

## Chapter 1 Multiple-Choice Review Exercises

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**Directions:** These review exercises are multiple-choice questions based on the content in Chapter 1: Limits and Continuity.

- 1.1: Defining a Limit
- 1.2: Evaluating Limits Analytically
- 1.3: Squeeze Theorem and Trigonometric Limits
- 1.4: Continuity
- 1.5: Formal Definition of a Limit
- 1.6: Limits with Infinity

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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## Limits and Continuity

Number of Questions—45

**NO CALCULATOR**

1. If  $\lim_{x \rightarrow 3} f(x) = 8$ , then which option is true?

(A)  $f(3) = 8$ .

(B) As  $x$  approaches 3,  $f(x)$  approaches 8.

(C)  $f(8) = 3$ .

(D) As  $x$  approaches 8,  $f(x)$  approaches 3.

(E)  $f$  is continuous at  $x = 3$ .

2.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$  is

- (A) 0                      (B)  $\frac{1}{3}$                       (C) 1                      (D) 3                      (E) nonexistent

3.  $\lim_{x \rightarrow -7} \frac{x+7}{49-x^2}$  is

- (A)  $-7$       (B)  $-\frac{1}{14}$       (C)  $-\frac{1}{49}$       (D)  $\frac{1}{14}$       (E)  $49$

4.  $\lim_{x \rightarrow 0} \frac{x}{\tan x}$  is

- (A)  $-1$       (B)  $0$       (C)  $1$       (D)  $\pi$       (E) nonexistent

5.  $\lim_{x \rightarrow \infty} \frac{3x^4 - 8x^3 + x^2 - 10}{2x^2 - 5x^3 - 10x^4}$  is

(A)  $-\frac{3}{10}$

(B) 0

(C)  $\frac{3}{10}$

(D)  $\frac{3}{2}$

(E) nonexistent

6. The horizontal asymptote of  $f(x) = \frac{3x^3 + x^2 - 4}{8 - 5x^3}$  is

(A)  $y = -\frac{5}{3}$

(B)  $y = -\frac{3}{5}$

(C)  $y = \frac{3}{8}$

(D)  $y = \frac{3}{5}$

(E)  $y = \frac{5}{3}$

7.  $\lim_{x \rightarrow \pi/2} \tan 2x$  is

- (A)  $-1$       (B)  $-\frac{1}{2}$       (C)  $0$       (D)  $1$       (E)  $\pi$

8. Function  $g$  is discontinuous at  $x = 5$ . Selected values of  $g$  are shown in the table below.

$x$	4.99	4.999	5	5.0001	5.001
$f(x)$	2.99	2.999	$-4$	3.001	3.01

A reasonable estimate for  $\lim_{x \rightarrow 5} g(x)$  is

- (A)  $-4$       (B)  $-3$       (C)  $2$       (D)  $3$       (E)  $5$

9. If  $f(x) = \begin{cases} 3x - 2 \cos x & x < \pi \\ x^2 & x \geq \pi, \end{cases}$  then  $\lim_{x \rightarrow \pi^-} f(x)$  is

- (A)  $-\pi^2$       (B)  $3\pi$       (C)  $3\pi - 2$       (D)  $3\pi + 2$       (E)  $\pi^2$

10.  $\lim_{x \rightarrow 3} \frac{\sqrt{x-2} - 1}{9-3x}$  is

- (A)  $-\frac{1}{2}$       (B)  $-\frac{1}{6}$       (C)  $\frac{1}{6}$       (D)  $\frac{1}{2}$       (E) nonexistent

11. Given that  $\lim_{x \rightarrow a} f(x)$  exists, which statements must be true?

I.  $f(x)$  is continuous at  $x = a$ .

II.  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ .

III.  $f(a)$  is defined.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

12. If  $\lim_{k \rightarrow 0} \frac{e^k - 1}{k} = 1$ , then  $\lim_{k \rightarrow 0} \frac{e^2 - e^{2+k}}{k}$  is

(A)  $-e^2$

(B)  $-e^{-2}$

(C)  $e^{-2}$

(D) 1

(E)  $e^2$

13.  $g(x) = \frac{x^2 - 5x + 6}{x - 3}$  has a removable discontinuity at

- (A)  $x = -3$       (B)  $x = -2$       (C)  $x = 0$       (D)  $x = 2$       (E)  $x = 3$

14.  $\lim_{x \rightarrow \infty} \frac{\sin(2x)}{x + 2}$  is

- (A)  $-1$       (B)  $0$       (C)  $1$       (D)  $2$       (E)  $\infty$



15. Let  $f(x) = \begin{cases} 2 - kx & x \leq 3 \\ kx^2 - 22 & x > 3. \end{cases}$  For what value of  $k$  is  $f$  continuous at  $x = 3$ ?

- (A)  $-4$       (B)  $0$       (C)  $1$       (D)  $2$       (E)  $3$

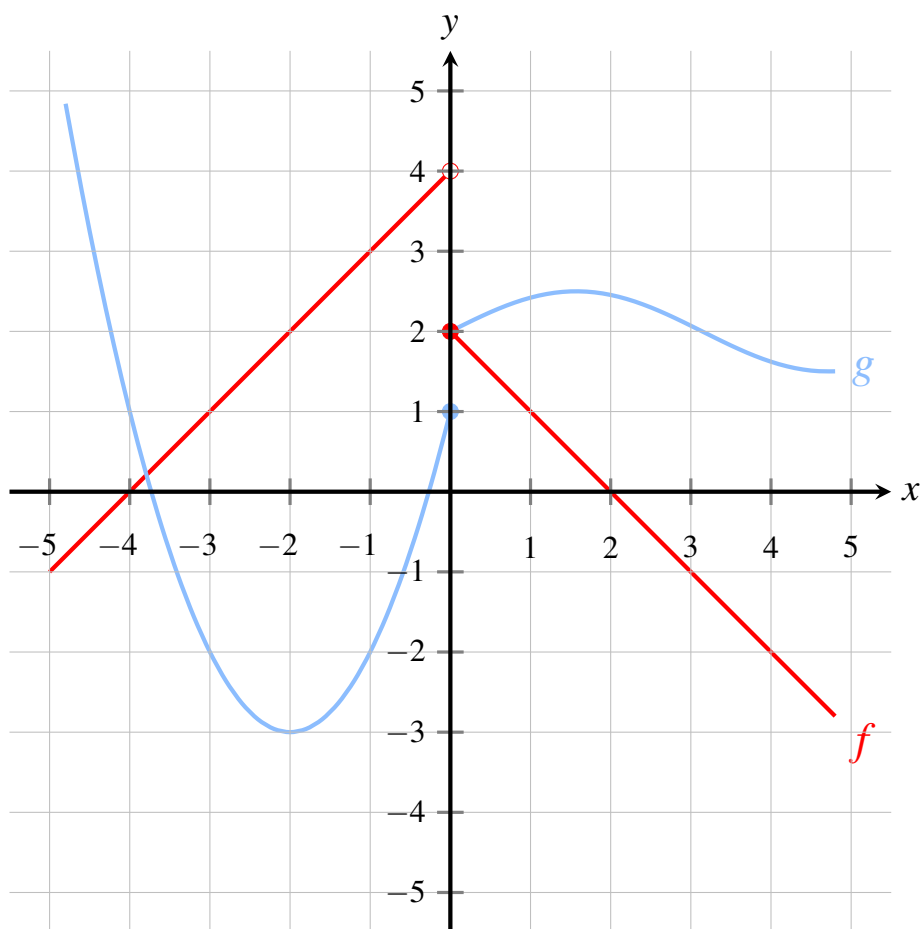
16. The oblique asymptote of  $f(x) = \frac{x^2 + 7x + 1}{x - 2}$  is

- (A)  $y = 9$       (B)  $y = x - 9$       (C)  $y = x$       (D)  $y = x + 5$       (E)  $y = x + 9$

17.  $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 1}}{x + 3}$  is

- (A)  $-4$       (B)  $-2$       (C)  $2$       (D)  $4$       (E) nonexistent

Questions 18–23 refer to the following graph.



18.  $\lim_{x \rightarrow 0} f(x)$  is

- (A) 0                      (B) 1                      (C) 2                      (D) 4                      (E) nonexistent

19.  $\lim_{x \rightarrow 0^+} g(x)$  is

- (A) 0                      (B) 1                      (C) 2                      (D) 4                      (E) nonexistent

20.  $\lim_{x \rightarrow -2} [f(x) - g(x)]$  is

- (A) -5                      (B) -1                      (C) 1                      (D) 5                      (E) nonexistent

21.  $\lim_{x \rightarrow -2} [g(x)]^2$  is

(A)  $-4$

(B)  $-2$

(C)  $2$

(D)  $4$

(E)  $9$

22.  $\lim_{x \rightarrow 0} [f(x)g(x)]$  is

(A)  $1$

(B)  $2$

(C)  $4$

(D)  $8$

(E) nonexistent

23.  $\lim_{x \rightarrow 0} f(-x^2)$  is

(A)  $0$

(B)  $1$

(C)  $2$

(D)  $4$

(E) nonexistent

24. On what interval of  $x$  is  $f(x) = \frac{\ln(x-2)}{x^2-9}$  continuous?

(A)  $(-\infty, -3) \cup (3, \infty)$

(B)  $(-\infty, -2)$

(C)  $(2, \infty)$

(D)  $(3, \infty)$

(E)  $(2, 3) \cup (3, \infty)$

25.  $\lim_{x \rightarrow -3} \frac{5 \sin(x+3)}{6+2x}$  is

(A) 0                      (B)  $\frac{5}{6}$                       (C) 1                      (D)  $\frac{5}{2}$                       (E) nonexistent

26. Function  $f$  is continuous and satisfies  $f(4) = 8$ . If  $\lim_{x \rightarrow 2} g(x) = 4$ , then  $\lim_{x \rightarrow 2} f(g(x))$  is

- (A)  $-8$       (B)  $-4$       (C)  $2$       (D)  $4$       (E)  $8$

27.  $\lim_{x \rightarrow 2} \frac{\frac{1}{2} - \frac{1}{x}}{2 - x}$  is

- (A)  $-\frac{1}{2}$       (B)  $-\frac{1}{4}$       (C)  $\frac{1}{4}$       (D)  $\frac{1}{2}$       (E) nonexistent

28.  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x^2}\right)$  is

- (A) 0                      (B)  $\frac{1}{4}$                       (C)  $\frac{1}{2}$                       (D) 1                      (E) nonexistent

29. Let  $f(x) = \begin{cases} x^2 + 1 & x < p \\ 2x & x \geq p \end{cases}$ . If  $f(x)$  is continuous at  $x = p$ , then  $p$  is

- (A) -1                      (B) 0                      (C) 1                      (D) 2                      (E) 4

30.  $\lim_{t \rightarrow \infty} \sin t$  is

- (A)  $-1$       (B)  $0$       (C)  $1$       (D)  $\pi$       (E) nonexistent

31. If  $\lim_{x \rightarrow 2} f(x) = -3$ , then  $\lim_{x \rightarrow 2} ([f(x)]^2 - 2x)$  is

- (A)  $-7$       (B)  $-3$       (C)  $4$       (D)  $5$       (E)  $9$



32. Function  $f$  is continuous. Selected values of  $f(x)$  are shown in the table below.

$x$	$-1$	$2$	$3$	$6$	$11$
$f(x)$	$2$	$1$	$1$	$1$	$2$

Following the Intermediate Value Theorem, which value of  $f(x)$  is guaranteed to exist for  $-1 \leq x \leq 11$ ?

- (A) 0                      (B) 3                      (C) 5                      (D) 6                      (E) 11

33.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 - 4x^2 + 1}}{3x^3 + 2}$  is

- (A)  $-\infty$                       (B)  $-\frac{2}{3}$                       (C)  $\frac{2}{3}$                       (D)  $\infty$                       (E) nonexistent

34.  $\lim_{x \rightarrow \infty} x \sin \left( \frac{1}{x} \right)$  is

- (A)  $-1$               (B)  $0$               (C)  $1$               (D)  $\pi$               (E) nonexistent

35. At  $x = 4$ , which choice about  $g(x) = \frac{12 + x - x^2}{x - 4}$  is true?

- (A)  $g(x)$  has a vertical asymptote at  $x = 4$ .  
(B)  $g(x)$  has a jump discontinuity at  $x = 4$ .  
(C)  $g(x)$  has a removeable discontinuity at  $x = 4$ .  
(D)  $\lim_{x \rightarrow 4} g(x)$  does not exist.  
(E)  $g(x)$  is continuous at  $x = 4$ .

36.  $\lim_{x \rightarrow 0^+} \ln(\sin x)$  is

- (A)  $-\infty$       (B) 0      (C) 1      (D)  $e$       (E)  $\infty$

37.  $\lim_{x \rightarrow 0} \frac{x^2}{\sin^2 2x}$  is

- (A) 0      (B)  $\frac{1}{4}$       (C) 1      (D) 4      (E) nonexistent

38.  $\lim_{x \rightarrow 0} \frac{x - x \cos(x)}{x^2}$  is

- (A)  $-\pi$       (B) 0      (C) 1      (D)  $\pi$       (E) nonexistent

39.  $\lim_{x \rightarrow 0^+} \arctan\left(\frac{1}{x}\right)$  is

- (A)  $-\infty$       (B)  $-\frac{\pi}{2}$       (C) 0      (D)  $\frac{\pi}{2}$       (E)  $\infty$

40.  $\lim_{x \rightarrow \infty} \frac{5e^x - x}{8e^x + 9}$  is

(A)  $-\frac{5}{8}$

(B) 0

(C)  $\frac{5}{8}$

(D) 1

(E)  $\infty$

41.  $\lim_{x \rightarrow \pi/4} \frac{\cos 2x}{\cos x - \sin x}$  is

(A) 0

(B)  $\frac{\sqrt{2}}{2}$

(C)  $\sqrt{2}$

(D)  $\pi$

(E) nonexistent

42. Functions  $g$  and  $h$  are continuous and satisfy  $g(1) = h(1) = 3$ . Function  $f$  satisfies  $g(x) \leq f(x) \leq h(x)$  for  $0 \leq x \leq 2$ . Which statements must be true?

I.  $\lim_{x \rightarrow 1} g(x) = \lim_{x \rightarrow 1} h(x) = 3$ .

II.  $\lim_{x \rightarrow 1} f(x) = 3$ .

III.  $f(x)$  is continuous at  $x = 1$ .

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

43. If  $f(1) = 2$ ,  $\lim_{x \rightarrow 1^-} f(x) = 4$ , and  $\lim_{x \rightarrow 1^+} f(x) = -1$ , then  $\lim_{x \rightarrow 1} f(\cos(x - 1))$  is

(A)  $-1$

(B)  $0$

(C)  $2$

(D)  $4$

(E) nonexistent

44. If  $\lim_{x \rightarrow 3} (2x + 4) = 10$ , then  $|(2x + 4) - 10| < \varepsilon$  and  $|x - 3| < \delta$ , where  $\delta =$

- (A)  $\frac{\varepsilon}{4}$       (B)  $\frac{\varepsilon}{2}$       (C)  $\varepsilon$       (D)  $2\varepsilon$       (E)  $4\varepsilon$

45. If  $\lim_{x \rightarrow a} f(x) = \infty$ , then which option is true?

- (A) For positive  $M$ , there exists a positive  $\delta$  such that  $f(x) > M$  for  $|x - a| > 0$ .
- (B) For positive  $M$ , there exists a positive  $\delta$  such that  $f(x) < M$  for  $|x - a| > \delta$ .
- (C) For positive  $M$ , there exists a positive  $\delta$  such that  $f(x) > M$  for  $|x - a| > \delta$ .
- (D) For positive  $M$ , there exists a positive  $\delta$  such that  $f(x) < M$  for  $0 < |x - a| < \delta$ .
- (E) For positive  $M$ , there exists a positive  $\delta$  such that  $f(x) > M$  for  $0 < |x - a| < \delta$ .

*This marks the end of the review exercises. The following page contains the answers to all the questions.*



- |       |       |
|-------|-------|
| 1. B  | 34. C |
| 2. D  | 35. C |
| 3. B  | 36. A |
| 4. C  | 37. B |
| 5. A  | 38. B |
| 6. B  | 39. D |
| 7. C  | 40. C |
| 8. D  | 41. C |
| 9. D  | 42. E |
| 10. B | 43. D |
| 11. B | 44. B |
| 12. A | 45. E |
| 13. E |       |
| 14. B |       |
| 15. D |       |
| 16. E |       |
| 17. C |       |
| 18. E |       |
| 19. C |       |
| 20. D |       |
| 21. E |       |
| 22. C |       |
| 23. D |       |
| 24. E |       |
| 25. D |       |
| 26. E |       |
| 27. B |       |
| 28. A |       |
| 29. A |       |
| 30. E |       |
| 31. D |       |
| 32. A |       |
| 33. A |       |