# Analysis of Implied Volatility Surface

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***Abstract*—This report investigates the volatility smile of PayPal Holdings Inc. call options and computes the Greek parameters as a function of time to maturity and strike prices. The report aims to provide insights into the presence of the volatility smile and the behavior of the Greek parameters over time. A chart of the volatility smile for PayPal Holdings Inc. call options is constructed, which clearly shows the presence of a volatility smile. The report then computes the delta, gamma, theta, vega and Rho parameters as a function of time to maturity and strike prices. The results demonstrate a smooth behavior of the Greek parameters as time to maturity increases, which is desirable for effective hedging in options markets.**

***Index Terms*—Implied volatility smile, Greeks parameters, skew**

## I. Company description

PayPal Holdings, Inc. is a technology company that provides digital payment solutions and financial services to businesses and consumers around the world [1]. Founded in December 1998, by Max Levchin, Peter Thiel, and Luke Nosek, initially, the company developed security software for handheld devices, but later shifted its focus to digital wallets. In 2002, PayPal was acquired by eBay, becoming the primary payment method for eBay transactions. PayPal went public on the NASDAQ in 2002 and was later spun off as an independent company in 2015. Since then, PayPal has continued to grow and diversify its services: it has become a leading provider of online payments, operating in more than 200 markets with over 400 million active user accounts.

PayPal's digital wallet enables customers to securely store their credit and debit card information, as well as their bank account details, and use it to make online transactions with merchants. The company's platform also supports online money transfers and serves as an electronic alternative to traditional paper methods like checks and money orders.

In addition to its digital wallet, PayPal offers a range of other financial services, including peer-to-peer payments, merchant services, working capital loans, and small business loans. The company also acquired Venmo, a mobile payment platform popular with millennials, in 2013.

PayPal's success has been driven by its commitment to innovation and security. The company has been a pioneer in digital payments, introducing features like One Touch checkout and the ability to pay with just a mobile phone number or email address. PayPal also prioritizes user privacy and has implemented advanced security measures like encryption, fraud detection, and two-factor authentication.

PayPal's impact on the financial industry has been significant, as the company has helped to revolutionize the way people make payments and conduct transactions. Its services have become essential for businesses of all sizes, particularly those operating in the e-commerce space. In 2020, PayPal announced that it would begin allowing users to buy, hold, and sell cryptocurrencies within the PayPal app, further expanding its services.

Overall, PayPal has established itself as a trusted and reliable provider of financial services, with a reputation for innovation and security. As the world becomes increasingly digitized and online transactions continue to grow, PayPal is well-positioned to continue its growth and impact in the years to come.

## II. Methods

Volatility smile is a term used in financial markets to describe the implied volatility of options with different strike prices on the same underlying asset [2], [3]. Implied volatility is the market's expectation of how much the price of an underlying asset will fluctuate in the future. Typically, implied volatility is higher for options that are either deep in-the-money or deep out-of-the-money, compared to those at-the-money.

The term "volatility smile" was introduced in the late 1980s, following the stock market crash of 1987. At that time, the Black-Scholes option pricing model, which assumes that volatility is constant across different strike prices, was found to be insufficient in accurately pricing options. The observed market prices of options were not consistent with the model's predictions, especially for those with low strike prices or far away from the current price of the underlying asset.

The volatility smile phenomenon reflects the fact that the market tends to price options with lower strike prices or further from the current price of the underlying asset with higher implied volatility. This suggests that market participants perceive a higher risk of large price movements in the underlying asset when it is trading at lower prices, or when it is far from the current price.

The volatility smile is used by traders and analysts to estimate the market's expectation of future price movements in the underlying asset. It can also be used to identify potential mispricings in options, which may present opportunities for arbitrage or hedging. The volatility smile is an important tool in risk management, as it can help traders and portfolio managers to estimate the potential losses or gains associated with different option strategies.

## III. Results

A first aim of this report consist in compute the implied volatility surface for call options; in order to do that, a set of European style options has been choosen from Paypal Holdings, inc. to compare the variation of implied volatilities as function of the Strike price K, and the time to maturity T. In the majority of cases, there were available options with a constant difference of strike price in the interval $[60, 110]\$$, and time to maturity is about vary from 0 up to 20 months with no constant interval steps at the very final T values. the graph can be see in fig 1.

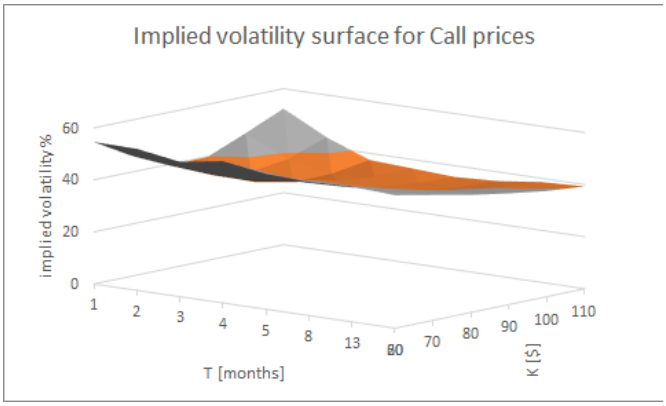


Fig. 1: Implied volatility as function of time to maturity and strike price.

A second goal of this report consists in analyze the Greek's behaviour as time to maturity increases: the Greeks parameter were computed with the same approach (using VBA code) as in the previous report: in this case the spot price $S$ was fixed at the value $S = 72,93\$$, taken in date 25-04-2023. Since PayPal Holdings, Inc. is a no Dividend-paying company the $q$ parameter was set equally null. The implied volatilities computed in the previous section were used to the corresponding maturity-strike $(T, K)$ couples, and the last parameter needed is the interest rate to the corresponding time to maturity T: it was computed exploiting the put-call parity formula which allow to obtain the discount factor $D(T)$ at each time to maturity, from which is trivial to obtain the corresponding interest rate:

$$r = -\frac{ln(D)}{T}. \tag{1}$$

More details about the options used from Yahoo Finance, for the put-call parity are available in table 1. In fig 2, are shown the Greeks surfaces computed.

## IV. Concluding Remarks

As expected from from a theoretical point of view, in fig. 1 The implied volatility shows the smile7skew effect which is more highlighted in the first part, with closer maturities, while for longer maturities the surface flattens. The lowest value is reached around $K \simeq 75$ which is close to the price of the asset, and tends to reach higher values for $K$ far away from $S$, as a direct consequence of the failure of Black-Scholes model. On the other hand, the Greeks chart shows extremely behaviours, as can be seen in fig. 2: it is true that as time to maturity increase ( 1 up to 20 months) all of them tend to become flat, but not all of them keep their expected shape.

## References


[1] Wikipedia contributors, "Paypal — Wikipedia, the free encyclopedia," 2023. [Online; accessed 27-April-2023].

[2] T. Bjork, *Arbitrage Theory in Continuous Time*. Oxford University Press, 2009.

[3] J. Hull, *Option, Future, and other Derivatives*. Pearson, 2006.

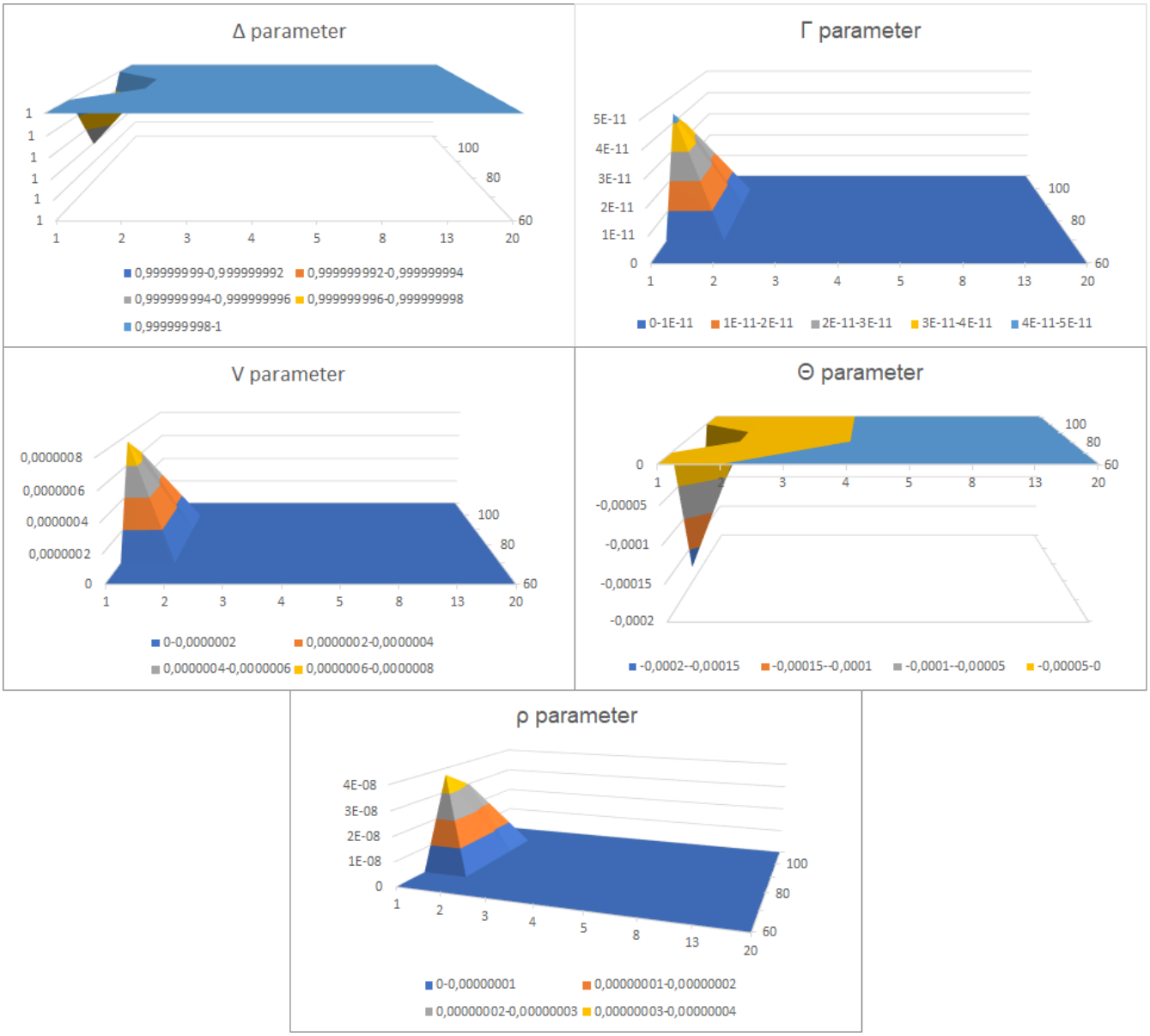


Fig. 2: Greeks parameter as function of time to maturity and strike price.

| T[months] | Option | Contract Name | last trade date | Strike | Last | Price | Bid | Ask |
|---|---|---|---|---|---|---|---|---|
| 1 | call | PYPL230519C00045000 | 2023-04-03 | 45 | 30,49 | 28,25 | 28,45 | 28,35 |
| 1 | call | PYPL230519C00110000 | 2023-04-20 | 110 | 0,01 | 0,01 | 0,02 | 0,015 |
| 1 | put | PYPL230519P00045000 | 2023-04-21 | 45 | 0,02 | 0 | 0 | 0 |
| 1 | put | PYPL230519P00110000 | 2023-03-20 | 110 | 37,78 | 34,45 | 34,95 | 34,7 |
| 2 | call | PYPL230616C00045000 | 2023-04-06 | 45 | 30,25 | 28,5 | 28,8 | 28,65 |
| 2 | call | PYPL230616C00110000 | 2023-04-25 | 110 | 0,05 | 0,04 | 0,05 | 0,045 |
| 2 | put | PYPL230616P00045000 | 2023-04-17 | 45 | 0,06 | 0,05 | 0,06 | 0,055 |
| 2 | put | PYPL230616P00110000 | 2023-04-24 | 110 | 35,6 | 36,8 | 36,95 | 36,875 |
| 3 | call | PYPL230721C00045000 | 2023-03-29 | 45 | 30 | 28,8 | 29,15 | 28,975 |
| 3 | call | PYPL230721C00110000 | 2023-04-25 | 110 | 0,08 | 0,08 | 0,09 | 0,085 |
| 3 | put | PYPL230721P00045000 | 2023-04-25 | 45 | 0,15 | 0,16 | 0,18 | 0,17 |
| 3 | put | PYPL230721P00110000 | 2023-03-24 | 110 | 37,61 | 35,55 | 36,1 | 35,825 |
| 4 | call | PYPL230915C00045000 | 2023-04-11 | 45 | 30,6 | 29,5 | 29,9 | 29,7 |
| 4 | call | PYPL230915C00110000 | 2023-04-24 | 110 | 0,32 | 0,31 | 0,32 | 0,315 |
| 4 | put | PYPL230915P00045000 | 2023-04-25 | 45 | 0,5 | 0,51 | 0,53 | 0,52 |
| 4 | put | PYPL230915P00110000 | 2023-04-24 | 110 | 35,45 | 36,8 | 37 | 36,9 |
| 5 | call | PYPL231020C00045000 | 2023-03-31 | 45 | 32,5 | 29,9 | 30,3 | 30,1 |
| 5 | call | PYPL231020C00110000 | 2023-04-24 | 110 | 0,48 | 0,45 | 0,48 | 0,465 |
| 5 | put | PYPL231020P00045000 | 2023-04-17 | 45 | 0,62 | 0,7 | 0,72 | 0,71 |
| 5 | put | PYPL231020P00110000 | 2023-04-10 | 110 | 35,45 | 36,7 | 36,95 | 36,825 |
| 8 | call | PYPL240119C00045000 | 2023-04-19 | 45 | 33,2 | 31 | 31,35 | 31,175 |
| 8 | call | PYPL240119C00110000 | 2023-04-25 | 110 | 1,25 | 1,23 | 1,27 | 1,25 |
| 8 | put | PYPL240119P00045000 | 2023-04-20 | 45 | 1,27 | 1,27 | 1,31 | 1,29 |
| 8 | put | PYPL240119P00110000 | 2023-04-25 | 110 | 36,5 | 36,85 | 37 | 36,925 |
| 13 | call | PYPL240621C00045000 | 2023-04-10 | 45 | 34,33 | 32,7 | 34,05 | 33,375 |
| 13 | call | PYPL240621C00110000 | 2023-04-24 | 110 | 3,1 | 2,86 | 2,98 | 2,92 |
| 13 | put | PYPL240621P00045000 | 2023-04-11 | 45 | 2,32 | 1,96 | 2,3 | 2,13 |
| 13 | put | PYPL240621P00110000 | 2023-04-20 | 110 | 35,7 | 36,75 | 37,3 | 37,025 |
| 20 | call | PYPL250117C00045000 | 2023-04-21 | 45 | 35,7 | 34,15 | 36,15 | 35,15 |
| 20 | call | PYPL250117C00110000 | 2023-04-25 | 110 | 5,43 | 5,3 | 5,5 | 5,4 |
| 20 | put | PYPL250117P00045000 | 2023-04-25 | 45 | 3,21 | 3,05 | 3,3 | 3,175 |
| 20 | put | PYPL250117P00110000 | 2023-04-24 | 110 | 37 | 37,05 | 37,65 | 37,35 |

TABLE 1: Paypal call option used for put-call parity