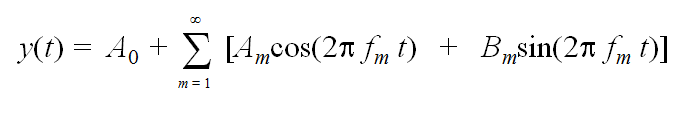
Signal & System Theory (CSD:5224)

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**Assignment 4**

Fourier Series

Fourier’s Theorem says that any periodic waveform can be written as the sum of a series of sine and cosine waves. Written in mathematical notation, it is often expressed



Suppose you have a discrete series consisting of the following sets of values

*f* = [1, 2, 3, 4]

*A* = [1, 3, 1, -1]

*B* = [2, 1, -1, 1],

where *f* is a vector describing four frequencies (in Hz), *A* is a vector describing the corresponding *A* coefficients in the series, and *B* is a vector describing the corresponding *B* coefficients in the series.

In the spaces below, draw the waveforms that will be added up for each frequency in the series.

A white paper with writing on it

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A paper with graph and numbers

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Recall from lecture the following definitions:

A vector is linearly independent iff there exists no constant a such that *aX*-*Y* = 0.

A vector is orthogonal iff *X* '*Y* = 0.

A vector is uncorrelated iff (*X*-*X*1)'(*Y*-*Y*1) = 0.

Consider the following two column vectors, *A* and *B*. Each represents 10 samples of a waveform.

*A* = [0.9877, 0.8910, 0.7071, 0.4540, 0.1564, -0.1564, -0.4540, -0.7071, -0.8910, -0.9877]'

*B* = [0.9511, 0.5878, 0.0000, -0.5878, -0.9511, -0.9511, -0.5878, -0.0000, 0.5878, 0.9511]'

**5) Sketch a plot of *A* and *B*.**

A drawing of a figure

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**6) Are *A* and *B* linearly independent? Justify your answer.**

Yes. There is no consistent proportion between A and B, which means that there is no value x such that Ax – B = 0, which fulfills the requirement for linear independence.

**7) Are *A* and *B* orthogonal? Justify your answer.**

Yes. A’B = 0, which fulfills the requirement for orthogonality.

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**8) Are *A* and *B* uncorrelated? Justify your answer.**

Yes. For completeness’s sake, I checked this is Matlab. However, because sinusoids always have mean 0 if there is no additive component, if they are orthogonal they will also be uncorrelated.

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*Note: Sinusoids having an integral number of cycles are used as basis functions for the Discrete Fourier Transform (the next topic in class). Sinusoids of length ½ cycle do not satisfy this condition. However, they still form part of a set of basis functions for decomposing signals in useful ways. The set starting with a half-cycle basis is related to decompositions used for compressing audio signals into MP3, video into MPEG, and images into JPEG formats. In case you are interested in learning more, this transform is often called the Discrete Cosine Transform (DCT).*

9) Write the equation for a sinusoid having a magnitude of 1 and a phase of pi/2.

F(t) = cos(2pi*ft +* pi/2)

10) Write the equation for a sinusoid that is *not* linearly independent of the expression in question 9.

F(t) = cos(2pi*ft -* pi/2)

11) Write the equation for two sinusoids that are orthogonal to the expression in question 9.

F(t) = cos(2pi*ft)*

F(t) = cos(2pi*ft +* pi)