Chapter 10 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 10: Infinite Sequences and Series.

10.1: Sequences
10.2: Infinite Series and Divergence Test
10.3: Integral Test
10.4: Comparison Tests
10.5: Alternating Series
10.6: Absolute Convergence and the Ratio and Root Tests
10.7: Power Series
10.8: Taylor and Maclaurin Series

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Infinite Sequences and Series

Number of Questions—20

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points	Points Available
Rapid Series		20
Short Questions		35
Question 18		15
Question 19		15
Question 20		15
TOTAL		**100**

Rapid Series

Determine whether the series converges or diverges. No partial credit is awarded for an incorrect conclusion.

1. $\sum_{n=1}^{\infty} 2$ *(2 pts.)*

2. $\sum_{n=0}^{\infty} 5(\frac{1}{4})^n$ *(2 pts.)*

3. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+11}}$ *(2 pts.)*

4. $\displaystyle\sum_{n=1}^{\infty} \frac{4n^2 - n + 3}{5n^3 - 7n^2 + 4}$ *(2 pts.)*

5. $\displaystyle\sum_{n=3}^{\infty} \frac{e^{7n}}{n!}$ *(2 pts.)*

6. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ *(2 pts.)*

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^4}{3n^4+8}$ *(2 pts.)*

8. $\displaystyle\sum_{n=9}^{\infty} \frac{\sin(\pi n)}{8-n}$ *(2 pts.)*

9. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{8n+6}{2n+3}\right)^{2n}$ *(2 pts.)*

10. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^3 n}{\sqrt{n^3}}$ *(2 pts.)*

Short Questions

11. Calculate the exact value of $\sum_{n=0}^{\infty} \frac{2}{n^2+8n+15}$. *(5 pts.)*

12. Write the first three terms of the Maclaurin series of $\sqrt[5]{4+x}$. *(5 pts.)*

13. Based on the Alternating Series Test, find the smallest integer N such that $S_N = \sum_{i=1}^{N} \frac{(-1)^i}{(i+4)^3}$ approximates $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+4)^3}$ with an error of less than 0.001. *(5 pts.)*

14. Find the third-degree Taylor polynomial for $f(x) = \cos 2x$ centered at $x = \dfrac{\pi}{3}$. *(5 pts.)*

15. Write a Maclaurin series for $\ln\left(1+\dfrac{x}{2}\right)$ and determine its interval of convergence. *(5 pts.)*

16. Based on the Integral Test, what is the maximum error with which $S_{10} = \displaystyle\sum_{i=1}^{10} \frac{1}{\sqrt[3]{2i+7}}$ approximates $S = \displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{2n+7}}$? *(5 pts.)*

17. Find a power series solution for the initial value problem $y' = \dfrac{2x^3}{x^7+1}$, $y(0) = 5$. *(5 pts.)*

Long Questions

18. Consider the sequence $\{a_n\}_1^\infty = \left\{2+\dfrac{\cos n}{n}\right\}_1^\infty$.

(a) Is $\{a_n\}_1^\infty$ bounded? Is it monotonic? *(3 pts.)*

(b) Find the limit of $\{a_n\}_1^\infty$ or show that it diverges. *(4 pts.)*

(c) Let $b_n = \dfrac{n^2+4n+6}{2n^2+7}$. What is the limit of the sequence $\left\{\dfrac{2a_n}{b_n}\right\}$? *(4 pts.)*

(d) Another sequence is defined by $\{c_n\}_1^\infty = \left\{-1, \frac{2}{5}, -\frac{4}{6}, \frac{8}{7}, \dots\right\}$. Determine c_n. *(4 pts.)*

19. A function f has the power series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+4} x^n$.

(a) Determine the interval of convergence of the power series. *(3 pts.)*

(b) What power series represents $f(2x)$? Find its interval of convergence. *(4 pts.)*

(c) Find a power series for $f'(x)$. *(4 pts.)*

(d) Let $g(x) = \displaystyle\int_0^x f(t)\,\mathrm{d}t$. Write a power series for g. *(4 pts.)*

20. The Taylor series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{4^n}{n}(x-5)^n$ converges to $f(x)$ for $|x-5|<R$, where R is the radius of convergence.

(a) Determine R. *(3 pts.)*

(b) Use the first three terms of the Taylor series to approximate $f(6)$. *(1 pt.)*

(c) Given $\left|f^{(4)}(x)\right| \leqslant 40$ for all $5 \leqslant x \leqslant 6$, use Taylor's Remainder Theorem to determine the maximum error in the estimate from part (b). *(4 pts.)*

(d) Find $f'(5)$ and $f''(5)$. *(3 pts.)*

(e) Let $g(x) = f(x)\cos x$. Write the third-degree Maclaurin polynomial for g. *(4 pts.)*

This marks the end of the review exercises.