Chapter 1 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 1: Limits and Continuity.

1.1: Defining a Limit
1.2: Evaluating Limits Analytically
1.3: Squeeze Theorem and Trigonometric Limits
1.4: Continuity
1.5: Formal Definition of a Limit
1.6: Limits with Infinity

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Limits and Continuity

Number of Questions—20

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points Earned	Points Available
Rapid Limits		20
Short Questions		35
Question 18		15
Question 19		15
Question 20		15
TOTAL		**100**

Rapid Limits

Evaluate the limit if it exists. No partial credit is awarded.

1. $\lim_{x\to\infty} \frac{2}{x^6}$ *(2 pts.)*

2. $\lim_{x\to 3} \sqrt{x^3+6x^2-8}$ *(2 pts.)*

3. $\lim_{x\to-\infty} \frac{5-x^3}{8x^4+x^2}$ *(2 pts.)*

4. $\lim\limits_{x\to -4} \dfrac{8x-2x^2}{x^4-16x^2}$ *(2 pts.)*

5. $\lim\limits_{x\to 2} \cos\left(\dfrac{1}{x-2}\right)$ *(2 pts.)*

6. $\lim\limits_{x\to 0} \dfrac{5-5\cos x}{10x}$ *(2 pts.)*

7. $\lim_{x\to\infty} \dfrac{e^x}{4e^x + e^{2x}}$ *(2 pts.)*

8. $\lim_{\theta\to\pi/2} \dfrac{\sin 2\theta}{9 - 9\cos^2\theta}$ *(2 pts.)*

9. $\lim_{x\to 3} \dfrac{3x - 9}{\sin(2x - 6)}$ *(2 pts.)*

10. $\lim\limits_{x\to 4} \dfrac{\sqrt{4x-7}-3}{8-2x}$ *(2 pts.)*

Short Questions

11. If $\lim_{x\to -2} f(x) = 6$ and $\lim_{x\to -2} g(x) = 8$, then find $\lim_{x\to -2}[g(x) - f(x)]$ and $\lim_{x\to -2} \dfrac{f(x)+2}{3g(x)}$. *(5 pts.)*

12. What type of discontinuity, if any, does $f(x) = \dfrac{6-2x}{9-x^2}$ have at $x = 3$? *(5 pts.)*

13. Determine the interval of continuity of $g(x) = \dfrac{\sqrt{8-x}}{2x^3 - 8x}$. *(5 pts.)*

14. Find all asymptotes of the rational function $f(x) = \dfrac{2x^2 - 5x + 3}{x^2 + x - 6}$. *(5 pts.)*

15. If $\lim\limits_{x \to 0^-} f(x) = 4$ and $\lim\limits_{x \to 0^+} f(x) = 7$, then evaluate $\lim\limits_{x \to 3^+} f(9 - x^2)$ if it exists. *(5 pts.)*

16. Show that $\lim\limits_{x \to 0} \left[6 + \dfrac{1}{2} x \cos\left(\dfrac{3}{x} \right) \right] = 6$. *(5 pts.)*

17. Construct a proof for $\lim_{x \to 5^+} \ln(x-5) = -\infty$. *(5 pts.)*

Long Questions

18. Let $f(x) = x^3 - 3x^2 + x + 1$ and $g(x) = \dfrac{x^2 - 4x + 3}{x - 1}$. The graphs of both functions intersect at the point $(2, -1)$. Note that $f(-1) = -4$ and $f(3) = 4$.

(a) Where does the graph of $y = g(x)$ contain a removable discontinuity? *(3 pts.)*

(b) Justify why f must have at least one zero on $(-1, 3)$. *(3 pts.)*

(c) Find $\lim_{x \to 0} g(f(x))$ and $\lim_{x \to 1} f(g(x))$. Show the work that leads to your answers. *(5 pts.)*

(d) Let h be a function that satisfies $g(x) \leqslant h(x) \leqslant f(x)$ for all $x \geqslant -1$. It is known that $h(2) = -1$. Show that $h(x)$ is continuous at $x = 2$. *(4 pts.)*

19. The population of cows on a farm is modeled by the continuous function $N(t)$ for $t \geqslant 0$, where t is measured in years. It is known that $N(0) = 2000$ and $N(3) = 2500$.

(a) Interpret the meaning of $\lim_{t\to\infty} N(t) = 2700$ in context. *(1 pt.)*

(b) Justify why there exists a value c in $(0,3)$ for which $N(c) = 2200$. *(2 pts.)*

(c) Around $t = 5$, it is observed that $N(t) = t^2 - 4t + 2100$. At that time, the population grows at a rate given by $\lim_{t\to 5} \dfrac{N(t) - N(5)}{t - 5}$. Find this rate of change. *(4 pts.)*

(d) For $t \neq 7$, the growth rate of the cow population is modeled by $r(t) = \dfrac{t^2 - 10t + 21}{3 - t} \sin\left(\dfrac{1}{t-7}\right)$. *(6 pts.)*

Find, and interpret in context, the value of $\lim_{t\to 7} r(t)$.

20. Let f be a piecewise function defined by

$$f(x) = \begin{cases} 2 + \sin\left(\frac{\pi}{2}x\right) & x \leqslant 1 \\ px + q & 1 < x \leqslant 2 \end{cases}.$$

It is known that $f(2) = 4$ and f is continuous at 1.

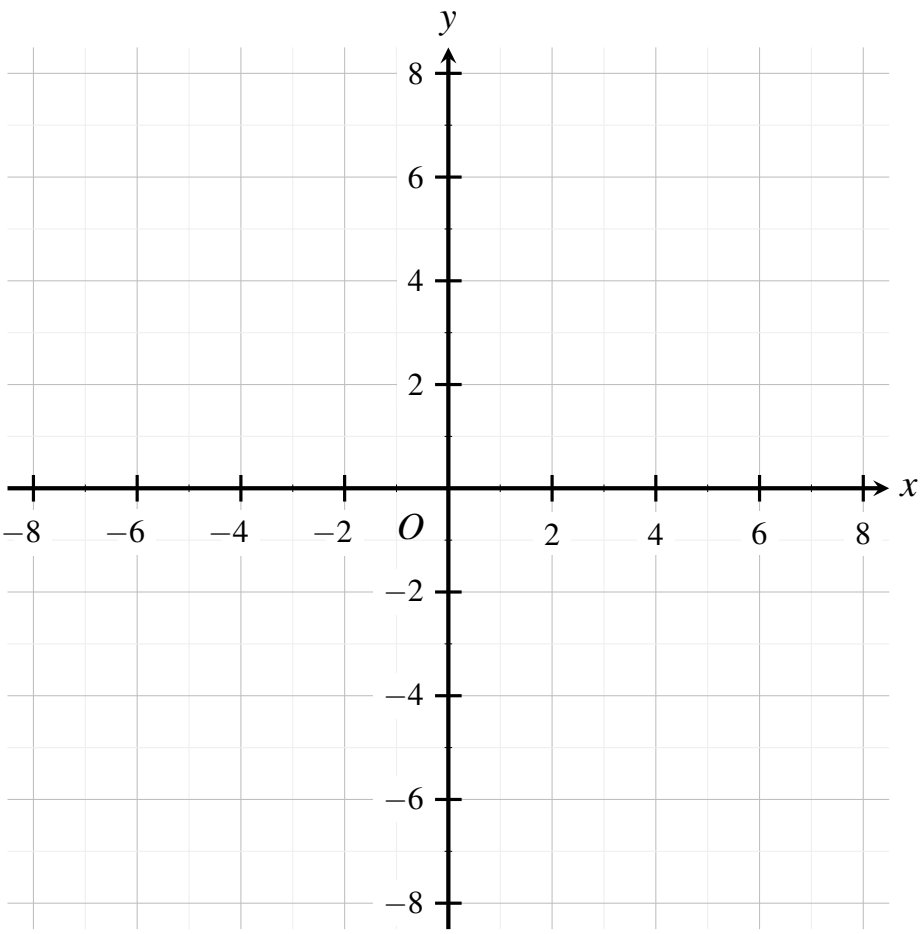
(a) Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1^-} f(x)$. *(2 pts.)*

(b) What are the values of p and q ? *(3 pts.)*

(c) Using the values of p and q from part (b), prove that $\lim_{x\to 1^+} (px+q) = p+q$ using the Delta-Epsilon definition of a limit. *(5 pts.)*

(d) It is known that $\lim_{x\to 4^-} f(x) = \infty$, $\lim_{x\to 4^+} f(x) = -\infty$, and $\lim_{x\to\infty} f(x) = 6$. Sketch a possible graph of $y = f(x)$. *(5 pts.)*

The following graph is for Problem 20.



This marks the end of the review exercises.