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Cherry Picking: Consumers' Choices in a Swarm Dynamics Considering Price and Quality of Goods

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Introduction

From the second half of the 1900s, the approach to Economics (and Social Sciences in general) moves towards the idea of accounting for collective behavior that emerges from the interaction among the singular individuals, developing new ways to study them. The single agent does not have a strategy based on a view of the entire dynamics to achieve its wellness, instead its strategy is limited and, so, not necessarily optimal. Its choices are local and, in a global sense, they can also appear completely irrational. The observations of the phenomenon of the irrational behaviour where made between the '60s and the '80s, in which the performances where compared on the stock market of computers, choosing randomly, and humans [1]. The two different groups had comparable results. While the idea of a whole dynamics depending on the local action of every individual was firstly developed by the economist Hayek in the first half of 1900s, in the so called "decentralized Hayekian market", a more structured and complete theory and approach were initiated in 1989 from the Santa Fe institute, with the idea of "Complexity Economics".

After that, different tools where developed to study the so called "soft sciences", through Computer Science, Mathemathics and Physics. The aim was (and still is) to capture the complexity of the collective dynamics, understanding the single, simple and limited action of the single agent. Some used methods to achieve this aim refers to the the kinetic theory and statistical mechanics. An example for this, is documented in [2], where this topic has been developed as a multiscale vision which leads to differential structures suitable to capture the complexity of living entities. Also, an important contribution to this approach is given by swarm theory, initiated by the celebrated paper of Cucker-Smale [3], in which the local interactions among the agents explains a regular collective trend of the swarm. Further developments starting from the Cucker-Smale model and called *behavioral swarms* will bring a crucial and flourishing contribute to the study of social and economic sciences. The main feature of the theory of behavioral swarms is that the micro-scale state of individuals on the swarms includes not only mechanical variable, but additional variables modeling their social-economical state. It follows that the collective dynamics is conditioned by this state.

In the First Chapter, we will see how the *theory of behavioral swarms* have been used not only in biological, but also in social and economic systems, for example in financial markets, as in [4]–[6] and, in a more specific way with respect to the work we are going to develop, in a competitive market in [7], which will be the basis of our work. In the chapter we will also see the economic background of the work, the idea of the decentralized Hayekian market and the importance of *stickiness* of sellers prices, in particular in the development of *Cherry-Picking* [8].

In Chapter 2, we will introduce our model, talking about the economic and mathematical aspects, and we will see how it is a further development of [7], to which we are adding the possibility for the seller to choose a buyer to interact with, under some conditions, instead of interacting with every seller in the market. The aim is to bring the model a bit more near to actual markets. Moreover we will present the computational results and we will see how the interaction among sellers is crucial and creates interesting behaviours, as the coordination of buyers in endogenous clusters. We will also analyze Pareto market efficiency and we will provide a statistical analysis on the split in clusters linked both to sellers' offered quality and buyers' market efficiency. We can see this work in two ways: both as an *explorative model* [9], to understand what kind of dynamics we can face in the created enviroment, and a further step to a *predictive model*, which will contain enough features to be assimilated to a current market.

The computational codes are reported in an Appendix.

Chapter 1

The Framework

The traditional Economics approach is based on the ideas of achieving market equilibrium and, often, of the "rational expectations", which states that the agent knows the model and acts consequently to achieve its aim. A different approach was developed already in the first half of 1900 by Hayek, who assumed an economy in which the agents simply react to the (often incomplete) information they receive about the market, through the market and refused the postulation of *a-priori* existing equilibrium.

The first evidences that questioned the idea of "rational expectations" were observed during the '60s and '80s when some experiments in the stock market let two groups compete: in the first one Economics students and, in the second one, computers acting randomly [1]. The two groups had comparable results. This suggested the idea of the *zero-intelligence*: the agent does not have all the informations about the model that it would need to make the best choice. Its acting is more similar to the systematic one, from the point of view of the global dynamics. That means that the agent has got a strategy, but, not having a view of the whole dynamics, the strategy its often not optimal and, in some cases, can also appear completely irrational.

A deep change in the economic approach happened in 1989, when the Santa Fe Institute gives birth to the idea of "Complexity Economics". The model is no more constructed to achieve a precise trend (the market equilibrium, for example), but puts its focus on the micro dynamics. That means we set the local behaviour of the singular agent (which has no idea of the general trend) and the way they interact among them. The whole dynamics is just the consequence of what happens in the "micro world".

From the birth of Complexity Economics, different methods have been used which encapsulate the idea of the micro interactions defining the whole trend. One of the approaches is Agent Based Models [10]–[13], a way based on computer science, in which agents are commonly implemented in software as objects, with internal rules. Worth to mention, Statistical Mechanics [14], [15] whose physical quantities as Entropy or Energy are used to describe or derive economic ones. Also used, the *kinetic theory of active particles*[16]–[21] in which the agents of the economical and social world are seen as interacting particles whose behaviour is assimilated to the social and economic one. Some contributes of it to the economic and social theory are in [4], [22]. For our work, we will use *theory of swarm* as our mathematical framework. Swarms theory and kinetic theory often interacts in approaches which might be based on Fokker-Plank methods [2] or stochastic evolutive games within the framework of the so-called kinetic theory of active particles [9]. An example of a kinetic theory approach to swarm modeling is given in [23]. Anyway, our choice of swarm dynamics instead of the kinetic theory approach is due to the fact that the second one presents a number of advantages, but also some withdraws. For instance, the derivation of models requires the assumption of continuity of the dependent variable, which is the probability distribution function over the activity variable, can be admissible only for very many active particles. Instead we will use a discrete approach and few agents acting in the system.

1.1 Market with decentralized prices

Swarm theory will be used in our model to describe a market with decentralized prices. In particular, our economic theoretical framework will be a paper of Bowles et al. [24]. The idea of a decentralized market comes from the economist Friedrich von Hayek (1899-1992). The decentralization derives from the fact that the information of supply and demand of a good is segmentated and not uniform and can be passed through the individuals acting in the economy through the changing of prices, acting like "signals".

This idea was in contrast with the Walrasian general equilibrium theory, which is the basis of the two fundamental theorems of welfare economics. Defining an allocation "Pareto efficient" if there is no alternative allocation where improvements can be made to at least one participant's well-being without reducing any other participant's well-being, the two theorems can be roughly stated as follows: a competitive price-taking market equilibrium will be Pareto-efficient, and any distributional concerns about the outcomes of such a market can be addressed through a redistribution of endowments.

Hayek's sharpest critique of the equilibrium model and the conception of competition on which it was built came in his 1948 paper "The Meaning of Competition" [25]. Here he argued that "the modern theory of competition deals almost exclusively with a state ... in which it is assumed that the data for the different individuals are fully adjusted to each other, while the problem which requires explanation is the nature of the process by which the data are thus adjusted." That is, "the modern theory of competitive equilibrium assumes the situation to exist which a true explanation ought to account for as the effect of the competitive process.". In other words, assuming the existence of equilibrium and *perfect* competition a-priori, precludes an analysis on the role of competition and of different distribution of information in the market, in a certain sense making them useless. Instead, Hayek saw the strength of the market economy as arising from the learning and diffusion of new information that it accomplishes in disequilibrium. Unforeseen changes in economic fundamentals that are initially recognized by only a small number of individuals would lead, through the messages conveyed by changes in prices, to adjustments across the entire economy. His case for competitive markets rested on the idea that competition was a "procedure for discovering facts which, if the procedure did not exist, would remain unknown or at least would not be used" (citing [26]).

After the change, a new order in economy comes from the passing of the information through prices and the response of the agents acting in it. The information is not uniform nor complete in every part of the economy and doesn't need to be, to be effective. Indeed, the disequilibrium created in this procedure, will create the right response to bring a new equilibrium. An approach that goes well beyond the passive price-taking of agents in the Walrasian model. Most of the criticism that Hayek made of the various approaches to analyzing the functioning of the market process turned on the idea that the coordination of individual actions and beliefs is taken as given and the process by which this happens is not discussed.

In [24] we see that, even highlighting that "Hayek's vision of a decentralized solution to a massive and perpetually changing coordination problem... will continue to shape the discipline well into the future", they disagree with the consequences that Hayek says derive from his own theory. Hayek says that this completely unregulated process, brings to a perfect situation of equilibrium in the market, brought by the information spreading through the system. But the error is exactly in the fact that (citing [24] again)"he did implicitly assume that the process of entrepreneurial discovery would be stabilizing on average - that the profit opportunities that arose in disequilibrium would be exploited in a manner that sustained coherence and order in the system". Instead "the same problems of stability that have plagued general equilibrium theory also arise in the context of entrepreneurial discovery: individually profitable activities can be destabilizing in the aggregate". More specifically, changes in asset prices can bring to short-term capital gains and losses and information about this changes will be actively searched. In particular, knowing that the information derives from informed individuals, other individuals could try to profit by buying the asset in anticipation, without waiting for further increases in price (what would be the "signal"). But this activity itself, brings to the break of the procedure that Hayek claimed to be stabilizing, because it is a new change in the economy, not a "natural adaptation" of prices and production.

Bowles et al. in [24] also underline how their critical observation do not obscure the important Hayek's contribute as "Hayek's economic vision and critique of equilibrium theory not only remain relevant, but apply with greater force as information has become ever more central to economic activity and the complexity of the information aggregation process has become increasingly apparent. Advances in computational capacity and the growth of online transactions and communication have made the collection and rapid processing of big data feasible and profitable.". Indeed, the idea of price adaptation in a decentralized market can be crucial in the building of computational models , especially agent-based one.

This relationship between ABM and Hayek's theory was firstly noted by Vriend [27], highlighting how ACE (agent-based computational economics) research in general can be seen as an application of Hayek's methodological insights. He shows how information-contagious behavior can emerge (computationally) in a coevolutionary process of interacting adaptive agents and how this is related to various Hayekian theme. He also obtains a similar result than Hayek's conclusion on the self-ordering free economy, because his model exhibits self-organization through information-contagious behavior. But it is not a simple monotonic process from disorder to order until the solution has been reached. Instead, the system continually moves back and forth between order and disorder. Lately, this link will be also seen by Rosser [28] and Axel [29], among others.

Our model will be based on the work that proposes a simplified version of the Hayek's decentralized market hypothesis with an agent based model in Appendix A of [8] . In it, elementary processes of price adaptation in exchanges of a unique good are considered among two kinds of agents: buyers and sellers. In particular, an "Atomistic Hayekian Version" is developed, such that sellers and buyers meet randomly and make the exchange under some circumstances. Then prices adaptation occures. Different version with some changes are also shown, with interesting consequences: the case in which seller and buyers axchange "no matter what", shows exploding sequences of the means of the prices, and exploding standard deviations. From that we can understand that missing the intelligence of the theory in the correction of the prices (implicitly propagating among all the agents), a system of pure random price corrections is far from being plausible. The versions studied after that will be crucial for our work: the case in which the number of buyers is considerably higher than the one of sellers and the case with *cherry-picking*. Starting from the first one, we see how in this case the presence of *sticking prices* is crucial to re-equilibrate the trend of price that, otherwise, would appear unstable. *Sticky* is a general term that, when applied to the sellers' prices, means that the firms are reluctant to change their prices also in the presence of modifications in input cost or demand quantities. With sellers carrying out *cherry-picking*, which is the act of carefully choosing the buyer (or the seller, if the buyer is the *cherry-picker*), they perform also a "sticking" effect on their own prices. That suggests how the effect of cherry-picking in absence of other destabilizing factors, must be re-equilibrated by sticking prices. But the stickyness of sellers prices are not only a way to stabilize the model, it can also be considered a realistic economic phenomenon [29] and will be crucial for the model we are going to develop.

1.2 Swarm dynamics

Theory of swarms was arguably initiated by the celebrated paper by Cucker and Smale [3], in which a population is considered living in $\mathbb{R}^3$. Starting from Vicksek's model [30], they introduce a second-order particle model which resembles Newton's equations in N-body system. They construct the interactions among the agents belonging to the population such that they interact according to their mutual distance and only with the individuals within a certain radius r from it. With this framework, the resulting dynamics is a flock in which every individual's velocity converges to the same speed of the others and the same happens to the mutual distance which becomes constant, so agents are all coordinated.

From this paper, a lot of literature has been created. A part of it is concentrated on the studying of the mathematical aspects of the model [31], [32], for example starting from a Boltzmann-type equation.

Worth to mention, especially considering the results of our model, is the study of clustering in swarm dynamics. Emergence of multi-cluster flocking is often observed in numerical simulations for the Cucker-Smale (C-S) model with short-range interactions. More in detail clustering dynamics of the model under the attractive-repulsive couplings was also discussed in [33]–[35]. However, the explicit computation of the number of emergent multi-clusters a-priori is a challenging problem for the C-S flocking model. In [36] the authors found that the one-dimensional C-S model can be converted into the first-order nonlinear consensus model with monotone coupling function on the real line. Then, for a given initial data and coupling strength, they provided an algorithm determining the number of asymptotic clusters and their group velocities which can be called "the complete cluster predictability" of the one-dimensional C-S model. Unfortunately, there is no multi-dimensional counterpart for this complete cluster predictability.

Figure 1.1: Two clusters in a real birds' swarm.

On the other side, we can see a literature devoted to the application of the theory, in particular to the living systems. In this case, while the original model describes the temporal evolution of the mechanical variables (positions and momentum of particles), in the modeling for the collective dynamics of biological and social complex systems, it's often needed to take into account internal variables such as temperature, spin and excitation [37]–[39]. A detailed description of the use of different possible internal variable, called *activity*, is in [5].

In [5], we can also find the construction of a general framework for the development of the theory of swarms as *behavioral swarms* and we will refer to it to analyze and understand the building of a model through swarm theory. In particular, the mechanical variables and the activity describing the single particle can be developed in the first or second order. In the last case, we can have more irregular changes in the trend of the variable itself. Taking the framework in [5] as an example, we can divide the functions used to describe the interactions of every i-particle acting in their *sensitivity domain* Ω_i (which defines the collection of particles the i-particle can interact with) as:

- η_i models the interaction rate of individual based interactions between i-particle with all particles in the sensitivity domain
- φ_i denotes the action, which occurs with rate η_i, over the activity variable over the i-particle by all particles in Ω_i
- ϕ_i denotes the acceleration, which occurs with rate η_i, over the mechanical variable by all particles in Ω_i

The structure of the model developed at the second order both for the machanical variable x_i and the activity u_i, making use of the functions above, would be:

$$
\begin{cases}
\dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\
\dfrac{d\mathbf{v}_i}{dt} = \displaystyle\sum_{j\in\Omega_i} \eta_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j)\, \psi_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j), \\
\dfrac{d\mathbf{u}_i}{dt} = \mathbf{z}_i, \\
\dfrac{d\mathbf{z}_i}{dt} = \displaystyle\sum_{j\in\Omega_i} \eta_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j)\, \varphi_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j),
\end{cases}
\tag{1.1}
$$

where the notation $j \in \Omega_i$ indicates that summation refers to all j-particles in the domain Ω_i. However, the modeling approach can consider hybrid frameworks which are first order for the activity variable and second order for the mechanical variables.

A detailed mathematical description of the functions above are possible only when we consider a specific model, but there are some common qualitative features that can be found in general in the modelling:

- **Hierarchy** in the decision making of the interaction of the particle as its seen in human crowds [40]
- **Sensitivity domain** Ω_i, a quantity that, as said, can be related to the particles that can have influence on the i-particle
- **Interaction rate** a distance between the interacting entities by a metric suitable to account both for the distance between the interacting entities and that of their statistical distribution
- **Social action** depends on the interaction of each particle with those in its sensitivity domain
- **Mechanical action** follow the rules of classical mechanics, but the parameters leading to accelerations depend on the social state by models to be properly defined

An important other feature of the modelling through swarm theory, is the possibility to consider different populations in the swarm. This subdivision accounts for different objectives or for a different strategy to pursue them. We will call this subdivision *functional subsystems*, in short FSs. Interactions occur either within each functional subsystem or across them. A general development of the system with different FSs, labeled by the subscripts $k = 1, ..., n$ can be written as

$$\begin{cases} \dfrac{d\mathbf{x}_{ik}}{dt} = \mathbf{v}_{ik}, \\ \dfrac{d\mathbf{v}_{ik}}{dt} = \displaystyle\sum_{jq\in\Omega_{ik}} \eta_{ik}(\mathbf{x}_{ik}, \mathbf{v}_{ik}, \mathbf{u}_{ik}, \mathbf{x}_{jq}, \mathbf{v}_{jq}, \mathbf{u}_{jq})\, \psi_i(\mathbf{x}_{ik}, \mathbf{v}_{ik}, \mathbf{u}_{ik}, \mathbf{x}_{jq}, \mathbf{v}_{jq}, \mathbf{u}_{jq}), \\ \dfrac{d\mathbf{u}_{ik}}{dt} = \mathbf{z}_{ik}, \\ \dfrac{d\mathbf{z}_{ik}}{dt} = \displaystyle\sum_{jq\in\Omega_{ik}} \eta_{ik}(\mathbf{x}_{ik}, \mathbf{v}_{ik}, \mathbf{u}_{ik}, \mathbf{x}_{jq}, \mathbf{v}_{jq}, \mathbf{u}_{jq})\, \varphi_{ik}(\mathbf{x}_{ik}, \mathbf{v}_{ik}, \mathbf{u}_{ik}, \mathbf{x}_{jq}, \mathbf{v}_{jq}, \mathbf{u}_{jq}), \end{cases} \tag{1.2}$$

Where q is the FSs to which the j particle belongs.

The application of the theory of swarms to social and economic systems can be seen in [4], [6], [7], [22], [35], in which the financial market is often studied for the precence of mechanisms with flocking aspects (for example in the case of volatility), but also social processes like the emerging of cultural differentations in a globalized world as ours. In particular [7] will be the basis of our work and consists in the application of the theory to a micro-economic system where buyers and sellers are two functional subsystems of the swarm.

1.3 Swarm Dynamics describing a decentralized Hayekian market

The model from which we start is [7]. In it we can find the construction of the framework describing a market with decentralized prices with elementary processes of prices adaptation through swarm dynamics. In particular, we are in the case of the "mall market" or the one of the selling web-site. That means that the price is exposed. Moreover,we are operating in a short term perspective as defined in economics: NB, what is relevant is not the length of the period under analysis, but the hypothesis that the productive factors are not changing (capital and labor, here represented by the number of sellers). So, we do not consider new sellers' entries or existent sellers' exits *in* or *from* the market. The consequence of this, is that any automatic price control mechanism is missing; instead, allowing the entry and exit mechanism, if prices go too high new sellers (firms) enter in the market increasing the offer side and lowering the prices, and vice versa.

In the dynamics we have two FSs in the swarm: the first one is the one of sellers made of N particles, the second one is the one of buyers with M particles. In general, M is of a higher order of magnitude with respect to N and the following parameter $\varepsilon = \frac{N}{M}$ expresses this ratio. We will not consider the mechanical variable of the particles, meaning that the position of particles will not be determinant to describe the state of the *a-particle*. What will define the state of the particle is only the *activity*. Indeed, it will be a second order system in which the economic variables used as the *activity* will be:

- u_s, where $s = 1, ..., N$ indicates the s-particle belonging to the first functional subsystem (sellers), and u_s is the price of the product (good) offered for sale by the s-seller.

- w_b, where $b = 1, ..., M$ indicates the b-particle belonging to the second functional subsystem (buyers), and w_b is the maximum price the b-buyer is willing to pay, also called *reservation price*.

The variables which define the activities of each of the two FSs are the vectors:

$$\mathbf{u} = (u_1, ..., u_s, ..., u_N) \qquad \text{and} \qquad \mathbf{w} = (w_1, ..., w_b, ..., w_M), \tag{1.3}$$

while the corresponding speeds are

$$\mathbf{v} = (v_1, ..., v_s, ..., v_N) \qquad \text{and} \qquad \mathbf{z} = (z_1, ..., z_b, ..., z_M), \tag{1.4}$$

Two kind of interaction can take place in the model:

- *Micro-micro* interaction: is the interaction that occurs between an $a-particle$ and another $a-particle$. In particular, in our case, is the interaction between a single buyer with a single seller (and viceversa)

- *Macro-micro* interaction: is the interaction that occurs between an $a-particle$ and a whole FS. In particular, in our case, is the interaction between the single seller with the sellers' FS and of the single buyer with buyers' FS

To define better the *macro-micro* interaction, we introduce the m-order moments in each FS:

$$\mathbb{E}_s^m = \frac{1}{N}\sum_{s=1}^{N} u_s^m \qquad \text{and} \qquad \mathbb{E}_b^m = \frac{1}{M}\sum_{b=1}^{M} w_b^m \tag{1.5}$$

In particular, the *macro-micro* interaction will be between the single particle and the first order moment of its own FS.

The dynamic is, at every cycle, every agent performing both the two kinds of interaction:

- Every buyer interacts with every seller (and vice versa). If the s-seller's price u_s is equal or higher than the b-buyer's one w_b, the exchange happens. It does not otherwise. The price adaptation consists in the fact that, if the exchange takes place, the b-buyer's reservation price tends to lower while the s-seller's price tends to get higher. The opposite happens if the exchange does not take place. We use the term *tend to* because the change after the interaction does not act on the price directly, but on its velocity, because we are building a second order system.

- Every agent interacts with its own FS. The price adaptation consists in comparing its price with the first order moment of its FS's prices, trying to move it closer to the 1-order moment.

Let us now introduce the following functions deemed to model interactions among particles and between particles and FSs:

- $\eta_s^b(u_s, w_b)$ models the rate at which a seller s interacts with a buyer b;

- $\eta_b^s(w_b, u_s)$ models the rate at which a buyer b interacts with a seller s;

- $\mu_s(u_s, \mathbb{E}_s)$ models the micro-macro interaction rate between a seller s and her/his own FS;

- $\mu_b(w_b, \mathbb{E}_b)$ models the micro-macro interaction rate between a buyer b and her/his own FS;

- $\varphi_s^b(u_s, w_b, v_s, z_b)$ denotes the micro-micro action, which occurs with rate η_s^b, of a buyer b over a seller s;

- $\varphi_b^s(w_b, u_s, z_b, v_s)$ denotes the micro-micro action, which occurs with rate η_b^s, of a seller s over a buyer b;

- $\psi_s(u_s, \mathbb{E}_s)$ denotes the micro-macro action, which occurs with rate μ_s of the FS of sellers over a seller s;

- $\psi_b(w_b, \mathbb{E}_b)$ denotes the micro-macro action, which occurs with rate μ_b of the FS of buyers over a buyer b.

Accordingly, the mathematical structure corresponding to the setting given by Eq. (2) in [7] is as follows:

$$
\begin{cases}
\dfrac{du_s}{dt} = v_s, \\
\dfrac{dw_b}{dt} = z_b, \\
\dfrac{dv_s}{dt} = \dfrac{1}{M} \displaystyle\sum_{q=1}^{M} \eta_s^q(u_s, w_q)\, \varphi_s^q(u_s, w_q, v_s, z_q) + \mu_s(u_s, \mathbb{E}_s)\, \psi_s(u_s, \mathbb{E}_s), \\
\dfrac{dz_b}{dt} = \dfrac{1}{N} \displaystyle\sum_{q=1}^{N} \eta_b^q(w_b, u_q)\, \varphi_b^q(w_b, u_q, z_b, v_q) + \mu_b(w_b, \mathbb{E}_b)\, \psi_b(u_b, \mathbb{E}_b),
\end{cases}
\tag{1.6}
$$

for $s = 1, \dots N$ and $b = 1, \dots, M$.

Once defined the structure of the model, we can define the interaction more specifically, giving a shape to the used functions.

Important to notice is that, assuming the price as exposed, brings to an important asymmetry in the behavior of the two types of agents. The buyer knows the seller's price, but not vice versa. That means that the buyer can make the adaptation of its reservation price basing also on the seller's price, while the seller can rely only on its own price.

The functions are defined as follows:

- The interaction rates for both micro-micro and macro-micro interactions asymmetrically decay with increasing metrics modeling the distance between the interacting entities starting from the same rates η_0 and μ_0.

$$
\begin{cases}
\eta_s^b \simeq \eta_s = \eta_0 \\
\eta_b^s = \eta_0 \, \exp\left(- \dfrac{1}{\varepsilon} \, \dfrac{|u_s - w_b|}{w_b} \right),
\end{cases}
\tag{1.7}
$$

  where $\varepsilon = N/M$ and ρ is a non negative parameter, and

$$
\begin{cases}
\mu_s = \mu_0, \\
\mu_b = \mu_0 \, \exp\left(- \dfrac{1}{\varepsilon} \, \dfrac{|w_b - \mathbb{E}_b|}{w_b} \right).
\end{cases}
\tag{1.8}
$$

- The actions φ and ψ correspond to a dynamics of consensus driven by the difference between the sellers' and buyers' prices, in the micro-micro interaction, and between the local price and the global one, in the micro-macro interaction. The following model of interaction is proposed

$$\begin{cases} \varphi_s^b = \alpha\, u_s \mathrm{sign}(w_b - u_s), \\ \varphi_b^s = \beta\,(u_s - w_b), \end{cases} \tag{1.9}$$

and

$$\begin{cases} \psi_s = \gamma\,({}_s{-}u_s), \\ \psi_b = \kappa\,({}_b{-}u_b), \end{cases} \tag{1.10}$$

where $\alpha, \beta, \gamma, \kappa$ are non negative parameters.

If the interactions terms introduced in Eqs. (1.7)–(1.10) are replaced into the general structure (1.11), we get a system of ODEs that the describe the whole dynamics. So it will be:

$$\begin{cases} \dfrac{du_s}{dt} = v_s, \\ \dfrac{dw_b}{dt} = z_b, \\ \dfrac{dv_s}{dt} = \dfrac{\alpha}{M}\displaystyle\sum_{q=1}^{M} u_s sign(w_b - u_s) + \epsilon_1(\mathbb{E}_s - u_s)\,\psi_s(u_s, \mathbb{E}_s), \\ \dfrac{dz_b}{dt} = \dfrac{\beta}{N}\displaystyle\sum_{q=1}^{N} \exp\left(-\dfrac{1}{\varepsilon}\,\dfrac{|u_s - w_b|}{w_b}\right)(u_s - w_b) + \epsilon_2 \exp\left(-\dfrac{1}{\varepsilon}\dfrac{|w_b - \mathbb{E}_b|}{w_b}\right)(E_b - w_b), \end{cases} \tag{1.11}$$

where time has been scaled with respect to η_0 and $\epsilon = \frac{\mu_0}{\eta_0}$ and $\epsilon_1 = \epsilon\gamma$ and $\epsilon_2 = \epsilon\delta$

Chapter 2

Model with Cherry Picking

The model we will construct (and that can also be found in [41]), ad said, is a further development of [7], presented above. The main characteristic is the adding of *cherry picking*. This means that an agent chooses a specific other agent to interact with, under some conditions instead of interacting with every seller at each cycle. In this scenario, each buyer chooses a specific seller basing its choice on the offered price and/or quality of the good.

Assume that a level of quality of the product, denoted by c_s, is assigned to each seller s. This quantity will remain constant during the whole process, which means that we are looking at it in the short term and thus the agent is not able to improve or worsen the product's quality. The buyers are now seen as "cherry pickers" because we start from a world in which each of them is aware of the seller's price, but not viceversa. That means that the buyer has more information than the seller (like in a mall or online shopping), so the seller does not know the buyer's "reservation quality" and "reservation price". The only thing it is aware of, is whether the buyer who visited the shop (or the online web site) bought or not.

After the buyer makes its choice, the price dynamics will remain: the buyer decides to buy if its reservation price is higher or equal to the one of seller it has chosen. To choose the seller, the buyer must "visit" its shop (or online shopping site) to check the quality and/or the price of the product offered by every seller and compare them. So the buyer's reservation price is not changed by every seller's price, because the buyer is just looking for the condition it made and then compares its price only with the price of the seller it has chosen. On the other hand, the seller is aware of the visit of every buyer, and if it is not chosen then its price will go down.

Summarizing the above reasonings, the whole dynamics can be described as follows:

1. Each buyer looks for the right seller (under the above mentioned conditions), visiting and comparing the prices and quality offered by all of them.

2. After choosing the right one, the buyer will compare their prices and buy if its reservation price is higher or equal than seller's price (or not if it is not).

3. If the buyer effectively makes the transaction, then its reservation price will go down (if not it will go up).

4. Each seller is visited by every buyer. If they buy, then it will increase the price of the product (if not it will decrease it).

2.1 Model 1

2.1.1 Derivation of the model

Let us first consider a scenario in which the buyer choice is conditioned by both features: quality and price. To make the model near the most to reality, we introduce three types of buyers:

1. Type of buyer B_1, numbered from 1 to a_1, who always chooses the seller offering the highest quality product.

2. Type of buyer B_2, numbered from $a_1 + 1$ to a_2, who always chooses the seller with the highest quality-price ratio.

3. Type of buyer B_3, numbered from $a_2 + 1$ to M, who always chooses the seller with the lowest price.

Another kind of buyer could have been the one choosing the seller with the highest price (for example in the case of a luxury good), but it is not present in every market and, most of all, it is a low percentage of it. For the size we are reproducing now, it is negligible, so we will not consider it.

Buyers belonging to each group B_1, B_2 and B_3 will act in different ways in the *micro-micro* interactions depending on their type. However, in the *macro-micro* interactions all buyers will behave in the same way.

Let us now define the following quantities:

- $s_{c_{max}} = \underset{s\in\{1,\ldots,N\}}{\arg\max}\, c_s$ (for sake of simplicity s_c) is the seller offering the highest quality.

- $s_{r_{max}} = \underset{s\in\{1,\ldots,N\}}{\arg\max}\, \frac{c_s}{w_s}$ (for sake of simplicity s_r) is the seller with the highest quality-price ratio.

- $s_{w_{min}} = \underset{s\in\{1,\ldots,N\}}{\arg\min}\, w_s$ (for sake of simplicity s_w) is the seller offering the lowest price.

Notice that the quantities identifying the three different seller presented above, are defined by the system itself and the condition determining it, and their definition does not depend on the choosing buyers' quantities.

Introducing the above defined types of buyers and sellers in Eq. (1.11), the system describing the dynamics under this scenario is:

$$
\begin{cases}
\dfrac{du_s}{dt} = v_s, \\
\dfrac{dw_b}{dt} = z_b, \\
\dfrac{dv_s}{dt} = \Bigg(\dfrac{1}{a_1}\displaystyle\sum_{q=1}^{a_1}\Big(\delta_s^{s_c}\left[\eta_s^q(u_s,w_q)\varphi_s^q(u_s,w_q,v_s,z_q)\right] + \big(\delta_s^{s_c}-1\big)(\eta_s^b\alpha u_s)\Big) + \\
\qquad + \frac{1}{a_2-a_1}\displaystyle\sum_{q=a_1+1}^{a_2}\Big(\delta_s^{s_r}\left[\eta_s^q(u_s,w_q)\varphi_s^q(u_s,w_q,v_s,z_q)\right] + \big(\delta_s^{s_r}-1\big)(\eta_s^b\alpha u_s)\Big) + \\
\qquad + \frac{1}{M-a_2}\displaystyle\sum_{q=a_2+1}^{M}\Big(\delta_s^{s_w}\left[\eta_s^q(u_s,w_q)\varphi_s^q(u_s,w_q,v_s,z_q)\right] + \big(\delta_s^{s_w}-1\big)(\eta_s^b\alpha u_s)\Big)\Bigg) + \\
\qquad + \mu_s(u_s,\mathbb{E}_s)\psi_s(u_s,\mathbb{E}_s), \\
\dfrac{dz_{b_1}}{dt} = \eta_{b_1}^{s_c}(w_{b_1},u_{s_c})\varphi_{b_1}^{s_c}(w_{b_1},u_{s_c},z_{b_1},v_{s_c}) + \mu_b(w_b,\mathbb{E}_b)\psi_b(w_b,\mathbb{E}_b), \\
\dfrac{dz_{b_2}}{dt} = \eta_{b_2}^{s_r}(w_{b_2},u_{s_r})\varphi_{b_2}^{s_r}(w_{b_2},u_{s_r},z_{b_2},v_{s_r}) + \mu_b(w_b,\mathbb{E}_b)\psi_b(w_b,\mathbb{E}_b), \\
\dfrac{dz_{b_3}}{dt} = \eta_{b_3}^{s_w}(w_{b_3},u_{s_w})\varphi_{b_3}^{s_w}(w_{b_3},u_{s_w},z_{b_3},v_{s_w}) + \mu_b(w_b,\mathbb{E}_b)\psi_b(w_b,\mathbb{E}_b),
\end{cases}
\tag{2.1}
$$

where δ_x^y denotes a delta Kronecker function, namely $\delta_x^y = 1$ if $x = y$ and $\delta_x^y = 0$ otherwise.

The functions used are:

- For the rate of interaction

$$\begin{cases} \eta_s^b \simeq \eta_s = \eta_0 \exp\Big(- \frac{\rho}{\varepsilon} u_s \Big) \\ \eta_b^s = \eta_0 \; \exp\Big(- \dfrac{1}{\varepsilon} \; \dfrac{|u_s - w_b|}{w_b} \Big), \end{cases} \tag{2.2}$$

where $\varepsilon = N/M$ and ρ is a non negative parameter, and

$$\begin{cases} \mu_s = \mu_0, \\ \mu_b = \mu_0 \; \exp\Big(- \dfrac{1}{\varepsilon} \; \dfrac{|w_b - \mathbb{E}_b|}{w_b} \Big). \end{cases} \tag{2.3}$$

- For the interaction

$$\begin{cases} \varphi_s^b = \alpha \, u_s \mathrm{sign}(w_b - u_s), \\ \varphi_b^s = \beta \, (u_s - w_b), \end{cases} \tag{2.4}$$

and

$$\begin{cases} \psi_s = \gamma \, (\mathbb{E}_s - u_s), \\ \psi_b = \kappa \, (\mathbb{E}_b - u_b), \end{cases} \tag{2.5}$$

where $\alpha, \beta, \gamma, \kappa$ are non negative parameters.

Some considerations:

- As it may be easily seen, the used functions are the same used in the original model [7], except for η_s^b , the one of the sellers. Indeed it is

$$\eta_s^b \simeq \eta_s = \eta_0 \exp\Big(- \frac{\rho}{\varepsilon} u_s \Big),$$

  where ρ is a parameter and $\varepsilon = \frac{N}{M}$. In this way we make the price of the sellers more "sticking", because, as said, *cherry picking* creates a *sticking effect* on the price of the picker (in this case the buyer). So, in this way, we can balance this not wanted effect.

- The Kronecker function δ aims to classify the seller: if it is the chosen one by the buyer, then the price dynamics is the same as in the original model without cherry picking. If not, the term $(\delta_s^{s_c}-1)(\eta_s^b\alpha u_s)$ will make the seller's price go down for every buyer of that type, following the same lower-price rule introduced in [7].

- Notice that the maximum (resp. minimum) value in the definitions of c_s and s_r (resp. w_s), above can be eventually reached by more than one seller. If this is the case, a random seller will be picked at random among those who attain the maximum (resp. minimum) value.

- In the simulation we will put $\kappa = 0$, which means that we are considering negligible the *macro-micro* interaction among buyers. So the buyers' dynamics will be:

$$\begin{cases} \dfrac{dz_{b_1}}{dt} = \epsilon_b \exp\left(-\dfrac{1}{\varepsilon}\dfrac{|u_{s_c}-w_{b_1}|}{w_{b_1}}\right)(w_{b_1}-u_{s_c}), \\ \dfrac{dz_{b_2}}{dt} = \epsilon_b \exp\left(-\dfrac{1}{\varepsilon}\dfrac{|u_{s_r}-w_{b_2}|}{w_{b_2}}\right)(w_{b_2}-u_{s_r}), \\ \dfrac{dz_{b_3}}{dt} = \epsilon_b \exp\left(-\dfrac{1}{\varepsilon}\dfrac{|u_{s_w}-w_{b_3}|}{w_{b_3}}\right)(w_{b_3}-u_{s_w}), \end{cases} \tag{2.6}$$

where $\epsilon_b = \eta_0\alpha$

2.1.2 Numerical results of Model 1

Let us firstly perform some numerical experiments by solving Eqs. (2.1) with $N = 10$ sellers and $M = 50$ buyers. In order to set initial conditions we consider that the initial prices, both for sellers and buyers, are taken randomly following a uniform distribution in the interval $[1000, 1005]$ while initial speeds are assumed to be all equal to 0. Figure 2.1 shows the temporal evolution of the system taking $\eta_0 = \mu_0 = \alpha = 1$, and $\beta = \gamma = 0.1$ and $\rho = 2$ for a short term of $T = 1000$ time steps. We can see that prices trend is made of regular waves maintaining same frequency and amplitude for each price, especially for the sellers' prices. The same behavior is observed in the prices' variances, both for buyers and sellers, as shown in Fig. 2.2.

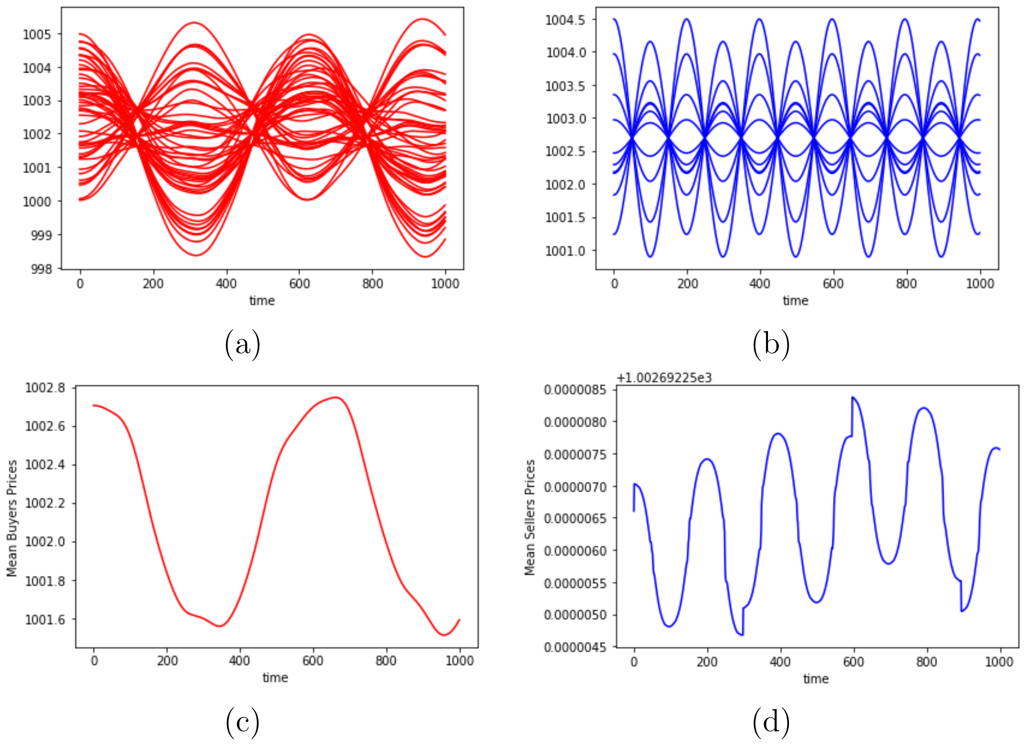


Figure 2.1: Sellers and buyers prices and mean prices for a short term $T = 1000$. (a) Buyers' prices, (b) sellers' prices, (c) buyers' mean price and (d) sellers' mean price.

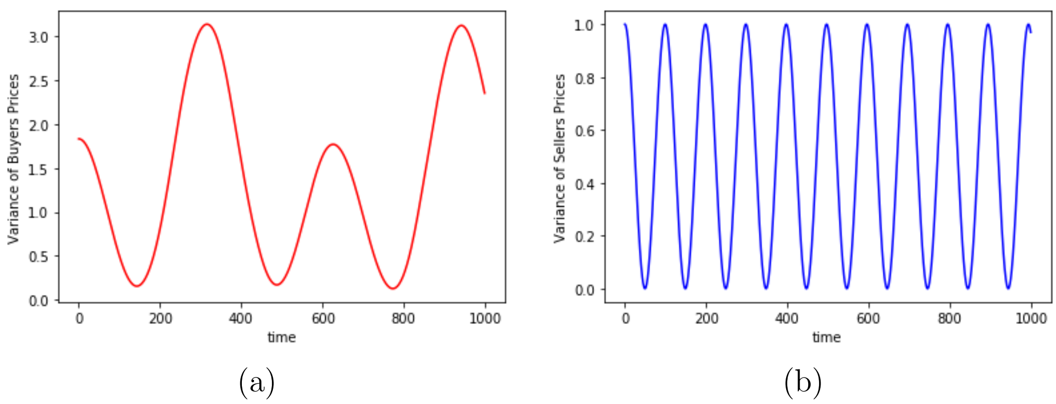


Figure 2.2: Variance of prices for a short term $T = 1000$ for (a) buyers and (b) sellers.

Figure 2.3 represents the corresponding Pareto market efficiencies for short and long terms, which are calculated as follows:

- Sellers' Pareto market efficiency is the sum, at every time t, of $P_s - I_c$ calculated at every exchange at a selling price P_s and for every seller with *initial* *cost* I_c, fixed from the beginning as $\frac{1}{10}$ of seller's price.

- Buyers' Pareto market efficiency is the sum, at every time t, of $R_p - P_s$ calculated at every exchange at a selling price P_s and for every buyer with *reservation price* R_p.

- The total Pareto market efficiency is the sum of the two above.

Notice that Pareto market efficiencies show a sort of regular and cyclical trend, where the benefits of the market are practically all on sellers, because of the *sticking prices* that were introduced for them and also for the choice made about the initial cost.

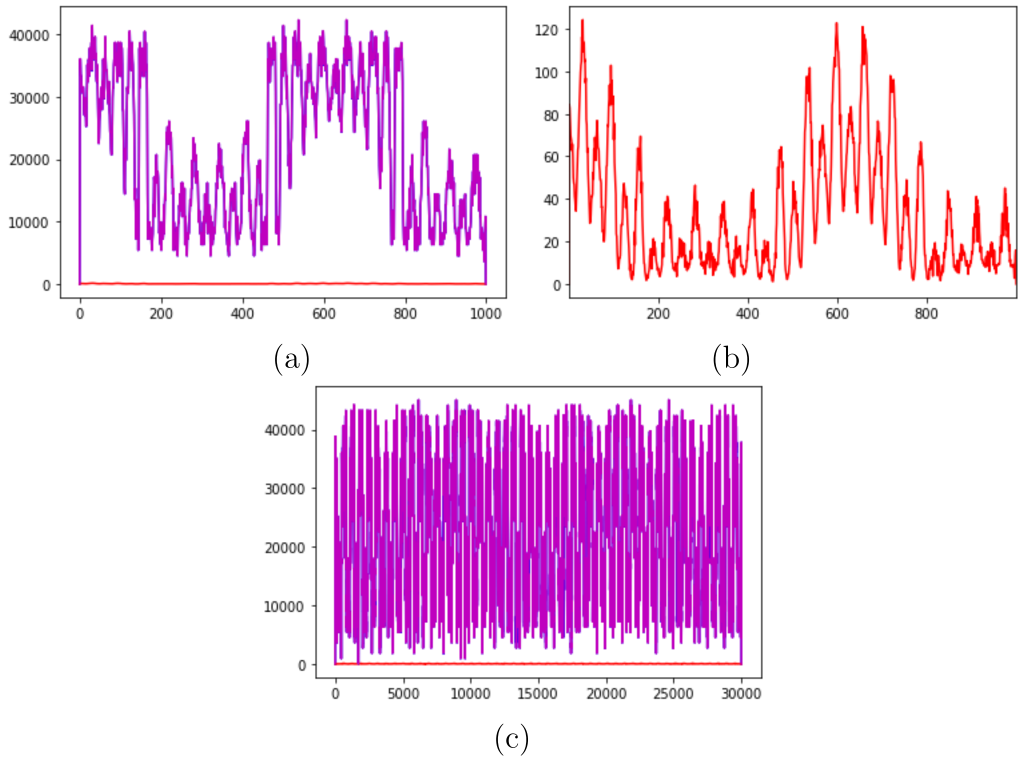
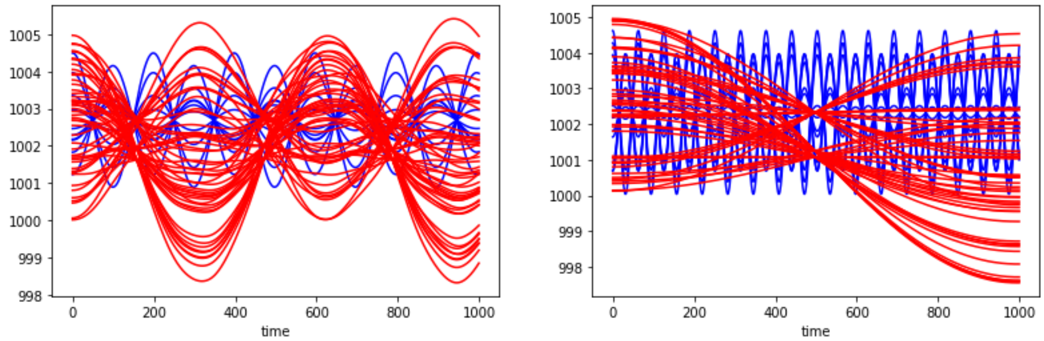


Figure 2.3: Red: buyers – blue: sellers – purple: total Pareto market efficiency. (a) Pareto market efficiency with $\rho = 0.1$, $\gamma = 0.1$, short term. (b) Buyers' Pareto market efficiency with $\rho = 0.1$, $\gamma = 0.1$, short term. (c) Pareto market efficiency with $\rho = 0.1$, $\gamma = 0.1$, long term.

In addition, we aim to investigate the influence of some of the model parameters on the overall dynamics. For instance, Fig. 2.4 shows the trend for two values of η_0, namely $\eta_0 = 1$ and $\eta_0 = 0.1$, while the other parameters keep the same value.

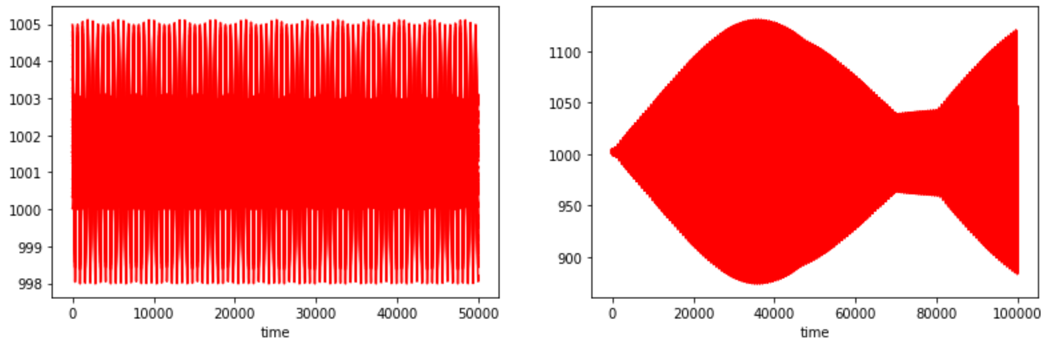
Notice that there is a change in the ratio between the frequencies of the prices of the two different types of agents.



(a) (b)

Figure 2.4: Buyers (red) and sellers (blue) trends for (a) $\eta_0 = 1$ and (b) $\eta_0 = 0.1$.

Both sellers' coordination and buyers' differentiation in type are crucial. In particular, when we decrease sellers' coordination through γ (which goes from 0.1 to 0.01), buyers' prices begin to change the amplitude of their waves during time and macro-waves appear in the long term.



(a) (b)

Figure 2.5: Buyers' prices trends for (a) $\gamma = 0.1$ and (b) $\gamma = 0.01$. In the first case prices range remains constant, in second case we can see macro-waves appearing

Taking a closer look to macro-waves, we can see that they are well differentiated depending on buyers' type. That means that a lower sellers' coordination, brings both to the formation of clusters in the buyers' functional subsystem, depending on their type, and to the aforementioned macro-waves. Recall that buyers B_1 only seek the best quality, buyers B_2 seek for the best quality-price ratio, while buyers B_3 always choose the lowest price. Figure 2.6 shows the dynamics for each type of buyer for different time intervals. In particular we use green for type B_1, purple for B_2 and yellow for B_3. While parameter γ was reduced to 0.01, all the other parameters keep the initially stated values. Three macro-waves emerge according to the type of buyer.

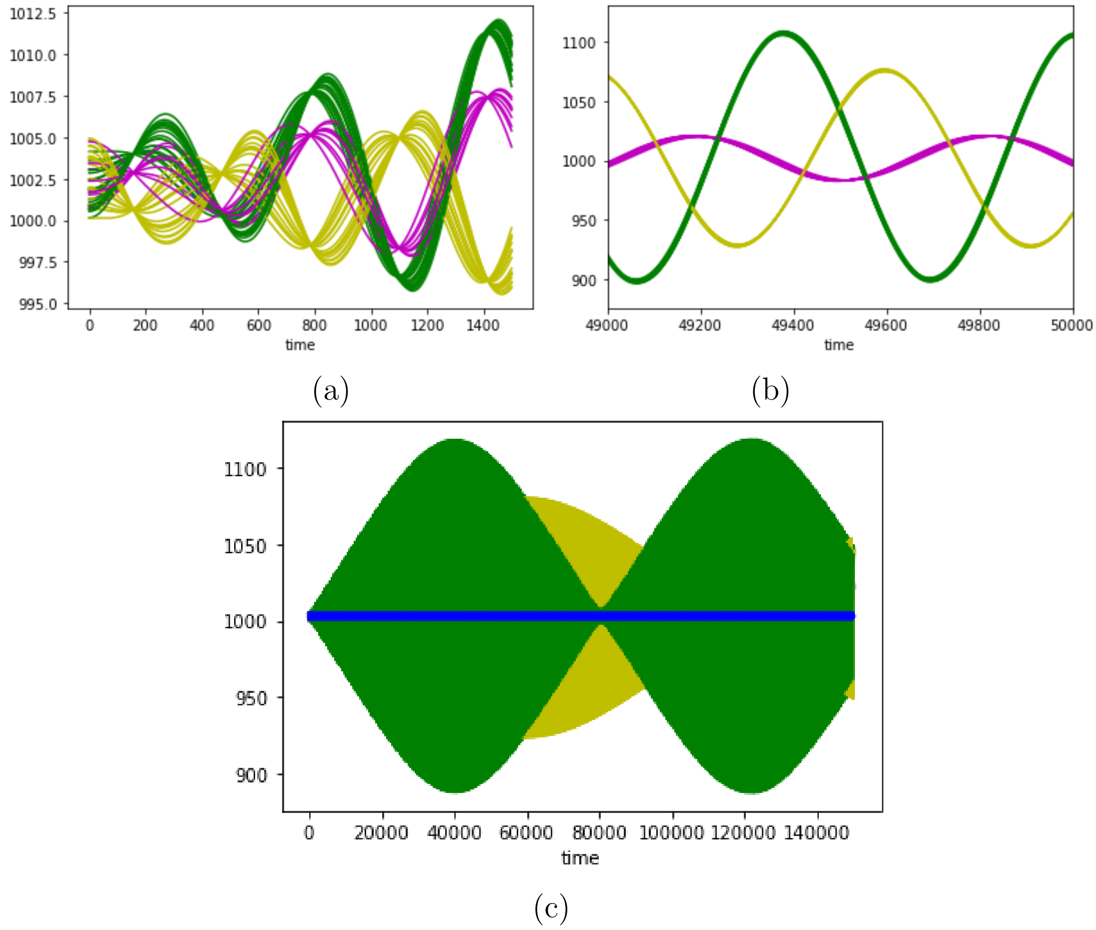
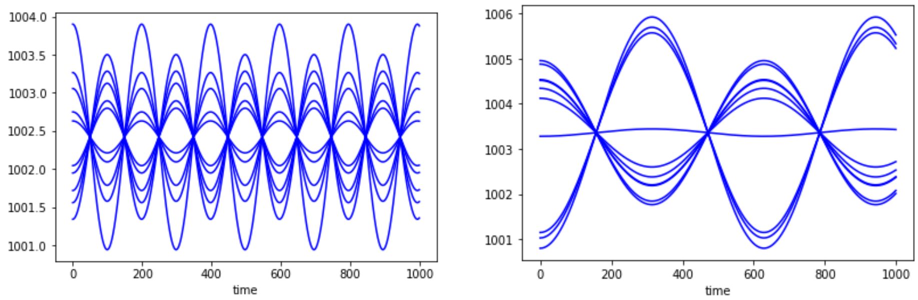


Figure 2.6: Dynamics of buyers' prices for different times intervals. Each color represents a buyer type, namely green B_1, purple B_2, yellow B_3. Blue in (c) is for sellers' prices, that remain in the same constant interval, as in the previous case

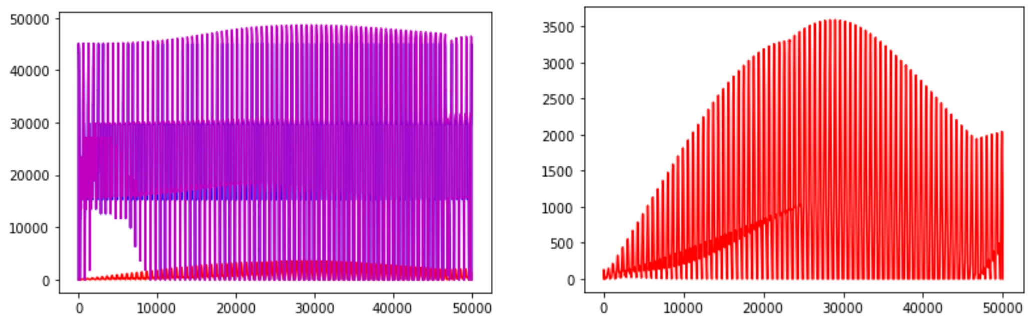
The stickiness of sellers' prices do not allow a visible change [for them] in their amplitude, as shown in Fig. 2.7.



(a) (b)

Figure 2.7: Comparing sellers prices trend for (a) $\gamma = 0.1$ and (b) $\gamma = 0.01$. Here with $\rho = 0.1$ and $\eta = 1$.

But even a small change in sellers' prices trend (which are more free to adapt to buyers' ones due to a lower coordination) brings to an amplified effect on buyers' prices, creating three different markets. Both the split and the macro-waves are a way for the buyer to reach (also creating it) the market they prefer. For example, macro-waves allow more often type B_1 to have higher probability of grabbing the best quality. In this way, they also reach a higher Pareto market efficiency, as shown in Fig. 2.8. A similar result deriving from buyers' coordination is also in [42].



(a) (b)

Figure 2.8: Red: buyers – blue: sellers – purple: total Pareto market efficiency. (a) Pareto market efficiency with $\rho = 0.1$, $\gamma = 0.01$, medium term. (b) Buyers' Pareto market efficiency with $\rho = 0.1$, $\gamma = 0.01$, medium term.

2.2 Model 2

2.2.1 Derivation of the Model

In this case, let us consider the "reservation quality" of the buyer, which is the minimum level of quality that it is willing to accept. We denote it as c_b. The *cherry picking* consists in choosing the seller offering the lowest price among those with $c_s \geq c_b$ (so, among sellers with quality high at least as its own "reservation quality", the buyer b will choose the one with lower price). We denote the chosen seller as:

$$s_{b_{min}} = \underset{s\in\{1,\dots,N\}}{\arg\min}\{w_s | c_s \geq c_b\}, \tag{2.7}$$

and for sake of simplicity we will refer to it as s_b. This time, the seller will not be the same for all the buyers of the same type (as in the previous case). In principle, it could be different for every buyer and that is why it depends on b. Indeed, this time, the choice of the seller will depend also on the choosing buyer's quantities, in particular every buyer is firstly taking into account its own c_b reservation quality to make the choice. That's why, in this case, we use s_b depending on b.

The overall dynamics are then described by the following system:

$$\begin{cases} \dfrac{du_s}{dt} = v_s, \\ \dfrac{dw_b}{dt} = z_b, \\ \dfrac{dv_s}{dt} = \left(\dfrac{1}{M}\displaystyle\sum_{q=1}^{M}\left(\delta_s^{s_q}\left[\eta_s^q(u_s,w_q)\varphi_s^q(u_s,w_q,v_s,z_q)\right] + \left(\delta_s^{s_q}-1\right)(\eta_0\alpha u_s)\right)\right)+ \\ \qquad +\mu_s(u_s,\mathbb{E}_s)\psi_s(u_s,\mathbb{E}_s), \\ \dfrac{dz_b}{dt} = \eta_b^{s_b}(w_b,u_{s_b})\varphi_b^{s_b}(w_b,u_{s_b},z_b,v_{s_b}) + \mu_b(w_b,\mathbb{E}_b)\psi_b(w_b,\mathbb{E}_b), \end{cases} \tag{2.8}$$

where all the interaction functions are the same than in Model 1.

Again, we are not considering the interaction among buyers, assuming it is negligible. So, the final buyers' dynamics will be:

$$\frac{dz_b}{dt} = \epsilon_b \exp\left(-\frac{1}{\varepsilon}\frac{|u_{s_b}-w_b|}{w_b}\right)(w_b - u_{s_b}) \tag{2.9}$$

where $\epsilon_b = \eta_0\alpha$

2.2.2 Numerical results for Model 2

Recall that Model 2 assumes that each buyer has a reservation quality, which is the minimum level of quality that it is willing to accept for the product. Among those sellers satisfying the quality requirement, the buyer will choose the option with lowest price.

Consider the same initial conditions than in the previous case. Fig. 2.9 shows the dynamics for the short term of individual and mean prices.

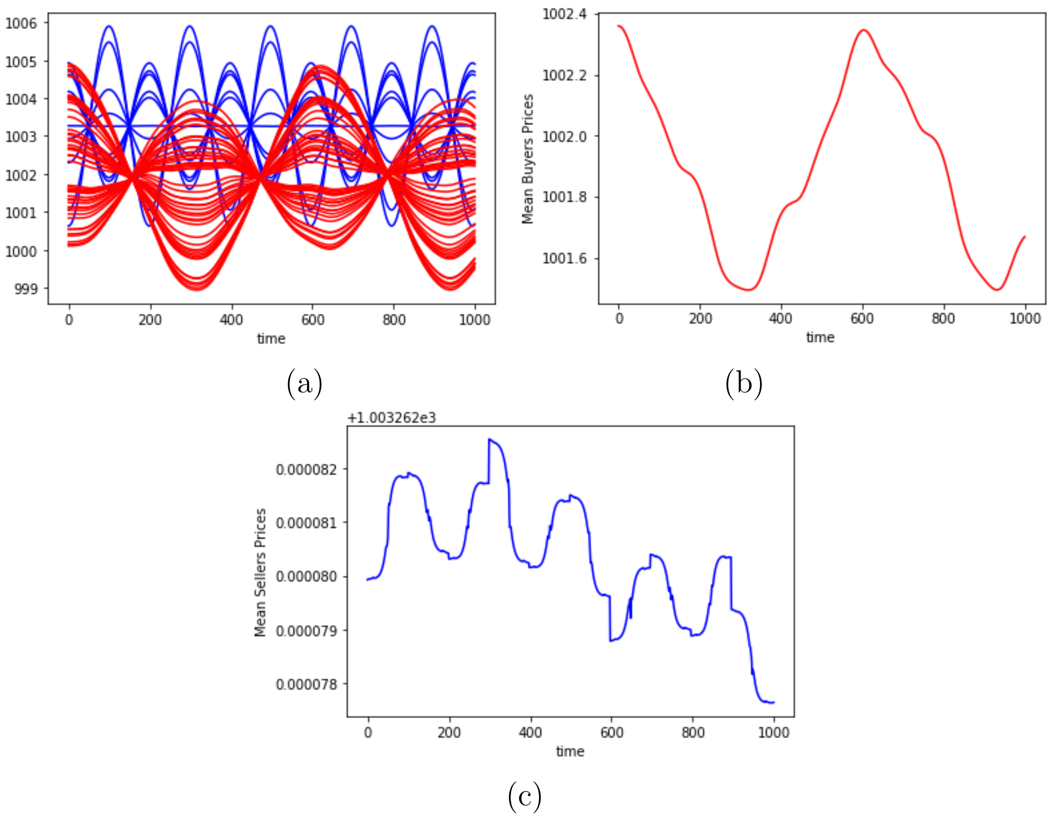
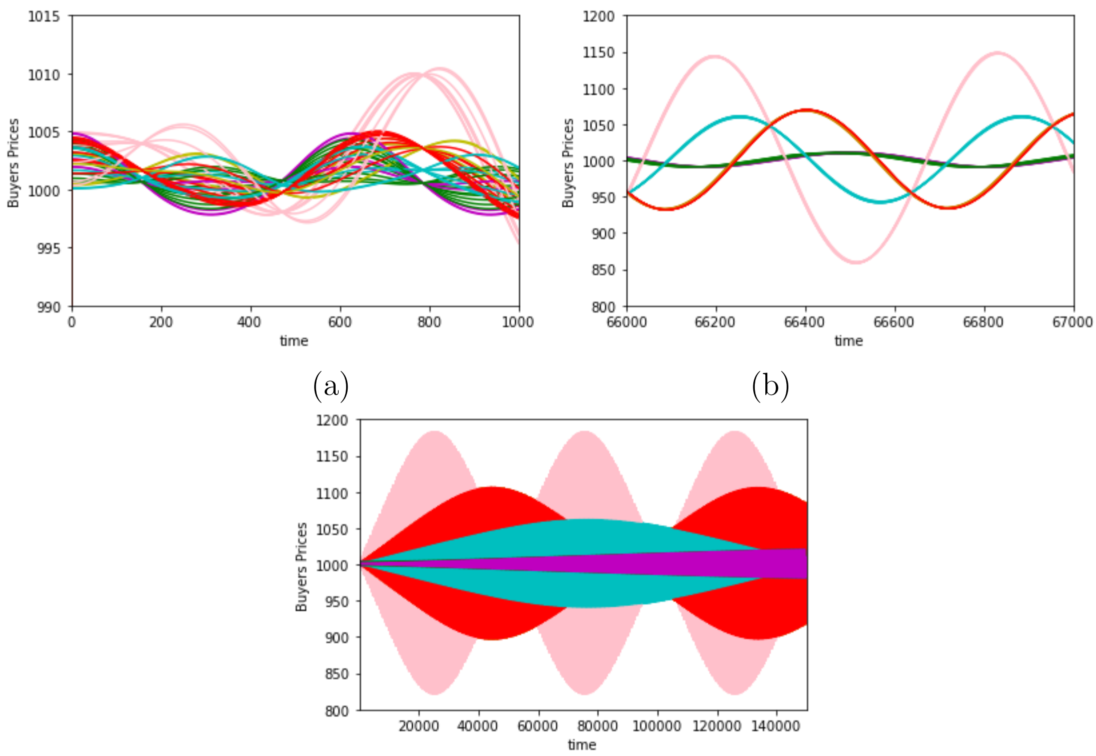


Figure 2.9: Sellers (blue) and buyers (red) prices and mean prices for a short term $T = 1000$, with $\rho = 2$. (a) individual prices (b) buyers' mean price and (c) sellers' mean price.

As in the first case, when the value of γ is changed, we can see macro-waves and a the formation of clusters depending on their (this time) reservation quality. Figure 2.10 shows the case in which the 50 buyers are divided into six reservation qualities that, ordered from larger to lower, will be represented in pink, red, yellow, cyan, green and magenta. It is clear that macro-waves emerge according to the reservation quality and this becomes especially clear for large times.

Notice that, as it usually happens in the simulations, in the short term (Fig. 2.10(a)) there are only three different trends for the six individuated groups, analogously to the first case (Fig. 2.6). In the medium term (Fig. 2.10(c)) we can see a slight differentiation in the frequency and, in the long term we can see that (in Fig. 2.10) we end up with four different clusters



(a) (b)

(c)

Figure 2.10: Evolution of buyers prices for different time intervals. Buyers are divided into 6 reservation qualities that, ordered from larger to lower, will be represented in pink, red, yellow, cyan, green and magenta. Here, $\rho = 0.5$, $\gamma = 0.01$, $\eta = 1$.

The split of the trends can also change depending on the (random) initial conditions of prices. Indeed, we can also see a smaller number of clusters and also a unique cluster, as shown in Fig. 2.12. That means that the formation of clusters is an endogenous effect. In this sense, we may state that the second model is a generalization of the first one, in the sense that buyers can organize both in three clusters as in the first case, but also in more or less as it is more convenient for them.

Also in this second version, the coordination of buyers allows them to gain Pareto market efficiency as in the first one.

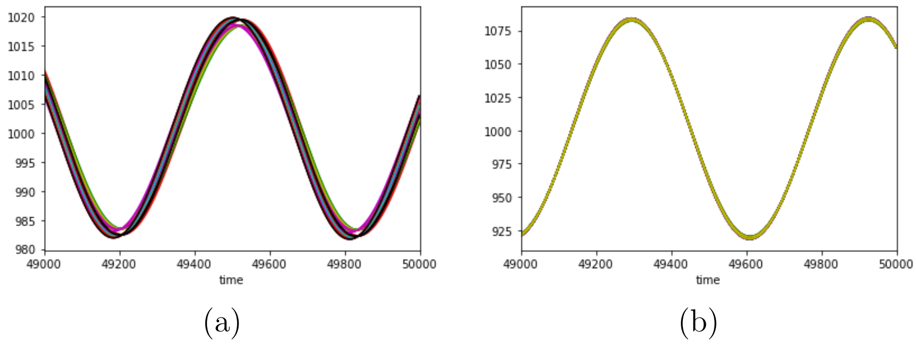


Figure 2.11: Two different simulations showing evolution of buyers' prices in which a unique cluster appears in the medium term. Here with $\rho = 0.5$, $\gamma = 0.01$, $\eta = 1$.

Analysis of clusters

To understand better the formation of clusters, we collected $\simeq 340$ data of the formation of prices and efficiency in the medium term ($5 \cdot 10^4$ time-steps).

First of all we tried to understand if the number of clusters was related to the sellers qualities. As mentioned before, sellers' quality are constant integer numbers created in the interval $[1000, 1005]$. For the first half of collected data we set always at least one seller's quality equal to 1005 and, for the second half, equal to 1004. We made this to allow every buyer to buy, even the one with the highest reservation quality (maximum buyer's quality is 1004) or, otherwise, their price would always get higher.

We identified 4 main scenarios: with 1, 2, 3 and 4 clusters. Also, but not statistically relevant, we found the case of 5 clusters (only one single time):

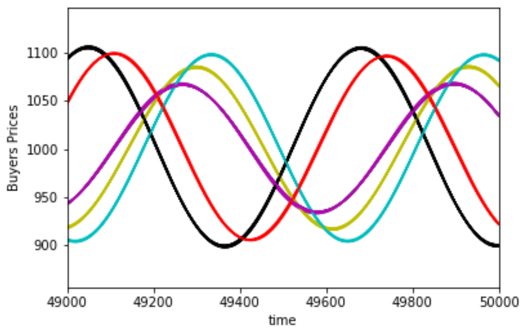
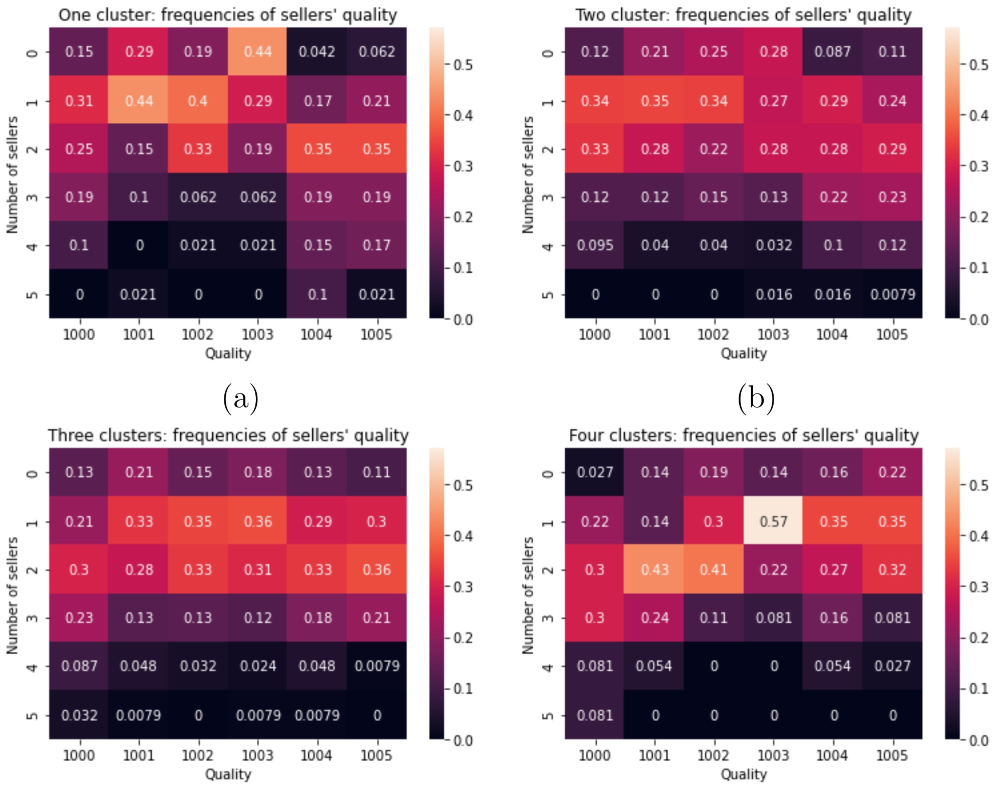


Figure 2.12: Formation of 5 clusters. Single case

What we have studied is, for every cluster scenario, the frequency a seller's quality appears in different values. Through heat maps, we have seen, for example, the frequency with which $0, 1, 2, 3, 4$ or 5 sellers offer a quality of 1000 (and this for every quality). In other words, we are representing the 2D distribution of numbers of sellers offering different levels of quality, for each number of clusters.



(a) (b) (c) (d)

Figure 2.13: Heatmaps representing the 2D distribution of numbers of sellers offering different levels of quality in the case of (a) one cluster, (b) two clusters, (c) three clusters and (d) four clusters.

- *One cluster* We can see a larger presence of high quality $(1004, 1005)$ and an important probability of having a high number of sellers (4 or 5 out of 10) offering this qualities. Less present are the low qualities (1000 and 1001) and more rare are the medium qualities (1002 and 1003). The most frequent number of sellers offering a quality of 1003 is even zero. Instead, the probability of not having 1004 or 1005 qualities is near to zero. This makes sense: a lot of sellers offering high quality means a better chance that every buyer buys from the same sellers (they are the only one that are accessible to every buyer) and high probability of finding the best price among them also for the buyer who accepts every quality. A frequent interaction of all the buyers with the same sellers, brings to the coordination in a unique cluster.

- *Two clusters* In the case of two clusters, the distribution becomes more uniform, even if we can still see a larger presence of high quality

- *Three clusters* Presence of high quality keeps on decreasing and the distribution becomes even more uniform

- *Four clusters* The uniformity breaks and we obtain quite the mirror situation of the case with one cluster: a large presence of low quality, less high quality and a lot of cases with few sellers offering medium quality. This allows buyers with low reservation quality, to have high probability of finding the best price in sellers offering 1000 or 1001 quality, sellers that are not accessible to the buyers with medium or high reservation quality.

We also analysed the relationship between buyers' Pareto market efficiency and clusters. In particular, we took into account the time average value of the buyers' Pareto market efficiency in the case of one, two, three and four clusters. Histograms are made with $\simeq 450$ data, containing repetitions of $5 \cdot 10^4$ cycles. So, if $TotBE$ is the total buyers' efficiency obtained in the complete simulation made of $5 \cdot 10^4$ cycles (called $TotTime$, the total time), we looked at the quantity

$$Mean\,Efficiency = \frac{TotBE}{TotTime} \tag{2.10}$$

First of all we report the frequency of mean efficiency considering all the samples:

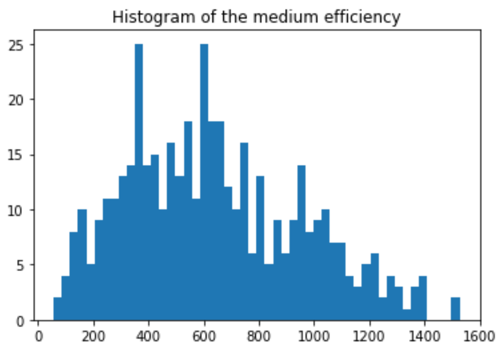


Figure 2.14: The frequency of mean efficiencies

Notice we have two intervals for the mode.

Dividing data considering the different numbers of clusters, we obtain the following histograms

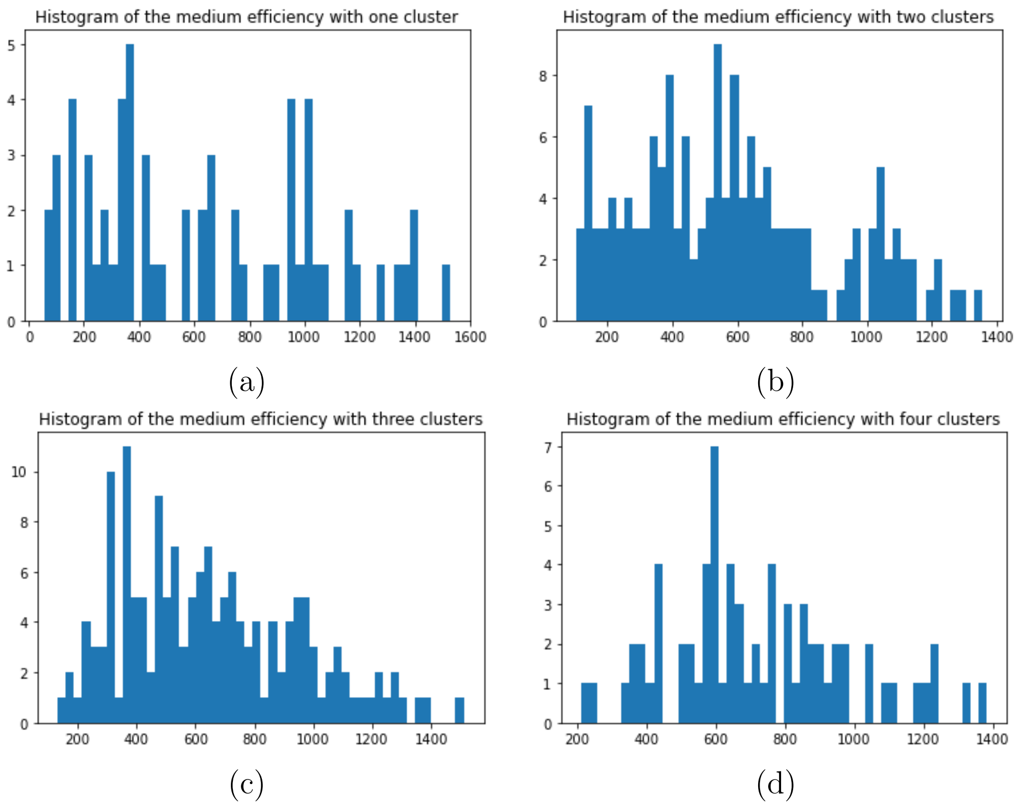


Figure 2.15: Histogram representing the frequency of mean efficiencies of buyers, in the case of (a) one cluster, (b) two clusters, (c) three clusters and (d) four clusters.

Moreover we report some statistics, considering that we used 50 bins to divide the total range, as made for histograms (note that for the mode we report the interval containing the higher number of mean efficiencies):

Number of clusters	1	2	3	4
Mean	636	585	653	723
Standard Deviation	405	303	298	260
Mode	$\simeq$ [350,380]	$\simeq$ [530,575]	$\simeq$ [350,380]	$\simeq$ [585,600]
Total Range	1471	1251	1376	1171
Number of samples	62	156	160	75

Without any doubt we can say that the number of data is too low to make a precise analysis (how the enormous value of the standard deviation also suggests).

The case with a unique cluster is the most rare, this means that its quantities are also the least reliable.

Standard deviation (StD) decreases with the number of clusters which is not only depending on the number of available data. Notice, indeed, that the case with four clusters has the least StD, but also fewer samples than the cases with two or three. This can suggest that a lower number of clusters tends to vary its mean efficiency more than other cases in which the value seems to be more defined. This could depend on the fact that even if every single group can gain every time a different efficiency, with a lot of clusters a high change in one group is less significant than the case with a unique cluster in which the variation regards all the buyers.

The mean of medium efficiency seems to grow with the number of clusters, apart for the case with one cluster (but we have to consider that the distance standing between the one cluster StD and the other StDs is relevant).

The mode divided in two values in the case with all the data, is now well defined for every number of clusters and, in particular the cases with an odd number of clusters have a lower mode about in the same interval. Also the ones with an even number of clusters have got the mode in a comparable interval, higher than the other cases. So, the first interval of the mode in the case with all data is given by the cases with an odd number of clusters, the second interval, by the cases with an even one.

Conclusions

In this project, using [7] as a base for our work, we developed the study of prices' dynamics, applying theory of swarms to describe the interactions of the particles living in our world. We introduced variables that can be seen as economic features. They are carried by particles that represent the agents divided in two different types: buyers and sellers.

We showed a system in which the asymmetry between behaviors of the two types is a fundamental characteristic and a crucial aspect for the obtained results. We study the dynamics of prices in a perfect competitive market where also the parameter of quality is crucial. We used the idea of *cherry picking* performed by buyers, which creates a more realistic behavior for our agents. In this context, the model explains the realistic behavior of markets besides the limits of the classical microeconomics models, with a unique price and a unique good; in the classical framework, goods with quality differences generate multiple markets. From that perspective, it is impossible to analyze the buyers' behavior in the face of quality differences. Considering micro-transactions with prices exposed by the sellers (so-called adhesion contracts), we can instead investigate the effects of the consumers' control about quality, *e.g.*, in food and beverage markets, while cherry picking the products. The relevance of quality is related to goods with a limited range of prices. If the range is enormous, the quality usually is consistent with the price levels.

In Chapter 1 we present the framework of our work and the model at the base of our world [7]. We introduce the decentralized Hayekian market and theory of swarms. We mainly focus on *behavioral swarms* and a generalization of the theory, pointing at the building of a basis structure for behavioral applications. Then, using this framework, we show how the Hayekian market can be described through the theory of swarms, reporting the work in [7]. We set agent's variables which are the price of the offered product for sellers and the reservation price for buyers. Prices' dynamics is based on the interaction (which affects the acceleration of prices) between two agents of the opposite type (*micro-micro* interaction) and the interaction between an agent and the whole group it is part of (the *macro-micro* interaction).

In Chapter 2 we add the main characteristics of our model: the quality variable and *cherry picking* (buyers choose seller to interact with, basing their choice on sellers' prices and qualities). We develop two models. In the first one we add quality as sellers' parameter and we distinguish three types of buyers on the different ways they choose sellers, every type basing its choice on different variables. In the second model we add also buyers' reservation quality and every buyer chooses seller basing both on seller's quality, with respect to its reservation one, and seller's price. In this further development, the asymmetry consists in *cherry picking*, in the *stickiness* of sellers prices and in the idea that the buyer knows seller's price and quality, but the seller does not know the buyer's reservation price and quality (and this is reason why it is the buyer to make the choice).

Computational results are also shown in Chapter 2. If sellers' *macro-micro* interaction is set sufficiently high, we obtain a regular oscillating trend for both sellers' and buyers' prices. Wave trend has a length of few interactions and the amplitude remains always constant in time. Otherwise, if we lower the interaction among sellers, we see a more interesting behavior, which is the main result of our work. Sellers' prices do not seem to have an important change, while the buyers' ones show a change in the amplitude of price's waves during time that, in the long term, creates macro-waves (with long wave-lenght). Moreover every price follows a different macro-cycle depending on buyer's type (for the first model) and buyers' reservation quality (for second model). In this sense, the second case appear to be a generalization of the first one, considering that in the second case we can see endogenous clusters. We can explain this trend saying that a higher freedom for sellers, not bounded by the medium sellers' price, creates a little change in their prices, which brings to an acceleration in buyers' ones. But, to understand better the economic reason behind this trend, we can see the effects on Pareto market efficiency, noticing that macro-waves are not only a way for buyers' to "create" different markets to reach the best choice for their condition, but also a way to increase their own Pareto market efficiency. Our results also suggest a concrete consequence on reality, especially considering the increasing of markets where the competition and the number of relevant sellers are getting lower: when sellers create a sort of "agreement" about their prices (in the model, when they have a high interaction among them), buyers suffer of a drawback. On the other hand, a more free market means a gain for buyers, without a significant loss for sellers.

Appendix

Here we report the code used both for the first and the second model. For a better explanation of the code you can visit the page https://github.com/Valer7a/Cherry-Picking-in-a-Decentralized-Hayekian-Market-with-Quality-described-through-Swam-Theory

where you can also find the link to reach the interacting version through Binder (http://mybinder.org).

2.2.3 First Model

```
# ## Lybraries
# Let's import all the useful libraries

import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
import random
import statistics as st


# ## Parameters
# Let's define the parameters shaping the dynamics



#Number of agents in the model
N = 10       #numbers of sellers
M = 50       #numbers of buyers
ratio = N/M  #ratio sellers/buyers


#Parameters for the model
#micro-micro interaction
eta_0 = 1        #intensity of rate of micro-micro interaction
alpha = 1        #intensity of mi-mi interaction for sellers
rho = 2          #intensity of exp argument for sellers rate
beta = 0.1       #intensity of mi-mi interaction for buyers
#macro-micro interaction
gamma_s = 0.01   #intensity of ma-mi int for sellers
mu_0 = 1         #intensity of rate of ma-mi int for sellers
```

```
#Number of cycles
n = 50000


# ## Quantities


#PRICES
b = np.full([n,M], 0.0) #buyers' prices during time-steps
s = np.full([n,N], 0.0) #sellers' prices during time-steps


#VELOCITIES of prices
x = np.full([n,M], 0.0) #buyers' prices velocities dTS
y = np.full([n,N], 0.0) #sellers' prices velocities dTS


#MEAN prices
MeanPrice_B = np.full([n,1], 0.0) #buyers' mean prices dTS
MeanPrice_S = np.full([n,1], 0.0) #sellers' mean prices dTS


#VARIANCE of prices
VariancePrice_S = np.full([n,1], 0.0) #buyers' variance dTS
VariancePrice_B = np.full([n,1], 0.0) #sellers' variance dTS


#PARETO MARKET EFFICIENCY
efficiency_S = np.full([n,1], 0.0)   #total buyers' eff dTS
efficiency_B = np.full([n,1], 0.0)   #total sellers' eff dTS
efficiency_Tot = np.full([n,1], 0.0) #total market eff dTS


#INITIAL CONDITIONS renewed at each TS (only for modelling)
z_b0 = [0,0] #initial cond. of every time-step for buyers
z_s0 = [0,0] #initial cond. of every time-step for sellers





#SELLERS

#Sellers Initial Cost
s_cost = np.full(N, 0.0)

#Every Seller's offered Quality
quality = np.full(N, 0.0)
```

```
#Sellers Quality/Price Ratio
ratio_qp = np.full(N, 0.0)



#BUYERS

#Buyers Type
b_type = np.full(M, 0.0)

#Buyers Chosen Seller at every timestep
chosen_seller = np.full([n,M], 0)

#Chosen Price and Seller renewed at each TS ( for modelling)
b_choices = [0,0]




#SETTING INITIAL CONDITIONS FOR ALL QUANTITIES

#Buyers initial Prices and Types
for i in range(0,M):
    #Price
    b[0,i] = random.random()*5 + 1000
    #Type
    b_type[i] = random.randint(1,3)
    print(b_type[i])


#Sellers initial Prices, Costs, Qualities, Quality/Price Ratio
for j in range(0,N):
    #Price
    s[0,j] = random.random()*5 + 1000
    #Cost
    s_cost[j] = (s[0,j])*0.1
    #Quality
    quality[j] = random.randint(1000,1005)
    print(quality[j])
    #Ratio
    ratio_qp[j] = quality[j]/s[0,j]


#Calculation of initial Mean Prices
MeanPrice_S[0] = st.mean(s[0]) #buyers initial mean prices
MeanPrice_B[0] = st.mean(b[0]) #sellers initial mean price


#Calculation of initial Variances of Prices
VariancePrice_S[0] = st.variance(s[0]) #sellers initial var
VariancePrice_B[0] = st.variance(b[0]) #buyers initial var
```

```
# ## Functions


#BUYERS FUNCTIONS


#Micro-Micro

#Function defining rate of micro-micro interaction
def eta_b(bb,ss,ratio,eta_0):
    if -0.01 < bb < 0.01:
        argument = (np.abs((ss-bb)/0.01))
    else:
        argument = (np.abs((ss-bb)/bb))
    xx = -(ratio)*(argument)
    etab = eta_0 * (np.exp(xx))
    return etab

#Function defining micro-micro interaction
def fi_b(bb,ss,beta):
    fib = beta * (ss-bb)
    return fib


#Function defining the seller
#with which the buyer will interact
def choice(s,b_type,quality,ratio_qp):
    chosen_price = s[0]
    b_choice = 0
    q = quality[0]
    value = ratio_qp[0]
    for j in range(0,N-1):
    #depending on the buyer's type,
    #best seller (with its offered price) is chosen
        if b_type == 1.0: #B1
            if quality[j+1] > q:
                chosen_price = s[j+1]
                b_choice = j+1
                q = quality[j+1]
            elif (quality[j+1] == q):
                b_choice = random.choice([b_choice, j+1])
                chosen_price = s[b_choice]
        elif b_type == 2.0: #B2
            if ratio_qp[j+1] > value:
                chosen_price = s[j+1]
                b_choice = j+1
                value = ratio_qp[j+1]
            elif (ratio_qp[j+1] == value):
                b_choice = random.choice([b_choice, j+1])
```

```
                chosen_price = s[b_choice]
        elif b_type == 3.0: #B3
            if chosen_price > s[j+1]:
                chosen_price = s[j+1]
                b_choice = j+1
            elif (chosen_price == s[j+1]):
                b_choice = random.choice([b_choice, j+1])
    #returns price of the chosen seller
    #and the chosen seller in this order
    return chosen_price,b_choice




#Whole Dynamics

#Function defining the whole dynamics
#of buyers' acceleration in ODE
def model_b(z_b,_,min_price,ratio,beta,N,eta_0):
    b = z_b[0]
    x = z_b[1]
    sumx = (eta_b(b,min_price,ratio,eta_0)) * fi_b(b,min_price,beta)
    dbdt = x
    dxdt = (1/N) * sumx
    return [dbdt,dxdt]



#SELLERS FUNCTIONS


#Micro-Micro

#Function defining rate of micro-micro interaction
def eta_s(s,ratio,eta_0,rho):
    xx = -(rho/ratio)*np.abs(s)
    etas = eta_0 * (np.exp(xx))
    return etas

#Function defining micro-micro interaction
def fi_s(b,s,alpha,chosen_s,s_counter):
    if (chosen_s == s_counter): #here seller knows if it's
                                #chosen seller for buyer i
        if (b<s):
            sig = -1
        else:
            sig = 1
    else:
        sig = -1
    fis = alpha*np.abs(s)*sig
    return fis
```

```
#Macro-Micro

#Function defining macro-micro interaction
def psi_s(s,E1,gamma_s):
    psis = gamma_s*(E1 - s)
    return psis




#Whole Dynamics

#Function defining the whole dynamics of sellers' acceleration in ODE
def model_s(z_s,_,b,E1,mu_0,eta_0,alpha,M,gamma_s,chosen_s,s_counter,rho):
    s = z_s[0]
    y = z_s[1]
    sumy = 0.0 #sum on buyers (interaction of the seller j with all buyers)
    for i in range(0,M):
        sumy = sumy + (eta_s(s,ratio,eta_0,rho)*fi_s(b[i],s,alpha,chosen_s[i],s_counter))
    dsdt = y
    dydt = (1/M)*(sumy) + mu_0 * psi_s(s,E1,gamma_s)
    return [dsdt,dydt]


# ## Whole Dynamics
#
# Now we are going to solve computationally the ODEs, obtaining prices, velocities,
#mean prices, variances and efficiencies both for buyers and sellers



tLeft = 0.1 #to define time interval


#SOLVING ODE

#iterating on Timesteps
for k in range(0,n-1):
    TimeInterval = [0,tLeft]
    #calculating first order moment of sellers prices at timestep k-1
    sumE1 = 0.0
    for jj in range(0,N):
        sumE1 = sumE1 + s[k,jj]
    E1 = sumE1/N

#iterating on Buyers
    for i in range(0,M):
        b_choices = choice(s[k],b_type[i],quality,ratio_qp) #every buyer chooses its best seller
```

```
        chosen_price = b_choices[0]
        chosen_seller[k,i] = b_choices[1]
        z_b0 = [b[k,i],x[k,i]] #initial conditions at time-step k+1
        z_b = odeint(model_b,z_b0,TimeInterval,args=(chosen_price,ratio,beta,N,eta_0)) #solving ODEs for buyer i
        b[k+1,i] = z_b[1,0] #price at time-step k+1 of i-buyer
        x[k+1,i] = z_b[1,1] #velocity at time-step k+1 of i-buyer
        if k > 0:
        #calculating buyers' and sellers' efficiency
            if b[k,i] >= chosen_price:
                efficiency_B[k] = efficiency_B[k] + (b[k,i] - chosen_price)
                efficiency_S[k] = efficiency_S[k] + (chosen_price - s_cost[chosen_seller[k,i]])
    #calculating mean and variance of buyers' prices and total efficiency for each TimeStep
    MeanPrice_B[k+1] = st.mean(b[k+1])
    VariancePrice_B[k+1] = st.variance(b[k+1])
    efficiency_Tot[k] = efficiency_B[k] + efficiency_S[k]

#iterating on Sellers
    for j in range(0,N):
        seller_counter = j #to detect if j-seller is the chosen one
        z_s0 = [s[k,j],y[k,j]] #initial conditions at time-step k+1
        z_s=odeint(model_s,z_s0,TimeInterval,args=(b[k],E1,mu_0,eta_0,alpha,M,gamma_s,chosen_seller[k],seller_counter,rho))
        s[k+1,j] = z_s[1,0] #price at time-step k+1 of j-seller
        y[k+1,j] = z_s[1,1] #velocity at time-step k+1 of j-seller
        ratio_qp[j] = (quality[j])/(s[k+1,j])
    #calculating mean and variance of sellers prices for each TimeStep
    MeanPrice_S[k+1] = st.mean(s[k+1])
    VariancePrice_S[k+1] = st.variance(s[k+1])
#to take trace of time-steps
    if (k % 1000) == 0:
        print(k)



# ## Plots



#BUYERS' AND SELLERS' PRICES PLOT
for i in range(0,M):
    if (b_type[i]) == 1:
        plt.plot(b[:,i],'g-') #first type buyers in green
    elif (b_type[i]) == 2:
        plt.plot(b[:,i],'m-') #second type buyers in magenta
    elif (b_type[i]) == 3:
        plt.plot(b[:,i],'y-') #third type buyers in yellow
plt.xlabel('time')
for j in range(0,N):
        plt.plot(s[:,j],'b-') #sellers prices in blue
plt.xlabel('time')
plt.show()

```

```
#SELLERS' PRICES PLOT
for j in range(0,N):
    plt.plot(s[:,j],'b-') #sellers prices in blue
plt.xlabel('time')
plt.show()



#BUYERS' PRICES PLOT WITH ZOOM IN [49000,50000] TIME-STEPS INTERVAL
for i in range(0,M):
    if (b_type[i]) == 1:
        plt.plot(b[:,i],'g-') #first type buyers in green
    elif (b_type[i]) == 2:
        plt.plot(b[:,i],'m-') #second type buyers in magenta
    elif (b_type[i]) == 3:
        plt.plot(b[:,i],'y-') #third type buyers in yellow
plt.xlabel('time')
plt.xlim([49000, 50000])
plt.show()




#BUYERS' PRICES VELOCITIES PLOT WITH ZOOM IN [49000,50000] TIME-STEPS INTERVAL
for i in range(0,M):
    if (b_type[i]) == 3:
        plt.plot(x[:,i],'y-') #third type buyers in yellow
    elif (b_type[i]) == 2:
        plt.plot(x[:,i],'m-') #second type buyers in magenta
    elif (b_type[i]) == 1:
        plt.plot(x[:,i],'g-') #first type buyers in green
plt.xlabel('time')
plt.ylabel('Buyers Velocities')
plt.xlim([49000, 50000])
plt.show()




#MEAN SELLERS' PRICES PLOT
plt.plot(MeanPrice_S,'b')
plt.xlabel('time')
plt.ylabel('Mean Sellers Prices')
plt.show()
```

```
#MEAN BUYERS' PRICES PLOT
plt.plot(MeanPrice_B,'r')
plt.xlabel('time')
plt.ylabel('Mean Buyers Prices')
plt.show()




#VARIANCE OF SELLERS' PRICES PLOT
plt.plot(VariancePrice_S,'b')
plt.xlabel('time')
plt.ylabel('Variance of Sellers Prices')
plt.show()




#VARIANCE OF BUYERS PRICES PLOT
plt.plot(VariancePrice_B,'r')
plt.xlabel('time')
plt.ylabel('Variance of Buyers Prices')
plt.show()




#ALL EFFICIENCIES PLOT
plt.plot(efficiency_S,'b')   #sellers' efficiency in blue
plt.plot(efficiency_B,'r')   #buyers' efficiency in red
plt.plot(efficiency_Tot,'m') #total efficiency in magenta
plt.show()




#BUYERS' EFFICIENCY PLOT
plt.plot(efficiency_B,'r')
plt.show()
```

2.2.4 Second Model

```
# ## Lybraries
# Let's import all the useful libraries


```

```python

import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
import random
import statistics as st


# ## Parameters
#
# Let's define the parameters shaping the dynamics



#Number of agents in the model
N = 10       #numbers of sellers
M = 50       #numbers of buyers
ratio = N/M  #ratio sellers/buyers #pt in fig 1 del paper capisco che il ratio Ã¨ M/N


#Parameters for the model
#micro-micro interaction
eta_0 = 1       #intensity of rate of micro-micro interaction
alpha = 1       #intensity of micro-micro interaction for sellers
rho = 2         #intensity of exponential argument in the rate of micro-micro interaction for seller
beta = 0.1      #intensity of micro-micro interaction for buyers
#macro-micro interaction
gamma_s = 0.01  #intensity of macro-micro interaction for sellers
mu_0 = 1        #intensity of rate of macro-micro interaction for sellers


#Number of cycles
n = 50000



# ## Quantities


#PRICES
b = np.full([n,M], 0.0) #buyers prices during time-steps
s = np.full([n,N], 0.0) #sellers prices during time-steps


#VELOCITIES of prices
x = np.full([n,M], 0.0) #buyers prices velocities during time-steps
y = np.full([n,N], 0.0) #sellers prices velocities during time-steps


#MEAN prices
MeanPrice_B = np.full([n,1], 0.0) #buyers mean prices during time-steps
```

```python
MeanPrice_S = np.full([n,1], 0.0) #sellers mean prices during time-steps


#VARIANCE of prices
VariancePrice_S = np.full([n,1], 0.0) #buyers variance of prices during time-steps
VariancePrice_B = np.full([n,1], 0.0) #sellers variance of prices during time-steps


#PARETO MARKET EFFICIENCY
efficiency_S = np.full([n,1], 0.0)   #total buyers efficiency during time-steps
efficiency_B = np.full([n,1], 0.0)   #total sellers efficiency during time-steps
efficiency_Tot = np.full([n,1], 0.0) #total market efficiency during time-steps


#INITIAL CONDITIONS renewed at every time-step (only for the modelling)
z_b0 = [0,0] #initial conditions of every time-step for buyers
z_s0 = [0,0] #initial conditions of every time-step for sellers


#QUALITIES
quality_s = np.full(N, 0.0) #sellers' offered quality
quality_b = np.full(M, 0.0) #buyers' reservation qualities



#SELLERS

#Sellers Initial Cost
s_cost = np.full(N, 0.0)




#BUYERS

#Buyers Chosen Seller at every timestep
chosen_seller = np.full([n,M], 0)

#Buyer Chosen Price and Seller renewed at each timestep (only for modelling)
b_choices = [0,0]




#SETTING INITIAL CONDITIONS FOR ALL QUANTITIES

#Buyers initial Prices, and Reservation Qualities
for i in range(0,M):
    #Price
    b[0,i] = random.random()*5 + 1000
    #Quality
```

```
    quality_b[i] = random.randint(999,1004)
    print('buyer '+ str(i) + ' quality ' + str(quality_b[i]) )

#to make sure the lowest quality offering seller will have a buyer
quality_b[1] = 999


#Sellers initial Prices, Costs, Qualities
for j in range(0,N):
    #Price
    s[0,j] = random.random()*5 + 1000
    #Cost
    s_cost[j] = (s[0,j])*0.1
    #Quality
    quality_s[j] = random.randint(1000,1005)
    print('seller '+ str(j) + ' quality ' + str(quality_s[j]) )

#to meke sure every buyers chooses a seller
quality_s[N-1] = 1005



#Calculation of initial Mean Prices
MeanPrice_S[0] = st.mean(s[0]) #buyers initial mean prices
MeanPrice_B[0] = st.mean(b[0]) #sellers initial mean price


#Calculation of initial Variances of Prices
VariancePrice_S[0] = st.variance(s[0]) #buyers initial variance of prices
VariancePrice_B[0] = st.variance(b[0]) #sellers initial variance of prices




# ## Functions


#BUYERS FUNCTIONS


#Micro-Micro

#Function defining rate of micro-micro interaction
def eta_b(bb,ss,ratio,eta_0):
    if -0.01 < bb < 0.01:
        argument = (np.abs((ss-bb)/0.01))
    else:
        argument = (np.abs((ss-bb)/bb))
    xx = -(ratio)*(argument)
    etab = eta_0 * (np.exp(xx))
    return etab
```

```

#Function defining micro-micro interaction
def fi_b(bb,ss,beta):
    fib = beta * (ss-bb)
    return fib

#Function defining the seller with which the buyer will interact
def choice(s,quality_b,quality_s):
    acc_s = [] #stands for acceptable sellers, the sellers with quality high enough for the buyer
    l = 0 #to count which are acceptable sellers
    for m in range(0,N):
        if quality_s[m] >= quality_b:
            acc_s.append(m) #registering all sellers with the right quality
            l = l + 1
    chosen_price = s[acc_s[0]]
    b_choice = acc_s[0]
    for j in range(0,l-1): #choosing the seller with the lowest price among the acceptable ones
            if (chosen_price > s[acc_s[j+1]]):
                chosen_price = s[acc_s[j+1]]
                b_choice = acc_s[j+1]
            elif (chosen_price == s[acc_s[j+1]]):
                chosen_price = s[acc_s[j]]
                b_choice = random.choice([s[acc_s[j]], s[acc_s[j+1]]])
    #returns price of the chosen seller and the chosen seller in this order
    return chosen_price,b_choice


#Whole Dynamics

#Function defining the whole dynamics of buyers' acceleration in ODE
def model_b(z_b,_,min_price,ratio,beta,N,eta_0):
    b = z_b[0]
    x = z_b[1]
    sumx = (eta_b(b,min_price,ratio,eta_0)) * fi_b(b,min_price,beta)
    dbdt = x
    dxdt = (1/N) * sumx
    return [dbdt,dxdt]




#SELLERS FUNCTIONS


#Micro-Micro

#Function defining rate of micro-micro interaction
def eta_s(s,ratio,eta_0,rho):
    xx = -(rho/ratio)*np.abs(s)
    etas = eta_0 * (np.exp(xx))
    return etas
```

```python

#Function defining micro-micro interaction
def fi_s(b,s,alpha,chosen_s,s_counter):
    if (chosen_s == s_counter): #here seller knows if it is the chosen seller for buyer i
        if (b<s):
            sig = -1
        else:
            sig = 1
    else:
        sig = -1
    fis = alpha*np.abs(s)*sig
    return fis


#Macro-Micro

#Function defining macro-micro interaction
def psi_s(s,E1,gamma_s):
    psis = gamma_s*(E1 - s)
    return psis




#Whole Dynamics

#Function defining the whole dynamics of sellers' acceleration in ODE
def model_s(z_s,_,b,E1,mu_0,eta_0,alpha,M,gamma_s,chosen_s,s_counter,rho):
    s = z_s[0]
    y = z_s[1]
    sumy = 0.0 #sum on buyers (interaction of the seller j with all buyers)
    for i in range(0,M):
        sumy = sumy + (eta_s(s,ratio,eta_0,rho)*fi_s(b[i],s,alpha,chosen_s[i],s_counter))
    dsdt = y
    dydt = (1/M)*(sumy) + mu_0 * psi_s(s,E1,gamma_s)
    return [dsdt,dydt]


# ## Whole Dynamics



tLeft = 0.1 #to define time interval


#SOLVING ODE

#iterating on Timesteps
for k in range(0,n-1):
    TimeInterval = [0,tLeft]
    #calculating first order moment of sellers prices at timestep k-1
```

```
    sumE1 = 0.0
    for jj in range(0,N):
        sumE1 = sumE1 + s[k,jj]
    E1 = sumE1/N

#iterating on Buyers
    for i in range(0,M):
        b_choices = choice(s[k],quality_b[i],quality_s) #every buyer chooses its best seller
        chosen_price = b_choices[0]
        chosen_seller[k,i] = b_choices[1]
        z_b0 = [b[k,i],x[k,i]] #initial conditions at time-step k+1
        z_b = odeint(model_b,z_b0,TimeInterval,args=(chosen_price,ratio,beta,N,eta_0)) #solving ODEs for buyer i
        b[k+1,i] = z_b[1,0] #price at time-step k+1 of i-buyer
        x[k+1,i] = z_b[1,1] #velocity at time-step k+1 of i-buyer
        if k > 0:
        #calculating buyers' and sellers' efficiency
            if b[k,i] >= chosen_price:
                efficiency_B[k] = efficiency_B[k] + (b[k,i] - chosen_price)
                efficiency_S[k] = efficiency_S[k] + (chosen_price - s_cost[chosen_seller[k,i]])
    #calculating mean and variance of buyers' prices and total efficiency for each TimeStep
    MeanPrice_B[k+1] = st.mean(b[k+1])
    VariancePrice_B[k+1] = st.variance(b[k+1])
    efficiency_Tot[k] = efficiency_B[k] + efficiency_S[k]

#iterating on Sellers
    for j in range(0,N):
        seller_counter = j
        z_s0 = [s[k,j],y[k,j]]
        z_s=odeint(model_s,z_s0,TimeInterval,args=(b[k],E1,mu_0,eta_0,alpha,M,gamma_s,chosen_seller[k],seller_counter,rho))
        s[k+1,j] = z_s[1,0]
        y[k+1,j] = z_s[1,1]
    #calculating mean and variance of sellers prices for each TimeStep
    MeanPrice_S[k+1] = st.mean(s[k+1])
    VariancePrice_S[k+1] = st.variance(s[k+1])
#to take trace of time-steps
    if (k % 1000) == 0:
        print(k)


# ## Plots


#BUYERS' AND SELLERS' PRICES PLOT
for i in range(0,M):
    if (quality_b[i]) == 999: #buyers divided in colors depending on their reservation quality
        plt.plot(b[:,i],'g-')
    elif (quality_b[i]) == 1000:
        plt.plot(b[:,i],'m-')
    elif (quality_b[i]) == 1001:
        plt.plot(b[:,i],'y-')
    elif (quality_b[i]) == 1002:
```

```
        plt.plot(b[:,i],'c-')
    elif (quality_b[i]) == 1003:
        plt.plot(b[:,i],'r-')
    elif (quality_b[i]) == 1004:
        plt.plot(b[:,i],'k-')
plt.xlabel('time')
for j in range(0,N):
        plt.plot(s[:,j],'b-') #sellers' prices in blue
plt.xlabel('time')
plt.show()



#SELLERS' PRICES PLOT
for j in range(0,N): #sellers divided in colors depending on their offered quality
    if (quality_s[j]) == 1005:
        plt.plot(s[:,j],'g-')
    elif (quality_s[j]) == 1000:
        plt.plot(s[:,j],'m-')
    elif (quality_s[j]) == 1001:
        plt.plot(s[:,j],'y-')
    elif (quality_s[j]) == 1002:
        plt.plot(s[:,j],'c-')
    elif (quality_s[j]) == 1003:
        plt.plot(s[:,j],'r-')
    elif (quality_s[j]) == 1004:
        plt.plot(s[:,j],'k-')
plt.xlabel('time')
plt.ylabel('Sellers Prices')
plt.show()



#BUYERS' PRICES PLOT WITH ZOOM IN [49000,50000] TIME-STEPS INTERVAL
for i in range(0,M):
    if (quality_b[i]) == 999:
        plt.plot(b[:,i],'g-')
    elif (quality_b[i]) == 1000:
        plt.plot(b[:,i],'m-')
    elif (quality_b[i]) == 1001:
        plt.plot(b[:,i],'y-')
    elif (quality_b[i]) == 1002:
        plt.plot(b[:,i],'c-')
    elif (quality_b[i]) == 1003:
        plt.plot(b[:,i],'r-')
    elif (quality_b[i]) == 1004:
        plt.plot(b[:,i],'k-')
plt.xlabel('time')
plt.xlim([49000, 50000])
plt.show()

```

```
#BUYERS' PRICES VELOCITIES PLOT WITH ZOOM IN [49000,50000] TIME-STEPS INTERVAL
for i in range(0,M):
    if (quality_b[i]) == 999:
        plt.plot(x[:,i],'g-')
    elif (quality_b[i]) == 1000:
        plt.plot(x[:,i],'m-')
    elif (quality_b[i]) == 1001:
        plt.plot(x[:,i],'y-')
    elif (quality_b[i]) == 1002:
        plt.plot(x[:,i],'c-')
    elif (quality_b[i]) == 1003:
        plt.plot(x[:,i],'r-')
    elif (quality_b[i]) == 1004:
        plt.plot(x[:,i],'k-')
plt.xlabel('time')
plt.ylabel('Buyers velocities')
plt.xlim([4000, 6000])
plt.show()




#MEAN SELLERS' PRICES PLOT
plt.plot(MeanPrice_S,'b')
plt.xlabel('time')
plt.ylabel('Mean Sellers Prices')
plt.show()




#MEAN BUYERS' PRICES PLOT
plt.plot(MeanPrice_B,'r')
plt.xlabel('time')
plt.ylabel('Mean Buyers Prices')
plt.show()




#VARIANCE OF SELLERS' PRICES PLOT
plt.plot(VariancePrice_S,'b')
plt.xlabel('time')
plt.ylabel('Variance of Sellers Prices')
plt.show()
```

```
#VARIANCE OF BUYERS PRICES PLOT
plt.plot(VariancePrice_B,'r')
plt.xlabel('time')
plt.ylabel('Variance of Buyers Prices')
plt.show()




#ALL EFFICIENCIES PLOT
plt.plot(efficiency_S,'b')   #sellers' efficiency in blue
plt.plot(efficiency_B,'r')   #buyers' efficiency in red
plt.plot(efficiency_Tot,'m') #total efficiency in magenta
plt.show()




#BUYERS' EFFICIENCY PLOT
plt.plot(efficiency_B,'r')
plt.show()
```

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