Signal & System Theory (CSD:5224)

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**Assignment 5**

DFT 1

*In all cases, be sure to show your work!*

1) Compute *X*, the DFT of *x* by hand, where *x* = [1, -1, 2, -4, -2, 3]. Show your work. Round all calculations to three decimal places. To help you get started, I’ve done the fundamental (*m*=1) below. You need to do all the rest. (Be sure to go back and do *m*=0 as well!)

**m = 0:**

Basis function:

1 – 0j for all values of n

Multiplying by x yields x back. Therefore X(0) = sum(x) = -1 + 0j

**m = 2:**

Basis function:

1.0000 + 0.0000i

-0.5000 - 0.8660i

-0.5000 + 0.8660i

1.0000 + 0.0000i

-0.5000 - 0.8660i

-0.5000 + 0.8660i

Multiply by x:

1 + 0i

0.5 + 0.866i

-1 + 1.732i

-4 + 0i

1 + 1.732i

-1.5 + 2.598i

Sum: -4 + 6.9282i

**m = 3:**

Basis function:

1.0000 + 0.0000i

-1.0000 - 0.0000i

1.0000 + 0.0000i

-1.0000 - 0.0000i

1.0000 + 0.0000i

-1.0000 - 0.0000i

Multiply by x and sum

1 + 1 + 2 + 4 – 2 – 3 = 3 + 0i

**m = 4:**

Take the complex conjugate of m=2, gives us -4 – 6.9282i

**m = 5:**

Take the complex conjugate of m=1, gives us 6 + 0i

Final value of *X*:

-1 + 0i

6 + 0i

-4 + 6.9282i

3 – 0i

-4 – 6.9282i

6 + 0i

2) Sketch the DFT magnitude spectrum and the DFT phase spectrum. Put frequency on the *x*-axis, and include the correct frequency values. Correctly label the y-axes and use appropriate magnitude and phase values. *NOTE 1: In this problem, the sampling rate was not specified. Therefore, you cannot know the frequency in Hz (cycles per second). However, you can know the frequency in cycles. Be sure to label appropriately. NOTE 2: Remember that when the real part of the DFT is negative, the standard, 2-quadrant arctangent will give you the wrong answer. You need to use a calculator or function that give the 4-quadrant arctangent. Alternatively, use the corrections shown in lecture slides.*

A graph on a piece of paper

Description automatically generated

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For the fundamental, *m*=1, so the basis function is cos(2**1*n*/*N*) –*j*sin(2**1*n*/*N*).

Since *N*=6, *n* = [0 1 2 3 4 5]. Therefore, the basis function is

1.0000 +0*j*

0.5000 - 0.8660*j*

-0.5000 - 0.8660*j*

-1.0000 - 0.0000*j*

-0.5000 + 0.8660*j*

0.5000 +0.8660*j*

The input is multiplied by the basis function, and the output is the sum of the products:



Remember your complex math:

To add complex numbers, simply add up the real and imaginary parts separately.

Example: (1.0 - 0.5*j*) + (0.5 - 0.8*j*) = 1.5 – 1.3*j*

To multiply complex numbers, you have to use the distributive property (“foil” method). This is very similar to multiplying two binomials.

Example: (1.0 - 0.5*j*) \* (0.5 - 0.8*j*) =

**f**irst: 1.0 \* 0.5 = 0.50

**o**utside: 1.0 \* -0.8*j* = - 0.80 *j*

**i**nside: -0.5*j* \* 0.5 = - 0.25 *j*

**l**ast: -0.5*j* \* -0.8 *j* = -0.40 (remember *j* = sqrt(-1), so *j* ^2 = -1)

= 0.10 - 1.05 *j*

In the present case, however, we are multiplying a real number by a complex number. The process therefore simplifies to multiplying both parts of the complex number by the real number.

Example: 2 \* (0.5 - 0.8 *j*) =

2 \* 0.5 = 1

2 \* -0.8 *j* = -1.6 *j*

= 1 -1.6 *j*