$$\alpha_s = \frac{g^2}{(4\pi)^2} \tag{0.1}$$

d dim integrals from P&S

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^n} = \frac{(-1)^n \mathrm{i}}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{0.2}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{0.3}$$

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} \mathrm{i}}{(4\pi)^{d/2}} \frac{\eta^{\mu\nu}}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{0.4}$$

Crucial formulae:

$$\frac{1}{A_1^{m_1}\ldots A_n^{m_n}} = \int_0^1 \mathrm{d}x_1 \ldots \mathrm{d}x_n \delta\left(\sum x_i - 1\right) \frac{\prod x_i^{m_i-1}}{\left(\sum x_i A_i\right)^{\sum m_i}} \frac{\Gamma\left(\sum m_i\right)}{\prod \Gamma(m_i)} \tag{0.5}$$

$$\overbracket{\psi(x)\bar{\psi}}(0) = G(x) = \frac{\mathrm{i}\not{x}}{2\pi^2 x^4} = \frac{\mathrm{i}\Gamma\left(\frac{d}{2}\right)\not{x}}{2\pi^2 \left(x^2\right)^{\frac{d}{2}}} \tag{0.6}$$

$$= \frac{\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{\frac{D+1}{2}}} \frac{\mathrm{i}\not{x}}{\left(-x^2+\mathrm{i}\epsilon\right)^{\frac{D+1}{2}}} \tag{0.7}$$

$$= \frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}} \frac{\mathrm{i}\not{x}}{\left(-x^2+\mathrm{i}\varepsilon\right)^{\frac{d}{2}}} \tag{0.8}$$

$$CHECK : \overbracket{A^{\mu,a}(x)A}^{\nu,b}(0) = \frac{-\mathrm{i}}{4\pi^2} \frac{\eta^{\mu\nu}}{x^2} (\text{must be wrong}) \tag{0.9}$$

$$= \frac{-\mathrm{i}}{4\pi^2} \frac{\Gamma\left(\frac{d}{2}-1\right)\eta^{\mu\nu}\delta^{ab}}{\left(x^2\right)^{\frac{d}{2}-1}} \tag{0.10}$$

$$= \frac{-1}{4\pi^{\frac{d}{2}}} \frac{\Gamma\left(\frac{d}{2}-1\right)\eta^{\mu\nu}\delta^{ab}}{\left(-x^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}-1}} \tag{0.11}$$

$$\not{A} = A^\mu \gamma_\mu = A^{\mu,a}\gamma_\mu t^a \tag{0.12}$$

$$t^a t^a = C_2 \mathbb{1} = C_F \mathbb{1} \tag{0.13}$$

$$\not{x}\not{x} = x^2 \tag{0.14}$$

$$\not{a}\not{b}\not{a} = -\not{b}a^2 + 2(a\cdot b)\not{a} \tag{0.15}$$

$$\gamma_\mu \gamma^\nu \gamma^\mu = -(d-2)\gamma^\nu \tag{0.16}$$

$$\gamma^\mu \gamma_\mu = d \tag{0.17}$$

work in gauges (q: quantum, c: classical fields)

$$x_\mu A^{\mu,c} = 0 \tag{0.18}$$

$$D_\mu^c A^{\mu,q} = 0 \tag{0.19}$$

$$G^{(1)}(x) = \int \mathrm{d}z \frac{\mathrm{i}(\not{x} - \not{z})}{2\pi^2 (x-z)^4} \mathrm{i}g \not{A}(z) \frac{\mathrm{i}\not{z}}{2\pi^2 z^4} \tag{0.20}$$

$$= \frac{-3\mathrm{i}g}{2\pi^4} \int \mathrm{d}z \int_0^1 \mathrm{d}u u \overline{u} \frac{(\overline{u}\not{x} - \not{z}) \not{A}(z+ux) (u\not{x} + \not{z})}{(z^2 + u\overline{u}x^2)^4} \tag{0.21}$$

To get there use formula eq. (0.5) and shift $z \to z + ux$. Next we should expand the field A assuming small z: $A^\mu(x+z) = A^\mu(x) + z^\alpha \partial_\alpha A^\mu(x) + z^\beta z^\alpha \partial_\alpha \partial_\beta A^\mu(x)$ Then i get

$$G^{(1)}(x) = \frac{-3\mathrm{i}g}{2\pi^4} \int \mathrm{d}z \int_0^1 \mathrm{d}u u \overline{u} \frac{(\overline{u}\not{x} - \not{z}) \gamma_\mu (u\not{x} + \not{z})}{(z^2 + u\overline{u}x^2)^4} \left(A^\mu(ux) + z^\alpha \partial_\alpha A^\mu(ux) + z^\beta z^\alpha \partial_\alpha \partial_\beta A^\mu(ux) \right) \tag{0.22}$$

I am actually not sure whether the expansion should be only applied to field A, or also the left (fraction) term. I could also shift derivatives by integration by parts. I start with the easiest term: $z = 0$: look at the numerator:

$$(\overline{u}\not{x} - \not{z}) \gamma_\mu (u\not{x} + z) = \overline{u}u\not{x}\gamma_\mu\not{x} + \overline{u}\not{x}\gamma_\mu\not{z} + u\not{z}\gamma_\mu\not{x} + \not{z}\gamma_\mu\not{z} \tag{0.23}$$

$$= 2\overline{u}ux_\mu - \overline{u}u\gamma_\mu x^2 + \overline{u}\not{x}\gamma_\mu\not{z} + u\not{z}\gamma_\mu\not{x} + 2z_\mu - \gamma_\mu z^2 \tag{0.24}$$

for A=const i have only the contributions terms 1, 2 and 6 (from left to right) due to symmetry in z.

$$T_0 = \frac{-3\mathrm{i}g}{2\pi^4} \int_0^1 \mathrm{d}u u \overline{u} \left(\frac{\mathrm{i}\pi^2}{6\overline{u}u} \frac{2x_\mu \not{x} - \gamma_\mu x^2}{x^4} + \frac{-2\mathrm{i}\pi^2}{6u\overline{u}} \frac{-\gamma_\mu}{x^2} \right) A^\mu(ux) \tag{0.25}$$

$$= \frac{-g}{4\pi^2} \int_0^1 \mathrm{d}u \left(\frac{2x_\mu \not{x}}{x^4} + \frac{\gamma_\mu}{x^2} \right) A^\mu(ux) \tag{0.26}$$

This expression i can also find in Vladimirs paper for the first x^{-4} term. HOWEVER i do not see the other term in x^{-2}.

I Intro

A diagram a

flavour nonsinglet string operator and wilson line

$$Op. = \overline{\psi}(x) \lambda^a \not{x} [x, 0] \psi(0) \tag{1.1}$$

$$[x, y] = P \exp \left[\mathrm{i}g \int_0^1 \mathrm{d}u (x-y)_\mu A^\mu (xu + y(1-u)) \right] \tag{1.2}$$

From diagram i get a similar (but not the same) expression. Also i should think about what this diagram is - is it sort of "VEV" of operator, with not vacuum but quark states?

Brauns expression:

$$\int_0^1 \mathrm{d}u \overline{\psi}_c(x) \mathrm{i} x_\mu g A^\mu_q \not{x} \lambda^a \psi_q(0) \left[\mathrm{i}g \int \mathrm{d}z \overline{\psi}_q(z) \not{A}_q(z) \psi(z) \right] \tag{1.3}$$

$$= \int_0^1 \mathrm{d}u \overline{\psi}_c(x) \mathrm{i} x_\mu g A^\mu_q \not{x} \lambda^a \psi_q(0) \mathrm{i}g \int \mathrm{d}z \overline{\psi}_q(z) \not{A}_q(z) \psi(z) \tag{1.4}$$

$$= -\frac{\mathrm{i}\alpha_s}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \not{x} \int_0^1 \mathrm{d}u \int \mathrm{d}z \frac{\Gamma(d/2)\Gamma(d/2-1)\not{z}\not{x}}{(-z^2)^{\frac{d}{2}} \left((-ux-z)^2 \right)^{\frac{d}{2}-1}} \psi_c(z) \tag{1.5}$$

I need to verify this by looking up propagators for d dimensional spacetime, and hereby check it. going on from this point we use integration tricks(Denominator + shift):

$$-\frac{\mathrm{i}\alpha_s}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \not{x} \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v v^{\frac{d}{2}-2} \overline{v}^{\frac{d}{2}-1} \int \mathrm{d}z \frac{\Gamma(d-1)(\not{z}+uv\not{x})\not{x}}{(-z^2 - v\overline{v}u^2x^2)^{d-1}} \psi_c(z+uvx) \tag{1.6}$$

Go on with again d dim integral formulas therefore we have to expand the field psi
so the numerator structure is

$$\not{x} \left(\not{z}\not{x} + uvx^2 \right) \left(\psi_c(uvx) + z^\alpha \partial_\alpha \psi_c(uvx) + ... \right) \tag{1.7}$$

$$= \left(-x^2 \not{z} + 2(x \cdot z)\not{x} + uvx^2 \not{x} \right) \left(\psi_c(uvx) + z^\alpha \partial_\alpha \psi_c(uvx) + ... \right) \tag{1.8}$$

numerator(dropping asym terms):

$$uvx^2 \psi_c(uvx) + \left(-x^2 \not{z} + 2(x \cdot z)\not{x} \right) z^\alpha \partial_\alpha \psi_c(uvx) \tag{1.9}$$

This means, combined with the denomoinator i have the three terms

$$1: \quad \int \mathrm{d}z \frac{uvx^2 \not{x} \psi_c}{(-z^2 - v\overline{v}u^2x^2)^{d-1}} \tag{1.10}$$

$$= v^{2-\frac{d}{2}} \overline{v}^{1-\frac{d}{2}} u^{3-d} v \mathrm{i}\pi^2 \frac{\Gamma(d/2-1)}{\Gamma(d-1)} \frac{\not{x}\psi_c}{(-x^2)^{\frac{d}{2}-2}} \tag{1.11}$$

$$2: \quad \int \mathrm{d}z \frac{-x^2 \gamma_\beta z^\beta z^\alpha \partial_\alpha \psi_c}{(-z^2 - v\overline{v}u^2x^2)^{d-1}} \tag{1.12}$$

$$= (v\overline{v})^{2-\frac{d}{2}} \frac{\mathrm{i}}{2} \pi^2 \frac{\Gamma(\frac{d}{2}-2)}{\Gamma(d-1)} \frac{x^2 \gamma^\alpha \partial_\alpha \psi_c}{(-u^2x^2)^{\frac{d}{2}-2}} \tag{1.13}$$

$$3: \quad \int \mathrm{d}z \frac{2x_\beta z^\beta z^\alpha \partial_\alpha \psi_c}{(-z^2 - v\overline{v}u^2x^2)^{d-1}} \tag{1.14}$$

$$= (v\overline{v})^{2-\frac{d}{2}} \mathrm{i}\pi^2 \frac{\Gamma\left(\frac{d}{2}-2\right)}{\Gamma\left(d-1\right)} \frac{x^\alpha \partial_\alpha \psi_c}{(-u^2x^2)^{\frac{d}{2}-2}} \tag{1.15}$$

All terms are to be multiplied with

$$-\frac{\mathrm{i}\alpha_s}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v v^{\frac{d}{2}-2} \overline{v}^{\frac{d}{2}-1} \Gamma(d-1) \tag{1.16}$$

Since the 2nd term is x^{-2} - power, it is dropped already.(?)
Remaining:

$$\frac{\alpha_s \pi^2}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \left(\frac{\overline{v}\Gamma\left(\frac{d}{2}-2\right)}{(-u^2x^2)^{\frac{d}{2}-2}} x^\alpha \partial_\alpha \psi_c(uvx) + \frac{\Gamma\left(\frac{d}{2}-1\right)}{(-x^2)^{\frac{d}{2}-2}} u^{3-d} \not{x} \psi_c(uvx) \right) \tag{1.17}$$

This agrees with Vladimir Braun. Then we go on, like it is written in the paper. We drop the 2nd term because it is UV divergent for $u \to 0$ in $d = 4$ dimensions. With the first term we can manipulate $x^\alpha \partial_\alpha \psi_c(uvx) = \frac{\mathrm{d}}{\mathrm{d}u} \psi_c(uvx)$ (typo in Brauns Paper(?)) and integrate by parts: Have (for $d = 4$):

$$\frac{\alpha_s \pi^2}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \frac{\overline{v}\Gamma\left(\frac{d}{2}-2\right)}{(-x^2)^{\frac{d}{2}-2}} u^{d-4} \frac{\mathrm{d}}{\mathrm{d}u} \psi_c(uvx) \tag{1.18}$$

$$= \frac{\alpha_s \pi^2}{2\pi} C_F \overline{\psi}_c(x) \lambda^a \int_0^1 \mathrm{d}v \frac{\overline{v}\Gamma\left(\frac{d}{2}-2\right)}{(-x^2)^{\frac{d}{2}-2}} \left(\psi_c(vx) - \psi_c(0)\right) \tag{1.19}$$

B diagram b

Now the second matrix element

$$\int_0^1 \mathrm{d}u \int \mathrm{d}z \overline{\psi}_c(z) \mathrm{i}g \not{A}_q(z) \psi_q(z) \overline{\psi}_q(0) \mathrm{i}g x_\mu A^\mu(\overline{u}x) \not{x} \lambda^a \psi_c(-x) \tag{1.20}$$

$$= \frac{-\alpha_s}{2\pi^3} \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \int \mathrm{d}z \overline{\psi}_c(z + \overline{u}vx) \lambda^a \Gamma(d-1) (v\overline{v})^{\frac{d}{2}-2} \overline{v} \frac{\not{x}\left(\not{z} + \overline{u}v\not{x}\right)\not{x}}{(z^2 + v\overline{v}u^2x^2)^{d-1}} \psi_c(-x) \tag{1.21}$$

Expand field psi + Numerator analysis:

$$\left(\overline{\psi}_c(\overline{u}vx) + z^\alpha \partial_\alpha \overline{\psi}_c(\overline{u}vx)\right) \not{x}\left(\not{z} + \overline{u}v\not{x}\right)\not{x} = \left(\overline{\psi}_c(\overline{u}vx) + z^\alpha \partial_\alpha \overline{\psi}_c(\overline{u}vx)\right)\left(-\not{z}x^2 + 2(z\cdot x)\not{x} + \overline{u}vx^2\not{x}\right) \tag{1.22}$$

$$= \overline{\psi}_c(\overline{u}vx) \overline{u}vx^2 \not{x} + z^\alpha \partial_\alpha \overline{\psi}_c(\overline{u}vx) \left(-\not{z}x^2 + 2(z\cdot x)\not{x}\right) \tag{1.23}$$

these terms yield

$$1: \quad \int \mathrm{d}z \frac{\overline{\psi}_c \overline{u}vx^2 \not{x}}{(z^2 + v\overline{v}u^2x^2)^{d-1}} \tag{1.24}$$

$$= -v^{2-\frac{d}{2}} \overline{v}^{1-\frac{d}{2}} \overline{u} u^{2-d} v \mathrm{i}\pi^2 \frac{\Gamma(d/2-1)}{\Gamma(d-1)} \frac{\not{x}\psi_c}{(-x^2)^{\frac{d}{2}-2}} \tag{1.25}$$

$$2: \quad \int \mathrm{d}z \frac{-x^2 z^\beta z^\alpha \partial_\alpha \overline{\psi}_c \gamma_\beta}{(z^2 + v\overline{v}u^2x^2)^{d-1}} \tag{1.26}$$

$$= dropped(1/x^2) \tag{1.27}$$

$$3: \quad \int \mathrm{d}z \frac{2x_\beta z^\beta z^\alpha \partial_\alpha \overline{\psi}_c}{(z^2 + v\overline{v}u^2x^2)^{d-1}} \tag{1.28}$$

$$= (v\overline{v})^{2-\frac{d}{2}} \mathrm{i}\pi^2 \frac{\Gamma\left(\frac{d}{2}-2\right)}{\Gamma(d-1)} \frac{x^\alpha \partial_\alpha \overline{\psi}_c}{(-u^2x^2)^{\frac{d}{2}-2}} \tag{1.29}$$

merge it together, using only the third term since 2nd is of higher power and first term UV divergent ($u \to 0$).

$$\frac{-\mathrm{i}\alpha_s}{2\pi}\int_0^1 \mathrm{d}u\int_0^1 \mathrm{d}v x^\alpha\partial_\alpha\overline{\psi}_c(\overline{u}vx)\lambda^a\overline{v}u^{4-d}\frac{\Gamma\left(\frac{d}{2}-2\right)}{(-x^2)^{\frac{d}{2}-2}}\psi_c(-x) \tag{1.30}$$

$$=\frac{-\mathrm{i}\alpha_s}{2\pi}\int_0^1 \mathrm{d}v\left(\overline{\psi}_c(0)-\overline{\psi}_c(-vx)\right)\lambda^a\overline{v}\frac{\Gamma\left(\frac{d}{2}-2\right)}{(-x^2)^{\frac{d}{2}-2}}\psi_c(-x) \tag{1.31}$$

factor of i missing somewhere in the beginning. why? every propagator gives one, then from loop integral 1, 2 from the start, 5 in total?

shifting back the arguments of the functions by $+x$, receive:

$$\frac{-\mathrm{i}\alpha_s}{2\pi}\int_0^1 \mathrm{d}v\left(\overline{\psi}_c(x)-\overline{\psi}_c(\overline{v}x)\right)\lambda^a\overline{v}\frac{\Gamma\left(\frac{d}{2}-2\right)}{(-x^2)^{\frac{d}{2}-2}}\psi_c(0) \tag{1.32}$$

C diagram c

start with formula:

$$[D:c]=\int \mathrm{d}^dz\int \mathrm{d}^dy\,\mathrm{i}g\overline{\psi}(z)\not{A}(z)\psi(z)\overline{\psi}(x)\lambda^a\not{x}\psi(0)\mathrm{i}g\overline{\psi}(y)\not{A}(y)\psi(y) \tag{1.33}$$

$$=-\mathrm{i}^5g^2\frac{\Gamma^2\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{\left(2\pi^{\frac{d}{2}}\right)^2 4\pi^{\frac{d}{2}}}\delta^{ab}\eta^{\mu\nu}\int \mathrm{d}^dz\int \mathrm{d}^dy \tag{1.34}$$

$$\overline{\psi}(z)\lambda^a t^a t^b\gamma_\mu\frac{\not{z}-\not{x}}{(-(z-x)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\not{x}\frac{-\not{y}}{(-y^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\gamma_\nu\frac{1}{(-(y-z)^2+\mathrm{i}\epsilon)^{\frac{d}{2}-1}}\psi(y) \tag{1.35}$$

$$=\frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}}C_F\int \mathrm{d}^dz\int \mathrm{d}^dy\overline{\psi}(z)\lambda^a\gamma_\mu\left(\not{z}-\not{x}\right)\not{x}\not{y}\gamma^\mu\psi(y) \tag{1.36}$$

$$\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\frac{\alpha^{\frac{d}{2}-1}\beta^{\frac{d}{2}-1}\gamma^{\frac{d}{2}-2}\Gamma\left(\frac{3d}{2}-1\right)}{(-\alpha(z-x)^2-\beta y^2-\gamma(y-z)^2+\mathrm{i}\epsilon)^{\frac{3d}{2}-1}} \tag{1.37}$$

$$\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]=\int_0^1\mathrm{d}\alpha\int_0^1\mathrm{d}\beta\int_0^1\mathrm{d}\gamma\delta(\alpha+\beta+\gamma) \tag{1.38}$$

Start denominator algebra with *Mathematica*(shift y,z by y,z plus ux,wx and solve u and w such that there is no mix term xz, xy). Find the following shifts:

$$z\to z+\frac{\alpha\beta+\gamma\alpha}{\Lambda}x \tag{1.39}$$

$$y\to y+\frac{\gamma\alpha}{\Lambda}x \tag{1.40}$$

$$\Lambda=\alpha\beta+\beta\gamma+\gamma\alpha \tag{1.41}$$

Transforms the denominator to

$$\left(-\alpha(z-x)^2-\beta y^2-\gamma(y-z)^2+\mathrm{i}\epsilon\right) \tag{1.42}$$

$$\to\left(\alpha y^2+\beta z^2+2\gamma yz-\frac{\alpha\beta\gamma}{\Lambda}x^2+\mathrm{i}\epsilon\right) \tag{1.43}$$

for the numerator(gamma structure) we only take terms which give us the correct power/dimension, meaning no x^2!

$$\gamma_\mu \left(\not{z} - \not{x}\right) \not{x} \not{y} \gamma^\mu \to \gamma_\mu \not{z} \not{x} \not{y} \gamma^\mu \tag{1.44}$$

Now we shift such that we can first integrate over y(isolate y^2) and then over z. now shift not by x but by z! again *Mathematica*

$$y \to y - \frac{\gamma}{\alpha} z \tag{1.45}$$

$$\left(\alpha y^2 + \beta z^2 + 2\gamma yz - \frac{\alpha\beta\gamma}{\Lambda} x^2 + \mathrm{i}\epsilon\right) \to \left(\frac{\Lambda}{\alpha} z^2 - \alpha y^2 - \frac{\alpha\beta\gamma}{\Lambda} x^2 + \mathrm{i}\epsilon\right) \tag{1.46}$$

$$= (-\alpha) \left(y^2 - \left(\frac{\Lambda}{\alpha^2} z^2 - \frac{\beta\gamma}{\Lambda} x^2\right) + \mathrm{i}\epsilon\right) \tag{1.47}$$

at the same time the numerator:

$$\gamma_\mu \not{z} \not{x} \not{y} \gamma^\mu \to \gamma_\mu \not{z} \not{x} \left(\not{y} - \frac{\gamma}{\alpha} \not{z}\right) \gamma^\mu \tag{1.48}$$

$$\to -\frac{\gamma}{\alpha} \gamma_\mu \not{z} \not{x} \not{z} \gamma^\mu \tag{1.49}$$

$$= -\frac{\gamma}{\alpha} \gamma_\mu \gamma_\tau \not{x} \gamma_\eta \gamma^\mu z^\tau z^\eta \tag{1.50}$$

So i have the following relevant integral(if i made no mistakes the numerator structure is not relevant.):

$$(-\alpha)^{1-\frac{3d}{2}} \int \mathrm{d}^d y \frac{1}{\left(y^2 - \left(\frac{\Lambda}{\alpha^2} z^2 - \frac{\beta\gamma}{\Lambda} x^2\right) + \mathrm{i}\epsilon\right)^{\frac{3d}{2}-1}} = \mathrm{i}\pi^{\frac{d}{2}} \alpha^{1-\frac{3d}{2}} \frac{\Gamma\left(d-1\right)}{\Gamma\left(\frac{3d}{2}-1\right)} \frac{1}{\left(\frac{\Lambda}{\alpha^2} z^2 - \left(\frac{\beta\gamma}{\Lambda} x^2 - \mathrm{i}\epsilon\right)\right)^{d-1}} \tag{1.51}$$

Now pull out coefficient of z^2 and perform same integral again, now watch numerator!

$$\int \mathrm{d}^d z \frac{-\frac{\gamma}{\alpha} \left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \gamma_\mu \gamma_\tau \not{x} \gamma_\eta \gamma^\mu z^\tau z^\eta}{\left(z^2 - \left(\frac{\alpha^2\beta\gamma}{\Lambda^2} x^2 - \mathrm{i}\epsilon\right)\right)^{d-1}} = +\frac{\gamma}{\alpha} \left(-\frac{\Lambda}{\alpha^2}\right)^{1-d} \gamma_\mu \gamma \tau \not{x} \gamma^\tau \gamma^\mu \frac{\mathrm{i}\pi^{\frac{d}{2}}}{2} \frac{\Gamma\left(\frac{d}{2}-2\right)}{\Gamma\left(d-1\right)} \frac{1}{\left(\frac{\alpha^2\beta\gamma}{\Lambda^2} x^2 - \mathrm{i}\epsilon\right)^{\frac{d}{2}-2}} \tag{1.52}$$

At this point realize that all the previous stuff is pathological, since i forgot to expand the fields. Bad. !!!!!!!!!! Collect :

$$[D:c] = \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int \left[\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma\right] \alpha^{\frac{d}{2}-1} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right) \tag{1.53}$$

$$\int \mathrm{d}^d z \int \mathrm{d}^d y \overline{\psi}(z + \frac{\alpha\beta + \gamma\alpha}{\Lambda} x) \lambda^a \frac{\gamma_\mu \left(\not{z} - \frac{\beta\gamma}{\Lambda} \not{x}\right) \not{x} \left(\not{y} + \frac{\gamma\alpha}{\Lambda} \not{x}\right) \gamma^\mu}{\left(\alpha y^2 + \beta z^2 + 2\gamma yz - \frac{\alpha\beta\gamma}{\Lambda} x^2 + \mathrm{i}\epsilon\right)^{\frac{3d}{2}-1}} \psi(y + \frac{\gamma\alpha}{\Lambda} x) \tag{1.54}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int \left[\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma\right] \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right) (-1)^{\frac{3d}{2}-1} \tag{1.55}$$

$$\int \mathrm{d}^d z \int \mathrm{d}^d y \overline{\psi}(z + \frac{\alpha\beta + \gamma\alpha}{\Lambda} x) \lambda^a \frac{\gamma_\mu \left(\not{z} - \frac{\beta\gamma}{\Lambda} \not{x}\right) \not{x} \left(\not{y} - \frac{\gamma}{\alpha} \not{z} + \frac{\gamma\alpha}{\Lambda} \not{x}\right) \gamma^\mu}{\left(y^2 - \left(\frac{\Lambda}{\alpha^2} z^2 - \frac{\beta\gamma}{\Lambda} x^2 - \mathrm{i}\epsilon\right)\right)^{\frac{3d}{2}-1}} \psi(y - \frac{\gamma}{\alpha} z + \frac{\gamma\alpha}{\Lambda} x) \tag{1.56}$$

the numerator analysis, again: first expand the fields

$$\psi(y-\frac{\gamma}{\alpha}z+\frac{\gamma\alpha}{\Lambda}x)=\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)+y^{\rho}\partial_{\rho}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)-\frac{\gamma}{\alpha}z^{\rho}\partial_{\rho}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)+\ldots \tag{1.57}$$

$$\overline{\psi}(z+\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)=\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)+z^{\rho}\partial_{\rho}\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)+\ldots \tag{1.58}$$

Numerator:

$$\left(\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)+z^{\rho}\partial_{\rho}\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)+z^{\rho}z^{\zeta}\partial_{\rho}\partial_{\zeta}\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\right)\lambda^{a}\gamma_{\mu}\left(\not{z}-\frac{\beta\gamma}{\Lambda}\not{x}\right)\not{x}\left(\not{y}-\frac{\gamma}{\alpha}\not{z}+\frac{\gamma\alpha}{\Lambda}\not{x}\right)\gamma^{\mu} \tag{1.59}$$

$$\left(\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)+y^{\sigma}\partial_{\sigma}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)-\frac{\gamma}{\alpha}z^{\sigma}\partial_{\sigma}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)+\left(y^{\sigma}z^{\xi}+y^{\sigma}y^{\xi}+z^{\sigma}z^{\xi}\right)\partial_{\sigma}\partial_{\xi}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right)\right) \tag{1.60}$$

this are in total $3\cdot 2\cdot 3\cdot 4=72$ terms. Then some immediately cancel by symmetry. Still a lot terms. I will not write down all of them. now select only following terms This restriction only approves the term with constant fields...

$$\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^{a}\gamma_{\mu}\not{z}\not{x}\frac{\gamma}{\alpha}\not{z}\gamma^{\mu}\psi\left(\frac{\gamma\alpha}{\Lambda}x\right) \tag{1.61}$$

which corresponds to the previous calculation where i did not expand the fields. Do it again: have the calculation

$$[D:c] = \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right) (-1)^{\frac{3d}{2}-1} \tag{1.62}$$

$$\int \mathrm{d}^d z \int \mathrm{d}^d y \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{-\frac{\gamma}{\alpha}\gamma_\mu \not{z}\not{x}\not{z}\gamma^\mu}{\left(y^2-\left(\frac{\Lambda}{\alpha^2}z^2-\frac{\beta\gamma}{\Lambda}x^2-\mathrm{i}\epsilon\right)\right)^{\frac{3d}{2}-1}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.63}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right) (-1)^{\frac{3d}{2}-1} \tag{1.64}$$

$$\int \mathrm{d}^d z \frac{(-1)^{\frac{3d}{2}-1}\mathrm{i}\pi^{\frac{d}{2}}\Gamma(d-1)}{\Gamma\left(\frac{3d}{2}-1\right)} \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{-\frac{\gamma}{\alpha}\gamma_\mu \not{z}\not{x}\not{z}\gamma^\mu}{\left(\frac{\Lambda}{\alpha^2}z^2-\left(\frac{\beta\gamma}{\Lambda}x^2+\mathrm{i}\epsilon\right)\right)^{d-1}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.65}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right) (-1)^{\frac{3d}{2}-1} \left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.66}$$

$$\int \mathrm{d}^d z \frac{(-1)^{\frac{3d}{2}-1}\mathrm{i}\pi^{\frac{d}{2}}\Gamma(d-1)}{\Gamma\left(\frac{3d}{2}-1\right)} \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{-\frac{\gamma}{\alpha}\gamma_\mu \gamma_\tau \not{x}\gamma_\theta\gamma^\mu z^\tau z^\theta}{\left(z^2-\left(\frac{\alpha^2\beta\gamma}{\Lambda^2}x^2+\mathrm{i}\epsilon\right)\right)^{d-1}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.67}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.68}$$

$$\frac{(-1)^{4d-1}\mathrm{i}^2\Gamma\left(\frac{d}{2}-2\right)}{2} \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{-\frac{\gamma}{\alpha}\gamma_\mu \gamma_\theta \not{x}\gamma^\theta\gamma^\mu}{\left(\frac{\alpha^2\beta\gamma}{\Lambda^2}x^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}-2}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.69}$$

$$= \frac{+\mathrm{i}g^2}{16\pi^{\frac{d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \Lambda^{-3}\alpha^1\beta^1\gamma^1 \tag{1.70}$$

$$(-1)^{4d-1}\Gamma\left(\frac{d}{2}-2\right) \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{(d-2)^2\not{x}}{(x^2+\mathrm{i}\epsilon)^{\frac{d}{2}-2}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.71}$$

$$= \frac{+\mathrm{i}g^2}{16\pi^{\frac{d}{2}}} C_F \int [\mathrm{d}\alpha' \mathrm{d}\beta' \mathrm{d}\gamma'] \tag{1.72}$$

$$(-1)^{4d-1}\Gamma\left(\frac{d}{2}-2\right) \overline{\psi}(\overline{\alpha}'x)\lambda^a \frac{(d-2)^2\not{x}}{(x^2+\mathrm{i}\epsilon)^{\frac{d}{2}-2}} \psi(\beta'x) \tag{1.73}$$

I can also take the term (as an example)

$$-\frac{\alpha\beta\gamma^2}{\Lambda^2}\overline{\psi}\lambda^a\gamma_\mu\not{z}\not{x}\not{z}\gamma^\mu\psi \rightarrow \tag{1.74}$$

$$[D:c] = \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}}C_F\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\alpha^{-d}\beta^{\frac{d}{2}-1}\gamma^{\frac{d}{2}-2}\Gamma\left(\frac{3d}{2}-1\right)(-1)^{\frac{3d}{2}-1} \tag{1.75}$$

$$\int\mathrm{d}^dz\int\mathrm{d}^dy\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a\frac{-\frac{\alpha\beta\gamma^2}{\Lambda^2}\gamma_\mu x^2\not{z}\gamma^\mu}{\left(y^2-\left(\frac{\Lambda}{\alpha^2}z^2-\frac{\beta\gamma}{\Lambda}x^2-\mathrm{i}\epsilon\right)\right)^{\frac{3d}{2}-1}}\psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.76}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}}C_F\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\alpha^{-d}\beta^{\frac{d}{2}-1}\gamma^{\frac{d}{2}-2}(-1)^{3d}\left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.77}$$

$$\int\mathrm{d}^dz\mathrm{i}\pi^{\frac{d}{2}}\Gamma\left(d-1\right)\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a\frac{-\frac{\alpha\beta\gamma^2}{\Lambda^2}\gamma_\mu x^2\not{z}\gamma^\mu}{\left(z^2-\left(\frac{\alpha^2\beta\gamma}{\Lambda^2}x^2+\mathrm{i}\epsilon\right)\right)^{d-1}}\psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.78}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{d}{2}}}C_F\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\alpha^{-d}\beta^{\frac{d}{2}-1}\gamma^{\frac{d}{2}-2}(-1)^{4d-1}\left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.79}$$

$$\mathrm{i}^2\Gamma\left(\frac{d}{2}-1\right)\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a\frac{-\frac{\alpha\beta\gamma^2}{\Lambda^2}\gamma_\mu x^2\not{z}\gamma^\mu}{\left(\frac{\alpha^2\beta\gamma}{\Lambda^2}x^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}-1}}\psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.80}$$

$$= \frac{-\mathrm{i}g^2}{8\pi^{\frac{d}{2}}}C_F\int[\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma]\,\alpha^1\beta^1\gamma^1(-1)^{4d-1}\Lambda^{-3} \tag{1.81}$$

$$\Gamma\left(\frac{d}{2}-1\right)\overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a\frac{(d-2)\not{x}}{(x^2+\mathrm{i}\epsilon)^{\frac{d}{2}-2}}\psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.82}$$

$$= \frac{-\mathrm{i}g^2}{8\pi^{\frac{d}{2}}}C_F\int[\mathrm{d}\alpha'\mathrm{d}\beta'\mathrm{d}\gamma']\,(-1)^{4d-1} \tag{1.83}$$

$$\Gamma\left(\frac{d}{2}-1\right)\overline{\psi}(\overline{\alpha}'x)\lambda^a\frac{(d-2)\not{x}}{(x^2+\mathrm{i}\epsilon)^{\frac{d}{2}-2}}\psi(\beta'x) \tag{1.84}$$

and this is of same power with x^2 in denominator.
other possibility which now should give different order:

$$\frac{\gamma\alpha}{\Lambda}z^\rho\partial_\rho\overline{\psi}\lambda^a\gamma_\mu\not{z}\not{z}\not{z}\gamma^\mu\psi \tag{1.85}$$

then

$$[D:c] = \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} \Gamma\left(\frac{3d}{2}-1\right)(-1)^{\frac{3d}{2}-1} \tag{1.86}$$

$$\int \mathrm{d}^d z \int \mathrm{d}^d y \partial_\rho \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{\frac{\gamma\alpha}{\Lambda} z^\rho \gamma_\mu \not{z} x^2 \gamma^\mu}{\left(y^2 - \left(\frac{\Lambda}{\alpha^2}z^2 - \frac{\beta\gamma}{\Lambda}x^2 - \mathrm{i}\epsilon\right)\right)^{\frac{3d}{2}-1}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.87}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{3d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} (-1)^{\frac{6d}{2}-2} \left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.88}$$

$$\int \mathrm{d}^d z \mathrm{i}\pi^{\frac{d}{2}} \Gamma(d-1)\, \partial_\rho \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{\frac{\gamma\alpha}{\Lambda} z^\rho z^\theta \gamma_\mu \gamma_\theta x^2 \gamma^\mu}{\left(z^2 - \left(\frac{\beta\gamma}{\Lambda}x^2 + \mathrm{i}\epsilon\right)\right)^{d-1}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.89}$$

$$= \frac{\mathrm{i}g^2}{8\pi^{\frac{d}{2}}} C_F \int [\mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma]\, \alpha^{-d} \beta^{\frac{d}{2}-1} \gamma^{\frac{d}{2}-2} (-1)^{4d-1} \left(\frac{\Lambda}{\alpha^2}\right)^{1-d} \tag{1.90}$$

$$\frac{\mathrm{i}^2 \Gamma\left(\frac{d}{2}-2\right)}{2} \partial_\rho \overline{\psi}(\frac{\alpha\beta+\gamma\alpha}{\Lambda}x)\lambda^a \frac{\frac{\gamma\alpha}{\Lambda}\gamma_\mu \gamma^\rho x^2 \gamma^\mu}{\left(\frac{\beta\gamma}{\Lambda}x^2 + \mathrm{i}\epsilon\right)^{\frac{d}{2}-2}} \psi(\frac{\gamma\alpha}{\Lambda}x) \tag{1.91}$$

And this term is now of order x^2 in total, which is a correction only. this term will be dropped. So i think i do not care about derivatives on quantum fields, but every x,y,z pair on top decreases the power of the denominator.
Schematically

$$\int dx^d z \int \mathrm{d}^d y \frac{z^{2k} y^{2l} x^{2m}}{(y^2 - z^2 + x^2)^{\frac{3d}{2}-1}} \to \int dx^d z \frac{z^{2k} x^{2m}}{(z^2 + x^2)^{d-1-l}} \tag{1.92}$$

$$\to \frac{x^{2m}}{(x^2)^{\frac{d}{2}-1-l-k}} \tag{1.93}$$

$$= (x^2)^{m+l+k+1-\frac{d}{2}} \tag{1.94}$$

We are looking for $2 = m+l+1+k$ and i omitt the 1 slashed x, which is always there from the middle term, where i have no choice. (means I can have $x^2 (m=1)$, $x^2 \not{x} (m=1)$ or $\not{x} (m=0)$ only)
Applying this analysis I i only have the two terms which are the first two examples. All other terms with give higher power in x^2 and are hence a correction

II twist2

have operator (fraction):

$$O_q^1(z) = [-\infty n, zn]\, q(zn) \tag{2.1}$$

For one loop I have to take linear term of wilson line and inlcude first order of interaction Hamiltionian. Thus

$$A = \left[-\mathrm{i}g \int_{-\infty}^{z} \mathrm{d}\sigma B^+(\sigma n)\right] q(zn) \left(\mathrm{i}g \int \mathrm{d}^d x \overline{q}(x) \not{B}(x) q(x)\right) \tag{2.2}$$

I do not write the contractions since they are clear and I am lazy. result: we shift z to 0 because it does not matter and makes it easier. ACTUALLY:: this is a dangerous move. doing it carefully and not much more difficult, one shifts integration variable x such that z is dropped in propagators. However it remains in integration in wilson line AND also in quark field shift. thus care and understand that in the end the quark field position is shifted by $+z$. Important mistake I made at some point: the integration limit 0 is NOT SHIFTED! it is 0 for any z! Mid calculation shift : $x = x + \overline{\alpha}\sigma n$ note that n is a light-like vector

$$A = -(\mathrm{i}g)^2 C_F \int_{-\infty}^{z} \mathrm{d}\sigma \int \mathrm{d}^d x n^\mu \frac{-1}{4\pi^{\frac{d}{2}}} \frac{\Gamma\left(\frac{d}{2}-1\right)}{\left(-(x-\sigma n)^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}-1}} \frac{-\mathrm{i}\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}} \frac{\not{x}}{\left(-x^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}}} \gamma_\mu q(x) \tag{2.3}$$

$$= -(\mathrm{i}g)^2 C_F \int_{-\infty}^{z} \mathrm{d}\sigma \int \mathrm{d}^d x \frac{\mathrm{i}\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}\right)}{8\pi^d} \frac{\not{x}}{\left(-(x-\sigma n)^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}-1}\left(-x^2+\mathrm{i}\epsilon\right)^{\frac{d}{2}}} \gamma^+ q(x) \tag{2.4}$$

$$= -(\mathrm{i}g)^2 C_F \int_{-\infty}^{z} \mathrm{d}\sigma \int \mathrm{d}^d x \int_0^1 \mathrm{d}\alpha \frac{\mathrm{i}\Gamma\left(d-1\right)}{8\pi^d} \frac{\alpha^{\frac{d}{2}-1}\overline{\alpha}^{\frac{d}{2}-2}\not{x}\gamma^+}{\left(-\overline{\alpha}(x-\sigma n)^2-\alpha x^2+\mathrm{i}\epsilon\right)^{d-1}} q(x) \tag{2.5}$$

$$= -(\mathrm{i}g)^2 C_F \int_{-\infty}^{z} \mathrm{d}\sigma \int \mathrm{d}^d x \int_0^1 \mathrm{d}\alpha \frac{\mathrm{i}\Gamma\left(d-1\right)}{8\pi^d} \frac{\alpha^{\frac{d}{2}-1}\overline{\alpha}^{\frac{d}{2}-2}\not{x}\gamma^+}{\left(-x^2+\mathrm{i}\epsilon\right)^{d-1}} q(x+\overline{\alpha}\sigma n) \tag{2.6}$$

here use the formula (copied from Alexeys notes)

$$\int \mathrm{d}^d x \frac{x^\mu x^\nu}{\left(-x^2+\mathrm{i}\epsilon\right)^{d-1}} = \frac{-\mathrm{i}\pi^{\frac{d}{2}}}{2(2-\frac{d}{2})} \frac{\eta^{\mu\nu}}{\Gamma\left(d-1\right)} \tag{2.7}$$

own adaption for four elements in numerator:

$$\int \mathrm{d}^d x \frac{x^\mu x^\nu x^\rho x^\sigma}{\left(-x^2+\mathrm{i}\epsilon\right)^{d}} = \frac{\mathrm{i}\pi^{\frac{d}{2}}}{4(2-\frac{d}{2})\Gamma\left(d\right)} \left(\eta^{\mu\nu}\eta^{\rho\sigma}+\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}\right) \tag{2.8}$$

expand quark field and have

$$A = -(\mathrm{i}g)^2 C_F \int_{-\infty}^{z} \mathrm{d}\sigma \int_0^1 \mathrm{d}\alpha \frac{1}{8\pi^{\frac{d}{2}}} \frac{\alpha^{\frac{d}{2}-1}\overline{\alpha}^{\frac{d}{2}-2}\gamma^\nu\gamma^+}{2\left(2-\frac{d}{2}\right)} \partial_\nu q(\overline{\alpha}\sigma n) \tag{2.9}$$

A left quark

$$\overline{O}_q^1(z) = \overline{q}(zn)\left[zn, -\infty n\right] \tag{2.10}$$

First order corr:Mid calculation shift : $x = x + \alpha\sigma n$

$$\overline{A} = \left(\mathrm{i}g\int \mathrm{d}^d x\overline{q}(x)\not{B}(x)q(x)\right)\overline{q}(zn)\,[zn,-\infty n]\left[\mathrm{i}g\int_{-\infty}^{z}\mathrm{d}\sigma B^{+}(\sigma n)\right] \tag{2.11}$$

$$= (\mathrm{i}g)^2 C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int \mathrm{d}^d x\overline{q}(x)\gamma^{+}\frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}}\frac{\mathrm{i}\not{x}}{(-x^2+\mathrm{i}\varepsilon)^{\frac{d}{2}}}\frac{-1}{4\pi^{\frac{d}{2}}}\frac{\Gamma\left(\frac{d}{2}-1\right)}{(-(x-\sigma n)^2+\mathrm{i}\epsilon)^{\frac{d}{2}-1}} \tag{2.12}$$

$$= -\mathrm{i}(\mathrm{i}g)^2 C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int \mathrm{d}^d x\int_0^1\mathrm{d}\alpha\overline{q}(x)\frac{\Gamma\left(d-1\right)}{8\pi^d}\gamma^{+}\not{x}\frac{\alpha^{\frac{d}{2}-2}\overline{\alpha}^{\frac{d}{2}-1}}{(-\overline{\alpha}x^2-\alpha(x-\sigma n)^2+\mathrm{i}\epsilon)^{d-1}} \tag{2.13}$$

$$= -\mathrm{i}(\mathrm{i}g)^2 C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int \mathrm{d}^d x\int_0^1\mathrm{d}\alpha\overline{q}(x+\alpha\sigma n)\frac{\Gamma\left(d-1\right)}{8\pi^d}\gamma^{+}\not{x}\frac{\alpha^{\frac{d}{2}-2}\overline{\alpha}^{\frac{d}{2}-1}}{(-x^2+\mathrm{i}\epsilon)^{d-1}} \tag{2.14}$$

$$= -(\mathrm{i}g)^2 C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int_0^1\mathrm{d}\alpha\partial_\mu\overline{q}(\alpha\sigma n)\frac{1}{8\pi^{\frac{d}{2}}}\gamma^{+}\gamma^{\mu}\frac{\alpha^{\frac{d}{2}-2}\overline{\alpha}^{\frac{d}{2}-1}}{2\left(2-\frac{d}{2}\right)} \tag{2.15}$$

$$= \frac{\alpha_s}{4\pi}C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int_0^1\mathrm{d}\alpha\frac{\alpha^{\frac{d}{2}-2}\overline{\alpha}^{\frac{d}{2}-1}}{\left(2-\frac{d}{2}\right)}\partial_\mu\overline{q}(\alpha\sigma n)\gamma^{+}\gamma^{\mu} \tag{2.16}$$

$$= \frac{2\alpha_s}{4\pi}C_F\int_{-\infty}^{z}\mathrm{d}\sigma\int_0^1\mathrm{d}\alpha\frac{\overline{\alpha}}{\epsilon}\partial_{+}\overline{q}(\alpha\sigma n) \tag{2.17}$$

perform the last integral is not trivial for me:

$$f(x) = \int\frac{\mathrm{d}^d p}{(2\pi)^d}e^{\mathrm{i}(p|x)}\int \mathrm{d}^d x' e^{-\mathrm{i}(p|x')}f(x') \tag{2.18}$$

$$\partial_{+}\overline{q}(x) = \int\frac{\mathrm{d}^d p}{(2\pi)^d}e^{\mathrm{i}(p|x)}\int \mathrm{d}^d x' e^{-\mathrm{i}(p|x')}\partial_{+}\overline{q}(x') \tag{2.19}$$

$$= \int\frac{\mathrm{d}^d p}{(2\pi)^d}\int \mathrm{d}^d x'\left(-\mathrm{i}p_{+}\right)e^{\mathrm{i}(p|x-x')}\overline{q}(x') \tag{2.20}$$

$$= \int\frac{\mathrm{d}^d p}{(2\pi)^d}\left(-\mathrm{i}p_{+}\right)e^{\mathrm{i}(p|x)}\overline{q}(p) \tag{2.21}$$

then

$$\int_0^1\mathrm{d}\alpha\int_{-\infty}^{0}\mathrm{d}\sigma\overline{\alpha}\partial_{+}\overline{q}((\alpha\sigma+z)n) = \int\frac{\mathrm{d}^d p}{(2\pi)^d}e^{\mathrm{i}p_{+}z}\overline{q}(p)\int_0^1\mathrm{d}\alpha\int_{-\infty}^{0}\mathrm{d}\sigma\overline{\alpha}\left(-\mathrm{i}p_{+}\right)e^{\mathrm{i}\alpha\sigma p_{+}} \tag{2.22}$$

to deal with oscillating integral : introduce regulator for the integral, δ

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{\mathrm{i}p_+ z} \overline{q}(p) \int_0^1 \mathrm{d}\alpha \int_{-\infty}^0 \mathrm{d}\sigma \overline{\alpha} \left(-\mathrm{i}p_+\right) e^{\mathrm{i}\alpha\sigma p_+} e^{\delta\sigma} \tag{2.23}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{\mathrm{i}p_+ z} \overline{q}(p) \int_0^1 \mathrm{d}\alpha \frac{\overline{\alpha}\left(-\mathrm{i}p_+\right)}{\mathrm{i}\alpha p_+ + \delta} \tag{2.24}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{\mathrm{i}p_+ z} \overline{q}(p) \int_0^1 \mathrm{d}\alpha \frac{\alpha - 1}{\alpha + \frac{\delta}{\mathrm{i}p_+}} \tag{2.25}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{\mathrm{i}p_+ z} \overline{q}(p) \int_{\frac{\delta}{\mathrm{i}p_+}}^{1+\frac{\delta}{\mathrm{i}p_+}} \mathrm{d}\alpha \frac{\alpha - 1}{\alpha} \tag{2.26}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{\mathrm{i}p_+ z} \overline{q}(p) \left(1 - \ln\left(\frac{\mathrm{i}p_+}{\delta}\right)\right) \tag{2.27}$$

CARE global minus difference to Alexeys result...

B gluons

A better working method is maybe to parallel write serious exressions and then drop from them from time to time parts into "prefactor" expression

Let me do parallel computing.

I take color red for right field, color blue for left field. Common factors i put out in front will be black, no color. Thus here we have the following diagrams.

$$\left(-\mathrm{ig}\int \mathrm{d}^d x A_\nu^D(x)\partial_{x^\alpha} B_\beta^E(x) B_\gamma^F(x)\right) v_{DEF}^{\nu\alpha\beta\gamma} \mathrm{ig}\int_{-z}^{-\infty} \mathrm{d}\sigma B_+^{B'}(\sigma n) T_{CA'}^{B'} \partial_\mu B_\rho^{A'}(-zn) \tag{2.28}$$

$$+\left(-\mathrm{ig}\int \mathrm{d}^d x A_\nu^D(x)\partial_{x^\alpha} B_\beta^E(x) B_\gamma^F(x)\right) v_{DEF}^{\nu\alpha\beta\gamma} \mathrm{ig}\int_{-z}^{-\infty} \mathrm{d}\sigma B_+^{B'}(\sigma n) T_{CA'}^{B'} \partial_\mu B_\rho^{A'}(-zn) \tag{2.29}$$

$$\partial_\mu B_\rho^{A'}(zn)\mathrm{ig}\int_{-\infty}^{z} \mathrm{d}\sigma B_+^{B'}(\sigma n) T_{A'C}^{B'}\left(-\mathrm{ig}\int \mathrm{d}^d x A_\nu^D(x)\partial_{x^\alpha} B_\beta^E(x) B_\gamma^F(x)\right) v_{DEF}^{\nu\alpha\beta\gamma} \tag{2.30}$$

$$+\partial_\mu B_\rho^{A'}(zn)\mathrm{ig}\int_{-\infty}^{z} \mathrm{d}\sigma B_+^{B'}(\sigma n) T_{A'C}^{B'}\left(-\mathrm{ig}\int \mathrm{d}^d x A_\nu^D(x)\partial_{x^\alpha} B_\beta^E(x) B_\gamma^F(x)\right) v_{DEF}^{\nu\alpha\beta\gamma} \tag{2.31}$$

For the colors:

$$f^{DEF} T_{A'C}^{B'}(\delta^{A'E}\delta^{B'F} + \delta^{A'F}\delta^{B'E}) = -\mathrm{i}(f^{DEF} f^{B'A'C}\delta^{A'E}\delta^{B'F} + f^{DEF} f^{B'A'C}\delta^{A'F}\delta^{B'E}) \tag{2.32}$$

$$= -\mathrm{i}(-f^{DEF} f^{CEF} + f^{DEF} f^{CEF}) \tag{2.33}$$

$$= \mathrm{i}C_A \delta^{CD}(+1-1) \tag{2.34}$$

$+1-1$ means, the first line has $+$, the second line has -1.

$$f^{DEF}T^{B'}_{CA'}(\delta^{A'E}\delta^{B'F}+\delta^{A'F}\delta^{B'E}) = -\mathrm{i}(f^{DEF}f^{B'CA'}\delta^{A'E}\delta^{B'F}+f^{DEF}f^{B'CA'}\delta^{A'F}\delta^{B'E}) \tag{2.35}$$
$$= -\mathrm{i}(f^{DEF}f^{FCE}+f^{DEF}f^{ECF}) \tag{2.36}$$
$$= -\mathrm{i}f^{DEF}f^{FCE}(+1-1) \tag{2.37}$$
$$= -\mathrm{i}C_A\delta^{CD}(+1-1) \tag{2.38}$$
$$= -\mathrm{i}C_A\delta^{CD}(+1-1) \tag{2.39}$$

so from color the two diagrams differ by a total sign First common factors:

$$+\mathrm{i}^3C_Ag^2\int \mathrm{d}^d v^{\nu\alpha\beta\gamma}\delta^{CD} \tag{2.40}$$

$$\int_{-\infty}^{z}\partial_{zn^\mu}\partial_{x^\alpha}\Delta(zn-x)_{\rho\beta}\Delta(x-\sigma n)_{\gamma+}-\partial_{zn^\mu}\Delta(zn-x)_{\rho\gamma}\partial_{x^\alpha}\Delta(x-\sigma n)_{\beta+} \tag{2.41}$$
$$-\int_{-z}^{-\infty}\partial_{zn^\mu}\partial_{x^\alpha}\Delta(zn+x)_{\rho\beta}\Delta(x-\sigma n)_{\gamma+}-\partial_{zn^\mu}\Delta(zn+x)_{\rho\gamma}\partial_{x^\alpha}\Delta(x-\sigma n)_{\beta+} \tag{2.42}$$

Then we have the to compute as usual, derivatives on propagators. Since i know that the metric computation will kill the additional term from double derivative on one propagator (additional term $\tilde{\eta}_{\mu\alpha}$ I IGNORE IT HERE.)

$$+\,\mathrm{i}^3C_Ag^2\int \mathrm{d}^d\delta^{CD}\Gamma^2\left(\frac{d}{2}-1\right)\frac{1}{4^2\pi^d} \tag{2.43}$$

$$\int_{-\infty}^{z}\mathrm{d}\sigma v^{\nu\alpha\beta\gamma} \tag{2.44}$$
$$\frac{4(\frac{d}{2}-1)\frac{d}{2}(zn-x)_\mu(x-zn)_\alpha}{(-(x-zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}+1}}\frac{1}{(-(x-\sigma n)^2+\mathrm{i}\epsilon)^{\frac{d}{2}-1}}\eta_{\rho\beta}\eta_{\gamma+} \tag{2.45}$$
$$-\frac{-2(\frac{d}{2}-1)(zn-x)_\mu}{(-(x-zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\frac{-2(\frac{d}{2}-1)(x-\sigma n)_\alpha}{(-(x-\sigma n)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\eta_{\rho\gamma}\eta_{\beta+} \tag{2.46}$$

$$-\int_{-z}^{-\infty}\mathrm{d}\sigma v^{\nu\alpha\beta\gamma} \tag{2.47}$$
$$\frac{4(\frac{d}{2}-1)\frac{d}{2}(-zn-x)_\mu(x+zn)_\alpha}{(-(x+zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}+1}}\frac{1}{(-(x-\sigma n)^2+\mathrm{i}\epsilon)^{\frac{d}{2}-1}}\eta_{\rho\beta}\eta_{\gamma+} \tag{2.48}$$
$$-\frac{-2(\frac{d}{2}-1)(-zn-x)_\mu}{(-(x+zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\frac{-2(\frac{d}{2}-1)(x-\sigma n)_\alpha}{(-(x-\sigma n)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\eta_{\rho\gamma}\eta_{\beta+} \tag{2.49}$$

shifts:

$$
\begin{aligned}
x \to & x + uzn + \overline{u}\sigma n && (2.50)\\
\sigma \to & \sigma + z && (2.51)\\
HENCE & && (2.52)\\
x \to & x + zn + \overline{u}\sigma n && (2.53)\\
x - zn \to & x + \overline{u}\sigma n && (2.54)\\
x - \sigma n \to & x - u\sigma n && (2.55)
\end{aligned}
$$

$$
\begin{aligned}
x \to & x - uzn + \overline{u}\sigma n && (2.56)\\
\sigma \to & \sigma - z && (2.57)\\
HENCE & && (2.58)\\
x \to & x - zn + \overline{u}\sigma n && (2.59)\\
x + zn \to & x + \overline{u}\sigma n && (2.60)\\
x - \sigma n \to & x - u\sigma n && (2.61)
\end{aligned}
$$

Lorentz index computation is the same for both, red and blue. it is

$$
\begin{aligned}
v^{\nu\alpha\beta\gamma} &= 2\eta^{\nu\beta}\eta^{\alpha\gamma} - \eta^{\nu\alpha}\eta^{\beta\gamma} - 2\eta^{\nu\gamma}\eta^{\alpha\beta} && (2.62)\\
v^{\nu\alpha\beta\gamma}\eta_{\mu\alpha}\eta_{\rho\beta}\eta_{+\gamma} &= 0 (\text{that is why there is no additional term from the double derivative on prop.}) && (2.63)\\
v^{\nu\alpha\beta\gamma}\eta_{\rho\beta}\eta_{+\gamma} &= 2(\eta^{\nu\rho}\eta^{\alpha+} - \eta^{\nu+}\eta^{\alpha\rho}) && (2.64)\\
v^{\nu\alpha\beta\gamma}\eta_{\rho\gamma}\eta_{+\beta} &= -2(\eta^{\nu\rho}\eta^{\alpha+} - \eta^{\nu+}\eta^{\alpha\rho}) && (2.65)
\end{aligned}
$$

and results in relative minus sign between the respective lines plus a common factor:

$$
+ \mathrm{i}^3 C_A g^2 \int \mathrm{d}^d \delta^{CD} \Gamma^2\left(\frac{d}{2} - 1\right) \frac{1}{4^2\pi^d} 2(\eta^{\nu\rho}\eta^{\alpha+} - \eta^{\nu+}\eta^{\alpha\rho}) \tag{2.66}
$$

The minus sign from this computation annihilates the relative minus from color (the minus between the diagrams! this has nothing to do with the minus between red and blue!)

$$
\begin{aligned}
&\int_{-\infty}^{0}\int_{0}^{1} \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)} u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}(-4x_\mu)\frac{u(x+\overline{u}\sigma n)_\alpha + \overline{u}(x - u\sigma n)_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} && (2.67)\\
&= -4\int_{-\infty}^{0}\int_{0}^{1} \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)} u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\frac{x_\mu x_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} && (2.68)
\end{aligned}
$$

$$-\int_0^{-\infty}\int_0^1 \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)}u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}(-4x_\mu)\frac{u(x+\overline{u}\sigma n)_\alpha+\overline{u}(x-u\sigma n)_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} \tag{2.69}$$

$$=+4\int_0^{-\infty}\int_0^1 \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)}u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\frac{x_\mu x_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} \tag{2.70}$$

$$=-4\int_{-\infty}^{0}\int_0^1 \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)}u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\frac{x_\mu x_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} \tag{2.71}$$

at this point it looks like both computations are identical, but there is still the free field $A_\nu(x)$, which after shift is

$$A(x+zn+\overline{u}\sigma n) \qquad A(x-zn+\overline{u}\sigma n) \tag{2.72}$$

This has to be kept in mind. Further the integration can be done in for both terms identically. I write $A(x+yn+\overline{u}\sigma n)$, and $y=\pm z$, blue or red case. It follows the following computation:

$$+\mathrm{i}^3 C_A g^2\delta^{CD}\Gamma^2\left(\frac{d}{2}-1\right)\frac{1}{4^2\pi^d}2(\eta^{\nu\rho}\eta^{\alpha+}-\eta^{\nu+}\eta^{\alpha\rho})(-4)\int_{-\infty}^{0}\int_0^1 \mathrm{d}\frac{\Gamma(d)}{\Gamma^2\left(\frac{d}{2}-1\right)}u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2} \tag{2.73}$$

$$\int \mathrm{d}^d\frac{x_\mu x_\alpha x_\tau x_\sigma}{(-x^2+\mathrm{i}\epsilon)^d}\partial^\tau\partial^\sigma A(yn+\overline{u}\sigma n)=\frac{\mathrm{i}\pi^{\frac{d}{2}}}{4(2-\frac{d}{2})\Gamma(d)}\left(\eta_{\mu\alpha}\eta_{\tau\sigma}+\eta_{\mu\tau}\eta_{\alpha\sigma}+\eta_{\mu\sigma}\eta_{\alpha\tau}\right)A_\nu(yn+\overline{u}\sigma n) \tag{2.74}$$

Together metric and fields give

$$\left(\eta_{\mu\alpha}\eta_{\tau\sigma}+\eta_{\mu\tau}\eta_{\alpha\sigma}+\eta_{\mu\sigma}\eta_{\alpha\tau}\right)A_\nu(yn+\overline{u}\sigma n)-(\mu\leftrightarrow\rho)[\rho=+] \tag{2.75}$$

$$=2\partial_+\left(\partial_\mu A_+-\partial_+A_\mu\right)(yn+\overline{u}\sigma n) \tag{2.76}$$

Together with prefactors and so on

$$\frac{(-4)2^2\mathrm{i}^4C_Ag^2}{4^3\pi^{\frac{d}{2}}\epsilon}\int_{-\infty}^{0}\mathrm{d}\sigma\int_0^1\mathrm{d}u u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\partial_+\left(\partial_\mu A_+^C-\partial_+A_\mu^C\right)(yn+\overline{u}\sigma n) \tag{2.77}$$

now integration:

$$\int_0^{-\infty} \mathrm{d}\sigma \int_0^1 \mathrm{d}u u \partial_+ f(yn + \overline{u}\sigma n) \tag{2.78}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^4} \int_0^{-\infty} \mathrm{d}\sigma \int_0^1 \mathrm{d}u u(-\mathrm{i}p_+) e^{\mathrm{i}(y+\overline{u}\sigma n|p)+\delta\sigma} f(p) \tag{2.79}$$

$$= -\int \frac{\mathrm{d}^d p}{(2\pi)^4} \int_0^1 \mathrm{d}u u \frac{-\mathrm{i}p_+}{\mathrm{i}\overline{u}p_+ + \delta} e^{\mathrm{i}(y|p)} f(p) \tag{2.80}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^4} \int_0^1 \mathrm{d}\overline{u} \frac{1-\overline{u}}{\overline{u} + \frac{\delta}{\mathrm{i}p_+}} e^{\mathrm{i}(y|p)} f(p) \tag{2.81}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^4} \int_{\frac{\delta}{\mathrm{i}p_+}}^{1+\frac{\delta}{\mathrm{i}p_+}} \mathrm{d}\overline{u} \frac{1-\overline{u}-0}{\overline{u}} e^{\mathrm{i}(y|p)} f(p) \tag{2.82}$$

$$= \int \frac{\mathrm{d}^d p}{(2\pi)^4} \left(-1 + \ln\left(\frac{\mathrm{i}p_+}{\delta}\right)\right) e^{\mathrm{i}(y|p)} f(p) \tag{2.83}$$

note that i have in my previous computation a different integration order, this minus sign. From the two cases $y = \pm zn$ there are two results

$$\int_{-\infty}^0 \mathrm{d}\sigma \int_0^1 \mathrm{d}u u \partial_+ f(\pm zn + \overline{u}\sigma n) = -\left(-1 + \ln\left(\frac{\pm \mathrm{i}\hat{p}_+}{\delta}\right)\right) f(\pm zn) \tag{2.84}$$

then the final result is

$$\frac{g^2 C_A}{4\pi^2 \epsilon} \left(-1 + \ln\left(\frac{\pm \mathrm{i}\hat{p}_+}{\delta}\right)\right) F^C_{\mu+}(\pm zn) \tag{2.85}$$

Other diagram.

$$g f^{A'B'C'} B^{B'}_\mu(zn) B^{C'}_+(zn) 1 \left(-\mathrm{i}g \int \mathrm{d}^d x A^D_\nu(x) \partial_\alpha B^E_\beta(x) B^F_\gamma(x)\right) v^{\nu\alpha\beta\gamma}_{DEF} \tag{2.86}$$

$$\to g f^{A'B'C'} B^{B'}_\mu(zn) B^{C'}_+(zn) 1 \big(-\mathrm{i}g \int \mathrm{d}^d x A^D_\nu(x) \partial_\alpha B^E_\beta B^F_\gamma(x)\big) v^{\nu\alpha\beta\gamma}_{DEF} \tag{2.87}$$

$$+ g f^{A'B'C'} B^{B'}_\mu(zn) B^{C'}_+(zn) 1 \big(-\mathrm{i}g \int \mathrm{d}^d x A^D_\nu(x) \partial_\alpha B^E_\beta B^F_\gamma(x)\big) v^{\nu\alpha\beta\gamma}_{DEF} \tag{2.88}$$

here i compute both terms/diagrams, but again split common factor. color computation:

$$f^{A'B'C'} f^{DEF} (\delta^{B'E}\delta^{C'F} + \delta^{B'F}\delta^{C'E}) \tag{2.89}$$

$$= f^{A'EF} f^{DEF} (+1 + (-1)) \tag{2.90}$$

$$= C_A \delta^{A'D} (+1 + (-1)) \tag{2.91}$$

$$-\mathrm{i}g^2 \int \mathrm{d}^d x A_\nu(x) C_A \delta^{A'D} v^{\nu\alpha\beta\gamma} \tag{2.92}$$

$$\partial_{x^\alpha} \Delta_{\mu\beta}(zn - x) \Delta_{\gamma+}(zn - x) - \Delta_{\mu\gamma}(zn - x) \partial_{x^\alpha} \Delta_{\beta+}(zn - x) \tag{2.93}$$

$$\frac{-\mathrm{i}g^2\Gamma^2\left(\frac{d}{2}-1\right)}{4^2\pi^d}\int \mathrm{d}^d x A_\nu^{A'}(x) C_A v^{\nu\alpha\beta\gamma} \tag{2.94}$$

$$\frac{-2(\frac{d}{2}-1)(x-zn)_\alpha}{(-(x-zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\frac{1}{(-(x-zn)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}-1}(\eta_{\gamma+}\eta_{\mu\beta}-\eta_{\beta+}\eta_{\mu\gamma}) \tag{2.95}$$

$$=\int_0^1 \mathrm{d}u u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\frac{\Gamma(d-1)}{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}\right)}\frac{-2(\frac{d}{2}-1)(x-zn)_\alpha}{(-(x-zn)^2+\mathrm{i}\epsilon)^{d-1}}(\eta_{\gamma+}\eta_{\mu\beta}-\eta_{\beta+}\eta_{\mu\gamma}) \tag{2.96}$$

$$=\int_0^1 \mathrm{d}u u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}\frac{\Gamma(d-1)}{\Gamma^2\left(\frac{d}{2}-1\right)}\frac{-2(x-zn)_\alpha}{(-(x-zn)^2+\mathrm{i}\epsilon)^{d-1}}(\eta_{\gamma+}\eta_{\mu\beta}-\eta_{\beta+}\eta_{\mu\gamma}) \tag{2.97}$$

The shift is $x \to x + zn$. The metric can be contracted:

$$(\eta_{\gamma+}\eta_{\mu\beta}-\eta_{\beta+}\eta_{\mu\gamma})v^{\nu\alpha\beta\gamma} = 4(\eta_\mu^\nu\eta^{\alpha+}-\eta_\mu^\alpha\eta_{\nu+}) \tag{2.98}$$

$$\frac{-\mathrm{i}g^2 C_A\Gamma(d-1)}{4^2\pi^d}\int_0^1 \mathrm{d}u u^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-2}4(\eta_\mu^\nu\eta^{\alpha+}-\eta_\mu^\alpha\eta_{\nu+}) \tag{2.99}$$

$$\int \mathrm{d}^d x\frac{-2(x-zn)_\alpha x_\tau}{(-(x)^2+\mathrm{i}\epsilon)^{d-1}}\partial^\tau A_\nu^{A'}(zn) \tag{2.100}$$

$$=\frac{+2\mathrm{i}\pi^{\frac{d}{2}}}{2\epsilon}\frac{1}{\Gamma(d-1)}\partial_\alpha A_\nu^{A'}(zn) \tag{2.101}$$

Have in total

$$\frac{g^2 C_A}{4\pi^2\epsilon}(\partial_+ A_\mu-\partial_\mu A_+)^{A'}(zn) \tag{2.102}$$

$$=-\frac{g^2 C_A}{4\pi^2\epsilon}F_{\mu+}^{A'}(zn) \tag{2.103}$$

Looks ok but not sure about factor. So this has to be checked again. But it does not contribute to the pole with regulator δ. CHECKED IT, CORRECTED IT AND I THINK FACTOR IS OK NOW.
For other diagram, meaning other side of wilson line there is NO difference in color and also nothing else that makes me think there would be a difference. So it is the same result (of course at other space-time point.)

Next diagram:

$$gf^{A'B'C'}B_\mu^{B'}(zn)B_+^{C'}(zn)\int_{-\infty}^z \mathrm{d}\sigma \mathrm{i}g B_+(\sigma n) \tag{2.104}$$

To the outgoing field has to be classical, so I have to choose which one it should be. However Once needs to

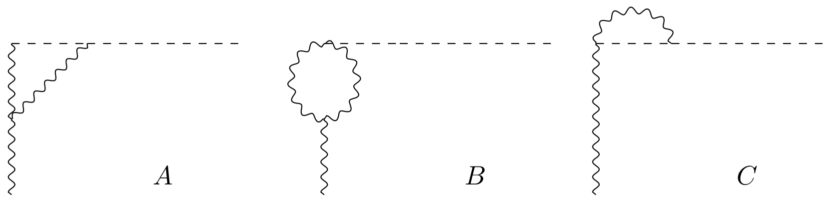


Table 1: diagrams for gluon TMD opertator on twist 1

understand that already here the suspicion that the diagram is null can be made. Two contractions:

$$gf^{A'B'C'}B_{\mu}^{B'}(zn)A_{+}^{C'}(zn)\int_{-\infty}^{z}\mathrm{d}\sigma \mathrm{i}gB_{+}^{D}(\sigma n)T_{A'C}^{D} \tag{2.105}$$

$$+gf^{A'B'C'}A_{\mu}^{B'}(zn)B_{+}^{C'}(zn)\int_{-\infty}^{z}\mathrm{d}\sigma \mathrm{i}gB_{+}^{D}(\sigma n)T_{A'C}^{D} \tag{2.106}$$

color is of course a relative minus. But the second diagram is null since n^2 is 0. First diagram should be null in light cone gauge, and because $g_{\mu+}$ is 0. So anyway it is 0.

Now the results: The diagrams are in talbe table 1:

The results are

$$A = \frac{g^2 C_A}{4\pi^2\epsilon}\left(-1+\ln\left(\frac{\mathrm{i}\hat{p}_+}{\delta}\right)\right)F_{\mu+}^{C}(zn) \tag{2.107}$$

$$B = -\frac{g^2 C_A}{4\pi^2\epsilon}F_{\mu+}^{A'}(zn) \tag{2.108}$$

$$C = 0 \tag{2.109}$$

1-loop diagrams for gluon pdf: twist2:

operators with this twist should look have the form

$$\int \mathrm{d}\sigma F_{\mu+}\left[zn,\sigma n\right]F_{\nu+}\left[\sigma,-\infty n\right] \tag{2.110}$$

Problably it is ok to ignore the integraion which should be kept until the end anyway? Like i have learned before the color of the whole object is connected, and just one index is open here. so I need to know what to do for:

no disturbance disturbance in first path disturbance in second path

$$F_{\mu+}^{A}F_{\nu+}^{B}T_{AC}^{B} \tag{2.111}$$

$$\mathrm{i}g\int_{\sigma}^{z}\mathrm{d}\tau F_{\mu+}^{A}B_{+}^{B}(\tau n)T_{AC}^{B}F_{\nu+}^{D}T_{CE}^{D} \tag{2.112}$$

$$\mathrm{i}g\int_{-\infty}^{\sigma}\mathrm{d}\tau F_{\mu+}^{A}F_{\nu+}^{B}T_{AC}^{B}B_{+}^{D}(\tau n)T_{CE}^{D} \tag{2.113}$$

Note: I will compute for antisymmetric part of tensor: (i know it is fully as) $partial_{\mu}A_M$, since M is capital μ. at some convenient point in computation i will subract interchanged term and replace M by $+$. Same for $\nu \leftrightarrow N$. with this:

Diagram A:

$$F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.114}$$

$$\to \tag{2.115}$$

$$F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.116}$$

$$+ F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.117}$$

I claim that the computation is - after computing color - 100% identical to the computation for twist 1. The intermediate field is simply ignored. (one of course has to add the diagram A'). color is

$$(\delta^{AB'}\delta^{BC'} - \delta^{AC'}\delta^{BB'}) f^{A'B'C'} T^B_{AC} T^D_{CE} = -\mathrm{i} f^{A'AB} f^{BAC} T^D_{CE}(1-1) \tag{2.118}$$

$$= \mathrm{i} C_A T^D_{A'E} \tag{2.119}$$

so if i trust my intuition i end with the expression

$$\pm \int_{-\infty}^{\pm z} \mathrm{d}\sigma \frac{g^2 C_A}{4\pi^2 \epsilon} \left(-1 + \ln\left(\frac{\pm \mathrm{i}\hat{p}_+}{\delta} \right) \right) F^{A'}_{\mu +}(\pm zn) F^D_{\nu +}(\sigma n) T^D_{A'E} \tag{2.120}$$

all $+$-es are for field at zn, and all $-$-es are for other case. Minuses i am not so sure, should check. Most likely there is a sign flip in this expression. I need to check it.

Diagram B: here i show full computation. we start with the same expression as for diagram A.

$$F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.121}$$

$$\to \tag{2.122}$$

$$= F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.123}$$

$$+ F^A_{\mu M} \mathrm{i} g B^B_+ T^B_{AC} F^D_{\nu N} T^D_{CE} \Big(-\mathrm{i}g \int \mathrm{d}^d x A^{A'}_\iota(x) \partial_{x^\alpha} B^{B'}_\beta(x) B^{C'}_\gamma(x) \Big) v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.124}$$

unfortunately i forgot to put arguments, on the fields of the operator, but they are clear.

the color factor requires the identity for 3 structure function product. it can de easily derived from Jacobi-identity (for structure functions) plus identity for product of two structure functions. the formula reads

$$f^{DEA} f^{EFB} f^{FDC} = \frac{1}{2} C_A f^{ABC} \tag{2.125}$$

The color factor compuation for this diagrams is

$$T^B_{AC} T^D_{CE} f^{A'B'C'} (\delta^{BC}\delta^{DB'} + \delta^{BB'}\delta^{DC'}) = -(-\mathrm{i})^2 f^{CBA} f^{BDA'} f^{DCE}(1-1) \tag{2.126}$$

$$= \frac{-C_A}{2} (-\mathrm{i})^2 f^{AA'E}(1-1) \tag{2.127}$$

again the minus is between different possibilities of contracting the fields. We can remove the remaining structure function back to the color matrix to restore the initial expression.

$$T^B_{AC}T^D_{CE}f^{A'B'C'}(\delta^{BC}\delta^{DB'}+\delta^{BB'}\delta^{DC'})=\frac{-\mathrm{i}C_A}{2}T^{A'}_{AE}(1-1) \tag{2.128}$$

The terms yield

$$-\mathrm{i}^2g^2F^A_{\mu M}\int \mathrm{d}^dA^{A'}_\iota(x)\frac{-\mathrm{i}C_A}{2}T^{A'}_{AE}v^{\iota\alpha\beta\gamma} \tag{2.129}$$

$$\Delta_{+\gamma}(\tau n-x)\partial_\nu\partial_\alpha\Delta_{N\beta}(\sigma n-x)-\partial_\alpha\Delta_{+\beta}(\tau n-x)\partial_\nu\Delta_{N\gamma}(\sigma n-x) \tag{2.130}$$

always keep in mind to which argument the derivatives are related. I do omit the extra term arising from double derivative on the propagator, since its contribution is null (Lorentz contracted is 0)

$$-\mathrm{i}^2g^2F^A_{\mu M}\int \mathrm{d}^dA^{A'}_\iota(x)\frac{-\mathrm{i}C_A}{2}T^{A'}_{AE}v^{\iota\alpha\beta\gamma}\Gamma^2\left(\frac{d}{2}-1\right)\frac{1}{4^2\pi^{\frac{d}{2}}} \tag{2.131}$$

$$\frac{1}{(-(\tau n-x)^2+\mathrm{i}\epsilon)^{\frac{d}{2}-1}}\frac{4\left(\frac{d}{2}-1\right)\frac{d}{2}(\sigma n-x)_\nu(x-\sigma n)_\alpha}{(-(\sigma n-x)^2+\mathrm{i}\epsilon)^{\frac{d}{2}+1}}\eta_{\gamma+}\eta_{N\beta} \tag{2.132}$$

$$-\frac{-2\left(\frac{d}{2}-1\right)(x-\tau n)_\alpha}{(-(\tau n-x)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\frac{-2\left(\frac{d}{2}-1\right)(\sigma n-x)_\nu}{(-(\sigma n-x)^2+\mathrm{i}\epsilon)^{\frac{d}{2}}}\Gamma(d)\eta_{\beta+}\eta_{N\gamma} \tag{2.133}$$

$$-\mathrm{i}^2g^2F^A_{\mu M}\int \mathrm{d}^dA^{A'}_\iota(x)\frac{-\mathrm{i}C_A}{2}T^{A'}_{AE}v^{\iota\alpha\beta\gamma}\Gamma^2\left(\frac{d}{2}-1\right)\frac{1}{4^2\pi^{\frac{d}{2}}}4\frac{1}{\Gamma^2\left(\frac{d}{2}-1\right)} \tag{2.134}$$

$$\int_0^1 \mathrm{d}uu^{\frac{d}{2}-2}\overline{u}^{\frac{d}{2}}\frac{-x_\nu(x-\sigma n)_\alpha}{(-u(\tau n-x)^2-\overline{u}(\sigma n-x)^2+\mathrm{i}\epsilon)^d}\eta_{\gamma+}\eta_{N\beta} \tag{2.135}$$

$$+\int_0^1 \mathrm{d}uu^{\frac{d}{2}-1}\overline{u}^{\frac{d}{2}-1}\frac{x_\nu(x-\tau n)_\alpha}{(-u(\tau n-x)^2-\overline{u}(\sigma n-x)^2+\mathrm{i}\epsilon)^d}\eta_{\beta+}\eta_{N\gamma} \tag{2.136}$$

The shifts in this setup are:

$$\tau \to \tau+\sigma \text{ first shift} \tag{2.137}$$

$$x \to x+u\tau n+\sigma n \tag{2.138}$$

$$x-\sigma n \to x+u\tau n \tag{2.139}$$

$$x-\tau n \to x-\overline{u}\tau n \tag{2.140}$$

compute metric

$$\eta_{\gamma+}\eta_{N\beta}v_{\iota\alpha\beta\gamma}=2(\eta_{\iota N}\eta_{\alpha+}-\eta_{\iota+}\eta_{\alpha N}) \tag{2.141}$$

$$=-\eta_{\beta+}\eta_{N\gamma}v_{\iota\alpha\beta\gamma} \tag{2.142}$$

Hence another relative sign between term and identical metric term Have:

$$-\mathrm{i}^2 g^2 F^A_{\mu M} \int \mathrm{d}^d A^{A'}_{\iota}(x+u\tau n+\sigma n) \frac{-\mathrm{i}C_A}{2} T^{A'}_{AE} \Gamma^2\left(\frac{d}{2}-1\right) \frac{1}{4^2\pi^{\frac{d}{2}}} 4 \frac{1}{\Gamma^2\left(\frac{d}{2}-1\right)} \int_0^1 \mathrm{d}u u^{\frac{d}{2}-2} \overline{u}^{\frac{d}{2}-1} 2(\eta_{\iota N}\eta_{\alpha +} - \eta_{\iota +}\eta_{\alpha N}) \tag{2.143}$$

$$\frac{-\overline{u}x_\nu(x+u\tau n)_\alpha - u x_\nu (x-\overline{u}\tau n)_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} = \frac{-x_\nu x_\alpha}{(-x^2+\mathrm{i}\epsilon)^d} \tag{2.144}$$

Then from integration we have double derivative

$$\int \mathrm{d}^d x \frac{-x_\nu x_\alpha x_\rho x_\kappa}{(-x^2+\mathrm{i}\epsilon)^d} \partial_\rho \partial_\kappa A^{A'}(u\tau n+\sigma n) = \frac{-\mathrm{i}\pi^{\frac{d}{2}}}{4(2-\frac{d}{2})\Gamma(d)} \left(\eta_{\nu\alpha}\eta_{\rho\kappa} + \eta_{\nu\rho}\eta_{\alpha\kappa} + \eta_{\nu\kappa}\eta_{\alpha\rho}\right) \partial^\kappa \partial^\rho A^{A'}_{\iota}(\sigma n + u\tau n) \tag{2.145}$$

now again contract metrics:

$$\left(\eta_{\nu\alpha}\eta_{\rho\kappa} + \eta_{\nu\rho}\eta_{\alpha\kappa} + \eta_{\nu\kappa}\eta_{\alpha\rho}\right) \partial^\kappa \partial^\rho A^{A'}_{\iota}(\sigma n + u\tau n)(\eta_{\iota N}\eta_{\alpha +} - \eta_{\iota +}\eta_{\alpha N}) = 2\partial_+ \left(\partial_\nu A_+ - \partial_+ A_\nu\right)^{A'} (\sigma n + u\tau n) \tag{2.146}$$

Total , combination of last two steps

$$-\mathrm{i}^2 g^2 F^A_{\mu M} \frac{-\mathrm{i}C_A}{2} T^{A'}_{AE} \Gamma^2\left(\frac{d}{2}-1\right) \frac{1}{4^2\pi^{\frac{d}{2}}} 4 \frac{1}{\Gamma^2\left(\frac{d}{2}-1\right)} \tag{2.147}$$

$$\int_0^1 \mathrm{d}u u^{\frac{d}{2}-2} \overline{u}^{\frac{d}{2}-1} 2\ 2\partial_+ \left(\partial_\nu A_+ - \partial_+ A_\nu\right)^{A'} (\sigma n + u\tau n) \frac{-\mathrm{i}\pi^{\frac{d}{2}}}{4(2-\frac{d}{2})\Gamma(d)} \tag{2.148}$$

$$= \int_0^1 \mathrm{d}u \overline{u} \frac{(-1)^3 \mathrm{i}^4 g^2 C_A 2^2}{4^3 2^1 \pi^2 \epsilon} F^A_{\mu M}(zn) T^{A'}_{AE} \partial_+ F^{A'}_{\nu +}(\sigma n + u\tau n) \tag{2.149}$$

of course, there are the two integrals, over τ and σ. first is necessary for the computation, the other one belongs to the operator itself. i already shifted the τ integral by $+\sigma$. Now i want to subtract σ again but add z. I also show the integral over σ now, which is from $-\infty$ to z, and i also shift this one. I end up

$$\int_{-\infty}^0 \mathrm{d}\tau \int_0^1 \mathrm{d}u \overline{u} \partial_+ f(y+u\tau n) = -\int \frac{\mathrm{d}^d p}{(2\pi)^d} \left(\ln\left(\frac{\mathrm{i}p_+}{\delta}\right) - 1\right) e^{\mathrm{i}(p|y)} f(p) \tag{2.150}$$

$$= \left(1 - \ln\left(\frac{\mathrm{i}p_+}{\delta}\right)\right) f(\sigma n) \tag{2.151}$$

so in my case here:

$$\frac{-g^2 C_A}{2(4\pi)^2\epsilon} F^A_{\mu M}(zn) \int_{-\infty}^z \mathrm{d}\sigma \left(1 - \ln\left(\frac{\mathrm{i}p_+(\sigma)}{\delta}\right)\right) F^{A'}_{\nu +}(\sigma n) T^{A'}_{AE} \tag{2.152}$$

the argument of p_+ indicates on which field its acting.

The next diagram i want to compute is diagram AB. I think its the last out three diagrams for twist 2 which are not absolutely the same. Probably this is the most interesting. the first two are the same computation as for twist 1, essentially.

the therm is

$$F^A_{\mu+}F^D_{\nu+}T^D_{AE} \tag{2.153}$$
$$\to gf^{ABC}B^B_\mu B^C_+ gf^{DFH}B^F_\nu B^H_+ T^D_{AE} \tag{2.154}$$

I only have this contraction, since all other 3 possibilities yield zero Color computation yields:

$$+\mathrm{i}g^2 f^{DAE}f^{ABC}f^{BDH}\Delta_{\mu\nu}(zn-\sigma n) \tag{2.155}$$
$$= +\frac{g^2C_A}{2}f^{ECH}\Delta_{\mu\nu}(zn-\sigma n) \tag{2.156}$$
$$= \frac{-g^2C_A}{2}T^C_{HE}\Delta_{\mu\nu}(zn-\sigma n)A^C_+A^H_+ \tag{2.157}$$

Maybe one can argue that these drop since the outgoing fields are classical so they drop in lc-gauge?

0 diagrams from metric: the following diagrams are 0 for the reason $\eta_{++}=\eta_{\mu+}=\eta_{\nu+}=0$: $A1, A1', B1, B1', E1, E2, E3$ The remaining computation is for the diagrams A2 and B2. It should be very similar to what i have seen for twist 1:

$$gf^{ABC}B^B_\mu B^C_+(zn)F^D_{\nu+}T^D_{AE}\bigg(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\bigg)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.158}$$
$$\to gf^{ABC}B^B_\mu B^C_+(zn)F^D_{\nu+}T^D_{AE}\bigg(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\bigg)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.159}$$
$$+\,gf^{ABC}B^B_\mu B^C_+(zn)F^D_{\nu+}T^D_{AE}\bigg(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\bigg)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.160}$$

apart from color it is identical to twist one, so i will not do perform the calculation in all details. color gives

$$f^{ABC}T^D_{AE}f^{A'B'C'}(\delta^{BB'}\delta CC' + \delta^{BC'}\delta CB') = -\mathrm{i}f^{ABC}f^{DAE}f^{A'BC}(1-1) \tag{2.161}$$
$$= -\mathrm{i}C_A f^{DA'E} \tag{2.162}$$
$$= C_A T^D_{A'E} \tag{2.163}$$

So color is identical (gives factor C_A and connects color indices) Apart from that i see no difference, either. So the result is the same:

$$-\frac{g^2C_A}{4\pi^2\epsilon}F^{A'}_{\mu+}(zn)\int_{-\infty}^{z}\mathrm{d}\sigma F^D_{\nu+}(\sigma n)T^D_{A'E} \tag{2.164}$$

last diagram: B2

diagram	correction	diagram	correction
$A+A'$	incorrectResult	$B+B'$	$\frac{-g^2 C_A}{2(4\pi)^2\epsilon}\left(1-\ln\left(\frac{\mathrm{i}p_+(\sigma)}{\delta}\right)\right)$
$A1, A1', B1, B1'$	0	$E1, E2, E3$	0
$A2$	$-\frac{g^2 C_A}{4\pi^2\epsilon}$	$B2$	$-\frac{g^2 C_A}{4\pi^2\epsilon}$

Table 2: gluon twist 2; the "correction" is the factor to the "default" expression. Together it is the result for the one loop diagram with the corresponding Name. For example, diagram $B+B' = \frac{-g^2 C_A}{2(4\pi)^2\epsilon} F^A_{\mu M}(zn)\int_{-\infty}^{z}\mathrm{d}\sigma\left(1-\ln\left(\frac{\mathrm{i}p_+(\sigma)}{\delta}\right)\right) F^{A'}_{\nu+}(\sigma n)T^{A'}_{AE}$. In blue the "default" expression, for clarification.

$$F^A_{\mu+}(zn)gf^{DBC}B^B_\nu B^C_+(\sigma n)T^D_{AE}\Big(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\Big)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.165}$$

$$\to F^A_{\mu+}(zn)gf^{DBC}B^B_\nu B^C_+(\sigma n)T^D_{AE}\Big(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\Big)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.166}$$

$$+F^A_{\mu+}(zn)gf^{DBC}B^B_\nu B^C_+(\sigma n)T^D_{AE}\Big(-\mathrm{i}g\int \mathrm{d}^d x A^{A'}_\iota(x)\partial_{x^\alpha}B^{B'}_\beta(x)B^{C'}_\gamma(x)\Big)v^{\iota\alpha\beta\gamma}_{A'B'C'} \tag{2.167}$$

COLOR:

$$f^{DBC}T^D_{AE}f^{A'B'C'}(\delta^{BB'}\delta^{CC'}+\delta^{BC'}\delta^{CB'}) = \delta^{A'D}T^D_{AE}C_A(1-1) \tag{2.168}$$

again, the rest of the computation must be exactly the same. here the point for the fixed field is σn and not zn but it is not integrated over (here) so it must be identical. the color gives the same result (just connects and C_A) FINAL:

$$-\frac{g^2 C_A}{4\pi^2\epsilon}F^{A'}_{\mu+}(zn)\int_{-\infty}^{z}\mathrm{d}\sigma F^D_{\nu+}(\sigma n)T^D_{A'E} \tag{2.169}$$

Result table:

III Tipps and Tricks

- $\int_0^1 \mathrm{d}u f(u,\overline{u}) = \int_0^1 \mathrm{d}\overline{u} f(\overline{u},u)$
- Gamma function properties
- index μ is transverse!!!! has more implications than one might think!

Aux:

$$\partial_{x^\alpha}\frac{1}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^n} = \frac{-2n(x-z)_\alpha}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^{n+1}} \tag{3.1}$$

$$\partial_{z^\mu}\partial_{x^\alpha}\frac{1}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^n} = \partial_{z^\mu}\frac{-2n(x-z)_\alpha}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^{n+1}} \tag{3.2}$$

$$= \frac{4n(n+1)(z-x)_\mu(x-z)_\alpha}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^{n+2}} + \frac{2n\eta_{\mu\alpha}}{\left(-(z-x)^2+\mathrm{i}\epsilon\right)^{n+1}} \tag{3.3}$$

Even better:

$$\partial_{y^\alpha} \frac{1}{\left(-(y-w)^2+\mathrm{i}\epsilon\right)^n} = \frac{-2n(y-w)_\alpha}{\left(-(y-w)^2+\mathrm{i}\epsilon\right)^{n+1}} \tag{3.4}$$

$$\partial_{w^\beta}\partial_{y^\alpha} \frac{1}{\left(-(y-w)^2+\mathrm{i}\epsilon\right)^n} = \frac{4n(n+1)(y-w)_\alpha(w-y)_\beta}{\left(-(y-w)^2+\mathrm{i}\epsilon\right)^{n+2}} + \frac{2n\eta_{\alpha\beta}}{\left(-(y-w)^2+\mathrm{i}\epsilon\right)^{n+1}} \tag{3.5}$$

$$\int \mathrm{d}^d x \frac{x^\mu x^\nu}{\left(-x^2+\mathrm{i}\epsilon\right)^{d-1}} = \frac{-\mathrm{i}\pi^{\frac{d}{2}}}{2(2-\frac{d}{2})} \frac{\eta^{\mu\nu}}{\Gamma\left(d-1\right)} \tag{3.6}$$

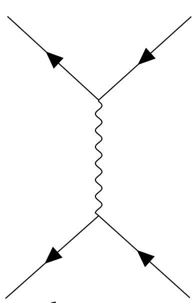
own adaption for four elements in numerator:

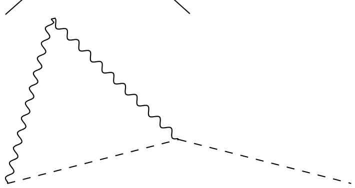
$$\int \mathrm{d}^d x \frac{x^\mu x^\nu x^\rho x^\sigma}{\left(-x^2+\mathrm{i}\epsilon\right)^{d}} = \frac{\mathrm{i}\pi^{\frac{d}{2}}}{4(2-\frac{d}{2})\Gamma\left(d\right)} \left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right) \tag{3.7}$$

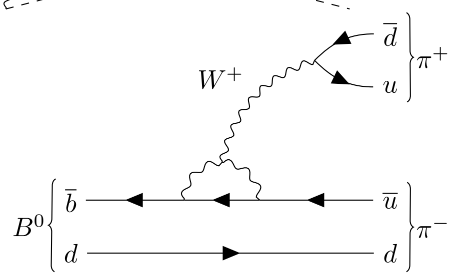
$$\int_{-\infty}^0 \mathrm{d}\tau \int_0^1 \mathrm{d}u\overline{u}\partial_+ f(y+u\tau n) = -\int \frac{\mathrm{d}^d p}{(2\pi)^d} \left(\ln\left(\frac{\mathrm{i}p_+}{\delta}\right) - 1\right) e^{\mathrm{i}(p|y)} f(p) \tag{3.8}$$

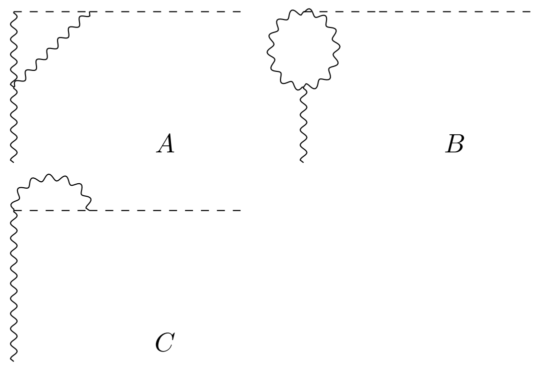
$$= \left(1 - \ln\left(\frac{\mathrm{i}p_+}{\delta}\right)\right) f(\sigma n) \tag{3.9}$$

IV Diagrams.









A
B
C