Analysis of Greeks parameter for Black-Scholes model

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Abstract—**This report explores the behavior of Greek parameters using VBA codes, with a focus on time to maturity and stock prices. The Greek parameters under consideration included Delta, Gamma, Theta, Vega and Rho. A volatility shock of $\pm 50\%$ was introduced to observe how the parameters changed. Additionally, the report examined the impact of different dividend rates on these parameters. The analysis sheds light on the sensitivity of Greeks to changes in various factors, which can help investors make informed decisions in the financial markets. Overall, the report provides valuable insights into the behavior of Greeks parameters and their significance in finance.**

Index Terms—**Greeks, implied volatily, dividend rate**

I. Introduction

In finance, Greek parameters refer to a set of statistical measures used to assess the risk and potential return of various financial instruments, such as options and futures contracts [1], [2]. These measures are named after Greek letters, such as Delta, Gamma, Theta, Vega, and Rho, and provide investors and traders with a way to quantify how changes in different factors will affect the value of their investments.

The above-mentioned parameters have their origins in the Black-Scholes model, a mathematical formula developed by Fischer Black and Myron Scholes in the 1970s: this model is widely used in options pricing and has been instrumental in the development of modern quantitative finance. Over time, the Greeks have become a standard part of the toolkit for options traders and are used by financial institutions around the world. In addition to the original five parameters, other Greeks parameters have been developed to further refine options pricing models and better capture the complexity of financial markets. Nevertheless today, the Greeks parameters remain a critical tool for anyone trading or investing in options and other financial instruments. By understanding these parameters and how they interact with each other, traders can make more informed decisions about their investments and better manage their risks.

II. Methods

A. *Delta*

Delta is one of the Greek parameters used in finance to measure the sensitivity of an option's price to changes in the underlying asset's price. Delta is expressed as a number between 0 and 1 for call options, or between -1 and 0 for puts. For example, a delta of 0.5 indicates that for every $\$1$ increase in the underlying asset's price, the option's price will increase by $\$0.50$. Similarly, a delta of -0.5 for a put option indicates that for every $\$1$ decrease in the underlying asset's price, the option's price will increase by $\$0.50$. According to its intuition, Delta is defined as the first-order derivative of the option's price with respect to the underlying asset price.

$$\Delta := \frac{\partial p}{\partial S}, \tag{1}$$

where p is the portfolio's price, and S represents the underlying asset. Recalling the B.S. formula for the price of a European call option:

$$C = S \cdot N(d_1) - e^{-rT} K \cdot N(d_2), \tag{2}$$

where N is the cumulative standard normal distribution, d_1 and d_2 are defined as:

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{(\sigma\sqrt{T})}, \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

σ is the volatility of the underlying asset, r is the risk-free interest rate, K is the strike price of the option, and T is the time to expiration: the first derivative of the Black-Scholes formula with respect to the underlying asset price become:

$$\Delta(Call) = N(d_1), \tag{4}$$

$$\Delta(Put) = N(d_1) - 1.$$

The delta of an option is not a fixed value, but instead varies depending on a number of factors, including of course the price of the underlying asset, but also the time to expiration, volatility, and interest rates. In general, the delta of a call option will increase as the underlying asset price increases, while the delta of a put option will increase as the underlying asset price decreases. This relationship is due to the fact that as the underlying asset price moves closer to the option's strike price, the option becomes more valuable and the probability of it expiring in the money increases.

B. *Gamma*

Gamma is another important Greek parameter used to measure the sensitivity of an option's price to changes in the price of the underlying asset; it quantify the rate of change of an option's Delta in response to changes in the price of the underlying asset. In other words, it measures the second derivative of the option price with respect to the underlying asset price. Mathematically, gamma is expressed as:

$$\Gamma := \frac{\partial^2 p}{\partial S^2} = \frac{\partial \Delta}{\partial S}, \tag{5}$$

where p is the price of the portfolio and S is the price of the underlying asset. Intuitively, tells us how fast the Delta of an option changes as the underlying asset price changes so it tell us how often we need to adjust the hedging portfolio in order to make it delta-neutral, i.e. to make it immune against small changes in the underlying asset price S. Gamma is greatest approximately at-the-money (ATM) and diminishes the further out you go either in-the-money (ITM) or out-of-the-money (OTM). Using, as price function, the BS formula again, can be obtained the explicit expression:

$$\Gamma = \frac{N(d_1)}{S\sigma\sqrt{T}}. \tag{6}$$

C. *Rho*

Rho measures the sensitivity of an option's price to changes in interest rates. It is usually denoted by the Greek letter rho, ρ, and is expressed as a positive or negative number, depending on the type of option. In other words, it measures how much an option's price will change in response to a change in the risk-free interest rate. Mathematically, rho is expressed as:

$$\rho := \frac{\partial p}{\partial r}, \tag{7}$$

where p is the price of the portfolio and r is the risk-free interest rate. In practical, as interest rates increase, the present value of future cash flows decreases, which can reduce the value of an option. Conversely, as interest rates decrease, the present value of future cash flows increases, which can increase the value of an option. Except under extreme circumstances, the value of an option is less sensitive to changes in the risk-free interest rate than to changes in other parameters. For this reason, rho is the least used of the first-order Greeks. Rho is typically expressed as the amount of money, per share of the underlying, that the value of the option will gain or lose as the risk-free interest rate rises or falls by 1.0% per annum (100 basis points). So Rho quantifies this effect by measuring the rate at which an option's price changes as interest rates change. Its explicit expression can be written as:

$$\rho(Call) = KTe^{-rT}N(d_2), \tag{8}$$

$$\rho(Put) = -KTe^{-rT}N(-d_2)$$

D. *Vega*

Vega is another important Greek parameter used in finance to measure the rate of change of an option's price with respect to changes in volatility. In other words, it measures how much an option's price will change in response to a change in the implied volatility of the underlying asset. Mathematically, Vega is expressed as:

$$V := \frac{\partial p}{\partial \sigma}, \tag{9}$$

Intuitively, Vega reflects the impact of changes in market sentiment on an option's value. As the market becomes more volatile and uncertainty increases, option prices tend to increase because the probability of the underlying asset moving in a favorable direction (i.e., in-the-money) also increases so Vega quantifies this effect by measuring the rate at which an option's price changes as implied volatility changes. Vega is typically expressed as the amount of money per underlying share that the option's value will gain or lose as volatility rises or falls by 1 percentage point. All options (both calls and puts) will gain value with rising volatility. Again applying the Black-Scholes model as pricing formula, it can be shown that the explicit expression is:

$$V = \sqrt{T} S \cdot N(d_1). \tag{10}$$

E. *Theta*

Theta measures the rate of change of an option's price with respect to the passage of time. In other words, it measures how much the option's price will decrease as time passes, all else being equal. Mathematically, theta is expressed as:

$$\Theta := -\frac{\partial p}{\partial t}, \tag{11}$$

where p is the price of the portfolio and t is time. With the B.S. Formula as pricing formula for the option, the Theta parameter for a call or a put option can be recovered as:

$$\Theta(Call) = -\frac{S \cdot N(d_1)\sigma}{2\sqrt{T}} - Kre^{-rT}N(d_2), \tag{12}$$

$$\Theta(Put) = -\frac{S \cdot N(d_1)\sigma}{2\sqrt{T}} + Kre^{-rT}N(-d_2).$$

Intuitively, theta reflects the time decay of an option's value. As an option gets closer to its expiration date, its value decreases because there is less time for the underlying asset price to move in a favorable direction, and this is the reason why we would expect negative results for this parameter. (Theta is an important parameter since can be used in different strategies to make profit or investments; for example by selling options with high theta values, traders can capture the time decay effect and earn a profit if the options expire out of the money.)

III. RESULTS

The aim of this report is to analyze the behaviour of Greeks parameters, as functions of the Spot price S and the time to maturity T: in order to do that, the needed parameter to apply the previous analytical results were fixed as reported in Table 1:

K [$]	r	vol
100	1%	20%

TABLE 1: Set of fixed parameters for computations

On the other hand, as said before, the S price, and the time to maturity have been left free to vary, taking values in following intervals:

S[$]	T[months]
60	0,1
65	0,2
70	0,3
75	0,4
80	0,5
85	0,6
90	0,7
95	0,8
100	0,9
105	1
110	2
115	3
120	4
125	5
130	/
135	/
140	/

TABLE 2: Analyzed values for stock price and time to maturity

And finally it was considered a shock of the volatility σ leading to values $\sigma' = 5\%$ and $\sigma'' = 75\%$, with the purpose of showing how much they can impact on Greeks surfaces.

Using eqs. 4, 6, 8, 10 and 12 implemented as VBA codes, it have been calculated the surfaces of Greeks parameters for different stock prices (that correspond to x-axis), time to maturity (y-axis) and volatilities as reported figure 1.

IV. Concluding Remarks

As can be seen from fig. 1, all graphs tends to follow the analytical predictions, in particular: The Delta parameter shows the typical S shape with values in range $[0, 1]$ which tends to be smoother as time to maturity grows, from the comparison between different values of volatility it can be recognize that the S shape tends to be sharper for small sigmas and become more sparse for greater sigmas. The Gamma parameter has the expected Dirac-delta's shape with maximum values relatively low; also in this case the smoothness of graphs result to be proportional to sigmas values and in particular it can be observed that the maximum value is high as the graph tends to be sharper. The Vega parameter should have a surface quite similar to the Gamma's one, Dirac-Delta's like, but this is appreciable only for $\sigma = 5\%$, for greater values, the surface become so sparse that the the peak is almost indistinguishable; the main difference with the second Greek parameter can be observed from the maximum value reached, which is $>> 1$, in contrast with the previous one. Also the Theta parameter, as expected from theory, has a Dirac-Delta's like shape, but with the major difference that it has negative values so the peak results to be inverted; in contrast to the previous case as sigma (and of course the smoothness of the surfaces) grows, the maximum value result to be higher in modulo. Finally the Rho parameter should have a shape similar to the Delta's one, which can be recognized from the graph of $\sigma = 5\%$, but then, for higher values of σ, it start to become more altered; on the other side it mainly differ from the first Greek parameter since it is not bounded between $[0, 1]$.

The graphs shown if fig. 2 display that the behaviour of Greeks tends to be similar to the case of no dividend paying asset when q has low values: this is in accordance with the expectation since $q \simeq 1 - 2\%$ is a realistic value; while stressing it, and leading to unrealistic rates tends to have repercussions on graphs, in particular it is interesting how in all the charts the maximum value, in modulo, tends to migrate toward the higher stock prices.

References

[1] T. Bjork, *Arbitrage Theory in Continuous Time*. Oxford University Press, 2009.

[2] J. Hull, *Option, Future, and other Derivatives*. Pearson, 2006.

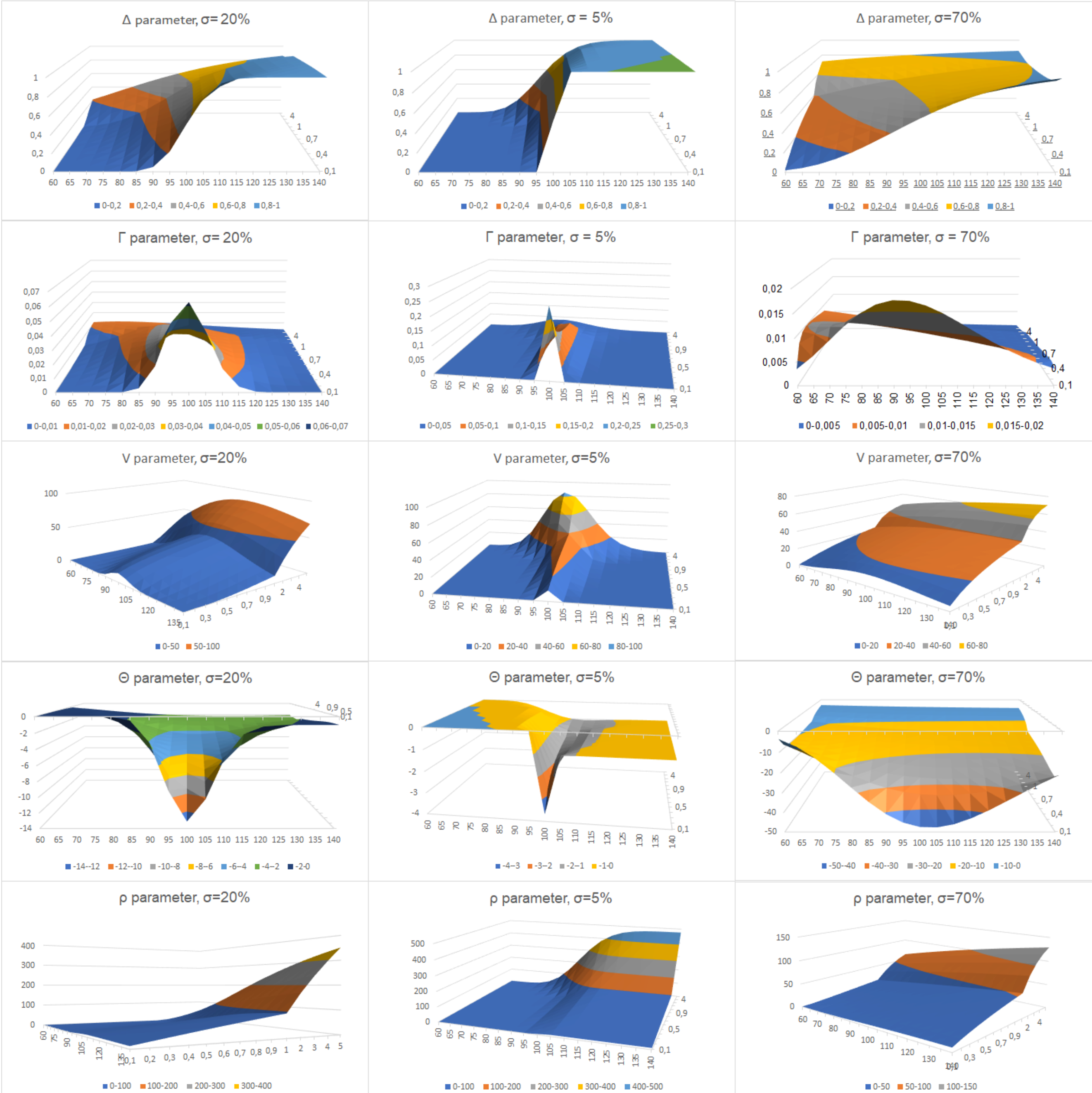

Fig. 1: Greeks parameter as function of time to maturity and stock price and volatilities.

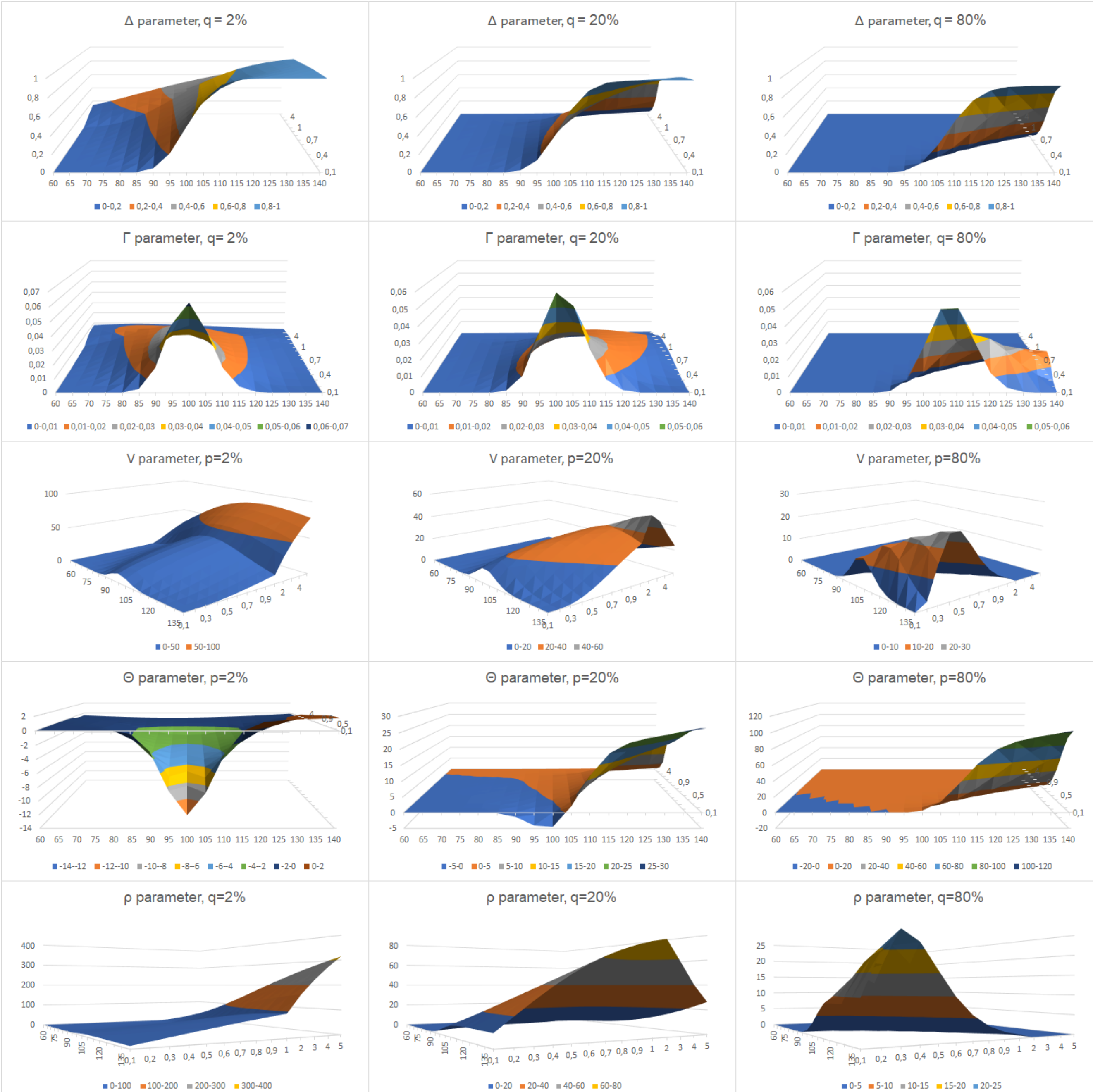

Fig. 2: Greeks parameter as function of time to maturity, stock price and percentage of dividend rate.