# ARS: Analisi Reti Sociali

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Name ______________________________
Student ID ______________________________

**Note**: Whenever an exercise requires the application of a formula/algorithm both the methodology and the solution must be *reported* and *discussed*.

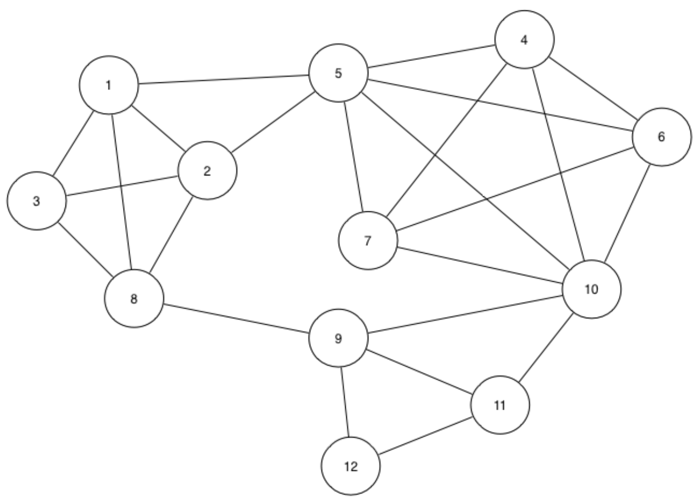


Figure 1:

**Exercise 1: Community Evaluation** [6 points]
Given the Graph $\mathcal{G}$ shown in Figure 1 compare the following partitions:

**P1** = [(1, 2, 3, 5), (4, 6, 7, 10), (8, 9, 11, 12)]

**P2** = [(2, 3, 8), (1, 5, 4, 7), (6, 9, 10, 11, 12)]

**P3** = [(1, 2, 3, 8), (4, 5, 6, 7, 10), (9, 11, 12)]

Which partition is the best with respect to the *modularity* score? and for *conductance*?

**Solution.**

Modularity:

**P1:** $wP1 = min(mod(C1), mod(C2), mod(C3)) = mod(C3)$

- mod(C1) = $(\frac{4}{24} - \frac{17}{2*24})^2 \simeq 0{,}035$
- mod(C2) = $(\frac{4}{24} - \frac{18}{2*24})^2 \simeq 0{,}043$
- mod(C3) = $(\frac{4}{24} - \frac{13}{2*24})^2 \simeq \mathbf{0{,}011}$

**P2:** $wP2 = min(mod(C1), mod(C2), mod(C3)) = mod(C1)$

- mod(C1) = $(\frac{3}{24} - \frac{11}{2*24})^2 \simeq \mathbf{0{,}011}$
- mod(C2) = $(\frac{4}{24} - \frac{18}{2*24})^2 \simeq 0{,}043$
- mod(C3) = $(\frac{5}{24} - \frac{19}{2*24})^2 \simeq 0{,}035$

**P3:** $wP3 = min(mod(C1), mod(C2), mod(C3)) = mod(C3)$

- mod(C1) = $(\frac{4}{24} - \frac{15}{2*24})^2 \simeq 0{,}021$
- mod(C2) = $(\frac{5}{24} - \frac{24}{2*24})^2 \simeq 0{,}085$
- mod(C3) = $(\frac{3}{24} - \frac{9}{2*24})^2 \simeq \mathbf{0{,}003}$

Best partitions w.r.t. modularity: P1, P2
$max(wP1, wP2, wP3) = wP1 = wP2$

Conductance:

**P1:** $wP1 = max(cond(C1), cond(C2), cond(C3)) = cond(C1)$

- cond(C1) = $\frac{2*7}{2*5+7} \simeq \mathbf{0{,}823}$
- cond(C2) = $\frac{2*6}{2*8+6} \simeq 0{,}545$
- cond(C3) = $\frac{2*5}{2*4+5} \simeq 0{,}769$

**P2:** $wP2 = max(cond(C1), cond(C2), cond(C3)) = cond(C2)$

- cond(C1) = $\frac{2*5}{2*3+5} \simeq 0{,}909$
- cond(C2) = $\frac{2*10}{2*4+10} \simeq \mathbf{1{,}111}$
- cond(C3) = $\frac{2*7}{2*6+7} \simeq 0{,}736$

**P3:** $wP3 = max(cond(C1), cond(C2), cond(C3)) = cond(C3)$

- cond(C1) = $\frac{2*3}{2*6+3} \simeq 0{,}4$
- cond(C2) = $\frac{2*4}{2*10+4} \simeq 0{,}333$
- cond(C3) = $\frac{2*3}{2*3+3} \simeq \mathbf{0{,}666}$

Best partition w.r.t. conductance: P3
$min(wP1, wP2, wP3) = wP3$

**Exercise 2: Community Discovery** [6 points]
Given the Graph $\mathcal{G}$ shown in Figure 1 compute its communities applying k-clique for k=3,4,5.

**Solution.** The communities identified by k-clique are:

**k=3** : {1, 2, 3, 5, 8}, {4, 5, 6, 7, 10}, {9, 10, 11, 12}

**k=4** : {1, 2, 3, 8}, {4, 5, 6, 7, 10}

**k=5** : {4, 5, 6, 7, 10}

**Exercise 3: Threshold Model** [8 points]
Given the graph $\mathcal{G}$ shown in Figure 1 apply the Threshold model considering the following two scenarios:

**S1**
- Set of initial infected nodes: I = $\{5\}$;
- Node threshold $\tau$: 1/4 for even nodes, 2/3 for odd nodes.

**S2**
- Set of initial infected nodes: I = $\{6, 9\}$;
- Node threshold $\tau$: 1/4
- Set of blocked nodes (i.e. nodes that are not allowed to change their status): B = $\{10\}$

Consider a node infected at time $t$ iff at least $\tau$% of its neighbors were already infected at time $t-1$.

**Solution.** The nodes infected in the two scenarios during each iteration are:

**S1**

$I_0$ : {5}
$I_1$ : {2, 4, 6}
$I_2$ : {7, 8, 10}
$I_3$ : {1, 3}

**S2**

$I_0$ : {6, 9}
$I_1$ : {4, 7, 8, 11, 12}
$I_2$ : {1, 2, 3, 5}

**Exercise 4: Resilience** [6 points]
Given the graph $\mathcal{G}$ shown in Figure 1 remove 6 nodes, one at a time, applying the following selection criteria:

- random selection;
- decreasing degree centrality.

Compare how each removal impacts on the graph structure in terms of number of connected components and average component size. Specify the node removed - and obtained partition - at each iteration: if multiple nodes generate the same score solve the tie removing the one with lowest identifier.

**Solution.**

**Decreasing Degree Centrality**

$it_1$ : node 5, degree 6, components 1, avg. size 11
$it_2$ : node 10, degree 5, components 2, avg. size 5
$it_3$ : node 8, degree 4, components 3, avg. size 3.3
$it_4$ : node 1, degree 2, components 3, avg. size 2.6
$it_5$ : node 4, degree 2, components 3, avg. size 2.3
$it_6$ : node 9, degree 2, components 3, avg. size 2

**Random**

$it_1$ : node 9, degree 4, components 1, avg. size 11
$it_2$ : node 12, degree 1, components 1, avg. size 10
$it_3$ : node 6, degree 4, components 1, avg. size 9
$it_4$ : node 4, degree 3, components 1, avg. size 8
$it_5$ : node 1, degree 4, components 1, avg. size 7
$it_6$ : node 5, degree 3, components 2, avg. size 2.5

**Exercise 5: Multiple-choice Questions** [4 points]
Identify the correct answer(s), among the ones proposed, for to the following questions:

**Q1:** Which of the following Community Discovery methods produce non-overlapping communities?

- **(A)** Girvan-Newman
- (B) DEMON
- (C) k-Clique
- **(D)** Greedy Modularity/Louvain

**Q2:** In which of the following network topologies we define a vanishing epidemic threshold?

- (A) Random network
- **(B)** Scale-free network
- (C) Regular grid network
- (D) Small-world network

**Q3:** Is a scale-free network more resilient to *random* node failures than a random graph?

- **(A)** Yes
- (B) No

**Q4:** Is a scale-free network more resilient to *targeted* node removal than a random graph?

- (A) Yes
- **(B)** No

**Exercise 6: Open Question** [2 points]
Consider the SIS epidemic model with infection rate $\beta$ and recovery rate $\mu$: which are the possible equilibrium the model can reach? in which condition the epidemic dies out?

**Solution.** The epidemic dies out for $\lambda = \frac{\beta}{\mu} < 1$. $\lambda$ is the reproductive number: it identifies the average number of infectious individuals generated by a single infected node in a fully susceptible population.

The equilibrium states are then:

- Die out: $\lambda < 1$, as described before.
- Outbreak: $\lambda > 1$. In this scenario the recovery process is slower than disease spreading.

Once reached the stationary state the fraction of infected nodes is $1 - \frac{\mu}{\beta}$