Report 6

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1 Introduction

This report aims to determine the Value at Risk (VaR) for a balanced portfolio consisting of assets from McDonald and Yum Brands, as well as examining the potential additivity or non-additivity of VaR. The VaR is calculated for both individual assets and the portfolio itself. Various methods are employed to compute VaR. Initially we calculated Normal VaR at 99%, 99.5%, and 95% confidence levels, over horizons ranging from 1 to 100 days, using a flat estimation of sigma. Then Normal VaR at the same confidence levels and horizons, but with sigma computed via Riskmetrics EWMA with a decay parameter of 0.94. We determined VaR also through Monte Carlo simulation at confidence levels of 99%, 99.5%, and 95%. Then we compute the historical VaR according to the historical value of the returns and lastly, we found VaR using historical simulation method.
The report also demonstrates the lack of subadditivity in the VaR for the portfolio. Subadditivity implies that the VaR of a combined portfolio is less than the sum of the VaRs of its constituent parts. However, it's noted that VaR can be non-subadditive in certain instances. This occurs with some distribution such as the normal one.

2 Company description

2.1 McDonald

McDonald's Corporation is an American multinational fast food chain, founded in 1940 as a restaurant operated by Richard and Maurice McDonald, in San Bernardino, California, United States. They rechristened their business as a hamburger stand, and later turned the company into a franchise, with the Golden Arches logo being introduced in 1953 at a location in Phoenix, Arizona. In 1955, Ray Kroc, a businessman, joined the company as a franchise agent and in 1961 bought out the McDonald brothers. Previously headquartered in Oak Brook, Illinois, it moved to nearby Chicago in June 2018. McDonald's is also a real estate company through its ownership of around 70% of restaurant buildings and 45% of the underlying land (which it leases to its franchisees).

McDonald's is the world's largest fast food restaurant chain, serving over 69 million customers daily in over 100 countries in more than 40,000 outlets as of 2021. McDonald's is best known for its hamburgers, cheeseburgers, and french fries, although their menu also includes other items like chicken, fish, fruit, and salads. Their bestselling licensed item is their french fries, followed by the Big Mac. The McDonald's Corporation revenues come from the rent, royalties, and fees paid by the franchisees, as well as sales in company-operated restaurants. McDonald's is the world's second-largest private employer with 1.7 million employees (behind Walmart with 2.3 million employees), the majority of whom work in the restaurant's franchises. As of 2022, McDonald's has the sixth-highest global brand valuation.

McDonald's has been subject to criticism over the health effects of its products, its treatment of employees, and its participation in various legal cases.

2.2 Yum Brands

Yum! Brands, Inc., formerly Tricon Global Restaurants, Inc., is an American multinational fast food corporation listed on the Fortune 1000. Yum! operates the brands KFC, Pizza Hut, Taco Bell, and The Habit Burger Grill, except in China, where the brands are operated by a separate company, Yum

China. Yum! previously also owned Long John Silver's and AW Restaurants. The company was created as a spin-off of PepsiCo in 1997.

Based in Louisville, Kentucky, Yum! is one of the world's largest fast food restaurant companies in terms of system units. In 2016, Yum! had 43,617 restaurants, including 2,859 that were company-owned and 40,758 that were franchised, in 135 nations and territories worldwide.

The company's history began in 1977 when PepsiCo entered the restaurant business by acquiring Pizza Hut from co-founders Dan and Frank Carney. A year later, PepsiCo purchased Taco Bell from founder Glen Bell. In 1986, R. J. Reynolds sold KFC to PepsiCo to pay off debt from its recent purchase of Nabisco.

In 1990, Hot 'n Now was acquired via Taco Bell from William Van Domelen, but the company was sold in 1996. In 1992, PepsiCo acquired California Pizza Kitchen. In 1993, it acquired Chevys Fresh Mex, D'Angelo Grilled Sandwiches, and the American division of Canadian chain East Side Mario's. These chains were later sold when PepsiCo exited the restaurant business and spun off KFC, Pizza Hut, and Taco Bell. In 1997, PepsiCo sold PepsiCo Food Systems restaurant-supply unit to Ameriserve Food Distribution Inc.

Yum! was created in 1997 as Tricon Global Restaurants, Inc. from PepsiCo's fast food division as the parent corporation of the KFC, Pizza Hut, and Taco Bell restaurant companies. In 2000, Tricon Global tested multi-branded locations with Yorkshire Global Restaurants. In 2001, KFC started test restaurants in Austin, Texas, called "Wing Works".

Yorkshire merged with Tricon Global Restaurants to form Yum! Brands in 2002. Yum! began testing co-branding locations in 2002 and launched WingStreet in 2003. In 2004, East Dawning test cafeteria-style restaurant was opened in Shanghai.

3 Theory

3.1 Value at Risk (VaR)

Value at Risk (VaR) serves as a crucial statistical metric for evaluating the financial risk associated with a portfolio, indicating the maximum potential loss within a specified confidence level over a defined time frame.

Mathematically, VaR is denoted as:

$$\mathrm{VaR}_\alpha(X) = -\inf\{x : F_X(x) \geq \alpha\}$$

Here, $\mathrm{VaR}_\alpha(X)$ signifies the portfolio's Value at Risk at confidence level α, with $F_X(x)$ representing the cumulative distribution function (CDF) of the portfolio returns, and α representing the significance level, typically corresponding to a certain percentile of the distribution (e.g., 95%, 99%).

VaR conveys a statement of confidence like, "I am X percent certain there will not be a loss of more than V dollars in the next N days." The VaR value, V, depends on the time horizon (N days) and confidence level (X%). It represents the loss level over N days with a probability of only $1 - \frac{\alpha^2}{100}$ of being exceeded. For instance, with X = 99 and N = 10, the expected shortfall is the average amount the company loses over a 10-day period when the loss surpasses the 10-day 99% VaR.

VaR encompasses two parameters: the time horizon (N, measured in days) and the confidence level (X). In practical estimation for market risk, analysts often set N = 1 initially due to insufficient data for longer periods. The assumption is often made that:

$$N\text{-day VaR} = 1\text{-day VaR} \times \sqrt{N}$$

This formula holds true when changes in the portfolio's value on consecutive days follow independent identical normal distributions with mean zero. In other scenarios, it serves as an approximation.

3.2 Riskmetrics EWMA (Exponentially Weighted Moving Average)

Riskmetrics EWMA is a widely used method in finance for estimating the volatility of financial assets, particularly in the calculation of Value at Risk (VaR). The EWMA model assigns exponentially decreasing weights to past observations, with more recent observations receiving higher weights. This is achieved through the formula:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2$$

where: σ_t^2 is the estimated variance of returns at time t, λ is the decay factor, usually a value close to 1 indicating a slower decay of past observations' influence, σ_{t-1}^2 is the estimated variance of returns at time $t-1$, and r_{t-1} is the return at time $t-1$.

By iteratively applying this formula to historical return data, the EWMA model provides a dynamic estimate of volatility that gives more weight to recent observations while still considering past data. This makes it well-suited for capturing changing market conditions and is often favored in risk management practices.

3.3 Monte Carlo Approach

The Monte Carlo approach is a powerful method used in finance for estimating the Value at Risk (VaR) and assessing the risk associated with financial portfolios.

In this method, random simulations are performed to model the behavior of financial assets and their potential future returns. These simulations involve generating random scenarios based on specified distributions for asset prices and other relevant factors.

Each simulated scenario represents a possible future outcome for the portfolio. By running a large number of simulations, a distribution of potential portfolio value daily change is generated. From this distribution, the VaR can be estimated by identifying the portfolio value corresponding to the desired confidence level.

The Monte Carlo approach allows for the incorporation of complex factors and dependencies into the risk assessment process.

3.4 Historical VaR

The historical VaR is computed directly from historical data without making any assumptions about the distribution of returns. It involves sorting historical returns from worst to best and selecting the α-th percentile of the sorted returns as the VaR level. For example, if $\alpha = 0.05$, the 5% worst historical returns would be used to estimate the VaR.

3.5 VaR with Historical Simulation

The historical simulation method is a risk management technique used to estimate the Value at Risk (VaR) of a portfolio or investment. Instead of relying on theoretical models or assumptions about the probability distribution of returns, this method uses historical data to simulate potential future outcomes. Because historical simulation uses real data, it can capture unexpected events and correlations that would not necessarily be predicted by a theoretical model. Historical simulation uses the actual distribution of risk factors. This means that the estimation of the actual distribution of changes in the risk factors is not required. If we consider with ΔP the Portfolio value of tomorrow minus the one of today, we can construct it from the historical data and than take samples from its distribution for a simulation. SO we create a pdf from historical data and use it for the simulation. Finally, we calculate VaR by identifying the appropriate quantile of the distribution of sampled ΔP values.

3.6 Model-Building Approach

The main alternative to historical simulation is the model-building approach. It is appropriate to mention one issue concerned with the units for measuring volatility. In option pricing, time is usually measured in years, and the volatility of an asset is usually quoted as a "volatility per year". In VaR calculations, we measure volatility "per day" as $\sigma_{\text{day}} = \frac{\sigma}{\sqrt{252}}$. Strictly speaking, σ_{day} should be defined as the standard deviation of the continuously compounded return in one day. However, in practice, we assume that it is the standard deviation of the percentage change in one day.

Parametric approaches assume a specific distribution for the changes in asset prices. One common assumption is that the distribution of the returns follows a Normal distribution.

In this case, since ΔP is linear in the returns, it follows a Normal distribution with mean $\mu_P = \alpha_1\mu_1 + \alpha_2\mu_2$ and standard deviation $\sigma_P = \sqrt{\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2 cov_{12}}$, where cov_{12} is the covariance between assets S_1 and S_2.

The Parametric Normal method approach provides a simple and convenient way to estimate VaR under the assumption of Normality. However, it's important to note that real-world financial data often deviates from the Normal distribution assumption, especially during extreme market conditions.

4 Analysis and results

4.1 Variance, average and Model-Building

I compute average and variance of an equibalanced portfolio of 2 assets, McDonald and Yum Brands, with a 6 months time window of daily returns, selected in Refinitiv.
In particular i chose a Portfolio of two assents composed as follow: Suppose that we have a portfolio worth P consisting of n=2 assets with an amount a_i being invested in asset i for $i = 1, \ldots, n$. Define Δx_i as the return on asset i in one day. The dollar change in the value of the investment in asset i in one day is $a_i \Delta x_i$, and

$$\Delta P = \sum_{i=1}^{n} a_i \Delta x_i$$

where ΔP is the dollar change in the value of the whole portfolio in one day. In my case α_i are both equal to 1.
A standard result in statistics tells us that, if two variables X and Y have standard deviations equal to s_X and s_Y with the coefficient of correlation between them equal to r, the standard deviation of $X + Y$ is given by

$$s_{X+Y} = \sqrt{s_X^2 + s_Y^2 + 2 r s_X s_Y}$$

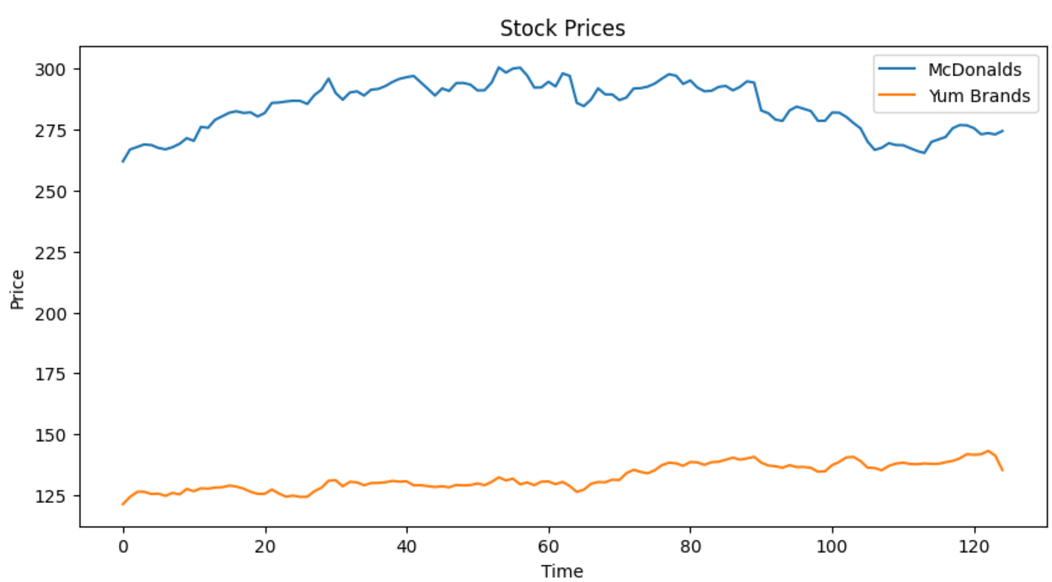
Using matrix notation, the equation for the variance of the portfolio just given becomes

$$s_P^2 = \alpha^T \mathbf{C} \alpha$$

where α is the (column) vector whose ith element is α_i , C is the variance–covariance matrix, and α^T is the transpose of α.
The results obtained are

Average Return of Portfolio: 0.0013 Variance of Portfolio: 0.0002

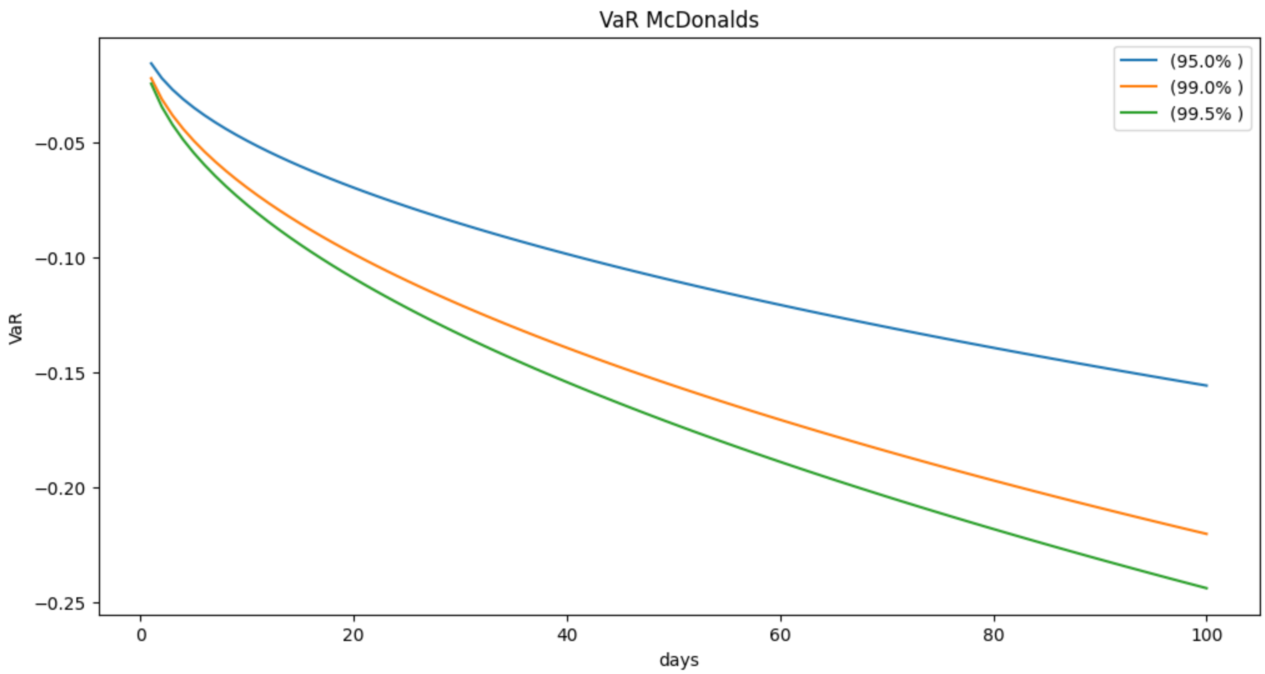
Then I plot the closing prices for the two stocks

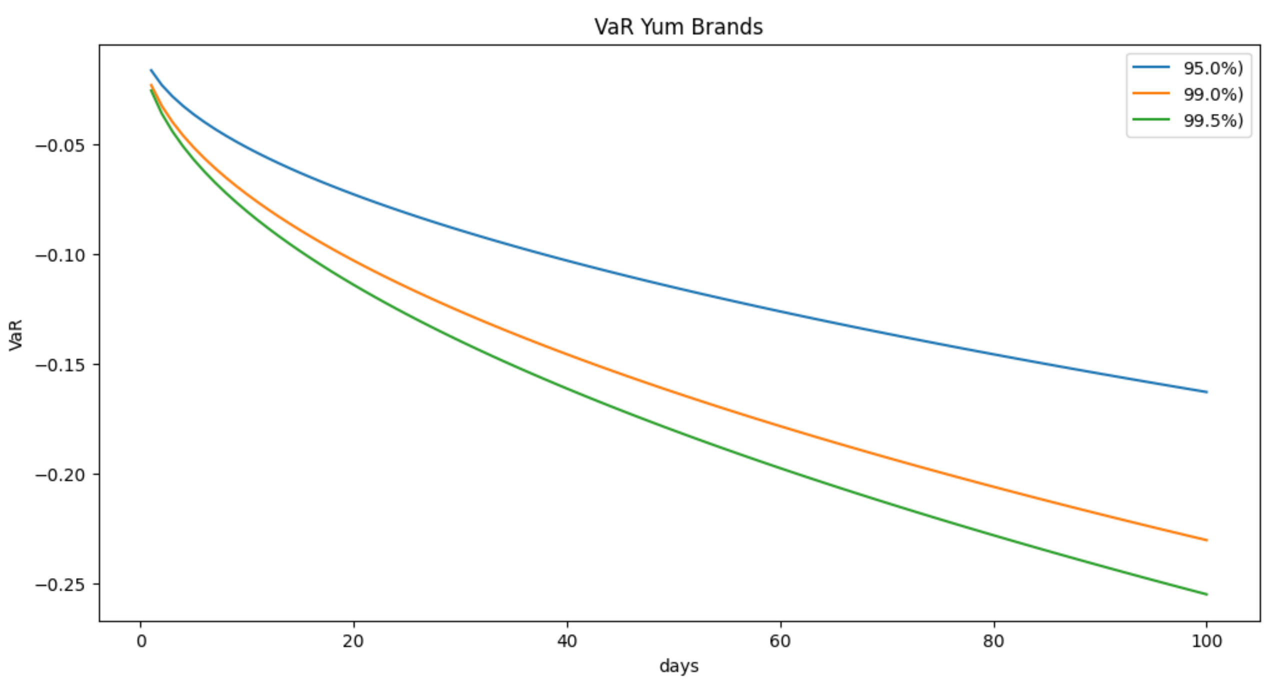


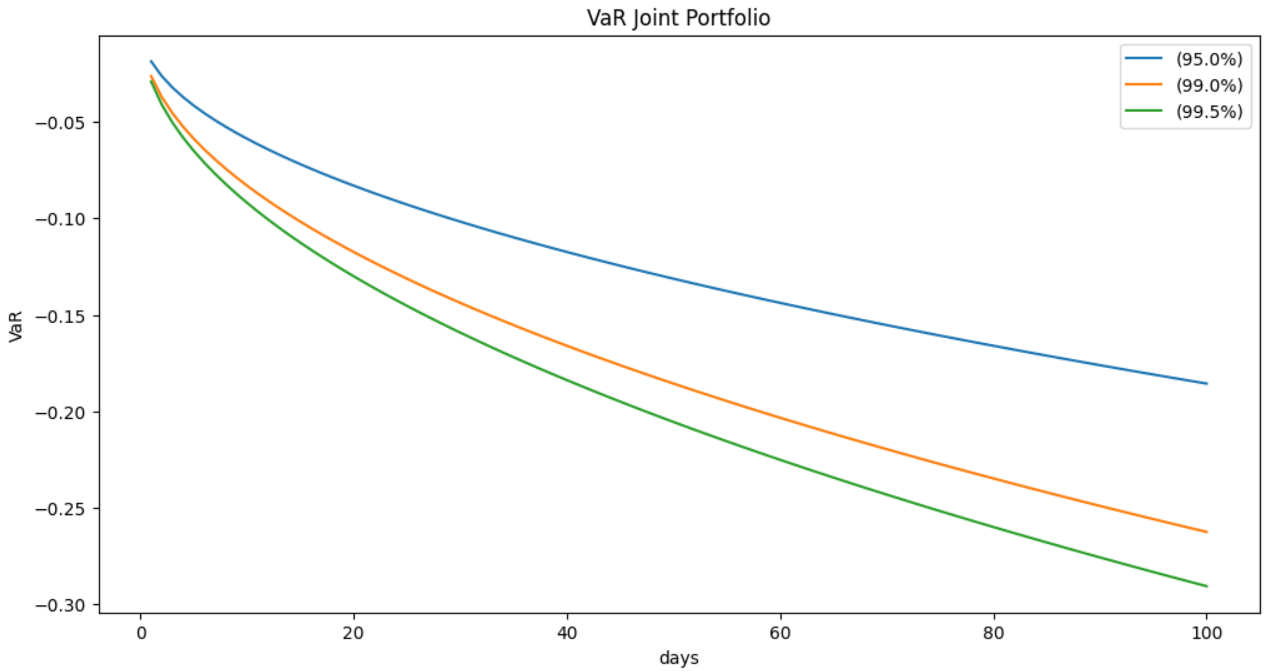
The correlation matrix is :

$$\begin{bmatrix} 1 & 0.54069327 \\ 0.54069327 & 1 \end{bmatrix}$$

that is symmetric and with 1 in the main diagonal as expected.
Additionally I compute the parametric single and joint Normal VaR at (95%, 99% , 99,5%) with T=1,..,100 days horizon (estimation of sigma flat). So using the model-building approach with Gaussian hypothesis, in particular the fact that a normal distribution can be converted to the standard normal distribution (z) with the formula z = (x-mean) / standard deviation, we find the VaR:



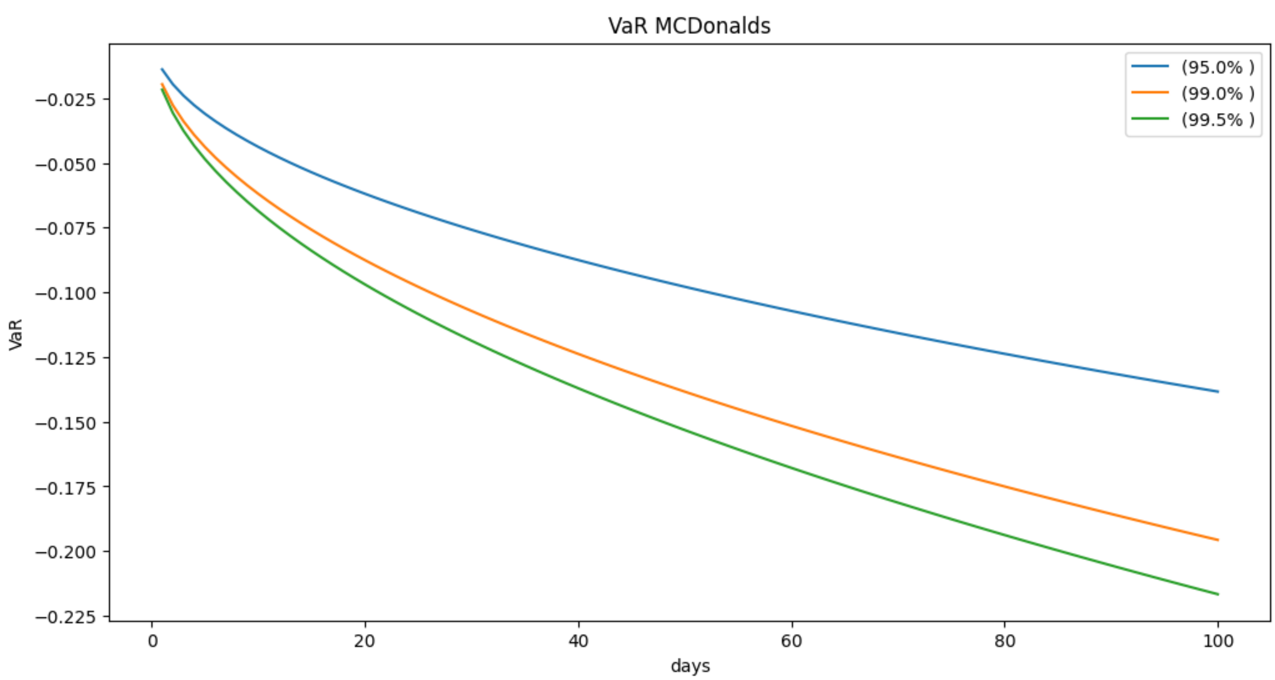


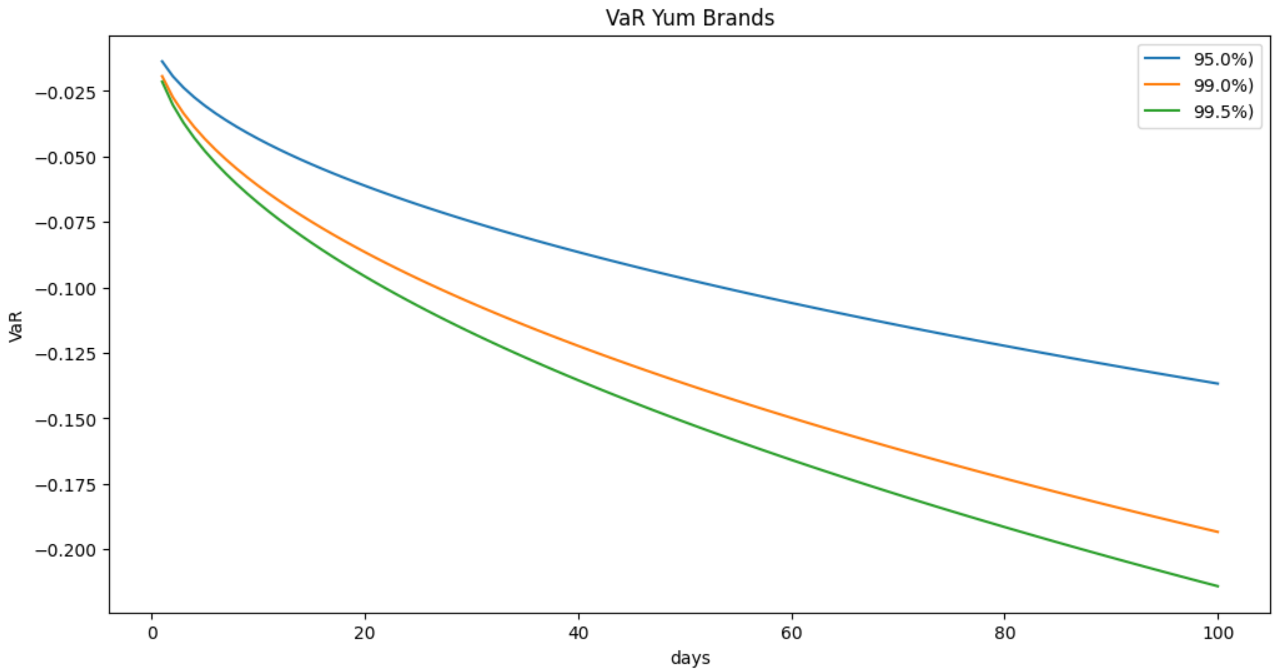


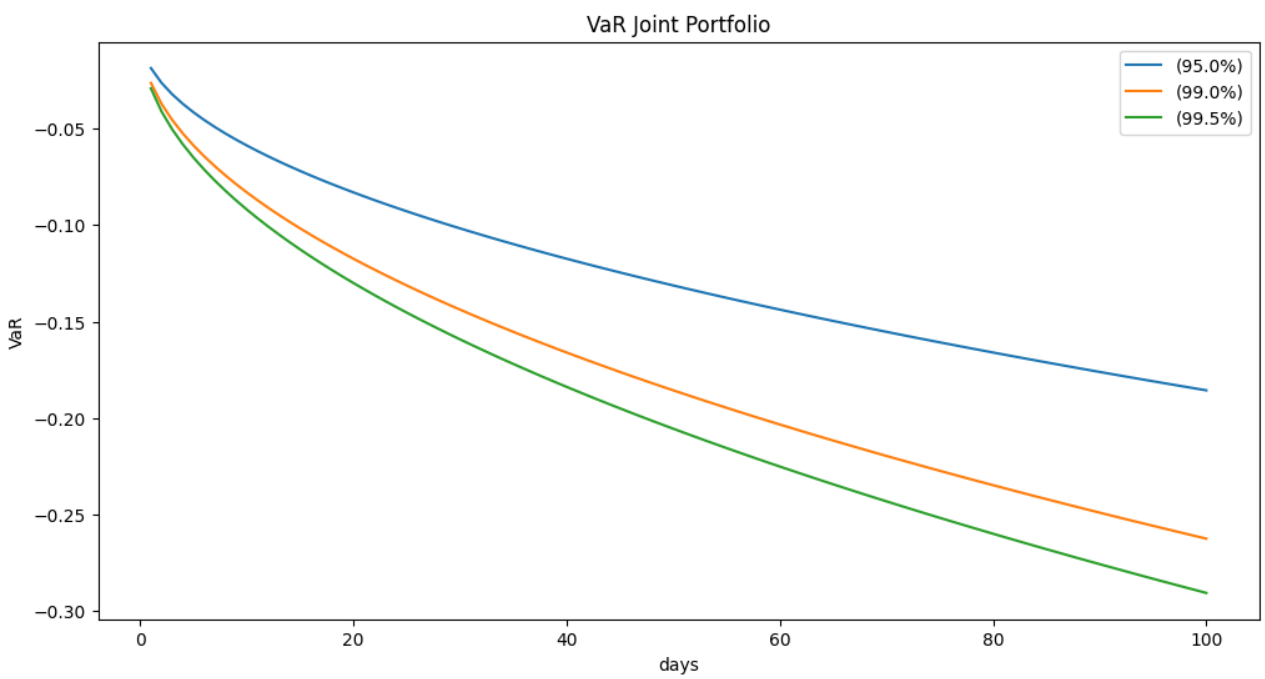
For the additivity, check the last section.

4.2 EWMA method

I continued the analysis by computing the VaR using EWMA, following the method described above. I did the some steps as before using sigma estimated following Riskmetrics EWMA with lambda=0,94.

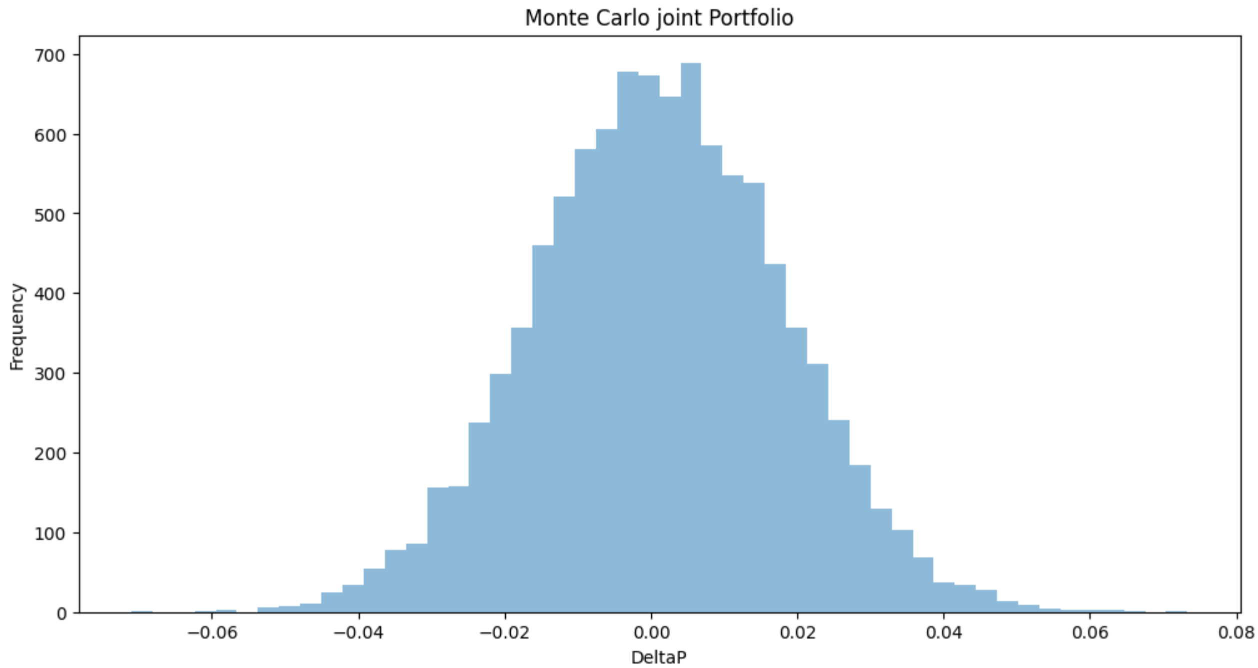


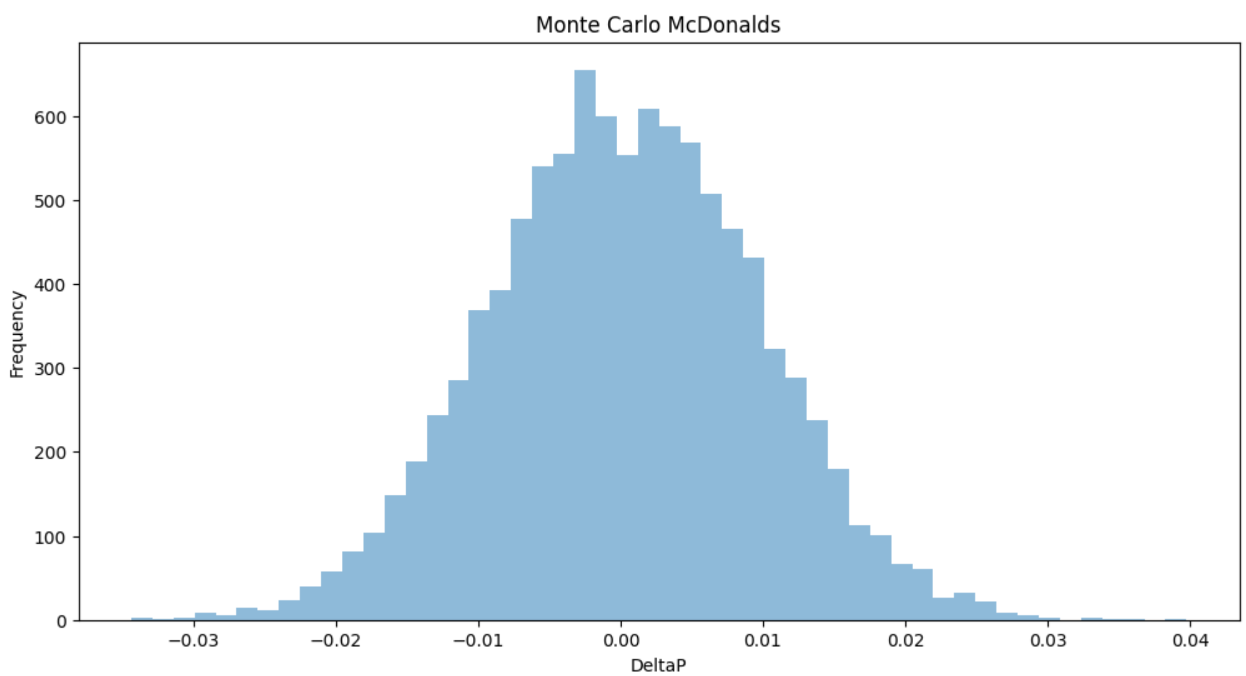


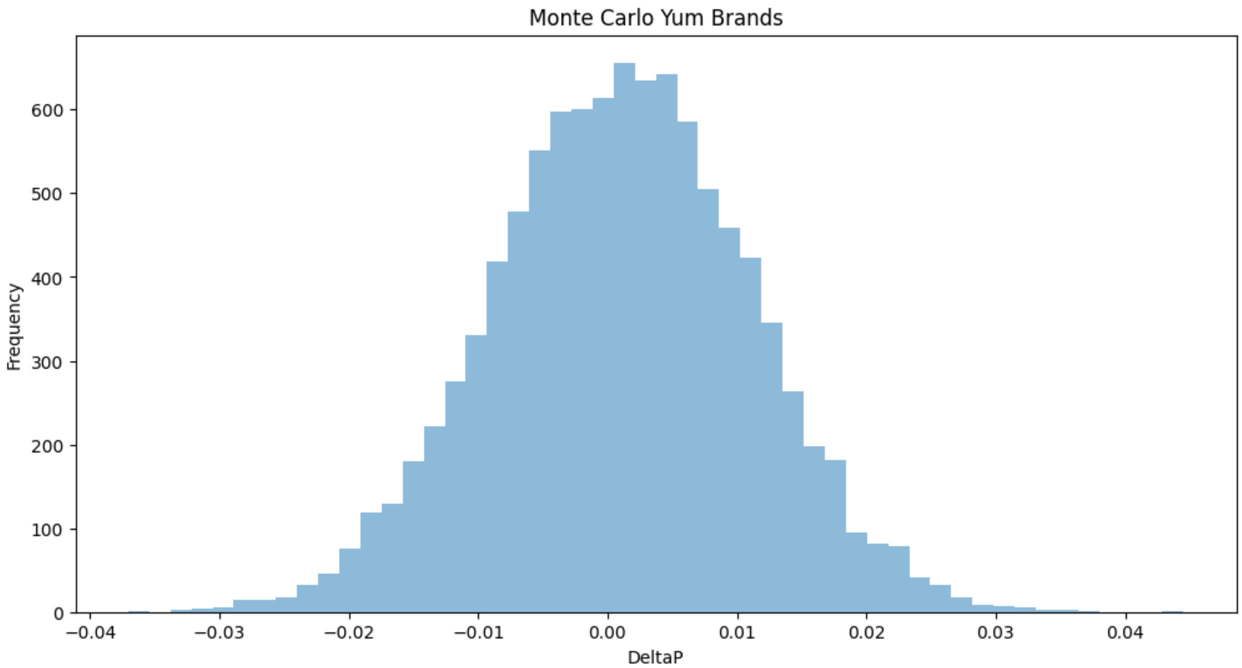


4.3 Monte-Carlo method

For Montecarlo method, I used a multivariate distribution to semple the returns, from this return I used the percentile to find the VaR. I did these calculations for the two single assets and the joint Portfolio. The simulations for N=1000 and T= one day are shown in the following histograms:


Monte Carlo joint Portfolio
Frequency
DeltaP
0
100
200
300
400
500
600
700
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06
0.08


Monte Carlo McDonalds
Frequency
DeltaP
0
100
200
300
400
500
600
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
0.04

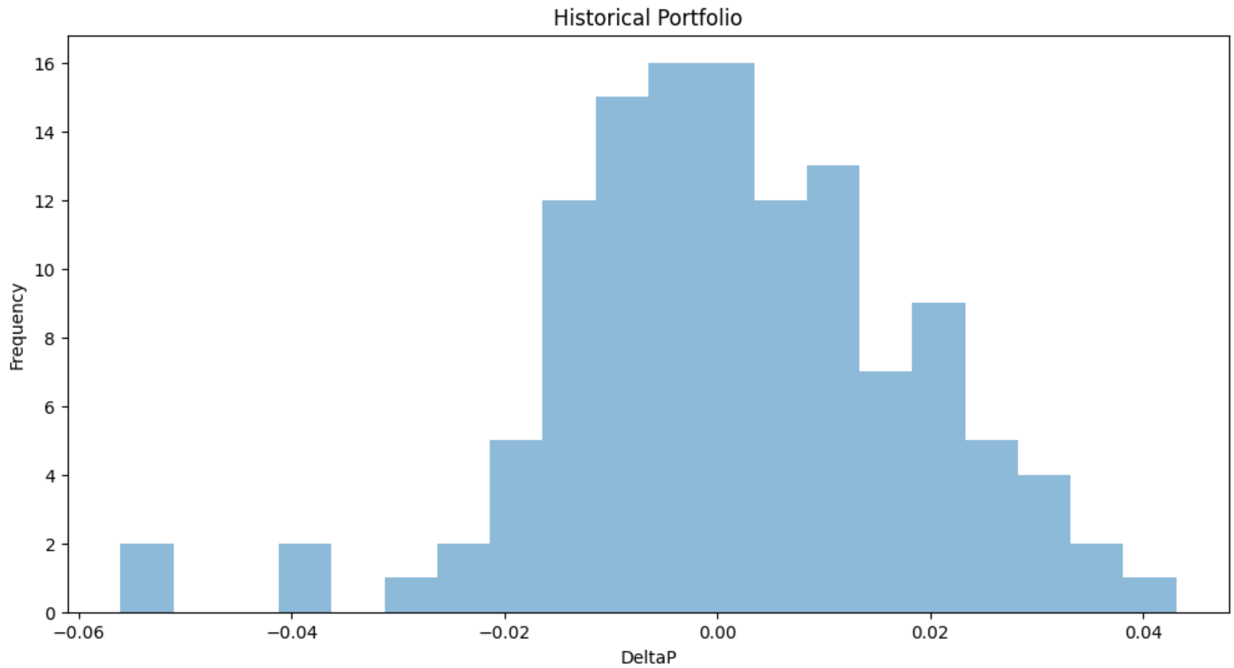


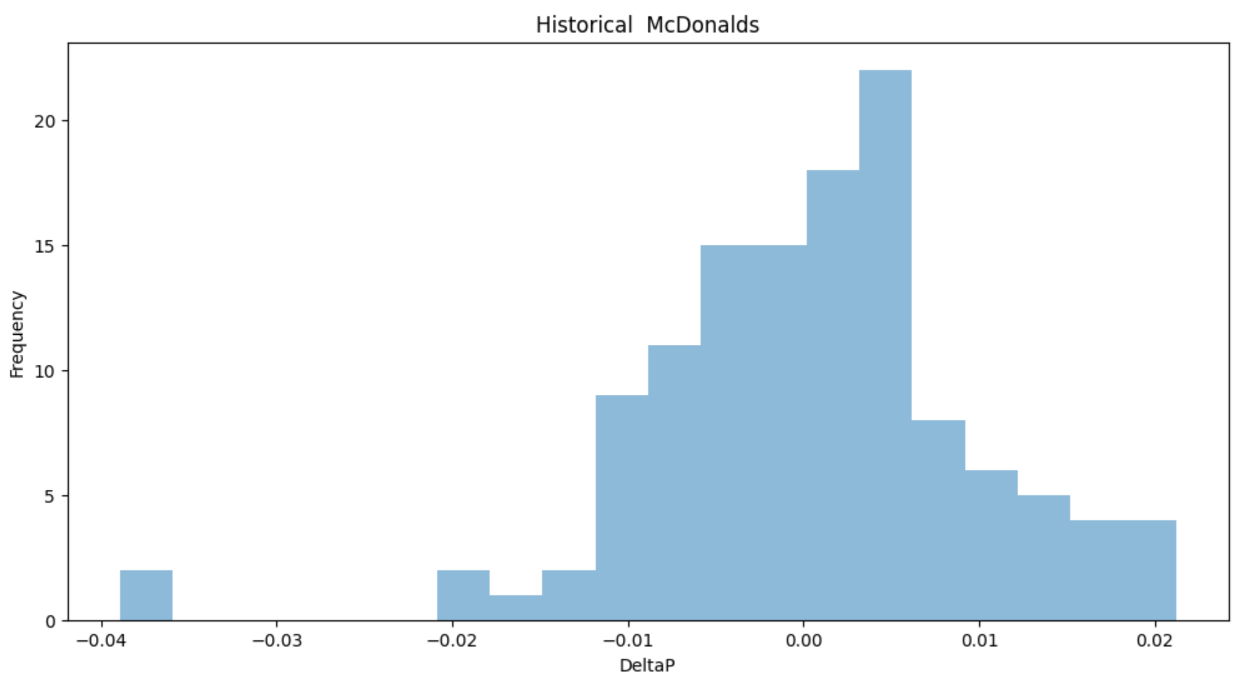
The VaR for different CL are :

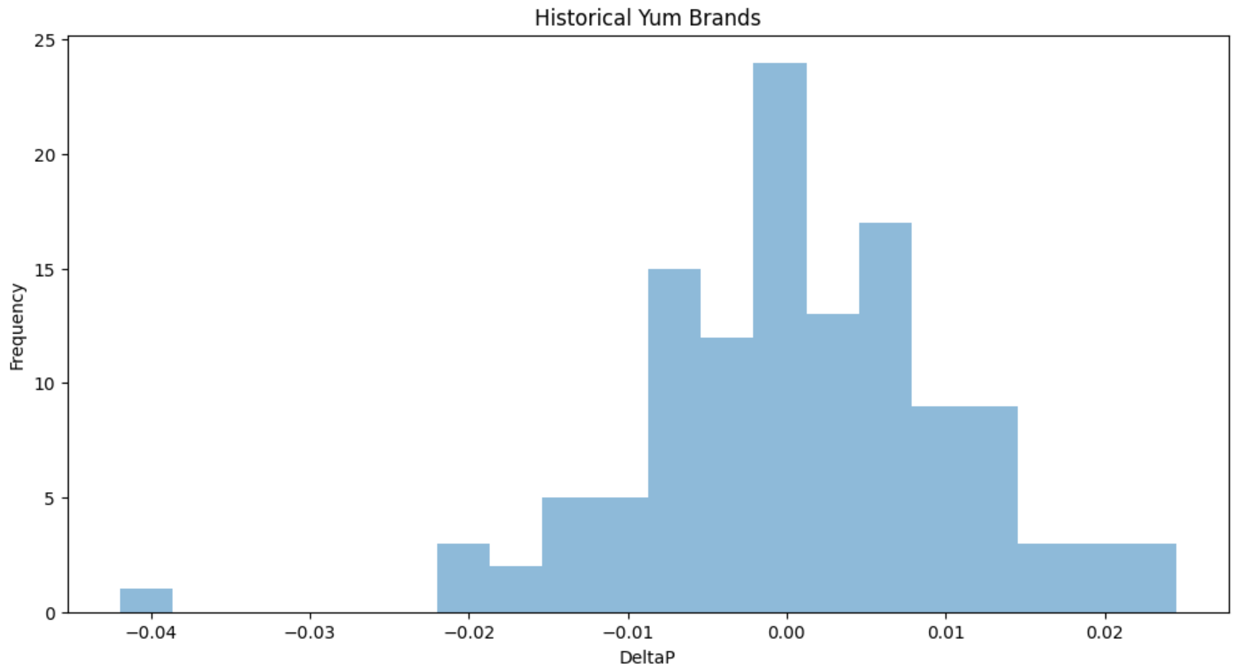
VaR	95%	99%	99.5%
joint	0.028	0.040	0.046
McDonald	0.015	0.022	0.025
Yum Brands	0.017	0.024	0.026

4.4 Historical VaR

As explained in the theoretical section we calculated the historical VaR. The histogram for historical Var are the following


Historical Portfolio
Frequency
DeltaP
0
2
4
6
8
10
12
14
16
−0.06
−0.04
−0.02
0.00
0.02
0.04


Historical McDonalds
Frequency
DeltaP
0
5
10
15
20
−0.04
−0.03
−0.02
−0.01
0.00
0.01
0.02

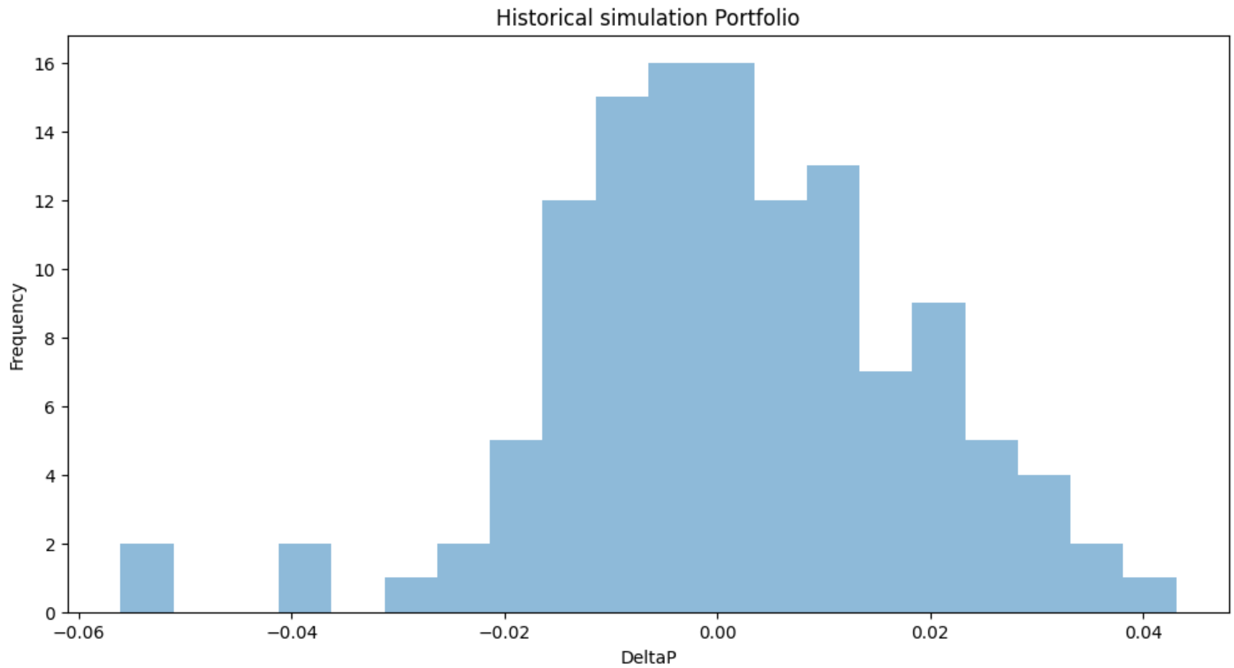


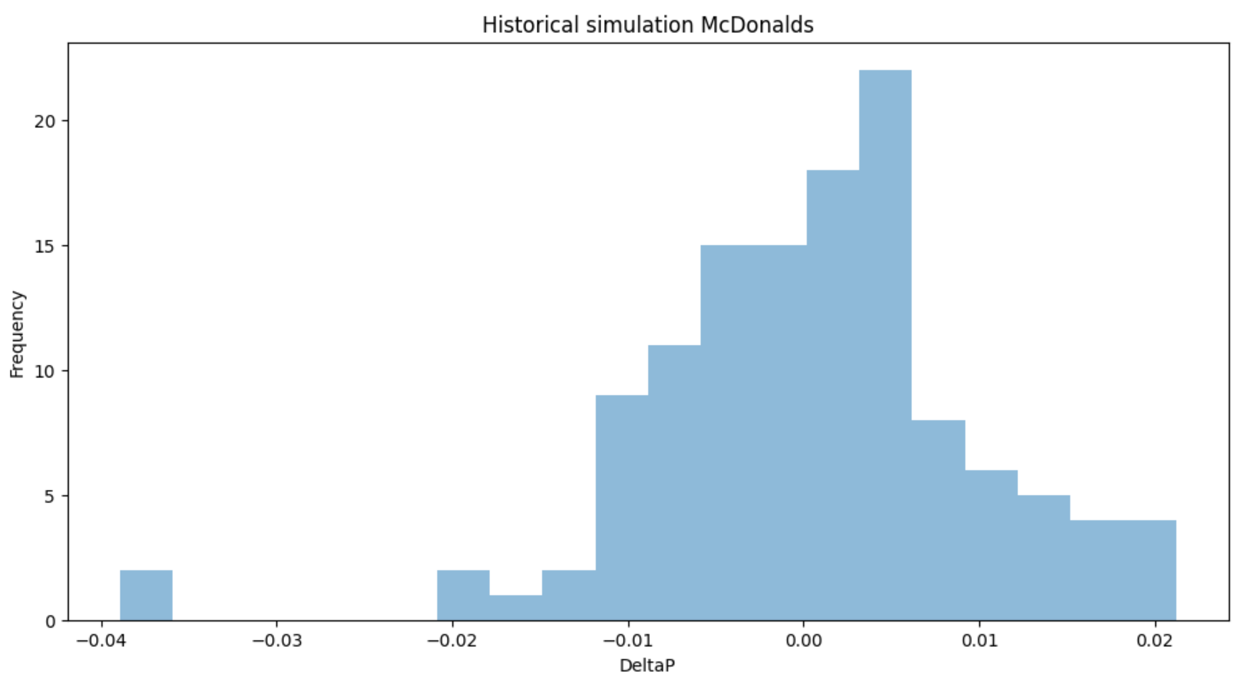
The VaR for different CL are :

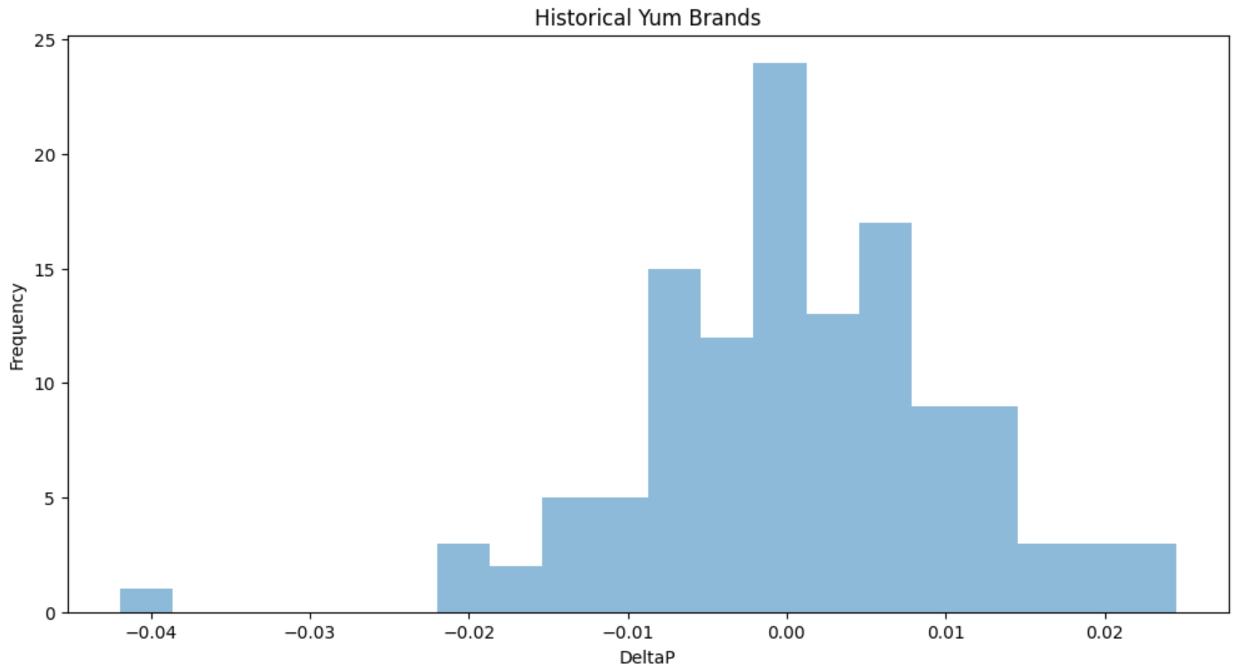
VaR	95%	99%	99.5%
joint	0.028	0.037	0.039
McDonald	0.015	0.020	0.021
Yum Brands	0.017	0.022	0.023

4.5 Historical simulation method

Following the method explained in the previous section, I Compute the empirical cumulative distribution function and use the inverse transform sampling to obtain the VaR. The histograms are shown below:


Historical simulation Portfolio
Frequency
DeltaP
0
2
4
6
8
10
12
14
16
−0.06
−0.04
−0.02
0.00
0.02
0.04


Historical simulation McDonalds
Frequency
DeltaP
0
5
10
15
20
−0.04
−0.03
−0.02
−0.01
0.00
0.01
0.02



The VaR are:

VaR	95%	99%	99.5%
joint	0.028	0.033	0.035
McDonald	0.015	0.018	0.019
Yum Brands	0.021	0.023	0.024

4.6 Additivity

Subadditivity implies that the VaR of a combined portfolio is less than the sum of the VaRs of its constituent parts. However, it's noted that VaR can be non-subadditive in certain instances. This occurs with some distribution such as the normal one. In out study cases the error are shown in the following table:

Additivity errror	95%	99%	99.5%
Monte Carlo VaR	13.41	10.51	10.5
Historical VaR	15.13	10.94	10.9
Historical simul. VaR	20.39	1.24	1.24
Parametric VaR	13.46	71.68	13.46
EWMA VaR	48.15	48.15	48.15

5 Conclution

We calculated at the beginning the average and variance of an equilibrate portfolio. Then we compute the VaR for the two single assets separately and the joint one with different methods: Gaussian model building approach, EWMA, Monte Carlo method, historical VaR, and finally historical simulation. Chosen a CL the VaR that are obtained are very similar, because the market was non-oscillating in an unconventional way. In all cases, there is no additivity as shown by the error in the previous section.