Value at risk evaluation in an equi-balanced portfolio

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***Abstract*—This report presents an evaluation of Value at Risk (VaR) in an equally balanced portfolio consisting of two stocks. Four different methods were employed to calculate VaR: sigma-flat estimation, RiskMetrics EWMA for volatility estimation, Monte Carlo simulations, and historical VaR. The study also examined the non-additivity property of VaR. Furthermore, the report examined the non-additivity property of VaR, which states that the VaR of a portfolio should not exceed the sum of the VaRs of its individual assets. Analyzing the absolute difference between the sum of single asset VaRs and the portfolio VaR allowed for an assessment of diversification benefits and potential violations of the non-additivity property.**

By comparing and contrasting these different methods, this report provides a comprehensive evaluation of VaR. The findings contribute to a deeper understanding of the strengths, limitations, and applicability of each method in assessing portfolio risk.

***Index Terms*—VaR, MonteCarlo simulations, equi-balanced portfolio, non-additivity**

I. Introduction

Value at Risk (VaR) is a widely used risk management metric in the field of finance. It is a statistical measure that estimates the potential loss an investment portfolio or financial institution may face over a specified time period, with a given level of confidence. VaR helps assess and manage the downside risk associated with investment positions or the overall portfolio.

VaR is expressed as a dollar amount or percentage, representing the potential loss relative to the portfolio value. For instance, a one-day 5% VaR of $1 million indicates that there is a 5% probability of the portfolio losing $1 million or more within a single trading day. It can also be calculated for longer timeframes, such as weeks, months, or years; the usual assumption is:

$$VaR_{N^{th}} = Var_{1^{st}} \times \sqrt{N} \tag{1}$$

where $VaR_{N^{th}}$ represent the VaR at N-th day, and $Var_{1^{st}}$ is the 1-day VaR. This formula is exactly true when the changes in the value of the portfolio on successive days have independent identical normal distributions with mean zero. In other cases, it is an approximation [1].

II. Companies description

A. Alibaba Group

Alibaba Group is a multinational conglomerate and one of the world's largest e-commerce companies. It was founded by Jack Ma and a group of 17 individuals in Hangzhou, China, in 1999. The company's journey from a small start-up to a global powerhouse is a testament to its visionary leadership, innovative business models, and relentless pursuit of growth. Alibaba started as a business-to-business (B2B) online marketplace, connecting Chinese manufacturers with international buyers. Its initial platform, Alibaba.com, quickly gained popularity and became a trusted platform for global trade. Leveraging the growing internet penetration and the rise of e-commerce, Alibaba expanded its services to include business-to-consumer (B2C) and consumer-to-consumer (C2C) platforms.

In 2003, Alibaba launched Taobao, a C2C online marketplace that offered a wide range of products, challenging eBay's dominance in China. With a focus on fostering trust and providing a seamless user experience, Taobao became immensely popular among Chinese consumers. To further diversify its offerings, Alibaba introduced Tmall in 2008, a B2C platform catering to brands and retailers seeking to reach Chinese consumers directly. Alibaba's financial growth has been remarkable. In 2014, the company conducted its record-breaking initial public offering (IPO) on the New York Stock Exchange, raising over $25 billion. This IPO was the largest in history at that time and valued Alibaba at around $231 billion. It provided the company with substantial capital to invest in its various ventures and expand its global footprint.

Alibaba's revenue primarily comes from its core commerce business, which includes online marketplaces, digital advertising, and cloud computing services. The company also ventured into sectors such as digital entertainment, logistics, fintech, and healthcare. Its affiliate, Ant Group, launched Alipay, a popular digital payment platform that revolutionized financial services in China. Alibaba's financial success is reflected in its annual revenues. In the fiscal year 2021, the company generated approximately $109.5 billion in revenue, representing a significant increase from its early years. Its net income in the same period amounted to around $21.9 billion, showcasing its profitability and sustainable growth.

Despite its achievements, Alibaba has faced regulatory challenges and scrutiny from Chinese authorities. In recent years, the company has been subject to antitrust investigations and fines, which have impacted its operations and stock value. However, Alibaba remains resilient and is actively working to address regulatory concerns while seeking opportunities for continued growth.

B. Sea Limited

Sea Limited, also known as Sea Group, is a leading technology company based in Singapore. Established in 2009 by Forrest Li, Sea has experienced significant growth and emerged as a key player in the Southeast Asian digital landscape. The company operates three main businesses: Garena, Shopee, and SeaMoney, each contributing to its success and financial achievements.

Sea began its journey as Garena, an online gaming platform that quickly gained popularity across Southeast Asia. Garena offered a diverse range of games and built a strong user base by focusing on localizing content and providing immersive gaming experiences. The success of Garena paved the way for Sea's expansion into other areas of digital commerce and finance. In 2015, Sea launched Shopee, an e-commerce platform that aimed to revolutionize online shopping in Southeast Asia. Shopee differentiated itself by offering a mobile-first approach, social commerce features, and a seamless user experience. The platform quickly gained traction, appealing to a wide range of consumers and merchants in the region. Shopee's growth has been impressive, capturing a significant share of the Southeast Asian e-commerce market. SeaMoney, another important arm of Sea, focuses on digital financial services. Initially, SeaMoney started as an in-house payment system for Garena, but it expanded its services to become a comprehensive digital financial platform. SeaMoney offers a range of financial products, including mobile wallet services, digital payments, remittances, and micro-loans. The company's commitment to providing accessible financial services has helped it gain traction in underserved markets within Southeast Asia.

From a financial perspective, Sea has witnessed substantial growth. In 2017, the company went public on the New York Stock Exchange and raised approximately $884 million. Since then, Sea's market capitalization has soared, and it has become one of the most valuable companies in Southeast Asia. In 2020, the company reported total revenue of $4.4 billion, representing a staggering 101.4% increase compared to the previous year. Moreover, Sea's gross merchandise value (GMV) on the Shopee platform reached $35.4 billion in 2020, demonstrating the platform's significant impact on the region's e-commerce sector.

Sea's success can be attributed to its ability to capture the growth potential of the Southeast Asian market, which boasts a young and increasingly tech-savvy population. The company's strategic focus on mobile-first solutions, localized content, and fostering a strong community has resonated well with consumers in the region. While Sea has achieved remarkable success, it faces competition from regional and global players. The Southeast Asian market remains highly dynamic and competitive, with evolving consumer preferences and regulatory challenges. However, Sea's strategic investments, strong brand presence, and focus on innovation position it well for sustained growth in the digital economy of Southeast Asia.

III. Methods

In this report it has been used an equibalanced portfolio between two assets. An equibalanced portfolio is an investment portfolio allocation strategy where each asset or investment within the portfolio is assigned an equal weight or allocation. In other words, the portfolio is divided equally among the selected assets, regardless of their individual market capitalization, sector, or other factors. The concept behind this type of strategy is to provide equal exposure to each investment, with the goal of diversifying risk across various assets. It this specific case, it has been chosen to divide the portfolio between Alibaba Group and Sea Limited's stocks. There are different methods to compute VaR, based on historical data or statistical models. The most common ones are:

A. Parametric Normal VaR with σ-flat estimation

This is a simplified approach that relies on a fixed estimate of the asset's volatility or standard deviation, σ to calculate VaR. It assumes that the returns of the asset follow a normal distribution, and VaR value corresponds to the quantile associated to the choosen confidence level, eventually, multiplied by the investment, to obtain the absolute value of VaR rather then the percentage value. It's important to note that the sigma-flat estimation method assumes that returns are normally distributed and that the asset's volatility remains constant over time. These two assumptions may not hold true in practice, especially during periods of high market volatility or during non-normal market conditions.

B. Parametric Normal VaR with EWMA σ estimation

This is a technique that incorporates a weighted average of historical volatility to calculate VaR. It is a more dynamic approach compared to the sigma-flat estimation method; it consist in assign weights to the historical returns, giving more weight to recent returns and less weight to older returns. The weights are typically calculated using an exponentially decreasing function:

$$r_i^{weighted} = \sqrt{\lambda^i(1-\lambda)}r_i. \tag{2}$$

The EWMA sigma estimation method assigns more importance to recent returns, reflecting the assumption that recent volatility is a better indicator of future volatility than older observations. By incorporating this dynamic approach, the EWMA method adapts to changes in volatility levels and can capture shifts in market conditions more effectively.

C. MonteCarlo VaR

The Monte Carlo method is a simulation-based approach used to estimate Value at Risk (VaR). Unlike traditional methods that assume specific distributions or rely on historical data, this method generates multiple possible scenarios by randomly sampling from input parameters. The procedure is as follows [1]:

1) Value the portfolio today in the usual way using the current values of market variables.
2) Sample once from the multivariate normal probability distribution of the Δx_i.
3) Use the values of the Δx_i that are sampled to determine the value of each market variable at the end of one day.
4) Revalue the portfolio at the end of the day in the usual way.
5) Subtract the value calculated in Step 1 from the value in Step 4 to determine a sample ΔP.
6) Repeat Steps 2 to 5 many times to build up a probability distribution for ΔP.

The VaR is calculated as the appropriate percentile of the probability distribution of ΔP.

D. Historical VaR

The historical method is a straightforward approach used to compute Value at Risk (VaR) by relying on historical data. It is based on the following formula:

$$VaR = I\sigma\sqrt{T}q(1-X) \tag{3}$$

where I represent the initial investment on the portfolio. The formula derives from the assumption that asset returns follow a normal distribution. By using historical data, the standard deviation, σ, based on the observed variability of returns. The inverse cumulative distribution function, $q(\cdot)$, helps determine the appropriate threshold value (negative return) that corresponds to the desired confidence level.

IV. Results

A. Balanced portfolio

After collecting all six-months historical data from Yahoo Finance about Baba Group and SE limited, for each of them it was calculated the daily returns, mean of daily returns and standard deviation. The initial investment on the portfolio was setted equal to, $I = 500\$$, and each asset's stock price in the portfolio has been daily calculated as:

$$S_1 = Ip = I\frac{P_{SE}}{P_{SE} + P_{Baba}}, \quad (4)$$

$$S_2 = I(1-p) = I\frac{P_{Baba}}{P_{SE} + P_{Baba}},$$

where $P_{Baba/SE}$ stands for the Adj Closing price of each asset. The trend of S_1 and S_2 is displayed in Fig. 1, in such a way that the expected value of the portfolio should always be equal to the initial investment I. On the other hand the daily returns where computed as

$$r^i_{portfolio} = p\, r^i_{Baba} + (1-p)\, r^i_{SE}, \quad (5)$$

and the standard deviation of the portfolio can be evaluated as:

$$\sigma_{portfolio} = \sqrt{\sigma^2_{Baba} + \sigma^2_{SE} + 2\rho\sigma_{Baba}\sigma_{SE}} \quad (6)$$

The results are presented in table 1:

	Baba	SE	portfolio
mean [$]	0,001134	0,003259	0,002196
std [$]	0,03321	0,03899	0,02979

TABLE 1: Mean and standard deviation of returns.

B. Normal VaR, flat σ

As explained in section III.A the parametric normal VaR with flat σ can be obtained considering normal distributions $\mathcal{N}(\mu, \sigma)$, where means and standard deviations coincides with the ones in table 1. In Figure 2 can be seen the surfaces representing the VaR values obtained for each asset and the portfolio as functions of the confidence level, in a range between $[0,9 : 9,995]$, and time horizon $[1 : 100]$ days, knowing that the value of VaR at the N-th day is obtained according to eq. 1.

C. Normal VaR, EWMA σ

To estimate the weighted returns it was imposed a factor $\lambda = 0,94$, and following a procedure similar to the one of the previous section it was possible to obtain the results in Figure 3.

D. Montecarlo VaR

The Montecarlo simulations were computed using a VBA excel code, each simulation consists in $N_{MC} = 1500$ time steps, Simulated for $T = 100$ days. Results are shown in Figure 4.

E. Historical Var

According to eq. 3 the VaR value have been calculated as function of the same ranges of confidence level and time horizon as in previous sections; results can be compared in Figure 5.

V. Concluding Remarks

As expected, the VaR results are negative values expressed in percentage: they represent the maximum expected loss at the specified confidence level and time horizon. Remembering that the confidence level determines the probability associated with the VaR estimate, from each chart obtained, Figs. 2, 3, 4, 5, it can clearly be seen that when the confidence level increases, the VaR decreases: in other words, as there is more confidence that will not be a loss greater then **VaR** dollars in the future (e.g., moving from a 90% confidence level to a 99% confidence level), the estimated potential loss becomes smaller. This makes intuitive sense because a higher confidence level implies a lower tolerance for risk, and therefore the estimated maximum loss is reduced. It's also important to note that the relationship between confidence level and VaR is not a linear one, and it can vary depending on the characteristics of the portfolio and the underlying assets. Another important dependence is the one related to the time horizon; according to eq. 1 is can be seen a negative square-root trend. It is interesting to notice that in the MonteCarlo simulation method, Fig. 4, the surfaces result to be less smooth compared to the other ones, this reflect the random nature of the simulation method and introduces inherent variability. Overall, in all cases the non-additivity property of the VaR emerges: the difference between, the sum of the two asset's VaR, and the portfolio VaR it's negative, according to the inequality 7:

$$VaR_{porfolio} > VaR_{Baba} + VaR_{SE}, \quad (7)$$

which suggests that the portfolio's risk is lower than the combined risk of its individual assets. In this case, the portfolio exhibits a diversification benefit.

REFERENCES

[1] J. Hull, *Option, Future, and other Derivatives*. Pearson, 2006.

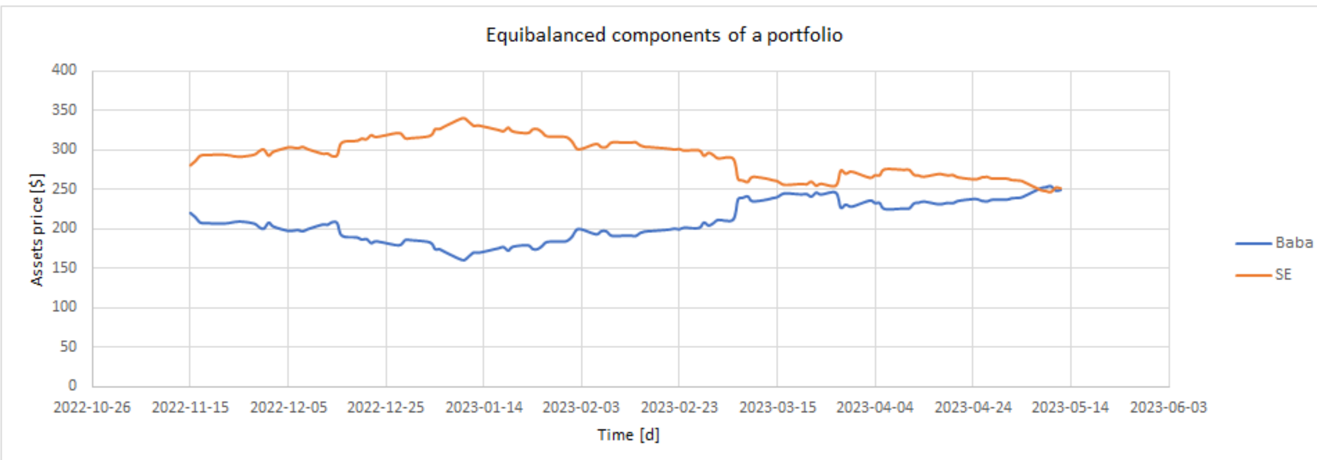

Fig. 1

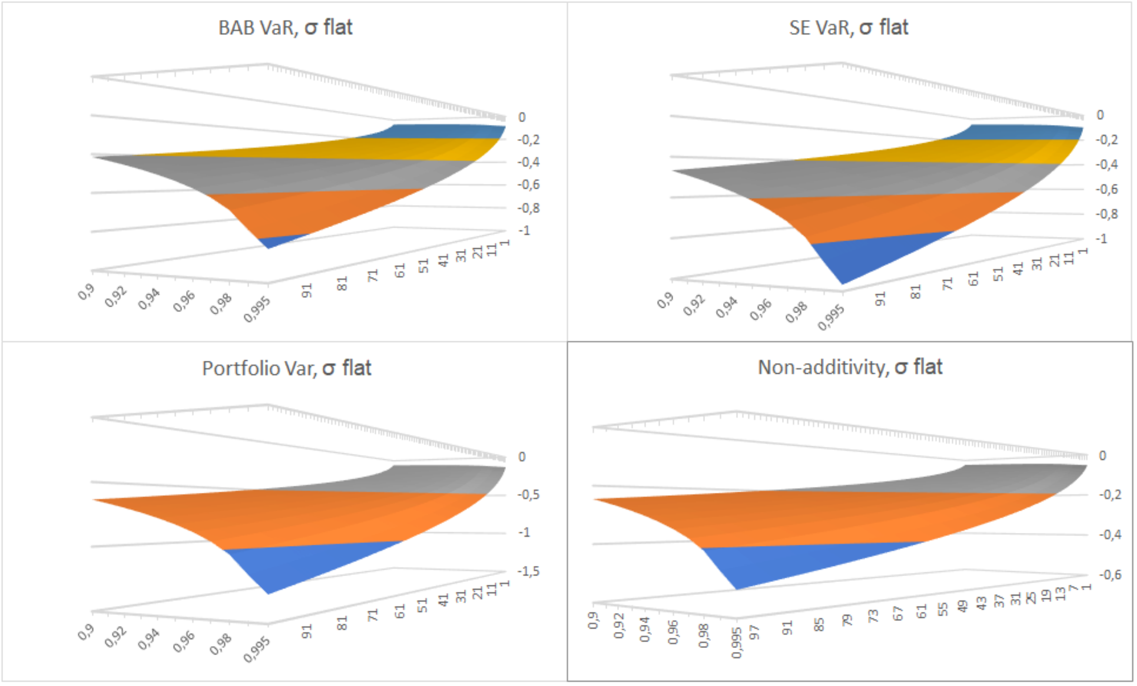

Fig. 2: Value at Risk for Normal case with σ flat

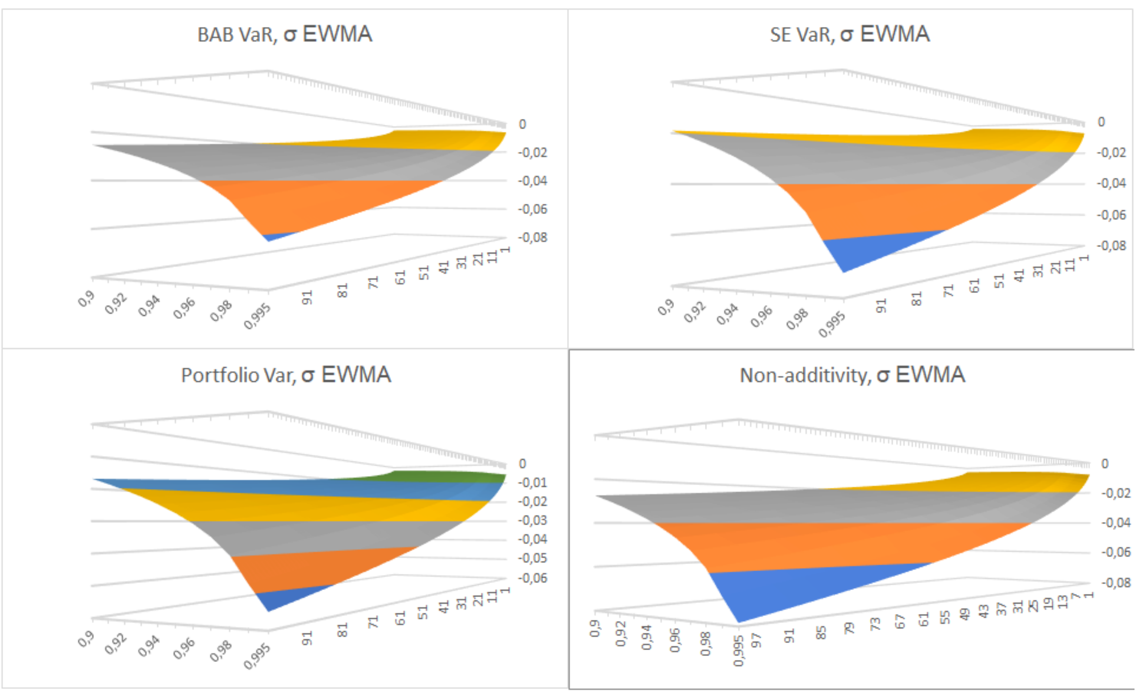

Fig. 3: Value at Risk for Normal case with using the RiskMetrics EWMA

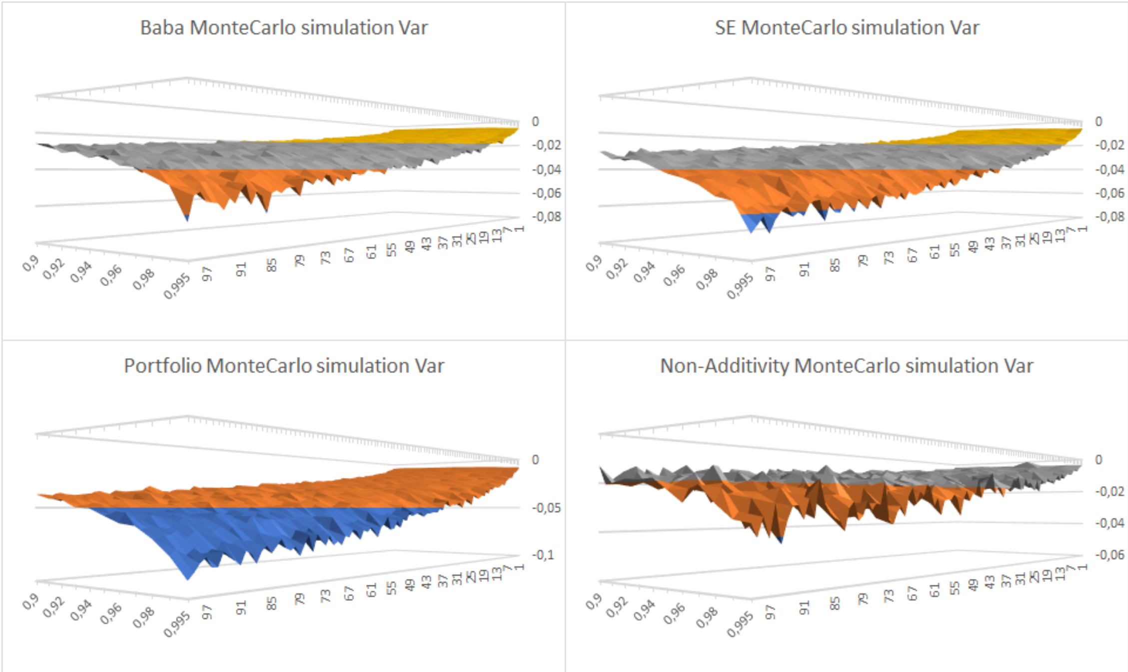

Fig. 4: Value at Risk for Normal case with using the MonteCarlo method

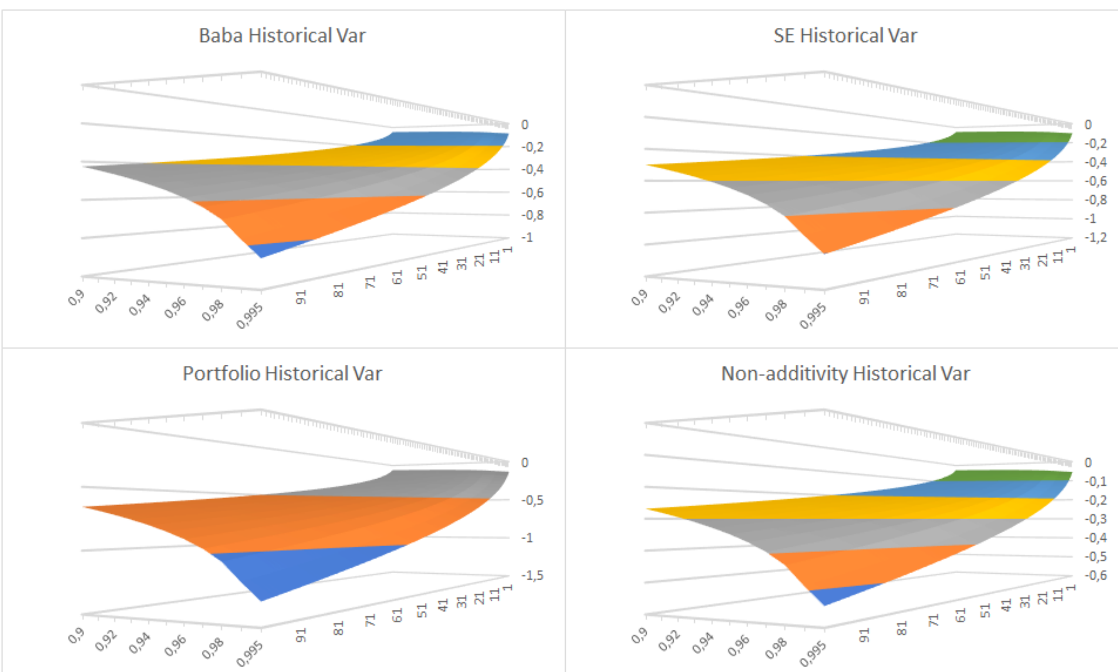

Fig. 5: Value at Risk for Normal case with using the historical method