VALCALC

Chapter 5 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 5: Applications of Integration.

- **5.1**: Areas between Curves
- **5.2**: Volumes with Cross Sections
- **5.3**: Solids of Revolution
- **5.4**: Shell Method
- **5.5**: Work
- **5.6**: Average Value of a Function

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Applications of Integration

Number of Questions—50

NO CALCULATOR

1. The area bounded by $y = 3 - x^2$ and $y = 2x$ is

(A) $\frac{5}{3}$ (B) $\frac{8}{3}$ (C) $\frac{32}{3}$ (D) 8 (E) 9

2. Which of the following quantities can be negative?

I. The area between curves

II. A function's average value

III. The work done by a force

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

3. The average value of $f(x) = e^x + \cos 3x$ over $0 \leqslant x \leqslant 2$ is

(A) $\dfrac{e^2}{2} + \dfrac{\sin 6}{3}$

(B) $e^2 - 1 + \dfrac{\cos 6 - 1}{3}$

(C) $\dfrac{e^2 - 1}{2} + \dfrac{\cos 6 - 1}{6}$

(D) $e^2 - 1 + \dfrac{\sin 6}{3}$

(E) $\dfrac{e^2 - 1}{2} + \dfrac{\sin 6}{6}$

4. The force along the x-axis is given by $F(x) = x^3 + 6x^2 + 8$. How much work is done by the force in moving an object from $x = 1$ to $x = 4$?

(A) $\dfrac{41}{4}$ (B) $\dfrac{393}{4}$ (C) $\dfrac{855}{4}$ (D) 88 (E) 224

5. The region enclosed by $y = x^3$, the x-axis, the y-axis, and the line $x = 1$ is revolved around the x-axis to generate a solid of volume

(A) $\frac{\pi}{14}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{7}$ (D) $\frac{\pi}{4}$ (E) $\frac{3\pi}{5}$

6. A force of 200 pounds is required to stretch a spring from its natural length of 1 foot to an elongated length of 5 feet. The work, in foot-pounds, done to stretch the spring from 3 feet long to 4 feet long is

(A) 125 (B) 175 (C) 200 (D) 225 (E) 375

7. A sample of gas rests under a piston. The gas initially has a volume of 10 cubic meters and exerts a pressure of 30 newtons per square meter. The gas then doubles in volume. The work, in joules, done by the gas as it expands to push up the piston is

(A) $3\ln 2$ (B) $3\ln 20$ (C) $30\ln 20$ (D) $300\ln 2$ (E) $300\ln 20$

8. A hydraulic press exerts a force that varies with the position according to the function $F(x) = 6x\sqrt{16-x^2}$. The work done by the press for $0 \leqslant x \leqslant 4$ is

(A) 16 (B) 32 (C) 64 (D) 128 (E) 256

9. If f_{avg} is the average value of f on $[-6,4]$, then $\int_{-4}^{6} f(x)\,\mathrm{d}x =$

(A) $-10f_{\text{avg}}$ (B) $-6f_{\text{avg}}$ (C) $-2f_{\text{avg}}$ (D) $6f_{\text{avg}}$ (E) $10f_{\text{avg}}$

10. The area of the region bounded by the curves $y = \sqrt[3]{x}$ and $y = 2\sqrt[3]{x} - 2$ and the line $x = 1$ is

(A) $\frac{11}{8}$ (B) $\frac{11}{4}$ (C) $\frac{11}{2}$ (D) 4 (E) 8

11. At each $x \in [1,5]$, a solid's cross-sectional area is given by $A(x) = \dfrac{5+2\ln x}{x}$. The solid's volume is

(A) $\dfrac{1}{2}(5+2\ln 5)^2 - \dfrac{25}{2}$

(B) $\dfrac{2}{5}\ln 5 - 4$

(C) $\dfrac{1}{4}(5+2\ln 5)^2$

(D) $\dfrac{1}{4}(5+2\ln 5)^2 - \dfrac{25}{4}$

(E) $\dfrac{1}{2}(5+2\ln 5)^2$

12. The area of the region bounded by the line $y = x+4$ and the parabola $x = y^2 - 6$ is

(A) $\dfrac{9}{2}$ (B) $\dfrac{21}{2}$ (C) $\dfrac{10}{3}$ (D) 9 (E) 21

13. The area of the region bounded by the curves $y = 2\sqrt{x}$ and $y = 4\sqrt{x} - 4$ is

(A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) $\frac{16}{3}$ (D) $\frac{32}{3}$ (E) $\frac{64}{3}$

14. The region bounded by the curves $y = e^x$ and $y = 2e^x - 3$ and the y-axis is rotated about the line $x = -2$. Which integral expression gives the volume of the solid generated?

(A) $2\pi \int_0^{\ln 3} x(3 - e^x)\,\mathrm{d}x$

(B) $2\pi \int_0^{\ln 3} (x-2)(3 - e^x)\,\mathrm{d}x$

(C) $2\pi \int_0^{\ln 3} (x-2)(e^x - 3)\,\mathrm{d}x$

(D) $2\pi \int_0^{\ln 3} (x+2)(3 - e^x)\,\mathrm{d}x$

(E) $2\pi \int_0^{\ln 3} (x+2)(e^x - 3)\,\mathrm{d}x$

15. If $f(x) = 4x^3 + 3$, then what value of c satisfies the Mean Value Theorem for Integrals for f on $[-1, 1]$?

(A) 0 (B) $\sqrt[3]{\frac{3}{4}}$ (C) $\sqrt[3]{\frac{3}{2}}$ (D) $\sqrt{\frac{1}{2}}$ (E) $\sqrt{\frac{3}{2}}$

16. The semicircle $y = \sqrt{36 - x^2}$ is the base of a solid such that at each x, cross sections perpendicular to the x-axis are squares. The solid's volume is

(A) 36 (B) 72 (C) 144 (D) 288 (E) 576

17. Which integral gives the volume of the solid generated when the region bounded by $x = 8 - y^2$ and $x = 4$ is rotated about the y-axis?

(A) $\pi \int_4^8 (4 - y^2)^2 \, dy$

(B) $\pi \int_4^8 \left[(8 - y^2)^2 - 16\right] \, dy$

(C) $\pi \int_{-2}^{2} (4 - y^2)^2 \, dy$

(D) $\pi \int_{-2}^{2} [(8 - y^2)^2 + 16] \, dy$

(E) $\pi \int_{-2}^{2} \left[(8 - y^2)^2 - 16\right] \, dy$

18. A square tank has sides of length 4 meters. Water fills the tank to a height of 2 meters. If $\delta = 98000$, then the work, in joules, required to pump out all the water to the top of the tank is

(A) 14δ (B) 28δ (C) 56δ (D) 96δ (E) 192δ

19. The volume of the solid generated upon rotating the region bounded by $y = \dfrac{1}{x^2}$ and the x-axis from $x = 1$ to $x = 2$ about the y-axis is

(A) π (B) $\dfrac{3\pi}{4}$ (C) $\pi \ln 2$ (D) $2\pi \ln 2$ (E) $4\pi \ln 2$

20. A triangle is formed by the x-axis and the lines $y = \dfrac{x}{2}$ and $y = 6 - x$. The solid generated upon rotating this triangle about the y-axis has a volume of

(A) 6π (B) 9π (C) 40π (D) 50π (E) 54π

21. A pond's base is the region bounded by the graph of $y = \cos x$, the x-axis, and the lines $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. At each x, the pond has a height of $2 + \sin x$. The pond's volume is

(A) 1 (B) 2 (C) 4 (D) π (E) 2π

22. What value of c satisfies the Mean Value Theorem for Integrals for $f(x) = 6x^2 - 2x$ over $0 \leqslant x \leqslant 2$?

(A) $\frac{2+\sqrt{76}}{12}$ (B) $\frac{2+\sqrt{148}}{12}$ (C) $\frac{2+\sqrt{292}}{12}$ (D) $\frac{1}{6}$ (E) $\frac{1}{2}$

23. The region bounded by the parabola $y = 6 - x^2$ and the line $y = 2$ is rotated about the line $y = 2$. The volume of the solid generated is

(A) $\frac{256\pi}{15}$ (B) $\frac{512\pi}{15}$ (C) $\frac{232\pi}{5}$ (D) $\frac{384\pi}{5}$ (E) $\frac{464\pi}{5}$

24. Let R be the region bounded by the curve $y = x^3$ and the lines $y = 2 - x$ and $x = y - 6$. The area of R is

(A) $\frac{51}{4}$ (B) $\frac{87}{4}$ (C) $\frac{115}{4}$ (D) 16 (E) 24

25. Region R is enclosed by $x = e^y$, the line $x = 4$, and the x-axis. Which integral equals the volume of the solid generated by rotating R about the line $y = 5$?

(A) $2\pi \int_1^4 (y+5)(4-e^y)\,\mathrm{d}y$

(B) $2\pi \int_1^4 (5-y)(4-e^y)\,\mathrm{d}y$

(C) $2\pi \int_0^{\ln 4} (5-y)(e^y-4)\,\mathrm{d}y$

(D) $2\pi \int_0^{\ln 4} (y+5)(4-e^y)\,\mathrm{d}y$

(E) $2\pi \int_0^{\ln 4} (5-y)(4-e^y)\,\mathrm{d}y$

26. A cylindrical tank has a radius of 5 meters and a height of 10 meters. Water fills the tank to a water level of 3 meters. Take $\delta = 98000$. The work, in joules, needed to pump out all the water to the top of the tank is

(A) $\frac{225\pi}{2}\delta$ (B) $\frac{1225\pi}{2}\delta$ (C) $\frac{1275\pi}{2}\delta$ (D) $\frac{2275\pi}{2}\delta$ (E) $500\pi\delta$

27. The region bounded by the parabola $y = x^2 - 4$ and the x-axis is the base of a solid whose cross sections perpendicular to the y-axis are equilateral triangles. The solid's volume is

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (C) $8\sqrt{3}$ (D) $12\sqrt{3}$ (E) $24\sqrt{3}$

28. A spring has a stiffness of $k = 80$ newtons per meter. If 16000 joules of work is done to stretch the spring, then how far (in meters) is the spring elongated past its natural length?

(A) 10 (B) 20 (C) 40 (D) 200 (E) 400

29. A mountain's height y is modeled by the semicircle $y = \sqrt{25 - x^2}$. Its average height is

(A) $\dfrac{5\pi}{4}$ (B) $\dfrac{5\pi}{8}$ (C) $\dfrac{25\pi}{4}$ (D) $\dfrac{25\pi}{8}$ (E) $\dfrac{25\pi}{16}$

30. A sample of gas initially has a pressure of 20 pounds per square inch and a volume of 30 cubic inches. The gas exerts 20 inch-pounds of work as it expands. Its new volume, in cubic inches, is

(A) $20e^{1/20}$

(B) $20e^{1/30}$

(C) $30e^{1/20}$

(D) $30e^{1/30}$

(E) $600e^{1/600}$

31. The total area enclosed by the graphs of $y = 2x^3 + 1$ and $2x - y + 1 = 0$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) $\frac{7}{2}$

32. A chain weighs 120 pounds and is 40 feet long. If the chain is dangling off a mountain, then how much work (in foot-pounds) is required to retract it fully upward?

(A) 600 (B) 1200 (C) 2400 (D) 4800 (E) 21600

33. The average value of $\dfrac{1}{x\ln x}$ on $\left[e, e^2\right]$ is

(A) $\dfrac{1}{e^2-e}$ (B) $\dfrac{\ln 2}{e^2-e}$ (C) $\dfrac{\frac{1}{2e^2}-\frac{1}{e}}{e^2-e}$ (D) e^2-e (E) $(e^2-e)\ln 2$

34. The region bounded by the line $y = 4-x$ in the first quadrant is the base of a solid whose cross sections perpendicular to the x-axis are semicircles. The solid's volume is

(A) $\dfrac{4\pi}{3}$ (B) $\dfrac{8\pi}{3}$ (C) $\dfrac{16\pi}{3}$ (D) $\dfrac{32\pi}{3}$ (E) $\dfrac{64\pi}{3}$

35. The area of the region bounded by $y = 4$, $y = x^2$, and $y = 6 - x$ is

(A) $\frac{25}{2}$ (B) $\frac{25}{3}$ (C) $\frac{32}{3}$ (D) $\frac{61}{6}$ (E) $\frac{125}{6}$

36. The area of the region bounded by the x-axis, the line $x = 4$, and the curve $y = \ln x$ is

(A) $15 - e^4$

(B) $4\ln 4 - 3$

(C) $4\ln 4 - e$

(D) $4\ln 4 + e$

(E) $4\ln 4 - 4 + e$

37. Which integral equals the volume of the solid generated upon rotating the region bounded by $y = \cos x$ and $y = \sin x$, $-\dfrac{3\pi}{4} \leqslant x \leqslant \dfrac{\pi}{4}$, about the line $y = 3$?

(A) $\pi \displaystyle\int_{-3\pi/4}^{\pi/4} \left[(3-\sin x)^2 - (3-\cos x)^2\right] \mathrm{d}x$

(B) $\pi \displaystyle\int_{-3\pi/4}^{\pi/4} \left[(3-\cos x)^2 - (3-\sin x)^2\right] \mathrm{d}x$

(C) $\pi \displaystyle\int_{-3\pi/4}^{\pi/4} (3-\cos x+\sin x)^2 \,\mathrm{d}x$

(D) $\pi \displaystyle\int_{-3\pi/4}^{\pi/4} (3+\cos x-\sin x)^2 \,\mathrm{d}x$

(E) $\pi \displaystyle\int_{-3\pi/4}^{\pi/4} \left[9-(\cos x-\sin x)^2\right] \mathrm{d}x$

38. The region bounded by $y = \sqrt{x}$, $x = 6 - y$, and the x-axis is rotated about the x-axis to generate a solid of volume

(A) $\dfrac{32\pi}{3}$ (B) 18π (C) 72π (D) 80π (E) 144π

39. Region R is enclosed by the parabola $x = 5 - 2y^2$ and the line $x + y = 4$. The volume of the solid generated by rotating R about the line $x = 7$ is

(A) $\frac{9\pi}{8}$ (B) $\frac{63\pi}{10}$ (C) $\frac{73\pi}{15}$ (D) $\frac{43\pi}{30}$ (E) $\frac{143\pi}{30}$

40. Which integral equals the volume of the solid generated upon rotating the region enclosed by $y = 2x$ and $y = 8 - x^2$ about the line $y = -10$?

(A) $\pi \int_{-4}^{2} \left[(2 - x^2)^2 - (10 + 2x)^2\right] \, \mathrm{d}x$

(B) $\pi \int_{-4}^{2} \left[(2 - x^2)^2 - (10 - 2x)^2\right] \, \mathrm{d}x$

(C) $\pi \int_{-4}^{2} \left[(18 - x^2)^2 - (10 - 2x)^2\right] \, \mathrm{d}x$

(D) $\pi \int_{-4}^{2} \left[(18 + x^2)^2 - (10 + 2x)^2\right] \, \mathrm{d}x$

(E) $\pi \int_{-4}^{2} \left[(18 - x^2)^2 - (10 + 2x)^2\right] \, \mathrm{d}x$

41. The region bounded by $x = y^2$ and $y = x - 6$ is rotated about the line $x = -2$ to generate a solid of volume

(A) $\dfrac{385\pi}{6}$ (B) $\dfrac{250\pi}{3}$ (C) $\dfrac{500\pi}{3}$ (D) $\dfrac{415\pi}{2}$ (E) 250π

42. A particle traveling along a straight line has an average velocity of 6 over $2 \leqslant t \leqslant 5$, where t is time. Its displacement over this interval is

(A) 2 (B) 6 (C) 12 (D) 18 (E) 30

43. The region bounded by $y = \dfrac{1}{x^2+1}$, the x-axis, and the lines $x = 1$ and $x = 4$ is the base of a solid. At each x, the solid's height is $2x$. The solid's volume is

(A) $\ln \dfrac{17}{2}$

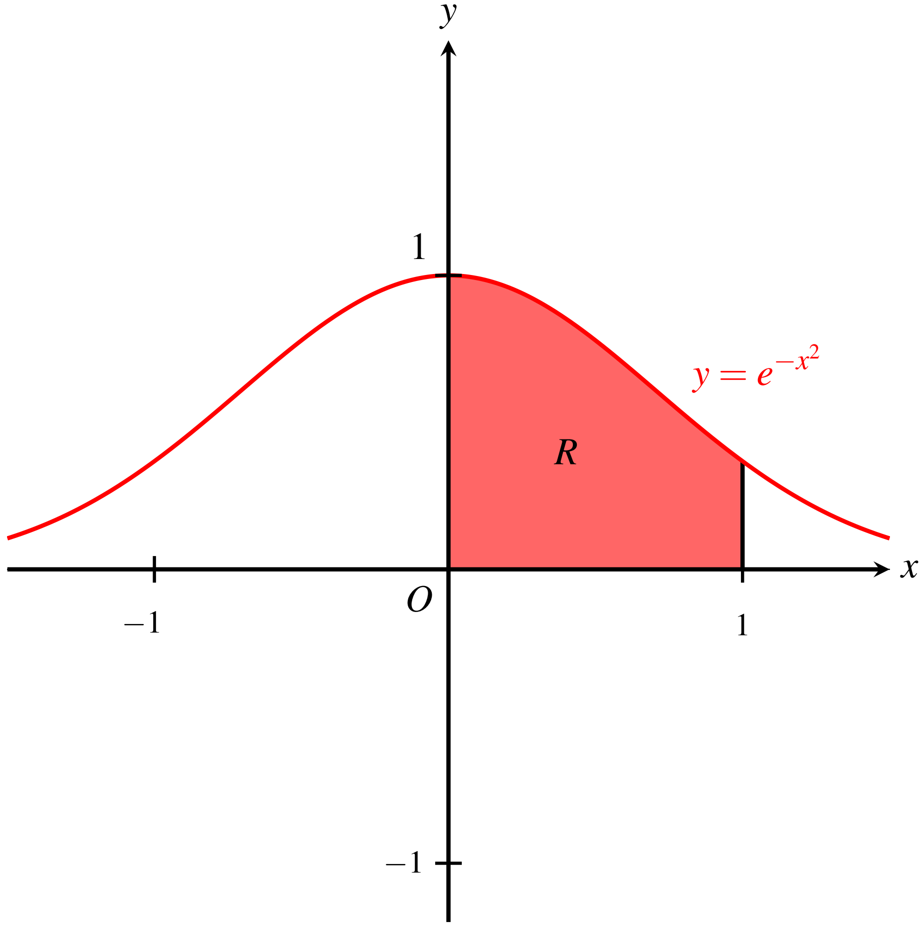
(B) $\ln 4$

(C) $\tan^{-1} 4 - \dfrac{\pi}{4}$

(D) $6\tan^{-1} 4 - \dfrac{3\pi}{2}$

(E) $8\tan^{-1} 4 - 2\pi$

Questions 44–47 refer to the following region.



44. Which integral equals the volume of the solid generated upon rotating R about the x-axis?

(A) $\pi \int_0^1 e^{-x^2}\,\mathrm{d}x$

(B) $\pi \int_0^1 e^{-2x^2}\,\mathrm{d}x$

(C) $\pi \int_0^1 e^{2x^2}\,\mathrm{d}x$

(D) $\pi \int_0^1 e^{-x^4}\,\mathrm{d}x$

(E) $\pi \int_0^1 e^{x^4}\,\mathrm{d}x$

45. The volume of the solid generated upon rotating R about the y-axis is

(A) $\frac{\pi}{2}(e-1)$

(B) $\pi(e-1)$

(C) $\frac{\pi}{2}\left(1-\frac{1}{e}\right)$

(D) $\pi\left(1-\frac{1}{e}\right)$

(E) $\frac{\pi}{e}$

46. A horizontal line $y=K$, where $0<K<1$, divides the region R into two subregions. Which integral gives the area of the top subregion?

(A) $\int_0^K e^{-x^2}\,\mathrm{d}x$

(B) $\int_0^K \left(K-e^{-x^2}\right)\mathrm{d}x$

(C) $\int_0^K \left(e^{-x^2}-K\right)\mathrm{d}x$

(D) $\int_0^{\sqrt{-\ln K}} \left(K-e^{-x^2}\right)\mathrm{d}x$

(E) $\int_0^{\sqrt{-\ln K}} \left(e^{-x^2}-K\right)\mathrm{d}x$

47. Which integral gives the volume of the solid generated by rotating R about the line $y = 1$?

(A) $\pi \int_0^1 \left(1 - e^{-x^2}\right)^2 \, \mathrm{d}x$

(B) $\pi \int_0^1 \left[\left(1 + e^{-x^2}\right)^2 - 1\right] \mathrm{d}x$

(C) $\pi \int_0^1 \left[\left(1 - e^{-x^2}\right)^2 - 1\right] \mathrm{d}x$

(D) $\pi \int_0^1 \left[1 - \left(1 + e^{-x^2}\right)^2\right] \mathrm{d}x$

(E) $\pi \int_0^1 \left[1 - \left(1 - e^{-x^2}\right)^2\right] \mathrm{d}x$

48. On $[3,5]$, f and g are positive functions whose average values are $f_{\text{avg}} = 6$ and $g_{\text{avg}} = 2$. If $f(x) \geqslant g(x)$, then the area bounded by the curves of f and g and the lines $x = 3$ and $x = 5$ is

(A) 2 (B) 3 (C) 4 (D) 8 (E) 16

49. In an inverted conical tank of radius 1 foot and height 2 feet, water is filled up to a height of 1 foot. The work, in foot-pounds, needed to pump out all the water to the top is

(A) $62.5\left(\dfrac{\pi}{12}\right)$

(B) $62.5\left(\dfrac{7\pi}{12}\right)$

(C) $62.5\left(\dfrac{\pi}{3}\right)$

(D) $62.5\left(\dfrac{5\pi}{48}\right)$

(E) $62.5\left(\dfrac{7\pi}{48}\right)$

50. The region enclosed by the curves $y = \cos x$ and $y = \sin x$ and the y-axis in the first quadrant is the base of a solid whose cross sections perpendicular to the x-axis are rectangles. If each rectangle's height is twice the length of its base in the enclosed region, then the solid's volume is

(A) $\frac{\pi}{4} - \frac{1}{2}$

(B) $\frac{\pi}{2} - 1$

(C) $2\left(\sqrt{2} - 1\right)^2$

(D) $4\left(\sqrt{2} - 1\right)^2$

(E) $8\left(\sqrt{2} - 1\right)^2$

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. C
2. D
3. E
4. C
5. C
6. A
7. D
8. D
9. E
10. B
11. D
12. A
13. C
14. D
16. D
17. E
18. D
19. D
20. C
21. C
22. B
23. B
24. A
25. E
26. C
27. C
28. B
29. A
30. D
31. C
32. C
33. B
34. B
35. D
36. B
37. A
38. A
39. B
40. E
41. E
42. D
43. A
44. B
45. D
46. E
47. E
48. D
49. D
50. B