Chapter 3 Multiple-Choice Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 3: Applications of Differentiation.

3.1: Minimum and Maximum Values
3.2: Mean Value Theorem and Rolle's Theorem
3.3: Using the First and Second Derivatives
3.4: Particle Motion
3.5: Indeterminate Forms and L'Hopital's Rule
3.6: Optimization
3.7: Applications of Differentiation in Economics
3.8: Newton's Method

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Applications of Differentiation

Number of Questions—50

NO CALCULATOR

1. Suppose that $f(x)$ satisfies the Mean Value Theorem over $a \leqslant x \leqslant b$. Which of the following statements must be true?

I. $f(x)$ is continuous over $a \leqslant x \leqslant b$.

II. $f(x)$ is differentiable over $a < x < b$.

III. A value c exists in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

(A) I only

(B) III only

(C) I and II only

(D) I and III only

(E) I, II, and III

2. The function $f(x) = x^2 - 4x + 3$ has a critical point at

(A) $x = -2$ (B) $x = 0$ (C) $x = 1$ (D) $x = 2$ (E) $x = 4$

3. If $f'(x) = (x-6)(x+9)$, then f is increasing for x in

(A) $(-\infty, -9] \cup [6, \infty)$ only

(B) $(-\infty, -9]$ only

(C) $[-9, 6]$ only

(D) $[-6, 9]$ only

(E) $[6, \infty)$ only

4. Suppose that $g'(x) > 0$ for $x < 4$ and $g'(x) < 0$ for $x > 4$. By the First-Derivative Test,

(A) g has a relative minimum at $x = 4$.

(B) g has a relative maximum at $x = 4$.

(C) g is decreasing at $x = 4$.

(D) g is increasing at $x = 4$.

(E) g has an inflection point at $x = 4$.

5. If the twice-differentiable function g satisfies $g'(a) = 0$ and $g''(a) = 6$, then the Second-Derivative Test says

(A) g is increasing at $x = a$.

(B) g is decreasing at $x = a$.

(C) g has an inflection point at $x = a$.

(D) g has a relative maximum at $x = a$.

(E) g has a relative minimum at $x = a$.

6. If $f'(x) = (x-3)(x+7)$, then f has a relative minimum at

(A) $x = -7$ (B) $x = -3$ (C) $x = -2$ (D) $x = 3$ (E) $x = 7$

7. If $f'(c) = 0$, then which of the following must be true?

I. The graph of f has a horizontal tangent at c.

II. The graph of f has a critical point at c.

III. f has an extremum at c.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

8. $\lim_{x\to\infty} \frac{\ln x}{2x} =$

(A) $-\infty$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) ∞

9. The Mean Value Theorem is satisfied for $y = 2x^2$ on $0 \leqslant x \leqslant 3$ if

(A) $c = 0$ (B) $c = \sqrt{3}$ (C) $c = \sqrt{6}$ (D) $c = 3$ (E) $c = 6$

10. Rolle's Theorem is satisfied for $y = 4 - x^2$ on $-2 \leqslant x \leqslant 2$ if

(A) $c = -2$ (B) $c = -\sqrt{2}$ (C) $c = 0$ (D) $c = \sqrt{2}$ (E) $c = 2$

11. The function $y = 3\sin x$ satisfies Rolle's Theorem on $0 \leqslant x \leqslant \pi$ if

(A) $c = 0$ (B) $c = \dfrac{\pi}{4}$ (C) $c = \dfrac{\pi}{2}$ (D) $c = \pi$ (E) $c = 3$

12. Which function does *not* have a critical point at $x = 0$?

(A) $y = \dfrac{1}{x}$

(B) $y = x^2$

(C) $y = \sqrt[3]{x}$

(D) $y = \cos x$

(E) $y = 3x^4 - 5x^3 + 7x^2$

13. The graph of $x^3 - 3x^2 - 9x$ is concave up for

(A) $x < 1$

(B) $-1 < x < 3$

(C) $-1 < x < 0$

(D) $x > 1$

(E) $x > 3$

14. Let $h'(x) = x^4 - 5\ln x$. The graph of h has an inflection point at

(A) $x = \sqrt[4]{\frac{1}{4}}$ (B) $x = \sqrt[4]{\frac{5}{4}}$ (C) $x = \sqrt[3]{\frac{1}{4}}$ (D) $x = \sqrt[3]{\frac{5}{4}}$ (E) $x = \frac{5}{4}$

15. If $g''(x) = (x-2)(x+1)$, then the graph of g is concave down for x in

(A) $(-\infty, -1) \cup (2, \infty)$

(B) $(-\infty, -1)$

(C) $(-2, 1)$

(D) $(-1, 2)$

(E) $(2, \infty)$

16. If the graph of f is concave up over (a,b), then which of the following must be true?

I. All tangents to the graph of f on (a,b) lie below the graph.

II. The slopes of the tangents to f on (a,b) are increasing.

III. The function f' is increasing on $[a,b]$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

17. Which expression is *not* an indeterminate limit form?

(A) $\infty \times \infty$ (B) $\infty - \infty$ (C) $0 \times \infty$ (D) 0^0 (E) 1^∞

18. The function $y = 4\sin^2 x + 8\cos x$, $-2\pi < x < 2\pi$, has critical points when

(A) $x = 0$ only

(B) $x = -\pi$ and $x = \pi$ only

(C) $x = -\dfrac{2\pi}{3}$ and $x = \dfrac{2\pi}{3}$ only

(D) $x = -\dfrac{2\pi}{3}$, $x = 0$, and $x = \dfrac{2\pi}{3}$ only

(E) $x = -\pi$, $x = 0$, and $x = \pi$ only

19. The function $x^3 - 3x^2 + 1$ has a relative minimum when

(A) $x = -3$ (B) $x = -1$ (C) $x = 0$ (D) $x = 1$ (E) $x = 2$

20. Let f be a second-differentiable function such that $f'(a) < 0$ and $f''(a) > 0$. Which choice best describes the behavior of $f(x)$ when $x = a$?

(A) $f(x)$ is decreasing and concave down at $x = a$.

(B) $f(x)$ is decreasing and concave up at $x = a$.

(C) $f(x)$ is increasing and concave down at $x = a$.

(D) $f(x)$ is increasing and concave up at $x = a$.

(E) It cannot be determined.

21. Suppose that g is a second-differentiable function such that $g'(c) > 0$ and $g''(c) > 0$. Which choice best describes the behavior of $g(x)$ when $x = c$?

(A) $g(x)$ is increasing and concave up when $x = c$.

(B) $g(x)$ is increasing and concave down when $x = c$.

(C) $g(x)$ is decreasing and concave up when $x = c$.

(D) $g(x)$ is decreasing and concave down when $x = c$.

(E) It cannot be determined.

22. If $y = 2x^2$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$, then $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 3$ is

(A) 2 (B) 9 (C) 12 (D) 24 (E) 36

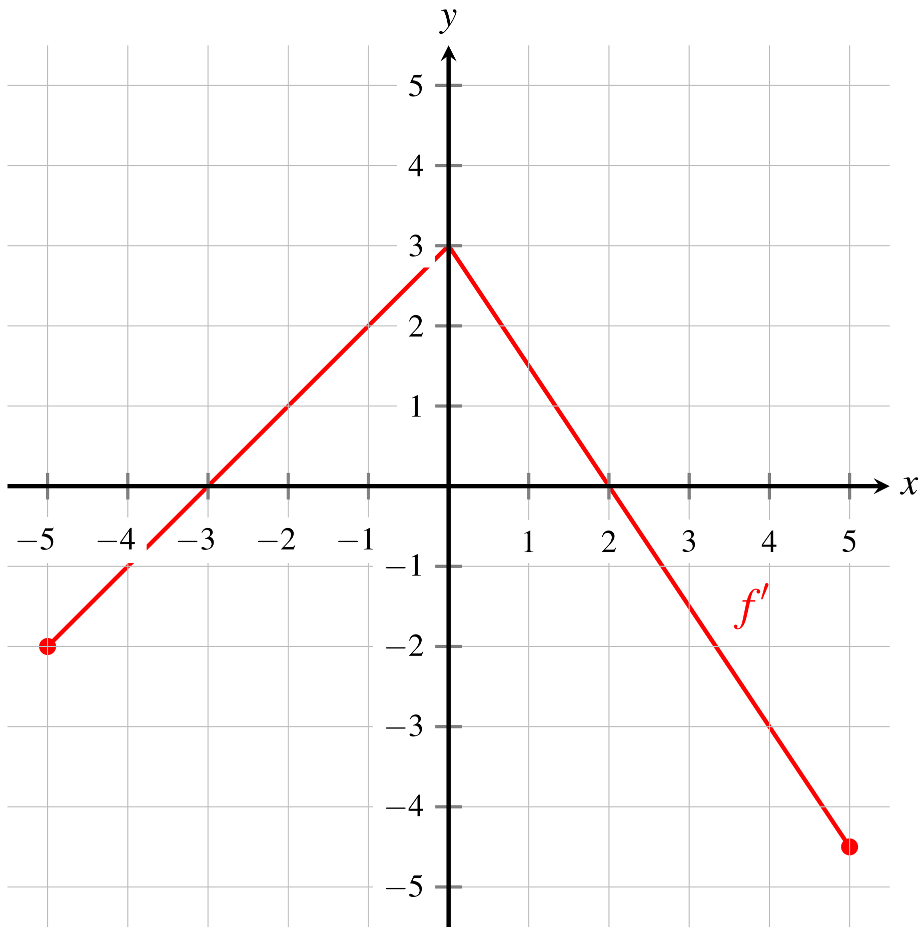
23. Two numbers sum to 50. Their maximum possible product is

(A) 25 (B) 50 (C) 100 (D) 625 (E) 1250

24. Two positive numbers sum to 10. What is the smallest possible sum of their squares?

(A) 5 (B) 10 (C) 25 (D) 50 (E) 100

Questions 25–30 refer to the following graph of f', the derivative of f.



25. The function f is increasing on

(A) $[-5,0]$ (B) $[-3,0]$ (C) $[-3,2]$ (D) $[0,2]$ (E) $[0,5]$

26. $f(x)$ has a relative *maximum* when

(A) $x = -5$ (B) $x = -3$ (C) $x = 0$ (D) $x = 2$ (E) $x = 5$

27. $f(x)$ has a relative *minimum* when

(A) $x = -5$ (B) $x = -3$ (C) $x = 0$ (D) $x = 2$ (E) $x = 5$

28. Where does the graph of $f(x)$ have an inflection point?

(A) $x = -5$

(B) $x = -3$

(C) $x = 0$

(D) $x = 2$

(E) The graph of $f(x)$ has no inflection points.

29. The graph of $f(x)$ is concave down for x in

(A) $(-5, -3)$ (B) $(-5, 0)$ (C) $(0, 2)$ (D) $(0, 5)$ (E) $(2, 5)$

30. Suppose that $f(-1) = 3$ and $f(1) = 6$. According to the graph of f', does the Mean Value Theorem guarantee a value c in $(-1, 1)$ such that $f'(c) = \dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{3}{2}$, and why?

(A) Yes, all the conditions of the Mean Value Theorem on $(-1, 1)$ are satisfied.

(B) No, f may not be continuous on $[-1, 1]$.

(C) No, f may not be differentiable at the endpoints $x = -1$ and $x = 1$.

(D) No, f is not differentiable on $(-1, 1)$ because $f'(0)$ does not exist.

(E) No, f' is not differentiable on $(-1, 1)$.

31. The absolute minimum value of $y = (4x - 4)\sqrt{3x}$ over $0 \leqslant x \leqslant 3$ is

(A) $-\dfrac{8}{3}$ (B) 0 (C) $\dfrac{1}{3}$ (D) 1 (E) 24

32. An object's position along a straight line varies with time $t \geqslant 0$ as modeled by $s(t) = t^3 - 2t^2 - 4t + 3$. The object changes direction when

(A) $t = 0$ (B) $t = \frac{2}{3}$ (C) $t = 1$ (D) $t = 2$ (E) $t = 3$

33. A function g is defined by

$$g(x) = \begin{cases} 3x^2 & x \neq 0 \\ \text{undefined} & x = 0. \end{cases}$$

Which of the following must be true?

I. The absolute maximum of g on $[-2, 2]$ is 12.

II. g has no absolute minimum.

III. g has a critical point when $x = 0$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

34. A car drives along the x-axis; its position as a function of time t is $x(t)$. When $x'(t) < 0$ and $x''(t) > 0$, the car is

(A) slowing down and moving to the left

(B) slowing down and moving to the right

(C) speeding up and moving to the left

(D) speeding up and moving to the right

(E) stationary

35. $\lim\limits_{x \to -\infty} \dfrac{-x}{e^{-x}} =$

(A) $-\infty$ (B) 0 (C) 1 (D) e (E) ∞

36. $\lim_{x\to\infty}(5x - \ln x) =$

(A) $-\infty$ (B) 0 (C) 1 (D) 5 (E) ∞

37. A square's sides are each decreasing at a constant rate of 3 feet per minute. When each side is 4 feet long, how fast is the square's area *decreasing*, in square feet per minute?

(A) 3 (B) 8 (C) 12 (D) 16 (E) 24

38. The area of the largest rectangle that can be inscribed in the semicircle $y = \sqrt{16 - x^2}$ is

(A) $2\sqrt{2}$ (B) 4 (C) $4\sqrt{2}$ (D) 8 (E) 16

39. $\lim\limits_{x \to \infty} xe^{-x} =$

(A) $-\infty$ (B) 0 (C) 1 (D) e (E) ∞

40. $\lim_{x\to\infty} x\ln\left(1+\frac{7}{x}\right) =$

(A) $-\infty$ (B) 0 (C) 1 (D) 7 (E) ∞

41. $\lim_{x\to 0} \frac{\sin 5x}{x^2}$ is

(A) $-\frac{5}{2}$ (B) 0 (C) $\frac{5}{2}$ (D) 5 (E) nonexistent

42. Which choice is true regarding the growth rates of $f(x) = \sqrt[3]{x}$ and $g(x) = \ln x$?

(A) Since $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \infty$, $f(x) = \sqrt[3]{x}$ grows faster than $g(x) = \ln x$.

(B) Since $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \infty$, $g(x) = \ln x$ grows faster than $f(x) = \sqrt[3]{x}$.

(C) Since $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = 0$, $f(x) = \sqrt[3]{x}$ grows faster than $g(x) = \ln x$.

(D) Since $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = 0$, $g(x) = \ln x$ grows faster than $f(x) = \sqrt[3]{x}$.

(E) Since $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)}$ equals a nonzero number, $f(x) = \sqrt[3]{x}$ and $g(x) = \ln x$ grow at the same rate.

43. A hose pumps water into a cylinder at a rate of 7 cubic feet per minute. The circular base of the cylinder has an area of 40 square feet. How quickly is the water level increasing, in feet per minute, when the volume of water is 300 cubic feet?

(A) $\dfrac{7}{40}$ (B) $\dfrac{40}{7}$ (C) $\dfrac{15}{2}$ (D) 40 (E) 280

44. An 18-foot ladder leaning against a wall slides away from the wall at a rate of 2 feet per second. When the top of the ladder is 9 feet above the floor, how fast (in feet per second) is the ladder sliding down the wall?

(A) $\frac{18}{\sqrt{243}}$ (B) $\frac{486}{81}$ (C) $\frac{\sqrt{243}}{9}$ (D) $\frac{2\sqrt{243}}{9}$ (E) $\sqrt{243}$

45. A balloon is inflated at a rate of 10 cubic meters per minute such that it maintains a spherical shape. When the balloon's volume is 36π cubic meters, how fast is its radius increasing (in meters per minute)? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{1}{36\pi}$ (B) $\frac{1}{12\pi}$ (C) $\frac{5}{18\pi}$ (D) 1 (E) 36

46. $\lim\limits_{x \to 6^+} \left(\sqrt[3]{x-6}\right)^{(x-6)} =$

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) e (E) ∞

47. $\lim\limits_{x \to \infty} (e^x + 3x)^{7/x} =$

(A) 1 (B) e (C) 7 (D) e^7 (E) ∞

48. What point on the line $y = 3 - 2x$ is closest to the point $(1, -4)$?

(A) $(0,3)$ (B) $(2,-1)$ (C) $(3,-3)$ (D) $(3.5,-4)$ (E) $(4,-5)$

49. Every month, a store sells 50 keyboards if they are priced at \$30 each. The sell stores 4 fewer keyboards for each \$12 the price is raised. The store's revenue is maximized when each keyboard is sold at a price of

(A) \$25 (B) \$27.50 (C) \$30 (D) \$35 (E) \$42.50

50. A company sells headphones for \$101 each. Its cost to produce x headphones is given by the function $C(x) = x^2 + x + 900$. What is the maximum profit the company can make?

(A) \$0 (B) \$50 (C) \$90 (D) \$1600 (E) \$5050

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. E
2. D
3. A
4. B
5. E
6. D
7. C
8. B
9. B
10. C
11. C
12. A
13. D
14. B
15. D
16. E
17. A
18. E
19. E
20. B
21. A
22. D
23. D
24. D
25. C
26. D
27. B
28. C
29. D
30. A
31. A
32. D
33. C
34. A
35. B
36. E
37. E
38. E
39. B
40. D
41. E
42. A
43. A
44. D
45. C
46. C
47. D
48. C
49. D
50. D