VALCALC

Chapter 8 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 8: Differential Equations.

8.1: Slope Fields and Euler's Method
8.2: Separation of Variables
8.3: Exponential Models
8.4: Logistic Models
8.5: First-Order Linear Differential Equations

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Differential Equations

Number of Questions—50

NO CALCULATOR

1. Which differential equations are first-order?

I. $16xy\dfrac{dy}{dx} = \cos y$

II. $6\dfrac{dy}{dx} = x - \dfrac{d^2y}{dx^2}$

III. $\left(\dfrac{dy}{dx}\right)^4 + y^7 = \dfrac{3}{x^2}$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

2. Which situation is best modeled by a logistic function?

(A) The concentration of a reactant in a first-order chemical reaction

(B) The temperature of a cup of hot coffee as it cools

(C) The mass of carbon-14 in a decomposing organism

(D) The number of views on a social media post

(E) The balance in a bank account compounded continuously

3. Which differential equation is *not* separable?

(A) $\frac{dy}{dx} + x \sin y = 8x$

(B) $\frac{dy}{dx} \sin x - 12y = 0$

(C) $y^3 \frac{dy}{dx} + x^4 = 5x$

(D) $\frac{dy}{dx} = \frac{y^6 + 6y}{2x^4 + x + 3}$

(E) $\frac{dy}{dx} - 7x = \cos y$

4. The general solution to $\dfrac{dy}{dx} = 2x^4\sqrt{y}$ is

(A) $y = \dfrac{x^5}{5} + C$

(B) $y = \left(\dfrac{x^5}{5} + C\right)^2$

(C) $y = \left(\dfrac{x^5}{5} + C\right)^4$

(D) $y = \sqrt{\dfrac{x^5}{5} + C}$

(E) $y = \sqrt[4]{\dfrac{x^5}{5} + C}$

5. A function $y = f(x)$ satisfies the differential equation $\dfrac{dy}{dx} = x^4 + 2y^6$ and initial condition $f(2) = 5$.

Using Euler's Method with which step size leads to the most accurate approximation of $f(5)$?

(A) $h = 0.1$ (B) $h = 0.5$ (C) $h = 1$ (D) $h = 2$ (E) $h = 3$

6. Which choice lists the correct properties of $y = \dfrac{600}{1+5e^{-0.4x}}$?

(A) It represents logistic growth with a carrying capacity of 100.

(B) It represents logistic growth with a carrying capacity of 120.

(C) It represents logistic growth with a carrying capacity of 600.

(D) It represents logistic decay with a carrying capacity of 100.

(E) It represents logistic decay with a carrying capacity of 600.

7. Which differential equation yields a slope field that has *no* horizontal line segments?

(A) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -8$

(B) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x+2$

(C) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos^2 x$

(D) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4y$

(E) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 y^2$

8. The rate of change of a function is directly proportional to the square of itself and inversely proportional to the cube of its input. Which differential equation models the function?

(A) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = ky^2x^3$

(B) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{ky^2}{x^3}$

(C) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{kx^3}{y^2}$

(D) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\sqrt{y}\,\sqrt[3]{x}$

(E) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k\sqrt{y}}{\sqrt[3]{x}}$

9. The carrying capacity of the logistic model given by $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 3y(200-4y)$ is

(A) 0 (B) 50 (C) 100 (D) 200 (E) 800

10. Which functions are solutions to the differential equation $y'' - 6y' + 8y = 4x$?

I. $y = e^{2x}$

II. $y = -6e^{2x} + \dfrac{x}{2} + \dfrac{3}{8}$

III. $y = 2e^{2x} + e^{4x} + \dfrac{x}{2} + \dfrac{3}{8}$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

11. A function f grows at a rate proportional to half of itself and satisfies $f(0) = 40$. The function's identity is

(A) $f(x) = 40e^{x/2}$

(B) $f(x) = 40e^{-x/2}$

(C) $f(x) = 40e^{x}$

(D) $f(x) = 40e^{2x}$

(E) $f(x) = 40e^{-2x}$

12. The solution to the initial value problem $\frac{dy}{dx} = \frac{4x}{y^4}$, $y(1) = 1$, is

(A) $y = \sqrt[4]{4x - 3}$

(B) $y = \sqrt[4]{2x^2 - 1}$

(C) $y = \sqrt[5]{20x - 19}$

(D) $y = \sqrt[5]{2x^2 - 1}$

(E) $y = \sqrt[5]{10x^2 - 9}$

13. A principal amount of \$500 is deposited into a bank account with a 2% annual interest rate, compounded four times per year. After 3 years, the balance is

(A) $500(1.005)^3$

(B) $500(1.005)^{12}$

(C) $500(1.02)^3$

(D) $500(1.02)^{12}$

(E) $500(1.06)^4$

14. Which statement about logistic functions is *false*?

(A) There are two horizontal asymptotes to the graph of a logistic function.

(B) A logistic function increases or decreases most rapidly at its inflection point.

(C) A logistic function has at least one critical point.

(D) The graph of a logistic function is concave down above its inflection point.

(E) The graph of a logistic function is concave up below its inflection point.

15. Which family of curves is an orthogonal trajectory to $y = k\sqrt{x}$?

(A) $y = C\sqrt{x}$

(B) $y = x^2 + C$

(C) $y = -x^2 + C$

(D) $y^2 = 2x^2 + C$

(E) $y^2 = -2x^2 + C$

16. A function f grows exponentially with x such that $f(1) = 60$ and $f(3) = 540$. The identity of f is

(A) $f(x) = 20(3)^x$

(B) $f(x) = 20(9)^x$

(C) $f(x) = 60(3)^x$

(D) $f(x) = 60(9)^x$

(E) $f(x) = 540(9)^x$

17. The solution to the initial value problem $\frac{dy}{dt} = 6 - 5y$, $y(3) = 1$, is

(A) $y = \frac{6}{5} + \frac{1}{5}e^{5t-15}$

(B) $y = \frac{6}{5} + \frac{1}{5}e^{-5t+15}$

(C) $y = \frac{6}{5} - \frac{1}{5}e^{5t-15}$

(D) $y = \frac{6}{5} - \frac{1}{5}e^{-5t+15}$

(E) $y = \frac{6}{5} - \frac{1}{5}e^{-5t-15}$

18. A disease begins to spread at a school with 2000 students. The rate at which students are infected is directly proportional to the number of students already infected and to those not yet infected. The disease spreads fastest when the number of students infected is

(A) 250 (B) 500 (C) 1000 (D) 1250 (E) 1500

19. The solution to the initial value problem $y' \cos y = 5x^4 + 1$, $y(1) = 0$, is

(A) $\sin y = 5x^4 - 5$

(B) $\sin y = x^5 + x$

(C) $\sin y = x^5 + x - 2$

(D) $\cos y = x^5 + x$

(E) $\cos y = x^5 + x - 2$

20. If $\dfrac{dy}{dx} = 2x + 4y$ and $y(0) = 1$, then Euler's Method with a step size of $h = 0.5$ estimates $y(1)$ to be

(A) 5 (B) 9.5 (C) 11 (D) 13 (E) 21

21. The doubling time of $y = 6^x$ is

(A) $\ln 2$ (B) $\ln 6$ (C) $\dfrac{1}{\ln 2}$ (D) $\dfrac{\ln 2}{\ln 6}$ (E) $\dfrac{\ln 6}{\ln 2}$

22. The general solution to the differential equation $y' + y = 8x$ is

(A) $y = Ce^{-x} - 8x$

(B) $y = Ce^{-x} + 8x + 8$

(C) $y = Ce^{-x} + 8x - 8$

(D) $y = Ce^{-x} - 8x + 8$

(E) $y = Ce^{-x} - 8x - 8$

23. A chemical compound has a half-life of 300 years. The time, in years, for a sample of the compound to decay to 30% of its initial amount is

(A) $300\log_{1/2} 0.3$

(B) $300\log_{1/2} 0.7$

(C) $300\ln 0.3$

(D) $300\ln 0.7$

(E) $300\ln 300$

24. The integrating factor necessary to solve the differential equation $2xy' - 8x^4y = 9x$ is

(A) e^{-8x} (B) e^{8x} (C) $e^{-2x^5/5}$ (D) e^{-x^4} (E) e^{x^4}

25. For the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 3y - 1$ and initial value $y(1) = 2$, Euler's Method with step size $h = 2$ approximates $y(5)$ to be

(A) 39 (B) 41 (C) 53 (D) 122 (E) 164

26. If $\frac{\mathrm{d}y}{\mathrm{d}x} x^4 \ln 3y = y$, then

(A) $\frac{1}{6}(\ln 3y)^2 = \frac{1}{x^4} + C$

(B) $\frac{1}{6}(\ln 3y)^2 = -\frac{1}{x^4} + C$

(C) $\frac{1}{6}(\ln 3y)^2 = -\frac{1}{3x^3} + C$

(D) $\frac{3}{2}(\ln 3y)^2 = \frac{1}{x^4} + C$

(E) $\frac{3}{2}(\ln 3y)^2 = -\frac{1}{3x^3} + C$

27. A logistic decay function satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -50y(500 - y)$ and initial condition $y(0) = 400$. Its identity is

(A) $y = \dfrac{400}{1 - \frac{1}{4}e^{-0.1x}}$

(B) $y = \dfrac{500}{1 + \frac{1}{4}e^{-0.1x}}$

(C) $y = \dfrac{500}{1 + \frac{1}{4}e^{0.1x}}$

(D) $y = \dfrac{500}{1 - \frac{1}{4}e^{0.1x}}$

(E) $y = \dfrac{500}{1 - \frac{1}{4}e^{-0.1x}}$

28. A principal amount of \$600 is compounded continuously at a rate of 3% per year. After how many years does the balance grow to \$1800 ?

(A) $\dfrac{\log_{1.03} 3}{0.03}$ (B) $\dfrac{\log_{1.03} 9}{0.03}$ (C) $\dfrac{\ln 3}{0.03}$ (D) $\dfrac{\ln 6}{0.03}$ (E) $\dfrac{\ln 9}{0.03}$

29. The general solution to $y' = x^2y - 3y + x^2 - 3$ is

(A) $y = Ce^{\frac{1}{3}x^3 - 3x}$

(B) $y = Ce^{\frac{1}{3}x^3 - 3x} + 1$

(C) $y = Ce^{\frac{1}{3}x^3 - 3x} - 1$

(D) $y = Ce^{\frac{1}{3}x^3 - 3x - 1}$

(E) $y = Ce^{\frac{1}{3}x^3 - 3x - 1} - 1$

30. For $y \geqslant 0$, which family of functions is an orthogonal trajectory to $y^6 = k\sin x$?

(A) $y = \dfrac{C}{(\sin x)^{1/6}}$

(B) $y = \dfrac{1}{(C\sin x)^{1/6}}$

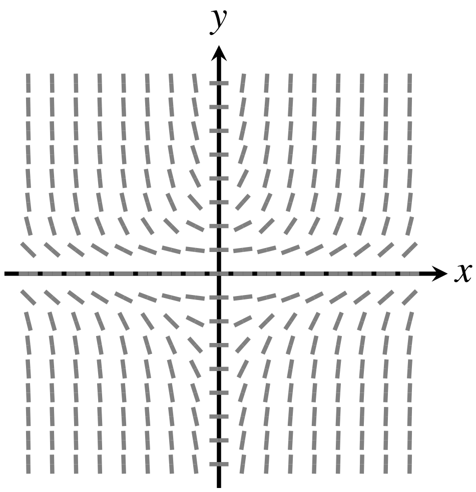
(C) $y = \sqrt{C + 12\ln|\sec x|}$

(D) $y = \sqrt{C - 12\ln|\sec x|}$

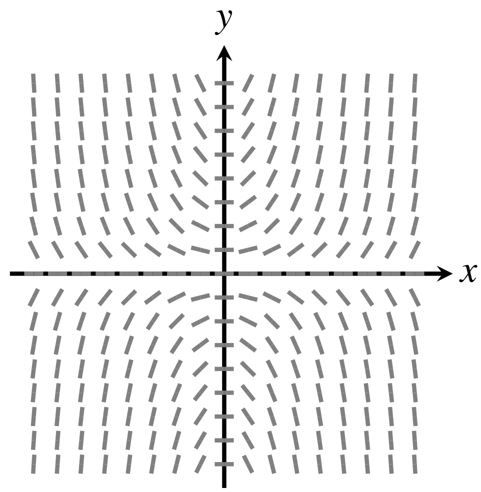
(E) $y = C + \sqrt{12\ln|\sec x|}$

31. Which choice is the slope field of $\dfrac{dy}{dx} = xy^2$?

(A)



(B)



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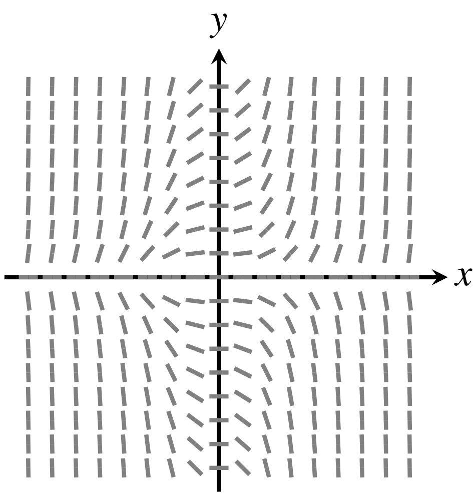

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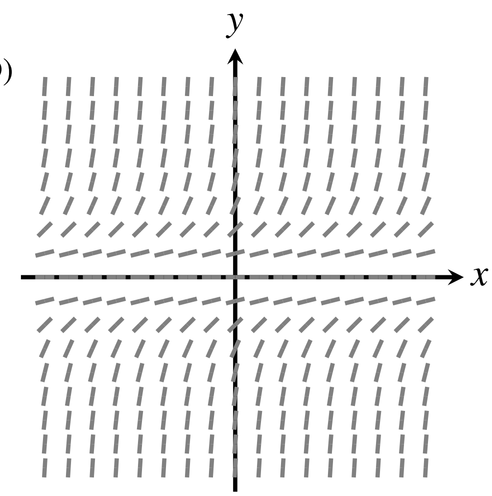

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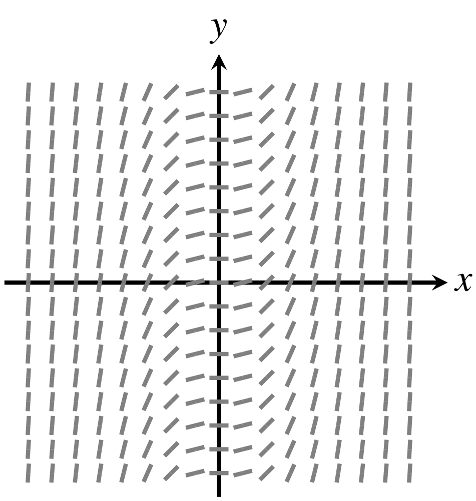


32. What is the general solution to $y' + \frac{5}{x}y = \ln x$?

(A) $y = x^5 \ln x + C$

(B) $y = -x^5 \ln x + C$

(C) $y = \ln x + \frac{C}{x^5}$

(D) $y = \frac{x}{6}\ln x + \frac{x}{36} + \frac{C}{x^5}$

(E) $y = \frac{x}{6}\ln x - \frac{x}{36} + \frac{C}{x^5}$

33. A tank initially contains 50 kilograms of salt dissolved in 1000 liters of water. Brine with a concentration of 0.2 kilogram of salt per liter is then pumped into the tank at a rate of 10 liters per hour, during which the tank is mixed and drained to maintain a constant volume of 1000 liters. The mass of salt, in kilograms, in the tank after 2 hours is

(A) $20+\dfrac{30}{e^{1/500}}$

(B) $20+\dfrac{30}{e^{1/50}}$

(C) $200-\dfrac{150}{e^{1/500}}$

(D) $200-\dfrac{150}{e^{1/50}}$

(E) $2000-\dfrac{1950}{e^{1/500}}$

34. In the first-order chemical reaction A $\longrightarrow$ B, the initial concentration of reactant A is 60 grams. After 2 minutes, the concentration of A decreases to 20 grams. The integrated rate law for the reaction is

(A) $[A] = 60\left(\frac{1}{\sqrt{2}}\right)^t$

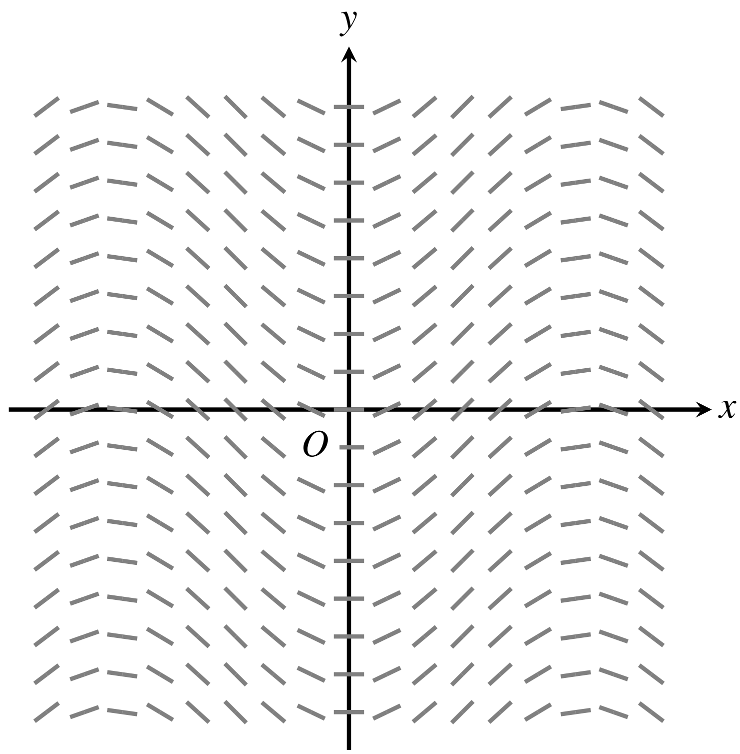
(B) $[A] = 60\left(\frac{1}{\sqrt{3}}\right)^t$

(C) $[A] = 60\left(\frac{1}{2}\right)^t$

(D) $[A] = 60\left(\frac{1}{3}\right)^t$

(E) $[A] = 60\left(\frac{1}{9}\right)^t$

35. Which differential equation yields the following slope field?



(A) $\dfrac{dy}{dx} = -\sin x$

(B) $\dfrac{dy}{dx} = -\cos x$

(C) $\dfrac{dy}{dx} = \sin x$

(D) $\dfrac{dy}{dx} = \cos x$

(E) $\dfrac{dy}{dx} = 1 + \sin x$

36. What is the solution to the initial value problem

$$t^3\frac{\mathrm{d}y}{\mathrm{d}t} - 3t^2y = t^6\sec^2 t\,, \quad y(\pi) = 2\ ?$$

(A) $y = t^3\tan t$

(B) $y = t^3\tan t + 2$

(C) $y = t^3 + t^3\tan t$

(D) $y = \frac{2}{\pi^3}t^3 + \tan t$

(E) $y = \frac{2}{\pi^3}t^3 + t^3\tan t$

37. An ecologist discovers an initial population of 50 rhinoceros and concludes that the environment contains enough resources to support 500 rhinoceros. Two years later, the population has grown to 100 rhinoceros. Which logistic growth function models the number of rhinoceros, N, as a function of time in years, t, after the population was discovered?

(A) $N(t) = \dfrac{500}{1+9\left(\frac{2}{3}\right)^t}$

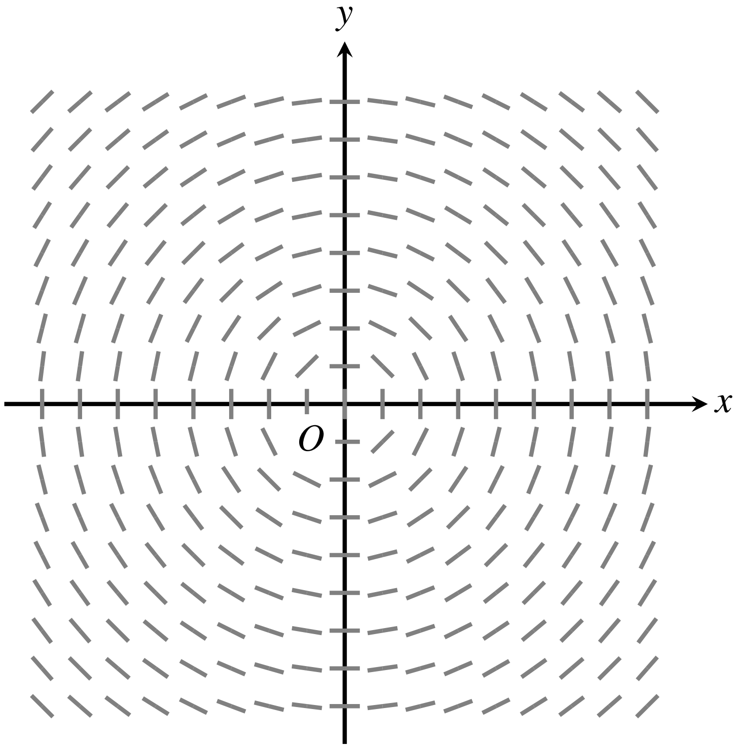
(B) $N(t) = \dfrac{500}{1+9\left(\frac{3}{2}\right)^t}$

(C) $N(t) = \dfrac{500}{1-9\left(\frac{3}{2}\right)^t}$

(D) $N(t) = \dfrac{500}{1+9\left(\frac{4}{9}\right)^t}$

(E) $N(t) = \dfrac{500}{1+9\left(\frac{9}{4}\right)^t}$

38. Which family of curves is the solution to the differential equation with the following slope field?



(A) $x = Cy^2$

(B) $y = C - x^2$

(C) $x^2 - y^2 = C^2$

(D) $y^2 - x^2 = C^2$

(E) $x^2 + y^2 = C^2$

39. As water flows out of a tank, the water level decreases at a rate proportional to the square root of itself. The initial water level is 25 feet, and the water level after 1 minute is 16 feet. The water level, in feet, after 2 minutes is

(A) 1 (B) 3 (C) 4 (D) 6 (E) 9

40. The differential equation $\frac{dv}{dt} = 9.8 - 0.02v^2$ models the velocity v (in meters per second) of an object in free fall under the influence of a drag force. The object is stationary at $t = 0$, where t is time in seconds. Which statements are true?

I. The object's terminal velocity (the maximum velocity it attains) is $\sqrt{490}$ meters per second.

II. The object speeds up and then slows down.

III. The object's initial acceleration is 9.8 meters per second squared.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

41. A capacitor with a capacitance of 5×10^{-6} farad initially holds a charge of $\dfrac{1}{1000}$ coulomb and is connected to an appliance with a resistance of 2000 ohms. After $\dfrac{1}{200}$ second, the charge remaining on the capacitor, in coulombs, is

(A) $\dfrac{e^{-1/200}}{1000}$ (B) $\dfrac{e^{1/2}}{1000}$ (C) $\dfrac{e^{-1/2}}{1000}$ (D) $\dfrac{e^{2}}{1000}$ (E) $\dfrac{e^{-2}}{1000}$

42. An object's acceleration a is related to its velocity v by the equation $a = -\dfrac{1}{2}v^2$. If its initial velocity is 4, then its velocity at $t = 1$ is

(A) -4 (B) $-\dfrac{4}{3}$ (C) $-\dfrac{1}{3}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{4}{3}$

43. The solution to the initial value problem $y' + 8x^3y = x^7$, $y(0) = \frac{1}{4}$, is

(A) $y = x^7 + \frac{1}{4}e^{-2x^4}$

(B) $y = x^7 - \frac{1}{4}e^{-2x^4}$

(C) $y = \frac{5}{16}e^{-2x^4} + \frac{x^4}{8} - \frac{1}{16}$

(D) $y = \frac{5}{16}e^{-2x^4} - \frac{x^4}{8} + \frac{1}{16}$

(E) $y = \frac{5}{16}e^{-2x^4} - \frac{x^4}{8} - \frac{1}{16}$

44. A freshly baked pastry is removed from the oven to cool in a 60° F room. The pastry's initial temperature is 300° F; two minutes later, the pastry cools down to 180° F. By Newton's Law of Cooling, what is the pastry's temperature 4 minutes after it is removed from the oven?

(A) 80° F (B) 90° F (C) 100° F (D) 120° F (E) 140° F

45. What is the general solution to $y' + xy^2 = x$?

(A) $y = -1 + \dfrac{2}{Ce^{x^2} - 1}$

(B) $y = 1 + \dfrac{2}{Ce^{x^2} + 1}$

(C) $y = 1 + \dfrac{2}{Ce^{x^2} - 1}$

(D) $y = 1 - \dfrac{2}{Ce^{x^2} + 1}$

(E) $y = 1 - \dfrac{2}{Ce^{x^2} - 1}$

46. Which family of curves is mutually orthogonal to $\cos^2 y = kx^3$?

(A) $\sin y = Ce^{x^2/3}$

(B) $\sin y = Ce^{-x^2/3}$

(C) $\cos y = Ce^{x^2/3}$

(D) $\cos y = Ce^{-x^2/3}$

(E) $\cos y = Ce^{-x^2/6}$

47. The general solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{y}{x} = \cos 2x$ is

(A) $y = \dfrac{C}{x} + \cos 2x$

(B) $y = \dfrac{C}{x} + \dfrac{1}{2}\sin 2x + \dfrac{1}{4x}\cos 2x$

(C) $y = \dfrac{C}{x} + \dfrac{1}{2}\sin 2x - \dfrac{1}{4x}\cos 2x$

(D) $y = \dfrac{C}{x} - \dfrac{1}{2}\sin 2x + \dfrac{1}{4x}\cos 2x$

(E) $y = \dfrac{C}{x} - \dfrac{1}{2}\sin 2x - \dfrac{1}{4x}\cos 2x$

48. A tank contains 50 liters of water. A solution whose salt concentration is 0.5 kilogram per liter is pumped into the tank at a rate of 4 liters per minute. The solution is kept mixed and drained at a rate of 3 liters per minute. Which function models the mass of salt, y (in kilograms), as a function of time, t (in minutes)?

(A) $y = \frac{t}{2} - 25 - \frac{1}{(t+50)^3}$

(B) $y = \frac{t}{2} - 25 + \frac{1}{2(t+50)^3}$

(C) $y = \frac{t}{2} - 25 - \frac{1}{2(t+50)^3}$

(D) $y = \frac{t}{2} + 25 + \frac{50^4}{2(t+50)^3}$

(E) $y = \frac{t}{2} + 25 - \frac{50^4}{2(t+50)^3}$

49. A function $y = f(x)$ satisfies the differential equation $y' = y^2 - 4$. If $f(0) = 0$, then

(A) $\lim_{x\to-\infty} f(x) = -2$ and $\lim_{x\to\infty} f(x) = 0$

(B) $\lim_{x\to-\infty} f(x) = -2$ and $\lim_{x\to\infty} f(x) = 2$

(C) $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 2$

(D) $\lim_{x\to-\infty} f(x) = 2$ and $\lim_{x\to\infty} f(x) = -2$

(E) $\lim_{x\to-\infty} f(x) = 2$ and $\lim_{x\to\infty} f(x) = 2$

50. A tank contains 8 kilograms of salt dissolved in 2000 liters of water. A solution whose salt concentration is 0.5 kilogram of salt per liter is pumped into the tank; at the same time, the tank is mixed and drained to maintain a constant volume of 2000 liters. If $y(t)$ is the mass (in kilograms) of salt in the tank as a function of time, then $\lim_{t\to\infty} y(t)$ is

(A) 0 (B) 250 (C) 500 (D) 1000 (E) 2000

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. D
2. D
3. E
4. B
5. A
6. C
7. A
8. B
9. B
10. D
11. A
12. E
13. B
14. C
15. E
16. A
17. D
18. C
19. C
20. B
21. D
22. C
23. A
24. D
25. D
26. E
27. C
28. C
29. C
30. D
31. A
32. E
33. D
34. B
35. C
36. E
37. A
38. E
39. E
40. D
41. C
42. E
43. C
44. D
45. C
46. A
47. B
48. E
49. D
50. D