Stylized Social Security Reforms and Labor Market Frictions

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Abstract

Typical overlapping generations models used in studying Social Security reforms assume a perfectly competitive labor market. By endogenizing the labor market as in Mortensen and Pissarides (1994, 1999), I analyze additional labor market channels through which the Social Security reforms affect employed, unemployed and retired agents' behavior and what ramifications it has for the rest of the population. While the results, in general, are in line with models in previous research, I find a reinforcing feedback effect between (1) interest rate and employment, as well as (2) job search behavior and job openings. These additional labor market channels magnify the long-term beneficial outcomes of policies encouraging private saving and labor force participation. The negative long-term outcomes of labor force participation and saving disincentivizing policies, such as increased income or capital gains taxation, are also magnified with a more dynamic labor market present.

In addition, the model allows unemployed agents of different ages to compete for job openings and suggests that worker substitutability may be affected by the fiscal reform. In the long run, increased labor market participation of older workers may be accompanied by higher employment of the young when private saving increases sufficiently, thereby decreasing the interest rate. When a reform incentivizes elderly labor market participation but not private saving, it may have a negative effect on the employment of the young.

JEL Classification: D15, E13, H55, J11, J26, J64
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1 Introduction

Typical overlapping generation (OLG) models, popularized by Samuelson (1958) and Diamond (1965), abstract from directly modeling the job search behavior of the unemployed and instead either assume perfectly competitive labor market or rely on exogenous transition matrix describing the flow of agents between states. On the other hand, labor market models such as search-and-matching focus on the flows between states and the behavior of the unemployed but lack other important agents' decisions such as consumption, asset accumulation or social security claiming. Based on De la Croix, Pierrard, and Sneessens (2013), I develop a general equilibrium overlapping generations model that endogenizes the labor market flows using search-and-matching features as in Mortensen and Pissarides (1994, 1999). I use the model to study the effects of stylized social security reforms in the US on the behavior of the employed, unemployed, retired and the firm, and analyze reforms' implications on the overall economy.

I find that the existence of a richer and more dynamic labor market magnifies the effects of the reforms on aggregate capital, labor supply and consumption compared to other models. Due to interaction and feedback responses between the employed, unemployed and the firm in labor and capital markets, private saving and labor force participation encouraging policies, such as decrease in old age benefits, increase aggregate capital, employment and consumption in the long run by more than is found in models without search-and-matching labor market. Similarly, policies that disincentivize private saving or labor force participation are found to have more negative effects on aggregate capital, employment and consumption than in similar models with simpler labor market.

Inclusion of search-and-matching labor market into otherwise typical overlapping generations models allows the labor supply to also change on the extensive margin and creates additional labor market channels through which policy reforms affect the economy. Policies that incentivize private saving increase the savings rate and depress the interest rate. Lower interest rate increases investment and firm's labor demand which manifests through an increase in job openings and a higher wage rate. This incentivizes more active job search across the heterogeneous groups of unemployed. Increase in aggregate job search increases the value of a marginal vacancy and firms further increases job openings. Policies that incentivize labor force participation through higher aggregate job search effort increases the value of a marginal vacancy and number of job

openings the firm creates. An increase in the number of job openings increases the probability for unemployed agent to get hired incentivizing further labor force participation by those on the margin. These feedback mechanisms between the employed, unemployed and the firm implies that a particular fiscal reform may have a relatively larger effect on the economy than previously found using models unable to capture this due to simplifications regarding the labor market.

This paper is among many that study fiscal reform effects on the economy. Among the most notable contributions, De Nardi, Imrohoroğlu, and Sargent (1999), İmrohoroğlu, İmrohoroğlu, and Joines (1995, 1999) develop general equilibrium overlapping generations models and study the fiscal reforms with various simplifying assumptions.[1] Imrohoroğlu and Kitao (2012), Kitao (2014) relax some of the previous assumptions by introducing endogenous labor supply and benefit claiming. Braun and Joines (2015) and Kitao (2015) incorporate population aging into heterogeneous-agent OLG models to study population aging in Japan. Nishiyama (2015) considers policy reforms in a similar model calibrated for the US.

The above mentioned models consider the labor market supply only on the intensive margin (number of hours worked). As discussed by Lucas (2007) and De la Croix, Pierrard, and Sneessens (2013), labor market imperfections on the extensive margin (employed or not) may have significant implications on the long-term outcomes of fiscal reforms. Building on De la Croix, Pierrard, and Sneessens (2013), I develop an otherwise typical general equilibrium overlapping generations model but with a labor market defined by search-and-matching as in Mortensen and Pissarides (1994, 1999). Instead of exogenous assumptions on the labor market or a perfectly competitive labor market, the model considers a multitude of complex interdependent endogenous household and firm decisions that drive the labor market flows and endogenously determine the distribution of agents across labor market states. Compared to previous overlapping generations labor market models such as Lugauer (2012) and Hairault, Sopraseuth, and Langot (2010), the model of this paper considers asset accumulation allowing interaction between agents of the model not only in the labor but also the capital markets. De la Croix, Pierrard, and Sneessens (2013) develop a similar model in which they discuss the importance of inclusion of labor market frictions in an aging economy and when studying pension reforms that affect the interest rate. Relative to their paper, I allow agents to endogenously adjust their job search intensity making the

[1] De Nardi, Imrohoroğlu, and Sargent (1999) introduce allow income shocks but set an exogenous retirement age. İmrohoroğlu, İmrohoroğlu, and Joines (1995) allow for unemployment using a Markov process; later land is introduced as another factor of production (İmrohoroğlu, İmrohoroğlu, and Joines, 1999).

labor market distribution completely endogenous. Instead of assuming a representative household, agents are heterogeneous in age, labor market experiences and accumulated assets creating a more dynamic interaction between the employed and unemployed as well as highlighting the frictions between the unemployed of different ages. Relative to their early retirement and shift from pay-as-you-go to a fully funded pension, I study the stylized reforms typically associated with social security in the US: increase in taxation, reduction and delay of old age benefits.

In the next sections, I describe the model (Section 2) and the benchmark economy (Section 3), simulate the fiscal reforms (Section 4), discuss the results (Section 5) and conclude (Section 6).

2 Model

I build a discrete time general equilibrium overlapping generations (OLG) model with search-and-matching features. The model consists of a large number of overlapping generations of agents, a single firm with constant returns to scale technology and government providing transfers to non-employed and social security to retired agents. There exists a labor market in which the unemployed match with the firm's vacancies according to a Cobb-Douglas matching function.

2.1 Agents

The model is populated by overlapping generations of risk-averse agents who enter the model at the model age of 1 (age 15) and face lifespan uncertainty. The conditional probability of surviving from age j to $j+1$ is denoted λ_j with terminal age being T (age 100).

In their first period, all agents are non-employed. In a given time period, agents are heterogeneous in age, asset holdings and current labor market state. Agents can be in one of the three labor market states: non-employed, employed and retired. Number of agents in a particular age j, with assets a, in labor market state m in period t is $P_t^{j,a,m}$. Total population of agents across all possible states is as follows.

$$P_t = \sum_j \sum_a \sum_m P_t^{j,a,m} \tag{1}$$

All agents make consumption ($c_t^{j,a,m}$), saving or next period asset ($a_{t+1}^{j,a,m}$) and retirement ($Ret_t^{j,a,m}$) decisions. Depending on the labor market state, agents either choose

optimal labor supply ($n_t^{j,a,E}$) if employed, adjust job search intensity ($\epsilon_t^{j,a,U}$) which affects the probability of getting a job ($p(\epsilon_t^{j,a,U})$) if unemployed, or neither if retired. Agents face time-separable CRRA utility function:

$$U(c,l) = \frac{(c^{\gamma_1} l^{1-\gamma_1})^{1-\eta} - 1}{1-\eta} \tag{2}$$

where l is leisure or non-market activity time.[2] This function implies that agents utility is increasing in consumption and leisure ($U_c > 0, U_l > 0$), and exhibits diminishing returns ($U_{cc} < 0, U_{ll} < 0$).

Agents enter the model without any assets. To smooth unemployment shocks and in preparation of retirement, agents save in asset a earning interest r_t.

Job search intensity can be thought of as number of job applications posted. Increase in job search intensity increases the chances of getting matched with a vacancy. An agent who is not employed receives a small non-employment transfer (nb) that does not depend on job search or work history.[3]

All employed agents are of the same productivity. Agents can adjust their income by choosing optimal labor supply. Employed agents face an exogenous probability of losing a their job (χ) and moving to non-employment before the next period.

Starting at age j_n, agents qualify for early social security or old-age benefits (ss_n), starting j_l they qualify for larger benefits (ss_l). To receive these benefits agents must be in retired state. They become retired by choosing $Ret = 1$ in the previous period. Depending on retirement timing, agents receive lower or higher benefits for the entire retirement. Retirement is a consuming state.

Due to lifespan uncertainty, agents may die with positive asset holdings which are confiscated by the government.[4] Agents pay labor income (τ_n), capital gains (τ_a) and consumption taxes (τ_c).

Non-employed agents' value function:

$$V^{j,a,U}(c,\tilde{a},\epsilon,Ret) = \max_{c,\tilde{a},\epsilon,Ret} \Big\{ U(c,0,\epsilon) + \beta\lambda_j([p(\epsilon)\tilde{V}^{j+1,\tilde{a},E} + \\ + (1-p(\epsilon))\tilde{V}^{j+1,\tilde{a},U}][Ret=0] + [\tilde{V}^{j+1,\tilde{a},R}][Ret=1]) \Big\} \tag{3}$$

[2] $l = (1-n)$ if employed, $l = (1-\epsilon^{\gamma_2})$ if unemployed. More discussion on the utility function is provided in Appendix 1.

[3] While standard unemployment benefits, dependent on previous employment history, may be more appropriate in this situation, it creates a lot of computational burden without a significant addition to the dynamics of the model.

[4] Equivalent to 100% inheritance tax.

subject to

$$[1 + r(1 - \tau_a)]a + (nb) = (1 + \tau_c)(c) + \tilde{a} \tag{4}$$

Employed agents' value function:

$$V^{j,a,E}(c, \tilde{a}, n, Ret) = \max_{c,\tilde{a},n,Ret} \Big\{ U(c, n, 0) + \beta\lambda_j([(1 - \chi)\tilde{V}^{j+1,\tilde{a},E} + \\ + (\chi)\tilde{V}^{j+1,\tilde{a},U}][Ret = 0] + [\tilde{V}^{j+1,\tilde{a},R}][Ret = 1]) \Big\} \tag{5}$$

subject to

$$[1 + r(1 - \tau_a)]a + (1 - \tau_n)(wn) = (1 + \tau_c)(c) + \tilde{a} \tag{6}$$

Retired agents' value function:

$$V^{j,a_t,R_i}(c, \tilde{a}) = \max_{c,\tilde{a}} \Big\{ U(c, 0, 0) + \beta\lambda_j \tilde{V}^{j+1,\tilde{a},R_i} \Big\} \tag{7}$$

subject to

$$[1 + r(1 - \tau_a)]a + ss_i(j, Ret^j = 1) = (1 + \tau_c)(c) + \tilde{a} \tag{8}$$

where $_i$ stands for the retirement state in terms of its timing (normal, late) and ss_i are the benefits associated with retirement state.

$$ss_i(j, Ret^j = 1) = \begin{cases} 0 & \text{if } j < j_e | Ret^j = 0 \\ ss_n & \text{if } j \geq j_n \wedge Ret^j_{j_n} = 1 \\ ss_l & \text{if } j \geq j_l \wedge Ret^j_t = 1 \text{ for } t \geq j_l \end{cases} \tag{9}$$

2.2 Firm

There exists one profit-maximizing firm that employs all of the workers and hires all of the capital. Firm's production follows Cobb-Douglas production function:

$$Y_t = F(K_t, N_t) = A_P K_t^{\alpha} N_t^{1-\alpha} \tag{10}$$

where K_t is capital holdings of all agents and N_t is the total labor supply in period t, and α is output elasticity parameter.

$$K_t = \sum_j \sum_a \sum_m P_t^{j,a,m} a_t^{j,a,m} \tag{11}$$

where $a_t^{LM,j}$ is the asset holdings of agent at age j in labor market state LM in period t.

$$N_t = \sum_j \sum_a P_t^{j,a,E} n_t^{j,a,E} \tag{12}$$

where $n_t^{j,a,m}$ is the labor supply of an employed agent at age j with asset holdings a, in employed state, $m = E$, in period t.

The firm rents all of the available capital paying the rental rate r_t and chooses the number of vacancies to post (V_t). The firm exhibits increasing vacancy posting costs (vV_t^{ζ}).[5]

Firm's profit maximization problem can be stated as follows.

$$W_t = \max_{V_t, K_t} \left\{ F(K_t, N_t) - r_t K_t - w_t N_t - vV_t^{\zeta} + R_t^{-1} E_t W_{t+1} \right\} \tag{13}$$

First order conditions for the firm in terms of the two choice variables are as follows.

$$r_t = \frac{\partial F(K_t, N_t)}{\partial K_t} \tag{14}$$

$$\zeta v V_t^{\zeta - 1} = \frac{\partial E_t F(K_{t+1}, N_{t+1})}{\partial E_t N_{t+1}} \frac{\partial E_t N_{t+1}}{\partial V_t} - R_{t+1}^{-1} q_t E_t w_{t+1} \overline{n}_{new,t} \tag{15}$$

where q_t is the probability to fill a posted vacancy in period t and $\overline{n}_{new,t}$ is the average hours worked by a newly hired worker in period t.[6].

The firm pays the marginal product of capital for the capital it rents. It rents all the capital agents supply for the firm. The firm keeps increasing the number of vacancies until the cost of an additional vacancy equals the expected benefit of that vacancy.

Firm's optimal decisions depend on next period's states: labor, capital, wages. The firm expects next period's capital and wages to be in equilibrium and thus not change.

$$E_t K_{t+1} = K_t \tag{16}$$

$$E_t w_{t+1} = w_t \tag{17}$$

The amount of labor in the next period depends on the exogenous separation shock

[5] While this assumption is not necessary, it reduces firm's vacancy posting responsiveness to changing aggregates K and N.

[6] The firm makes an assumption that newly hired workers in the next period behave the same as newly hired workers in the current period.

rate (χ), endogenous exit rate from employment (H_t), the number of vacancies the firm posts (V_t) and the probability to fill a posted vacancy (q_t).

$$E_t N_{t+1} = (1 - \chi - H_t)N_t + q_t V_t \overline{n}_{new,t} \tag{18}$$

The wage paid to the workers is a fraction of their marginal product of labor.[7]

$$w_t = \gamma_w (1 - \alpha) A_P K_t^{\alpha} N_t^{-\alpha} \tag{19}$$

2.3 Government

The government receives labor income tax revenue ($\tau_n w_t N_t$), capital gains tax revenue ($\tau_a r_t A_t$), consumption tax revenue ($\tau_c C_t$) levied on the workers. The model abstracts from setting a taxable maximum and earnings test.

Other tax revenues (OTR) include taxing accidental bequests and firm's profits. Since some agents die without depleting their assets, government collects those assets which is equivalents of imposing 100% tax rate on accidental bequests. In addition, the government owns the firm so the after-tax profits are also collected by the government.[8]

$$GR_t = \tau_n w_t N_t + \tau_a r_t A_t + \tau_c C_t + OTR_t \tag{20}$$

Using tax revenues, the government provides Social Security benefits to the retirees, non-employment transfers to the non-employed, pays for fixed costs (CG) and other government expenses (OG_t).[9]

$$GE_t = PR(ss) + PU(ub) + OG_t + CG \tag{21}$$

where PR is the size of the retired population and PU is the size of the non-employed population.

Keeping other policies fixed, the government balances the budget by adjusting the consumption tax rate.

[7] Standard search and matching models use Nash bargaining to find the equilibrium level of wages. However, this form is close to equivalent with γ_w representing workers' bargaining power. Similarly, Lugauer (2012) assumes that worker's wage is a fraction of firm's output. See Appendix X for more details.

[8] Alternatively, accidental bequests and after-tax profits can be equally distributed among the living agents. Under either assumption, the results of the model are quantitatively equivalent.

[9] Other government expenses are simply equal to other government revenues ($OG_t = OTR_t$). Under this formulation of the model, the government budget is not affected when accidental bequests or firm's profits change. Under alternative version of the model, accidental bequests and firm's profits are distributed equally among the living agents making both, other government expenses and other government revenues equal to zero.

$$GR_t = GE_t \tag{22}$$

2.4 Labor Market

Instead of assuming a competitive equilibrium without unemployment or unemployment based on exogenous transition matrix, the labor market in this model is driven by a search-and-matching mechanism. Total number of matches follow a Cobb-Douglas function:

$$M_t = A_M V_t^{\varphi} S_t^{1-\varphi} \tag{23}$$

where A_M is matching technology, V_t is number of posted vacancies, and S_t is the sum of job applications posted by all of the unemployed[10] in period t.

The probability for a single application in time period t to be successful at connecting an unemployed agent with a vacancy is defined as:

$$p(1) = \frac{A_M V_t^{\varphi} S_t^{1-\varphi}}{S_t} = A_M V_t^{\varphi} S_t^{-\varphi} \tag{24}$$

The probability for an unemployed agent of age j with assets a and search intensity $\epsilon_t^{j,a,U}$ to match with a vacancy is defined as:

$$p(\epsilon_t^{j,a,U}) = \epsilon_t^{j,a,U} \frac{A_M V_t^{\varphi} S_t^{1-\varphi}}{S_t} = \epsilon_t A_M V_t^{\varphi} S_t^{-\varphi} \tag{25}$$

The probability for a firm to fill a vacancy depends on the number of vacancies and the total matches in period t.

$$q_t = \frac{M_t}{V_t} = \frac{A_M V_t^{\varphi} S_t^{1-\varphi}}{V_t} = A_M V_t^{\varphi-1} S_t^{1-\varphi} \tag{26}$$

Stock of employed E_t, non-employed (U_t), early and late retired ($R_{n,t}$, $R_{l,t}$) agents obey the following dynamics.

$$E_t = \sum_j \sum_a \lambda_j (1-\chi)(1-Ret_{t-1}^{j,a,E}) P_{t-1}^{j,a,E} + \sum_j \sum_a \lambda_j (p(\epsilon_t^{j,a,U}))(1-Ret_{t-1}^{j,a,U}) P_{t-1}^{j,a,U} \tag{27}$$

$$U_t = \sum_j \sum_a \lambda_j (\chi)(1-Ret_{t-1}^{j,a,E}) P_{t-1}^{j,a,E} + \sum_j \sum_a \lambda_j (1-p(\epsilon_t^{j,a,U}))(1-Ret_{t-1}^{j,a,U}) P_{t-1}^{j,a,U} \tag{28}$$

[10] Non-employed agent with above zero job search is referred to as unemployed.

$$R_{n,t} = \sum_{j=1}^{j=j_n-1} \sum_{a} \lambda_j (Ret_{t-1}^{j,a,E}) P_{t-1}^{j,a,E} + \sum_{j=1}^{j=j_n-1} \sum_{a} \lambda_j (Ret_{t-1}^{j,a,U}) P_{t-1}^{j,a,U} + \lambda_j P_{t-1}^{j,a,R_n} \quad (29)$$

$$R_{n,t} = \sum_{j=j_l-1}^{j=J} \sum_{a} \lambda_j (Ret_{t-1}^{j,a,E}) P_{t-1}^{j,a,E} + \sum_{j=1}^{j=j_n-1} \sum_{a} \lambda_j (Ret_{t-1}^{j,a,U}) P_{t-1}^{j,a,U} + \lambda_j P_{t-1}^{j,a,R_l} \quad (30)$$

2.5 Stationary Equilibrium

Given the demographic, labor market and policy variables, an intertemporal equilibrium with labor market frictions is such that:

1. consumption, saving, job search and retirement decisions maximize agents' utility (eq. 3, 5, 7) subject to their budget constraints (eq. 4, 6, 9);
2. posted vacancies maximize firms profits (eq. 15);
3. the labor market clears; (eq. 23, 26, 27, 28, 29, 29)
4. the goods market clears;

$$Y_t + (1-\delta)K_t = C_t + G_t + K_{t+1} \quad \text{where } G_t = OG_t + CG \quad (31)$$

5. consumption tax is such that government budget is balanced; (eq. 22)
6. the population, capital satisfy and total labor aggregation conditions; (eq. 1, 11, 12)
7. the aggregate variables and distribution of agents across states is stationary.

$$\{P\}_{t+1}^{j,a,m} = \{P\}_{t}^{j,a,m} \quad (32)$$

2.6 Calibration

This paper studies the labor market channels through which fiscal reforms affect the economy in the long run. I calibrate the employment by age group and social security to broadly match the US and examine how changes in taxation and social security benefits affect the economy through changes in the labor market. Most of the parameters used are similar to related literature with job destruction and vacancy cost parameters calibrated to match US employment rates by age group.

Agents enter the the model at the age of 15 (model age 1) as unemployed without any assets. Each model period represents 5 years with the maximum being model period 18 (equivalent to age of 100). The fertility is kept constant and conditional survival probabilities are set according to Bell and Miller (2005) for the year 2000 for the initial steady state, and for the year 2100 for the new steady state. Subjective period discount factor is 0.96 which is equivalent to yearly discount factor of 0.992. As in Imrohoroğlu and Kitao (2012), γ_1, or the weight on consumption in the utility function, is calibrated such that agents, on average, spend a third of their time on market activities (labor supply or job search). The curvature parameter, η, in the utility function is set to 2 which yields a coefficient of relative risk aversion equal to 1.33 which is between typically used values for CRRA.[11]

The firm's technology parameters are similar to values used in previous literature. The income share parameter in the production function is 0.4 and depreciation rate, is such that capital to output ration on a yearly basis is 3. Production technology scale parameter (A_P) is set to unity.

As in De la Croix, Pierrard, and Sneessens (2013), the elasticity of matches w.r.t. vacancies as well as worker's bargaining power is set to 0.5.[12]. Matching technology scale parameter (A_M) is set to unity. Being novel parameters, job search curvature parameter (γ_2) is set to 2. Vacancy posting cost curvature ($zeta_1$) parameter is set to unity.

Government runs a simplified pay-as-you-go social security program in which average retirement benefits at full retirement age are approximately 40% of average earnings. Early retirees face a 20% benefits penalty consistent with the current Social Security regulations.[13] Labor income taxes are set to 12.4%[14]. Other tax rates are set to zero for the initial calibration. Government transfers to non-employed (nb) are set to to be around 11% of average wages to prevent zero consumption. As mentioned earlier, the model abstracts from setting a taxable maximum and earnings test.

Job destruction (χ) and vacancy cost parameters (v) are set to broadly match employment by age group in the US in the year 2000 adjusted to exclude population that is never active in the labor market.[15]

[11] While most macroeconomists use the values between 1 and 5, Chetty (2006) argues that CRRA is closer to 1.

[12] Note, that this model uses a simplified worker's bargaining process.

[13] Maximum penalty under current SS regulations is approximately up to 30% depending on individual's full retirement age citepSSA.online.handbook.

[14] This represents the actual 6.2% tax rate for the employer and the employee.

[15] Since all agents in the model are interested in work, data for employment to population in year 2000 is adjusted to exclude population that is never active in the labor market. Population size of each age group is reduced by such amount that unemployment to population for prime age workers would match the estimated unemployment rate, U6.

Table 1: Parameters

Environment		
T	Maximum Age	18 (100)
Production Technology		
A_P	Prod. technology par.	1.00
α	Prod elasticity par, CRS	0.40
δ	Depreciation par	0.45 (0.11 yearly)
Government		
τ_n	Labor Income Tax	12.4%
j_r	Normal Retirement Age	12 (70)
j_{er}	Early Retirement Age	11 (65)
ss	Social security par.	0.145
ss_e	Early SS benefits penalty	0.80
nb	Non-employment benefits par.	0.04

Matching Technology		
A_M	Matching technology scale par.	1.00
φ	Matching elasticity par.	0.50
γ_w	Workers' bargaining power	0.50
Firm Specific		
χ	Job destruction par.	0.045
v	Vacancy cost par.	0.0747
ζ_1	Vacancy cost curvature par.	1.00
Agent Specific		
β	Discount factor	0.96
γ_1	Utility par.	0.36
γ_2	Utility par.	2.00
η	Utility par.	2.00

Calibrated parameters for the reform simulations in Sections 4.1 through **??**.

2.6.1 Sensitivity Analysis

The results from policy reforms presented below may depend on the calibration and specific parameter values chosen. In order to show that the results are robust, I rerun the experiments using different values of some of the key parameters.

Matching elasticity parameter, φ, workers' share of MPL or bargaining power, γ_w, were each tested using values 0.25 and 0.75. Explain if results are the same, and what are some, if any, differences.

Value of utility job search curvature parameter, γ_2, was replaced with 1 and 3. The results are qualitatively identical.

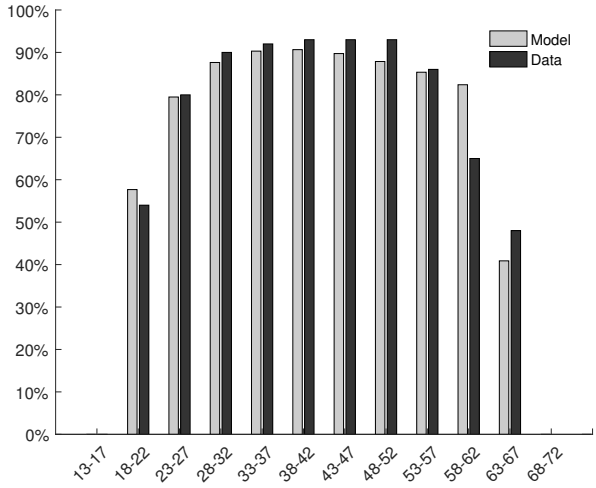
3 The Benchmark

As seen in Figure 1, even with many simplifying assumptions such as not allowing to work after claiming benefits, no productivity evolution in terms of age or productivity shocks, and only two retirement states, the model replicates the employment rates by age group fairly well.

Agents accumulate assets in employment and decumulate assets in retirement. Knowing that the terminal model age period is 18 (100) and having no bequest motive, the small fraction of agents who survive to the their final period, consume all their assets.

The emphasis of the model is on the agents' endogenous job search and retirement decisions. In Figure 2, we see that the young put more effort into job search than older agents. This is due to the fact that the young have a longer work horizon making the value of employment significantly greater over non-employment. For older agents, the remaining work horizon is shorter making the transition from non-employment to employment less utility-improving than for the young. Surprisingly, optimal job

Figure 1: Employment by Age Group

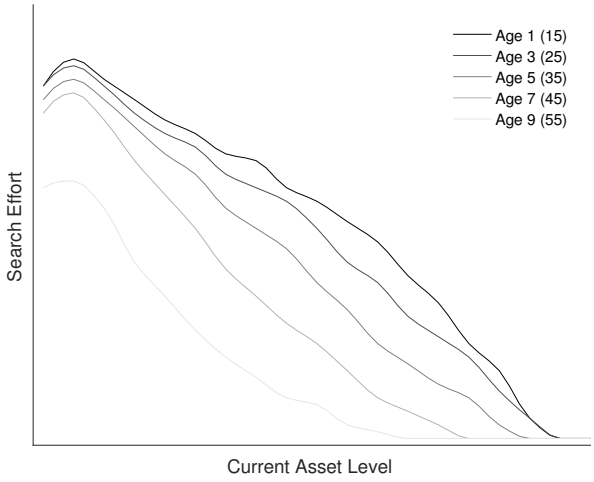


Employment by age group in the model and year 2000 adjusted for population that never joins the labor force.

search intensity for agents with no assets at all is lower than for those who have slightly more assets as can be seen in Figure 2. The utility function explains such behavior of unemployed agents. Due to multiplicative consumption and leisure terms in the utility function, at very low levels of consumption, small increase in consumption significantly decreases the marginal disutility of search and, thus, the agent does not mind searching for work with slightly more effort. However, as level of consumption keeps rising, a further increase in consumption has only a small effect on marginal disutility of job search. Agents with high levels of assets, are able to sustain higher levels of consumption (at which marginal utility of consumption is low) throughout their lives making it less important to find work. Across all ages, agents with higher assets search for work with less effort.

The policy functions for saving, labor supply, job-search, retirement and consumption indicate that agents are smoothing consumption over time by working more and saving in the first half of their lives and dissaving in retirement. Agents with lower asset holdings choose to work more to increase savings and consumption. Unemployed whose alternative to retirement is uncertain job search, retire sooner than those who are employed. Since policy functions are as in traditional OLG models, they are not presented in the paper.

Figure 2: Job Search Policy Functions



Job search effort decisions of non-employed agents depend on current asset holdings and age. For presentation purposes, the job search policy functions are smoothed.

4 Aging, Increasing Fiscal Expenses and Stylized Reforms

To stylistically replicate population aging and evaluate how various fiscal reforms affect budget deficit, employment and the overall economy, mortality rate is simulated to match the estimated survival function for the year 2100.[16]

Significantly higher survival rates of older individuals (see Figure 3) greatly increases the old-age dependency ratio. Without changes to taxation or benefits, this leads to inevitable budget deficit.

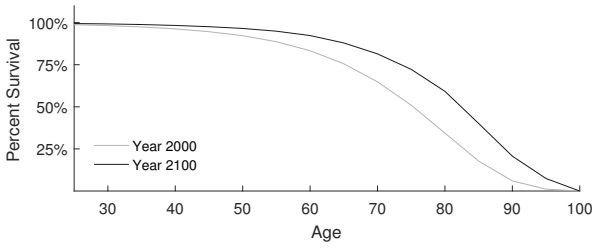
De la Croix, Pierrard, and Sneessens (2013), using a similar model calibrated to French data, discuss the effects of early retirement and a shift to fully funded pension scheme. I, on the other hand, consider two reforms that involve increasing taxation and two reforms that reduce government social security program commitments.

Reform 1 increases the labor income tax rate by 3.33 percentage point. Under Reform 2 capital gains tax is raised by 9.5 percentage points. Assuming no change in agents' behavior, these tax increases would eliminate the deficit. Since agents adjust their decisions when facing taxation, the budget deficit is eliminated using the consumption tax.

Reforms 3 and 4 deal with reducing government's spending on social security program. Reform 3 reduces social security benefits by 18.61 percent which would eliminate

[16] Fertility is not changed.

Figure 3: Survival Function



Survival function for the initial steady state (year 2000) and the new steady state (year 2100) based on estimates by Bell and Miller (2005).

the surplus assuming no change in agents' behavior. As with previous reforms, since agents adjust their behavior, consumption tax balances the remaining budget deficit or surplus. Reform 4 simply delays benefits by one model period (5 years).[17]

Table 2 shows the main results for each of the reforms relative to increase in labor income tax (Reform 1). While any reform can be chosen as the benchmark, income taxation is widely believed to have significant efficiency costs (Keane, 2011) and, at the same time, is one of the more likely reforms.[18]

To highlight the value added of introducing more dynamic labor market and the contribution of this paper, I run the exact reforms in two additional variations of the model with simplified labor markets. The first uses simple search frictions (SF) only based on agents' job search effort,[19] and the second assumes an exogenous labor market flows (EX). The parameters ζ_2 and ζ_3 are calibrated to match employment-to-population by age.

$$\text{SF model: } p(\epsilon_t^{j,a,U}) = (\epsilon_t^{j,a,U})^{\zeta_2} \tag{33}$$

$$\text{EX model: } p = \zeta_3 \tag{34}$$

4.1 Reform 1: Increase in Income Tax

Under Reform 1, labor income tax is increased by 3.33 percentage points. This creates many direct and indirect channels through which it affects employed and unemployed agents' behavior.

[17] The magnitude of the Reform 4 is limited by the model time increments.

[18]

[19] Ljungqvist and Sargent (1998), Alvarez and Veracierto (2001), Cozzi and Fella (2016) and Kitao, Ljungqvist, and Sargent (2017) use search frictions solely based on agent's search behavior.

Table 2: Results of Stylized Policy Reforms

	Reform 1: $\tau_n \uparrow$	Reform 2: $\tau_a \uparrow$	Reform 3: $SS_{ben} \downarrow$	Reform 4: $SS_{age} \uparrow$
K	-	-0.21%	7.70%	6.56%
N	-	1.90%	4.25%	5.41%
C	-	2.17%	3.57%	3.35%
W	-	0.42%	1.27%	1.40%
E	-	0.25%	1.46%	9.13%
V	-	0.02%	5.29%	2.57 %
$\overline{S}$	-	2.76%	3.84%	-3.91%#
w	-	-0.84%	1.30%	0.43 %
r	-	4.74%	-7.18%	-2.37 %
τ_c	-	-10.90%	-13.50%	-27.26%

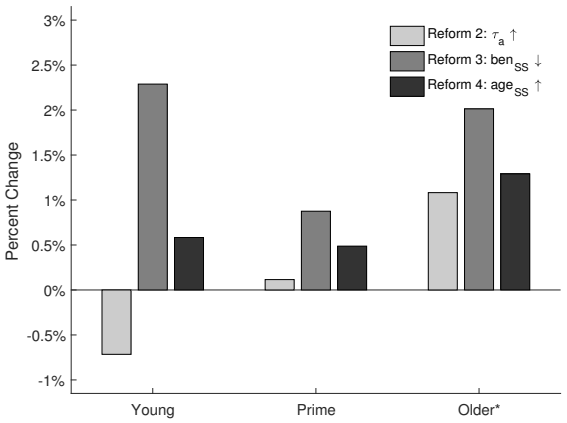
Policy simulation results.
K - aggregate asset holdings; N - aggregate labor supply; C - aggregate consumption; W - welfare measured as a sum of equally weighted living agents' utilities; E - total employment; V - number of vacancies posted by the firm; $\overline{S}$ is the average job search effort of the unemployed; w - equilibrium wage rate; r - equilibrium interest rate. Reform 1 - labor income tax; Reform 2 - capital gains tax; Reform 3 - decrease in benefits; Reform 4 - increase in retirement age.

In an exogenously determined labor market model (EX), the labor supply and asset accumulation behavior is affected. As discussed in many other policy analyses, higher labor income tax reduces after-tax income from labor which directly dis-incentivizes labor supply. Under this reform, agents labor supply is the lowest of all reforms which negatively affects agent's asset accumulation further reducing the wage and incentive to supply labor.

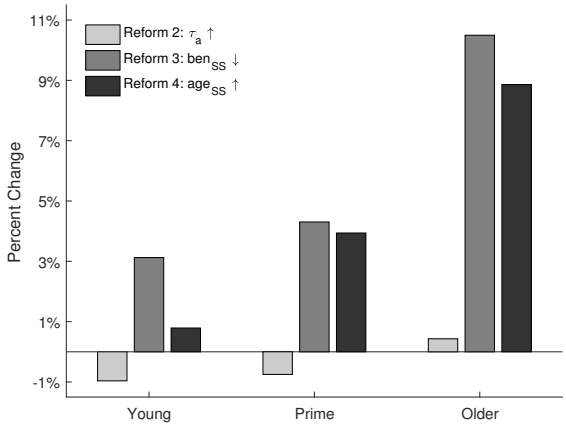
In the two models with search effort choice (SM and SF), lower after-tax returns from a unit of labor decreases the value of employment. Under this reform, agents across all age and wealth groups exert less job search effort, and both employed and unemployed agents retire earlier.

Allowing a more dynamic labor market (SF), as in the model of this paper, creates two additional labor market mechanisms. First, due to lower savings rate and resulting higher interest rate, the firm rents less capital. Decrease in rental capital leads to lower marginal product of labor and lower profit value of a potential hire negatively affecting the firm's vacancy posting. Second, the decrease in the average job search and earlier retirement by the unemployed makes it more difficult for the firm to fill a given vacancy. Decreased probability of hiring a worker with a marginal vacancy lowers the value of a vacancy and leads to decrease the number of new job openings posted even further. This creates a feedback effect: a decrease in firm's posted vacancies reduces the job finding rate from a given application which further dis-incentivizes job search and incentivizes earlier retirement.

Figure 4: Employment and Asset Holdings



(a) Employment by age group. Young, prime and older workers represent 20-25, 30-50, 55-65 year-old agents, respectively. Results are relative to Reform 1. Labor supply is qualitatively identical.



(b) Asset holdings by age group. Young, prime and older workers represent 20-25, 30-50, 55-100 year-old agents, respectively. Results are relative to Reform 1.

Under this reform, average job search, aggregate labor supply and employment is the lowest among the reforms leading to the lowest levels of consumption and welfare.

4.2 Reform 2: Increase in Capital Gains Tax

Under Reform 2, capital gains tax is increased by 9.5 percentage points and consumption tax covers the remaining budget deficit.

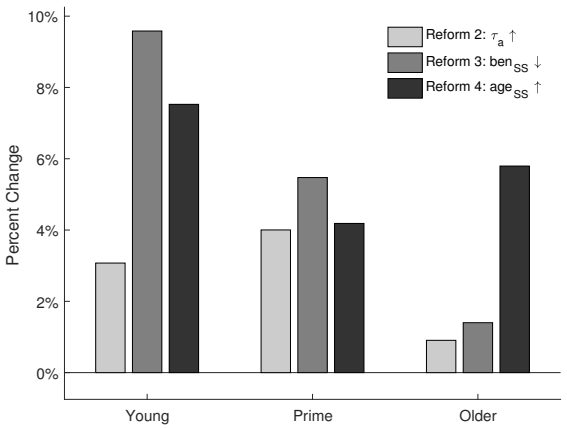
Absent of a any dynamics in the labor market, the agents reduce the savings rate due to lower after-tax returns and, in order to maintain consumption, increase labor supply, especially among older workers.

In the models with search frictions (SM and SF), agents also increase job search as the lower returns increases the importance of employment. In both models, agents increase job search effort. This has a positive effect on employment, and other aggregates as is seen by comparing model SF and EX for Reform 2.

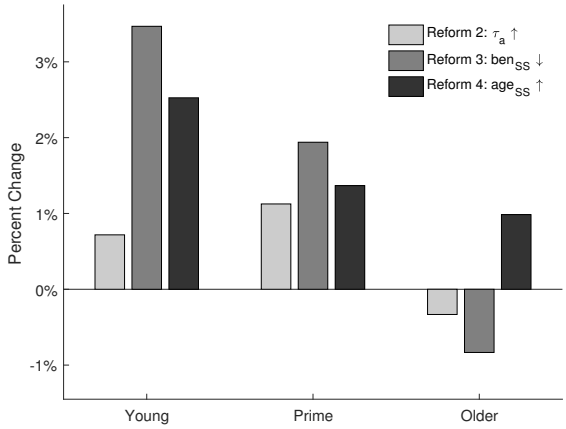
In the model of this paper, lower saving-investing reduces the marginal product of labor and firm's profitability of a marginal vacancy. This induces firms to post fewer vacancies causing employment among the younger population to be lower by around -0.75% as is seen in Figure 4a.

The main difference with the Reform 1 is that agents' search effort remains high due to the fact that the after-tax wage is higher. Increased average job search intensity despite fewer vacancies allows for higher employment among prime and older age

Figure 5: Consumption and Utility



(a) Consumption by age group. Young, prime and older workers represent 20-25, 30-50, 55-100 year-old agents, respectively. Results are relative to Reform 1.



(b) Welfare by age group. Young, prime and older workers represent 20-25, 30-50, 55-100 year-old agents, respectively. Results are relative to Reform 1.

workers resulting in higher aggregate employment. Moreover, older employed agents choose to keep labor supply higher and defer retirement in order to replace lower capital income. This leads to overall slightly higher aggregate consumption compared to Reform 1 but since the employment and consumption is slightly lower among younger agents, it leaves a scaring effect on the agents and the aggregate welfare is almost the same under each of these two reforms.

Relaxing assumptions on the labor market introduces additional dynamics. The more simplistic models, EX and SM, overstates the benefits of moving from labor taxation to capital taxation. Once higher interest rate is taken into account, the benefits of capital gains taxation seem to be diminished.

4.3 Reform 3: Decrease in Social Security Benefits

The stylized Reform 3 decreases the fiscal expenses instead of increasing revenues through taxation. Under this reform, the social security or old-age benefits[20] are reduced by 18.61%.

Lower benefits in older age incentivize employed agents to increase labor supply and save more during prime working age. Larger increase in aggregate asset holdings than in aggregate labor supply works to increase the marginal product of labor and the wage rate. Under this reform, workers sustain higher labor supply into older age.

[20]The benefit is not conditional on agent's work history or income.

Government's burden is reduced due to higher labor income and consumption tax revenues, as well as lower benefits. This is what happens in the model variation EX.

If unemployed are allowed to adjust search decisions as in SM and SF model variations, they increase job search which boosts employment. While the young unemployed are not that significantly affected, the decisions of the elderly are significantly different compared to the benchmark reform. Lower old age benefits increase the value difference between employment and retirement. In other words, lower benefits incentivize more people to stay in the labor force, accumulate savings and put significantly more effort into job search, even at older ages. Among all of the reforms, under Reform 3 the agents' average job search effort is the highest.

Introduction of search-matching labor market as in SM model, creates an additional feedback effect. Higher savings rate and increased rental capital of the firm increases labor demand. In response, the firm increases the number of posted vacancies by 5.5%. Increase in average job search and a number of posted vacancies allows the employment to increase by 1.4%. Even though, the young unemployed face increased competition from older unemployed who stay in the labor force longer and put more job search effort, the increase in vacancies is large enough to increase employment among all groups of workers. Increased employment allows for higher asset accumulation and consumption across all age groups as is seen in Figures 4 and 5a.

Private saving and labor supply incentvizing policies, such as decrease in old age benefits, create positive feedback effects between the unemployed and the firm which further boosts the employment compared to models without search and matching labor market.

Table 3: Results of Stylized Policy Reforms. Model Comparison

	Reform 1: $\tau_n \uparrow$	Reform 2: $\tau_a \uparrow$			Reform 3: $SS_{ben} \downarrow$			Reform 4: $SS_{age} \uparrow$		
		SM	SF	EX	SM	SF	EX	SM	SF	EX
K	-	-0.2%	-0.3%	-1.8%	7.7%	6.7%	6.4%	6.6%	6.3%	6.0%
N	-	1.9%	1.7%	1.0%	4.2%	2.8%	2.5%	5.4%	5.3%	5.4%
E	-	0.2%	0.3%	0%	1.5%	0.6%	0%	9.1%	9.2%	9.7%
C	-	2.1%	2.1%	1.4%	3.6%	1.9%	1.9%	5.3%	5.3%	5.5%
W	-	0.4%	0.6%	0.5%	1.3%	0.6%	0.6%	1.4%	1.3%	1.4%

Aggregate results from policy simulations.
K - aggregate asset holdings; N - aggregate labor supply; C - aggregate consumption; W - welfare measured as a sum of equally weighted agents' utilities; E - total employment.
Reform 1 - labor income tax; Reform 2 - capital gains tax; Reform 3 - decrease in benefits; Reform 4 - increase in retirement age.
.

While aggregate welfare is higher due to higher consumption, consumption increases by much more than agents' welfare. This is because a small group of agents who do not accumulate any assets prior to retirement must live on significantly lower old age benefits and/or increase job search effort, both of which negatively affect their utility. As is seen in Figure 5b, older agents' utility is lower compared to the benchmark reform.

4.4 Reform 4: Delay of Old Age Benefits

The final stylized policy reform delays the social security benefits by one model period (equivalent to 5 years) and, as with all other reforms, consumption tax fills the remaining budget imbalance.

Under Reform 4, across all specifications of the model, employment increases significantly due to agents being effectively forced[21] to postpone retirement by 5 years. Aggregate results are qualitatively similar to reduction of benefits. In the model without any endogenous search frictions (EX), agents maintain higher labor supply and savings rates throughout their careers the additional period of work.[22]

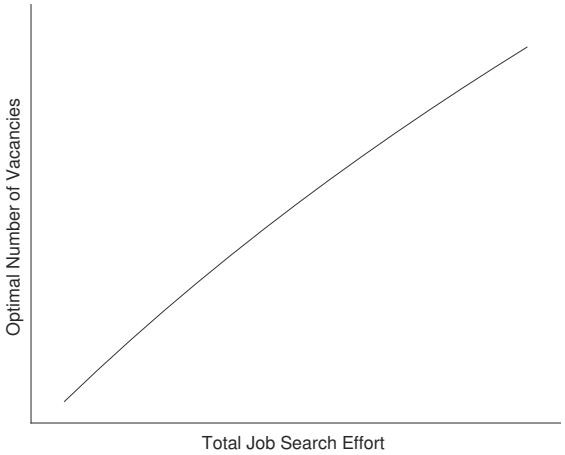
With a longer working horizon, the value difference between employment and unemployment increases incentivizing agents to increase job search effort. Without any additional dynamics, due to higher job search effort, the employment across the age groups increases (SF).

In the model of the paper, there are additional effects. Postponed retirement by the older workers decreases the need to post as many vacancies which negatively affects all unemployed. However, increase in capital and, thus, the marginal product of labor more than makes up for delayed retirement of the older. The overall number of vacancies posted by the firm increases. Finally, increased competition for vacancies due to more older unemployed staying in the labor force longer makes it more difficult for young to get employed. In the main model of the paper, employment and capital accumulation of the young is lower under Reform 4 compared to Reform 3 as is seen in Figure 4.

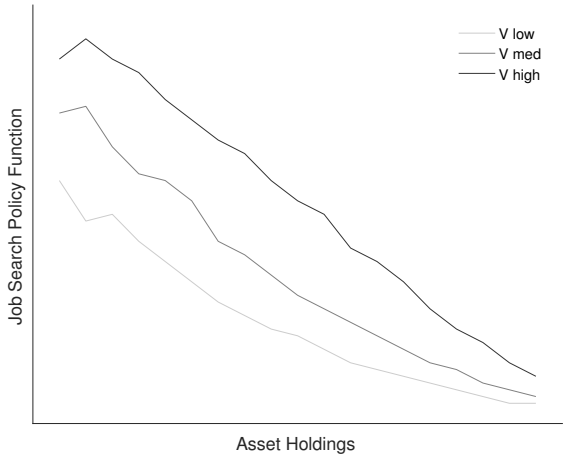
[21]

[22] Exogenous matching probabilities make the old unemployed just as likely to gain employment as the young. In the models with job search, the older unemployed never find it optimal to exert as much job search effort to have such higher probabilities of matching with a vacancy. Thus, in the model with exogenous LM flows, higher increase in the employment of the old pushes the overall employment higher compared to the other two model variations.

Figure 6: Job Openings and Job Search



(a) All else held constant, number of posted vacancies increase when total job search effort increases.



(b) All else held constant, job search effort of an unemployed agent increases when number of available vacancies increase.

5 Discussion

5.1 The Value of Search and Matching in OLG

The aggregate steady state results presented in this paper are very much aligned with the previous research. As in De Nardi, Imrohoroğlu, and Sargent (1999), Imrohoroğlu and Kitao (2012), Kitao (2014) among other, policies that encourage private saving labor force participation have significantly better outcomes in a long run compared to reforms involving higher taxation.

In this paper, I introduce search-matching-inspired labor market in which work, saving and retirement decisions of the employed, job search and retirement decisions of the unemployed, and the firm's profit maximization creates numerous additional effects.

Previous research already hinted at long-term benefits of reducing old-age or social security benefits, but the model developed in this paper suggests that aggregate effects from such policies on the economy may be larger than estimated using models with perfectly competitive labor markets. Policies that incentivize private saving (Reforms 3 and 4), decrease interest rate and increase firm's rental capital. This, in turn, increases the marginal product of labor or, in other words, the labor demand. This manifests in more job openings created and higher wages paid. Moreover, such policies increase value difference between employment and unemployment, making it more

important for unemployed workers to obtain employment as soon as possible. This increases their job search effort which in turn with higher vacancy posting leads to significantly increased employment. In addition, as seen in Figure 6, there are positive feedback effects between vacancy posting and job search decisions. Increase in number of vacancies, increases the probability of a given unemployed agent to match with a vacancy. As it is easier to find work, more unemployed participate in job search and with higher effort levels.Increased number of unemployed and overall job search effort levels, increases the number of matches for a given vacancy. This, in turn, makes a marginal vacancy more profitable and making the profit maximizing firm to increase number of vacancies. This looping feedback mechanism in the labor market results in significantly higher employment, labor supply, capital accumulation and consumption under Reform 3.

Similarly, this above discussed effect works the other way when enacted policies discourage labor force participation and private saving. Labor income tax discourages labor supply and makes it optimal for more elderly out-of-work agents to retire instead of attempting to rejoin the workforce. This also negatively affects the savings rate and asset accumulation of agents across the age distribution. Similarly, capital gains tax directly reduces private savings rate. Both of these types of taxation contribute to lower aggregate asset holdings, higher interest rate, decreased capital renting by the firm and lower labor demand. Wages and the number of job openings decreases incentivizing more unemployed to leave the labor force early which works to further decrease the optimal number of job openings. This feedback effect results in significantly lower aggregate employment and asset holdings under Reforms 1 and 2.

5.1.1 Substitutability of Young and Old

The analysis brings some insights into the discussion about the lump of labor idea stating that the number of jobs or the amount of work to be done is fixed. It implies that increased participation of elderly may crowd out younger people from employment. In many European countries, favorable policies towards early retirement, in part due to such arguments, contributed to falling average retirement age (Fenge, 2001).

While there is substantially more research pointing against such claims of fixed amount of work and younger and older worker substitutability,[23] some[24] find cases

[23] See Gruber and Wise (2010), Walker (2007), Hebbink (1993), Banks, Blundell, Bozio, and Emmerson (2010), Börsch-Supan and Schnabel (2010), Kalwij, Kapteyn, and De Vos (2010) among others.

[24] For example, see discussion by Hamermesh (1993), Card and Lemieux (2001), Fitzenberger and Kohn (2006), Boeri, Garibaldi, and Moen (2017).

with some degree of substitutability between the young and the elderly employment.

The framework of the model by construction assumes perfect substitutability of younger and older workers. The firm does not discriminate against young or old since their productivity, health and wages are homogeneous. The only difference between workers of different age that does exist: the remaining horizon of work[25] is assumed away as age discrimination is illegal in most developed economies.[26] I find that in the long run substitutability between the young and elderly employment may be affected by the implemented fiscal reforms.

Under all simulated reforms, the relative employment by age groups was noticeably different. Under Reform 2, increased employment of the elderly displaced some of the younger workers while under Reforms 3 (and Reform 4), it was accompanied by increased (almost unchanged) employment of the young (see Figure 4a). These findings suggest that in the long run it is important to focus on the incentives reforms create not only to labor supply but also to capital accumulation. Sufficient increase in aggregate capital and reduction in interest rate, increases marginal product of labor and the overall number of job openings allowing increased employment across all age groups. However, promoting elderly labor force participation without additional incentives for private saving may lead to some degree of crowding out of younger people from employment.

Relaxing worker homogeneity assumptions would affect the results in multiple ways including the substitutability of different age workers. Allowing for differing knowledge/skills and productivity due to longer work experience would likely separate the older workers from the young.[27] Similarly, older workers may be more costly to the employers not only due to higher wages but also lower health, limited flexibility and lower interest in work related training or education (Boeri and van Ours, 2013). Allowing the firm to discriminate by age when hiring,[28] that is allowing the firm to use directed rather than random search, may as well diminish the substitutability of younger and older workers.

5.1.2 Robustness to Other Assumptions

Results of policy simulations rest on the number of explicit and implicit assumptions. As mentioned in the previous section, the workers are homogeneous in their productiv-

[25] Shorter horizon refers to older workers having fewer years in which they can work before retirement.

[26] For example, "The Age Discrimination in Employment Act of 1967" prohibits age discrimination in the workplace in the US.

[27]

[28] Age discrimination in hiring is found to exist in many countries (Baert, 2018).

ity (wage) and health. Another version of the model allows for transitory productivity shocks; however, since the main results are almost identical, transitory productivity shocks are omitted from the main model. Work experience or tenure based productivity would likely make policies increasing employment of the young more welfare-improving. However, due to computational complexity, interactions between productivity and experience are omitted.

In the above simulations, unemployment insurance and old age benefits are tied to the wage rate. As average wage rate changes, so do the benefits to non-working (unemployed and retired) agents. However, policies such as unemployment insurance and social security are typically not tied to any real variables of an economy, besides being adjusted for inflation, and instead are an outcome of various political processes. To examine what the effects of reforms are when public good, unemployment insurance and social security benefits are fixed, I re-run the above reforms that are identical in every way except benefits are fixed to the level they are under Reform 1. In either case, unemployment insurance and retirement benefits stay relatively similar and very small relative to the wage rate. As a result, either benefit formula (fixed or fixed to the wage) leads to almost identical results not only qualitatively but also quantitatively.

As wealth inequality is not of primary importance of this paper, bequest motive is assumed away. The existence of bequest motive would increase the asset accumulation among younger agents and decrease dis-saving in old age significantly increasing aggregate asset holdings in the long run equilibrium, the wage rate, the number of vacancies and thus employment under each reform. The difference between the reforms is most likely be unchanged.

For simplicity, the government collects and consumes all of the profits and all of the remaining assets of the agents who die before reaching the terminal age. This way, any effect the reform has on profits does not affect the economy. In an alternative specification, corporate profits and the assets left by the deceased agents are equally distributed to all living agents. This raises the aggregate asset holdings and private consumption across the reforms. The notable difference is that under Reform 3 (reduction in benefits), the firms posts more vacancies and earn higher profits. If these profits are distributed back to the agents, the long run equilibrium aggregate asset holdings, consumption, employment and welfare under Reform 3 would likely be even slightly greater compared to other reforms.

The model economy is assumed to be closed. A small open economy with free capital flows may deliver differing results. Open capital flows would not allow the

interest rate to be as affected by the changing aggregate conditions in the economy. As discussed by De la Croix, Pierrard, and Sneessens (2013), lower interest rates due to increased saving in a closed economy have additional increased vacancy posting effect which would not exist if interest rate stayed constant. Knowing that under Reforms 3 and 4, the interest rate decreases, having open capital flows would have diminished firm's job openings and the looping feedback effect described above.

6 Conclusion

I analyze stylized social security reforms in a novel overlapping generations (OLG) model with search-and-matching features and compare the results to models with differently modeled labor markets. Consistent with other research, I find that increasing taxation to maintain unfunded social security benefits brings long-term consumption and welfare losses while decreasing benefits deliver welfare-enhancing outcomes in the long run. Relative to previous research using OLG models with less dynamic labor markets, this paper suggests that there may be additional mechanisms through which the reforms affect the economy. First, I find that policies that promote private saving cause a decrease in interest rate and increase labor demand. The firm increases the wage rate and the number of job openings posted which, in turn, increases employment. Second, policies that incentivize labor force participation increase the number of unemployed, by bringing some people on the fringes of the labor force, and their job search intensity. Higher aggregate job search increases the marginal value of a vacancy and the firm creates more job openings. More job openings bring more people out of the labor force continuing this looping feedback effect. Third, fiscal policy reforms may affect the displacement of younger by older workers. When a decrease in interest rate is sufficiently large, as in the reform that reduces old age benefits, the model allows for increased older worker participation with increased employment of the younger workers contributing to greater long-term economic outcomes. However, when a policy incentivizes older worker participation without also causing a sufficient decrease in interest rate, the increase in employment among older workers may cause either no change or a slight decrease in employment of the younger workers.

The results of the policy reforms rest on a number of simplifying assumptions. Relaxing some of the assumptions and allowing, for example, for random health shocks, disability, varying productivity, directed search by the firm, would provide insights into other mechanisms through which the reforms affect the well-being of the employed,

unemployed and retirees of all ages.

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Appendix A: Discussion about the Utility Function

The multiplicative CRRA utility function in terms of consumption and leisure assumed in the model is standard in the macroeconomic literature.

$$U(c_t, l_t) = \frac{(c_t^{\gamma_1} l_t^{1-\gamma_1})^{1-\eta} - 1}{1-\eta} \tag{35}$$

Here l_t is leisure or all non-market activity. Intratemporal elasticity of substitution between consumption and leisure for employed agents is equal to one. For employed agents, market activity is labor supply while for non-employed agents, that is job search.

$$l_t = \begin{cases} 1 - n_t & \text{if LM = Employed} \\ 1 - \epsilon_t^{\gamma_2} & \text{if LM = Unemployed} \end{cases} \tag{36}$$

Here, ϵ_t represents number of job applications submitted. While each application increases the probability of matching with a firm linearly, I choose to have an option of setting increasing time costs of finding additional vacancies to apply for ($\gamma_2 \geq 1$). The reasoning for such assumption is similar to Hotelling's (1929) linear and Salop's (1979) circular city model concepts. Assuming that the firm posts a number of vacancies in different locations (either physical or virtual) to maximize its hiring rate, some are posted to locations that are known and/or closer to a specific agent while others are further away or less known locations. Agent first looks, finds and applies to vacancies posted in locations that are known or closer to her, followed by further away locations in which due to distance or unfamiliarity, search takes longer. It follows that the time it takes to post an additional vacancy depends on the number of applications already posted. While in the main analysis I use $\gamma_2 = 2$, in sensitivity analysis I experiment with $\gamma_2 = 1$ and $\gamma_2 = 3$. The results of the model qualitatively are identical.

$$\frac{\partial l_t}{\partial \epsilon_t} = -\gamma_2 \epsilon_t^{\gamma_2 - 1} \tag{37}$$

Appendix B: Solution Technique

Solution is found by making initial guesses on aggregate variables and solving agent's problem backwards from $j = T$ (agent's last period before exiting the model) to $j = 1$ using value function iterations with discretized grids over state and choice variables.

1. Given the calibrated parameters and specific government policies, make a guess for aggregate capital (K_t), labor(N_t), total search effort (S_t), fraction of labor supply to decrease due to retirement (H_t), average labor supply by a newly hired worker ($\bar{n}_{new,t}$) and the tax rate for consumption (τ_c) that would balance the government's budget.

2. Solve for total output by the firm, interest and wage rate.

$$Y_t = A_P K_t^{\alpha} N_t^{1-\alpha}$$
$$r_t = \alpha A_P K_t^{\alpha-1} N_t^{1-\alpha} - \delta$$
$$w_t = \gamma_w (1-\alpha) A_P K_t^{\alpha} N_t^{-\alpha}$$

3. Solve the firm's problem for optimal number of vacancies (V_t). See Equation 15, which is also reproduced below, and Figure 7a

$$\zeta(vc) V_t^{\zeta-1} = R_t^{-1}[(1-\alpha)(A_P K_{t+1}^{\alpha}[(1-\chi-H_t)N_t + (A_M V_t^{\varphi-1} S_t^{1-\varphi}) V_t \overline{n}_{new,t}]^{-\alpha}) * \\ * (A_M V_t^{\varphi-1} S_t^{1-\varphi}) \overline{n}_{new,t} - [A_M V_t^{\varphi-1} S_t^{1-\varphi}](w_t) \overline{n}_{new,t}]$$

4. Solve agent's problem to get optimal policies for a_{t+1}, c_t, Ret_t and n_t if employed, ϵ_t if unemployed using value function iterations with discrete grids over choice and state variables.

5. Aggregate the assets, labor supply and search effort across all states and find the voluntary retirement rate.

$$K_t = \sum_j \sum_a \sum_m P_t^{j,a,m} a_t^{j,a,m}$$
$$N_t = \sum_j \sum_a P_t^{j,a,m} n_t^{j,a,E}$$
$$S_t = \sum_j \sum_a P_t^{j,a,m} \epsilon_t^{j,a,U}$$
$$H_t = \sum_j \sum_a P_t^{j,a,m} n_t^{j,a,E} (Ret_t^{j,a,E} = 1)$$
$$\overline{n}_{new,t} = ((\sum_j \sum_a P_t^{j,a,E} n_t^{j,a,E}) / (\sum_j \sum_a P_t^{j,a,E})) I(m_{t-1}^{j,a,E} = U)$$

where j, i, m represent agent's age, asset holdings and labor market state. $P_t^{i,j,m}$ denotes the number of agents of age j with asset holdings i in labor market state m at time t. $I(m_{t-1}^{j,a,E} = U)$ is an indicator function equal to one if an agent was unemployed in the previous period.

6. Check if the aggregates are as guessed and tax rate on consumption balances the budget. If not, update the guesses using a smoothing parameter in Step 1 and repeat until convergence.

Accuracy of the solution is checked using Euler errors. Simplified[29] example Euler equations for employed and unemployed agents of age 3 are shown below.

$$U_c^{j,a,E} = (1 + \tilde{r}(1 - \tau^a))\beta\lambda_3[(1 - \chi)U_c^{\tilde{j},\tilde{a},E} + (\chi)U_c^{\tilde{j},\tilde{a},U}] \tag{38}$$

$$U_c^{j,a,U} = (1 + \tilde{r}(1 - \tau^a))\beta\lambda_3[(p(\epsilon_t^{j,a,U}))U_c^{\tilde{j},\tilde{a},E} + (1 - p(\epsilon_t^{j,a,U}))U_c^{\tilde{j},\tilde{a},U}] \tag{39}$$

Appendix C: Firm's Decision

$$W_t = \max_{V_t,K_t} \left\{F(K_t, N_t) - r_t K_t - (1 + \tau^s)w_t N_t - vV_t^{\zeta} + R_t^{-1}E_t W_{t+1}\right\} \tag{40}$$

Taking first order condition with respect to vacancies yields the following.

$$\frac{\partial W_t^F}{\partial V_t} = -\zeta(vc)V_t^{\zeta-1} + R^{-1}[\frac{\partial F(E_tK_{t+1}, E_tN_{t+1})}{\partial E_tN_{t+1}}\frac{\partial E_tN_{t+1}}{\partial V_t} - (1+\tau^e)E_tw_{t+1}\frac{\partial E_tN_{t+1}}{\partial V_t}] = 0 \tag{41}$$

The firm assumes that economy is in a stationary steady state implying the following aggregate capital, labor supply, wage and interest rate.

$$\begin{aligned} E_tK_{t+1} &= K_t \\ E_tN_{t+1} &= (1 - \chi - H_t)N_t + q_tV_t\overline{n}_{new,t} \end{aligned} \tag{42}$$

$$\begin{aligned} E_tr_{t+1} &= r_t \\ E_tw_{t+1} &= w_t \end{aligned} \tag{43}$$

Plugging in these values and taking the derivatives.

$$\zeta(vc)V_t^{\zeta-1} = R_t^{-1}[\frac{\partial F(E_tK_{t+1}, E_tN_{t+1})}{\partial E_tN_{t+1}}\frac{\partial E_tN_{t+1}}{\partial V_t} - \frac{\partial E_tW_{t+1}^F}{\partial V_t}]$$

where

[29] Since under no circumstances, agents choose to retire at age 3, the value function can be simplified by elimination of the retirement choice.

$$\frac{\partial F(E_tK_{t+1}, E_tN_{t+1})}{\partial E_tN_{t+1}}\frac{\partial E_tN_{t+1}}{\partial V_t} =$$
$$= \frac{\partial F(E_tK_{t+1}, E_tN_{t+1})}{\partial E_tN_{t+1}}\frac{\partial E_tN_{t+1}}{\partial V_t} = (1-\alpha)(A_P E_tK_{t+1}^{\alpha} E_tN_{t+1}^{-\alpha})(q_t\overline{n}_{new,t}) =$$
$$= (1-\alpha)(A_P K_{t+1}^{\alpha} N_{t+1}^{-\alpha})(\frac{M_t}{V_t}) = (1-\alpha)(A_P K_{t+1}^{\alpha} N_{t+1}^{-\alpha})(\frac{A_M V_t^{\varphi} S_t^{1-\varphi}}{V_t})\overline{n}_{new,t} =$$
$$= (1-\alpha)(A_P E_tK_{t+1}^{\alpha} N_{t+1}^{-\alpha})(A_M V_t^{\varphi-1} S_t^{1-\varphi})\overline{n}_{new,t} =$$
$$= (1-\alpha)(A_P K_t^{\alpha}[(1-\chi-H_t)N_t + q_tV_t\overline{n}_{new,t}]^{-\alpha})(A_M V_t^{\varphi-1} S_t^{1-\varphi})\overline{n}_{new,t} =$$
$$= (1-\alpha)(A_P K_t^{\alpha}[(1-\chi-H_t)N_t + (A_M V_t^{\varphi-1} S_t^{1-\varphi})V_t\overline{n}_{new,t}]^{-\alpha})(A_M V_t^{\varphi-1} S_t^{1-\varphi})\overline{n}_{new,t} \quad (44)$$

and where

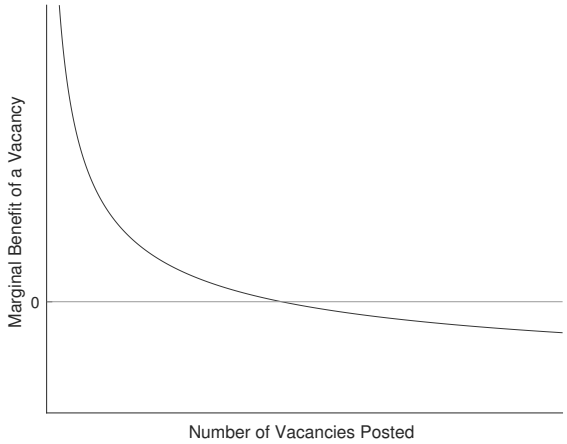
$$\frac{\partial E_tW_{t+1}^F}{\partial V_t} = [q_t](E_tw_{t+1}) = [q_t][\overline{n}_{new,t}](w_t) = [\frac{A_M V_t^{\varphi} S_t^{1-\varphi}}{V_t}][\overline{n}_{new,t}](w_t) =$$
$$= [A_M V_t^{\varphi-1} S_t^{1-\varphi}][\overline{n}_{new,t}](w_t) \quad (45)$$

After all substitution and simplification, firm's optimal vacancy posting decision is as follows.

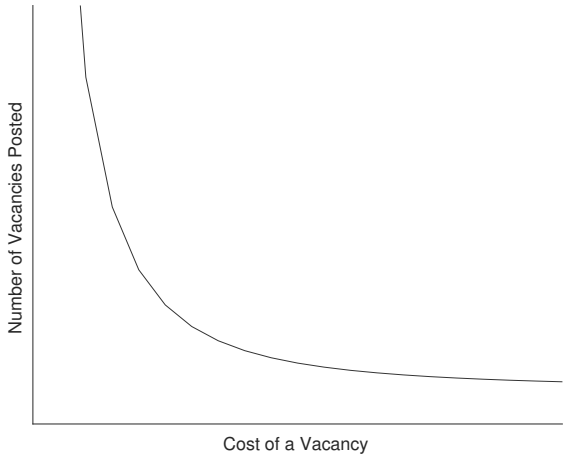
$$0 = R_t^{-1}[(1-\alpha)(A_P K_t^{\alpha}[(1-\chi-H_t)N_t + (A_M V_t^{\varphi-1} S_t^{1-\varphi})V_t\overline{n}_{new,t}]^{-\alpha})*$$
$$*(A_M V_t^{\varphi-1} S_t^{1-\varphi})\overline{n}_{new,t} - (1+\tau^f)[A_M V_t^{\varphi-1} S_t^{1-\varphi}](\overline{n}_{new,t})(w_t)] - \zeta(vc)V_t^{\zeta-1} \quad (46)$$

As is seen in Figure 7b, as the vacancy cost parameter increases, the firm chooses to post fewer vacancies.

Figure 7: Firm's Vacancies



(a) Marginal Benefit of a vacancy depends on vacancies already posted.



(b) As the cost of a vacancy parameter vc increases, the firm's posted vacancies decrease.