

Chapter 6 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 6: Integration Techniques.

- **6.1**: Integration by Parts
- **6.2**: Trigonometric Integrals
- **6.3**: Trigonometric Substitution
- 6.4: Integration by Partial Fractions
- **6.5**: Improper Integrals

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

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Integration Techniques

Number of Questions—12

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points Earned	Points Available
Short Integrals		40
Question 9		15
Question 10		15
Question 11		15
Question 12		15
TOTAL		100

Short Integrals

1.
$$\int (3t+2)\cos 5t \, dt$$
 (5 pts.)

$$2. \int_0^{\pi/6} \sin^2\theta \cos^3\theta \,d\theta \tag{5 pts.}$$

3.
$$\int \sin 5\theta \cos 9\theta \, d\theta$$
 (5 pts.)

4.
$$\int_{-\infty}^{\infty} \frac{2}{x^2 + 16} dx$$
 (5 pts.)

5.
$$\int x^2 e^{-x/4} dx$$
 (5 pts.)

6.
$$\int_{5}^{10} \frac{x^2}{\sqrt{x^2 - 25}} \, \mathrm{d}x$$
 (5 pts.)

7.
$$\int \frac{1}{\sqrt{30-6x-2x^2}} dx$$
 (5 pts.)

8.
$$\int_{1}^{4} e^{\sqrt{x}} dx$$
 (5 pts.)

Long Questions

9. Evaluate $\int \sec^m x \tan^n x \, dx$ for each case.

(a)
$$m = 3, n = 5$$
 (5 pts.)

(b)
$$m = 4, n = 2$$
 (5 pts.)

(c)
$$m = 4, n = 9$$
 (5 pts.)

10. Evaluate
$$\int_0^1 \frac{3x^2 - 8}{x^4 - 81} \, \mathrm{d}x.$$

(15 pts.)

11. Calculate $\int_{-1}^{0} \frac{1-x}{2x^2+kx+6} dx$ for each value of k.

(a)
$$k = 7$$
 (5 pts.)

(b)
$$k = -13$$
 (5 pts.)

(c)
$$k = 8$$
 (5 pts.)

- 12. Consider the family of improper integrals $I = \int_1^\infty \frac{1}{x^{3c+6}} dx$, where c is a constant.
 - (a) Find all values of c for which the integral converges. (2 pts.)

(b) For $c = -\frac{1}{3}$, show that $I = \frac{1}{4}$. (4 pts.)

(c) Does $\int_{1}^{\infty} \frac{|\cos x|}{x^5} dx$ converge or diverge? (3 pts.)

(d) For $c = -\frac{17}{9}$, evaluate $\int_0^1 \frac{1}{x^{3c+6}} dx$ or show that it diverges. (6 pts.)

This marks the end of the review exercises.