Chapter 2 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 2: Differentiation Rules.

2.1: Defining a Derivative
2.2: Differentiating Power, Exponential, and Sinusoidal Functions
2.3: Product Rule and Quotient Rule
2.4: Chain Rule
2.5: Implicit Differentiation and Differentiating Inverse Functions
2.6: Logarithmic Differentiation
2.7: Linearization and Differentials
2.8: Hyperbolic Functions
2.9: Related Rates

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Differentiation Rules

Number of Questions—50

NO CALCULATOR

1. A tree's height as a function of time is modeled by $h(t)$, where $h(t)$ is measured in feet and t is measured in years since the tree was planted. What is the best interpretation of the equation $h'(5) = 2$?

(A) When the tree is 2 years old, it is 5 feet tall.

(B) When the tree is 2 years old, it is growing at a rate of 2 feet per year.

(C) When the tree is 5 years old, it is 2 feet tall.

(D) When the tree is 5 years old, it is growing at a rate of 2 feet per year.

(E) When the tree is 5 years old, it is growing at a rate of 5 feet per year.

2. If $y=\sin(2x+1)$, then $\dfrac{dy}{dx}=$

(A) $\cos(2x+1)$

(B) $-2\cos(2x+1)$

(C) $2\cos(2x+1)$

(D) $2\sin(2x+1)$

(E) $-2\sin(2x+1)$

3. $\dfrac{d}{dx}\left(\dfrac{2}{x^3}\right)=$

(A) $-\dfrac{8}{x^4}$ (B) $-\dfrac{6}{x^4}$ (C) $-\dfrac{6}{x^2}$ (D) $\dfrac{6}{x^2}$ (E) $\dfrac{6}{x^4}$

4. If $f(x) = x^2 \ln x$, then $f'(x) =$

(A) 2

(B) x

(C) $2x\ln x$

(D) $2x\ln x + 2$

(E) $2x\ln x + x$

5. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sec^2 x\right) =$

(A) $-2\sec^2 x\tan x$

(B) $-2\sec^2 x$

(C) $-2\sec x$

(D) $2\sec x$

(E) $2\sec^2 x\tan x$

6. An equation of the line tangent to the graph of f is $y+8=3(x-7)$. Which choice lists the correct properties of f ?

(A) $f(-7)=-8$ and $f'(-7)=3$

(B) $f(-7)=8$ and $f'(-7)=3$

(C) $f(7)=-8$ and $f'(7)=-3$

(D) $f(7)=-8$ and $f'(7)=3$

(E) $f(7)=8$ and $f'(7)=3$

7. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^4\right)=$

(A) 0 (B) $-e^4$ (C) $-4e^3$ (D) $4e^3$ (E) e^4

8. $\dfrac{\mathrm{d}}{\mathrm{d}t}(\log_8 t)\Big|_{t=8} =$

(A) $\dfrac{1}{8\ln 8}$ (B) $\dfrac{1}{\ln 8}$ (C) $\dfrac{1}{\log_8 e}$ (D) $\dfrac{1}{8}$ (E) 1

9. Given that f is differentiable at a, which of the following must be true?

I. f is continuous at a.

II. The graph of $y = f(x)$ does not contain a vertical tangent or sharp turn at $x = a$.

III. $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists.

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

10. If $g(x) = \sqrt{\sin 3x}$, then $g'\left(\frac{\pi}{6}\right)$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{2}$ (E) undefined

11. Which function is *not* differentiable over the set of all real numbers?

(A) $|x+3|$

(B) 6

(C) $\cos 2x$

(D) $5 + \sinh x$

(E) $x^4 - 2x^3 + 6$

12. What is an equation of the line tangent to the graph of $y = e^{2x}$ at $x = 2$?

(A) $y - e^4 = e^4(x-2)$

(B) $y - e^4 = 2e^4(x-2)$

(C) $y - e^4 = 2e^4(x+2)$

(D) $y + e^4 = e^4(x-2)$

(E) $y + e^4 = 2e^4(x+2)$

13. It is known that $f(-1) = -3$ and $f'(-1) = 6$. If $a(x) = x^2 f(x)$, then $a'(-1) =$

(A) -15 (B) -12 (C) -6 (D) 6 (E) 12

14. The line $y = 4x + 8$ is parallel to the line tangent to the graph $y = x^2 - 4x - 4$ when

(A) $x = -3$ (B) $x = -2$ (C) $x = 2$ (D) $x = 4$ (E) $x = 5$

15. For the curve $2x^2 + 3xy - y = 5$, $\dfrac{dy}{dx} =$

(A) $\dfrac{3y - 4x}{1 - 3x}$ (B) $\dfrac{4x + 3y}{3x}$ (C) $\dfrac{4x - 3y}{1 - 3x}$ (D) $\dfrac{4x + 3y}{1 - 3x}$ (E) $\dfrac{4x + 3y}{1 + 3x}$

16. The slope of the line normal to the graph of $y = \sqrt{x}$ at $x = 9$ is

(A) -6 (B) $-\dfrac{1}{6}$ (C) $-\dfrac{1}{3}$ (D) $\dfrac{1}{6}$ (E) 6

17. If $y = \dfrac{1}{t^2+4}$, then $\mathrm{d}y =$

(A) $\dfrac{-2t}{t^2+4}\,\mathrm{d}t$

(B) $\dfrac{-2t}{(t^2+4)^2}\,\mathrm{d}t$

(C) $\dfrac{1}{t^2+4}\,\mathrm{d}t$

(D) $\dfrac{2t}{t^2+4}\,\mathrm{d}t$

(E) $\dfrac{2t}{(t^2+4)^2}\,\mathrm{d}t$

18. $\sinh(\ln 2) =$

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{5}{4}$ (D) 1 (E) 2

19. If $y = \sin^6 5x$, then $\dfrac{dy}{dx} =$

(A) $6\sin^5 5x$

(B) $6\sin^5 5x\cos 5x$

(C) $30\sin^5 5x$

(D) $30\sin^5 5x\cos 5x$

(E) $30\sin^6 5x\cos 5x$

20. A linearization of $\sqrt{x}$ centered at $x = 1$ approximates $\sqrt{1.4}$ to be

(A) 0.6 (B) 0.8 (C) 1 (D) 1.2 (E) 1.4

21. If $v(x) = x^2\sqrt{x}\sin x$, then $v'(x) =$

(A) $\frac{5}{2}x\sqrt{x}\cos x$

(B) $2x\sqrt{x}\cos x$

(C) $-\frac{5}{2}x\sqrt{x}\sin x - x^2\sqrt{x}\cos x$

(D) $\frac{5}{2}x\sqrt{x}\sin x - x^2\sqrt{x}\cos x$

(E) $\frac{5}{2}x\sqrt{x}\sin x + x^2\sqrt{x}\cos x$

22. A cube's side lengths are all 1 inch. By using differentials, how much does the cube's volume (in cubic inches) *decrease* when its side lengths are all shrunk to 0.9 inch?

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.5 (E) 0.6

23. If $y = \sin^{-1}\left(x^2+4\right)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-1}{\sqrt{1-(x^2+4)^2}}$

(B) $\dfrac{1}{\sqrt{1-(x^2+4)^2}}$

(C) $\dfrac{-2x}{\sqrt{1-(x^2+4)^2}}$

(D) $\dfrac{2x}{\sqrt{1-(x^2+4)^2}}$

(E) $\dfrac{2x}{\sqrt{1+(x^2+4)^2}}$

24. $\lim\limits_{h\to 0} \dfrac{\ln(1+h)}{h}$ is

(A) -1 (B) 0 (C) 1 (D) e (E) nonexistent

25. If $y = \tan^{-1} 3x$, then $y'' =$

(A) $\dfrac{-54x}{\sqrt{1-9x^2}}$ (B) $\dfrac{-54x}{9x^2-1}$ (C) $\dfrac{-54x}{(1-9x^2)^2}$ (D) $\dfrac{-54x}{(1+9x^2)^2}$ (E) $\dfrac{54x}{(1+9x^2)^2}$

26. If f and g are inverse functions of each other, with $f(2) = 4$ and $f'(2) = 7$, then $g'(4) =$

(A) $-\dfrac{1}{7}$ (B) $\dfrac{1}{7}$ (C) $\dfrac{1}{4}$ (D) 4 (E) 7

27. If $y = 8^x$, then $y' =$

(A) 8^x (B) $8^x \ln 8$ (C) $8^x \log_8 e$ (D) $e^x \ln 8$ (E) $e^x \log_8 e$

28. $\dfrac{d}{dx}\ln\left(x^2+4x-8\right)=$

(A) $\dfrac{1}{2x+4}$

(B) $\dfrac{1}{x^2+4x-8}$

(C) $\dfrac{-2x-4}{x^2+4x-8}$

(D) $\dfrac{2x+4}{x^2+4x-8}$

(E) $\dfrac{2x+4}{\ln(x^2+4x-8)}$

29. The range of $4\operatorname{sech}2x+5$ is

(A) $(0,4]$ (B) $(0,5]$ (C) $(4,5]$ (D) $(4,9]$ (E) $(5,9]$

30. If $f(x) = \cos x$, then $f^{(66)}(x) =$

(A) $-\cos x$ (B) $-\sin x$ (C) $\cos x$ (D) $\sin x$ (E) $\tan x$

31. If $e^{2x} - \sin y = y^2$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

(A) $\dfrac{e^{2x}}{2y + \cos y}$

(B) $\dfrac{e^{2x}}{2y - \cos y}$

(C) $\dfrac{2e^{2x}}{2y - \cos y}$

(D) $\dfrac{2e^{2x}}{\cos y - 2y}$

(E) $\dfrac{2e^{2x}}{2y + \cos y}$

32. $\lim\limits_{x \to \pi/3} \dfrac{\frac{1}{2} - \cos x}{x - \frac{\pi}{3}}$ is

(A) $-\dfrac{\sqrt{3}}{2}$ (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{2}$ (E) nonexistent

33. $\dfrac{d}{dx}\left(\dfrac{\tan x}{x^2}\right) =$

(A) $\dfrac{\sec^2 x}{2x}$

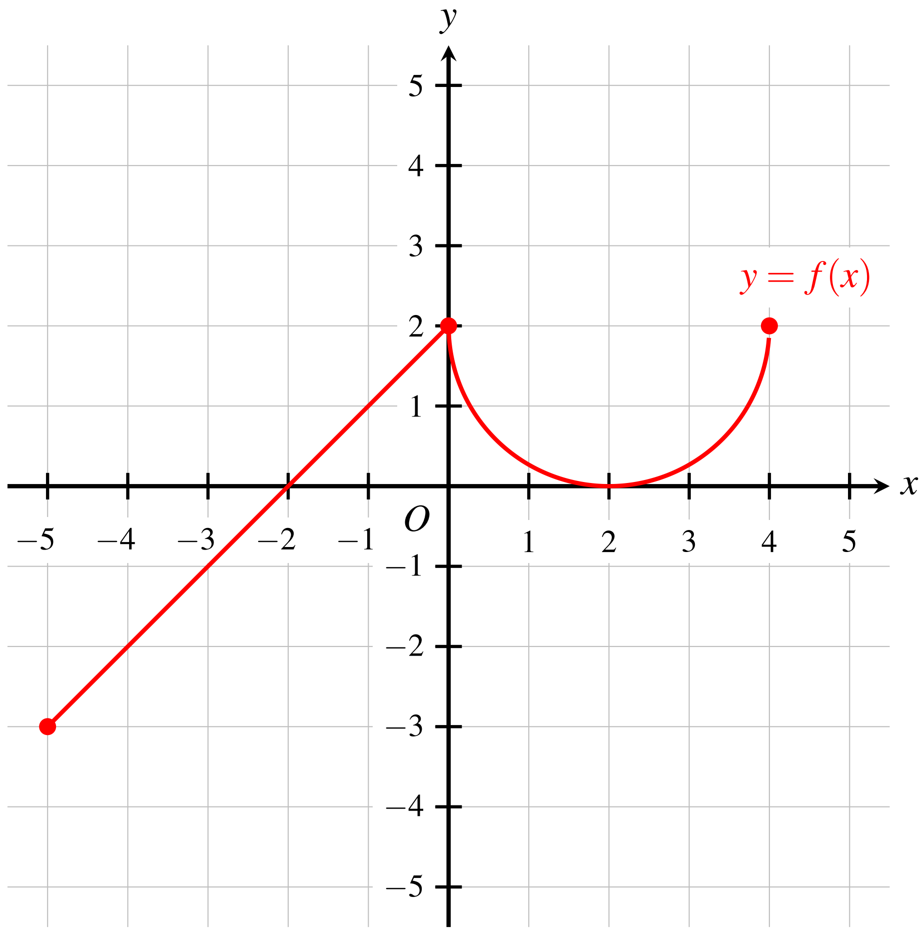
(B) $\dfrac{2x\tan x - x^2\sec^2 x}{x^2}$

(C) $\dfrac{2x\tan x - x^2\sec^2 x}{x^4}$

(D) $\dfrac{x^2\sec^2 x - 2x\tan x}{x^2}$

(E) $\dfrac{x^2\sec^2 x - 2x\tan x}{x^4}$

Questions 34–36 refer to the following graph.



34. At which value of x is f *not* differentiable?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 3

35. $\lim_{x \to -5^+} \dfrac{f(x)+3}{x+5}$ is

(A) -5 (B) -3 (C) 1 (D) 3 (E) nonexistent

36. Let h be a differentiable function such that $h(-2) = 2$ and $h'(-2) = 3$. If $p(x) = \dfrac{f(2x)}{3h(x)}$, then $p'(-2) =$

(A) $-\dfrac{5}{6}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{5}{6}$ (E) $\dfrac{5}{3}$

37. A particle travels along the x-axis with time t according to the function $x(t) = \ln(t+1)$. At $t = 2$, the particle's acceleration is

(A) $-\frac{1}{9}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{27}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

38. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$ is

(A) e^{-2} (B) e^{-1} (C) e (D) e^{2} (E) nonexistent

39. A circle's radius is increased from 10 to 10.1. By using differentials, the change in the circle's area is approximately

(A) 2π (B) 4π (C) 10π (D) 20π (E) 40π

40. At which values of x does $y = \frac{1}{3}x^3 + 2x^2 - 5x + 6$ have a horizontal tangent?

I. $x = -5$.

II. $x = -1$.

III. $x = 1$.

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

41. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\sin^2 x}{e^{4x}}\right) =$

(A) $\sin 2x - 4\sin^2 x$

(B) $\dfrac{\sin 2x + 4\sin^2 x}{e^{4x}}$

(C) $\dfrac{\sin 2x - 4\sin^2 x}{e^{4x}}$

(D) $\dfrac{\sin 2x + 4\sin^2 x}{e^{8x}}$

(E) $\dfrac{\sin 2x - 4\sin^2 x}{e^{8x}}$

42. $\dfrac{\mathrm{d}}{\mathrm{d}x}\tanh^{-1}(\cos x) =$

(A) $-\csc x$ (B) $-\sec x$ (C) $\cos x$ (D) $\csc x$ (E) $\sec x$

43. For the curve $y^3 - y = x + 8$, $\dfrac{d^2y}{dx^2} =$

(A) $-\dfrac{6y}{(3y^2-1)^2}$

(B) $\dfrac{6y}{(3y^2-1)^2}$

(C) $-\dfrac{6y}{(3y^2-1)^3}$

(D) $\dfrac{6y}{(3y^2-1)^3}$

(E) $-\dfrac{6y}{(3y^2-1)^4}$

44. $\dfrac{d}{dx}\left(x^{\sec x}\right) =$

(A) $(\sec x)x^{\sec x - 1}$

(B) $(\sec x)x^{\sec x}$

(C) $(\sec x \tan x)\, x^{\sec x}$

(D) $\left(\dfrac{\sec x}{x} - \sec x \tan x \ln x\right) x^{\sec x}$

(E) $\left(\dfrac{\sec x}{x} + \sec x \tan x \ln x\right) x^{\sec x}$

45. The curve $2x^3y + 16x^2y^2 + 1 = 0$ has a vertical tangent when

I. $x = -2$.

II. $x = 0$.

III. $x = 2$.

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II, and III

46. If $y = \sin^4\left(\sqrt{x}\right)$, then $y' =$

(A) $4\sin^3\left(\sqrt{x}\right)$

(B) $4\sin^3\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$

(C) $\dfrac{2\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}$

(D) $\dfrac{2\sin^3\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)}{\sqrt{x}}$

(E) $\dfrac{2\cos^3\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}$

47. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^4 e^{2x}\sec x\right) =$

(A) $4x^3e^{2x}\sec x + x^4e^{2x}\sec x + x^4e^{2x}\sec x$

(B) $4x^3e^{2x}\sec x + 2x^4e^{2x}\sec x + x^4e^{2x}\sec x$

(C) $4x^3e^{2x}\sec x + x^4e^{2x}\sec x + x^4e^{2x}\sec x\tan x$

(D) $4x^3e^{2x}\sec x + 2x^4e^{2x}\sec x + x^4e^{2x}\sec x\tan x$

(E) $x^4e^{2x}\sec x + 2x^4e^{2x}\sec x + x^4e^{2x}\sec x$

48. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{e^{\sin x}\tan x}{x^5\sin 5x}\right) =$

(A) $\dfrac{e^{2\sin x}\tan x + e^{\sin x}\tan x\sec^2 x - 5x^4 e^{\sin x} - 5e^{\sin x}\cos 5x}{x^5\sin 5x}$

(B) $\dfrac{5x^4 e^{\sin x} + 5e^{\sin x}\cos 5x - e^{2\sin x}\tan x - e^{\sin x}\tan x\sec^2 x}{x^5\sin 5x}$

(C) $\dfrac{xe^{\sin x}\sin x + xe^{\sin x}\sec^2 x - 5e^{\sin x}\tan x - 5xe^{\sin x}\tan x\cot 5x}{x^6\sin 5x}$

(D) $\dfrac{5e^{\sin x}\tan x + 5xe^{\sin x}\tan x\cot 5x - xe^{\sin x}\sin x - xe^{\sin x}\sec^2 x}{x^6\sin 5x}$

(E) $\dfrac{e^{2\sin x}\tan x + e^{\sin x}\tan x\sec^2 x - 5x^4 e^{\sin x} - 5e^{\sin x}\cos 5x}{x^{10}\sin^2 5x}$

49. A projectile's height, in feet, above the ground as a function of time, in seconds, is given by the function $h(t) = -16t^2 - 64t + 80$. The projectile's speed, in feet per second, upon striking the ground is

(A) 32 (B) 64 (C) 96 (D) 128 (E) 192

50. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{\ln(\ln(\ln x))}\right) =$

(A) $\dfrac{-1}{\ln(\ln x)[\ln(\ln(\ln x))]^2}$

(B) $\dfrac{1}{x(\ln x)\ln(\ln x)\ln(\ln(\ln x))}$

(C) $\dfrac{-1}{x(\ln x)\ln(\ln x)\ln(\ln(\ln x))}$

(D) $\dfrac{1}{x(\ln x)\ln(\ln x)[\ln(\ln(\ln x))]^2}$

(E) $\dfrac{-1}{x(\ln x)\ln(\ln x)[\ln(\ln(\ln x))]^2}$

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. D
2. C
3. B
4. E
5. E
6. D
7. A
8. A
9. E
10. B
11. A
12. B
13. E
14. D
15. D
16. A
17. B
18. B
19. D
20. D
21. E
22. C
23. D
24. C
25. D
26. B
27. B
28. D
29. E
30. A
31. E
32. B
33. E
34. C
35. C
36. D
37. A
38. D
39. A
40. C
41. C
42. A
43. C
44. E
45. C
46. D
47. D
48. C
49. C
50. E