Chapter 5 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 5: Applications of Integration.

5.1: Areas between Curves
5.2: Volumes with Cross Sections
5.3: Solids of Revolution
5.4: Shell Method
5.5: Work
5.6: Average Value of a Function

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Applications of Integration

Number of Questions—14

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points	Points Available
Short Questions		55
Question 12		15
Question 13		15
Question 14		15
TOTAL		**100**

Short Questions

1. Find the average value of $f(x) = x^3 - 6x^2$ on $[0, 2]$. *(5 pts.)*

2. An object traveling along the x-axis experiences a force given by $F(x) = x^4 e^{-x^5}$. Calculate the work done on the object as it moves from $x = 0$ to $x = 1$. *(5 pts.)*

3. Find the area of the region bounded by $y = x^2 + 6$ and $y = 9 - 2x$. *(5 pts.)*

4. Calculate the volume of the solid generated by rotating the region bounded by $y = \sqrt[3]{x}$, the x-axis, and the line $x = 2$ about the x-axis. *(5 pts.)*

5. Calculate c to satisfy the Mean Value Theorem for Integrals for $f(x) = \dfrac{1}{\sqrt{4x}}$ over $1 \leqslant x \leqslant 4$. *(5 pts.)*

6. The region enclosed by $y = \sqrt{9 - x^2}$, $x = 1$, $x = 2$, and the x-axis is the base of a solid whose cross sections perpendicular to the x-axis are rectangles of height $2x$. Calculate the solid's volume. *(5 pts.)*

7. A sample of gas under a pressure of 30 pounds per square inch occupies 2 cubic inches of volume. Assuming the gas's temperature remains constant, calculate the work done by the gas as its volume doubles. *(5 pts.)*

8. Calculate the area of the region bounded by $y = 3\sin x$ and $y = 3\cos x$ for $\frac{\pi}{4} \leqslant x \leqslant \pi$. *(5 pts.)*

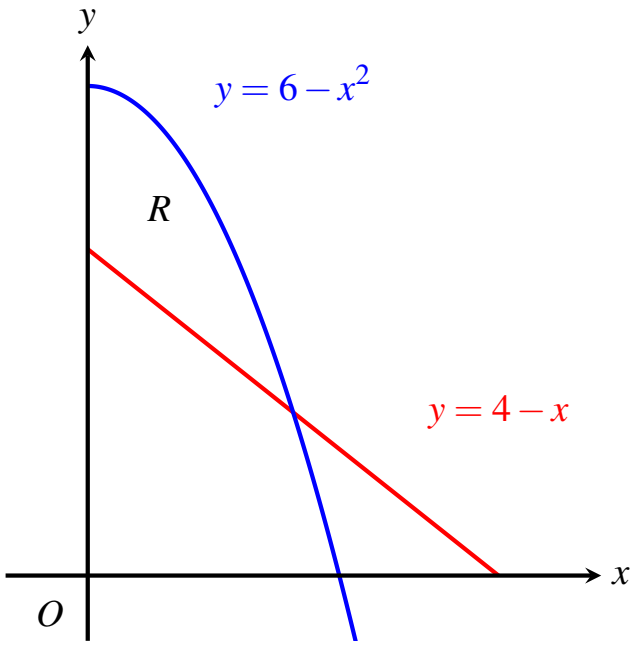
9. It takes 15 pounds of force to stretch a spring from its natural length of 1 foot to an elongated length of 1.5 feet. Calculate the work needed to pull the spring 1 foot past its natural length. *(5 pts.)*

10. A tank is 5 meters tall and has a cross-sectional area of 10 square meters. If water fills the tank up to a height of 3 meters, then how much work is needed to pump all the water to the top of the tank? *(5 pts.)*

11. A log's radius r as a function of distance x from the left end is $r(x) = 6 - x^2$ for $0 \leqslant x \leqslant 2$. Calculate the log's volume on this interval. *(5 pts.)*

Long Questions

12. The region R is bounded by $y = 6 - x^2$, $y = 4 - x$, and the y-axis, as in the following figure.



(a) Calculate the area of R. *(2 pts.)*

(b) Write, but do not solve, an integral expression that equals the volume of the solid generated by rotating R about the x-axis. *(3 pts.)*

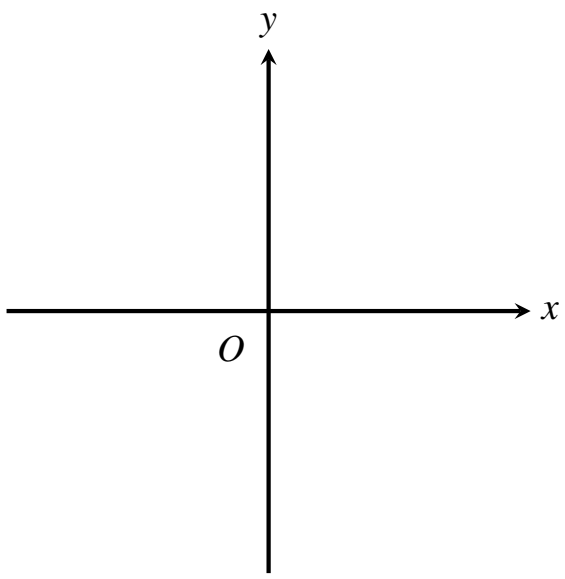
(c) Write, but do not solve, an integral expression that equals the volume of the solid generated by rotating R about the y-axis. *(3 pts.)*

(d) What is the average distance from the line $y = -1$ to a point on the upper boundary of R ? *(3 pts.)*

(e) Calculate k such that the vertical line $x = k$ divides R into two subregions with equal areas. *(4 pts.)*

13. Region S is enclosed by $y = x^2$ and $y = x$ in the first quadrant.

(a) On the following graph, sketch the two functions and shade in the region S. *(1 pt.)*

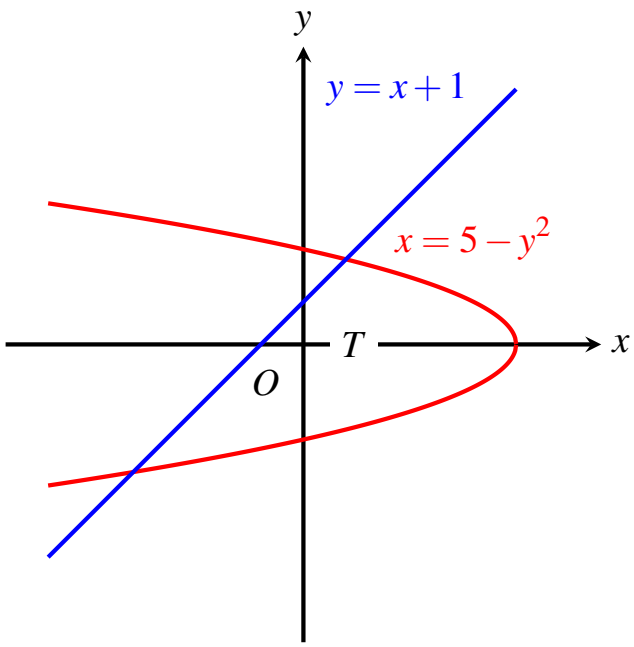


(b) Set up, but do not evaluate, an integral whose value equals the volume of the solid generated by rotating S about the line $y = -2$. *(4 pts.)*

(c) Calculate the volume of the solid generated by rotating S about the line $x = -1$. *(5 pts.)*

(d) Region S is the base of a solid whose cross sections perpendicular to the y-axis are squares. Calculate this solid's volume. *(5 pts.)*

14. Let T be the region bounded by the graphs of $y = x + 1$ and $x = 5 - y^2$, as in the following figure.



(a) Determine the area of T. *(2 pts.)*

(b) Region T is a thin plate whose mass, in kilograms, equals twice the area found in part (a). Calculate the work needed to lift the plate 3 meters high. *(3 pts.)*

(c) Calculate the volume of the solid generated by rotating T about the line $y = 5$. *(4 pts.)*

(d) Calculate the volume of the solid generated by rotating T about the line $x = -6$. *(4 pts.)*

(e) Set up, but do not evaluate, an integral that equals the volume of the solid generated by rotating T about the line $x = -4$. *(2 pts.)*

This marks the end of the review exercises.