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Homework Assignment 2
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1. Logical Identities

1. $\neg(p \to (q \to p))$

- **Solution**
1. $\neg(p \to (q \to p))$
2.$\neg(\neg p \vee (\neg q \vee p)) - Implication$
3.$\neg(\neg p) \wedge \neg(\neg q \vee p) - Demorgan$
4.$\neg(\neg p) \wedge \neg(\neg q) \wedge \neg p - Demorgan$
5.$p \wedge q \wedge \neg p - Double.Negation$
6.$p \wedge \neg p \wedge q - rearranged$
We know $\neg p \wedge p \equiv False$
$also\ \ False \wedge q$
$\therefore False \equiv q$

2. $\neg((p \wedge q) \to (q \vee p))$

- **Solution**
1. $\neg((p \wedge q) \to (q \vee p))$
2.$\neg(p \wedge q) \vee (q \vee p) - Implication$
3.$\neg p \vee \neg q \vee (q \vee p) - Demorgan$
4.$(\neg p \vee p) \vee (\neg q \vee q) - Associative$
We know p $\vee \neg p \equiv T$
$\therefore True \equiv True$

2. Logical Consequence

1. Jimmy is smart
   Smart people are rich

Jimmy is rich

- **Solution**
1. Jimmy is smart
2. Jimmy $\to Smart$
3. $Smart\ \ people\ \ are\ \ rich$
4. $Smart \to Rich$
5. $Jimmy \to Rich$

2. Islands are surrounded by water
   Puerto Rico is surrounded by water

Puerto Rico is an island

- **Solution**
  1. Islands are surrounded by water
  2. Island $\rightarrow$ *Surrounded by water*
  3. *Puerto Rico is surrounded by water*
  4. $PuertoRico \rightarrow Surroundedbywater$
  5. $PuertoRico \rightarrow Island$

3. Translating English Sentences into Formulas

S(x): x is a student in CSE015
M(x): x plays a musical instrument
Domain: Set of all people

1. Not every student in CSE015 plays a musical instrument.

   Solution: $\neg(\forall x(S(x) \rightarrow M(x)))$

2. A person is either a student in CSE015 or plays a musical instrument, but not both..

   Solution: $\exists x(S(x) \oplus M(x))$

3. There exists at least one student in CSE015 who does not play a musical instrument..

   Solution: $\exists((S(x) \wedge \neg M(x))$

4. Logical Equivalence

$\forall x(A(x) \wedge B(x)) \equiv \forall x(A(x) \rightarrow B(x))$

Solution

$A(x)$	$B(x)$	$A(x) \wedge B(x)$	$A(x) \rightarrow B(x)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

A(x)$\wedge B(x) \neq A(x) \rightarrow B(x)$

5. Nested Quantifiers

A(x,y) is the statement xy=0;
B(x,y) is the statement x+y = 0.

a. $\exists x \forall y A(x, y)$

Solution:
$if\ \ x = 0$
$then\ \forall y A(x, y) \equiv True$
$\therefore True$

b. $\exists x \forall y A(x, y)$

Solution:
$\exists x \exists y B(x, y)$
for y=-x
if y = 2, then x = -2
then B(x,y)$\equiv$ True

c. $\forall x \exists y A(x, y)$

Solution:
if y = 0
then $\forall$xA(x,y)$\equiv$True
$\therefore$True

d. $\exists x \forall y (A(x, y) \wedge B(x, y))$

Solution:
A(x,y) is true true, when x=0, y=0 or x and y=0
B(x,y) is true true, when x=0, y=0 or x=-y
however when x=-y, A(x,y) $\neq$true
but this is not $\forall$y
$\therefore \exists x \forall y (A(x, y) \wedge B(x, y)) \equiv False$

e. $\exists x \forall y (A(x, y) \wedge \neg B(x, y))$

Solution:
for x or y = 0, then A(x,y) and ¬B(x,y)≡True
$\therefore \exists x \forall y (A(x, y) \wedge \neg B(x, y)) \equiv True$