\frac{d}{2})}{8\pi^4} \overline{q}^X (x) \tag{3.50}$$

$$\blacktriangle (z_1 - x) \partial_{z_1^\mu} \Delta_{\nu M} (z_2 - x) \gamma^\nu \tag{3.51}$$

Color factor

$$C_F T^L_{XC} \tag{3.52}$$

Output: prefactor:

$$\frac{-\mathrm{i} C_F g}{32\pi^2 \epsilon} \tag{3.53}$$

integral term

$$\int_0^1 \mathrm{d}u u \overline{u} \overline{q}^C (uz_1 n + \overline{u} z_2 n) \not{\partial} \partial_+ \gamma^\mu \tag{3.54}$$

Further evolution:

$$\int_0^1 \mathrm{d}u u \overline{u} \overline{q}^C(uz_1 n + \overline{u} z_2 n) \not{\partial} \partial_+ \gamma^\mu \tag{3.55}$$

$$= -\int_0^1 \mathrm{d}u (1-2u) \overline{q}^C(uz_1 n + \overline{u} z_2 n) \not{\partial} \gamma^\mu \tag{3.56}$$

$$= \mathrm{i}g \int_0^1 \mathrm{d}u (1-2u) \overline{q}^C(uz_1 n + \overline{u} z_2 n) \not{A}(uz_1 n + \overline{u} z_2 n) \gamma^\mu \tag{3.57}$$

$$= \mathrm{i}g \int_0^1 \mathrm{d}u (1-2u) \overline{q}^C(uz_1 n + \overline{u} z_2 n) \int_{-\infty}^{uz_1 + \overline{u} z_2} \mathrm{d}\beta F_{+\nu}(\beta n) \gamma^\nu \gamma^\mu \tag{3.58}$$

$$= -\mathrm{i}g \int_0^1 \mathrm{d}u (1-2u) \overline{q}^C(uz_1 n + \overline{u} z_2 n) \left(\int_{-\infty}^{z_2} + \int_{z_2}^{uz_1 + \overline{u} z_2} \right) \mathrm{d}\beta F_{\nu +}(\beta n) \gamma^\nu \gamma^\mu \tag{3.59}$$

H Diagram B

$$\overline{q}^A(z_1) F^D_{\mu +}(z_2) T^D_{AC} (+\mathrm{i}g \int \mathrm{d}^d x \overline{q}^X \gamma^\nu B^Z_\nu T^Z_{XY} q^Y)(+\mathrm{i}g \int \mathrm{d}^d y \overline{q}^O \gamma^\nu B^P_\nu T^P_{OV} q^V) \tag{3.60}$$

common prefactor:

$$\int \mathrm{d}^d x \int \mathrm{d}^d y \mathrm{i}^2 g^2 \frac{\Gamma(\frac{d}{2}-1)\Gamma^2(\frac{d}{2})}{16\pi^6} \tag{3.61}$$

$$(1) = \blacktriangle(z_1 - x) \partial_{z_1^\mu} \Delta_{\eta M}(z_2 - y) \blacktriangle(x - y) \overline{q}^O(y) \gamma^\eta \gamma^\nu \tag{3.62}$$

$$(2) = \blacktriangle(z_1 - y) \partial_{z_1^\mu} \Delta_{\nu M}(z_2 - x) \blacktriangle(y - x) \overline{q}^X(x) \gamma^\nu \gamma^\eta \tag{3.63}$$

Color factor

$$(1) = \frac{C_F}{2} T^Z_{OC} \tag{3.64}$$

$$(2) = \frac{C_F}{2} T^P_{XC} \tag{3.65}$$

Output: prefactor(blue factor 2 is symmetry factor):

$$2 \frac{-C_F g^2 \Gamma(-\epsilon)}{32\pi^2} = \frac{C_F \alpha_s}{\epsilon} \tag{3.66}$$

integral term

$$\int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \frac{\alpha\beta\gamma}{(\alpha\beta+\beta\gamma+\gamma\alpha)^5} \bar{q}^O(\frac{z_1\gamma\alpha + z_2\beta(\gamma+\alpha)}{\alpha\beta+\beta\gamma+\gamma\alpha} n) T^Z_{OC} \tag{3.67}$$

$$\Big\{(z_1 - z_2)\alpha\beta\gamma(\beta+\gamma)\gamma^\mu\gamma^\nu \overrightarrow{\partial_+}^2 + (z_1 - z_2)\alpha\beta\gamma(\alpha+\gamma)\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}^2 \tag{3.68}$$

$$+ (\beta\gamma + \alpha(\beta+\gamma))((\alpha(\beta - 2\gamma) + \beta\gamma)\gamma^\mu\gamma^\nu - \alpha\gamma\gamma^\nu\gamma^\mu)\overleftarrow{\partial_+} \tag{3.69}$$

$$+ (z_1 - z_2)\alpha\beta(\alpha(\beta+\gamma) + \gamma(\beta+2\gamma))\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}\overrightarrow{\partial_+} \tag{3.70}$$

$$- (\beta\gamma + \alpha(\beta+\gamma))((-\beta\gamma + 2\alpha(\beta+\gamma))\gamma^\mu\gamma^\nu - \beta\gamma\gamma^\nu\gamma^\mu)\overrightarrow{\partial_+}\Big\} A^Z_\nu(\frac{z_1(\gamma+\beta)\alpha + z_2\gamma\beta}{\alpha\beta+\beta\gamma+\gamma\alpha}) \tag{3.71}$$

$$= \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \, \frac{\alpha\beta\gamma\Lambda^2}{\Lambda^5} \bar{q}^O(z^{\beta'}_{21} n) T^Z_{OC} \tag{3.72}$$

$$\Big\{(z_1 - z_2)\alpha'\overline{\alpha'}\gamma^\mu\gamma^\nu \overrightarrow{\partial_+}^2 + (z_1 - z_2)\beta'\overline{\beta'}\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}^2 \tag{3.73}$$

$$+ (1 - 3\beta')\gamma^\mu\gamma^\nu - \beta'\gamma^\nu\gamma^\mu \overleftarrow{\partial_+} \tag{3.74}$$

$$+ (z_1 - z_2)(\gamma' + 2\beta'\alpha')\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}\overrightarrow{\partial_+} \tag{3.75}$$

$$+ (3\alpha' - 2)\gamma^\mu\gamma^\nu + \alpha'\gamma^\nu\gamma^\mu \overrightarrow{\partial_+}\Big\} A^Z_\nu(z^{\alpha'}_{12}) \tag{3.76}$$

with

$$z^u_{ij} = z_i + u(z_j - z_i) \tag{3.77}$$

"from i to j along variable u"

change to primed variables:

$$(\alpha', \beta', \gamma') \to (\beta, \alpha, \gamma) \tag{3.78}$$

prefactor

$$\frac{C_F \alpha_s}{\epsilon} \tag{3.79}$$

integral term

$$\int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \bar{q}^O(z^\alpha_{21} n) T^Z_{OC} \Big\{(z_1 - z_2)\beta\bar{\beta}\gamma^\mu\gamma^\nu \overrightarrow{\partial_+}^2 + (z_1 - z_2)\alpha\bar{\alpha}\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}^2 \tag{3.80}$$

$$+ ((1 - 3\alpha)\gamma^\mu\gamma^\nu - \alpha\gamma^\nu\gamma^\mu)\, \overleftarrow{\partial_+} + (z_1 - z_2)(\gamma + 2\alpha\beta)\gamma^\mu\gamma^\nu \overleftarrow{\partial_+}\overrightarrow{\partial_+} \tag{3.81}$$

$$+ ((3\beta - 2)\gamma^\mu\gamma^\nu + \beta\gamma^\nu\gamma^\mu)\, \overrightarrow{\partial_+}\Big\} A^Z_\nu(z^\beta_{12}) \tag{3.82}$$

useful:

$$\partial_+ f(z_{ij}^\lambda n) = -(z_i - z_j)^{-1}\partial_\lambda f(z_{ij}^\lambda n) \tag{3.83}$$

$$A_\alpha(z_{ij}^\lambda n) = (z_i - z_j)\int_\Pi^\lambda \mathrm{d}\sigma F_{\alpha+}(z_{ij}^\sigma n) \tag{3.84}$$

$$x \tag{3.85}$$

Π would be a Πoint of reference, usually $\pm\infty$ refine the result term by term:

$$\int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,(z_1 - z_2)\beta\overline{\beta}\overrightarrow{\partial_+}^2 A_\nu^Z(z_{12}^\beta) = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,(1-2\beta)F_{\nu+}^Z(z_{12}^\beta) \tag{3.86}$$

$$\int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\overline{q}(z_{21}^\alpha n)\overleftarrow{\partial_+}^2\alpha\overline{\alpha}(z_1 - z_2)A_\nu(z_{12}^\beta) = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,(2\overline{q}(z_{21}^\alpha n) - \overline{q}(z_1 n) - \overline{q}(z_2 n))\int_\Pi^\beta F_{\nu+}(z_{12}^\sigma n) \tag{3.87}$$

$$\int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\overline{q}(z_{21}^\alpha n)\overleftarrow{\partial_+}(-\alpha)A_\nu(z_{12}^\beta) = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,(\overline{q}(z_1 n) - \overline{q}(z_{21}^\alpha n))\int_\Pi^\beta F_{\nu+}(z_{12}^\sigma n) \tag{3.88}$$

$$\int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\overline{q}(z_{21}^\alpha n)\overleftarrow{\partial_+}(1-3\alpha)A_\nu(z_{12}^\beta) = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,(\overline{q}(z_2 n) + 2\overline{q}(z_1 n) - 3\overline{q}(z_{21}^\alpha n))\int_\Pi^\beta F_{\nu+}(z_{12}^\sigma n) \tag{3.89}$$

$$\int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\overline{q}(z_{21}^\alpha n)\overleftarrow{\partial_+}(z_1 - z_2)(1-\alpha-\beta+2\alpha\beta) = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\left(-\overline{q}(z_2 n)\overline{\beta} + \overline{q}(z_1 n)\beta - \overline{q}(z_{21}^\alpha n)(2\beta - 1)\right) \tag{3.90}$$

the other terms are already in good form by derivative acting on gluon field. resumming

$$B = \int [\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\left\{\gamma_\mu\gamma^\nu\left[(1-2\beta)\right]\right\} \tag{3.91}$$

I Diagram A

$$\overline{q}^A(z_1)F_{\mu+}^D(z_2)T_{AC}^D(+\mathrm{i}g\int \mathrm{d}^d x\overline{q}^X\gamma^\nu B_\nu^Z T_{XY}^Z q^Y)(-\mathrm{i}g\int \mathrm{d}^d y A_\iota^{A'}\partial_\alpha B_\beta^{B'} D_\gamma^{C'})v_{A'B'C'}^{\iota\alpha\beta\gamma} \tag{3.92}$$

common prefactor:

$$\int \mathrm{d}^d x\int \mathrm{d}^d y - \mathrm{i}^2 g^2\frac{\Gamma^2(\frac{d}{2}-1)\Gamma(\frac{d}{2})}{32\pi^6}\overline{q}^X(x)\gamma^\theta A_\iota^{A'}(y) \tag{3.93}$$

$$\blacktriangle(z_1 - x)\partial_{y^\alpha}\partial_{z_2^\mu}\Delta_{\beta M}(z_2 - y)\Delta_{\theta\gamma}(x-y) \tag{3.94}$$

$$+\blacktriangle(z_1 - x)\partial_{z_2^\mu}\Delta_{\gamma M}(z_2 - y)\partial_{y^\alpha}\Delta_{\theta\beta}(x-y) \tag{3.95}$$

Color factor

$$-\mathrm{i}\frac{C_F}{2}T_{XZ}^{A'}(1-1) \tag{3.96}$$

Output(main term from main computation, there is an additional term right below): prefactor:

$$\frac{g^2 C_F \Gamma(-\epsilon)}{32\pi^2} = -\frac{C_F \alpha_s}{2\epsilon} \tag{3.97}$$

integral term

$$\int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \frac{\alpha\beta\gamma}{\Lambda^3} \overline{q}^X(z_{12}^{\alpha'}) \Big\{ \Big[\big(-\frac{\beta(\alpha+\gamma)}{\Lambda} \overleftarrow{\partial_+} - \frac{\beta\gamma}{\Lambda} \overrightarrow{\partial_+} \big) \gamma_\mu \gamma^\nu + \overleftarrow{\partial_+} \gamma^\nu \gamma_\mu \Big] \tag{3.98}$$

$$+ \Big[\frac{3\beta\gamma + \alpha(3\beta+\gamma)}{\Lambda} \overleftarrow{\partial_+} + \frac{(\beta+\gamma)(3\beta\gamma + \alpha(3\beta+\gamma))}{\gamma\Lambda} \overrightarrow{\partial_+} \Big] \eta_\mu^\nu \Big\} A_\nu^{A'}(z_{21}^{\beta'}) \tag{3.99}$$

$$= \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \frac{\alpha\beta\gamma}{\Lambda^3} \overline{q}^X(z_{12}^{\alpha'}) \Big\{ \Big[\big(-\overline{\beta'} \overleftarrow{\partial_+} - \alpha' \overrightarrow{\partial_+} \big) \gamma_\mu \gamma^\nu + \overleftarrow{\partial_+} \gamma^\nu \gamma_\mu \Big] \tag{3.100}$$

$$+ \Big[(3\overline{\beta'} + \beta') \overleftarrow{\partial_+} + \frac{(\beta+\gamma)(3\beta\gamma + \alpha(3\beta+\gamma))}{\gamma\Lambda} \overrightarrow{\partial_+} \Big] \eta_\mu^\nu \Big\} A_\nu^{A'}(z_{21}^{\beta'}) \tag{3.101}$$

$$= \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \, \overline{q}^X(z_{12}^{\beta}) \Big\{ \Big[\big(-\overline{\alpha} \overleftarrow{\partial_+} - \beta \overrightarrow{\partial_+} \big) \gamma_\mu \gamma^\nu + \overleftarrow{\partial_+} \gamma^\nu \gamma_\mu \Big] \tag{3.102}$$

$$+ \Big[(3\overline{\alpha} + \alpha) \overleftarrow{\partial_+} + X_{undefined} \overrightarrow{\partial_+} \Big] \eta_\mu^\nu \Big\} A_\nu^{A'}(z_{21}^{\alpha}) \tag{3.103}$$

Then here is a bonus term, which can not be neglected. It is $\propto \eta^{\mu\alpha}$ (inside the computations first steps). The contribution is prefactor:

$$\frac{g^2 C_F \Gamma(-\epsilon)}{32\pi^2} T_{XC}^{A'} = \frac{-\alpha_s C_F}{2\epsilon} T_{XC}^{A'} \tag{3.104}$$

integral term

$$\int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \, \overline{q}^X(z_{12}^{\alpha'}) \frac{-3\alpha\beta}{(\alpha+\gamma)\Lambda^2} \overrightarrow{\partial_+} A_\mu^{A'}(z_{21}^{\beta'}) \tag{3.105}$$

we can use the Anticommutator to rewrite the metric tensor:

$$\eta^{\vartheta\eta} = \frac{1}{2} \left(\gamma^\vartheta \gamma^\eta + \gamma^\eta \gamma^\vartheta \right) \tag{3.106}$$

Thus in Total we have

$$A = \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma] \, \overline{q}(z_{12}^{\beta}) \Big\{ \gamma_\mu \gamma^\nu \big[+ \left(\frac{1}{2}\overline{\alpha} + \alpha \right) \overleftarrow{\partial_+} + (-\beta + XY) \overrightarrow{\partial_+} \big] \tag{3.107}$$

$$+ \gamma^\nu \gamma_\mu \big[\left(1 + \frac{3}{2}\overline{\alpha} + \frac{1}{2}\alpha \right) \overleftarrow{\partial_+} + XY \overrightarrow{\partial_+} \big] \Big\} A_\nu^{A'}(z_{21}^{\alpha}) \tag{3.108}$$

IV Identities

For color factor computations:

$$f^{ade}f^{bcd}+f^{bde}f^{cad}+f^{cde}f^{abd}=0 \tag{4.1}$$
$$f^{heb}(f^{ade}f^{bcd}+f^{bde}f^{cad}+f^{cde}f^{abd})=0 \tag{4.2}$$
$$f^{ade}f^{heb}f^{bcd}+f^{bde}f^{heb}f^{cad}+f^{cde}f^{heb}f^{abd}=0 \tag{4.3}$$
$$f^{ade}f^{heb}f^{bcd}+f^{bde}f^{heb}f^{cad}+f^{cde}f^{heb}f^{abd}=0 \tag{4.4}$$
$$-f^{ade}f^{heb}f^{cbd}+f^{deb}f^{heb}f^{cad}-f^{adb}f^{hbe}f^{ced}=0 \tag{4.5}$$
$$-2f^{ade}f^{heb}f^{cbd}+C_G\delta_{dh}f^{cad}=0 \tag{4.6}$$
$$f^{ade}f^{heb}f^{cbd}=\frac{C_G}{2}f^{ahc} \tag{4.7}$$

for 3 integration variables: There exists the following transformation

$$(\alpha,\beta,\gamma)\to(\alpha',\beta',\gamma')=(\frac{\beta\gamma}{\Lambda},\frac{\gamma\alpha}{\Lambda},\frac{\alpha\beta}{\Lambda}),\quad \Lambda=\alpha\beta+\beta\gamma+\gamma\alpha \tag{4.8}$$

with the additional constraint

$$\alpha+\beta+\gamma=1 \tag{4.9}$$

it is indeed a two dimensional transformation. These variables arise naturally in the computation. To change the variables, the Jacobi Determinant is needed to use the Transformation theorem. It should be noted that also the transformed variables naturally fulfill the condition

$$\alpha'+\beta'+\gamma'=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\Lambda}=1 \tag{4.10}$$

and preserve the domain for each variable:

$$\alpha,\beta,\gamma,\in[0,1]\to\alpha',\beta',\gamma',\in[0,1] \tag{4.11}$$

The Jacobian of this (two dimensional) transformation is

$$|\det D\Phi|=\frac{\alpha\beta\gamma}{\Lambda^3} \tag{4.12}$$

which is checked in Mathematica. I also checked this by hand and found the same.