Report 4

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1 Introduction

This report is divided into three distinct sections, each serving its purpose. Firstly, I utilize a VBA script to compute the Greek letters for a Black-Scholes model. Moving on to the second section, I show the volatility surface and the Greeks specifically for ADYEN NV (ADYEN.AS). Lastly, in the third section, I again employ the Black-Scholes formula to determine the Greeks of at-the-money (ATM) options with a maturity of 3 months. Furthermore, I conduct a comparison between the Greeks derived from implied volatility and the value provided by Refinitiv. The overarching objective of this report is to juxtapose the theoretical computations with empirical findings, aiming to validate the expected results based on theoretical underlying.

2 Theoretical section

2.1 The Greeks for a Black-Scholes Model

In mathematical finance, the Greeks are the quantities (known in calculus as partial derivatives) representing the sensitivity of the price of a derivative instrument such as an option to changes in one or more underlying parameters on which the value of an instrument or portfolio of financial instruments is dependent. A financial institution selling an option in the over-the-counter markets faces the challenge of managing its risk. When the option matches those traded on exchanges, hedging is straightforward. However, custom-tailored options present a more complex risk management task. One approach to this problem is using the "Greek letters" (or "Greeks").

Each Greek letter measures a distinct dimension of risk in an option position. The objective for a trader is to manage the Greeks to maintain acceptable risk levels.

2.1.1 Delta

The delta (Δ) of an option portfolio is defined as:

$$\Delta = \frac{\delta p}{\delta S} = \Phi(d_1)$$

where p is the portfolio price, S is the underlying asset price, and $\Phi(x) = \int_{-\infty}^{x} e^{-y^2/2}\, dy$. The variable d_1 is calculated using the Black-Scholes formula.

2.1.2 Gamma

The gamma (Γ) of an option portfolio is defined as:

$$\Gamma = \frac{\delta^2 p}{\delta S^2} = \frac{\delta \Delta}{\delta S} = \frac{1}{S\sigma\sqrt{T-t}} \cdot \phi(d_1)$$

where σ is the volatility, $\phi(x) = e^{-x^2/2}/\sqrt{2\pi}$ is the Gaussian density distribution, and d_1 is calculated using the Black-Scholes formula.

2.1.3 Rho

The rho (ρ) of an option portfolio is defined as:

$$\rho = \frac{\delta p}{\delta R} = K \cdot (T-t) \cdot e^{-R(T-t)} \cdot \Phi(d_2)$$

where K is the strike price, T is the maturity time, R is the interest rate, and $d_2 = d_1 - \sigma\sqrt{T-t}$. Both d_1 and d_2 are calculated using the Black-Scholes formula.

2.1.4 Vega

The vega (ν) of an option portfolio is defined as:

$$\nu = \frac{\delta p}{\delta \sigma} = S \cdot (T-t) \cdot \phi(d_1)$$

where S is the price of the underlying asset, $\phi(x)$ is the Gaussian density distribution, and d_1 is calculated using the Black-Scholes formula.

2.1.5 Theta

The theta (Θ) of an option portfolio is defined as:

$$\Theta = \frac{\delta p}{\delta t} = -\frac{S\phi(d_1)\sigma}{2\sqrt{T-t}} - RKe^{-R(T-t)}\Phi(d_2)$$

where S is the price of the underlying asset, σ is the volatility, K is the strike price, R is the interest rate, and d_1 and d_2 are calculated using the Black-Scholes formula.

2.1.6 Calculation and results

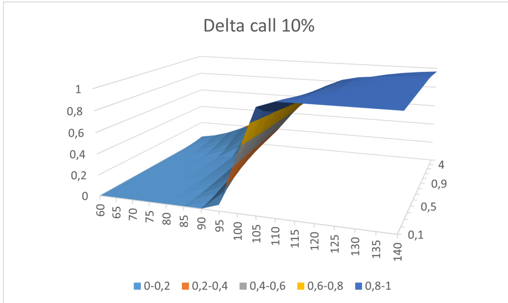
I provide a VBA script that computes the Greeks for call and put options in the B&S model. I compute the values of Greek letters for different values of S, maturity T, and volatility σ, keeping constant strike price K and risk-free rate r. The variables and constants used are shown below:

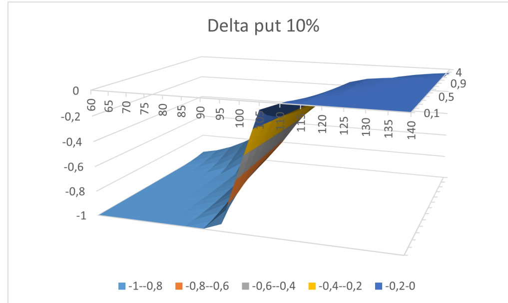
Parameter	Values
K	100
r	0.1
σ	$0.2, 0.1, 0.3$
S	$60, 65, \ldots, 135, 140$
T	$0.1, 0.2, \ldots, 1, 2, \ldots, 4, 5$

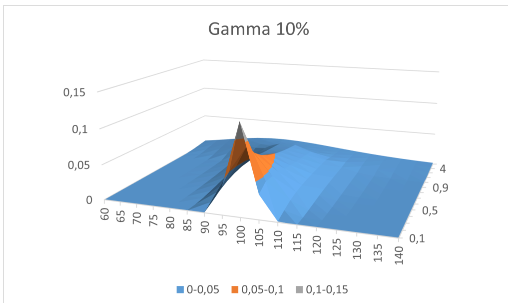
Table 1: Parameters and Values

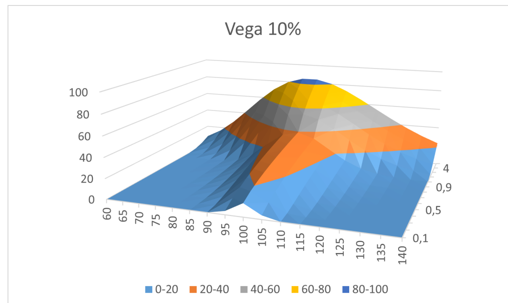
For all the combination of volatility and Greeks I created a 3D graph corresponding to the surfaces of the latter. Here is an example of this calculation and all the 3D Greeks graphs (Greeks in function of T and K).

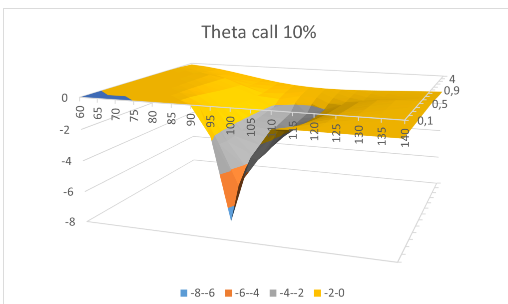
	60	65	70	...
0,1	4,8964E-16	6,71899E-12	1,12175E-08	...
0,2	8,30131E-09	1,02103E-06	4,41525E-05	...
0,3	2,31357E-06	5,89785E-05	0,000752226	...
0,4	4,014E-05	0,000465199	0,0032161	...
0,5	0,000227433	0,001640907	0,007845886	
0,6	0,000733393	0,003855117	0,014402779	
0,7	0,00170977	0,007164237	0,022425911	
0,8	0,003249929	0,011483425	0,031463321	
0,9	0,005386379	0,016663451	0,041145993	
	...	...	...	...

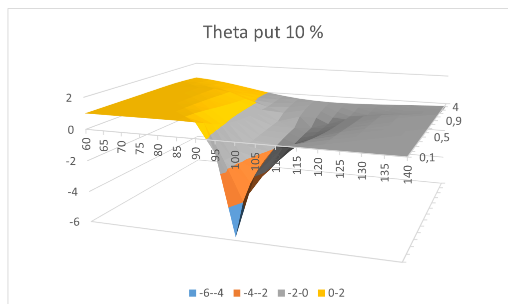

Delta call 10%
0-0,2 0,2-0,4 0,4-0,6 0,6-0,8 0,8-1

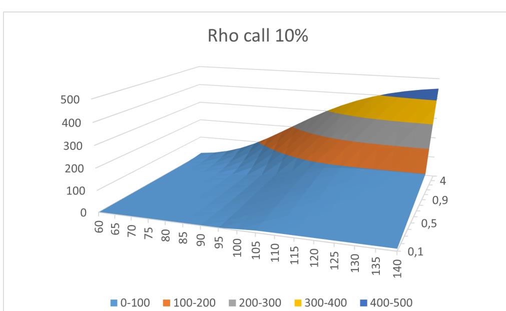

Delta put 10%
-1--0,8 -0,8--0,6 -0,6--0,4 -0,4--0,2 -0,2-0

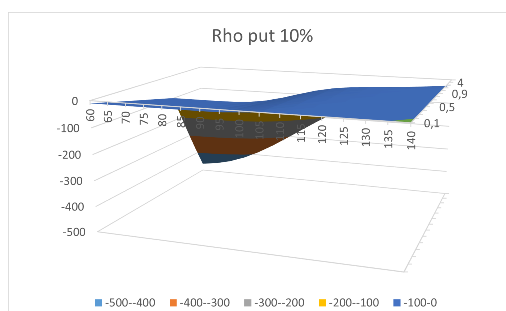

Gamma 10%
0-0,05 0,05-0,1 0,1-0,15

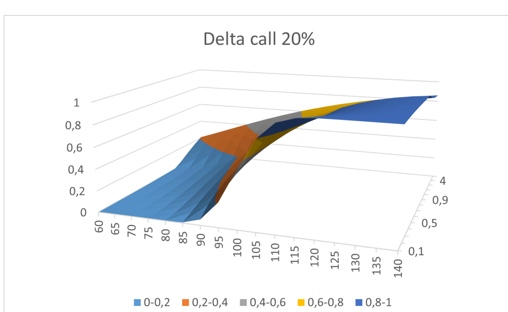

Vega 10%
0-20 20-40 40-60 60-80 80-100

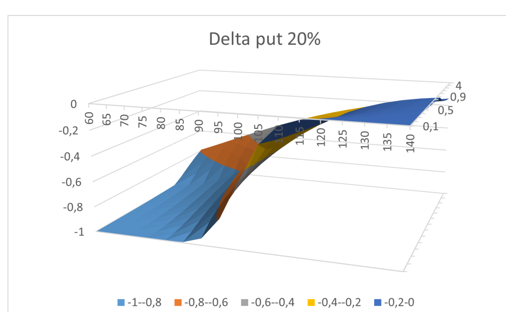

Theta call 10%
-8--6 -6--4 -4--2 -2-0

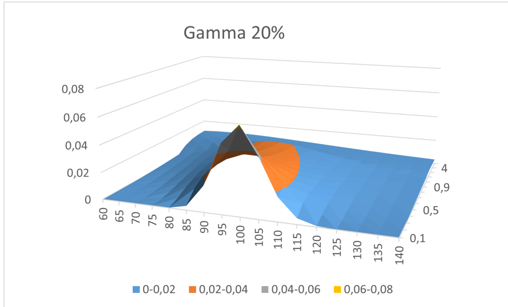

Theta put 10 %
-6--4 -4--2 -2-0 0-2

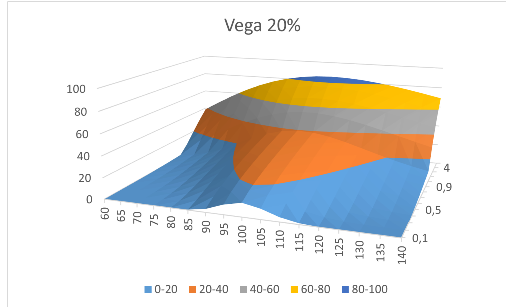

Rho call 10%
0-100 100-200 200-300 300-400 400-500

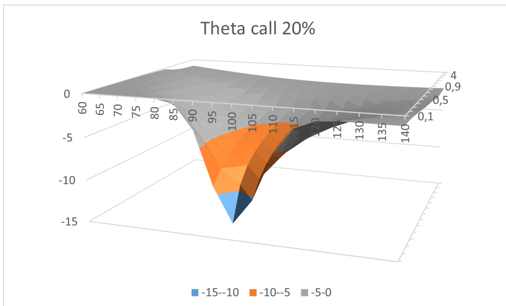

Rho put 10%
-500--400 -400--300 -300--200 -200--100 -100-0

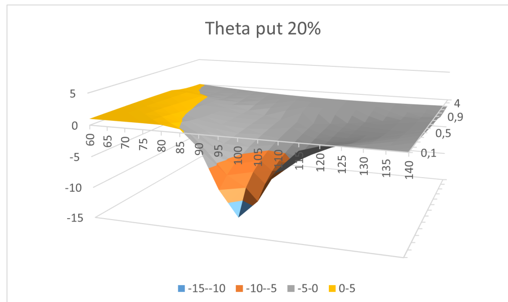

Delta call 20%
0-0,2 0,2-0,4 0,4-0,6 0,6-0,8 0,8-1

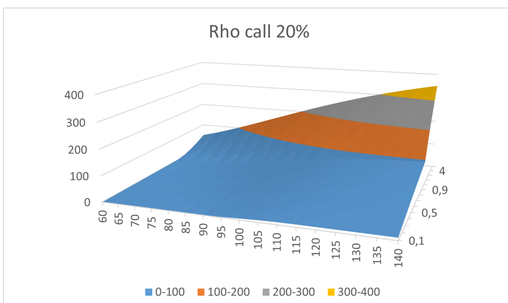

Delta put 20%
-1--0,8 -0,8--0,6 -0,6--0,4 -0,4--0,2 -0,2-0

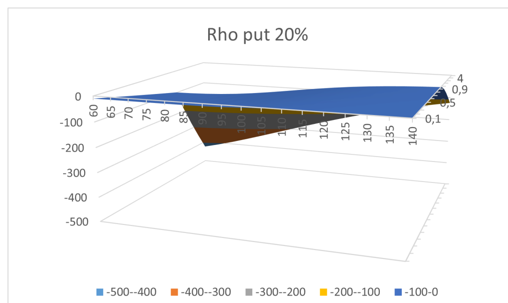

Gamma 20%
0,08
0,06
0,04
0,02
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-0,02 0,02-0,04 0,04-0,06 0,06-0,08

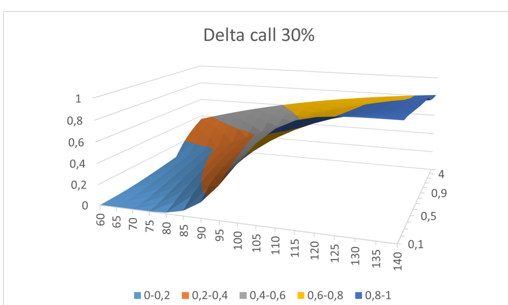

Vega 20%
100
80
60
40
20
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-20 20-40 40-60 60-80 80-100

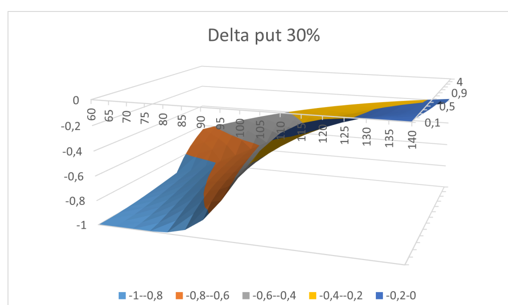

Theta call 20%
0
-5
-10
-15
60 65 70 75 80 85 90 95 100 105
4
0,9
0,5
0,1
-15--10 -10--5 -5-0

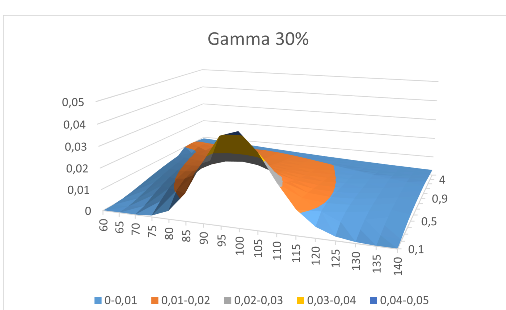

Theta put 20%
5
0
-5
-10
-15
60 65 70 75 80 85 90 95 100 120 125 130 135 140
4
0,9
0,5
0,1
-15--10 -10--5 -5-0 0-5

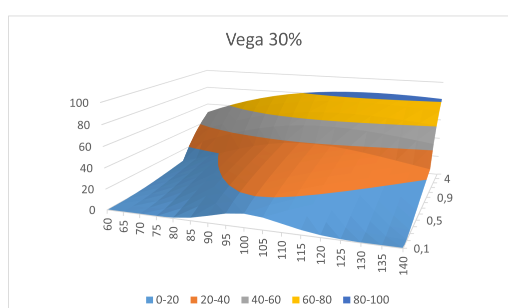

Rho call 20%
400
300
200
100
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-100 100-200 200-300 300-400

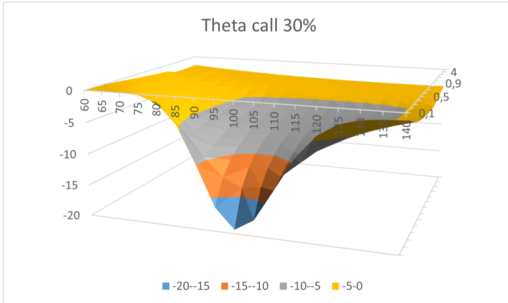

Rho put 20%
0
-100
-200
-300
-400
-500
60 65 70 75 80 85 120 125 130 135 140
4
0,9
0,1
-500--400 -400--300 -300--200 -200--100 -100-0

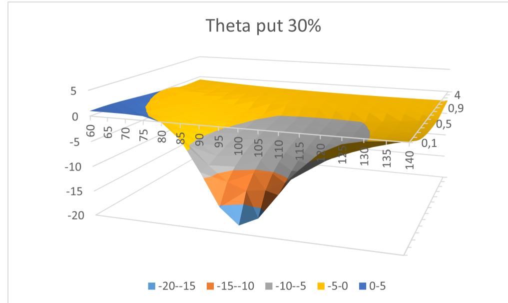

Delta call 30%
1
0,8
0,6
0,4
0,2
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-0,2 0,2-0,4 0,4-0,6 0,6-0,8 0,8-1

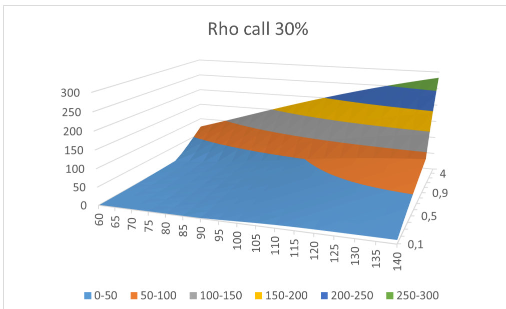

Delta put 30%
0
-0,2
-0,4
-0,6
-0,8
-1
60 65 70 75 80 85 90 125 130 135 140
4
0,9
0,5
0,1
-1--0,8 -0,8--0,6 -0,6--0,4 -0,4--0,2 -0,2-0

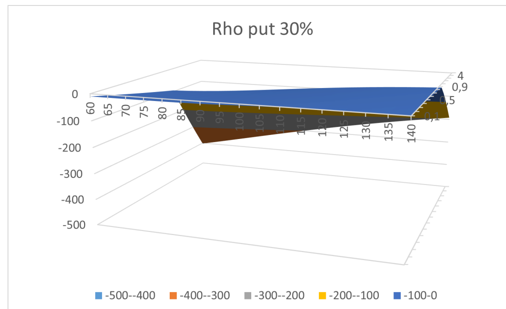

Gamma 30%
0,05
0,04
0,03
0,02
0,01
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-0,01 0,01-0,02 0,02-0,03 0,03-0,04 0,04-0,05


Vega 30%
100
80
60
40
20
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-20 20-40 40-60 60-80 80-100









3 Empirical section

3.1 Adyen

Adyen is a Dutch payment company listed on the Euronext Amsterdam stock exchange. Founded in 2006 by Pieter van der Does and Arnout Schuijff, it serves as an acquiring bank facilitating e-commerce, mobile, and point-of-sale payments for businesses.

Adyen offers online services for merchants to accept electronic payments through various methods including credit cards, debit cards, wire transfers, and real-time bank transfers. It connects merchants to international and local payment methods, acting as a payment gateway and service provider.

The company expanded globally in 2012 and obtained a pan-European acquiring license. By 2015, Adyen achieved a valuation of 2.3 billion and became profitable since 2011. In 2017, it surpassed 100 billion in processed volume and obtained a European banking license.

Adyen went public in 2018 with an IPO in Amsterdam. It became the primary payments processing partner for eBay in 2018. In 2021, Adyen achieved net revenue of 1 billion, and in 2022, it exceeded 1.3 billion in revenue.

3.2 Implied Volatility Smile

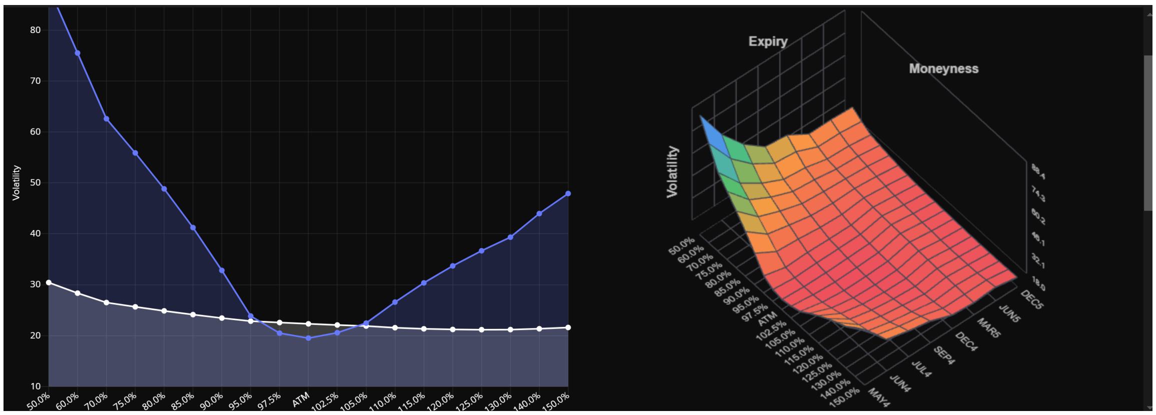
The implied volatility smile/skew is a term used in options trading to describe the phenomenon where the implied volatility of options with different strike prices but the same expiration date deviates from what would be expected under the Black-Scholes model.

Under the Black-Scholes model, implied volatility is assumed to be constant across all strike prices and maturities. However, in reality, options markets often exhibit a smile or skew shape when plotted against strike prices.
A smile refers to a situation where the implied volatility is higher for options with at-the-money (ATM) strike prices compared to options with out-of-the-money (OTM) or in-the-money (ITM) strike prices. This typically occurs when traders are willing to pay higher premiums for options that protect against extreme price movements, leading to higher implied volatilities for ATM options.
A skew, on the other hand, occurs when the implied volatility is higher for either OTM or ITM options compared to ATM options. A skew can be either positive or negative, depending on whether the implied volatility is higher for OTM or ITM options. Skews often arise due to market factors such as supply and demand dynamics, sentiment, or changes in market conditions.
The presence of an implied volatility smile/skew provides valuable information to options traders

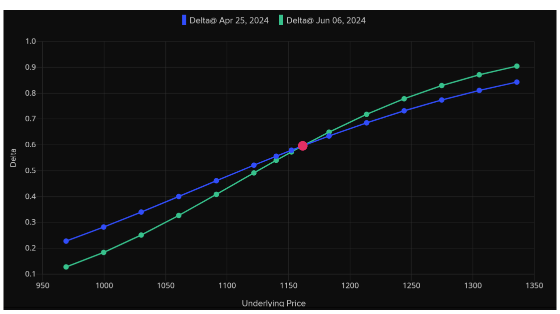
and investors. It suggests that market participants have different expectations for future volatility depending on the option's strike price. Traders may use the information from the smile/skew to adjust their trading strategies, hedge their positions, or assess market sentiment.

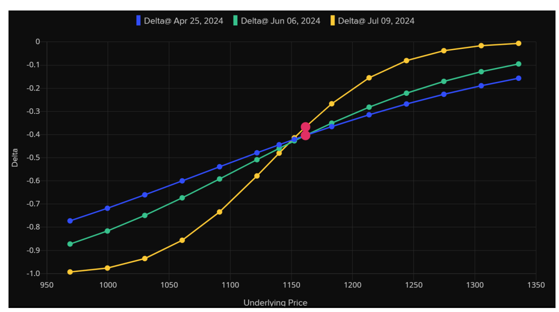
3.3 Data and comments

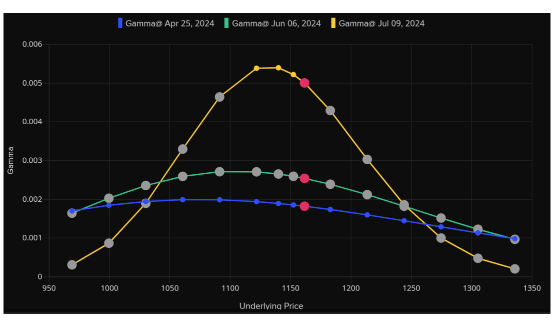
I chose in Refinitiv an asset that does not pay dividends and on which there is a book of European options. For the 3D picture that shows the implied volatility surface (as a function of the strikes and time to maturities), I checked in the Refinitiv options window Adyen implied volatility surface.

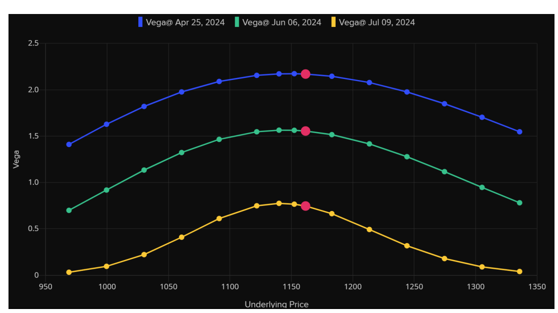


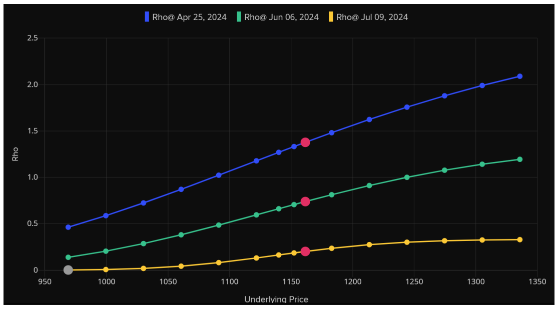
We can observe that it creates a sort of parabola, so as expected it follows the smile shape.
I checked for the smoothness of the Greeks for different times (different colors) and call or put option, except for vega and gamma which are the same in both cases, as shown in the following images (call on the left, put on the right):

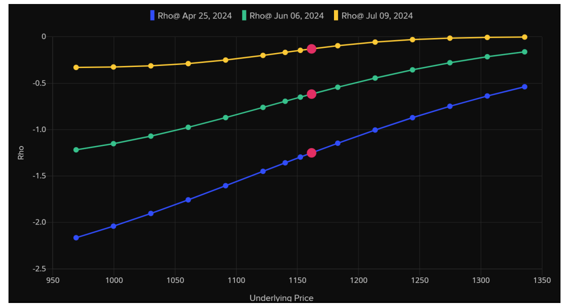


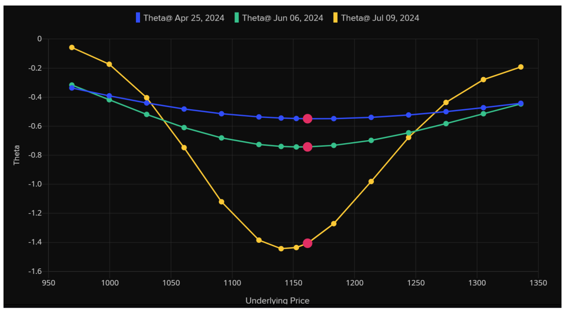


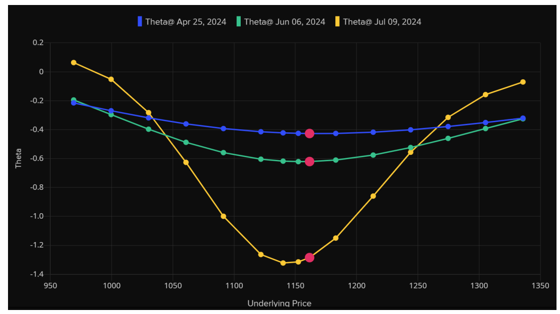












4 Merge section

In this last section, I select the asset chosen in the second part of the homework and consider the book of European options. I use the VBA code in the first part to compute the Greeks of an ATM option with a maturity of 3 months. I then compared it with the Greeks provided by Refinitiv. The data are shown in the following table.

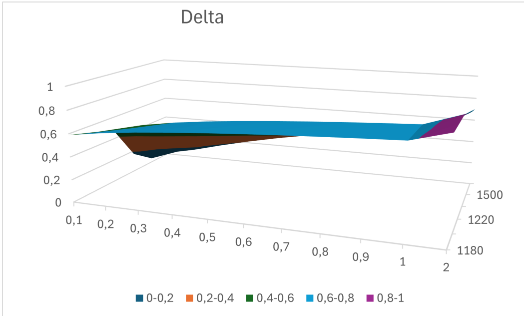
	Parameters call	*Parameters put*
S	1177	1177
K	1180	1180
T	0,25	0,25
sigma	0,37937	0,381107
r	0,223442	0,223442
div	0	0

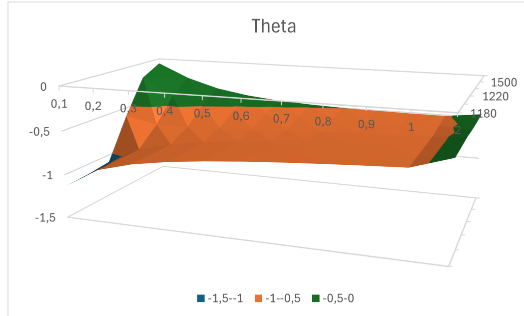
Results	*my call*	*my put*	*Refinitiv call*	*Refinitiv put*
Delta	0,646509	-0,35381	0.54	-0.46
Theta	-0,84677	-0,16542	-0.57	-0.56
Vega	2,187617	2,187617	2.24	2.23
Gamma	0,001665	0,001665	0.00	0.00
Rho	1,600934	-1,19067	1.26	-1.48

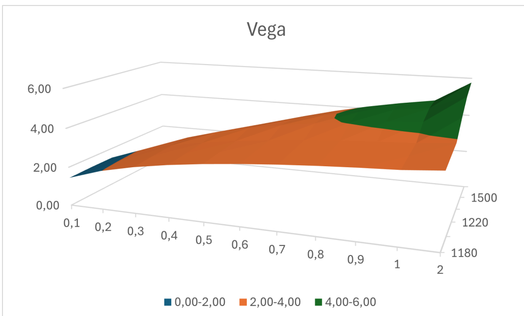
I then calculated the relative error for both call and put:

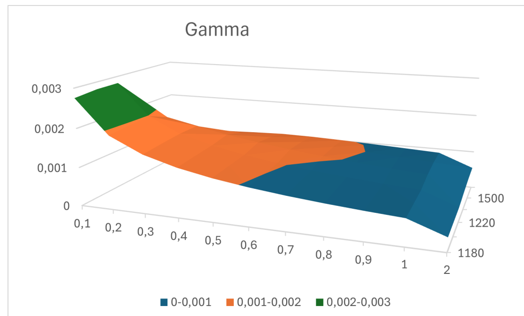
errors call	*errors put*
0,164745	0,30015
0,326851	2,38536
0,02395	0,01937
10ˆ(-2)	10ˆ(-2)
0,21296	0,243

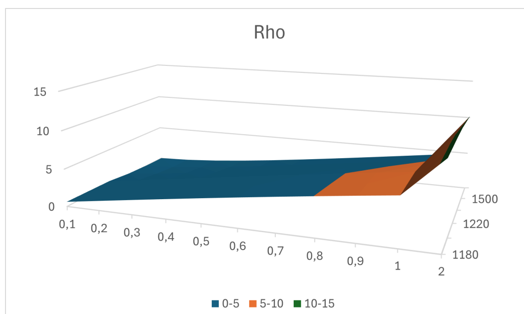
Finally I change the maturity and illustrate the surface of the Greeks:











5 Conclusion

In conclusion, in the first part of the homework, I wrote a VBA script valid for the B&S method and computed the calculation of the Greeks for various volatility, strike prices, and times. In the second section, using data from Refinitive, I show the smile shape of the implied volatility 3D graph, as expected and explained above, for empirical data and I check the smoothness of the Greeks. Finally, in the last section, I calculate the Greeks for an ATM option with maturity at 3 months and compare the results with the theoretical one from Refinitive. Lastly, I graph the Greeks for various times.