



CSE 015: Discrete Mathematics

Homework 3

Spring 2021

Introduction

The purpose of this assignment is to give you more practice with mathematical proofs, specifically direct proofs, proofs by contraposition and proofs by contradiction. There are also exercises on basic counting principles and set theory. As always, you will also be practicing your \LaTeX skills.

Exercises

Create a new \LaTeX document and type out the solutions to the following exercises. Your document should include an appropriate title, your name, as well as a date. Please number your solutions appropriately. Upload your `.tex` and your `.pdf` files under the relevant CatCourses assignment.

1 Mathematical Proofs

Prove (or disprove) the following results, showing all steps of your argument (0.2 points each).

1. The square of an even number is even.
2. The product of two odd integers is odd.
3. If $n^3 + 5$ is odd then n is even, for any $n \in \mathbb{Z}$.
4. If $3n + 2$ is even then n is even, for any $n \in \mathbb{Z}$.
5. The sum of a rational number and an irrational number is irrational.

2 Basic Counting Principles

Answer the following questions. Show all steps of your solutions (0.2 points each).

1. How many different arrangements of the English alphabet are there?

- There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
- A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
- A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student answers every question?
- Suppose we have the same multiple choice test as described in question 4, but we relax the assumption that the student has to answer all questions. In other words, how many ways are there for a student answer the questions on the test if the student can leave answers blank?

3 Set Theory

- Are these pairs of sets equal or not: $\{5, 1, 3\}$, $\{1, 3, 5\}$. You just have to answer Yes or No (0.2 points).
- What is the cardinality of (the number of elements in) the set $\{a, \{a\}, \{a, \{a\}\}$ (0.2 points)?
- Find the power set of $\{a, \{a, b\}\}$, where a and b are distinct elements. Your answers must be sets. That is, they should start with $\{$ and end with $\}$ (0.2 points).
- Let $A = \{a, b, c\}$ and $B = \{y, z\}$. Determine the following set. Your answers must be sets. That is, they should start with $\{$ and end with $\}$ (0.2 points each).
 - $A \times B$
 - $B \times A$
- Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Determine the following sets. Note that your answers must be sets. That is, they should start with $\{$ and end with $\}$ (0.2 points each).
 - $A \cup B$
 - $A \cap B$
 - $B - A$
- Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Determine the following sets. Your answers must be sets. That is, they should start with $\{$ and end with $\}$ (0.2 points each).
 - $A \cap B \cap C$
 - $(A \cup B) \cap C$
- Determine the set $\bigcup_{i=1}^{\infty} A_i$ if, for every positive integer i (0.25 points each),
 - $A_i = \{-i, -i + 1, \dots, -1, 0, 1, \dots, i - 1, i\}$
 - $A_i = \{-i, i\}$

Note that here you don't necessary need to provide the answer as an explicit set of elements. You can also describe the set if that seems simpler.

- Determine the set $\bigcap_{i=1}^{\infty} A_i$ if, for every positive integer i (0.25 points each),

- (a) $A_i = \{-i, -i + 1, \dots, -1, 0, 1, \dots, i - 1, i\}$
- (b) $A_i = \{-i, i\}$

Note that here you don't necessary need to provide the answer as an explicit set of elements. You can also describe the set if that seems simpler.