Take Home Exam
Computational Finance
Implied Volatility

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Sections and Chapters

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Preliminary assumptions

Before diving into this pricing simulation case study, it's crucial to set some fundamental assumptions. For the sake of simplicity, both interest rates and dividends are set to zero. The models are based on the data provided in the assignment, I'll only detail the time grid and strike price used to compute call prices.

$$t_n = \left[\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{6}{12}, \frac{12}{12}, \frac{18}{12}\right]$$

$$K = [0.8, 0.9, 0.95, 0.975, 1, 1.025, 1.05, 1.1, 1.2]$$

Every table presented in this project presents the strike prices on the first row and every t_n on the first column. Furthermore, Monte Carlo simulations are structured around 360 days per year, using $dt = \frac{T}{360}$ to precisely track daily market movements. This consistent approach is applied across all models.

In this paper, the emphasis will be placed on analyzing the outcomes, examining the tests conducted, and interpreting these results.

1 Constant Elasticity of Variance Model

The starting point involves simulating asset price movements via the Monte Carlo method, under the Constant Elasticity of Variance model 1.

$$dS(t) = \sigma S^{(\beta)}(t) dW_t, \quad \beta > \frac{1}{2},$$
$$S(t_0) = S_0 =! \tag{1}$$

The simulation generates paths for the asset's price over a given time-frame, applying formula 2 at each step.

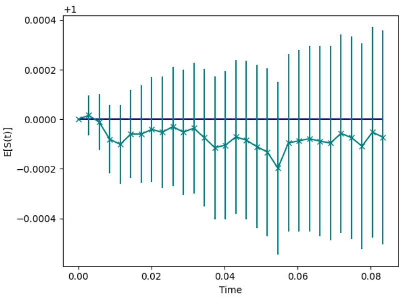
$$S_{n+1} = S_n \exp\left(-\frac{1}{2}\sigma_x^2 S_n^{2(\beta-1)} dt + \sigma_x S_n^{\beta-1} \sqrt{dt}\, \eta_n\right) \tag{2}$$

In this study, the regularized version of the Constant Elasticity of Variance model is employed [1], ensuring that the stochastic process never reaches zero. Indeed, by setting a small positive threshold ($\varepsilon > 0$), two volatility regimes are defined. For $S > \varepsilon$ the volatility is according to the CEV specification given by the formula 1. For $S \leq \varepsilon$, the volatility is fixed at the constant level ε . The value chosen here for ε is 0.01, a very small value that does not impact the results. From now on, the parameters selected for the Constant Elasticity of Variance (CEV) model are specified as $\sigma_x = 0.4$ and $\beta = 0.6$, the results for the other possible values given by other parameters are presented in Appendix A. The simulations are configured with 100,000 iterations.

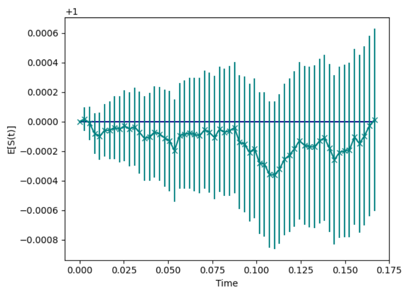
1.1 Martingality Check

In this context, a new function is created to test the compliance with the martingale property over the time grid t_n of S, which is generated under the CEV model. The function ensures a consistent temporal resolution by adjusting the number of steps proportionally to the time span. This standardization of the time increment allows for uniformity in simulation across varying lengths of time.

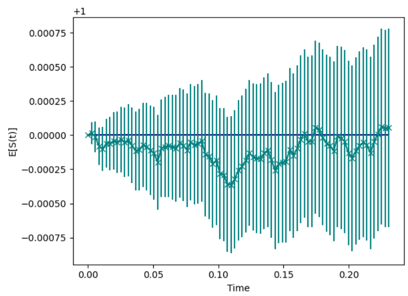
Figures 1a, 1b, 1c, 2a, 2b and 2c showcase the maintenance of the martingale property across all the periods, expressed in months. Notice that an error margin of 1.6 has been employed in the analysis.



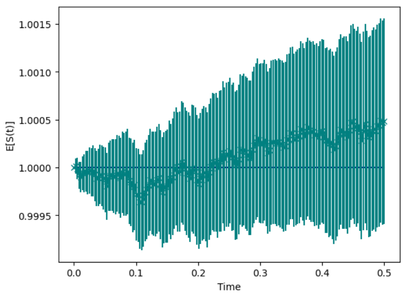
(a) Month 1 - Martingale Check



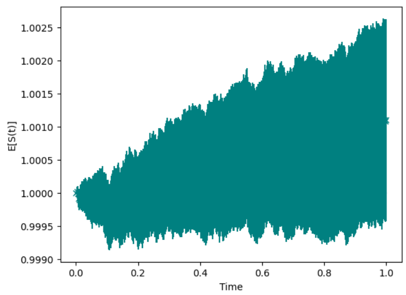
(b) Month 2 - Martingale Check



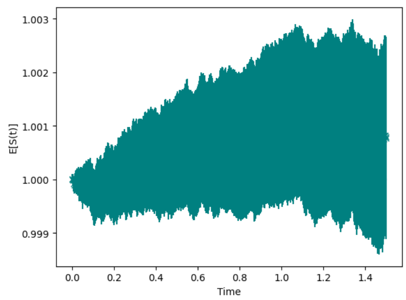
(c) Month 3 - Martingale Check



(a) Month 6 - Martingale Check



(b) Month 12 - Martingale Check



(c) Month 18 - Martingale Check

Figure 2: Martingale Check

1.2 Monte Carlo technique vs Analytic formula

After simulating 100,000 thousand possible paths for the underlying and assessing that our paths simulation satisfy the martingality test, we can compute the call option prices, varying both the strike prices (top row) and the time to maturity (first column) 1.

Table 1: Prices and Errors CEV- MC simulation - 100.000 trajectories - $\sigma_x = 0.4$, $\beta = 0.6$

	0.80000	0.90000	0.95000	0.97500	1.00000
0.08333	0.20132±0.00057	0.11138±0.00050	0.07470±0.00044	0.05932±0.00040	0.04608±0.00035
0.16667	0.20691±0.00077	0.12555±0.00067	0.09251±0.00060	0.07821±0.00056	0.06545±0.00051
0.25000	0.21215±0.00091	0.13479±0.00079	0.10315±0.00072	0.08924±0.00068	0.07666±0.00064
0.50000	0.23510±0.00122	0.16702±0.00108	0.13829±0.00100	0.12526±0.00096	0.11311±0.00092
1.00000	0.27091±0.00167	0.21027±0.00152	0.18387±0.00144	0.17160±0.00140	0.15993±0.00135
1.50000	0.30019±0.00203	0.24340±0.00187	0.21832±0.00179	0.20657±0.00175	0.19532±0.00171

	1.02500	1.05000	1.10000	1.20000
0.08333	0.03450±0.00031	0.02595±0.00027	0.01325±0.00019	0.00260±0.00008
0.16667	0.05420±0.00047	0.04440±0.00043	0.02885±0.00035	0.01076±0.00021
0.23077	0.06538±0.00059	0.05535±0.00055	0.03883±0.00047	0.01756±0.00032
0.50000	0.10182±0.00088	0.09139±0.00083	0.07301±0.00075	0.04499±0.00059
1.00000	0.14887±0.00131	0.13841±0.00127	0.11928±0.00119	0.08750±0.00103
1.50000	0.18457±0.00167	0.17430±0.00163	0.15517±0.00155	0.12209±0.00179

To verify if the model and if the Monte Carlo simulations forecast sensible values, the analytic formula is implemented through equation 3 for $0 < \beta < 1$. [1]

$$S_{n+1} = S_n \left(1 - F_{\mathrm{ncx}^2}(a, b+2, c)\right) - K F_{\mathrm{ncx}^2}(c, b, a) \tag{3}$$

$$a = \frac{K^{2(1-\beta)}}{(1-\beta)^2 \sigma_x^2 T}, b \ = \frac{1}{1-\beta}, c = \frac{S_0^{2(1-\beta)}}{(1-\beta)^2 \sigma_x^2 T}.$$

Where $F_{\mathrm{ncx}^2}(x, k, \lambda)$ is the cumulative distribution function of the non-central chi-squared distribution with x as the value, k as the degrees of freedom, and λ as the non-centrality parameter.

Table 2: Call Prices - Analytic formula CEV

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.20136	0.11136	0.07466	0.05928	0.04604	0.03496	0.02592	0.01326	0.00259
0.16667	0.20683	0.12529	0.09217	0.07786	0.06509	0.05383	0.04404	0.02854	0.01052
0.23077	0.21203	0.13468	0.10301	0.08913	0.07656	0.06528	0.05524	0.03870	0.01738
0.50000	0.23423	0.16627	0.13760	0.12462	0.11252	0.10130	0.09093	0.07264	0.04480
1.00000	0.26951	0.20891	0.18253	0.17029	0.15868	0.14768	0.13729	0.11822	0.08648
1.50000	0.29869	0.24199	0.21686	0.20507	0.19379	0.18301	0.17277	0.15352	0.12038

Table 2 presents the analytic results for call options, which are compared with the Monte Carlo derived prices adjusted for respective error (1). The analytic prices fall within the range MC price +/- error for all combinations of strike price (K) and maturity (T). This result confirm the validity of the model.

1.3 CEV vs Black & Scholes

An alternative approach to ascertain the model's accurate implementation involves utilizing a specific attribute of the CEV model: when β is set to 1, it converges with the Black & Scholes model. This condition allows to compare the prices given by the CEV model and those which comes from Black & Scholes via its closed-form expression, facilitating a comparative analysis.

Observations from this comparison reveal that the analytical prices remain within the Monte Carlo price range, further validating the model's integrity.

[1] bonus

Table 3: Call Prices - MC simulations - CEV - $\beta = 1$, $\sigma_x = 0.4$

	0.80000	0.90000	0.950000	0.97500	1.00000
0.08333	0.20101 ± 0.00041	0.11077 ± 0.00036	0.07428 ± 0.00032	0.05909 ± 0.00029	0.04607 ± 0.00026
0.16667	0.20588 ± 0.00056	0.12453 ± 0.00049	0.09192 ± 0.00043	0.07792 ± 0.00041	0.06547 ± 0.00038
0.25000	0.21224 ± 0.00066	0.13617 ± 0.00058	0.10544 ± 0.00053	0.09202 ± 0.00050	0.07989 ± 0.00047
0.50000	0.23170 ± 0.00091	0.16485 ± 0.00081	0.13710 ± 0.00076	0.12461 ± 0.00073	0.11300 ± 0.00070
1.00000	0.26550 ± 0.00128	0.20726 ± 0.00118	0.18229 ± 0.00112	0.17077 ± 0.00110	0.15987 ± 0.00107
1.50000	0.29336 ± 0.00159	0.23995 ± 0.00149	0.21671 ± 0.00144	0.20590 ± 0.00141	0.19560 ± 0.00138

	1.02500	1.05000	1.10000	1.20000
0.08333	0.03520 ± 0.00023	0.02635 ± 0.00020	0.01386 ± 0.00014	0.00308 ± 0.00007
0.16667	0.05453 ± 0.00035	0.04502 ± 0.00032	0.02993 ± 0.00026	0.01211 ± 0.00017
0.25000	0.06900 ± 0.00044	0.05930 ± 0.00041	0.04321 ± 0.00036	0.02173 ± 0.00025
0.50000	0.10227 ± 0.00067	0.09239 ± 0.00065	0.07501 ± 0.00059	0.04847 ± 0.00048
1.00000	0.14960 ± 0.00104	0.13993 ± 0.00101	0.12232 ± 0.00096	0.09202 ± 0.00085
1.50000	0.18580 ± 0.00136	0.17646 ± 0.00133	0.15912 ± 0.00128	0.12928 ± 0.00137

Table 4: Call Prices - Analytic BS - σ =0.4

	0.80000	0.90000	0.95000	0.97500	1.000	1.025	1.05000	1.10000	1.20000
0.08333	0,20105	0,11074	0,07424	0,05905	0,04604	0,03518	0,02634	0,01390	0,00309
0.16677	0,20575	0,12422	0,09154	0,07753	0,06507	0,05414	0,04465	0,02961	0,01185
0.25000	0,21186	0,13589	0,10520	0,09179	0,07966	0,06876	0,05906	0,04292	0,02147
0.50000	0,23083	0,16411	0,13644	0,12400	0,11246	0,10180	0,09197	0,07468	0,04831
1.00000	0,26391	0,20571	0,18081	0,16935	0,15852	0,14830	0,13867	0,12108	0,09188
1.50000	0,29147	0,23798	0,21468	0,20384	0,19350	0,18367	0,17430	0,15693	0,12709

1.4 Volatility Surface

To construct volatility surfaces, the methodology outlined here is applied across all models discussed in this project.

Implied volatility, as shown in Equation 4, is derived through a CFLib function [2], using as inputs the CEV results as market prices, t_n and K.

This result is implemented to assembly an implied volatility matrix (see 4), offering insight into the interplay between strike prices and time to expiration.

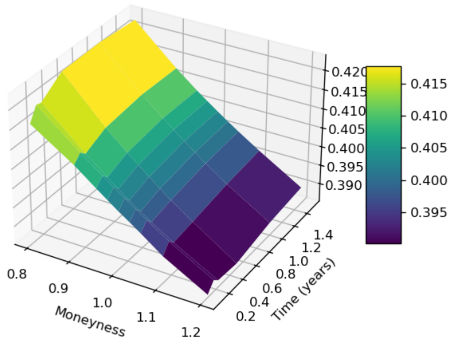
$$\Pi_{\text{BS}}(\kappa, t, \sigma) = \Pi(\kappa, t), \tag{4}$$

Table 5: Call Volatilities - CEV Model - $\sigma = 0.4$, $\beta = 0.6$

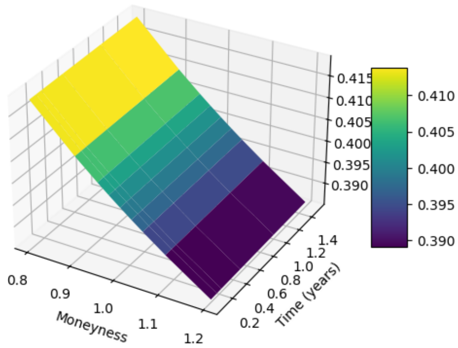
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.40901	0.40652	0.40311	0.40120	0.39931	0.3976	0.39611	0.39343	0.38645
0.16667	0.41512	0.40861	0.40484	0.40271	0.40061	0.39857	0.39651	0.39285	0.38563
0.25000	0.42151	0.41175	0.40712	0.40498	0.40291	0.40081	0.39875	0.39481	0.38784
0.50000	0.42200	0.41143	0.40664	0.40431	0.40212	0.40000	0.39800	0.39421	0.38734
1.00000	0.42314	0.41243	0.40762	0.40531	0.40318	0.40104	0.39891	0.39503	0.38783
1.50000	0.42383	0.41340	0.40878	0.40654	0.40441	0.40221	0.40023	0.39633	0.38910

The data is then plotted on a three-dimensional surface where the x-axis indicates the moneyness, the y-axis shows time to expiration in years, and the z-axis represents the implied volatility.

[2]the library provide the implied volatility calculation for put options, a new function specifically designed to calculate the implied volatility for call options is added



(a) Volatility Surface, MC Simulation 100.000 - σ_x 0.4 and β 0.6

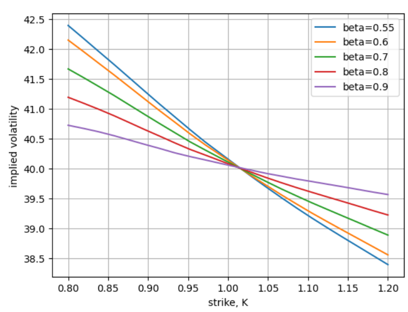


(b) Volatility Surface, Analytic - σ_x 0.4 and β 0.6

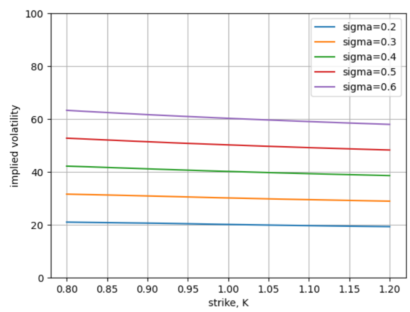
Figure 3: Volatility Surfaces CEV model

In the depicted volatility surfaces, implied volatility exhibits a decline as moneyness increases, highlighting a characteristic skewness effect. The first one 3a represent the volatility surface considering as market prices the ones computed through MC simulation, the second 3b represents the volatility surface using the analytic prices.

Setting sigma at 0.4 and letting beta varying β = [0.55, 0.6, 0.7, 0.8,0.9] the impact of changing β on the implied volatility can be observed in 4a. Since we are considering a $\beta < 0.9$ and $\beta > 0.55$ our result matches with those presented in [2]. It is observed that all lines intersect at approximately the same point, a phenomenon known as "pinning," typically around the forward price level for the underlying asset at option expiry. This intersection suggests that despite the changes in β, there's a point where its impact on implied volatility is the same for all β values. After this point, the curves diverge significantly, indicating that β has a considerable effect on the shape of the implied volatility curve, thus affecting the observed market volatility skew.



(a) Effect of β, $\sigma_x = 0.4$



(b) Effect of σ_x, $\beta = 0.6$

Figure 4: Effect of Variables

The graph 4b presents the impact σ_x on the implied volatility. It highlights how an increase in σ_x results in a pronounced steepening of the implied volatility skew, reflecting greater expected volatility for options that are significantly in or out of the money.

2 Displaced Diffusion Model

This model introduces a conceptual advancement to the classic Black & Scholes model by applying a shift to the underlying price, giving rise to a shifted lognormal process, with shift parameter equal to Δ. This displacement can be seen as an adjustment for scenarios where the underlying asset's market conditions are altered.

$$dY = \sigma_Y Y dW_t, \tag{5}$$
$$S_t = Y_t - \Delta, \tag{6}$$

where the initial point is

$$Y_0 = S_0 + \Delta \tag{7}$$

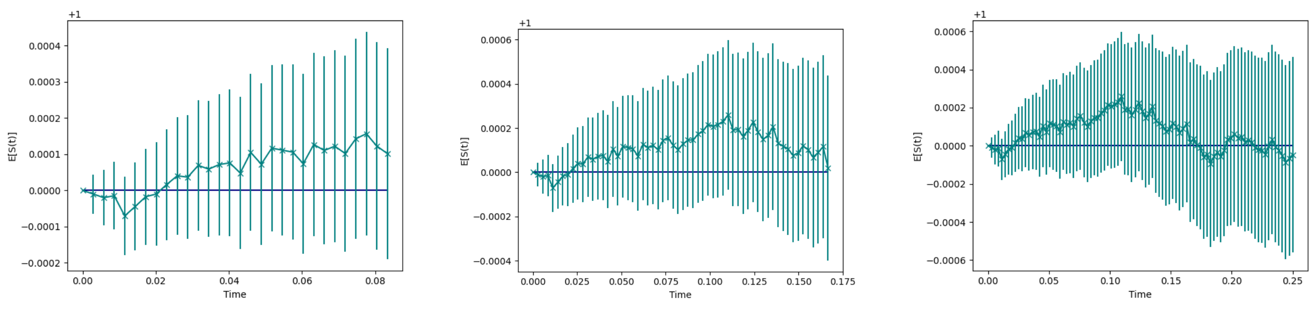
For the Constant Elasticity of Variance (CEV) model, the principal parameters Δ and σ are set to 0.6 and 0.4, respectively, which will be used to compute and present the results regarding this model. For the results obtained by implementing other parameters choices please refer to Appendix B.

2.1 Martingale Check

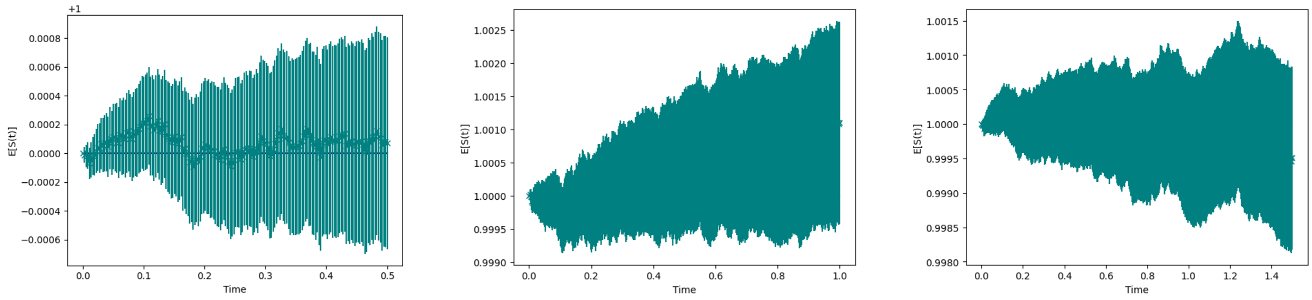
Figures 5a, 5b, 5c, 6a, 6b and 6c confirm the maintenance of the martingale property for S (5) in the displaced diffusion model across all periods considered.

The same method calibrating the number of steps to match the uniform temporal resolution for various durations used in the previous model, is employed here.

The analysis is conducted employing 1,000,000 iterations and adopting an error margin of 1.6. This model allows for more iterations to be computed due to its lower computational cost compared to the CEV model.



(a) Month 1 - Martingale Check (b) Month 2 - Martingale Check (c) Month 3 - Martingale Check



(a) Month 6 - Martingale Check (b) Month 12 - Martingale Check (c) Month 18 - Martingale Check

Figure 6: Martingale Check

2.2 Monte Carlo vs Analytic Formula

Through Monte Carlo simulations and utilizing formula 7, call option prices for a range of time-to-maturity and strike price combinations can be computed. The analytic prices are computed via the Black & Scholes closes formula, incorporating a shift represented by Δ set to 0.6 (8). Table 6 and Table 7 display the prices given by the two methodologies. By including the errors, it can be observed that the analytic price falls within the range determined by the MC simulation price plus/minus the error, determining a good implementation of the Displaced Diffusion model.

Table 6: Call Prices - MC simulations - Displaced Diffusion Model - 1.000.000 simulation - $\sigma = 0.4$, $\Delta = 0.6$

	0.80000	0.90000	0.95000	0.97500	1.00000
0.08333	0.21039±0.00027	0.13208±0.00023	0.10019±0.00021	0.08624±0.00020	0.07365±0.00018
0.16667	0.22872±0.00036	0.15887±0.00032	0.12961±0.00029	0.11647±0.00028	0.10428±0.00027
0.25000	0.24852±0.00043	0.18060±0.00039	0.15278±0.00036	0.14006±0.00035	0.12814±0.00034
0.50000	0.28831±0.00060	0.22993±0.00055	0.20427±0.00053	0.19320±0.00051	0.18088±0.00050
1.00000	0.35205±0.00086	0.30014±0.00081	0.27669±0.00079	0.26556±0.00077	0.25481±0.00076
1.50000	0.40125±0.00108	0.35299±0.00104	0.33091±0.00101	0.32036±0.00100	0.31012±0.00099

	1.02500	1.05000	1.10000	1.20000
0.08333	0.06242±0.00017	0.05251±0.00016	0.03630±0.00013	0.01580±0.00009
0.16667	0.09305±0.00026	0.08274±0.00024	0.06475±0.00022	0.03814±0.00017
0.25000	0.11698±0.00032	0.10659±0.00031	0.08795±0.00028	0.05852±0.00023
0.50000	0.17000±0.00049	0.15965±0.00048	0.14051±0.00045	0.10796±0.00040
1.00000	0.24444±0.00075	0.23445±0.00074	0.21558±0.00071	0.18192±0.00066
1.50000	0.30020±0.00098	0.29059±0.00097	0.27224±0.00094	0.23883±0.00090

Table 7: Call Prices - Analytic Prices - Displaced Diffusion Model

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.21058	0.13221	0.10023	0.08627	0.07366	0.06241	0.05245	0.03682	0.01570
0.16667	0.22842	0.15862	0.12942	0.11629	0.10412	0.09290	0.08261	0.06467	0.03812
0.25000	0.24504	0.17982	0.15203	0.13934	0.12745	0.11633	0.10596	0.08740	0.05813
0.50000	0.28699	0.22880	0.20324	0.19131	0.17994	0.16912	0.15883	0.13978	0.10740
1.00000	0.35071	0.29885	0.27544	0.26434	0.25363	0.24330	0.23335	0.21452	0.18091
1.50000	0.40074	0.35248	0.33040	0.31984	0.30961	0.29968	0.29005	0.27176	0.23822

The analytic formula is given by [3]

$$C(S_0, K, T, r, \sigma, \Delta) = (S_0 + \Delta)\Phi(d_1) - (K + \Delta)e^{-rT}\Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0+\Delta}{K+\Delta}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T} \tag{8}$$

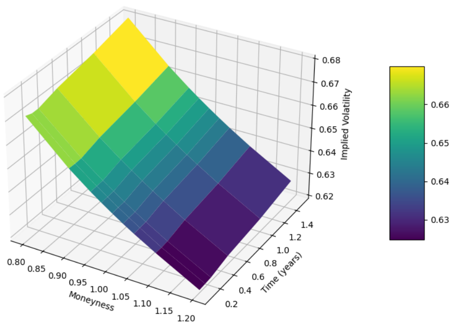
2.3 Volatility Surface

The extraction of implied volatilities is carried out through a function from the CFLib library, facilitating the construction of an implied volatility matrix.

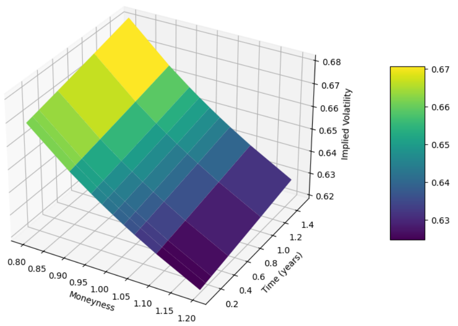
Table 8: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.4$, $\Delta = 0.6$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.67279	0.65611	0.64886	0.64559	0.64258	0.63948	0.63648	0.63073	0.62082
0.16667	0.67272	0.65606	0.64859	0.64533	0.64212	0.63890	0.63586	0.63013	0.61986
0.25000	0.67298	0.65641	0.64947	0.64611	0.64291	0.63981	0.63688	0.63128	0.62131
0.50000	0.67264	0.65669	0.64971	0.64640	0.64329	0.64016	0.63729	0.63169	0.62169
1.00000	0.67715	0.66064	0.65343	0.65001	0.64673	0.64355	0.64057	0.63474	0.62458
1.50000	0.67835	0.66209	0.65484	0.65143	0.64814	0.64501	0.64205	0.63633	0.62613

From this outcome, a volatility surface emerges, as illustrated in Figure 7a. Figure 7b displays the surface volatility computed using the analytic formula of the Displaced Diffusion Model, the substantial similarity between the two graphs indicates the accurate implementation of the model. This surface volatility has the same behaviour of the one computed trough the CEV model, showing a volatility skew [3].



(a) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.6$
150,000 MC simulation



(b) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.6$
Analytic Approach

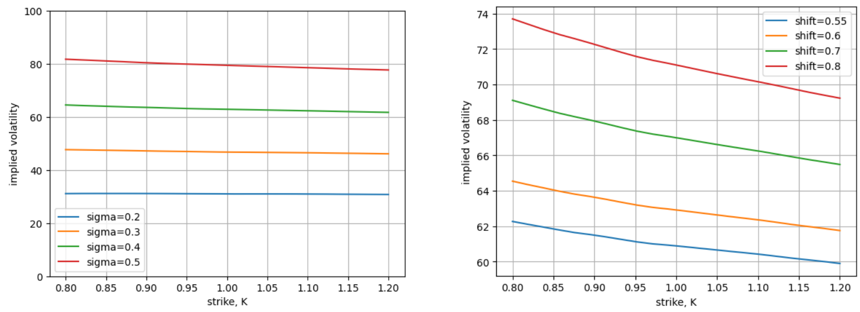
Figure 7: Volatilities Surfaces - Displaced Diffusion Model

Figure 8a and 8b shows the effect of β and σ on the implied volatility.

This confirms that the volatility generated by this model exhibits a skew and does not capture the volatility smile surface. Furthermore, it can be observed that it displays the same behavior as the CEV model when sigma varies. Even if the displaced diffusion model introduces a displacement parameter still has limitations in capturing the full shape of the volatility smile. The primary reason for this limitation

[3] bonus

lies in its linearly modified structure, which is not flexible enough to adapt to the complex market dynamics that give rise to the volatility smile.



(a) Effect of σ

(b) Effect of Δ

Figure 8: Effect of Variables

3 Heston Model

The Heston model ([4]) provides a stochastic differential equation framework that captures the dynamics of both asset prices and volatility simultaneously (formula 9, given by the CIR model).

$$S_t = S_0 e^{X_t}$$

$$dX_t = -\frac{\nu_t}{2} dt + \sqrt{\nu_t} dW_t, \quad X_0 = 0 \tag{9}$$

$$d\nu_t = \lambda(\bar{\nu} - \nu_t)dt + \eta\sqrt{\nu_t} dY_t, \quad \nu_0 = \sigma^2 \tag{10}$$

This model is particularly renowned for its ability to address volatility smile phenomena observed in financial markets.

3.1 Feller condition

First of all, the Feller condition (see Equation 11) needs to be checked. Based on the data provided, it is found to be violated.

$$\eta^2 < 2\lambda\bar{\nu} \tag{11}$$

Given the violation of the Feller condition, Euler discretization method can lead to scenarios where variables such as variance could incorrectly assume negative values.

A more efficient approach involves the adoption of the Quadratic Exponential ([5]) algorithm for implementing volatility dynamics. Incorporated within the CFLib library, the QE method stands out for its ability to maintain numerical stability and efficiency.

Thus, it emerges as the preferred choice.

Afterwards, for each of these simulations, denoted as N_v, a number of N_s simulations for the underlying are generated, this approach creates a 3D array, with dimension N_v, t_n (time-steps) and N_s.

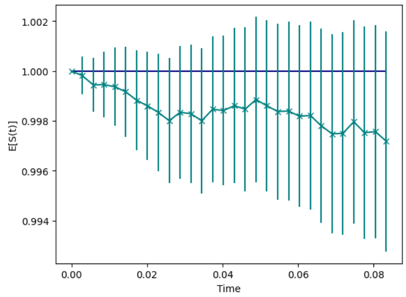
N_s simulation are set equal to 4,000 and N_v to 3,000.

3.2 Martingality Check

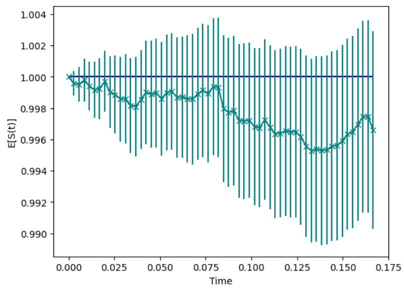
Since we are generating N_v volatility trajectories and N_s underlying trajectories, we have $N_v N_s$ values. To compute the statistical error and check if the martingality test is not violated the best approach is the following: for each block of N_s trajectory is computed the variable

$$m^j := \frac{1}{N_s} \sum_{k=1}^{N_s} G(S_k^j) \tag{12}$$

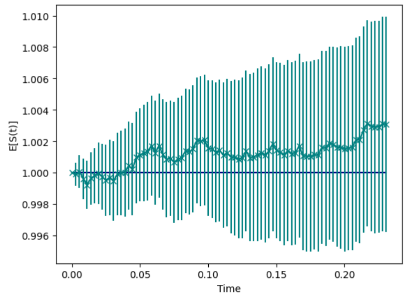
and N_s variables m_j are used compute the statistical error in the usual way. Then is possible to test the martingality condition for each period, using a marginal error equal to 1.8.



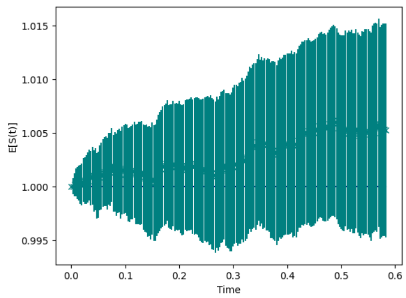
(a) Martingale First Period



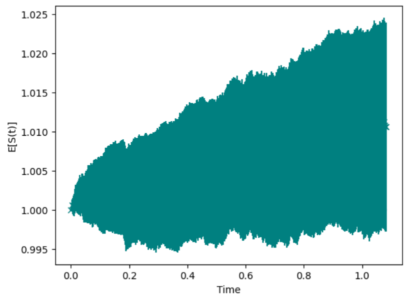
(b) Martingale Second Period



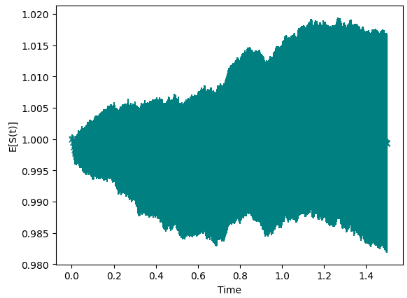
(c) Martingale Third Period



(a) Martingale Fourth Period



(b) Martingale Fifth Period



(c) Martingale Sixth Period

Figure 10: Martingale Check

3.3 Monte Carlo vs Analytic formula

To proceed in order to compute the prices of the call options, it is necessary to extract the final asset prices for all scenarios across all simulated paths at the last time point, which is essential for computing option payoffs.

Table 9: Call Prices - Heston MC

	0.80000	0.90000	0.95000	0.97500	1.000
0.08333	0.20096 ± 0.00169	0.10354 ± 0.00143	0.05899 ± 0.00116	0.03818 ± 0.00099	0.02133 ± 0.00078
0.16677	0.20304 ± 0.00221	0.10896 ± 0.00183	0.06647 ± 0.00151	0.04777 ± 0.00130	0.02999 ± 0.00110
0.25000	0.20386 ± 0.00255	0.11293 ± 0.00207	0.07167 ± 0.00173	0.05333 ± 0.00152	0.03607 ± 0.00130
0.50000	0.21015 ± 0.00355	0.12623 ± 0.00290	0.08845 ± 0.00249	0.07026 ± 0.00227	0.05582 ± 0.00204
1.00000	0.22327 ± 0.00485	0.14959 ± 0.00408	0.11199 ± 0.00364	0.09654 ± 0.00341	0.08272 ± 0.00317
1.50000	0.23692 ± 0.00592	0.16523 ± 0.00508	0.13331 ± 0.00463	0.11878 ± 0.00439	0.10503 ± 0.00415

	1.02500	1.05000	1.10000	1.20000
0.08333	0.00903 ± 0.00054	0.00318 ± 0.00034	0.00033 ± 0.00012	0.00001 ± 0.00002
0.16677	0.01683 ± 0.00086	0.00823 ± 0.00064	0.00182 ± 0.00034	0.00014 ± 0.00010
0.25000	0.02234 ± 0.00107	0.01262 ± 0.00084	0.00331 ± 0.00048	0.00033 ± 0.00017
0.50000	0.04143 ± 0.00179	0.03008 ± 0.00155	0.01394 ± 0.00110	0.00236 ± 0.00050
1.00000	0.07039 ± 0.00292	0.05766 ± 0.00267	0.03724 ± 0.00217	0.01293 ± 0.00131
1.50000	0.09257 ± 0.00390	0.07951 ± 0.00364	0.05864 ± 0.00315	0.02842 ± 0.00221

Through the CFLib [4] it is possible to compute the analytic prices, using the so called SINC algorithm [6] and examine if those fall in between the MC price +/- the error, with a margin error of 1.8.

Table 10: Call Prices - Analytic Prices - Heston

	0.800000	0.900000	0.950000	0.975000	1.000000	1.025000	1.050000	1.100000	1.200000
0.083333	0.200866	0.104488	0.059759	0.039456	0.021836	0.009129	0.003075	0.000383	0.000009
0.166667	0.203330	0.109874	0.067039	0.047648	0.030552	0.016987	0.008194	0.001777	0.000135
0.230769	0.205274	0.113524	0.071830	0.052993	0.036247	0.022456	0.012507	0.003363	0.000330
0.500000	0.213054	0.127054	0.088826	0.071539	0.055817	0.041958	0.030255	0.013917	0.002385
1.000000	0.226602	0.147996	0.113456	0.097689	0.083051	0.069633	0.057515	0.037398	0.013123

The paper cited above computes the price of a put option as a Cash or Nothing (CoN hereafter) plus Asset or Nothing (AoN) options. Using the put call parity, a closed formula for a call option is the following:

$$
\begin{aligned}
C \approx & \frac{1}{2}(S_0 - K) + \frac{2}{\pi}\sum_{n=1}^{N/4} \frac{1}{2n-1}\Big[\sin(2\pi k k_{2n-1})\hat{f}(k_{2n-1}) \\
& - S_0 \hat{f}\left(k_{2n-1} - \frac{i}{2\pi}\right)\Big] - \cos(2\pi k k_{2n-1}) S_0 \Big[\hat{f}(k_{2n-1}) \\
& - \hat{f}\left(k_{2n-1} - \frac{i}{2\pi}\right)\Big] + S_0 - K e^{-rT}
\end{aligned} \tag{13}
$$

3.4 Volatility Surface

Table 11: Call Volatilities - Heston Model

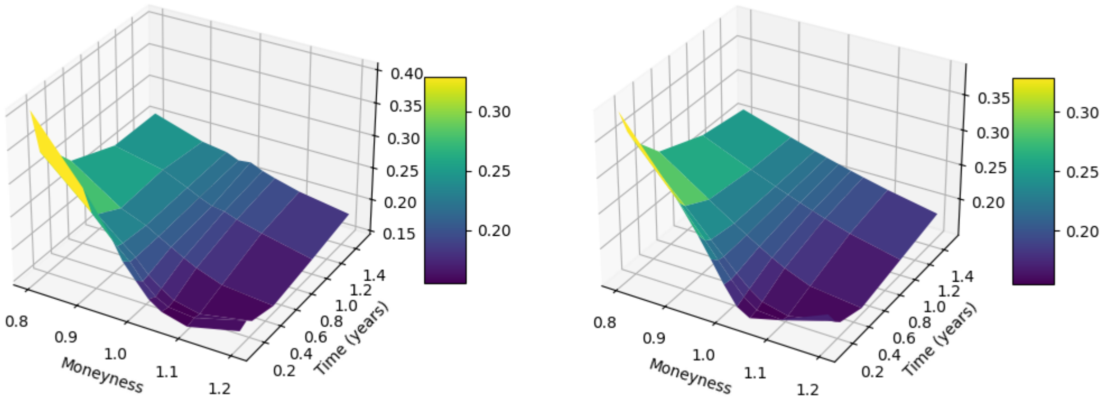
	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.39465	0.28167	0.23908	0.20759	0.18525	0.16268	0.15468	0.15912	0.19022
0.16667	0.34507	0.26479	0.22746	0.21032	0.18421	0.16710	0.15530	0.15317	0.17533
0.25000	0.30913	0.25811	0.22561	0.20926	0.18830	0.17198	0.16045	0.15044	0.16585
0.50000	0.26980	0.24014	0.21904	0.20440	0.19803	0.18570	0.17738	0.16346	0.15459
1.00000	0.25256	0.24087	0.21811	0.21223	0.20773	0.20407	0.19647	0.18457	0.16782
1.50000	0.24973	0.23354	0.22419	0.21998	0.21558	0.21202	0.20540	0.19720	0.18224

Using the CFLib is it possible to extract the implied volatilities shown in table 10. The Heston model is able to capture the volatility smile surface, contrary to the other model analyzed in the previous chapters.

The model is particularly adept at capturing this effect because it assumes that volatility is a random process, rather than a fixed parameter. This aligns more

[4] In the CFLib it is implemented a function HestonCall inside Heston.py cause the analytic formula was present just for the put

closely with observed market behavior, where volatility is not constant and can be influenced by a variety of market factors.



(a) Volatility Surface Call-MC simulation (b) Volatility Surface Call - Analytic Approach

Figure 11: Volatilities Surfaces - Heston Model

References

[1] Leif B.G Andersen. *Efficient Simulation of the Heston Stochastic Volatility Model*. 2007. URL: https://ssrn.com/abstract=946405 (cit. on p. 1).

[2] Cornelis W Oosterlee and Lech A Grzelak. "Mathematical Modeling and Computation in Finance". In: () (cit. on p. 5).

[3] R. Lee and D. Wang. "Displaced Lognormal Volatility Skews: Analysis and Applications to Stochastic Volatility Simulations". In: *The Review of Financial Studies* 25.5 (2012), pp. 1593–1636. DOI: 10.1093/rfs/hhs044. URL: https://doi.org/10.1007/s10436-009-0145-7 (cit. on p. 8).

[4] Steven L. Heston. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options". In: *The Review of Financial Studies* 6.2 (1993), pp. 327–343. URL: http://www.jstor.org/stable/2962057 (cit. on p. 9).

[5] Leif Andersen and Jesper Andreasen. "Volatility Skews and Extensions of the Libor Market Model". In: *Applied Mathematical Finance* 7.1 (2000), pp. 1–32. DOI: 10.1080/135048600450275. URL: https://ideas.repec.org/a/taf/apmtfi/v7y2000i1p1-32.html (cit. on p. 9).

[6] Fabio Baschetti et al. "The SINC Way: A Fast and Accurate Approach to Fourier Pricing". In: *Quantitative Finance* (2022) (cit. on p. 11).

A Appendix A

A.1 Call Prices CEV - 100,000 simulations

Table 1: Call Prices CEV - MC - sigma 0.3, beta 0.55

0.08333	0.20014 ± 0.00044	0.10482 ± 0.00040	0.06476 ± 0.00034	0.04827 ± 0.00030	0.03457 ± 0.00026	0.02373 ± 0.00022	0.01556 ± 0.00018	0.00580 ± 0.00011	0.00045 ± 0.00003
0.16667	0.20193 ± 0.00060	0.11344 ± 0.00053	0.07745 ± 0.00046	0.06227 ± 0.00042	0.04910 ± 0.00038	0.03794 ± 0.00034	0.02871 ± 0.00029	0.01543 ± 0.00021	0.00346 ± 0.00010
0.25000	0.20429 ± 0.00072	0.11958 ± 0.00062	0.08522 ± 0.00055	0.07051 ± 0.00051	0.05752 ± 0.00047	0.04628 ± 0.00042	0.03670 ± 0.00038	0.02209 ± 0.00030	0.00674 ± 0.00017
0.50000	0.21720 ± 0.00095	0.14222 ± 0.00083	0.11123 ± 0.00075	0.09751 ± 0.00071	0.08495 ± 0.00067	0.07354 ± 0.00063	0.06328 ± 0.00058	0.04602 ± 0.00050	0.02264 ± 0.00035
1.00000	0.24058 ± 0.00128	0.17372 ± 0.00114	0.14530 ± 0.00106	0.13236 ± 0.00102	0.12023 ± 0.00098	0.10890 ± 0.00093	0.09838 ± 0.00089	0.07971 ± 0.00081	0.05079 ± 0.00065
1.50000	0.26075 ± 0.00153	0.19825 ± 0.00139	0.17120 ± 0.00131	0.15872 ± 0.00126	0.14692 ± 0.00122	0.13580 ± 0.00118	0.12533 ± 0.00114	0.10627 ± 0.00106	0.07506 ± 0.00089

Table 2: Call Prices CEV - MC - sigma 0.3, beta 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.08333	0.20014 ± 0.00044	0.10482 ± 0.00040	0.06476 ± 0.00034	0.04827 ± 0.00031	0.03457 ± 0.00026	0.02373 ± 0.00022	0.01556 ± 0.00018	0.00580 ± 0.00011	0.00045 ± 0.00003
0.16667	0.20193 ± 0.00060	0.11344 ± 0.00053	0.07745 ± 0.00046	0.06227 ± 0.00042	0.04910 ± 0.00038	0.03794 ± 0.00034	0.02871 ± 0.00029	0.01543 ± 0.00021	0.00346 ± 0.00010
0.25000	0.20429 ± 0.00072	0.11958 ± 0.00062	0.08522 ± 0.00055	0.07051 ± 0.00051	0.05752 ± 0.00047	0.04628 ± 0.00043	0.03670 ± 0.00038	0.02209 ± 0.00030	0.00674 ± 0.00017
0.50000	0.21720 ± 0.00096	0.14222 ± 0.00083	0.11123 ± 0.00076	0.09751 ± 0.00072	0.08495 ± 0.00067	0.07354 ± 0.00063	0.06328 ± 0.00059	0.04602 ± 0.00051	0.02264 ± 0.00035
1.00000	0.24058 ± 0.00129	0.17372 ± 0.00115	0.14530 ± 0.00107	0.13236 ± 0.00103	0.12023 ± 0.00098	0.10890 ± 0.00094	0.09838 ± 0.00090	0.07971 ± 0.00082	0.05079 ± 0.00066
1.50000	0.26075 ± 0.00155	0.19825 ± 0.00140	0.17120 ± 0.00132	0.15872 ± 0.00128	0.14692 ± 0.00124	0.13580 ± 0.00119	0.12533 ± 0.00115	0.10627 ± 0.00107	0.07506 ± 0.00091

Table 3: Call Prices CEV - MC - sigma 0.3, beta 0.7

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.08333	0.20012 ± 0.00044	0.10469 ± 0.00040	0.06465 ± 0.00034	0.04821 ± 0.00031	0.03456 ± 0.00026	0.02379 ± 0.00022	0.01566 ± 0.00018	0.00593 ± 0.00011	0.00050 ± 0.00003
0.16667	0.20177 ± 0.00061	0.11319 ± 0.00053	0.07729 ± 0.00046	0.06218 ± 0.00042	0.04910 ± 0.00038	0.03803 ± 0.00034	0.02888 ± 0.00030	0.01569 ± 0.00022	0.00368 ± 0.00010
0.25000	0.20400 ± 0.00072	0.11924 ± 0.00063	0.08502 ± 0.00055	0.07040 ± 0.00051	0.05752 ± 0.00047	0.04638 ± 0.00043	0.03690 ± 0.00039	0.02243 ± 0.00031	0.00710 ± 0.00017
0.50000	0.21642 ± 0.00097	0.14164 ± 0.00084	0.11091 ± 0.00076	0.09733 ± 0.00072	0.08492 ± 0.00068	0.07367 ± 0.00064	0.06356 ± 0.00060	0.04656 ± 0.00052	0.02345 ± 0.00036
1.00000	0.23916 ± 0.00131	0.17286 ± 0.00116	0.14485 ± 0.00109	0.13212 ± 0.00104	0.12020 ± 0.00100	0.10909 ± 0.00096	0.09878 ± 0.00092	0.08053 ± 0.00084	0.05219 ± 0.00068
1.50000	0.25890 ± 0.00157	0.19721 ± 0.00143	0.17066 ± 0.00135	0.15845 ± 0.00131	0.14693 ± 0.00127	0.13607 ± 0.00122	0.12587 ± 0.00118	0.10731 ± 0.00110	0.07691 ± 0.00094

Table 4: Call Prices CEV - MC - sigma 0.3, beta 0.8

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.08333	0.20010 ± 0.00044	0.10461 ± 0.00040	0.06458 ± 0.00035	0.04817 ± 0.00031	0.03456 ± 0.00027	0.02382 ± 0.00022	0.01573 ± 0.00018	0.00601 ± 0.00011	0.00053 ± 0.00003
0.16667	0.20168 ± 0.00061	0.11303 ± 0.00053	0.07718 ± 0.00047	0.06213 ± 0.00043	0.04910 ± 0.00039	0.03809 ± 0.00034	0.02899 ± 0.00030	0.01587 ± 0.00022	0.00383 ± 0.00011
0.25000	0.20382 ± 0.00072	0.11924 ± 0.00063	0.08502 ± 0.00056	0.07040 ± 0.00052	0.05752 ± 0.00048	0.04645 ± 0.00043	0.03690 ± 0.00039	0.02265 ± 0.00031	0.00734 ± 0.00018
0.50000	0.21642 ± 0.00097	0.14164 ± 0.00085	0.11091 ± 0.00077	0.09733 ± 0.00073	0.08492 ± 0.00069	0.07376 ± 0.00065	0.06375 ± 0.00061	0.04692 ± 0.00053	0.02401 ± 0.00038
1.00000	0.23825 ± 0.00132	0.17230 ± 0.00118	0.14485 ± 0.00110	0.13212 ± 0.00106	0.12020 ± 0.00102	0.10923 ± 0.00098	0.09878 ± 0.00094	0.08109 ± 0.00086	0.05219 ± 0.00070
1.50000	0.25770 ± 0.00160	0.19654 ± 0.00146	0.17034 ± 0.00138	0.15845 ± 0.00134	0.14693 ± 0.00130	0.13628 ± 0.00126	0.12625 ± 0.00122	0.10803 ± 0.00114	0.07818 ± 0.00098

Table 5: Call Prices CEV - MC - sigma 0.4, beta 0.55

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.08333	0.20136 ± 0.00057	0.11146 ± 0.00050	0.07475 ± 0.00044	0.05935 ± 0.00040	0.04608 ± 0.00038	0.03497 ± 0.00031	0.02591 ± 0.00027	0.01317 ± 0.00019	0.00255 ± 0.00008
0.16667	0.20705 ± 0.00077	0.12568 ± 0.00067	0.09258 ± 0.00059	0.07825 ± 0.00055	0.06545 ± 0.00051	0.05416 ± 0.00047	0.04432 ± 0.00043	0.02872 ± 0.00034	0.01060 ± 0.00021
0.25000	0.21233 ± 0.00091	0.13495 ± 0.00079	0.10324 ± 0.00071	0.08930 ± 0.00067	0.07666 ± 0.00063	0.06534 ± 0.00059	0.05527 ± 0.00055	0.03867 ± 0.00046	0.01734 ± 0.00031
0.50000	0.23555 ± 0.00121	0.16730 ± 0.00107	0.13845 ± 0.00099	0.12535 ± 0.00095	0.11313 ± 0.00091	0.10177 ± 0.00087	0.09127 ± 0.00083	0.07278 ± 0.00074	0.04458 ± 0.00058
1.00000	0.27163 ± 0.00165	0.21068 ± 0.00150	0.14485 ± 0.00142	0.13212 ± 0.00138	0.12020 ± 0.00134	0.10923 ± 0.00130	0.09878 ± 0.00125	0.08109 ± 0.00117	0.05219 ± 0.00101
1.50000	0.30092 ± 0.00200	0.24390 ± 0.00185	0.21858 ± 0.00177	0.20671 ± 0.00173	0.19534 ± 0.00169	0.18447 ± 0.00164	0.17409 ± 0.00160	0.15473 ± 0.00152	0.12125 ± 0.00136

Table 6: Call Prices CEV - MC - sigma 0.4, beta 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.08333	0.20132 ± 0.00069	0.11138 ± 0.00060	0.07470 ± 0.00053	0.05932 ± 0.00049	0.04608 ± 0.00045	0.03500 ± 0.00041	0.02595 ± 0.00036	0.01325 ± 0.00028	0.00260 ± 0.00015
0.16667	0.20691 ± 0.00093	0.12555 ± 0.00081	0.09251 ± 0.00073	0.07821 ± 0.00069	0.06545 ± 0.00065	0.05420 ± 0.00061	0.04440 ± 0.00057	0.02885 ± 0.00048	0.01076 ± 0.00033
0.25000	0.21212 ± 0.00110	0.13479 ± 0.00097	0.10315 ± 0.00089	0.08924 ± 0.00085	0.07666 ± 0.00081	0.06538 ± 0.00076	0.05535 ± 0.00072	0.03883 ± 0.00064	0.01756 ± 0.00048
0.50000	0.23511 ± 0.00148	0.16702 ± 0.00134	0.13829 ± 0.00126	0.12526 ± 0.00122	0.11311 ± 0.00117	0.10182 ± 0.00113	0.09139 ± 0.00109	0.07302 ± 0.00101	0.04499 ± 0.00085
1.00000	0.27091 ± 0.00206	0.21027 ± 0.00191	0.18387 ± 0.00183	0.17160 ± 0.00179	0.15993 ± 0.00174	0.14887 ± 0.00170	0.13841 ± 0.00166	0.11928 ± 0.00158	0.08750 ± 0.00142
1.50000	0.30002 ± 0.00253	0.24340 ± 0.00238	0.21832 ± 0.00230	0.20657 ± 0.00226	0.19532 ± 0.00222	0.18457 ± 0.00218	0.17430 ± 0.00214	0.15517 ± 0.00206	0.12209 ± 0.00190

Table 7: Call Prices CEV - MC - sigma 0.4, beta 0.7

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20124 ±0.00057	0.11122 ±0.00050	0.07459 ±0.00044	0.05926 ±0.00040	0.04608 ±0.00036	0.03505 ±0.00031	0.02605 ±0.00027	0.01340 ±0.00019	0.00272 ±0.00008
0.166667	0.20664 ±0.00078	0.12529 ±0.00067	0.09236 ±0.00060	0.07814 ±0.00056	0.06546 ±0.00052	0.05428 ±0.00048	0.04455 ±0.00043	0.02912 ±0.00035	0.01108 ±0.00022
0.250000	0.21170 ±0.00092	0.13445 ±0.00080	0.10296 ±0.00073	0.08914 ±0.00068	0.07665 ±0.00064	0.06547 ±0.00060	0.05553 ±0.00056	0.03915 ±0.00048	0.01802 ±0.00033
0.500000	0.23423 ±0.00124	0.16646 ±0.00110	0.13798 ±0.00102	0.12508 ±0.00098	0.11307 ±0.00093	0.10192 ±0.00089	0.09163 ±0.00085	0.07350 ±0.00077	0.04584 ±0.00061
1.000000	0.26951 ±0.00170	0.20948 ±0.00155	0.18344 ±0.00147	0.17136 ±0.00143	0.15988 ±0.00139	0.14902 ±0.00135	0.13875 ±0.00131	0.12000 ±0.00123	0.08883 ±0.00107
1.500000	0.29826 ±0.00208	0.24246 ±0.00193	0.21784 ±0.00185	0.20633 ±0.00181	0.19532 ±0.00177	0.18480 ±0.00173	0.17477 ±0.00169	0.15608 ±0.00161	0.12380 ±0.00145

Table 8: Call Prices CEV - MC - sigma 0.5, beta 0.55

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20420 ±0.00069	0.11960 ±0.00060	0.08526 ±0.00053	0.07056 ±0.00049	0.05759 ±0.00045	0.04635 ±0.00040	0.03676 ±0.00036	0.02208 ±0.00028	0.00667 ±0.00015
0.166667	0.21497 ±0.00093	0.13925 ±0.00081	0.10809 ±0.00073	0.09432 ±0.00069	0.08179 ±0.00065	0.07046 ±0.00060	0.06030 ±0.00056	0.04330 ±0.00048	0.02066 ±0.00033
0.250000	0.22339 ±0.00110	0.15153 ±0.00096	0.12156 ±0.00088	0.10814 ±0.00084	0.09577 ±0.00080	0.08445 ±0.00076	0.07413 ±0.00072	0.05639 ±0.00063	0.03095 ±0.00048
0.500000	0.25668 ±0.00147	0.19318 ±0.00132	0.16577 ±0.00124	0.15314 ±0.00120	0.14120 ±0.00116	0.12995 ±0.00112	0.11939 ±0.00108	0.10027 ±0.00099	0.06924 ±0.00083
1.000000	0.30478 ±0.00204	0.24801 ±0.00188	0.22273 ±0.00180	0.21082 ±0.00176	0.19939 ±0.00172	0.18845 ±0.00168	0.17799 ±0.00164	0.15848 ±0.00155	0.12480 ±0.00139
1.500000	0.34261 ±0.00249	0.28954 ±0.00234	0.26557 ±0.00226	0.25421 ±0.00222	0.24323 ±0.00217	0.23266 ±0.00213	0.22246 ±0.00209	0.20319 ±0.00201	0.16880 ±0.00185

Table 9: Call Prices CEV - MC - sigma 0.5, beta 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20410 ±0.00069	0.11949 ±0.00060	0.08519 ±0.00053	0.07052 ±0.00049	0.05759 ±0.00045	0.04638 ±0.00041	0.03682 ±0.00036	0.02218 ±0.00028	0.00678 ±0.00015
0.166667	0.21473 ±0.00093	0.13907 ±0.00081	0.10799 ±0.00073	0.09427 ±0.00069	0.08179 ±0.00065	0.07051 ±0.00061	0.06039 ±0.00057	0.04347 ±0.00048	0.02092 ±0.00033
0.250000	0.22305 ±0.00110	0.15130 ±0.00096	0.12144 ±0.00088	0.10807 ±0.00084	0.09577 ±0.00080	0.08450 ±0.00076	0.07424 ±0.00072	0.05660 ±0.00064	0.03129 ±0.00048
0.500000	0.25605 ±0.00148	0.19281 ±0.00132	0.16556 ±0.00124	0.15301 ±0.00120	0.14116 ±0.00117	0.13000 ±0.00113	0.11952 ±0.00109	0.10056 ±0.00101	0.06979 ±0.00085
1.000000	0.30382 ±0.00206	0.24748 ±0.00191	0.22242 ±0.00183	0.21063 ±0.00179	0.19932 ±0.00174	0.18850 ±0.00170	0.17816 ±0.00166	0.15889 ±0.00158	0.12563 ±0.00142
1.500000	0.34143 ±0.00253	0.28890 ±0.00238	0.26523 ±0.00230	0.25400 ±0.00226	0.24318 ±0.00222	0.23275 ±0.00218	0.22270 ±0.00214	0.20370 ±0.00206	0.16983 ±0.00190

Table 10: Call Prices CEV - MC - sigma 0.5, beta 0.7

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20391 ±0.00069	0.11926 ±0.00060	0.08505 ±0.00053	0.07044 ±0.00049	0.05758 ±0.00045	0.04644 ±0.00041	0.03695 ±0.00036	0.02240 ±0.00028	0.00701 ±0.00015
0.166667	0.21427 ±0.00093	0.13872 ±0.00081	0.10780 ±0.00073	0.09417 ±0.00069	0.08179 ±0.00065	0.07061 ±0.00061	0.06058 ±0.00057	0.04382 ±0.00048	0.02143 ±0.00033
0.250000	0.22240 ±0.00110	0.15086 ±0.00096	0.12119 ±0.00088	0.10794 ±0.00084	0.09575 ±0.00080	0.08461 ±0.00076	0.07445 ±0.00072	0.05701 ±0.00064	0.03196 ±0.00048
0.500000	0.25482 ±0.00148	0.19208 ±0.00132	0.16515 ±0.00124	0.15277 ±0.00120	0.14109 ±0.00117	0.13010 ±0.00113	0.11979 ±0.00109	0.10116 ±0.00101	0.07091 ±0.00085
1.000000	0.30195 ±0.00206	0.24645 ±0.00191	0.22185 ±0.00183	0.21029 ±0.00179	0.19922 ±0.00174	0.18864 ±0.00170	0.17854 ±0.00166	0.15976 ±0.00158	0.12733 ±0.00142
1.500000	0.33915 ±0.00253	0.28770 ±0.00238	0.26460 ±0.00230	0.25367 ±0.00226	0.24314 ±0.00222	0.23299 ±0.00218	0.22324 ±0.00214	0.20480 ±0.00206	0.17197 ±0.00190

Table 11: Call Prices CEV - MC - sigma 0.5, beta 0.8

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20373 ±0.00070	0.11904 ±0.00061	0.08491 ±0.00054	0.07037 ±0.00050	0.05757 ±0.00046	0.04650 ±0.00041	0.03707 ±0.00037	0.02261 ±0.00029	0.00725 ±0.00016
0.166667	0.21382 ±0.00095	0.13838 ±0.00083	0.10761 ±0.00075	0.09408 ±0.00071	0.08180 ±0.00067	0.07071 ±0.00063	0.06078 ±0.00058	0.04418 ±0.00050	0.02195 ±0.00036
0.250000	0.22176 ±0.00112	0.15042 ±0.00099	0.12095 ±0.00091	0.10781 ±0.00087	0.09574 ±0.00083	0.08472 ±0.00079	0.07468 ±0.00075	0.05744 ±0.00067	0.03129 ±0.00051
0.500000	0.25362 ±0.00153	0.19137 ±0.00139	0.16476 ±0.00131	0.15255 ±0.00127	0.14103 ±0.00123	0.13022 ±0.00119	0.12008 ±0.00115	0.10178 ±0.00107	0.07205 ±0.00091
1.000000	0.30015 ±0.00217	0.24546 ±0.00202	0.22131 ±0.00195	0.20999 ±0.00191	0.19917 ±0.00187	0.18883 ±0.00183	0.17898 ±0.00179	0.16067 ±0.00171	0.12907 ±0.00156
1.500000	0.33699 ±0.00272	0.28660 ±0.00257	0.26408 ±0.00249	0.25344 ±0.00246	0.24319 ±0.00242	0.23335 ±0.00238	0.22387 ±0.00234	0.20598 ±0.00226	0.17420 ±0.00211

Table 12: Call Prices CEV - MC - sigma 0.6, beta 0.55

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20861 ±0.00081	0.12864 ±0.00070	0.09603 ±0.00062	0.08183 ±0.00058	0.06909 ±0.00054	0.05779 ±0.00050	0.04788 ±0.00045	0.03187 ±0.00037	0.01255 ±0.00023
0.166667	0.22480 ±0.00108	0.15353 ±0.00095	0.12377 ±0.00087	0.11042 ±0.00083	0.09811 ±0.00079	0.08679 ±0.00074	0.07645 ±0.00070	0.05855 ±0.00062	0.03268 ±0.00046
0.250000	0.23635 ±0.00128	0.16869 ±0.00114	0.14001 ±0.00106	0.12698 ±0.00102	0.11484 ±0.00098	0.10357 ±0.00093	0.09313 ±0.00089	0.07468 ±0.00081	0.04643 ±0.00065
0.500000	0.27927 ±0.00174	0.21938 ±0.00158	0.19307 ±0.00150	0.18081 ±0.00146	0.16912 ±0.00142	0.15801 ±0.00138	0.14747 ±0.00134	0.12805 ±0.00126	0.09533 ±0.00109
1.000000	0.33880 ±0.00243	0.28527 ±0.00228	0.26104 ±0.00220	0.24953 ±0.00216	0.23842 ±0.00212	0.22772 ±0.00208	0.21741 ±0.00203	0.19797 ±0.00195	0.16354 ±0.00179
1.500000	0.38465 ±0.00301	0.33477 ±0.00285	0.31194 ±0.00277	0.30102 ±0.00273	0.29044 ±0.00269	0.28017 ±0.00265	0.27022 ±0.00261	0.25122 ±0.00253	0.21666 ±0.00237

Table 13: Call Prices CEV - MC - sigma 0.6, beta 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20844 ±0.00081	0.12849 ±0.00070	0.09594 ±0.00063	0.08178 ±0.00058	0.06908 ±0.00054	0.05782 ±0.00050	0.04795 ±0.00046	0.03201 ±0.00038	0.01273 ±0.00023
0.166667	0.22446 ±0.00109	0.15331 ±0.00095	0.12365 ±0.00087	0.11036 ±0.00083	0.09811 ±0.00079	0.08685 ±0.00075	0.07656 ±0.00071	0.05877 ±0.00062	0.03302 ±0.00047
0.250000	0.23590 ±0.00129	0.16841 ±0.00115	0.13985 ±0.00107	0.12690 ±0.00103	0.11483 ±0.00098	0.10363 ±0.00094	0.09325 ±0.00090	0.07493 ±0.00082	0.04686 ±0.00066
0.500000	0.27847 ±0.00175	0.21891 ±0.00160	0.19280 ±0.00152	0.18064 ±0.00148	0.16906 ±0.00144	0.15805 ±0.00140	0.14761 ±0.00136	0.12840 ±0.00127	0.09600 ±0.00111
1.000000	0.33761 ±0.00247	0.28459 ±0.00232	0.26063 ±0.00224	0.24926 ±0.00220	0.23830 ±0.00216	0.22775 ±0.00212	0.21758 ±0.00208	0.19843 ±0.00199	0.16452 ±0.00183
1.500000	0.38320 ±0.00307	0.33398 ±0.00291	0.31149 ±0.00283	0.30075 ±0.00279	0.29034 ±0.00275	0.28024 ±0.00271	0.27046 ±0.00267	0.25179 ±0.00259	0.21786 ±0.00244

Table 14: Call Prices CEV - MC - sigma 0.6, beta 0.7

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20812 ±0.00081	0.12820 ±0.00071	0.09577 ±0.00063	0.08169 ±0.00059	0.06907 ±0.00055	0.05789 ±0.00051	0.04809 ±0.00046	0.03228 ±0.00038	0.01308 ±0.00024
0.166667	0.22380 ±0.00110	0.15287 ±0.00096	0.12342 ±0.00089	0.11024 ±0.00085	0.09811 ±0.00080	0.08696 ±0.00076	0.07679 ±0.00072	0.05920 ±0.00064	0.03372 ±0.00048
0.250000	0.23501 ±0.00130	0.16785 ±0.00116	0.13955 ±0.00108	0.12673 ±0.00104	0.11481 ±0.00100	0.10374 ±0.00096	0.09350 ±0.00092	0.07544 ±0.00084	0.04773 ±0.00068
0.500000	0.27689 ±0.00179	0.21800 ±0.00164	0.19228 ±0.00156	0.18032 ±0.00152	0.16894 ±0.00148	0.15814 ±0.00144	0.14791 ±0.00140	0.12909 ±0.00132	0.09736 ±0.00116
1.000000	0.33529 ±0.00255	0.28330 ±0.00240	0.25988 ±0.00232	0.24880 ±0.00228	0.23813 ±0.00224	0.22786 ±0.00220	0.21799 ±0.00216	0.19942 ±0.00209	0.16654 ±0.00193
1.500000	0.38043 ±0.00320	0.33252 ±0.00305	0.31071 ±0.00297	0.30031 ±0.00293	0.29024 ±0.00289	0.28049 ±0.00285	0.27105 ±0.00282	0.25303 ±0.00274	0.22035 ±0.00259

Table 15: Call Prices CEV - MC - sigma 0.6, beta 0.8

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20812 ±0.00082	0.12820 ±0.00071	0.09577 ±0.00064	0.08169 ±0.00060	0.06907 ±0.00055	0.05789 ±0.00051	0.04809 ±0.00047	0.03228 ±0.00039	0.01308 ±0.00025
0.166667	0.22380 ±0.00111	0.15287 ±0.00098	0.12342 ±0.00090	0.11024 ±0.00086	0.09811 ±0.00082	0.08696 ±0.00078	0.07679 ±0.00073	0.05920 ±0.00065	0.03372 ±0.00050
0.250000	0.23501 ±0.00132	0.16785 ±0.00118	0.13955 ±0.00110	0.12673 ±0.00106	0.11481 ±0.00102	0.10374 ±0.00098	0.09350 ±0.00094	0.07544 ±0.00086	0.04773 ±0.00070
0.500000	0.27689 ±0.00183	0.21800 ±0.00168	0.19228 ±0.00160	0.18032 ±0.00156	0.16894 ±0.00152	0.15814 ±0.00148	0.14791 ±0.00144	0.12909 ±0.00136	0.09736 ±0.00120
1.000000	0.33529 ±0.00265	0.28330 ±0.00250	0.25988 ±0.00242	0.24880 ±0.00238	0.23813 ±0.00234	0.22786 ±0.00230	0.21799 ±0.00227	0.19942 ±0.00219	0.16654 ±0.00204
1.500000	0.38043 ±0.00336	0.33252 ±0.00321	0.31071 ±0.00314	0.30031 ±0.00310	0.29024 ±0.00306	0.28049 ±0.00302	0.27105 ±0.00299	0.25303 ±0.00291	0.22035 ±0.00277

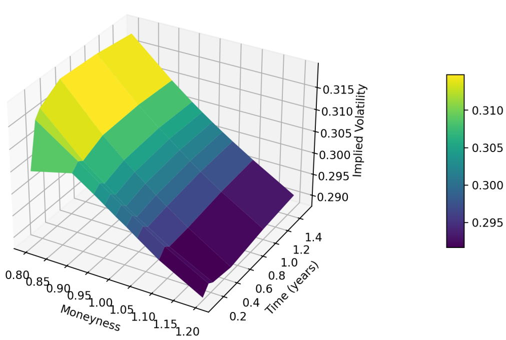
A.2 Call - Volatilities - CEV

Table 16: Call Volatilities - CEV Model - $\sigma = 0.3$, $\beta = 0.55$

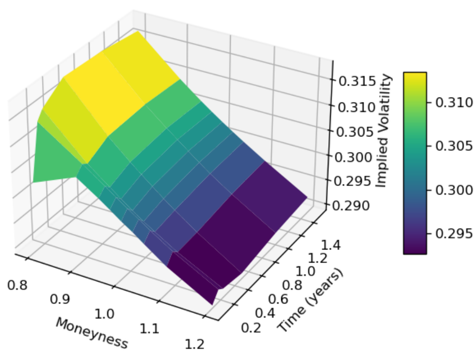
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.28591	0.30505	0.30252	0.30106	0.29941	0.29801	0.29700	0.29455	0.28941
0.16667	0.30911	0.30674	0.30382	0.30221	0.30054	0.2986	0.29701	0.29383	0.28801
0.25000	0.31773	0.30943	0.30561	0.30383	0.30202	0.30033	0.29851	0.29534	0.28988
0.50000	0.31881	0.30951	0.30545	0.30342	0.30159	0.29971	0.29800	0.29480	0.28901
1.00000	0.31921	0.31013	0.30601	0.30411	0.30227	0.30041	0.29878	0.29542	0.28915
1.50000	0.31955	0.31063	0.30661	0.30480	0.30301	0.30132	0.29951	0.29622	0.29017

Table 17: Call Volatilities - CEV Model - $\sigma = 0.3$, $\beta = 0.6$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.28263	0.30421	0.30229	0.30081	0.29940	0.29822	0.29733	0.29525	0.29071
0.16667	0.30731	0.30599	0.30357	0.30201	0.30044	0.29881	0.29746	0.29453	0.28922
0.25000	0.31604	0.30861	0.30524	0.30366	0.30201	0.30041	0.29897	0.29590	0.29111
0.50000	0.31711	0.30875	0.30507	0.30320	0.30141	0.29999	0.29839	0.29552	0.29036
1.00000	0.31744	0.30938	0.30561	0.30388	0.30214	0.30054	0.29901	0.29605	0.29040
1.50000	0.31770	0.30975	0.30621	0.30457	0.30291	0.30131	0.29983	0.29682	0.29146



(a) Volatility Surface Call $\sigma_x = 0.3$, $\beta = 0.55$



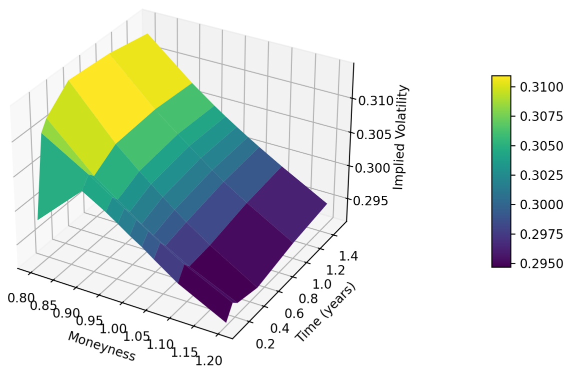
(b) Volatility Surface Call $\sigma_x = 0.3$, $\beta = 0.6$

Table 18: Call Volatilities - CEV Model - $\sigma = 0.3$, $\beta = 0.7$

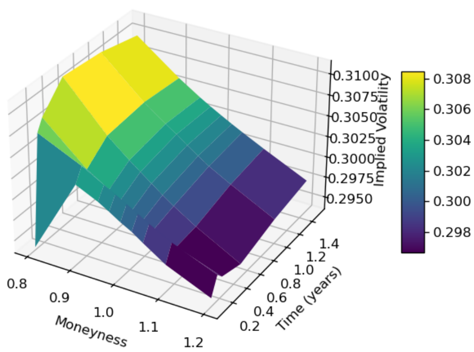
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.29826	0.30461	0.30259	0.30143	0.30022	0.29914	0.29787	0.29546	0.29194
0.16677	0.31015	0.30615	0.30391	0.30279	0.30167	0.30056	0.29943	0.29736	0.29396
0.25000	0.31457	0.30615	0.30328	0.30197	0.30079	0.29970	0.29872	0.29696	0.29305
0.50000	0.31460	0.30736	0.30437	0.30298	0.30161	0.30024	0.29896	0.29661	0.29247
1.00000	0.31410	0.30774	0.30505	0.30376	0.30245	0.30115	0.29992	0.29777	0.29382
1.50000	0.31284	0.30720	0.30470	0.30354	0.30243	0.30136	0.30033	0.29836	0.29465

Table 19: Call Volatilities - CEV Model - $\sigma = 0.3$, $\beta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.29386	0.30305	0.30183	0.30104	0.30020	0.29948	0.29856	0.29682	0.29454
0.16677	0.30700	0.30462	0.30317	0.30243	0.30169	0.30093	0.30015	0.29877	0.29664
0.25000	0.31130	0.30457	0.30252	0.30159	0.30079	0.30007	0.29945	0.29837	0.29571
0.50000	0.31122	0.30574	0.30357	0.30256	0.30157	0.30057	0.29965	0.29798	0.29512
1.00000	0.31075	0.30615	0.30428	0.30337	0.30242	0.30150	0.30063	0.29919	0.29650
1.50000	0.30953	0.30567	0.30400	0.30323	0.30250	0.30180	0.30113	0.29983	0.29738



(a) Volatility Surface Call $\sigma_x = 0.3$, $\beta = 0.7$



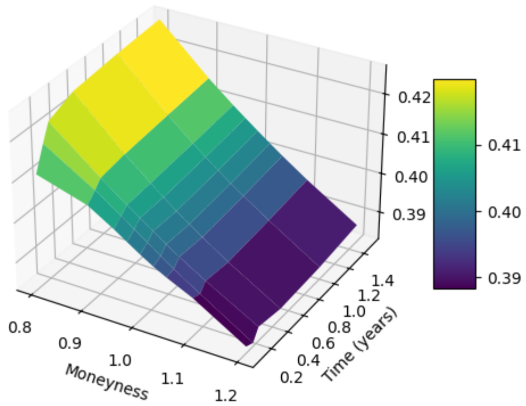
(b) Volatility Surface Call $\sigma_x = 0.3$, $\beta = 0.8$

Table 20: Call Volatilities - CEV Model - $\sigma = 0.4$, $\beta = 0.55$

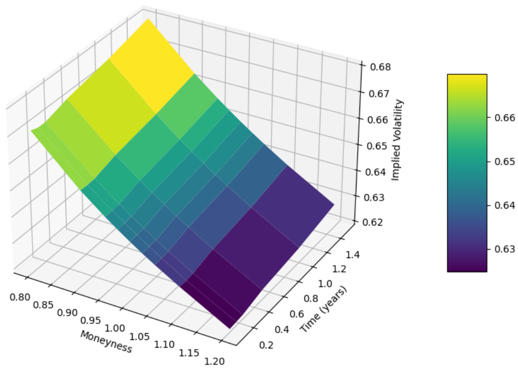
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.41141	0.40752	0.40363	0.40142	0.39931	0.39733	0.39560	0.39251	0.38470
0.16677	0.41742	0.40971	0.40533	0.40301	0.40072	0.39833	0.39612	0.39204	0.38391
0.25000	0.42371	0.41272	0.40763	0.40520	0.40299	0.40055	0.39821	0.39391	0.38614
0.50000	0.42443	0.41251	0.40722	0.40465	0.40211	0.39988	0.39751	0.39331	0.38551
1.00000	0.42551	0.41364	0.40827	0.405714	0.40323	0.40080	0.39861	0.39424	0.38610
1.50000	0.42623	0.41464	0.40941	0.406910	0.40455	0.40210	0.39981	0.39544	0.38748

Table 21: Call Volatilities - CEV Model - $\sigma = 0.4$, $\beta = 0.6$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.40901	0.40652	0.40311	0.40120	0.39931	0.3976	0.39611	0.39343	0.38645
0.16667	0.41512	0.40861	0.40484	0.40271	0.40061	0.39857	0.39651	0.39285	0.38563
0.25000	0.42151	0.41175	0.40712	0.40498	0.40291	0.40081	0.39875	0.39481	0.38784
0.50000	0.42200	0.41143	0.40664	0.40431	0.40212	0.40000	0.39800	0.39421	0.38734
1.00000	0.42314	0.41243	0.40762	0.40531	0.40318	0.40104	0.39891	0.39503	0.38783
1.50000	0.42383	0.41340	0.40878	0.40654	0.40441	0.40221	0.40023	0.39633	0.38910



(a) Volatility Surface Call $\sigma_x = 0.4$, $\beta = 0.55$



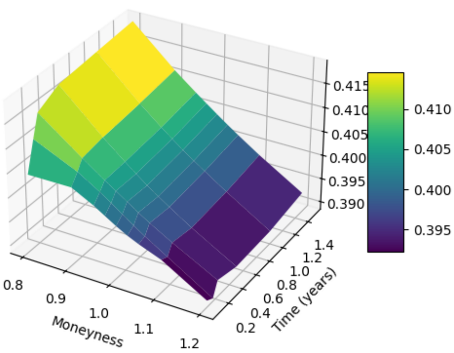
(b) Volatility Surface Call $\sigma_x = 0.4$, $\beta = 0.6$

Table 22: Call Volatilities - CEV Model - $\sigma = 0.4$, $\beta = 0.7$

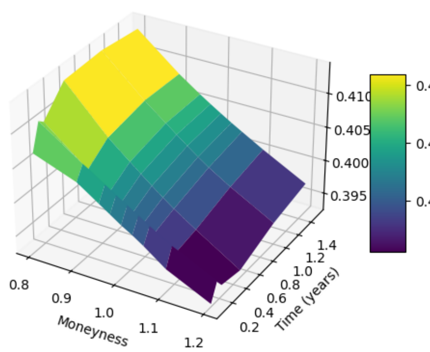
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.41143	0.40667	0.40346	0.40188	0.40030	0.39887	0.39734	0.39408	0.38944
0.16677	0.41509	0.40851	0.40540	0.40383	0.40234	0.40086	0.39937	0.39656	0.39183
0.25000	0.41712	0.40797	0.40438	0.40267	0.40115	0.39968	0.39831	0.39589	0.39105
0.50000	0.41835	0.40949	0.40577	0.40395	0.40219	0.40044	0.39878	0.39575	0.39004
1.00000	0.41858	0.41050	0.40697	0.40522	0.40348	0.40181	0.40020	0.39728	0.39209
1.50000	0.41770	0.41020	0.40689	0.40533	0.40383	0.40237	0.40096	0.39825	0.39321

Table 23: Call Volatilities - CEV Model - $\sigma = 0.4$, $\beta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.16677	0.41077	0.40647	0.40442	0.40335	0.40237	0.40136	0.40034	0.39844	0.39539
0.25000	0.41267	0.40586	0.40334	0.40215	0.40114	0.40017	0.39927	0.39776	0.39460
0.50000	0.41376	0.40728	0.40465	0.40335	0.40207	0.40083	0.39964	0.39754	0.39353
1.00000	0.41405	0.40836	0.40590	0.40465	0.40342	0.40224	0.40112	0.39915	0.39564
1.50000	0.41329	0.40817	0.40596	0.40492	0.40392	0.40295	0.40203	0.40022	0.39687



(a) Volatility Surface Call $\sigma_x = 0.4$, $\beta = 0.7$



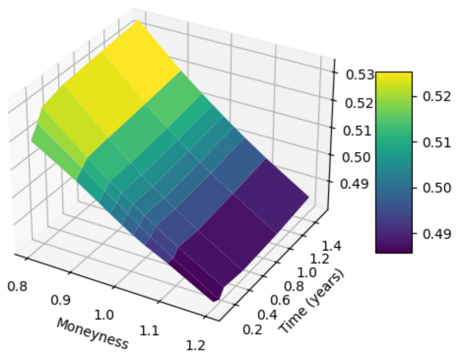
(b) Volatility Surface Call $\sigma_x = 0.4$, $\beta = 0.8$

Table 24: Call Volatilities - CEV Model - $\sigma = 0.5$, $\beta = 0.55$

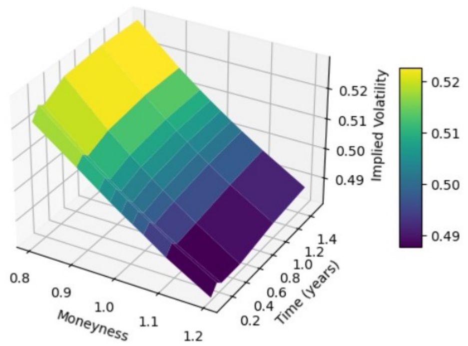
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.51941	0.50993	0.50460	0.50182	0.4992 5	0.4967	0.49442	0.49054	0.48171
0.16667	0.52384	0.51254	0.50671	0.50381	0.50090	0.49801	0.49525	0.49013	0.48034
0.25000	0.52992	0.51611	0.50974	0.50677	0.50385	0.50092	0.49801	0.49274	0.48293
0.50000	0.53041	0.51584	0.50910	0.50603	0.50294	0.50004	0.49722	0.49191	0.48227
1.00000	0.53201	0.51736	0.51061	0.50750	0.50440	0.5015	0.49871	0.49334	0.48323
1.50000	0.53342	0.51900	0.51251	0.50945	0.50641	0.50341	0.50052	0.49500	0.48502

Table 25: Call Volatilities - CEV Model - $\sigma = 0.5$, $\beta = 0.6$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.51661	0.50864	0.50391	0.50158	0.49923	0.49701	0.49500	0.49161	0.48383
0.16667	0.52091	0.51126	0.50610	0.50342	0.50091	0.49831	0.49576	0.49122	0.48245
0.25000	0.52711	0.51475	0.50917	0.50642	0.50388	0.50122	0.49864	0.49382	0.48511
0.50000	0.52753	0.51431	0.50844	0.50552	0.50281	0.50021	0.49774	0.49300	0.48431
1.00000	0.52881	0.51578	0.50973	0.50696	0.50421	0.50161	0.49914	0.49423	0.48521
1.50000	0.53031	0.51744	0.51161	0.50886	0.50610	0.50343	0.50091	0.49605	0.48701



(a) Volatility Surface Call $\sigma_x = 0.5$, $\beta = 0.55$



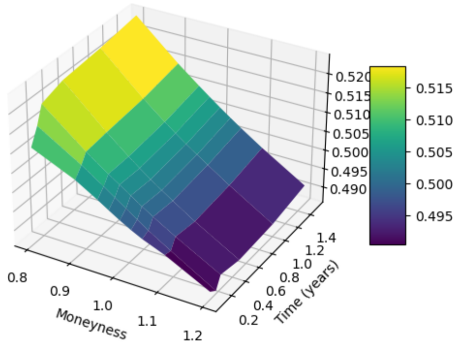
(b) Volatility Surface Call $\sigma_x = 0.5$, $\beta = 0.6$

Table 26: Call Volatilities - CEV Model - $\sigma = 0.5$, $\beta = 0.7$

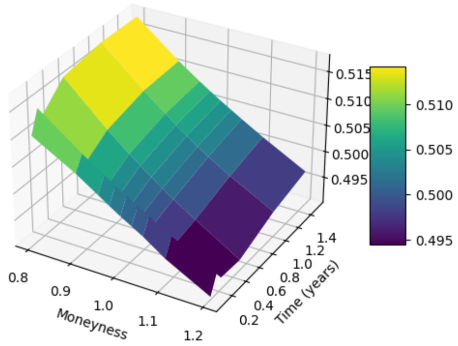
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.51586	0.50835	0.50433	0.50234	0.50040	0.49860	0.49680	0.49284	0.48636
0.16677	0.51981	0.51095	0.50692	0.50492	0.50309	0.50122	0.49937	0.49583	0.48951
0.25000	0.52031	0.50990	0.50550	0.50345	0.50157	0.49972	0.49798	0.49484	0.48885
0.50000	0.52226	0.51180	0.50718	0.50498	0.50279	0.50067	0.49864	0.49491	0.48779
1.00000	0.52333	0.51351	0.50901	0.50681	0.50468	0.50262	0.50064	0.49699	0.49050
1.50000	0.52296	0.51361	0.50947	0.50750	0.50559	0.50374	0.50196	0.49850	0.49209

Table 27: Call Volatilities - CEV Model - $\sigma = 0.5$, $\beta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.51028	0.50573	0.50304	0.50166	0.50035	0.49915	0.49792	0.49507	0.49066
0.16677	0.51439	0.50840	0.50570	0.50433	0.50312	0.50185	0.50058	0.49817	0.49396
0.25000	0.51469	0.50725	0.50420	0.50280	0.50154	0.50031	0.49916	0.49718	0.49327
0.50000	0.51643	0.50898	0.50571	0.50415	0.50259	0.50109	0.49966	0.49709	0.49208
1.00000	0.51759	0.51077	0.50759	0.50603	0.50453	0.50310	0.50174	0.49929	0.49491
1.50000	0.51743	0.51109	0.50830	0.50698	0.50570	0.50448	0.50328	0.50095	0.49665



(a) Volatility Surface Call $\sigma_x = 0.5$, $\beta = 0.7$



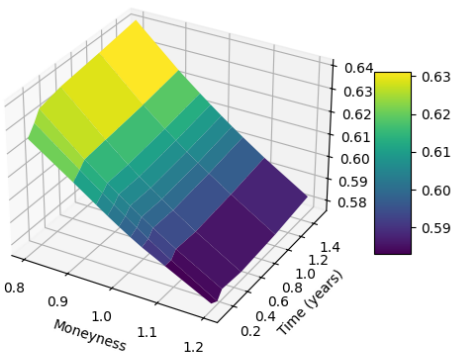
(b) Volatility Surface Call $\sigma_x = 0.5$, $\beta = 0.8$

Table 28: Call Volatilities - CEV Model - $\sigma = 0.6$, $\beta = 0.55$

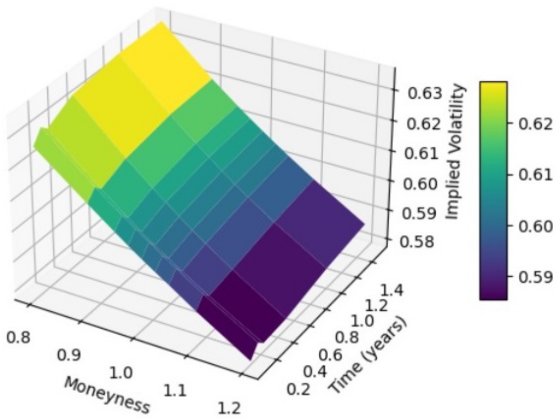
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.62541	0.61230	0.60564	0.60231	0.59911	0.59610	0.59331	0.58835	0.57832
0.16667	0.62981	0.61545	0.60823	0.60471	0.60121	0.59787	0.59444	0.58824	0.57662
0.25000	0.63633	0.61951	0.61206	0.60832	0.60490	0.60141	0.59800	0.59162	0.57971
0.50000	0.63663	0.61921	0.61128	0.60751	0.60399	0.60041	0.59710	0.59074	0.57901
1.00000	0.63883	0.62122	0.61331	0.60951	0.60598	0.60242	0.59902	0.59251	0.58057
1.50000	0.64121	0.62398	0.61601	0.61221	0.60860	0.60500	0.60163	0.59502	0.58305

Table 29: Call Volatilities - CEV Model - $\sigma = 0.6$, $\beta = 0.6$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.62201	0.61073	0.60483	0.60191	0.59910	0.59648	0.59393	0.58969	0.58082
0.16667	0.62633	0.61375	0.60743	0.60423	0.60115	0.59801	0.59500	0.58950	0.57912
0.25000	0.63292	0.61794	0.61115	0.60792	0.60481	0.60171	0.59864	0.59291	0.58233
0.50000	0.63301	0.61744	0.61030	0.60690	0.60377	0.60061	0.59764	0.59191	0.58150
1.00000	0.63483	0.61912	0.61219	0.60877	0.60551	0.60240	0.59941	0.59365	0.58282
1.50000	0.63721	0.62188	0.61481	0.61143	0.60820	0.60500	0.60191	0.59614	0.58531



(a) Volatility Surface Call $\sigma_x = 0.6$, $\beta = 0.55$



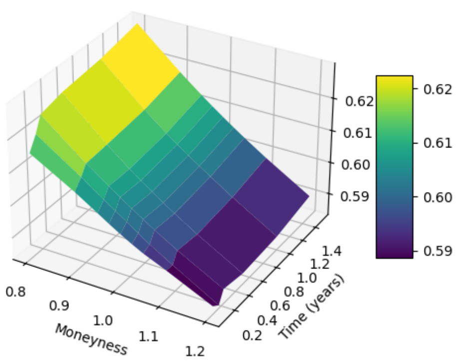
(b) Volatility Surface Call $\sigma_x = 0.6$, $\beta = 0.6$

Table 30: Call Volatilities - CEV Model - $\sigma = 0.6$, $\beta = 0.7$

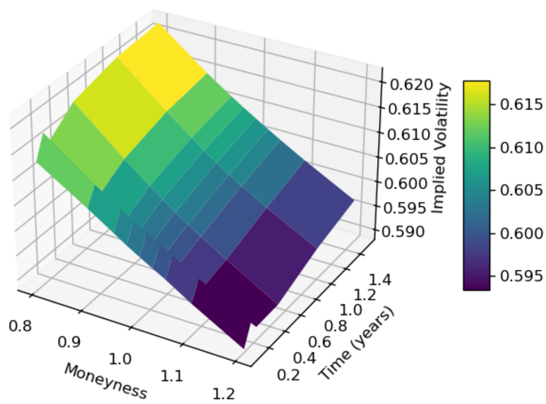
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.61976	0.61005	0.60521	0.60280	0.60052	0.59836	0.59625	0.59173	0.58351
0.16677	0.62411	0.61344	0.60850	0.60611	0.60390	0.60165	0.59944	0.59517	0.58746
0.25000	0.62372	0.61194	0.60670	0.60432	0.60205	0.59983	0.59772	0.59384	0.58666
0.50000	0.62622	0.61414	0.60865	0.60602	0.60344	0.60095	0.59854	0.59410	0.58571
1.00000	0.62837	0.61666	0.61119	0.60858	0.60605	0.60361	0.60127	0.59691	0.58909
1.50000	0.62869	0.61749	0.61247	0.61009	0.60777	0.60554	0.60335	0.59913	0.59132

Table 31: Call Volatilities - CEV Model - $\sigma = 0.6$, $\beta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.61305	0.60689	0.60362	0.60197	0.60044	0.59899	0.59757	0.59438	0.58865
0.16677	0.61758	0.61037	0.60703	0.60541	0.60393	0.60240	0.60089	0.59797	0.59278
0.25000	0.61693	0.60872	0.60511	0.60352	0.60199	0.60052	0.59912	0.59664	0.59195
0.50000	0.61911	0.61065	0.60680	0.60492	0.60310	0.60136	0.59969	0.59665	0.59077
1.00000	0.62140	0.61323	0.60938	0.60756	0.60580	0.60412	0.60253	0.59960	0.59432
1.50000	0.62206	0.61445	0.61107	0.60945	0.60792	0.60640	0.60491	0.60207	0.59678



(a) Volatility Surface Call $\sigma_x = 0.6$, $\beta = 0.7$



(b) Volatility Surface Call $\sigma_x = 0.6$, $\beta = 0.8$

B Appendix B

B.1 Call prices DD - 1,000,000 simulations

Table 32: Call Prices - MC - sigma 0.4, delta 0.2

	0.80000	0.90000	0.95000	0.97500	1.000	1.025	1.05000	1.10000	1.20000
0.083333	0.20319 ±0.00021	0.11736 ±0.00019	0.08277 ±0.00016	0.06812 ±0.00015	0.05528 ±0.00014	0.04424 ±0.00013	0.03491 ±0.00011	0.02085 ±0.00009	0.00632 ±0.00005
0.166667	0.21190 ±0.00029	0.13521 ±0.00025	0.10405 ±0.00023	0.09043 ±0.00021	0.07810 ±0.00020	0.06703 ±0.00019	0.05718 ±0.00018	0.04087 ±0.00015	0.01955 ±0.00010
0.250000	0.22149 ±0.00034	0.15007 ±0.00030	0.12065 ±0.00028	0.10756 ±0.00027	0.09554 ±0.00025	0.08454 ±0.00024	0.07454 ±0.00023	0.05735 ±0.00020	0.03265 ±0.00015
0.500000	0.24836 ±0.00047	0.18540 ±0.00043	0.15867 ±0.00040	0.14646 ±0.00039	0.13500 ±0.00038	0.12426 ±0.00036	0.11423 ±0.00035	0.09617 ±0.00033	0.06723 ±0.00028
1.000000	0.29171 ±0.00067	0.23637 ±0.00062	0.21212 ±0.00060	0.20081 ±0.00058	0.19003 ±0.00057	0.17976 ±0.00056	0.16998 ±0.00055	0.15183 ±0.00052	0.12073 ±0.00047
1.500000	0.32680 ±0.00084	0.27570 ±0.00079	0.25295 ±0.00077	0.24224 ±0.00075	0.23197 ±0.00074	0.22209 ±0.00073	0.21262 ±0.00072	0.19483 ±0.00070	0.16344 ±0.00065

Table 33: Call Prices - MC - sigma 0.4, delta 0.4

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.206424 ±0.000243	0.124593 ±0.000211	0.091468 ±0.000188	0.077200 ±0.000175	0.064493 ±0.000162	0.053323 ±0.000148	0.043641 ±0.000135	0.028344 ±0.000109	0.010592 ±0.000066
0.166667	0.219609 ±0.000326	0.146743 ±0.000286	0.116696 ±0.000261	0.103350 ±0.000248	0.091112 ±0.000235	0.079958 ±0.000222	0.069851 ±0.000208	0.052608 ±0.000183	0.020845 ±0.000135
0.250000	0.232704 ±0.000389	0.156436 ±0.000345	0.132629 ±0.000320	0.123412 ±0.000307	0.114458 ±0.000294	0.100391 ±0.000281	0.090183 ±0.000268	0.072215 ±0.000242	0.044972 ±0.000193
0.500000	0.267257 ±0.000535	0.207019 ±0.000489	0.180956 ±0.000464	0.168908 ±0.000451	0.157497 ±0.000438	0.146709 ±0.000426	0.136531 ±0.000413	0.117914 ±0.000387	0.087037 ±0.000338
1.000000	0.320761 ±0.000762	0.267420 ±0.000715	0.246369 ±0.000691	0.232443 ±0.000678	0.221699 ±0.000666	0.211394 ±0.000654	0.205512 ±0.000641	0.182971 ±0.000617	0.150446 ±0.000568
1.500000	0.363317 ±0.000961	0.313851 ±0.000914	0.291489 ±0.000890	0.280875 ±0.000878	0.270626 ±0.000866	0.260726 ±0.000855	0.251172 ±0.000843	0.233050 ±0.000819	0.200508 ±0.000772

Table 34: Call Prices - MC - sigma 0.4, shift 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.210698 ±0.000271	0.132293 ±0.000235	0.100291 ±0.000211	0.086317 ±0.000198	0.073706 ±0.000185	0.062442 ±0.000171	0.052488 ±0.000158	0.036224 ±0.000132	0.015707 ±0.000086
0.166667	0.228472 ±0.000363	0.158463 ±0.000320	0.129434 ±0.000295	0.116297 ±0.000282	0.104128 ±0.000269	0.092908 ±0.000256	0.082607 ±0.000242	0.064642 ±0.000216	0.038095 ±0.000167
0.250000	0.245014 ±0.000433	0.175195 ±0.000387	0.151965 ±0.000362	0.139279 ±0.000350	0.127380 ±0.000337	0.116259 ±0.000323	0.105893 ±0.000310	0.087328 ±0.000284	0.058058 ±0.000234
0.500000	0.287057 ±0.000593	0.228861 ±0.000538	0.203294 ±0.000503	0.191368 ±0.000514	0.179997 ±0.000501	0.169170 ±0.000488	0.158879 ±0.000475	0.139836 ±0.000450	0.107450 ±0.000403
1.000000	0.350493 ±0.000858	0.298627 ±0.000810	0.275196 ±0.000786	0.264086 ±0.000773	0.253370 ±0.000761	0.243039 ±0.000749	0.233079 ±0.000735	0.214236 ±0.000705	0.180614 ±0.000663
1.500000	0.400389 ±0.001085	0.352313 ±0.001038	0.330058 ±0.001014	0.319514 ±0.001002	0.309287 ±0.000990	0.299365 ±0.000978	0.289747 ±0.000966	0.271389 ±0.000943	0.237948 ±0.000895

Table 35: Call Prices - MC - sigma 0.4, shift 0.8

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.215821 ±0.000297	0.140315 ±0.000258	0.101999 ±0.000234	0.095455 ±0.000221	0.082919 ±0.000208	0.071582 ±0.000195	0.061413 ±0.000181	0.044373 ±0.000155	0.021502 ±0.000107
0.166667	0.238171 ±0.000398	0.158462 ±0.000354	0.129620 ±0.000329	0.112697 ±0.000316	0.111144 ±0.000303	0.105788 ±0.000289	0.095421 ±0.000276	0.076882 ±0.000250	0.038095 ±0.000200
0.250000	0.258101 ±0.000476	0.194961 ±0.000430	0.167712 ±0.000405	0.155158 ±0.000392	0.143303 ±0.000379	0.132139 ±0.000365	0.121649 ±0.000352	0.102613 ±0.000326	0.071696 ±0.000274
0.500000	0.307463 ±0.000663	0.250852 ±0.000614	0.225670 ±0.000589	0.213837 ±0.000577	0.202496 ±0.000564	0.191640 ±0.000551	0.181260 ±0.000538	0.161884 ±0.000505	0.128286 ±0.000403
1.000000	0.380670 ±0.000953	0.329939 ±0.000905	0.306779 ±0.000881	0.295736 ±0.000869	0.285041 ±0.000856	0.274689 ±0.000844	0.264668 ±0.000832	0.245589 ±0.000807	0.210614 ±0.000758
1.500000	0.437826 ±0.001209	0.390501 ±0.001162	0.368647 ±0.001138	0.358157 ±0.001126	0.347948 ±0.001114	0.338809 ±0.001102	0.328341 ±0.001090	0.309791 ±0.001066	0.275646 ±0.001019

Table 36: Call Prices - MC - sigma 0.5, beta 0.2

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.20788 ±0.00026	0.12788 ±0.00023	0.09557 ±0.00020	0.08159 ±0.00019	0.06908 ±0.00018	0.05800 ±0.00016	0.04831 ±0.00015	0.03271 ±0.00012	0.01366 ±0.00008
0.166667	0.22272 ±0.00035	0.15184 ±0.00031	0.12259 ±0.00028	0.10956 ±0.00027	0.09756 ±0.00026	0.08657 ±0.00025	0.07655 ±0.00023	0.05927 ±0.00020	0.03428 ±0.00016
0.250000	0.23704 ±0.00042	0.17116 ±0.00037	0.14350 ±0.00034	0.13098 ±0.00034	0.11930 ±0.00033	0.10845 ±0.00032	0.09840 ±0.00030	0.08056 ±0.00027	0.05290 ±0.00022
0.500000	0.27421 ±0.00059	0.21623 ±0.00054	0.19111 ±0.00050	0.17947 ±0.00050	0.16843 ±0.00049	0.15797 ±0.00049	0.14808 ±0.00047	0.12988 ±0.00044	0.09931 ±0.00039
1.000000	0.33084 ±0.00089	0.28011 ±0.00081	0.25749 ±0.00078	0.24684 ±0.00077	0.23660 ±0.00076	0.22676 ±0.00075	0.21731 ±0.00074	0.19954 ±0.00071	0.16813 ±0.00067
1.500000	0.37551 ±0.00110	0.32900 ±0.00105	0.30799 ±0.00102	0.29801 ±0.00102	0.28836 ±0.00101	0.27903 ±0.00101	0.27002 ±0.00099	0.25290 ±0.00096	0.22198 ±0.00092

Table 37: Call Prices - MC - sigma 0.5, beta 0.4

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.083333	0.21377 ±0.00029	0.13775 ±0.00026	0.10666 ±0.00023	0.09300 ±0.00022	0.08059 ±0.00021	0.06941 ±0.00020	0.05943 ±0.00018	0.04228 ±0.00015	0.02070 ±0.00011
0.166667	0.23434 ±0.00040	0.16689 ±0.00035	0.13856 ±0.00033	0.12574 ±0.00031	0.11382 ±0.00030	0.10276 ±0.00029	0.09254 ±0.00028	0.07449 ±0.00020	0.04698 ±0.00020
0.250000	0.25293 ±0.00048	0.19004 ±0.00040	0.16314 ±0.00040	0.15080 ±0.00039	0.13982 ±0.00038	0.12828 ±0.00037	0.11806 ±0.00035	0.09958 ±0.00033	0.06977 ±0.00028
0.500000	0.29933 ±0.00067	0.24360 ±0.00062	0.21901 ±0.00060	0.20750 ±0.00059	0.19651 ±0.00059	0.18600 ±0.00058	0.17599 ±0.00055	0.15773 ±0.00052	0.12519 ±0.00047
1.000000	0.36816 ±0.00098	0.31905 ±0.00094	0.29680 ±0.00091	0.28624 ±0.00090	0.27603 ±0.00089	0.26617 ±0.00089	0.25664 ±0.00086	0.23855 ±0.00084	0.20599 ±0.00079
1.500000	0.42185 ±0.00127	0.37666 ±0.00122	0.35595 ±0.00119	0.34604 ±0.00119	0.33642 ±0.00118	0.32707 ±0.00117	0.31799 ±0.00115	0.30062 ±0.00113	0.26878 ±0.00109

Table 38: Call Prices - MC - sigma 0.5, beta 0.6

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.22082 ±0.00033	0.14801 ±0.00029	0.11786 ±0.00026	0.10444 ±0.00025	0.09210 ±0.00024	0.08085 ±0.00022	0.07065 ±0.00021	0.05321 ±0.00018	0.02860 ±0.00013
0.16667	0.24705 ±0.00044	0.18224 ±0.00040	0.15459 ±0.00037	0.14195 ±0.00036	0.13008 ±0.00034	0.11897 ±0.00033	0.10859 ±0.00032	0.08995 ±0.00029	0.06044 ±0.00024
0.25000	0.26981 ±0.00053	0.20918 ±0.00048	0.18284 ±0.00046	0.17064 ±0.00045	0.15907 ±0.00043	0.14812 ±0.00042	0.13777 ±0.00041	0.11881 ±0.00038	0.08732 ±0.00033
0.50000	0.32521 ±0.00075	0.27114 ±0.00070	0.24696 ±0.00068	0.23555 ±0.00067	0.22458 ±0.00065	0.21405 ±0.00064	0.20394 ±0.00063	0.18496 ±0.00060	0.15157 ±0.00056
1.00000	0.40604 ±0.00111	0.35811 ±0.00106	0.33615 ±0.00104	0.32565 ±0.00103	0.31547 ±0.00101	0.30558 ±0.00100	0.29598 ±0.00099	0.27766 ±0.00097	0.24423 ±0.00092
1.50000	0.46864 ±0.00143	0.42442 ±0.00139	0.40394 ±0.00137	0.39408 ±0.00136	0.38448 ±0.00134	0.37511 ±0.00133	0.36599 ±0.00132	0.34842 ±0.00130	0.31587 ±0.00125

Table 39: Call Prices - MC - sigma 0.5, beta 0.8

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.22873 ±0.00036	0.15855 ±0.00032	0.12913 ±0.00029	0.11589 ±0.00028	0.10362 ±0.00026	0.09230 ±0.00025	0.08193 ±0.00024	0.06385 ±0.00021	0.03716 ±0.00016
0.16667	0.26052 ±0.00049	0.19780 ±0.00044	0.17068 ±0.00041	0.15817 ±0.00040	0.14634 ±0.00039	0.13518 ±0.00037	0.12468 ±0.00036	0.10559 ±0.00034	0.07445 ±0.00028
0.25000	0.28734 ±0.00059	0.22849 ±0.00054	0.20259 ±0.00051	0.19049 ±0.00050	0.17895 ±0.00049	0.16797 ±0.00047	0.15752 ±0.00046	0.13819 ±0.00044	0.10533 ±0.00039
0.50000	0.35158 ±0.00083	0.29881 ±0.00079	0.27493 ±0.00076	0.26359 ±0.00075	0.25265 ±0.00074	0.24209 ±0.00072	0.23192 ±0.00071	0.21267 ±0.00069	0.17830 ±0.00064
1.00000	0.44428 ±0.00124	0.39725 ±0.00119	0.37551 ±0.00117	0.36507 ±0.00115	0.35490 ±0.00114	0.34499 ±0.00113	0.33535 ±0.00112	0.31684 ±0.00109	0.28271 ±0.00105
1.50000	0.51573 ±0.00160	0.47225 ±0.00156	0.45195 ±0.00153	0.44213 ±0.00152	0.43254 ±0.00151	0.42316 ±0.00150	0.41399 ±0.00149	0.39628 ±0.00147	0.36317 ±0.00142

Table 40: Call Prices - MC - sigma 0.6, shift 0.2

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.214377 ±0.000297	0.139225 ±0.000258	0.108581 ±0.000234	0.095112 ±0.000221	0.082863 ±0.000208	0.071810 ±0.000195	0.061914 ±0.000181	0.045332 ±0.000155	0.022930 ±0.000107
0.166667	0.235440 ±0.000398	0.169075 ±0.000354	0.141265 ±0.000329	0.128691 ±0.000316	0.116979 ±0.000303	0.106108 ±0.000289	0.096045 ±0.000276	0.078226 ±0.000250	0.050807 ±0.000200
0.250000	0.253548 ±0.000476	0.194637 ±0.000430	0.166415 ±0.000405	0.154535 ±0.000392	0.142993 ±0.000379	0.132322 ±0.000365	0.121231 ±0.000352	0.104182 ±0.000326	0.074736 ±0.000274
0.500000	0.301367 ±0.000663	0.247254 ±0.000614	0.223445 ±0.000589	0.213036 ±0.000577	0.201662 ±0.000564	0.191500 ±0.000551	0.181803 ±0.000538	0.163735 ±0.000505	0.132472 ±0.000403
1.000000	0.370525 ±0.000953	0.323554 ±0.000905	0.302348 ±0.000881	0.292284 ±0.000869	0.282559 ±0.000856	0.273164 ±0.000844	0.264492 ±0.000832	0.246869 ±0.000807	0.215847 ±0.000758
1.500000	0.437826 ±0.001209	0.390501 ±0.001162	0.368647 ±0.001138	0.358157 ±0.001126	0.347948 ±0.001114	0.338809 ±0.001102	0.328341 ±0.001090	0.309791 ±0.001066	0.275646 ±0.001019

Table 41: Call Prices - MC - sigma 0.6, beta 0.4

	0.80000	0.90000	0.95000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.22305 ±0.00034	0.15163 ±0.00030	0.12204 ±0.00028	0.10884 ±0.00026	0.09667 ±0.00025	0.08554 ±0.00024	0.07540 ±0.00022	0.05793 ±0.00020	0.03275 ±0.00015
0.16667	0.25088 ±0.00047	0.18756 ±0.00042	0.16051 ±0.00040	0.14813 ±0.00038	0.13648 ±0.00037	0.12555 ±0.00036	0.11531 ±0.00035	0.09685 ±0.00032	0.06723 ±0.00027
0.25000	0.27476 ±0.00057	0.21572 ±0.00052	0.19005 ±0.00049	0.17813 ±0.00048	0.16683 ±0.00047	0.15611 ±0.00046	0.14596 ±0.00044	0.12730 ±0.00042	0.09601 ±0.00037
0.50000	0.33249 ±0.00081	0.28028 ±0.00077	0.25691 ±0.00074	0.24588 ±0.00073	0.23527 ±0.00072	0.22508 ±0.00071	0.21528 ±0.00069	0.19684 ±0.00067	0.16425 ±0.00062
1.00000	0.41586 ±0.00123	0.37023 ±0.00119	0.34934 ±0.00116	0.33935 ±0.00115	0.32965 ±0.00114	0.32023 ±0.00113	0.31109 ±0.00112	0.29361 ±0.00109	0.26165 ±0.00105
1.50000	0.48006 ±0.00162	0.43851 ±0.00158	0.41928 ±0.00156	0.41002 ±0.00155	0.40100 ±0.00154	0.39221 ±0.00153	0.38363 ±0.00152	0.36711 ±0.00150	0.33646 ±0.00145

Table 42: Call Prices - MC - sigma 0.6, beta 0.6

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.223054 ±0.000343	0.151631 ±0.000302	0.122041 ±0.000277	0.108836 ±0.000264	0.096674 ±0.000251	0.085537 ±0.000238	0.075397 ±0.000225	0.057931 ±0.000189	0.032747 ±0.000119
0.166667	0.238059 ±0.000467	0.168757 ±0.000403	0.141052 ±0.000367	0.126903 ±0.000349	0.115452 ±0.000331	0.103955 ±0.000313	0.095421 ±0.000298	0.076882 ±0.000250	0.050807 ±0.000200
0.250000	0.274757 ±0.000567	0.215717 ±0.000520	0.190045 ±0.000495	0.178134 ±0.000482	0.166825 ±0.000470	0.151606 ±0.000457	0.145956 ±0.000444	0.127298 ±0.000419	0.096010 ±0.000336
0.500000	0.332490 ±0.000815	0.280277 ±0.000768	0.256913 ±0.000744	0.245882 ±0.000731	0.235272 ±0.000719	0.225077 ±0.000707	0.215282 ±0.000695	0.196841 ±0.000670	0.164249 ±0.000622
1.000000	0.415863 ±0.001230	0.370228 ±0.001185	0.349344 ±0.001162	0.339353 ±0.001151	0.329652 ±0.001139	0.320234 ±0.001128	0.310924 ±0.001116	0.293610 ±0.001094	0.261648 ±0.001049
1.500000	0.480057 ±0.001624	0.438511 ±0.001581	0.419278 ±0.001560	0.410024 ±0.001549	0.401003 ±0.001538	0.392207 ±0.001528	0.383630 ±0.001517	0.367106 ±0.001496	0.336461 ±0.001453

Table 43: Call Prices - MC - sigma 0.6, beta 0.8

	0.800	0.900	0.950	0.975	1.000	1.025	1.050	1.100	1.200
0.083333	0.243505 ±0.000419	0.177374 ±0.000347	0.149198 ±0.000349	0.136341 ±0.000336	0.124295 ±0.000323	0.113045 ±0.000310	0.105276 ±0.000297	0.083891 ±0.000227	0.054635 ±0.000150
0.166667	0.284427 ±0.000516	0.225207 ±0.000438	0.199177 ±0.000403	0.187037 ±0.000390	0.175469 ±0.000377	0.164461 ±0.000364	0.153597 ±0.000351	0.134656 ±0.000282	0.097141 ±0.000212
0.250000	0.314852 ±0.000591	0.262584 ±0.000521	0.241041 ±0.000482	0.225731 ±0.000468	0.214489 ±0.000455	0.203707 ±0.000442	0.193369 ±0.000429	0.173991 ±0.000368	0.140115 ±0.000295
0.500000	0.396458 ±0.000801	0.346725 ±0.000709	0.323946 ±0.000663	0.313056 ±0.000647	0.302493 ±0.000631	0.292253 ±0.000616	0.282326 ±0.000602	0.263381 ±0.000536	0.228937 ±0.000429
1.000000	0.507761 ±0.001030	0.463877 ±0.000948	0.443401 ±0.000894	0.433508 ±0.000874	0.423838 ±0.000854	0.414389 ±0.000835	0.405158 ±0.000817	0.387324 ±0.000764	0.354054 ±0.000611
1.500000	0.592798 ±0.001209	0.552653 ±0.001126	0.533746 ±0.001068	0.524571 ±0.001047	0.515576 ±0.001026	0.506754 ±0.001006	0.498106 ±0.000987	0.476106 ±0.000935	0.439483 ±0.000748

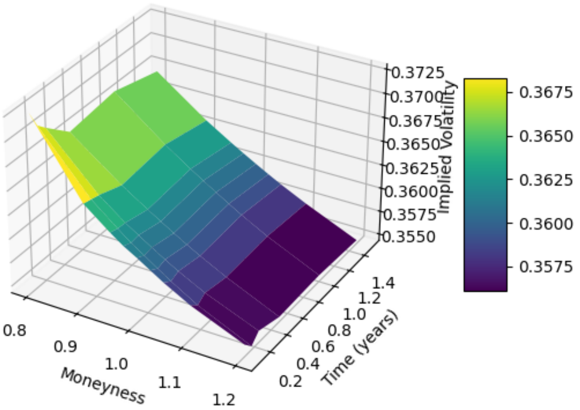
B.2 Call - Volatilities - DD

Table 44: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.3$, $\Delta = 0.2$

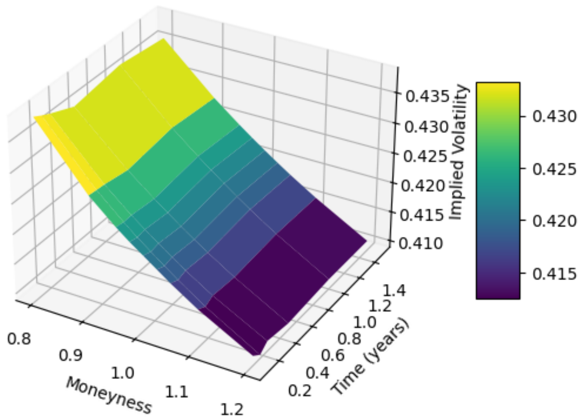
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.37408	0.36426	0.36204	0.36109	0.36025	0.35947	0.35879	0.35738	0.35519
0.16677	0.36806	0.36361	0.36174	0.36092	0.36013	0.35936	0.35857	0.35720	0.35494
0.25000	0.36772	0.36335	0.36155	0.36074	0.35997	0.35922	0.35850	0.35714	0.35479
0.50000	0.36777	0.36372	0.36198	0.36118	0.36041	0.35966	0.35895	0.35762	0.35515
1.00000	0.36750	0.36364	0.36193	0.36110	0.36030	0.35955	0.35882	0.35744	0.35498
1.50000	0.36759	0.36378	0.36208	0.36128	0.36051	0.35979	0.35908	0.35776	0.35535

Table 45: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.3$, $\Delta = 0.4$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.43882	0.42759	0.42369	0.42196	0.42036	0.41883	0.41744	0.41470	0.40997
0.16677	0.43515	0.42700	0.42346	0.42184	0.42029	0.41878	0.41729	0.41452	0.40981
0.25000	0.43498	0.42680	0.42331	0.42171	0.42017	0.41868	0.41725	0.41454	0.40972
0.50000	0.43544	0.42748	0.42405	0.42244	0.42089	0.41939	0.41797	0.41527	0.41037
1.00000	0.43566	0.42786	0.42440	0.42276	0.42118	0.41967	0.41823	0.41547	0.41052
1.50000	0.43630	0.42850	0.42504	0.42341	0.42185	0.42036	0.41893	0.41622	0.41130



(a) Volatility Surface Call $\sigma_x = 0.3$, $Delta = 0.2$



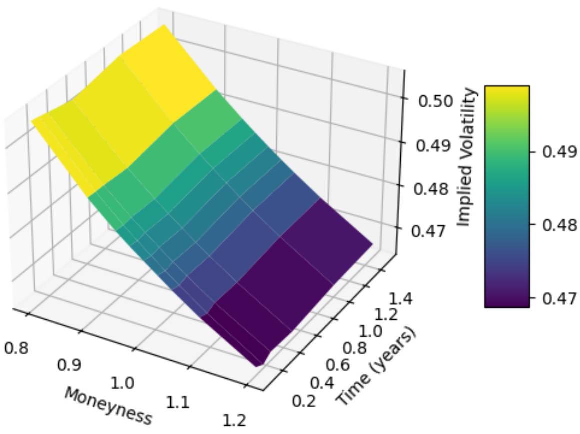
(b) Volatility Surface Call $\sigma_x = 0.3$, $Delta = 0.4$

Table 46: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.3$, $\Delta = 0.6$

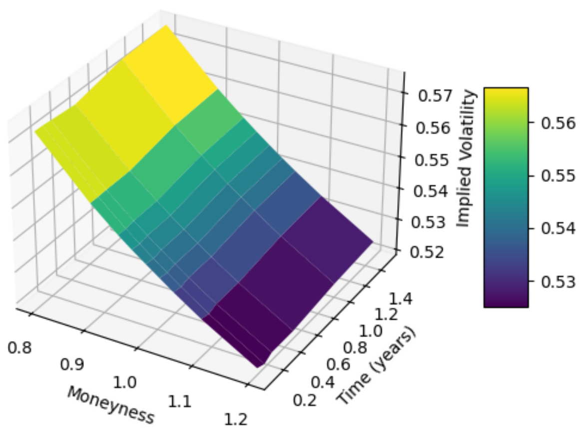
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.50497	0.49094	0.48538	0.48287	0.48050	0.47823	0.47612	0.47208	0.46476
0.16677	0.50242	0.49046	0.48525	0.48284	0.48051	0.47826	0.47606	0.47192	0.46473
0.25000	0.50233	0.49033	0.48517	0.48278	0.48047	0.47824	0.47609	0.47202	0.46470
0.50000	0.50331	0.49146	0.48631	0.48389	0.48156	0.47932	0.47716	0.47310	0.46573
1.00000	0.50431	0.49252	0.48730	0.48483	0.48245	0.48018	0.47800	0.47385	0.46637
1.50000	0.50577	0.49388	0.48863	0.48616	0.48379	0.48152	0.47934	0.47520	0.46772

Table 47: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.3$, $\Delta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.57145	0.55432	0.54711	0.54382	0.54068	0.53767	0.53484	0.52947	0.51958
0.16677	0.56983	0.55400	0.54712	0.54391	0.54081	0.53782	0.53490	0.52938	0.51969
0.25000	0.56980	0.55400	0.54716	0.54396	0.54087	0.53789	0.53503	0.52959	0.51978
0.50000	0.57142	0.55569	0.54881	0.54557	0.54246	0.53946	0.53657	0.53111	0.52124
1.00000	0.57354	0.55769	0.55068	0.54736	0.54418	0.54113	0.53820	0.53262	0.52256
1.50000	0.57614	0.56005	0.55297	0.54963	0.54643	0.54337	0.54041	0.53481	0.52469



(a) Volatility Surface Call $\sigma_x = 0.3$, $Delta = 0.6$



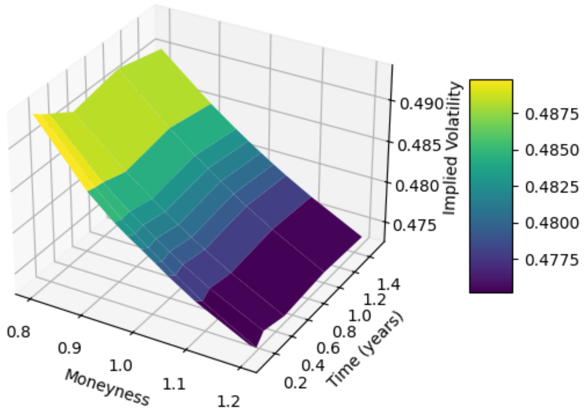
(b) Volatility Surface Call $\sigma_x = 0.3$, $Delta = 0.8$

Table 48: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.4$, $\Delta = 0.2$

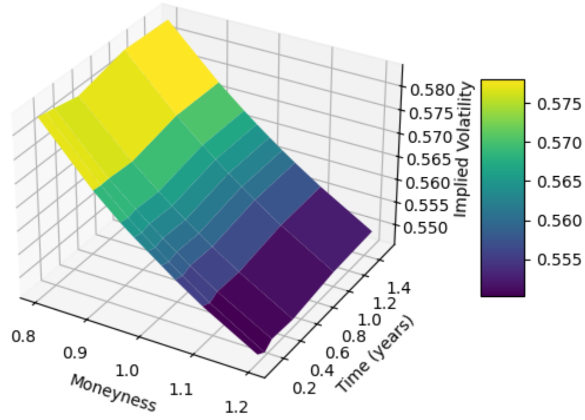
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.49379	0.48651	0.48387	0.48269	0.48151	0.48041	0.47930	0.47732	0.47427
0.16667	0.49298	0.48622	0.48341	0.48213	0.48092	0.47977	0.47861	0.47658	0.47297
0.25000	0.49235	0.48615	0.48351	0.48244	0.4812	0.48012	0.47913	0.47724	0.47419
0.50000	0.49051	0.48511	0.48281	0.48164	0.48066	0.47955	0.47850	0.47677	0.47342
1.00000	0.49163	0.48607	0.48361	0.4824	0.48137	0.48027	0.47921	0.47721	0.47390
1.50000	0.49002	0.48497	0.48268	0.48154	0.48058	0.47957	0.47855	0.47677	0.47354

Table 49: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.4$, $\Delta = 0.4$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.58311	0.57132	0.56623	0.56401	0.56204	0.55991	0.55780	0.55402	0.54740
0.16667	0.58280	0.57102	0.56592	0.56361	0.56141	0.55920	0.55711	0.55324	0.54633
0.25000	0.58242	0.57111	0.56638	0.56413	0.56190	0.55980	0.55781	0.55412	0.54770
0.50000	0.58133	0.57072	0.56600	0.56380	0.56161	0.55961	0.55762	0.55391	0.54733
1.00000	0.58371	0.57280	0.56801	0.56572	0.56350	0.56142	0.55949	0.55560	0.548812
1.50000	0.58320	0.57265	0.56790	0.56571	0.56365	0.56151	0.55961	0.55592	0.54920



(a) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.2$



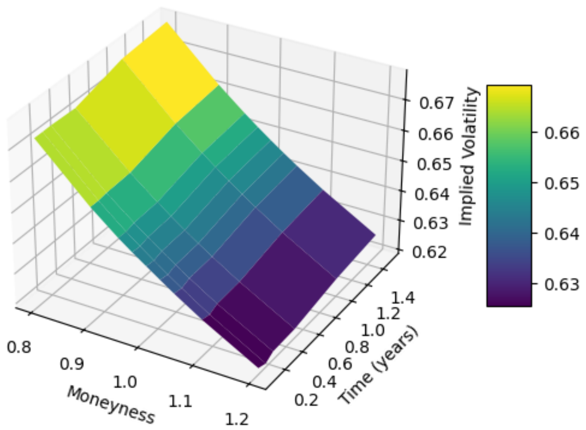
(b) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.4$

Table 50: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.4$, $\Delta = 0.6$

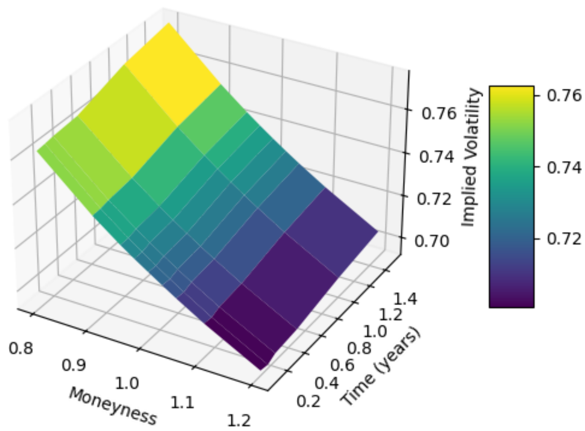
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.67279	0.65611	0.64886	0.64559	0.64258	0.63948	0.63648	0.63073	0.62082
0.16667	0.67272	0.65606	0.64859	0.64533	0.64212	0.63890	0.63586	0.63013	0.61986
0.25000	0.67298	0.65641	0.64947	0.64611	0.64291	0.63981	0.63688	0.63128	0.62131
0.50000	0.67264	0.65669	0.64971	0.64640	0.64329	0.64016	0.63729	0.63169	0.62169
1.00000	0.67715	0.66064	0.65343	0.65001	0.64673	0.64355	0.64057	0.63474	0.62458
1.50000	0.67835	0.66209	0.65484	0.65143	0.64814	0.64501	0.64205	0.63633	0.62613

Table 51: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.4$, $\Delta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.76275	0.74091	0.73156	0.72729	0.72307	0.71903	0.71507	0.70754	0.69422
0.16667	0.76316	0.74125	0.73162	0.72716	0.72296	0.71871	0.71477	0.70706	0.69345
0.25000	0.76378	0.74202	0.73272	0.72825	0.72417	0.72006	0.71609	0.70864	0.69533
0.50000	0.76456	0.74338	0.73397	0.72955	0.72520	0.72117	0.71710	0.70980	0.69646
1.00000	0.77196	0.74986	0.74007	0.73542	0.73103	0.72678	0.72273	0.71493	0.70108
1.50000	0.77562	0.75343	0.74358	0.73892	0.73458	0.73025	0.72618	0.71831	0.70449



(a) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.6$



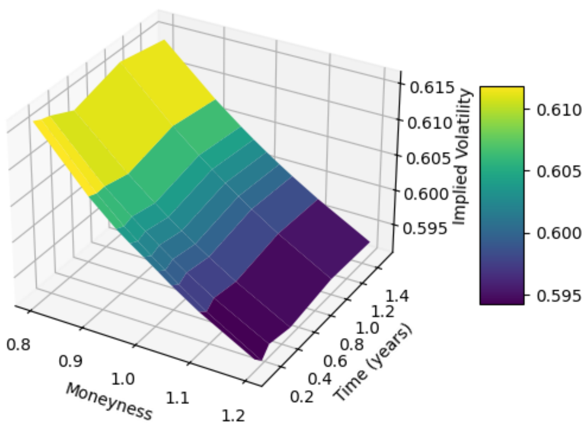
(b) Volatility Surface Call $\sigma_x = 0.4$, $Delta = 0.8$

Table 52: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.5$, $\Delta = 0.2$

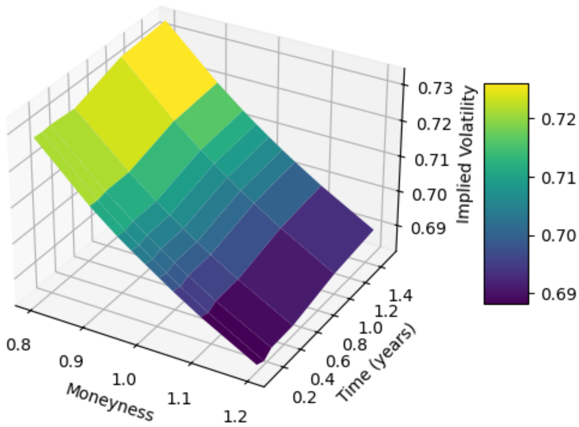
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.61610	0.60815	0.60484	0.60345	0.60211	0.60074	0.59935	0.59681	0.59274
0.16667	0.61555	0.60779	0.60424	0.60278	0.60122	0.59992	0.59852	0.59597	0.59142
0.25000	0.61529	0.6077	0.6046	0.60326	0.60184	0.60059	0.59922	0.5968	0.59286
0.50000	0.61359	0.60682	0.60386	0.60245	0.60111	0.59988	0.59858	0.59618	0.59216
1.00000	0.61535	0.60842	0.60528	0.60395	0.60252	0.60119	0.59987	0.59746	0.59318
1.50000	0.61391	0.60725	0.60442	0.60306	0.60172	0.60052	0.59931	0.59701	0.59292

Table 53: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.5$, $\Delta = 0.4$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.72841	0.71419	0.70809	0.7053	0.70276	0.70007	0.69768	0.69283	0.68445
0.16667	0.72824	0.71408	0.70774	0.70493	0.70224	0.69957	0.69692	0.69196	0.68341
0.25000	0.72847	0.71458	0.70867	0.70586	0.70308	0.70057	0.69799	0.69335	0.68514
0.50000	0.72799	0.71468	0.70874	0.70597	0.70321	0.70067	0.69828	0.69353	0.68527
1.00000	0.7325	0.71866	0.71254	0.70961	0.70683	0.70427	0.70165	0.69687	0.68821
1.50000	0.73349	0.71961	0.71367	0.71078	0.70809	0.70543	0.70297	0.69828	0.68972



(a) Volatility Surface Call $\sigma_x = 0.5$, $Delta = 0.2$



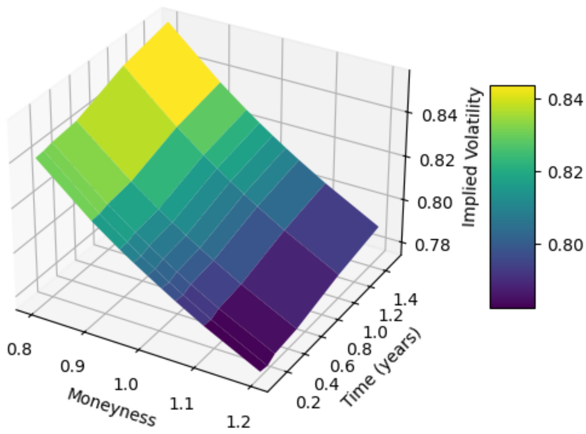
(b) Volatility Surface Call $\sigma_x = 0.5$, $Delta = 0.4$

Table 54: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.5$, $\Delta = 0.6$

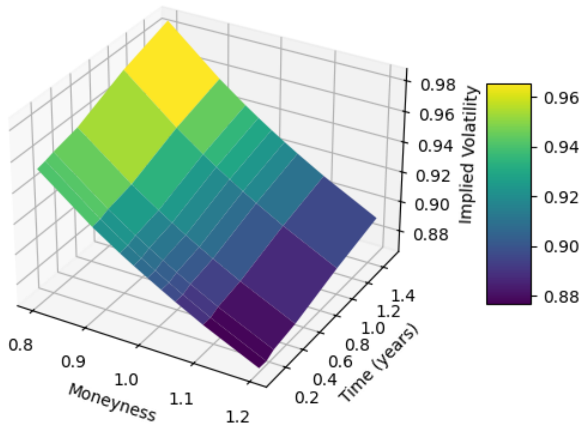
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.84090	0.82032	0.81154	0.80742	0.80357	0.79978	0.79595	0.78895	0.77638
0.16667	0.84132	0.82063	0.81157	0.80748	0.80333	0.79942	0.79564	0.78847	0.77564
0.25000	0.84229	0.82187	0.81297	0.80888	0.80485	0.80086	0.79729	0.79023	0.77783
0.50000	0.84350	0.82335	0.81447	0.8103	0.80622	0.80246	0.79873	0.79165	0.77918
1.00000	0.85215	0.83109	0.82171	0.81734	0.81313	0.80911	0.80524	0.79791	0.78484
1.50000	0.85683	0.83547	0.82607	0.82166	0.81744	0.81346	0.80951	0.80213	0.78894

Table 55: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.5$, $\Delta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.95376	0.92676	0.91512	0.90979	0.90458	0.89949	0.89451	0.88513	0.86848
0.16667	0.95491	0.92767	0.91573	0.91021	0.90487	0.89965	0.89466	0.88519	0.86811
0.25000	0.95669	0.92978	0.91792	0.91241	0.90707	0.90191	0.89691	0.88763	0.87095
0.50000	0.96041	0.93331	0.92144	0.91587	0.91046	0.90526	0.90020	0.89071	0.87379
1.00000	0.97489	0.94616	0.93354	0.92754	0.92180	0.91638	0.91116	0.90116	0.88335
1.50000	0.98531	0.95561	0.94268	0.93644	0.93066	0.92508	0.91963	0.90942	0.89121



(a) Volatility Surface Call $\sigma_x = 0.5$, $Delta = 0.6$



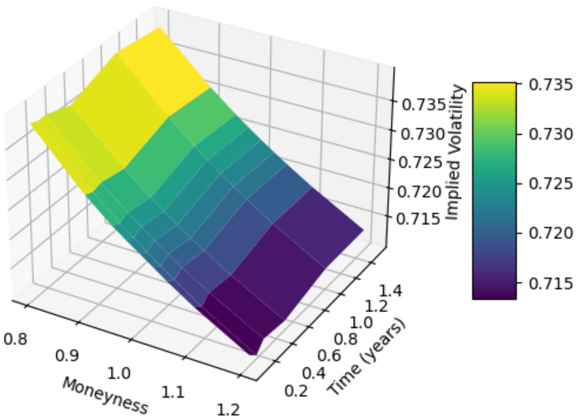
(b) Volatility Surface Call $\sigma_x = 0.5$, $Delta = 0.8$

Table 56: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.6$, $\Delta = 0.2$

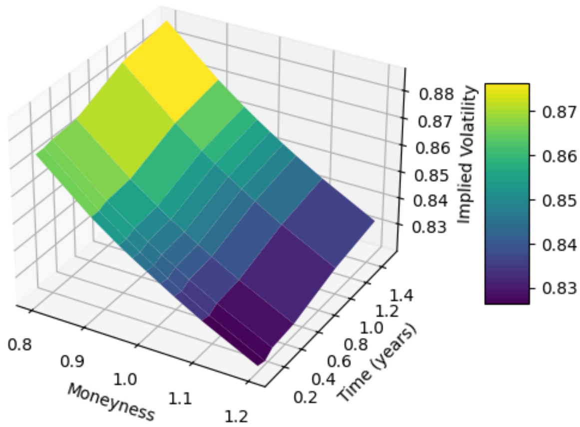
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.73692	0.72783	0.72411	0.72244	0.72082	0.71928	0.71787	0.71523	0.71024
0.16677	0.73572	0.72751	0.72404	0.72241	0.72084	0.71932	0.71783	0.71500	0.71016
0.25000	0.73539	0.72734	0.72392	0.72230	0.72074	0.71926	0.71781	0.71510	0.71021
0.50000	0.73746	0.72940	0.72589	0.72424	0.72266	0.72115	0.71971	0.71697	0.71204
1.00000	0.73884	0.73061	0.72704	0.72537	0.72376	0.72221	0.72072	0.71790	0.71286
1.50000	0.74201	0.73369	0.73008	0.72839	0.72677	0.72520	0.72370	0.72084	0.71573

Table 57: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.6$, $\Delta = 0.4$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	0.87182	0.85503	0.84794	0.84466	0.84150	0.83848	0.83562	0.83028	0.82044
0.16677	0.87134	0.85536	0.84846	0.84521	0.84208	0.83907	0.83614	0.83056	0.82075
0.25000	0.87166	0.85579	0.84891	0.84565	0.84253	0.83953	0.83664	0.83118	0.82134
0.50000	0.87624	0.86007	0.85300	0.84968	0.84649	0.84344	0.84050	0.83495	0.82496
1.00000	0.88218	0.86537	0.85807	0.85464	0.85134	0.84817	0.84512	0.83934	0.82898
1.50000	0.89060	0.87322	0.86565	0.86210	0.85870	0.85543	0.85228	0.84630	0.83559



(a) Volatility Surface Call $\sigma_x = 0.6$, $Delta = 0.2$



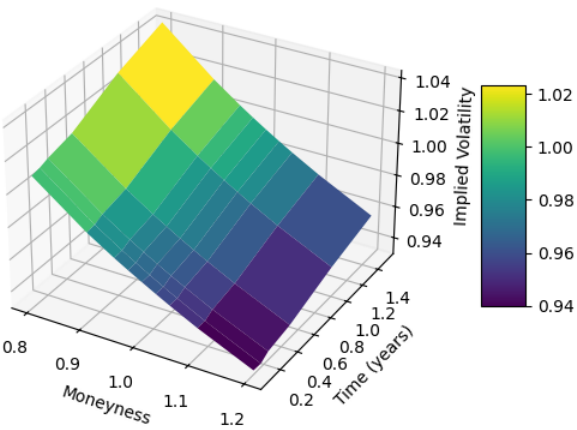
(b) Volatility Surface Call $\sigma_x = 0.6$, $Delta = 0.4$

Table 58: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.6$, $\Delta = 0.6$

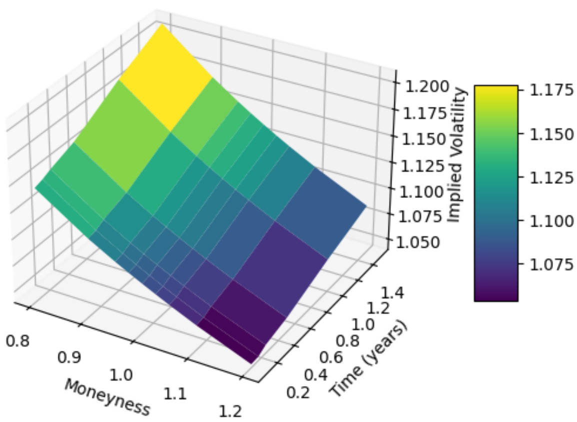
	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	1.00695	0.98252	0.97205	0.96715	0.96244	0.95793	0.95361	0.94553	0.93083
0.16677	1.00755	0.98380	0.97342	0.96854	0.96384	0.95932	0.95494	0.94659	0.93172
0.25000	1.00890	0.98512	0.97472	0.96981	0.96510	0.96057	0.95621	0.94795	0.93306
0.50000	1.01708	0.99258	0.98185	0.97681	0.97196	0.96731	0.96284	0.95438	0.93915
1.00000	1.03009	1.00418	0.99289	0.98758	0.98249	0.97758	0.97287	0.96395	0.94791
1.50000	1.04686	1.01942	1.00747	1.00187	0.99650	0.99133	0.98637	0.97697	0.96009

Table 59: Call Volatilities - Displaced Diffusion Model - $\sigma = 0.6$, $\Delta = 0.8$

	0.80000	0.90000	0.950000	0.97500	1.00000	1.02500	1.05000	1.10000	1.20000
0.08333	1.14240	1.11034	1.09647	1.08994	1.08368	1.07766	1.07188	1.06106	1.04144
0.16677	1.14452	1.11288	1.09902	1.09249	1.08620	1.08015	1.07430	1.06316	1.04318
0.25000	1.14731	1.11547	1.10150	1.09491	1.08858	1.08250	1.07664	1.06553	1.04548
0.50000	1.16043	1.12730	1.11277	1.10594	1.09938	1.09308	1.08701	1.07554	1.05485
1.00000	1.18380	1.14799	1.13238	1.12505	1.11800	1.11123	1.10472	1.09240	1.07026
1.50000	1.21331	1.17429	1.15734	1.14940	1.14180	1.13449	1.12747	1.11420	1.09040



(a) Volatility Surface Call $\sigma_x = 0.6$, $Delta = 0.6$



(b) Volatility Surface Call $\sigma_x = 0.6$, $Delta = 0.8$