# Social Network Analysis

2/04/2019

Name _____
Student ID _____

**Note**: Whenever an exercise requires the application of a known formula both the formula and its solution must be reported and discussed.

**Exercise 1: Graph Modelling** [4 points]
Given the matrix $\mathcal{G}$

$$\mathcal{G} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- Draw $\mathcal{G}$;
- Synthetically characterize the graph $\mathcal{G}$ describes (directedness, number of nodes/edges, density, components, max/min/avg degrees. . . ).

**Exercise 2: Projections**[4 Points]
Given the bipartite graph $\mathcal{B}$ in Fig. 1(left) compute:

- The adjacency matrixes of its projections (over the two node classes) and of the bipartite graph;
- The node having the lowest Closeness Centrality in both projections;
- The node having the highest Betweeness Centrality in both projections.

**Exercise 3: Synthetic graphs** [4 points]
Let $\mathcal{E}$ be an Erdos-Renyi graph having 3521 nodes and 16320 edges:

- What is the value of $p$ that allows to generate $\mathcal{E}$?
- What will be the average degree of $\mathcal{E}$ nodes? and the graph density?
- Describe the regime of $\mathcal{E}$.

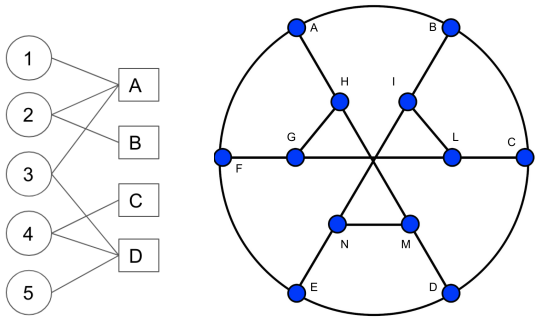


Figure 1: (left) the Bipartite graph. (rigth) The Franklin graph: named after Philip Franklin, who disproved the Heawood conjecture on the number of colors needed when a two-dimensional surface is partitioned into cells by a graph embedding.

**Exercise 4: Paths** [7 points]
Given the *Franklin graph* $\mathcal{G}$ shown in Fig. 1(right):

- Compute its diameter;
- List all the shortest paths among the pairs [F,L], [B,E], [C,H], [B,L];
- Compute the edge betweenness of (G,L), (A,F) and (I,N);
- Is it possible to identify an Hamiltonian cycle on $\mathcal{G}$? If not specify the minimum set of edges to needed to build it.

**Exercise 5: Indicators** [7 points]
Given *Franklin graph* $\mathcal{G}$ shown in Fig. 1(right) compute:

- Degree Centrality of all nodes;
- Closeness Centrality of A, G;
- Betweenness Centrality of I, E;
- Local Clustering Coefficient of F, L.

**Exercise 6: Graph Construction** [6 points]
Given 10 nodes and, at most, 20 edges build a graph such that all the following conditions hold:

- The graph is composed by a single components;
- There exists a path of length 5 between nodes 1 and 2;
- Node 2 has a clustering coefficient of $\frac{1}{3}$;
- The shortest path among 5 and 1 is equal to 2;
- Node 6 has the highest Degree Centrality;
- Node 7 has the lowest Closeness Centrality.