

# Mixed models

November 17, 2021

Answers for this lab must be submitted before **December 1st at 5pm on Moodle**. In your answer for each question, please include a copy of the R code used (if applicable) and the results obtained.

## 1. Wheat yield as a function of fertilizer and soil moisture

The `Wheat` data frame included in the `nlme` package presents the result of an agricultural experiment to measure the effect of fertilizer quantity and soil moisture on the yield of a wheat variety (`DryMatter`, the response variable). The plants were divided into 12 trays. Soil moisture is constant in a tray, but each tray is divided into 4 sections each receiving a different amount of fertilizer. There are therefore 48 individual measurements of the yield, i.e. 4 per tray.

```
library(nlme)
data(Wheat)
head(Wheat)
```

```
## Grouped Data: DryMatter ~ fertilizer | Tray
##   Tray Moisture fertilizer DryMatter
## 1     1         10          2   3.3458
## 2     1         10          4   4.3170
## 3     1         10          6   4.5572
## 4     1         10          8   5.8794
## 5     2         10          2   4.0444
## 6     2         10          4   4.1413
```

- Estimate the parameters of a linear model including the additive effects of fertilizer and moisture on yield. Is it correct to ignore the grouping of the measurements per tray? Justify your answer from a plot of residuals.
- Now fit a mixed model that includes the same fixed effects, as well as a random effect representing the grouped data structure. For each of the two fixed effects (fertilizer and moisture), how does the standard error of the coefficients vary compared with the model without random effects in (a)? Explain these differences in terms of the experimental design.
- From appropriate diagnostic plots, check if the mixed model residuals in (b) show no residual pattern and if they follow a normal distribution.
- Calculate the intra-class correlation for the mixed model in (b). What is the mathematical interpretation of this value?

## 2. Growth curves of spruce trees

The `Spruce` data frame included in the `nlme` package contains data on the growth of 79 spruce trees. The logarithm of the volume (`logSize`) for each spruce (identified by the `Tree` number) was measured at 13 different times from the beginning of the experiment (`days`). The trees are also divided between 4 plots.

```
data(Spruce)
head(Spruce)
```

```
## Grouped Data: logSize ~ days | Tree
##      Tree days logSize plot
## 1 01T01  152    4.51    1
## 2 01T01  174    4.98    1
## 3 01T01  201    5.41    1
## 4 01T01  227    5.90    1
## 5 01T01  258    6.15    1
## 6 01T01  469    6.16    1
```

- (a) Since the growth is not linear in time, we transform the number of days as a factor, which allows the model to independently estimate growth for each point in time.

```
Spruce$days <- as.factor(Spruce$days)
```

Estimate the parameters of a linear model of `logSize` as a function of the number of days, then answer the following questions: (i) How do you interpret the Intercept of that model? (ii) What is the mean variation of `logSize` between day 152 and day 201 and what is its standard error?

- (b) Now fit a mixed model that takes into account that these are repeated measurements on the same trees (*Note:* Ignore the plot for now.) and answer the following questions: (i) What do the standard deviation of the random effect and the residual standard deviation represent, respectively? (ii) Based on these results, why is it beneficial to measure the same trees on each sampling day to estimate a growth curve?
- (c) What is the intra-class correlation for the model in (b)? Based on this result, is the variation in size among trees at any point in the experiment due more to (i) initial differences in size between trees (on day 152) or (ii) the random variation in growth between trees for each time period?
- (d) Add to the model in (b) a random effect for the plot. According to this model, does tree size differ significantly between plots?