

Chapter 9 Multiple-Choice Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 9: Parametric Equations and Polar Coordinates.

- 9.1: Parametric Equations
- 9.2: Differentiating and Integrating Parametric Functions
- 9.3: Polar Coordinates and Functions
- 9.4: Differentiating Polar Functions
- 9.5: Areas with Polar Curves
- 9.6: Additional Calculus with Parametric and Polar

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Parametric Equations and Polar Coordinates

Number of Questions—50

NO CALCULATOR

1. A point given in polar coordinates by $(6, 300^\circ)$ is expressed in Cartesian coordinates by

(A) $(-3, 3\sqrt{3})$

(B) $(3, 3\sqrt{3})$

(C) $(3, -3\sqrt{3})$

(D) $(3\sqrt{3}, 3)$

(E) $(3\sqrt{3}, -3)$

2. The polar equation for the line $y = 12$ is

(A) $r = 12$

(B) $r = 12 \cos \theta$

(C) $r = 12 \sin \theta$

(D) $r = 12 \sec \theta$

(E) $r = 12 \csc \theta$

3. The polar curve $r = \cos 5\theta$ is

- (A) a limaçon with one loop
- (B) a circle of radius 1
- (C) a circle of radius 4
- (D) a rose with five petals
- (E) a rose with 10 petals

4. If $\omega \neq 0$, then which set of equations parameterizes the ellipse $\frac{x^2}{64} + \frac{y^2}{81} = 1$?

- (A) $x = 8 \sin \omega t, \quad y = 8 \sin \omega t$
- (B) $x = 8 \sin \omega t, \quad y = 8 \cos \omega t$
- (C) $x = 8 \sin \omega t, \quad y = 9 \cos \omega t$
- (D) $x = 9 \sin \omega t, \quad y = 8 \sin \omega t$
- (E) $x = 9 \sin \omega t, \quad y = 9 \cos \omega t$

5. For the polar curve $r = \cos^3 \theta$, $\frac{dx}{d\theta} =$

(A) $-4 \sin \theta \cos^2 \theta$

(B) $-4 \sin \theta \cos^3 \theta$

(C) $4 \sin \theta \cos^3 \theta$

(D) $\cos^4 \theta - 3 \sin^2 \theta \cos^2 \theta$

(E) $\cos^4 \theta + 3 \sin^2 \theta \cos^2 \theta$

6. If $x = t^2 + 4$ and $y = 3t^3 - 6t$, then $\frac{dy}{dx} =$

(A) $9t^2 - 6$ (B) $\frac{9t^2 - 6}{2t}$ (C) $\frac{2t}{9t^2 - 6}$ (D) $\frac{3t^3 - 6t}{t^2 + 4}$ (E) $\frac{t^2 + 4}{3t^3 - 6t}$

7. The line $y = 4x$ is represented in polar coordinates by

(A) $r = 4$

(B) $r = \tan^{-1} 4$

(C) $r = 4\theta$

(D) $\theta = 4$

(E) $\theta = \tan^{-1} 4$

8. If $r = \cot \theta$, then $\frac{d^2y}{d\theta^2} =$

(A) $-\sin \theta$

(B) $-\cos \theta$

(C) $\sin \theta$

(D) $\cos \theta$

(E) $2\csc^2 \theta \cot \theta$

9. If $x = \cos 2t$ and $y = t - 6$, then

(A) $x = \cos(y + 6)$

(B) $x = \cos(y - 6)$

(C) $x = \cos(y - 12)$

(D) $x = \cos(2y + 12)$

(E) $x = \cos(2y - 12)$

10. If $f(\theta) = 1 + 2\cos \theta$, then which statement is true about the polar curve $r = f(\theta)$?

(A) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving toward the pole when $\theta = \frac{\pi}{4}$.

(B) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

(C) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) > 0$, the graph is moving toward the pole when $\theta = \frac{\pi}{4}$.

(D) Since $f\left(\frac{\pi}{4}\right) < 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

(E) Since $f\left(\frac{\pi}{4}\right) < 0$ and $f'\left(\frac{\pi}{4}\right) > 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

11. On a platform 5 feet above the ground, a ball is thrown upward at an angle of 30° above the horizontal with a speed of 40 feet per second. If x is the ball's horizontal position from the platform and y is its height above the ground, then which set of parametric equations represents the ball's motion?

(A) $x = 20t$, $y = -16t^2 + 20t\sqrt{3} + 5$

(B) $x = 20t$, $y = -16t^2 - 20t\sqrt{3} + 5$

(C) $x = 20t\sqrt{3}$, $y = -16t^2 - 20t + 5$

(D) $x = 20t\sqrt{3}$, $y = -16t^2 + 20t + 5$

(E) $x = 40t$, $y = -16t^2 + 40t + 5$

12. Which limaçon has an inner loop?

(A) $r = 2 - \sin \theta$

(B) $r = 1 - \cos \theta$

(C) $r = 6 + 6\cos \theta$

(D) $r = 8 + 4\sin \theta$

(E) $r = 3 - 8\cos \theta$

13. If $x = 3t + 7$ and $y = \sqrt{t^3 - t - 5}$, then

(A) $y = \sqrt{\frac{x^3}{27} - \frac{x}{3} - 5}$

(B) $y = \sqrt{(x-7)^3 - (x-7) - 5}$

(C) $y = \sqrt{(3x+7)^3 - (3x+7) - 5}$

(D) $y = \sqrt{\frac{(x+7)^3}{27} - \frac{x+7}{3} - 5}$

(E) $y = \sqrt{\frac{(x-7)^3}{27} - \frac{x-7}{3} - 5}$

14. For the polar curve $r = 3 + \cos \theta$, $\frac{dy}{dx} =$

(A) $\frac{-3 \cos \theta - \cos 2\theta}{3 \sin \theta + \sin 2\theta}$

(B) $\frac{-3 \cos \theta - \cos 2\theta}{3 \cos \theta + \cos^2 \theta}$

(C) $\frac{3 \cos \theta + \cos^2 \theta}{3 \sin \theta + \sin \theta \cos \theta}$

(D) $\frac{3 \sin \theta + \sin 2\theta}{-3 \cos \theta - \cos 2\theta}$

(E) $\frac{3 \sin \theta + \sin \theta \cos \theta}{3 \cos \theta + \cos^2 \theta}$

15. A curve is parameterized by the equations $x = \sqrt{t+3}$ and $y = 3t^2 - 8t + 2$. When $x = 2$, $\frac{dy}{dx} =$

- (A) -8 (B) $-\frac{3}{2}$ (C) $-\frac{2}{\sqrt{5}}$ (D) 4 (E) $8\sqrt{5}$

16. What is the length of the curve parameterized by $x = \frac{2}{3}t^{3/2} + 8$ and $y = 2t^{3/2} - 4$ on the interval $0 \leq t \leq 2$?

- (A) $\frac{2}{3}\sqrt{10}$ (B) $\frac{2}{3}\sqrt{80}$ (C) $\frac{8}{3}\sqrt{8}$ (D) $\frac{8}{3}\sqrt{10}$ (E) $\frac{8}{3}\sqrt{80}$

17. Which forms of symmetry does the polar curve $r = 2 - 3 \sin \theta$ exhibit?

- I. Symmetry about the x -axis
- II. Symmetry about the y -axis
- III. Symmetry about the origin

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

18. The area enclosed by $r = 2 + \cos \theta$ is

- (A) 2π (B) 4π (C) $\frac{9\pi}{4}$ (D) $\frac{9\pi}{2}$ (E) 9π

19. Which equation represents the tangent line to the curve parameterized by $x = \sqrt[3]{t}$ and $y = t^3 - 4t^2 + 8$ at $t = 1$?

(A) $y - 5 = -15(x - 1)$

(B) $y - 5 = -5(x - 1)$

(C) $y - 5 = -\frac{1}{15}(x - 1)$

(D) $y - 5 = \frac{1}{3}(x - 1)$

(E) $y - 5 = 5(x - 1)$

20. A particle travels counterclockwise along a circle of radius 4 centered at $(5, -2)$. The particle begins at the top of the circle and completes one revolution over $0 \leq t \leq 3$. Which parametric equations describe the particle's motion?

(A) $x = 5 + 4 \cos\left(\frac{2\pi}{3}t\right), \quad y = -2 + 4 \sin\left(\frac{2\pi}{3}t\right)$

(B) $x = 5 - 4 \cos\left(\frac{2\pi}{3}t\right), \quad y = -2 + 4 \sin\left(\frac{2\pi}{3}t\right)$

(C) $x = 5 + 4 \sin\left(\frac{2\pi}{3}t\right), \quad y = -2 + 4 \cos\left(\frac{2\pi}{3}t\right)$

(D) $x = 5 - 4 \sin\left(\frac{2\pi}{3}t\right), \quad y = -2 + 4 \cos\left(\frac{2\pi}{3}t\right)$

(E) $x = 5 - 4 \sin\left(\frac{2\pi}{3}t\right), \quad y = -2 - 4 \cos\left(\frac{2\pi}{3}t\right)$

21. An equation of the line tangent to the polar curve $r = 4 \sin \theta$ at $\theta = \frac{\pi}{6}$ is

(A) $y - 1 = 2(x - \sqrt{3})$

(B) $y - 1 = \sqrt{3}(x - \sqrt{3})$

(C) $y - 1 = \frac{1}{\sqrt{3}}(x - \sqrt{3})$

(D) $y - \sqrt{3} = \sqrt{3}(x - 1)$

(E) $y - \sqrt{3} = \frac{1}{\sqrt{3}}(x - 1)$

22. When $\theta = \frac{\pi}{3}$, the distance between the polar curves $r = 5 \cos \theta$ and $r = 1 + 2 \cos \theta$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{5}{2}$

(E) $\frac{3\sqrt{3}}{2}$

23. The area of the region enclosed by the curve $x = \sin t$, $y = 2t$ for $0 \leq t \leq \pi$ is

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

24. If $x = 3 \cos 2t$ and $y = 2 \sin 2t$, then $\frac{d^2y}{dx^2} =$

- (A) $-\frac{2}{9} \csc^3 2t$
(B) $-\frac{2}{3} \cot 2t$
(C) $\frac{2}{3} \tan 2t$
(D) $\frac{4}{3} \csc^2 2t$
(E) $\frac{1}{3} \csc^2 2t \sec 2t$

25. The arc length of the curve parameterized by $x = 6t + 5$ and $y = 8 - \frac{16}{3}t^{3/2}$ for $0 \leq t \leq 1$ is

- (A) $\frac{1}{16}$ (B) 4 (C) $\frac{49}{6}$ (D) 10 (E) $\frac{1568}{3}$

26. The area of the region that is outside the cardioid $r = 4 - 4\cos \theta$ and also inside the circle $r = 4$ is

- (A) 4π (B) 8π (C) $16 - 2\pi$ (D) $32 - 4\pi$ (E) $64 - 8\pi$

27. If $x = 6 - e^{-8t}$ and $\frac{d^5y}{dx^5} = 3 \ln t$, then $\frac{d^6y}{dx^6} =$

- (A) $\frac{3}{8te^{-8t}}$ (B) $\frac{3}{6t - te^{-8t}}$ (C) $\frac{3 \ln t}{6 - e^{-8t}}$ (D) $\frac{8te^{-8t}}{3}$ (E) $\frac{6 - e^{-8t}}{3 \ln t}$

28. The area enclosed by $r^2 = 8 \cos 2\theta$ is

- (A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) 2 (D) 4 (E) 8

29. A cycloid has a peak at $(10\pi, 20)$. Parametric equations for the cycloid are

(A) $x = 10(1 - \cos \theta), \quad y = 10(\theta - \sin \theta)$

(B) $x = 10(\theta - \sin \theta), \quad y = 10(1 - \cos \theta)$

(C) $x = 20(1 - \cos \theta), \quad y = 20(1 - \sin \theta)$

(D) $x = 20(1 - \cos \theta), \quad y = 20(\theta - \sin \theta)$

(E) $x = 20(\theta - \sin \theta), \quad y = 20(1 - \cos \theta)$

30. The graph parameterized by $x = 6 - 4t^2$ and $y = t^4 - 2$ is concave up for

(A) $t < 0$

(B) $t < \frac{\sqrt{6}}{2}$

(C) $t > 0$

(D) $t > \sqrt[4]{2}$

(E) $-\infty < t < \infty$

31. Which integral equals the length of the polar curve $r = \cos^2 \theta$ over $0 \leq \theta \leq \frac{\pi}{2}$?

(A) $\int_0^{\pi/2} \sqrt{1 + 4 \sin^2 \theta \cos^2 \theta} \, d\theta$

(B) $\int_0^{\pi/2} \sqrt{\cos^2 \theta + 2 \sin \theta \cos \theta} \, d\theta$

(C) $\int_0^{\pi/2} \sqrt{\cos^2 \theta - 2 \sin \theta \cos \theta} \, d\theta$

(D) $\int_0^{\pi/2} \sqrt{\cos^4 \theta + 4 \sin^2 \theta \cos^2 \theta} \, d\theta$

(E) $\int_0^{\pi/2} \sqrt{\cos^4 \theta + 4 \sin^2 \theta \cos^2 \theta} \, d\theta$

32. Half of a petal on the curve $r = \cos 2\theta$ is revolved around the x -axis to generate a solid whose surface area is given by

(A) $2\pi \int_0^{\pi/4} \sin \theta \cos 2\theta \sqrt{1 + 4 \sin^2 2\theta} \, d\theta$

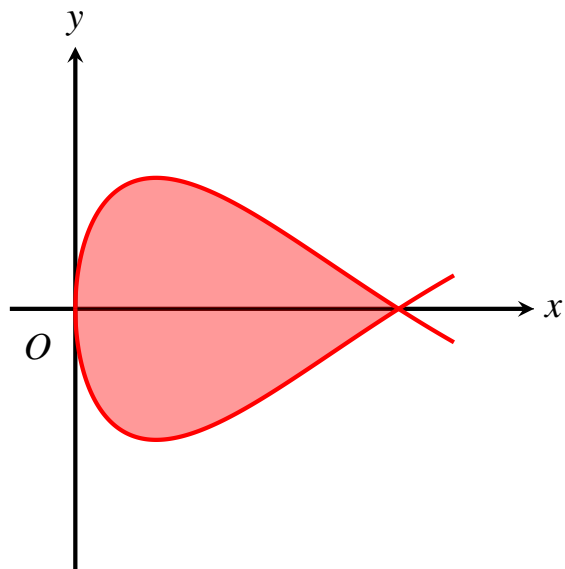
(B) $2\pi \int_0^{\pi/4} \sin \theta \cos 2\theta \sqrt{\cos^2 2\theta - 4 \sin^2 2\theta} \, d\theta$

(C) $2\pi \int_0^{\pi/4} \sin \theta \cos 2\theta \sqrt{\cos^2 2\theta + 4 \sin^2 2\theta} \, d\theta$

(D) $2\pi \int_0^{\pi/4} \cos \theta \cos 2\theta \sqrt{\cos^2 2\theta - 4 \sin^2 2\theta} \, d\theta$

(E) $2\pi \int_0^{\pi/4} \cos \theta \cos 2\theta \sqrt{\cos^2 2\theta + 4 \sin^2 2\theta} \, d\theta$

33. The graph parameterized by $x = t^2$ and $y = \sin 2t$ traces out a teardrop, as shown below.

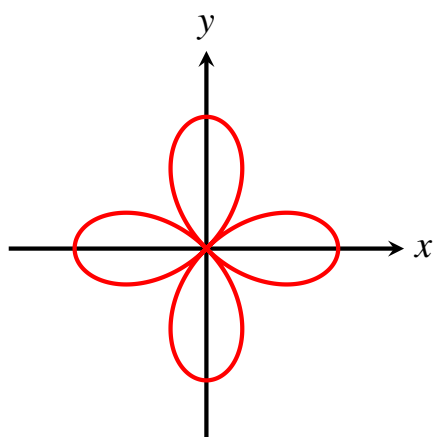


The area of the teardrop is

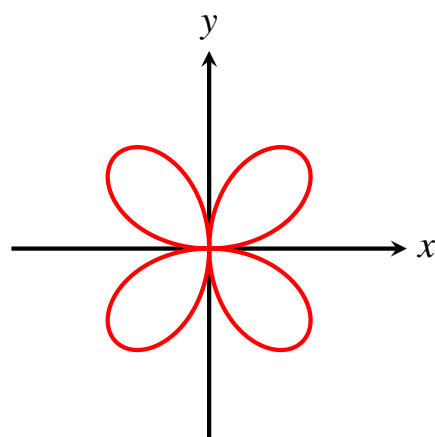
- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π
34. The area of the region enclosed by $r = 4 \cos \theta$ and $r = 4 \sin \theta$ is
- (A) 4 (B) $\pi + 4$ (C) $2\pi - 2$ (D) $2\pi - 4$ (E) $2\pi + 4$

35. Which choice is the graph of $r = \sin 4\theta$?

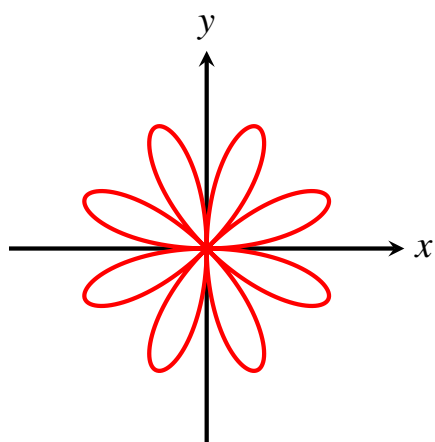
(A)



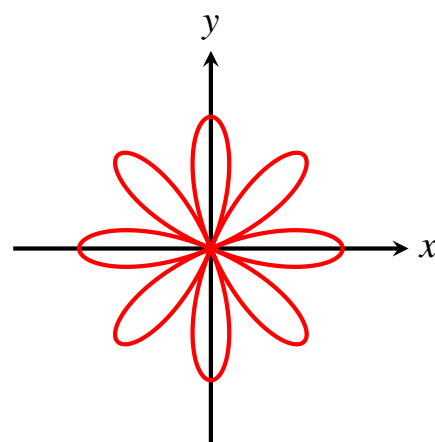
(B)



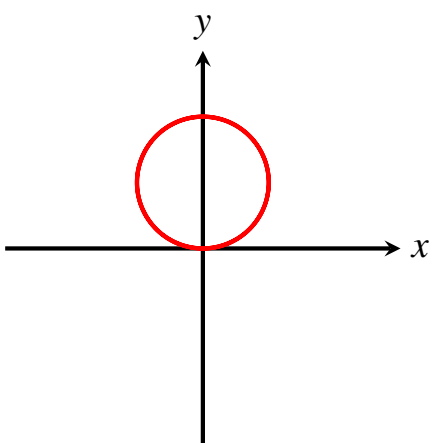
(C)



(D)



(E)



36. The area enclosed by the inner loop of the limaçon $r = 2 - 4\cos\theta$ is

(A) $2\pi + 3\sqrt{3}$

(B) $2\pi - 3\sqrt{3}$

(C) $4\pi + 6\sqrt{3}$

(D) $4\pi - 6\sqrt{3}$

(E) $8\pi - 12\sqrt{3}$

37. The graph of $(x^2 + y^2)^3 = 4x^2y^2$ is identical to which polar curve?

(A) $r = \sin\theta$

(B) $r = \sin 2\theta$

(C) $r = \cos 2\theta$

(D) $r = 2\sin 2\theta$

(E) $r = 2\cos 2\theta$

38. The area of the region bounded by $r = 4\theta$, where $0 \leq \theta \leq \pi$, and the x -axis is

- (A) $\frac{\pi^2}{2}$ (B) π^2 (C) $\frac{4\pi^3}{3}$ (D) $\frac{8\pi^3}{3}$ (E) $\frac{16\pi^3}{3}$

39. For $0 \leq \theta \leq \frac{\pi}{2}$, the tangent to the polar curve $r = 4 \sin \theta$ is 1 when

- (A) $\theta = \frac{\pi}{16}$ (B) $\theta = \frac{\pi}{8}$ (C) $\theta = \frac{\pi}{6}$ (D) $\theta = \frac{\pi}{4}$ (E) $\theta = \frac{\pi}{2}$

40. Curve C is parameterized by $x = 5 + 3 \cos t$ and $y = 3 \sin t$ for $0 \leq t \leq \frac{\pi}{2}$. The surface area of revolution in rotating C about the x -axis is

- (A) $6\pi\sqrt{18}$ (B) $12\pi\sqrt{18}$ (C) 9π (D) 18π (E) $15\pi^2$

41. For $0 \leq \theta \leq \pi$, the polar curve $r = 9 \cos \theta$ has a horizontal tangent when

- (A) $\theta = 0$ (B) $\theta = \frac{\pi}{4}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{3\pi}{4}$ (E) $\theta = \pi$

42. When $\theta = \frac{3\pi}{4}$, the polar curve $r = 9\cos 3\theta$ is getting

- (A) closer to the x -axis and closer to the y -axis
- (B) closer to the x -axis and farther away from the y -axis
- (C) farther away from the x -axis and closer to the y -axis
- (D) farther away from the x -axis and farther away from the y -axis
- (E) farther away from the x -axis, and neither closer to nor farther away from the y -axis

43. The area of *one* petal of the rose $r = \cos 8\theta$ is

- (A) $\frac{\pi}{64}$ (B) $\frac{\pi}{32}$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

44. A woman standing on a balcony 10 feet above the ground throws a flying disk upward with a speed of 20 meters per second at an angle of 45° to the horizontal. If x is the disk's horizontal distance from the woman and y is its height above the ground, then the disk's trajectory is modeled by

(A) $y = 10 + x + \frac{9.8x^2}{200}$

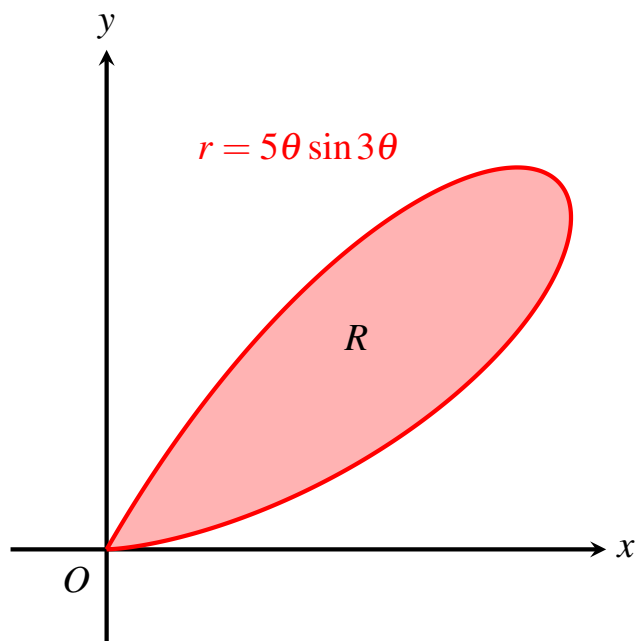
(B) $y = 10 + x + \frac{9.8x^2}{400}$

(C) $y = 10 + x + \frac{9.8x^2}{800}$

(D) $y = 10 + x - \frac{9.8x^2}{400}$

(E) $y = 10 + x - \frac{9.8x^2}{800}$

Questions 45 and 46 refer to the following region.



45. Which integral equals the area of R ?

(A) $\frac{1}{2} \int_{-\pi/2}^0 25\theta^2 \sin^2 3\theta \, d\theta$

(B) $\frac{1}{2} \int_0^{\pi/6} 25\theta^2 \sin^2 3\theta \, d\theta$

(C) $\frac{1}{2} \int_0^{\pi/3} 25\theta^2 \sin^2 3\theta \, d\theta$

(D) $\frac{1}{2} \int_0^{\pi/2} 25\theta^2 \sin^2 3\theta \, d\theta$

(E) $\frac{1}{2} \int_0^{\pi} 25\theta^2 \sin^2 3\theta \, d\theta$

46. The average distance from O to any point on the border of R is

- (A) $\frac{5}{9}$ (B) $\frac{5}{6}$ (C) $\frac{5}{3}$ (D) $\frac{5\pi}{9}$ (E) $\frac{5\pi}{3}$

47. The region S is bounded by the line $\theta = \frac{\pi}{2}$ and the polar curve $r = 7\theta$. Which integral equals the surface area of the solid generated by rotating S about the line $\theta = \frac{\pi}{2}$?

(A) $49\pi \int_0^{\pi/2} \theta \sin \theta \sqrt{\theta^2 + 1} \, d\theta$

(B) $49\pi \int_0^{\pi/2} \theta \cos \theta \sqrt{\theta^2 + 1} \, d\theta$

(C) $98\pi \int_0^{\pi/2} \sin \theta \sqrt{\theta^2 + 1} \, d\theta$

(D) $98\pi \int_0^{\pi/2} \theta \sin \theta \sqrt{\theta^2 + 1} \, d\theta$

(E) $98\pi \int_0^{\pi/2} \theta \cos \theta \sqrt{\theta^2 + 1} \, d\theta$

48. Which expression equals the area of the region that is inside $r = 6 - 4 \cos \theta$ and also inside $r = 4$?

(A) $\int_0^\pi (2 - 4 \cos \theta)^2 d\theta$

(B) $\int_0^{\pi/3} 16 d\theta + \int_{\pi/3}^\pi (6 - 4 \cos \theta)^2 d\theta$

(C) $\int_0^{\pi/3} (6 - 4 \cos \theta)^2 d\theta + \int_{\pi/3}^\pi 16 d\theta$

(D) $\int_0^\pi [(6 - 4 \cos \theta)^2 - 16] d\theta$

(E) $\int_0^\pi [16 - (6 - 4 \cos \theta)^2] d\theta$

49. If C is the curve parameterized by $x = \sin t$ and $y = e^t$ for $0 \leq t \leq 1$, then the area between C and the x -axis is

(A) $\frac{e \sin 1 + e \cos 1 - 1}{2}$

(B) $\frac{1 + e \sin 1 + e \cos 1}{2}$

(C) $\frac{1 + e \sin 1 - e \cos 1}{2}$

(D) $\frac{1 - e \sin 1 + e \cos 1}{2}$

(E) $\frac{1 - e \sin 1 - e \cos 1}{2}$

50. Let $A(\theta)$ be the area that is enclosed by the polar curves $r = 6 \cos \theta$ and $r = k \sin \theta$, where k is a positive constant. Then $\lim_{k \rightarrow \infty} A(\theta)$ is

- (A) $\frac{9\pi}{2}$ (B) 9π (C) 18π (D) 36π (E) nonexistent

This marks the end of the review exercises. The following page contains the answers to all the questions.

- | | |
|-------|-------|
| 1. C | 34. D |
| 2. E | 35. C |
| 3. D | 36. D |
| 4. C | 37. B |
| 5. B | 38. D |
| 6. B | 39. B |
| 7. E | 40. D |
| 8. B | 41. B |
| 9. D | 42. A |
| 10. A | 43. B |
| 11. D | 44. D |
| 12. E | 45. C |
| 13. E | 46. C |
| 14. A | 47. E |
| 15. A | 48. C |
| 16. B | 49. A |
| 17. B | 50. A |
| 18. D | |
| 19. A | |
| 20. D | |
| 21. B | |
| 22. B | |
| 23. D | |
| 24. A | |
| 25. C | |
| 26. D | |
| 27. A | |
| 28. E | |
| 29. B | |
| 30. E | |
| 31. D | |
| 32. C | |
| 33. C | |