Homework 4

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1 Question 1

Describe the buoyancy effect and why it is considered in underwater robotics while it is neglected in aerial robotics.

Buoyancy is a hydrostatic effect that occurs when a rigid body is submerged in a fluid under the effect of gravity. It doesn't depend on the relative motion between the body and the surrounding fluid, but it appear in stationary conditions. In fact, the fluid always exerts a force that opposes the weight of a submerged object. This effect is closely related to the fluid's density: if the fluid's density is lower than that of the object, the robot tends to sink, while in the opposite case it tends to float.
In aerial robotics, the buoyancy effect is negligible because the density of air is extremely lower than that of the robot. Consequently, the buoyancy force is small compared to other forces such as thrust and drag, and it can be ignored without significant consequences.
On the other hand, **in underwater robotics** the density of the robot and that of the water are comparable, so **the buoyancy effect must be considered**.
We can define the Bouyancy as:

$$b = \rho \Delta ||\overline{g}||$$

Where:

- $\overline{\boldsymbol{g}} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T \in \mathbb{R}^3$
- $\Delta \in \mathbb{R}$ is the volume of the robot
- $\rho \in \mathbb{R}$ is the density of the water

Different from the gravitational force, buoyancy does not act in the centre of mass of the robot but in the centre of buoyancy $r_b^b \in \mathbb{R}^3$ and it can be expressed as:

$$f_b^b = -R_b^T = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} = -R_b^T = \begin{bmatrix} 0 \\ 0 \\ \rho \Delta g \end{bmatrix}$$

We can notice that the sign $-$ is due to the fact that we consider the z-axis reference downwards. Indeed, we consider the positive gravitational force, so the buoyancy force will be have a sign $-$.
The buoyancy force acts at the center of buoyancy and not at the robot's center of mass. Often, these two centers do not coincide, which can create a torque that causes the body to rotate. This creates control problems for the robot, so it is preferable to align the two central vertical axes to avoid the generation of torque.

2 Question 2

Briefly justify whether the following expressions are true or false.

- The added mass effect considers an additional load to the structure. **False**

  To understand why this statement is false, it is necessary to describe this effect. The added mass and inertia effect is an effect that can be considered in the underwater robotic. When the robot is moving in a fluid, it modify the velocity of the fluid particles. Indeed, it creates an acceleration to the additional inertia of the fluid surrounding the robot. Due to this virtual effect the fluid apply a reaction force which is equal in magnitude and opposite in direction (added mass contribution). In fact, in order to move, the body must consider apply a greater force as if it had to react to a greater mass. It is easy to conclude that the robot does not actually have an additional mass because under static conditions this effect is not present.

- The added mass effect is considered in underwater robotics since the density of the underwater **True**

  The added mass effect depends on the density of the fluid because a robot accelerates a mass of fluid during motion. Although this effect is also present in drones, legged robots and wheeled robots, it is neglected because the density of air is extremely lower. This effect depends also on the body geometry of the robot because it determines the quantity of fluid accelerated during motion.

- The damping effect helps in the stability analysis.**True**

  In general, the viscosity of the fluid, can be causes the presence of damping effect that include drag and lift force acting on the robot. The drag force is parallel to the relative velocity between the body and the fluid, while the lift force is orthogonal to it. To semplificate the problem we can consider only quadratic damping terms and group them in a matrix (6x6) positive definite with constant elements, often denoted as D_{RB}.
  The damping effect is beneficial for stability because it acts as a disturbance opposing the motion of the underwater robot. This opposition helps in controlling the movement and contributes positively to the stability of the system.

  Specifically, in the context of Lyapunov stability analysis, the damping terms help make the derivative of the Lyapunov function more negative, which accelerates the convergence of the system's state to the desired equilibrium state. This is clearly outlined in the following expression of a Mixed Earth/Vehicle-fixed-frame-based model-based controller (Underwater Robots, G. Antonelli, p.72):

  $$\dot{V} = -s_v^T(K_D + D_{RB})s_v - k_p\lambda_p\tilde{p}_b^T\tilde{p}_b - k_o\epsilon^T\epsilon \leq 0$$

  In this expression, the term D_{RB} contributes to making $\dot{V}$ (the time derivative of the Lyapunov function) negative definite, thus ensuring the stability of the system.

- The Ocean current is usually considered as constant, and it is better to refer it with respect to the body frame.**False**

  It is important to remember that in the underwater robotics, it is common to assume that the ocean current is constant and irrotational to simplify control design. This allows for easier management of disturbances affecting the UUV's movement. To ensure that the ocean current can be considered constant and irrotational, it should be referred to the world frame.

$$\mathbf{v}_c = \begin{bmatrix} v_{c,x} \\ v_{c,y} \\ v_{c,z} \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^6, \quad \dot{\mathbf{v}}_c = \mathbf{0}_6 \tag{1}$$

On the other hand, if the ocean current is referred to the body frame of the UUV, it becomes time-varying due to the vehicle's movement through the water. This variability complicates the derivation of a control law for the UUV, as the disturbance is no longer constant (but time-varying and more difficult to control). In the dynamic model of a UUV, the effect of the ocean current can be incorporated by considering the relative velocity in the body-fixed frame. This involves calculating the relative velocity ν_r as follows:

$$\nu_r = \begin{bmatrix} \dot{p}_b \\ \omega_b \end{bmatrix} - R_b^T \nu_c$$

where ν_c is the ocean current velocity in the world frame, $\dot{p}_b$ is the translational velocity, ω_b is the angular velocity, and R_b is the rotation matrix.

Having observed that referring the ocean current in the body frame introduces time variability, complicating the control design, it is possible to conclude that the statement is false.

3 Question 3

Consider the Matlab files within the quadruped_simulation.zip file. Within this folder, the main file to run is MAIN.m. The code generates an animation and plots showing the robot's position, velocity, and z-component of the ground reaction forces. In this main file, there is a flag to allow video recording (flag_movie) that you can attach as an external reference or in the zip file you will submit. You must: a. implement the quadratic function using the QP solver qpSWIFT, located within the folder (refer to the instructions starting from line 68 in the file MAIN.m); b. modify parameters in the main file, such as the gait and desired velocity, or adjust some physical parameters in get_params.m, such as the friction coefficient and mass of the robot. Execute the simulation and present the plots you find most interesting: you should analyze them to see how they change with different gaits and parameters and comment on them.

We can use a control scheme called "whole-body control" to manage all types of legged robots. This scheme aims to achieve the desired values q_f^* and f_{gr}^* derived from a quadratic problem. The quadratic problem optimizes the tracking of the desired wrench for the center of mass (CoM) and a desired trajectory for the swing legs, subject to constraints such as dynamic consistency, non-sliding contact, torque limits, and swing leg tasks. In practice, this quadratic problem is represented by the function "qpSWIFT," which takes matrices representing constraints and objectives as input to solve the constrained optimization problem.

Quadratic Problem :

$$\min_{\zeta} f(\zeta)$$

subject to:

$$A\zeta = b,$$

$$D\zeta \leq c.$$

In the following, the different simulations performed will be shown and compared:

Trot gait

The **trot gait** consists of the diagonal movement of the legs (the right front leg moves with the left hind leg and vice versa). This gait is often employed when the robot needs to move on uncertain terrains while maintaining rapid movement, due to the fact that it always has two feet on the ground during movement. Using the parameters ($m = 5.5\,kg \quad \mu = 1 \quad v_{max} = 0.5\,m/s$), it can be observed that the robot correctly follows the position reference, achieving cyclic movement as can be seen from the velocity graph. The ground reaction forces along the z-axis are not too high with a maximum peak of 30 N.

Changing mass

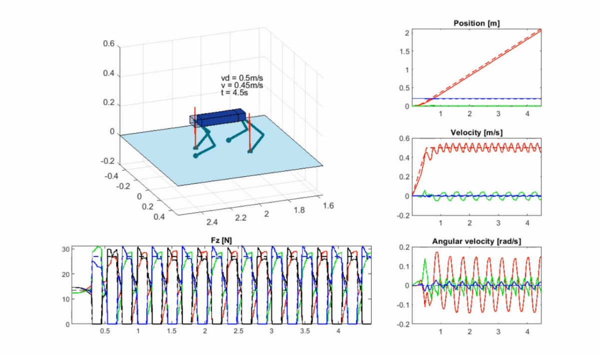
It has been noted that increasing the mass of the robot causes the ground reaction forces to increase considerably, reaching peaks of around $60\,N$ for a mass of $10\,kg$, but the gait behavior remains almost the same. Conversely, by decreasing the mass ($2\,kg$), it can be noted that the slipping effect decreases (the ground reaction forces decrease to about $10\,N$) but the instability parameters do not improve.

Changing speed

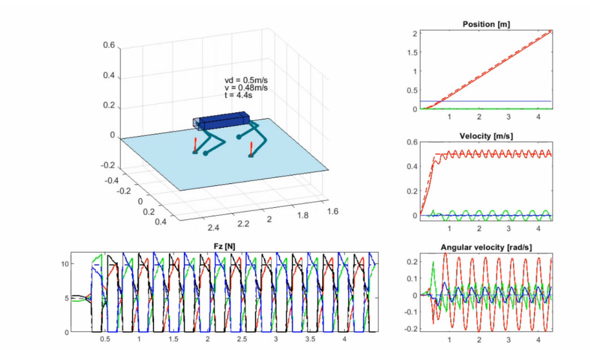
Increasing the speed, the position error starts to become greater, because the gait becomes more dynamic and therefore it is more difficult to follow the position and velocity references while also maintaining body balance.

Changing friction's coefficient

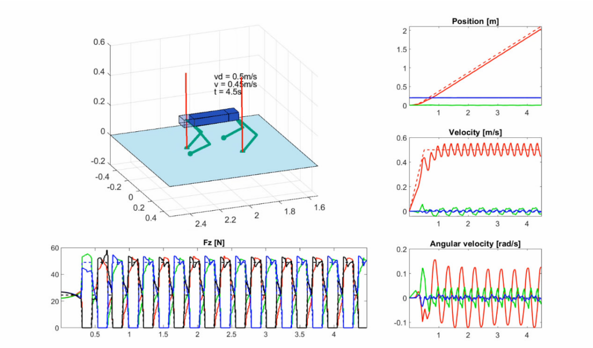
Reducing the coefficient of friction results in a greater loss of reference because the robot have difficult to mantain the equilibrium



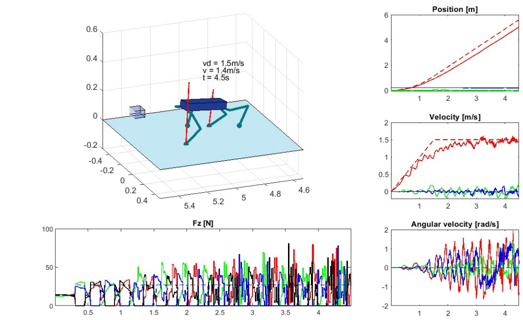
(a) Trot $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



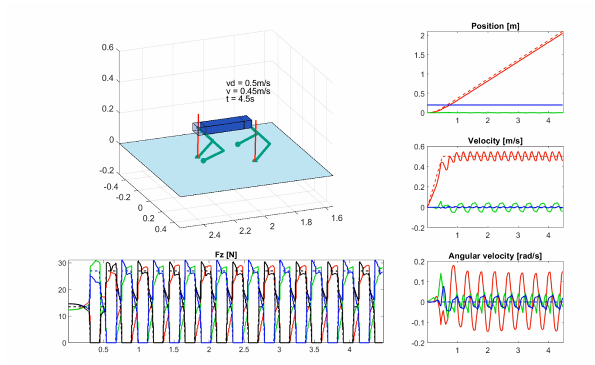
(b) Trot $m = 2Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(c) Trot $m = 10Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(d) Trot $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 1.5\,\mathrm{m/s}$



(e) Trot $m = 5.5Kg \quad \mu = 0.5 \quad v_{max} = 0.5\,\mathrm{m/s}$

Figure 1: Comparison of different trot parameters

Bound Gait

The Bound Gait involves the robot moving both front legs simultaneously followed by both hind legs. This gait, like the gallop, is much more dynamic and is used to achieve higher speeds or overcome obstacles. Unlike the trot, this gait is less stable, as can be observed in simulations with unchanged parameters ($m = 5.5\,kg \quad \mu = 1 \quad v_{max} = 0.5\,m/s$), where both angular velocity and linear velocity oscillations are greater. Additionally, the ground reaction forces are significantly higher compared to the trot, reaching about $100\,N$. Also we can observed that the position error is greater than the error that we can see during the simulation of trot gait.

Changing mass

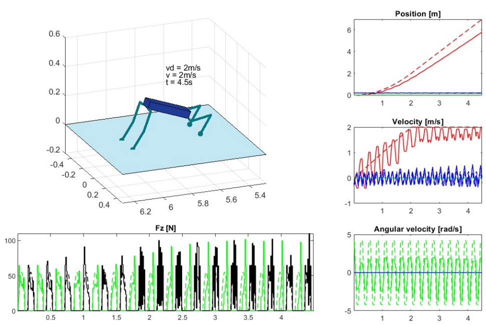
Reducing the mass decreases the ground reaction forces to $40\,N$, but does not improve stability. While Increasing the mass to around $10\,kg$ completely compromises system stability.

Changing speed

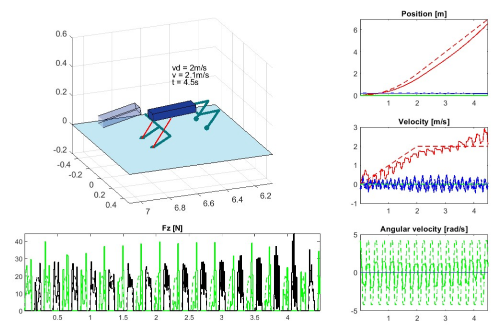
By increasing speed, the robot does not have much difficulty maintaining its trajectory, the results are similar to those obtained with the standard parameters with an increase in reaction force and a slight increase in position error. Compared to other gaits, it manages to maintain fairly high speeds before compromising its trajectory.

Changing friction's coefficient

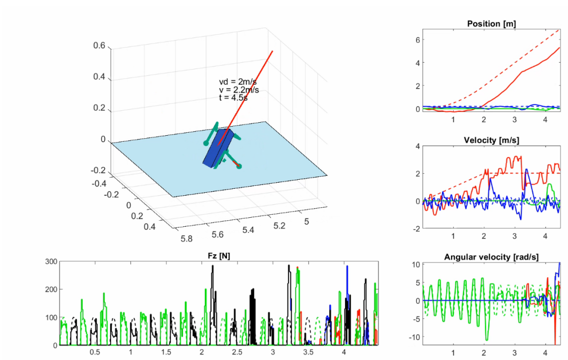
Reducing the coefficient of friction leads to significant instability. The robot struggles to maintain effective ground contact, resulting in peaks in reaction forces that indicate slipping problems and a greater loss of balance.



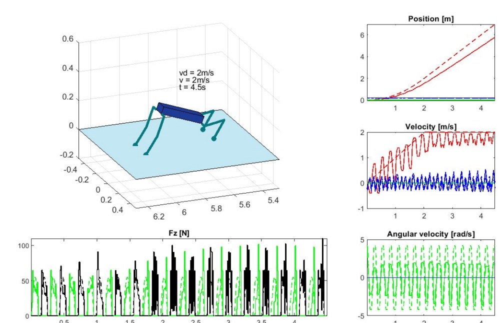
(a) Bound $m = 5.5Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



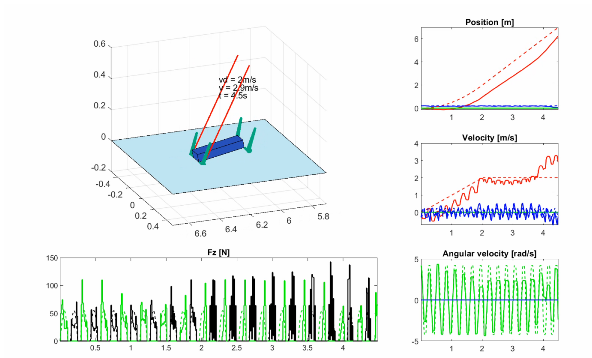
(b) Bound $m = 2Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



(c) Bound $m = 10Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



(d) Bound $m = 5.5Kg$ $\mu = 1$ $v_{max} = 1.5\,\text{m/s}$



(e) Bound $m = 5.5Kg$ $\mu = 0.5$ $v_{max} = 0.5\,\text{m/s}$

Figure 2: Comparison of different bound parameters

Pacing Gait

The pacing gait involves the simultaneous movement of the robot's lateral legs. Unlike the previous gait, this one is more stable ($m = 5.5$ kg, $\mu = 1$, $v_{\max} = 0.5$ m/s) and shows lower ground reaction forces below 40 N with almost constant error in the tracking of position.

Changing mass

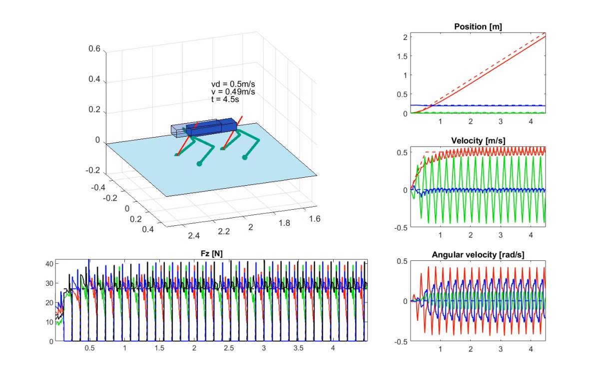
In this case, reducing the mass improves position tracking by decreasing inertia forces, making the robot's movement more precise. As can be seen, the ground reaction forces are more manageable, resulting in 15 N. And also the can see an improvement in the stability. However, increasing the mass to 10 kg leads to a deterioration in position error performance and a considerable decrease in stability.

Changing speed

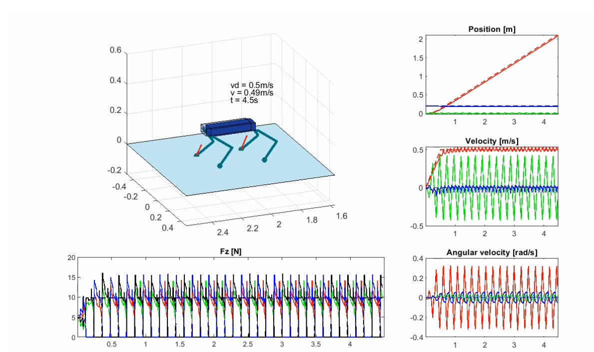
Increasing the speed results in greater tracking error. The robot have a reduction of stability. Additionally, the reaction forces become extremely impulsive as the result to react and stabilize the robot.

Changing friction's coefficient

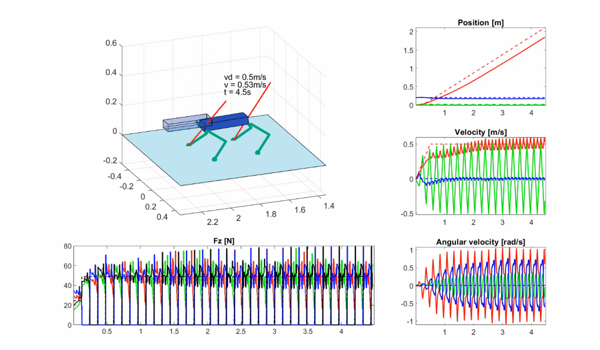
Decreasing the friction coefficient $\mu = 0.5$ makes it difficult to maintain position, causing slips and loss of control. This is evident from the image where the robot is shown to be completely overturned.



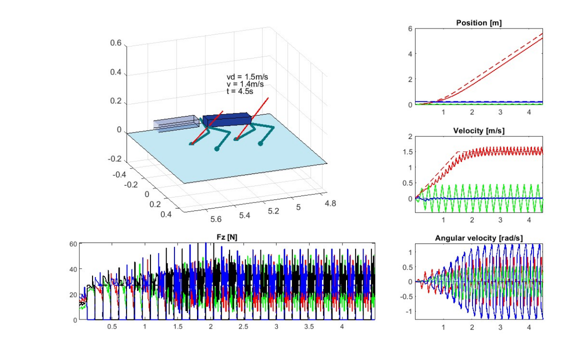
(a) Pacing $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



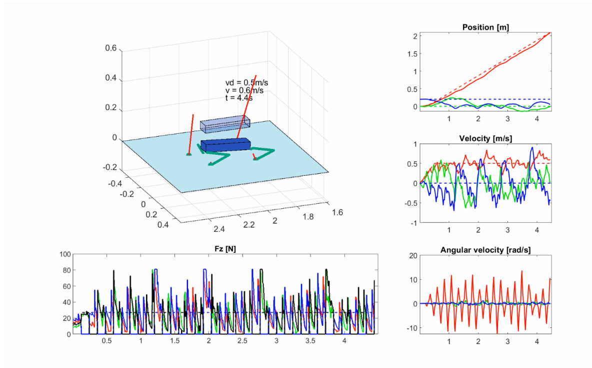
(b) Pacing $m = 2Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(c) Pacing $m = 10Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(d) Pacing $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 1.5\,\mathrm{m/s}$



(a) Pacing $m = 5.5Kg \quad \mu = 0.5 \quad v_{max} = 0.5\,\mathrm{m/s}$

Figure 4: Comparison of different pacing parameters

Gallop gait

The gallop gait is the fastest gait, typical of animals, but it has limited stability since at some moments only one leg is in contact with the ground. This makes it difficult for the robot to maintain a precise trajectory, especially in low-friction conditions and at high speeds.
It can be observed from the standard parameters that there is greater instability and an higher position error.

Changing mass

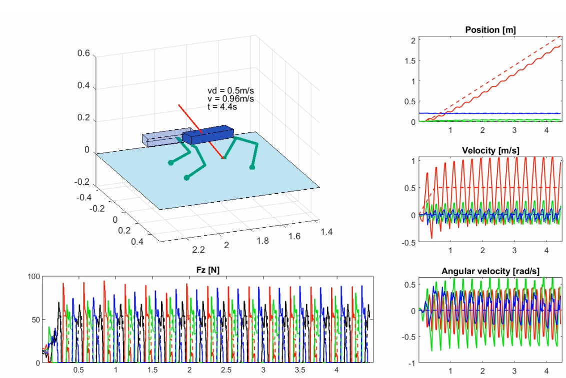
Reducing the mass slightly improves stability but is not enough to eliminate the issues completely.

Changing speed

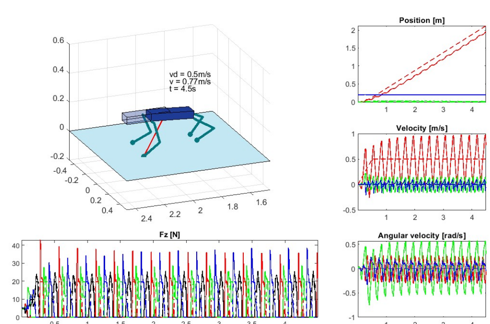
An increase in speed causes the gallop gait to degenerate. In fact, the oscillations in the robot's speed become more pronounced, making it difficult to maintain a stable trajectory. In the following figures, it can be seen that at high speeds, the gallop gait becomes unstable and exhibits unrealistic behaviors.

Changing friction's coefficient

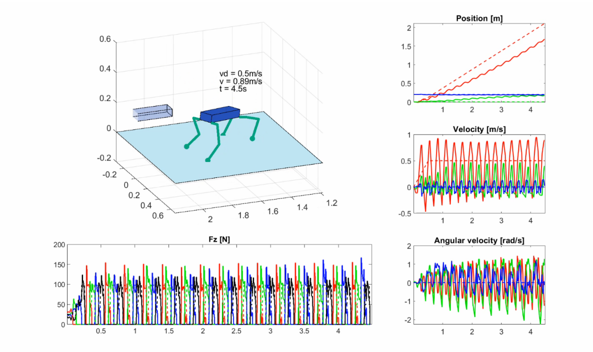
Finally, a decrease in the friction coefficient causes instability and oscillations. The robot finds it more challenging to maintain balance, leading to frequent slips and falls. An increase in position errors and instability is observed with the lowering of the friction coefficient.



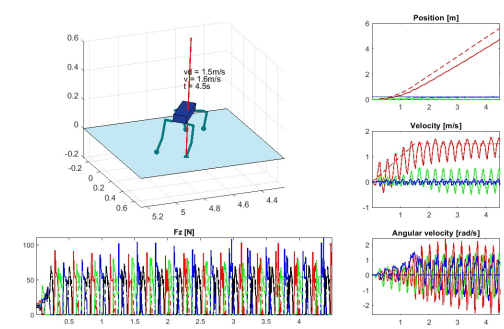
(a) Gallop $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



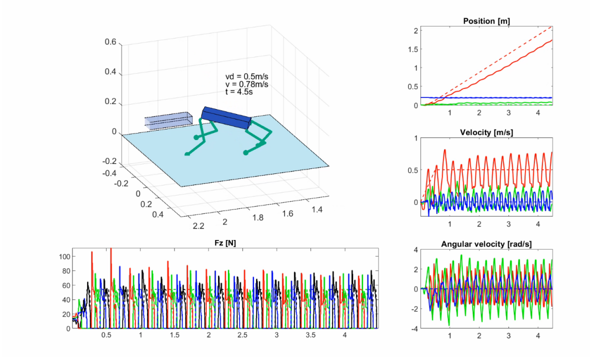
(b) Gallop $m = 2Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(a) Gallop $m = 10Kg \quad \mu = 1 \quad v_{max} = 0.5\,\text{m/s}$



(b) Gallop $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 1.5\,\text{m/s}$



Gallop $m = 5.5Kg \quad \mu = 0.5 \quad v_{max} = 0.5\,\text{m/s}$

Figure 6: Comparison of different gallop parameters

Trot-run

The trot-run gait is similar to the trot but with shorter flight phases. This means that, compared to trotting, the robot's legs remain off the ground for a shorter duration. This gait is less stable than trotting but more dynamic, with a higher risk of losing balance due to the brief flight phases.
It is immediately noticeable in simulations with standard values that the ground reaction forces peak at about 60 N, roughly double that of trotting, indicating greater mechanical stress during movement.
The speed reference is quickly reached with less oscillatory behavior, and the position error is quite reduced.

Changing mass

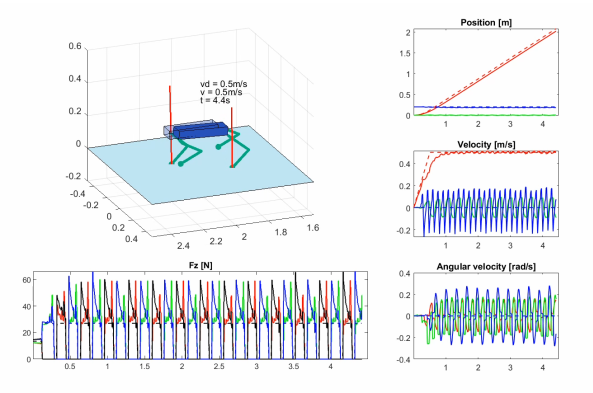
Reducing the mass improves position tracking. However, this is not sufficient to completely prevent instability. It can be observed that a reduced mass slightly improves stability but does not eliminate the instability issues.

Changing speed

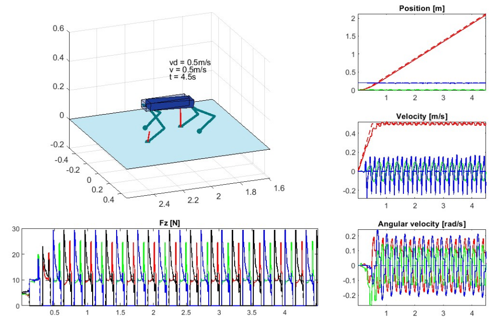
At high speeds, tracking errors become significant. Simulations show that the robot tends to deviate from the desired trajectory at higher speeds, indicating reduced stability.

Changing friction's coefficient

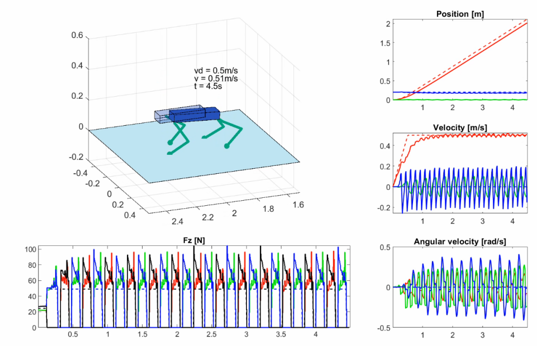
Decreasing the friction coefficient further increases instability and tracking error.



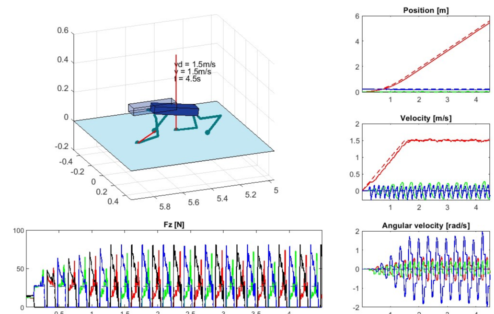
(a) Trot-run $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



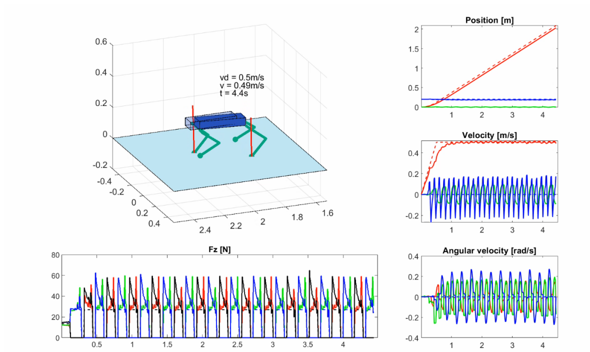
(b) Trot-run $m = 2Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(c) Trot-run $m = 10Kg \quad \mu = 1 \quad v_{max} = 0.5\,\mathrm{m/s}$



(d) Trot-run $m = 5.5Kg \quad \mu = 1 \quad v_{max} = 1.5\,\mathrm{m/s}$



(e) Trot-run $m = 5.5Kg \quad \mu = 0.5 \quad v_{max} = 0.5\,\mathrm{m/s}$

Figure 7: Comparison of different trot-run parameters

Crawl Gait

The Crawl gait is more stable as it always has three legs in contact with the ground, moving only one leg at a time. This type of movement is the slowest among the analyzed gaits.
Even from the first simulation, it is evident that the Crawl gait has a lower profile, making it suitable for slow and controlled movements. Despite its slower pace, it is significantly more stable compared to other gaits, with reaction forces peaking at 45N. Additionally, the position errors are extremely minimal.

Changing mass

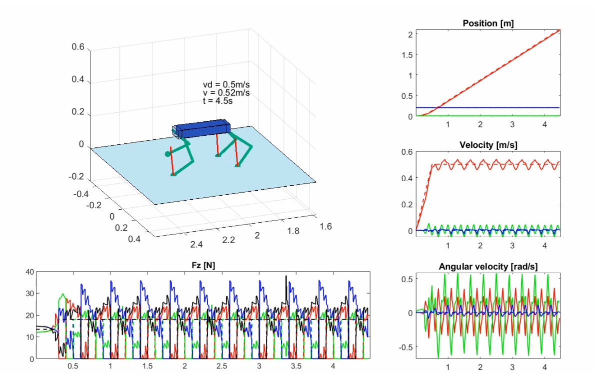
Reducing the mass further improves the robot's stability. It is observed that a lower mass leads to better position control.

Changing speed

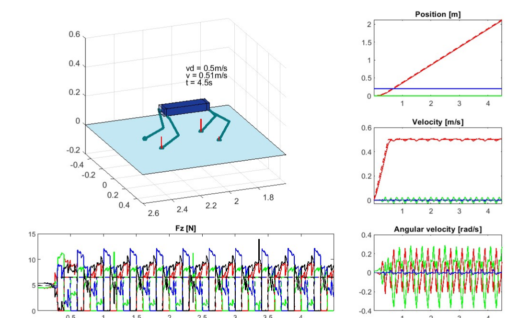
Increasing the robot's speed can lead to a more dynamic gait, reducing the inherent stability of the Crawl gait. As seen in the images, it maintains a certain level of stability only at low speeds.

Changing friction's coefficient

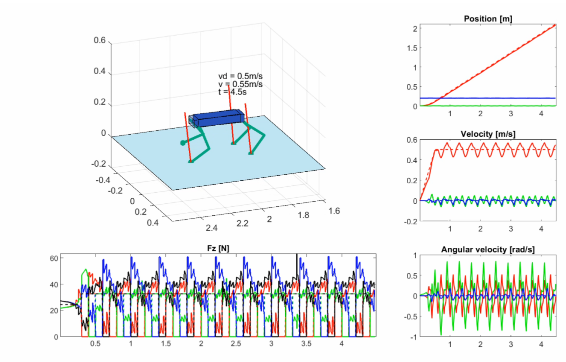
Reducing the friction coefficient results in less accurate position tracking, leading to control response delays, slight oscillations in angular velocities, and a loss of balance.



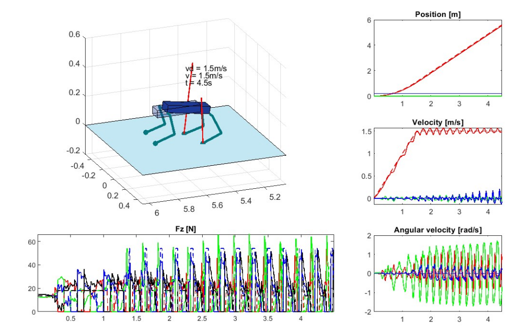
(a) Crawl $m = 5.5Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



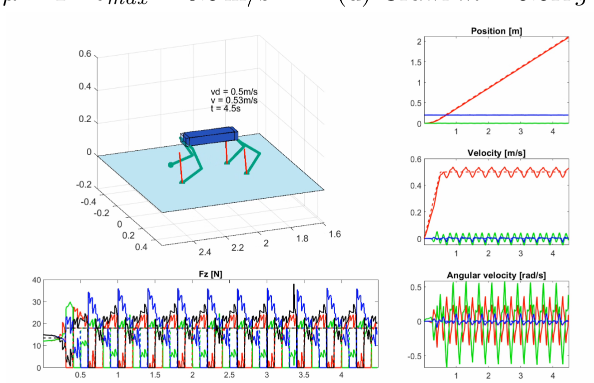
(b) Crawl $m = 2Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



(c) Crawl $m = 10Kg$ $\mu = 1$ $v_{max} = 0.5\,\text{m/s}$



(d) Crawl $m = 5.5Kg$ $\mu = 1$ $v_{max} = 1.5\,\text{m/s}$



(e) Crawl $m = 5.5Kg$ $\mu = 0.5$ $v_{max} = 0.5\,\text{m/s}$

Figure 8: Comparison of different Crawl parameters

Conclusion

Quadruped robot gaits vary significantly in terms of speed, stability, and ground reaction forces. It has been observed that the most dynamic gaits are the bound and gallop. These gaits allow the robot to reach very high speeds but offer limited stability and are prone to instability, especially on surfaces with low friction. In contrast, the crawl and trot gaits are extremely stable and perform well at low speeds.

The trot and trot-run gaits provide a balance between speed and stability, with the trot-run enabling higher speeds due to shorter flight phases, though with reduced stability compared to the trot. The pacing gait, while less dynamic, offers intermediate stability between the trot and crawl, with moderate speeds and ground reaction forces. It has been noted that reducing the friction coefficient does not compromise the performance of the crawl and trot gaits, whereas the bound gait faces some difficulties, leading to failures in the pacing and gallop gaits.
The choice of gait depends on operational requirements, terrain characteristics, and the need for speed and stability. Dynamic gaits are ideal for rapid movements on surfaces with good friction, while gaits like trot and crawl are preferable for controlled and stable movements on challenging terrains.

4 Question 4

Consider a legged robot as in the picture below. The foot and the leg are assumed to be massless. The point T represents the toe, the point H represents the heel, and the point P is the ankle. The value of the angle θ is positive counterclockwise and it is zero when aligned to the flat floor. Answer to the following questions by providing a brief explanation for your them.

- Without an actuator at the point P is the system stable at $\theta = \dfrac{\pi}{2} + \epsilon$?

  The robot shows in the picture before, without actuators, can be viewed as an inverse pendulum with an unstable equilibrium point at $\theta = \dfrac{\pi}{2}$. If we start in this position without any perturbation, the leg will maintain this position indefinitely because there is no torque acting on the pendulum. However, the system without an actuator at point P is not stable at $\theta = \dfrac{\pi}{2} + \epsilon$. The horizontal acceleration of the center of mass (CoM) results from a force pushing it away from the center of pressure (CoP), leading to an intrinsically unstable dynamic. Without actuation to correct for deviations, the robot cannot counteract the torque created by the force of gravity. Thus, if a small perturbation occurs, the gravitational torque will not be compensated, causing the system to move away from the equilibrium point. Therefore, the system is not stable in the $\theta = \dfrac{\pi}{2} + \epsilon$ configuration.

- Without an actuator at the point P (i.e.,$\ddot{\theta} \neq 0, \dot{\theta} \neq 0$), compute the zero-moment point on the ground as a function of θ and the geometric and constant parameters (if necessary).

  The Zero Moment Point (ZMP) coincide with the Center of Pressure (CoP). This alignment implies that the horizontal moments of the ground reaction forces relative to the CoP are zero. Considering that $\ddot{\theta} \neq 0, \dot{\theta} \neq 0$ and remembering that the location of the ZMP is function of the motion of th centre of mass p_c. We can compute the ZMP positionp_{xz} as:

$$p_z^{x,y} = p_c^{x,y} - \frac{p_c^z}{\ddot{p}_c^z - g_0 z}(\ddot{p}_c^{x,y} - g_0 x, y) + \frac{1}{m(\ddot{p}_c^z - g_0^z)}\dot{L}^{x,y}$$

  where we need to determine p_{xc}, p_{zc}, $\ddot{p}_{xc}$, and $\ddot{p}_{zc}$.

  In this case, the only mass is m, so obviously, the Center of Mass (CoM) of the robotic system in the z-x plane is simply the position of m in the plane given by the coordinates p_{xc} and p_{zc}. By computing twice the time derivative of these quantities, we can also determine $\ddot{p}_{xc}$ and $\ddot{p}_{zc}$.

$$p_c^x = l\cos(\theta), \quad \ddot{p}_c^x = -(l\sin(\theta)\dot{\theta} + l\cos(\theta)\dot{\theta}^2)$$

$$p_c^z = h + l\sin(\theta), \quad \ddot{p}_c^z = -l\sin(\theta)\dot{\theta}^2 + l\cos(\theta)\ddot{\theta}$$
$$\dot{L}^y = ml^2\ddot{\theta}$$
$$g_0^x = 0, \quad g_0^z = -g$$

We can now compute the ZMP in terms of θ and the given geometric and constant parameters:

$$p_z^x = l\cos(\theta) - \frac{h + l\sin(\theta)}{-l\sin(\theta)\dot{\theta}^2 + l\cos(\theta)\ddot{\theta} + g}\left(-(l\sin(\theta)\dot{\theta} + l\cos(\theta)\dot{\theta}^2)\right) + \frac{l^2\ddot{\theta}}{-l\cos(\theta)\ddot{\theta} + l\sin(\theta)\dot{\theta}^2 + g}$$

This expression provides the ZMP on the ground as a function of θ and the given geometric and constant parameters, fulfilling the condition of no actuator at point P ($\ddot{\theta} \neq 0, \dot{\theta} \neq 0$).

- Supposing to have an actuator at the ankle capable of perfectly cancelling the torque around P due to the gravity (i.e.,$\ddot{\theta} = 0, \dot{\theta} = 0$), what value of θ can you achieve without falling?

  In the static case ($\dot{\theta} = 0$ and $\ddot{\theta} = 0$), the robot falls when the projection of the Center of Mass (CoM) on the ground is external to the support polygon. In this treated case, the projection of the CoM is given by $p_c^x = l\cos(\theta)$.

  Considering the support polygon identified by the foot (which, in this case is the $\overline{HT}$ segment, we have to impose the following condition:

$$-L_2 \leq l\cos(\theta) \leq L_1$$

  The range of values of θ that the robot can reach without falling is regulated by the following inequalities:

$$\theta \geq \cos^{-1}\left(\frac{L_1}{l}\right) \tag{2}$$
$$\theta \leq \cos^{-1}\left(-\frac{L_2}{l}\right) \tag{3}$$