



CSE 015: Discrete Mathematics

Homework 2

Spring 2021

Introduction

This assignment is about propositional logic and its applications. As always, you will also be practicing your \LaTeX skills.

Exercises

Create a new \LaTeX document and type out the solutions to the following exercises. Your document should include an appropriate title, your name, as well as a date. Please number your solutions appropriately. Upload your `.tex` and your `.pdf` files under the relevant CatCourses assignment.

1 Logical Identities

Simplify the following propositions. Show all steps of your solutions.

1. $\neg(p \rightarrow (q \rightarrow p))$
2. $\neg((p \wedge q) \rightarrow (q \vee p))$

2 Logical Consequence

Determine if the following inferences are valid. Explain why, or why not.

1.
Jimmy is smart
Smart people are rich

Jimmy is rich
2.
Islands are surrounded by water
Puerto Rico is surrounded by water

Puerto Rico is an island

3 Translating English Sentences into Formulas

Consider the following two predicates:

$S(x)$: x is a student in CSE015;

$M(x)$: x plays a musical instrument.

Let the domain be the set of all people. Using the above predicates, translate the following sentences into formulas using the appropriate quantifiers and operators.

1. Not every student in CSE015 plays a musical instrument.
2. A person is either a student in CSE015 or plays a musical instrument, but not both.
3. There exists at least one student in CSE015 who does not play a musical instrument.

4 Logical Equivalence

Let $A(x)$ and $B(x)$ be two predicates defined over the same domain (non empty). Is this a valid logical equivalence or not?

$$\forall x(A(x) \wedge B(x)) \equiv \forall x(A(x) \rightarrow B(x))$$

You must justify your answer. Simply stating yes or no will give you no credit, even if your answer is correct.

5 Nested Quantifiers

The following two statements are defined for real numbers:

$A(x, y)$ is the statement $xy = 0$;

$B(x, y)$ is the statement $x + y = 0$.

For each of the following formulas determine their truth values. You must justify your answers. Simply stating true or false will give you no credit, even if your answer is correct.

1. $\exists x \forall y A(x, y)$
2. $\exists x \exists y B(x, y)$
3. $\forall x \exists y A(x, y)$
4. $\exists x \forall y (A(x, y) \wedge B(x, y))$
5. $\exists x \exists y (A(x, y) \wedge \neg B(x, y))$