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Homework Assignment 4
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1. Functions

1. Is f a function from $\mathbb{R}$ to $\mathbb{R}$ if

(a) f(x) = $\pm\sqrt{(x^2+1)}$?

- **Solution**
  This is not a function because it is not
  uniquely assigned to the point $x \in \mathbb{R}$

(b) f(x) = 1/x

- **Solution**
  This is not a function from $\mathbb{R}$ to $\mathbb{R}$
  because for the element x = 0, there is no image

(c) f(x) = $x - x^2$

- **Solution**
  This is not a function from $\mathbb{R}$ to $\mathbb{R}$
  because $x = 0$ and $x = 1$ are both mapped to element 0
  $\therefore$ it's condition is not unique

2. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

(a) The function that assigns to each positive integer the largest perfect square not exceeding this integer.

- **Solution**
  Domain = $\mathbb{N}$, all positive integers.
  Range = set of all perfect squares.
  - f is not one to one because if we take
  - f(2)=2 and$f$(3)=2, the largest perfect sq is ≤ 3
  - so we can say the elements have the same image
  $\therefore f$ is onto

(b) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string.

- **Solution**
  f(x) = (number of 1's in string) - (number of 0's in string)
  ex. f(1100) = 2-2 = 0
  Domain = set of all strings
  Range = set of all integers ($\mathbb{Z}$)
  - f is not one to one because if we take f(1100) = 0 and f(10) = 0
  - Different elements give us the same image
  $\therefore f$ is onto

(c) The function that assigns to each bit string twice the number of zeros in that string.

- **Solution**
  f(x) = 2*number of zeros
  ex. f(100) = 2*2=4
  Domain = set of all binary strings
  Range = $\{2 * n \colon n \in \{1, 2, \dots\}\}$
  .   = set of all even numbers
  - f is not one to one because if we take
  f(001) = 2*2 = 4 and f(100) = 2*2 = 4
  - we get the same image with different elements
  $\therefore f$ is onto

(d) The function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits).

- **Solution**
  f(x) = the num of bits of a string left over when a bit string is split into bytes
  ex. f(100001101) = 0
  f(0000000000) = 2
  Domain = set of all binary strings
  Range = set of all non-negative integers
  .   = $\{0,1,2,3,\dots\}$
  - f is not one to one because if we take
  - f(01) = 2 and f(10) = 2
  - we get the same image with different elements
  $\therefore f$ is onto

(e) The function that assigns to a bit string the number of one bits in the string. For example, the domain for the question is the set of bit strings (of any length). (Remember, a bit string is a string made up of just 0's and 1's.) The range is the set of nonnegative integers: 0, 1, 2, ... That is, the number of 1's in a bit string can be 0, 1, 2, ...

- **Solution**
  f(x) = the num 1's in x
  ex. f(00000) = 0
  . f(110) = 2
  Domain = set of all binary strings
  Range = set of all non-negative integers
  .   = $\{0,1,2,3,\dots\}$
  - f is not one to one because if we take
  - f(01) = 1 and f(10) = 1
  - we get the same image with different elements
  $\therefore f$ is onto

3. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z}$! $\mathbb{Z}$ is onto if

(a) f(m,n) =$m^2 - n^2$

- **Solution**
  - let $y \in$ Z | f$(m, n) = y$
  - so we have $m^2 - n^2 = y$
  - if n = 0 we get, $m^2 = y \longrightarrow$ m = $\pm\sqrt{y} \oplus \mathbb{Z}$
  - this tells us the preimage of y is nonexistent in $\mathbb{Z} \times \mathbb{Z}$
  $\therefore f$ is onto

(b) f(m,n) = m + n + 1

- **Solution**
  - let p $\in \mathbb{Z} \mid f$(m,n) = p
  - We can say that $p = m + n + 1$
  - Which implies that each of the elements of the given co-domain set $\mathbb{Z}$ has a preimage within the domain $\mathbb{Z} \times \mathbb{Z}$
  $\therefore f$ is onto

(c) f(m,n) =$|m|$ - $|n|$

- **Solution**
  let $y \in \mathbb{Z} \mid f$(m,n) = y
  - We can say that $y = |m|$ - $|n|$
  - Which illustrates that y has a preimage within $\mathbb{Z} \times \mathbb{Z}$
  - We know that y is an arbitrary value, which means each element in co-domain set $\mathbb{Z}$ has a preimage in $\mathbb{Z} \times \mathbb{Z}$
  $\therefore f$ is onto

(d) f(m,n) =$|m|$ - $|n|$

- **Solution**
  - let $y \in \mathbb{Z}$ be an element in codomain set in $\mathbb{Z}$ that f(m,n) = y
  - which means that $m^2 - 4 = y$
  $m^2 = y + 4$
  $m = \sqrt{y+4} \notin \mathbb{Z} \times \mathbb{Z}$
  - so y has no preimage in the domain of $\mathbb{Z} \times \mathbb{Z}$
  $\therefore f$ is onto

3. Consider these functions from the set of students in a discrete mathematics class. State under what conditions is the function one-to-one if it assigns to a student the following things below. Also state how likely this is in practice.

(a) Mobile phone number.

- **Solution**
  If each student is assigned a unique phone number than the function is considered one-to-one. so no two students must have the same number. which is the case since not one person can have the same number, but numbers can be reused for new users

(b) Student identification number.

- **Solution**
  If each student has a unique Student ID number, than the function is one-to-one.
  Because this is used to identify an individual numbers must not and are not the same to avoid confusion

(c) Final grade in the class.

- **Solution**
  If each student has a unique final grade than the function is one-to-one
  so no two students can have the same grade
  this is rarely the case because there are only 4 grades to choose from and classrooms have many students

(d) Home town.

- **Solution**
  If each student is from a unique home town, than the function is one-to-one.
  this is not the case at all because not everyone can be from seperate towns.

5. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$

(a) f(x) = $-3x+4$

- **Solution**
  - let x_1 and $x_2 \in \mathbb{R}$ | $x_1 \neq x_2$
  - we let $f\ (x_1) = -3x_1 + 4$ and $f\ (x_2) = -3x_2 + 4$
  - $f(x_1) \neq f(x_2)$
  - $-3x_1 \neq -3x_2$
  - Since $x_1 \neq x_2$
  $\therefore f(x)$ is one-to-one
  - Now $\forall y \in \mathbb{R}$ and $\exists x = 4 - y/3 \in \mathbb{R}$
  Such that $f(x) = y$
  - we know $f(4 - y/3) = y$
  - Then $f(x)$ is onto
  $\therefore f$ is a Bijection from $\mathbb{R}$ to $\mathbb{R}$

(b) f(x) = $-3^2 + 7$

- **Solution**
  - f(x) is not a one-to-one function
  - If we take f(1) = 4 = f(-1)
  - and 1 $\neq$-1 so it is not one-to-one
  $\therefore f(x)$ is not a Bijection

(c) f(x) = (x+1)/(x+2)

- **Solution**
  - This is not a function because there is not image of $x = -2 \in \mathbb{R}$
  - so it cant be one-to-one nor onto
  $\therefore f(x)$ is not a Bijection

(c) $f(x) = x^3$

- **Solution**
  - Let $f(x_1) \neq f(x_2)$
  - Also let x_1 and $x_2 \in \mathbb{R}$
  - We have $x_1^3 \neq x_2^3$
  - Then $x_1 \neq x_2$
  - So we can say $f(x)$ is one-to-one
  - Now $\forall y \in \mathbb{R}$ and $\exists x = y^1/3 \in \mathbb{R}$
  - such that $f(x) = y$
  - Then $f(x) = f(y^1/3) \longrightarrow$(y$^1/3)^3$
  - So $f(x) = y$ which makes it onto
  $\therefore f(x)$ is a Bijection