Report 5

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1 Introduction

This report aims to evaluate the effectiveness of Monte Carlo methods in pricing various types of options, encompassing Vanilla, Asian, and Lookback options. In the next section, we offer an introduction to the Monte Carlo Option model, explaining the Euler scheme's role in pricing both standard Vanilla options and path-dependent Exotic Options. Then we present the methodology and the analysis results. Finally, in the last section, we try a simplified model for pricing a certificate on a Worst Of written on 3 assets.

2 GBM and Types of option

2.1 Geometric Brownian Motion

The dynamics of the Geometric Brownian Motion (GBM) are encapsulated in the differential equation:

$$dS = \mu S dt + \sigma S dz$$

Here, dz represents a Wiener process, μ stands for the expected return in a risk-neutral environment, and σ denotes volatility. Given that this equation follows an Ito process, we can apply its lemma, which is defined as:

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2 + \frac{\partial G}{\partial t} \right) dt + \frac{\partial G}{\partial S}\sigma S dz$$

To simplify computations, we substitute $G = \ln S$ instead of $G = S$:

$$\frac{\partial G}{\partial S} = \frac{1}{S}, \quad \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2}, \quad \frac{\partial G}{\partial t} = 0$$

Thus, we obtain:

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz$$

To evaluate the value of $\ln S$ within a discrete time interval Δt, we approximate the equation as follows:

$$\ln S(t + \Delta t) - \ln S(t) = \left(\mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \epsilon \sqrt{\Delta t}$$

Here, ϵ is a random sample from a normal distribution with a mean of zero and a standard deviation 1.

2.2 Vanilla options

Vanilla options, also known as plain-vanilla options, are the simplest type of financial derivative contracts. These options give the holder the right, but not the obligation, to buy (in the case of a call option) or sell (in the case of a put option) an underlying asset at a predetermined price, known as the strike price or exercise price, on or before the expiration date.

The payoff of a vanilla call option at expiration is given by:

$$\text{Payoff}_{\text{Call}} = \max(0, S - K)$$

where S is the price of the underlying asset at expiration and K is the strike price.

Similarly, the payoff of a vanilla put option at expiration is given by:

$$\text{Payoff}_{\text{Put}} = \max(0, K - S)$$

where S and K have the same meanings as above.

The price of a vanilla option can be calculated using various pricing models, such as the Black-Scholes model or through Monte Carlo simulation methods.

2.3 Asian Options

An Asian option (or average value option) is a special type of option contract. For Asian options, the payoff is determined by the average underlying price over some pre-set period of time. This is different from the case of the usual European option and American option, where the payoff of the option contract depends on the price of the underlying instrument at exercise; Asian options are thus one of the basic forms of exotic options.

There are two types of Asian options: Average Price Option (fixed strike), where the strike price is predetermined and the averaging price of the underlying asset is used for payoff calculation; and Average Strike Option (floating strike), where the averaging price of the underlying asset over the duration becomes the strike price. There are numerous permutations of Asian options; the one used in this assignment is described below:

Fixed strike (or average rate) Asian options:

Asian call payoff:

$$C(T) = \max(A(0,T) - K, 0)$$

Asian put payout:

$$P(T) = \max(K - A(0,T), 0)$$

Asian options are a type of exotic option where the payoff is determined by the average underlying price over a specific period. They come in two forms: fixed strike (where the strike price is predetermined) and floating strike (where the strike price is based on the average price). Asian options offer advantages such as reduced risk of market manipulation and typically lower cost compared to European or American options due to their averaging feature.

2.4 Lookback Options

Lookback options, in financial terminology, are a type of exotic option with path dependency, among many other kinds of options. The payoff depends on the optimal (maximum or minimum) underlying asset's price occurring over the life of the option. The option allows the holder to "look back" over time to determine the payoff. There exist two kinds of lookback options: with floating strike and with fixed strike.

2.4.1 Lookback Option with Floating Strike

As the name suggests, the option's strike price is floating and determined at maturity. The floating strike is the optimal value of the underlying asset's price during the option's life. The payoff is the maximum difference between the market asset's price at maturity and the floating strike. For the call, the strike price is fixed at the asset's lowest price during the option's life, and for the put, it is fixed at the asset's highest price. The payoff functions for the lookback call and the lookback put, respectively, are given by:

$$LC_{float} = \max(S_T - S_{min}, 0) = S_T - S_{min}$$

and

$$LP_{float} = \max(S_{max} - S_T, 0) = S_{max} - S_T$$

where S_{max} is the asset's maximum price during the life of the option, S_{min} is the asset's minimum price during the life of the option, and S_T is the underlying asset's price at maturity T.

2.4.2 Lookback Option with Fixed Strike

Similar to standard European options, the option's strike price is fixed. The difference is that the option is not exercised at the price at maturity; the payoff is the maximum difference between the optimal underlying asset price and the strike. For the call option, the holder chooses to exercise when the underlying asset price is at its highest level. For the put option, the holder chooses to exercise at the underlying asset's lowest price. The payoff functions for the lookback call and the lookback put, respectively, are given by:

$$LC_{fix} = \max(S_{max} - K, 0)$$

and

$$LP_{fix} = \max(K - S_{min}, 0)$$

where S_{max} is the asset's maximum price during the life of the option, S_{min} is the asset's minimum price during the life of the option, and K is the strike price.

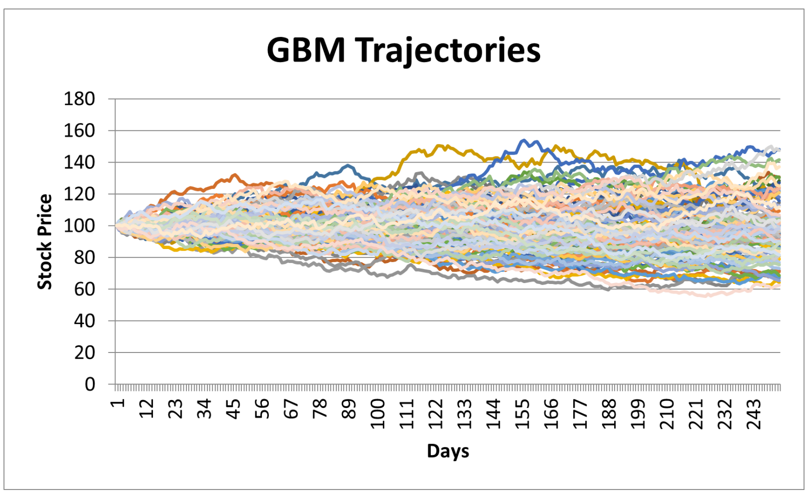
2.5 Worst Of

The "worst of" option is a financial derivative contract that pays out based on the performance of the worst-performing asset among a predefined set of assets over a specified period. It is essentially a form of option contract where the investor receives a payout based on the lowest value observed among the assets in the basket. The options are often considered a correlation trade since the value of the option is sensitive to the correlation between the various basket components. Our analysis of three assets is given in the last section.

3 Data analysis

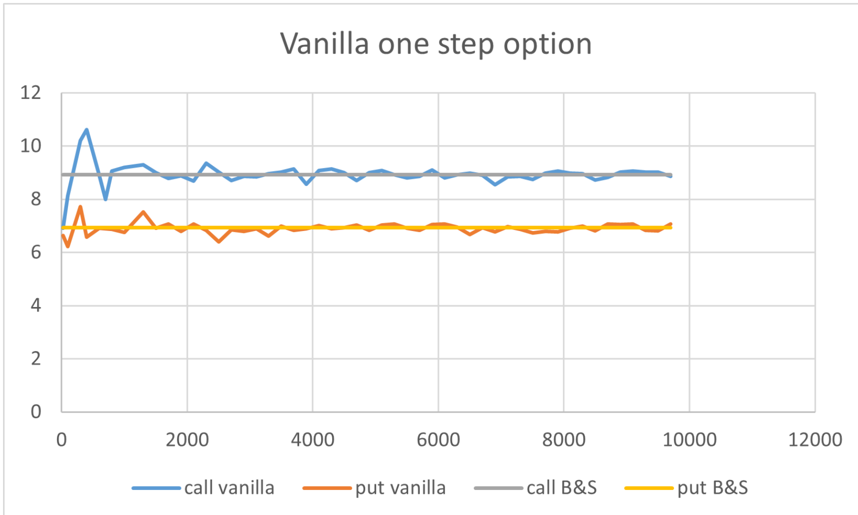
3.1 Simulating N Trajectories for the Geometric Brownian Motion (GBM)

We wrote a VBA code to simulate N trajectories using as parameters S_0=100, sigma=20%, r=1%, T=1 year, K=99, dt=1 day. The results are shown in the figure below:



3.2 Building a Pricer for Vanilla Options through Monte Carlo Simulation

To price vanilla options (both call and put), we employ Monte Carlo (MC) simulation techniques. In this method, we simulate the future price of the underlying asset (S_T) using a one-step simulation approach. This means that instead of simulating the entire path of the asset price, we directly simulate the future price (S_T) at the expiration time. We define the parameters of the option contract, including the current price of the underlying asset (S), the strike price (K), the time to expiration (T), the risk-free interest rate (r), and the volatility (σ) (the same used above).We simulate until $N = 9300$ values of the random variable S_T using the one-step simulation method. For each simulation, generate a random value from a standard normal distribution, denoted as Z, and use it to calculate the future price of the asset (S_T) based on the asset's current price (S), the risk-free interest rate (r), the volatility (σ), and the time to expiration (T). We repeat this process N times to obtain a set of S_T values. For each we simulated value of S_T, calculate the payoff of the vanilla option. We average the option payoffs obtained from the simulations to estimate the option price. Discount the average payoff back to present value using the risk-free interest rate (r) to obtain the option price. The results compared with the B-S formula are shown in the following graph.



By using a one-step simulation approach in Monte Carlo simulation, we can efficiently price vanilla options without the need to simulate the entire path of the asset price. This method provides a reliable estimate of option prices while reducing computational complexity.

3.3 Simulation of Vanilla Options using Multiple-Step Euler Scheme

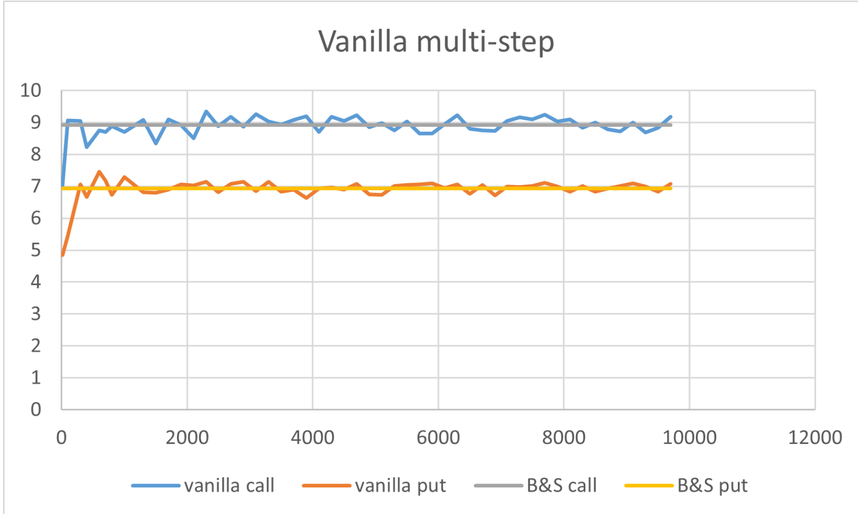
We define the parameters of the Vanilla option, including the current price of the underlying asset (S), the strike price (K), the time to expiration (T), the risk-free interest rate (r), and the volatility (σ). We choose a large number of steps N for the Euler scheme, ensuring $N > 500$ to achieve sufficient accuracy. We divide the time to expiration (T) into N equal time intervals. We calculate the length of each time interval, denoted as $\Delta t = \frac{T}{N}$. We initialize the simulation with the current price of the underlying asset (S_0). We iterate through each time step t_i from $t = 0$ to $t = T$. At each time step t_i, we generate a random number from a standard normal distribution, denoted as Z_i, to represent the random component of the asset price movement. We calculate the increment in the asset price (ΔS_i) using the Euler scheme formula:

$$\Delta S_i = S_{i-1} \cdot \left(r \cdot \Delta t + \sigma \cdot \sqrt{\Delta t} \cdot Z_i \right)$$

We update the asset price for the next time step:

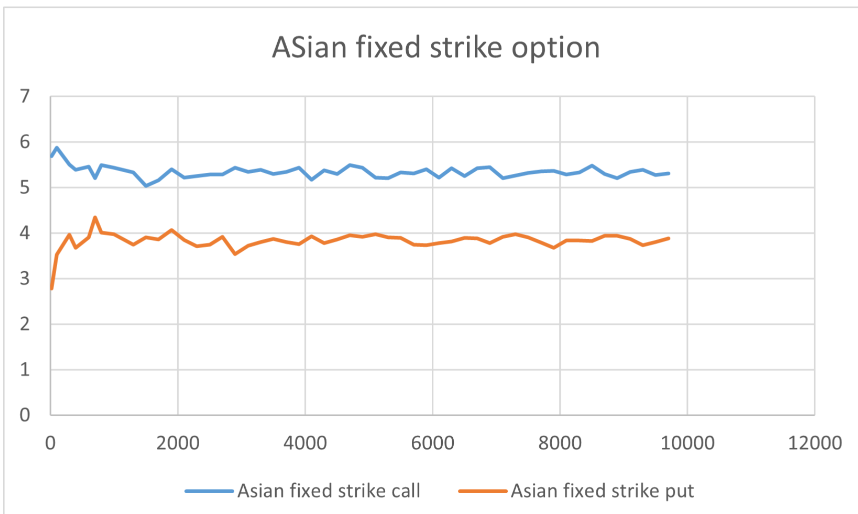
$$S_i = S_{i-1} + \Delta S_i$$

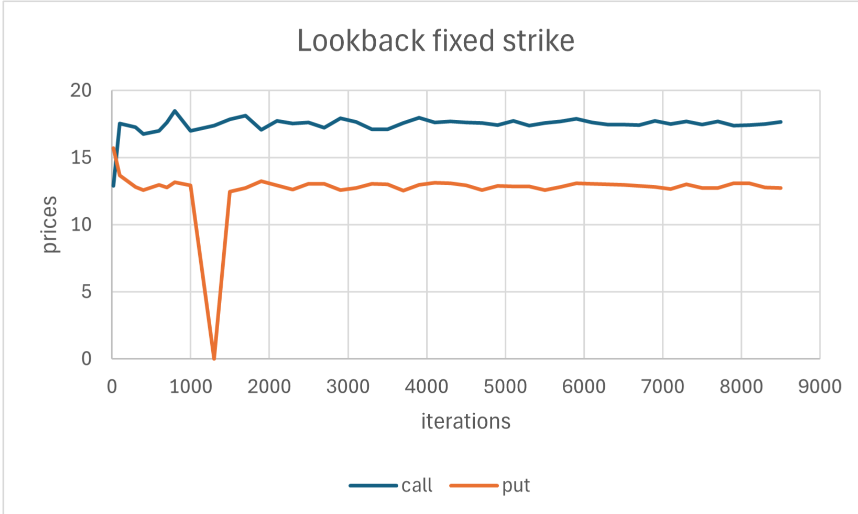
At the expiration time ($t = T$), we calculate the payoff of the Vanilla option based on the simulated asset prices: - For a call option: $\text{Payoff}_{\text{Call}} = \max(0, S_T - K)$ - For a put option: $\text{Payoff}_{\text{Put}} = \max(0, K - S_T)$ We repeat the simulation process multiple times (e.g., Monte Carlo simulation) to obtain a distribution of option payoffs.
We calculate the average of the option payoffs obtained from the simulations to estimate the option price. We discount the average payoff back to present value using the risk-free interest rate to obtain the option price.

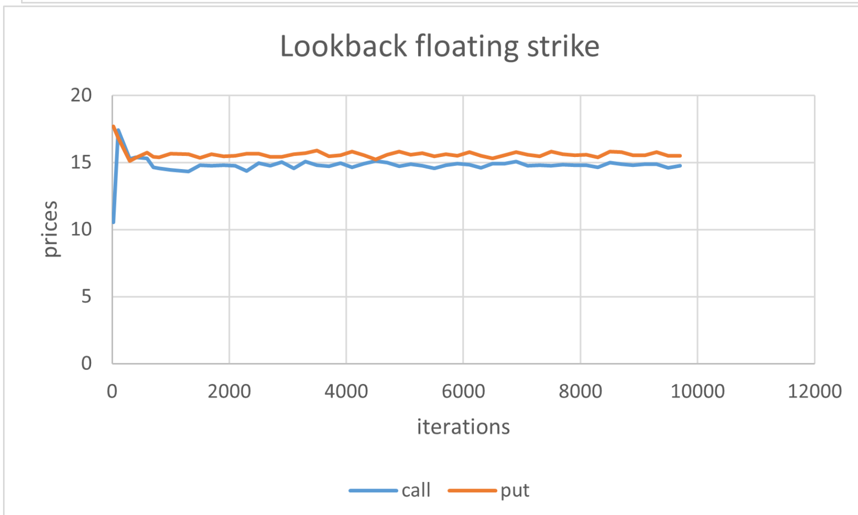


3.4 Simulating Asian and Exotic options using Monte Carlo

For the Asian and Lookback options, we simulate, for the fixed strike option, the pricing in the same way as the Multi-step vanilla option but changing their payoff (see in the first section for formulas). The results are shown in the following graphs:







The following table summarises the results from the Monte-Carlo simulation with the most N for every option.

Option type	call	put
vanilla one step	8.85	7,07
vanilla multistep	8,83	6,89
BS	8,91	6,93
Asian fixed strike	5,21	3,83
Lookback fixed	17,65	12,76
Lookback floating	14,97	15,50

4 Cash Collect Certificate Worst of

In our simplified framework, we analyze a basket comprising three stocks: MVDA, MSFT, and GOOGL. Strike prices are fixed at 300, 500, and 1000, respectively. Utilizing historical data from the preceding year, we extrapolate covariance, and probability distribution to conduct future guesses using Monte Carlo simulations. These simulations aim to price the Cash Collect Worst option, which has a maturity of one year and involves three observation points, each associated with a $25 coupon. The coupons are disbursed contingent on the poorest-performing stock surpassing its predefined strike price. Additionally, if this event occurs at maturity, the certificate guarantees a payoff of max(0, ST - K).

First assuming historical returns have Gaussian distributions we extract the data then average and covariance matrix. We generate multivariate normal numbers that mimic the distribution of our historical returns. We then calculate the payoffs for each simulation, discount them using a risk-free rate of $r = 0.01$, and subsequently average the Monte Carlo iterations.

Given this simplified model of the Cash Collect Worst Certificate, along with the predetermined parameters, we ascertain the certificate's price to be $f = 37.25$.