UNIVERSITY OF CALIFORNIA, MERCED

BERKELEY · DAVIS · IRVINE · LOS ANGELES · MERCED · RIVERSIDE · SAN DIEGO · SAN FRANCISCO SANTA BARBARA · SANTA CRUZ

CSE 015: Discrete Mathematics
Homework 4

Spring 2021

Introduction

The purpose of this assignment is to give you more practice with functions. As always, you will also be practicing your LaTeX skills. Create a new LaTeX document and type out the solutions to the following exercises. Your document should include an appropriate title, your name, as well as a date. Please number your solutions appropriately. Upload your `.tex` and your `.pdf` files under the relevant CatCourses assignment.

1 Functions (0.25 points each)

To prove that a function $f : A \to B$ is one-to-one, you need to show that $x_1 \neq x_2 \to f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in A$. To prove that such a function is onto, you need to show that $\forall y \in B \exists x \in A(f(x) = y)$.

1. Is f a function from R to R if
   (a) $f(x) = \pm\sqrt{(x^2 + 1)}$?
   (b) $f(x) = 1/x$
   (c) $f(x) = x - x^2$
2. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.
   (a) The function that assigns to each positive integer the largest perfect square not exceeding this integer.
   (b) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string.
   (c) The function that assigns to each bit string twice the number of zeros in that string.
   (d) The function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits).
   (e) The function that assigns to a bit string the number of one bits in the string.
   For example, the domain for the question is the set of bit strings (of any length). (Remember, a bit string is a string made up of just 0's and 1's.) The range is the set of nonnegative integers: $\{0, 1, 2, ...\}$. That is, the number of 1's in a bit string can be $0, 1, 2, ...$
3. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

(a) $f(m,n) = m^2 - n^2$
(b) $f(m,n) = m + n + 1$
(c) $f(m,n) = |m| - |n|$
(d) $f(m,n) = m^2 - 4$

Note: $\mathbb{Z} \times \mathbb{Z}$ is the set of pairs (m,n) where m and n are integers.

For example, the function in part (c) is onto because for any $p \in \mathbb{Z}$, you can find an $m \in \mathbb{Z}$ and an $n \in \mathbb{Z}$ such that $m + n + 1 = p$.

4. Consider these functions from the set of students in a discrete mathematics class. State under what conditions is the function one-to-one if it assigns to a student the following things below. Also state how likely this is in practice.

   (a) Mobile phone number.
   (b) Student identification number.
   (c) Final grade in the class.
   (d) Home town.

   For example, the function in part (c) is one-to-one if each student has a unique student identification number. In other words, if no two students share the same student identification number. This is usually the case in practice since identification numbers are usually used to identify a unique student.

5. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

   (a) $f(x) = -3x + 4$
   (b) $f(x) = -3x^2 + 7$
   (c) $f(x) = (x + 1)/(x + 2)$
   (d) $f(x) = x^3$

   Note: $\mathbb{R}$ is the set of real numbers. A bijection is a function that is both one-to-one and onto.

   For example, the function in part (b) is not a bijection because it is not onto. It is not onto because there exists at least one $y \in \mathbb{R}$ such that you cannot find an $x \in \mathbb{R}$ where $f(x) = y$. One such example is $y = 8$. (The function is also not one-to-one because $f(1) = 4 = f(-1)$.)