# Pricing of a Call Option using the Binomial Model

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***Abstract*—This report presents an analysis of historical data from Tesla, Inc. obtained from Yahoo Finance web site. Specifically, it has been calculated the price of a Call option using the static binomial model and compared it with the market quotes. The static binomial model is a commonly used tool [1], [2] for options pricing and is based on the assumption that the underlying asset's price can only move up or down by a certain amount over a given time period, and there are no arbitrage opportunities in the market considered. It has been found that the calculated price of the Call option using the binomial model closely matches the market quotes, indicating that the model provides a reasonable estimate of the option's value. This analysis has important implications for investors seeking to make informed decisions about options trading and highlights the usefulness of the binomial model in options pricing.**

***Index Terms*—Binomial model, Call option, TSLA,**

## I. Company description

Tesla, Inc. [3] is an American multinational automotive and clean energy company headquartered in Austin, Texas. Tesla designs and manufactures electric vehicles (electric cars and trucks), battery energy storage from home to grid-scale, solar panels and solar roof tiles, related products and services. Since 2021, the company had the most worldwide sales of battery electric vehicles and plug-in electric vehicles, capturing 21% of the battery-electric (purely electric) market and 14% of the plug-in market (which includes plug-in hybrids).

The company was founded in July 2003 by Martin Eberhard (CEO) and Marc Tarpenning (CFO) as *Tesla Motors* : the company's name is a tribute to inventor and electrical engineer Nikola Tesla.
The main idea behind Tesla was to build *"a car manufacturer that is also a technology company"*, with its core technologies as *"the battery, the computer software, and the proprietary motor"*. The first car the company produced was the Roadster, an all-electric vehicle. The bodyshell and most of the components were manufactured by Lotus Cars at the Hethel plant in England and shipped to the Tesla factory in California for final assembly. The Roadster has a range of 340 km and was the first production car to use lithium-ion cell batteries. In February 2004, via a $ 6.5 million investment, Elon Musk, co-founder of PayPal, became the largest shareholder of the company. Between 2008 and 2012, Tesla sold approximately 2,500 units worldwide. On the company's 5th anniversary, Elon Musk was named CEO. The second vehicle produced is a luxury sedan, the S model, followed by the X model which represents an innovation among large SUVs. Since 2014, the company has increasingly sought independence through the establishment and acquisition of various secondary companies for purposes ranging from the creation of individual mechanical parts to the increase in technological performance.

As mentioned, the initial investment came from Elon Musk, who managed the company's early challenges and later led partners at VantagePoint Capital Partners. Later, in 2009, further financing was added by Daimler, a German car manufacturer, in the amount of 50 million dollars in exchange for 10 % of the share capital. In June of the same year, Tesla receives an important funding from the United States Government equal to 500 million dollars. Tesla goes public on the New York Stock Exchange on June 29, 2010, raising more than $ 200 million in additional financing. Tesla's 2022 full year deliveries were around 1.31 million vehicles, a 40% increase over the previous year, and cumulative sales totaled 3 million cars as of August 2022. In October 2021, Tesla's market capitalization reached $1 trillion, the sixth company to do so in U.S. history.

## II. Methods

In order to price a Tesla's call option using the binomial model, the first thing to do is search for a call option: the used website is Yahoo Finance, where, searching for the specific acronym that represent the desired company, **TSLA** in this case, it can be found the list of all call and put options. First of all, it was selected a call option with maturity $T \simeq 3$ months, the exact expiration day result to be *June 16, 2023*; this call has been chosen to be with **strike at the money**, which means that the price of this specific options contract is close to the current market price of the underlying asset. Main feature of the contract are summarized in table 1:

| **Contract Name** | TSLA230616C0018330 |
|---|---|
| **Last Trade Date** | 2023-03-20 3:54PM |
| **Strike** | 183.33 |
| **Last Price** | 29.10 |
| **Bid** | 28.95 |
| **Ask** | 29.10 |

TABLE 1: Tesla's Call option for matuity $T_3 \simeq 3$ months.

As a second step, all the Hystorical daily Data of the last three months were considered, ranging from *Dec 14, 2022* to *Mar 13, 2023*, in order to calculate the **daily returns** $r_t$ as defined in 1:

$$r_t = \frac{S_t - S_{t-1}}{S_t}, \tag{1}$$

where $S_t$ represent the daily adjusted closing price at the open market day $t$. From the returns it can be obtained two important parameters: the **daily** and **annual volatilities**, they both represent a measure of the degree of fluctuation or variability of the price of a financial asset over time.

$$\sigma_d = \sqrt{\frac{\sum_t^N r_t - \bar{r}}{N-1}}, \tag{2}$$

$$\sigma_y = \sigma_d \cdot \sqrt{252}. \tag{3}$$

In a binomial model, it is supposed that during the option's life the stock price can either move up from initial price $S_0$ to a new level, $S_0 \cdot u$, where $u > 1$, or down to $S_0 \cdot d$, where $d < 1$. In particular the factors $u$ and $d$ depend on the contract's life as:

$$u = e^{+\sigma_y \sqrt{T_3}}, \qquad d = e^{-\sigma_y \sqrt{T_3}}; \tag{4}$$

Where $T$ is now expressed in unit of years, so three months corresponds to $T_3 = 3/12 = 0.25$.

On the other hand, the interest rate was taken from the global rates site: since Tesla belongs to the American market it has been used the **USD LIBOR interest rate** $R$ for maturity equal to three months $R = 4.87\%$; it has been used to compute the capitalisation factors through **simple compounding** $sc$ and **simple discounting** $sd$ as shown:

$$sc = 1 + RT_3, \qquad sd = \frac{1}{1 + RT_3}. \tag{5}$$

And as a final step, collecting all the elements together it has been possible to calculate the the **risk neutral probability weight** $q$

$$q = \frac{e^{RT} - d}{u - d}, \tag{6}$$

and price the call option applying the risk neutral pricing formula, considering the fact that Tesla does not provide any dividend so all the computation has been done without the dividend factor. In the binomial model the payoff of the option can be obtained as:

$$f_u = \max(0, S_0 u - K) \tag{7}$$
$$f_d = \max(0, S_0 d - K)$$

Where $S_0$ is the price of one Tesla share at 2023-03-21, 17:10PM EDT equal to $S_0$ = 194.15$ and $K$ represent the strike price. Finally the **price** of the option $p_0$ can be evaluated:

$$p_0 = e^{-RT}(q f^u + (1-q) f^d) \tag{8}$$

The whole procedure has been repeated for maturity $T_6 \simeq 6$ months, more details of the chosen option are shown in table 2.

## III. Results

All the previous results for maturity $T_3 \simeq 3$ months and $T_6 \simeq 6$ months has been reported in

| **Contract Name** | TSLA230915C00195000 |
|---|---|
| **Last Trade Date** | 2023-03-21 11:39PM |
| **Strike** | 195.00 |
| **Last Price** | 32.76 |
| **Bid** | 32.40 |
| **Ask** | 32.55 |

TABLE 2: Choosen option for maturity $T_6 = 6$ months.

table 3:

| **T [m]** | **3** | **6** |
|---|---|---|
| $\sigma_d$ | 0.049 | 0.044 |
| $\sigma_y$ | 0.783 | 0.703 |
| $u$ | 1.479 | 1.644 |
| $d$ | 0.676 | 0.608 |
| $R$ | 4.87% | 4.83% |
| $sc$ | 1.01 | 1.012 |
| $sd$ | 0.988 | 0.988 |
| $q$ | 0.419 | 0.402 |
| $f_u$ | 103.84 | 118.66 |
| $f_d$ | 0 | 0 |
| $p_{call}$ | 42.42 | 45.70 |

TABLE 3: Results of analysis for three and six months

The final estimations $p^3_{call} = 42.42\$$ and $p^6_{call} = 45.70\$$ have been compared with the **mid prices** $M_3 = 40.00\$$ and $M_6 = 32.46\$$, obtained as the average from bid and ask prices of the two chosen options, that should represent the target of our model.

## IV. Concluding Remarks

Both the call option prices for Tesla with maturity $T_3$ and $T_6$ are higher than the mid prices. Hypothetically this indicates that there is a higher demand for the call options, possibly due to a bullish sentiment in the market, but it should be remembered that the binomial tree model used to calculate the call option prices may not accurately reflect the actual market conditions, as there are many factors that can affect the prices of options such as changes in interest rates, market sentiment, and news events. Therefore, the results should be taken as an approximation and not as a precise indicator of the option's value.

## References


[1] T. Bjork, *Arbitrage Theory in Continuous Time*. Oxford University Press, 2009.

[2] J. Hull, *Option, Future, and other Derivatives*. Pearson, 2006.

[3] Wikipedia contributors, "Tesla, inc. — Wikipedia, the free encyclopedia," 2023. [Online; accessed 23-March-2023].