Chapter 2 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 2: Differentiation Rules.

2.1: Defining a Derivative
2.2: Differentiating Power, Exponential, and Sinusoidal Functions
2.3: Product Rule and Quotient Rule
2.4: Chain Rule
2.5: Implicit Differentiation and Differentiating Inverse Functions
2.6: Differentiating Logarithmic Functions
2.7: Related Rates
2.8: Linearization and Differentials
2.9: Hyperbolic Functions

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Differentiation Rules

Number of Questions—22

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points Earned	Points Available
Rapid Derivatives		20
Short Questions		50
Question 21		15
Question 22		15
TOTAL		**100**

Rapid Derivatives

Find the derivative of each function. Simplify your answers when possible. No partial credit is awarded.

1. $y = \sqrt[6]{x}$ (2 pts.)

2. $y = e^x \sec x$ (2 pts.)

3. $y = x \tan^{-1} 2x$ (2 pts.)

4. $y = \log_7 x$ *(2 pts.)*

5. $y = \dfrac{\cot x}{x^3 + x}$ *(2 pts.)*

6. $y = \cos\left(\dfrac{2}{x^3}\right)$ *(2 pts.)*

7. $y = \ln(x^4 - 6x^2 + 5)$ *(2 pts.)*

8. $y = \tan^{-1}(\text{sech}\, x)$ *(2 pts.)*

9. $y = \sin^2\left(\sqrt{9 - x^2}\right)$ *(2 pts.)*

10. $y = x\left(4 - e^{-x/2}\right)$ *(2 pts.)*

Short Questions

11. Let $f(x) = x^2 + x - 3$. Use the limit definition of a derivative to find $f'(1)$. *(5 pts.)*

12. If f is a continuous function such that $f(1) = 5$ and $f'(1) = 2$, then use linearization to approximate $f(1.2)$. *(5 pts.)*

13. For $xy^3 + y^2 = 8$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(5 pts.)*

14. Using differentials, estimate the amount by which a cube's volume increases as its side lengths increase from 5 inches to 5.1 inches. *(5 pts.)*

15. Prove that $\cosh 3x - \sinh 3x = e^{-3x}$. *(5 pts.)*

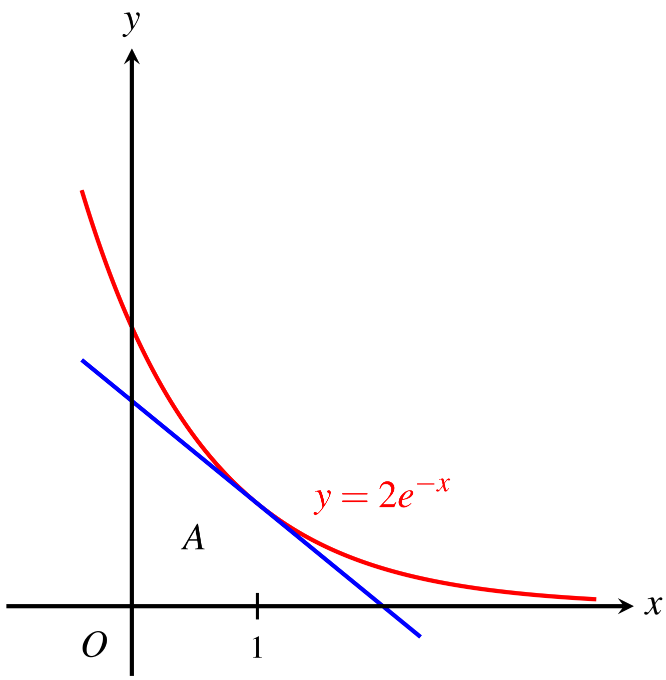
16. Write an equation of the line normal to the curve $y = x^5 - 2x^3 - 4$ at $x = 1$. *(5 pts.)*

17. If $g(x) = 3x^5 - x$, then calculate $\left(g^{-1}\right)'(2)$. (5 pts.)

18. A uniform tank has a cross-sectional area of 100 cubic centimeters. Water is pumped into the tank at a rate of 7 cubic centimeters per minute. How quickly is the tank's water level rising? (5 pts.)

19. Use Logarithmic Differentiation to find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt[3]{\frac{x^3 \sin 2x}{x^2 - 4}}$. (5 pts.)

20. The triangle A is bounded by the x-axis, the y-axis, and the line tangent to $y = 2e^{-x}$ at $x = 1$, as shown in the figure. Calculate the area of A. *(5 pts.)*



Long Questions

21. The graph C is given by the implicit equation $3x^2 + y^2 = 8 - y$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-6x}{2y+1}$. *(3 pts.)*

(b) Write an equation of the line tangent to C at the point $\left(\sqrt{\frac{2}{3}}, 2\right)$ in point-slope form. *(2 pts.)*

(c) Locate all the points at which C has a vertical tangent. *(5 pts.)*

(d) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of x and y. *(5 pts.)*

22. Particle P travels along the x-axis as modeled by the position function $x(t) = t^3 - 2t^2 + 3t$. Particle Q moves along the y-axis according to the position function $y(t) = 2t + 6$. Both $x(t)$ and $y(t)$ are measured in feet, and $t \geqslant 0$ is measured in seconds.

(a) At the moment when $t = 2$, how far apart are particles P and Q ? *(1 pt.)*

(b) Calculate both particles' velocities when $t = 2$. *(4 pts.)*

(c) When $t = 2$, calculate the rate at which the distance between particles P and Q is changing. *(6 pts.)*

(d) Find particle P's acceleration when $t = 2$. Is the particle speeding up or slowing down at this time? *(4 pts.)*

This marks the end of the review exercises.