Chapter 10 Multiple-Choice Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 10: Infinite Sequences and Series.

10.1: Sequences
10.2: Infinite Series and Divergence Test
10.3: Integral Test
10.4: Comparison Tests
10.5: Alternating Series
10.6: Absolute Convergence and the Ratio and Root Tests
10.7: Power Series
10.8: Taylor and Maclaurin Series

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Infinite Sequences and Series

Number of Questions—45

NO CALCULATOR

1. What is the difference between a sequence and a series?

(A) A sequence is the sum of a set of numbers, whereas a series is a set of numbers.

(B) A sequence is a set of numbers arranged in ascending order, whereas a series is the sum of a set of numbers arranged in ascending order.

(C) A sequence is a set of numbers, whereas a series is the sum of a set of numbers.

(D) A sequence always diverges, whereas a series may converge or diverge.

(E) A sequence always converges, whereas a series may converge or diverge.

2. Which sequence converges?

(A) $\{-1, 1, -1, 1, -1, \ldots\}$

(B) $\{-2, -1, 0, 1, 2, \ldots\}$

(C) $\{1, 2, 4, 8, 16, \ldots\}$

(D) $\left\{1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \frac{81}{256}, \ldots\right\}$

(E) $\left\{\frac{1}{2}, -\frac{3}{2}, \frac{5}{2}, -\frac{7}{2}, \frac{9}{2}, \ldots\right\}$

3. $\frac{\pi}{4} - \frac{\pi^2}{16} + \frac{\pi^3}{64} - \frac{\pi^4}{256} + \cdots + (-1)^{n+1}\left(\frac{\pi}{4}\right)^n + \cdots$ is

(A) $\frac{-4}{4+\pi}$ (B) $\frac{\pi}{4-\pi}$ (C) $\frac{\pi}{4+\pi}$ (D) $\frac{4}{4-\pi}$ (E) divergent

4. By the Binomial Theorem, $(3x-5)^4$ is

(A) $(3x)^4+\binom{4}{1}(3x)^3(-5)+\binom{4}{2}(3x)^2(-5)^2+\binom{4}{3}(3x)(-5)^3+(-5)^4$

(B) $-(3x)^4-\binom{4}{1}(3x)^3(-5)-\binom{4}{2}(3x)^2(-5)^2-\binom{4}{3}(3x)(-5)^3-(-5)^4$

(C) $(3x)^4+\binom{4}{1}(3x)^3(5)+\binom{4}{2}(3x)^2(5)^2+\binom{4}{3}(3x)(5)^3+(5)^4$

(D) $(3x)^4+\binom{4}{1}-(3x)^3(5)-\binom{4}{2}(3x)^2(5)^2-\binom{4}{3}(3x)(5)^3-(5)^4$

(E) $(3x)^4+\binom{4}{1}-(3x)^3(-5)-\binom{4}{2}(3x)^2(-5)^2-\binom{4}{3}(3x)(-5)^3-(-5)^4$

5. The Maclaurin series of $\cos x$ is

(A) $1+\dfrac{x^2}{2}+\dfrac{x^4}{4!}+\dfrac{x^6}{6!}+\cdots+\dfrac{x^{2n}}{(2n)!}+\cdots$

(B) $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\cdots+\dfrac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$

(C) $x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\dfrac{x^7}{7!}+\cdots+\dfrac{x^{2n+1}}{(2n+1)!}+\cdots$

(D) $1+x+\dfrac{x^2}{2}+\dfrac{x^3}{3!}+\cdots+\dfrac{x^n}{n!}+\cdots$

(E) $1-\dfrac{x^2}{2}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots+\dfrac{(-1)^n x^{2n}}{(2n)!}+\cdots$

6. Let $a_n = \dfrac{1}{\sqrt[3]{n^2}}$. Which option is true?

(A) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(B) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because it is a p-series with $p \leqslant 1$.

(C) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(D) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because it is a p-series with $p \leqslant 1$.

(E) It cannot be determined from the given information.

7. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}}$ (B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ (C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ (D) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ (E) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2\sqrt{n}}$

8. Suppose that $a_n \geqslant b_n > 0$ and $\{b_n\}$ diverges. Which of the following must be true?

I. $\{a_n\}$ diverges.

II. $\lim_{n \to \infty} a_n = \infty$.

III. $\lim_{n \to \infty} b_n = \infty$.

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

9. Which series is alternating?

(A) $\sum_{n=1}^{\infty}(-1)^{2n}$

(B) $\sum_{n=1}^{\infty}(-1)^{n}\cos(\pi n)$

(C) $\sum_{n=1}^{\infty}\frac{(-1)^{2n+3}}{n}$

(D) $\sum_{n=1}^{\infty}\frac{\sin(\pi n)}{\sqrt{n}}$

(E) All of the above

10. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Which of the following must be true?

I. $\lim_{n\to\infty} a_n = 0$

II. The sequence $\{a_n\}$ converges.

III. $\lim_{N\to\infty} \sum_{n=1}^{N} a_n = \pm\infty$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

11. The Maclaurin series $x^2 - \dfrac{x^4}{3!} + \dfrac{x^6}{5!} - \dfrac{x^8}{7!} + \cdots$ converges to which function?

(A) $\sin x$ (B) $\cos x$ (C) x^2e^x (D) $x^2 \sin x$ (E) $x \sin x$

12. Which series converges?

(A) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$ (B) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ (C) $\displaystyle\sum_{n=1}^{\infty}(-1)^n n$ (D) $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n}$ (E) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n}$

13. Suppose that $\displaystyle\sum_{n=1}^{\infty} b_n$ converges and $a_n \leqslant b_n$ for $n \geqslant 1$. Which choice must be true?

(A) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(B) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

(C) $\displaystyle\lim_{n\to\infty} a_n = 0$.

(D) $\displaystyle\lim_{n\to\infty} b_n \neq 0$.

(E) None of the above

14. Function g has derivatives of all orders. The table below shows selected values of the derivatives of $g(x)$ at $x = 3$, with $g(3) = 1$. What is the fourth-degree Taylor polynomial for $g(x)$ centered at $x = 3$?

n	1	2	3	4
$g^{(n)}(3)$	-2	2	$3/2$	$-4/3$

(A) $1-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(B) $1-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(C) $-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(D) $-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(E) $1-2(x+3)+(x+3)^2+\frac{1}{4}(x+3)^3-\frac{1}{18}(x+3)^4$

15. For what values of k does $\sum_{n=1}^{\infty} \frac{1}{n^{2k-3}}$ diverge?

(A) $k \leqslant 2$

(B) $k < \frac{3}{2}$

(C) $k > \frac{3}{2}$

(D) $k > 2$

(E) $k \geqslant 2$

16. Which choice correctly describes the convergence or divergence of $S = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}}$?

(A) S converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(B) S diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(C) S converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) S diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(E) It cannot be determined from the given information.

17. For all x, $1+\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+\cdots+\dfrac{x^{2n}}{(2n)!}+\cdots$ converges to

(A) $\sin x$ (B) $\cosh x$ (C) $\sinh x$ (D) $\cos x$ (E) e^{2x}

18. Let f be a function with derivatives of all orders. The fourth-degree Taylor polynomial of $f(x)$ centered at $x=2$ is $2+(x-2)-\dfrac{2}{5}(x-2)^2-\dfrac{3}{2}(x-2)^3+\dfrac{1}{7}(x-2)^4+\cdots$. What is the value of $f'''(2)$?

(A) -9 (B) $-\dfrac{9}{2}$ (C) $-\dfrac{3}{2}$ (D) $-\dfrac{1}{4}$ (E) 0

19. The coefficient of x^3 in the Maclaurin series of $e^{-x/3}$ is

(A) $-\dfrac{1}{27}$ (B) $-\dfrac{1}{162}$ (C) $\dfrac{1}{162}$ (D) $\dfrac{1}{27}$ (E) $\dfrac{1}{6}$

20. Which series converges conditionally?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+1}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+2}}$

21. Which choice about $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+3}}$ is correct?

(A) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ converges.

(B) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ diverges.

(C) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx > 0$.

(D) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ converges.

(E) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ diverges.

22. The binomial series expansion of $\frac{1}{2}\sqrt{4-x}$ is

(A) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(B) $1-\frac{1}{2}\left(-\frac{x}{4}\right)-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(C) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

(D) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(E) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

23. For which series is the Root Test inconclusive?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{8^n+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+3}{n+2}\right)^n$

(C) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+7}{3n-1}\right)^n$

(D) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6n^2+n-2}{4n^2+9}\right)^n$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{5^{3n+1}}$

24. Suppose that $\sum_{n=1}^{\infty} a_n$ converges and c is a finite constant. All the following series must converge *except*

(A) $c + \sum_{n=1}^{\infty} a_n$

(B) $c \sum_{n=1}^{\infty} a_n$

(C) $\sum_{n=1+|c|}^{\infty} a_n$

(D) $\sum_{n=1}^{\infty} [a_n]^c$

(E) $\sum_{n=1}^{\infty} [a_n + c\, a_n]$

25. The fourth-degree Taylor polynomial of e^x centered at $x=2$ is

(A) $1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^3$

(B) $1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^3$

(C) $e^2\left[1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^3\right]$

(D) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^3\right]$

(E) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3}(x-2)^3+\frac{1}{4}(x-2)^3\right]$

26. Which function does not have a Taylor series at the given center?

(A) e^x centered at $x=e$

(B) $\tan x$ centered at $x=\frac{\pi}{3}$

(C) $\sqrt{x}$ centered at $x=1$

(D) $\ln x$ centered at $x=1$

(E) $\sqrt{x-1}$ centered at $x=0$

27. The radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n n^2}{n!}$ is

(A) 0 (B) 1 (C) 2 (D) e (E) ∞

28. Suppose that $\sum_{n=1}^{\infty} a_n \geqslant \sum_{n=1}^{\infty} b_n > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges. Which of the following must be true?

I. $\sum_{n=1}^{\infty} a_n$ diverges.

II. $\lim_{N\to\infty} \sum_{i=1}^{N} a_i = \infty$.

III. $\lim_{N\to\infty} \sum_{i=1}^{N} b_i = \infty$.

(A) I only

(B) I and II only

(C) II and III only

(D) I and III only

(E) I, II, and III

29. For $-1 < x < 1$, which series is equivalent to $\dfrac{1}{1-x^2}$?

(A) $x^2 - x^4 + x^6 - x^8 + \cdots$

(B) $x^2 + x^4 + x^6 + x^8 + \cdots$

(C) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(D) $1 - x^2 + x^4 + -x^6 + x^8 + \cdots$

(E) $-1 - x^2 - x^4 - x^6 - x^8 - \cdots$

30. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^3+9}}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5}}{\sqrt{n^6+9}}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{3n^4+4}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+2}}$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{\sqrt{5n^4+3}}$

31. Let $a_n = \dfrac{n^2\sqrt{n^2+4}}{n^3}$. Which choice is correct?

(A) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| > 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(B) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| < 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(C) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| < 1$, $\sum\limits_{n=1}^{\infty} a_n$ converges by the Ratio Test.

(D) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(E) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test for $\sum\limits_{n=1}^{\infty} a_n$ is inconclusive.

32. Function f has derivatives of all orders. At $x = 0$, $f(x)$ is decreasing and concave up. Which choice could be the third-degree Maclaurin polynomial for f ?

(A) $2+x-\frac{1}{3}x^2+4x^3$

(B) $2+x+\frac{1}{3}x^2-4x^3$

(C) $-2-x-\frac{1}{3}x^2-4x^3$

(D) $-2-x+\frac{1}{3}x^2-4x^3$

(E) $2+x+\frac{1}{3}x^2+4x^3$

33. Let $a_n = \dfrac{1}{\sqrt{n+2}}$ for $n \geqslant 0$. Let f be a continuous function such that $f(n) = a_n$. Which choice must be true?

(A) $\displaystyle\sum_{n=2}^{\infty} a_n \leqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(B) $\displaystyle\sum_{n=2}^{\infty} a_n \geqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(C) $\displaystyle\sum_{n=2}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(D) $\displaystyle\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(E) It cannot be determined from the given information.

34. $\dfrac{d}{dx}\sum_{n=1}^{\infty}\dfrac{x^{3n+1}}{n!} =$

(A) $\dfrac{x^5}{5}+\dfrac{x^8}{8(2!)}+\dfrac{x^{11}}{11(3!)}+\cdots+\dfrac{x^{3n+2}}{(3n+2)n!}+\cdots$

(B) $4x+\dfrac{7}{2!}x^4+\dfrac{10}{3!}x^7+\cdots+(3n+1)\dfrac{3^{3n-2}}{n!}+\cdots$

(C) $\dfrac{x^7}{7}+\dfrac{x^{10}}{10(2!)}+\dfrac{x^{13}}{13(3!)}+\cdots+\dfrac{x^{3n+4}}{(3n+4)n!}+\cdots$

(D) $4x^3+\dfrac{1}{2!}x^6+\dfrac{1}{3!}x^9+\cdots+\dfrac{x^{3n}}{n!}+\cdots$

(E) $4x^3+\dfrac{7}{2!}x^6+\dfrac{10}{3!}x^9+\cdots+(3n+1)\dfrac{x^{3n}}{n!}+\cdots$

35. $\sum_{n=1}^{\infty}\left(\dfrac{1}{n^2+n}\right)$ is

(A) 0 (B) 1 (C) $\dfrac{3}{2}$ (D) 2 (E) divergent

36. Let $S = \sum_{n=1}^{\infty} \frac{\sin n}{n^2}$. Which of the following must be true?

I. S converges absolutely.

II. S converges conditionally.

III. S converges.

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

37. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!}$. What is the coefficient of the x^6 term in $\int_0^x f(t)\,\mathrm{d}t$?

(A) $-\frac{7}{6}$ (B) $-\frac{1}{12}$ (C) $-\frac{1}{48}$ (D) $\frac{1}{12}$ (E) $\frac{7}{6}$

38. What is the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$?

(A) The series diverges for all x.

(B) $-1 < x < 1$

(C) $-1 \leqslant x \leqslant 1$

(D) $0 < x < 1$

(E) $0 \leqslant x \leqslant 1$

39. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+4}}{n^2}$. The coefficient of the x^6 term in $f'(x)$ is

(A) $-\frac{7}{9}$ (B) $-\frac{1}{4}$ (C) $-\frac{1}{24}$ (D) $\frac{1}{6}$ (E) $\frac{7}{9}$

40. Function g has derivatives of all orders. The fourth-degree Maclaurin polynomial of $g(x)$ is $m(x)$. It is known that $|g^{(5)}(x)| \leqslant 0.7$ for $0 \leqslant x \leqslant 0.4$. Which choice must be true?

(A) $|g(0.4) - m(0.4)| \leqslant 0.7$

(B) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{4!}(0.4)^4$

(C) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{4!}(0.4)^4$

(D) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{5!}(0.4)^5$

(E) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{5!}(0.4)^5$

41. The power series $S = \sum_{n=0}^{\infty} c_n (x-5)^n$ converges at $x = 7$ and diverges at $x = 8$. Which of the following must be true?

I. S converges at $x = 4$.

II. S converges at $x = 3$.

III. S diverges at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

42. The power series $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ has a radius of convergence of R. The power series of which of the following must also have a radius of convergence of R ?

I. $f'(x)$

II. $\int f(x)\,\mathrm{d}x$

III. $f(2x)$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

43. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+8}}$. Using the Alternating Series Error Bound, what is the least number of terms must be summed to guarantee a partial sum that is within $\frac{1}{30}$ of S ?

(A) 22 (B) 30 (C) 891 (D) 900 (E) 908

44. The partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^3}$ is used to estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Based on the Integral Test, what is the smallest value of k such that the error in S_k is no more than $\frac{1}{200}$?

(A) 2 (B) 10 (C) 50 (D) 100 (E) 200

45. Let $f(x) = \frac{x}{1+x^6}$. Let $g(x) = \int f(x)\,\mathrm{d}x$ and $g(0) = 1$. What is the Maclaurin series expansion of g ?

(A) $\frac{x^2}{2} - \frac{x^8}{8} + \frac{x^{14}}{14} - \frac{x^{20}}{20} + \cdots + \frac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(B) $x - x^7 + x^{13} - x^{19} + \cdots + (-1)^n x^{6n+1} + \cdots$

(C) $1 + x - x^7 + x^{13} + \cdots + (-1)^n x^{6n+1} + \cdots$

(D) $1 + \frac{x^2}{2} - \frac{x^8}{8} + \frac{x^{14}}{14} + \cdots + \frac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(E) $1 - 7x^6 + 13x^{12} - 19x^{18} + \cdots + (-1)^n (6n+1) x^{6n} + \cdots$

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. C
2. D
3. C
4. A
5. E
6. D
7. C
8. A
9. D
10. C
11. E
12. B
13. E
14. A
15. A
16. D
17. B
18. A
19. B
20. C
21. E
22. A
23. B
24. D
25. D
26. E
27. E
28. E
29. C
30. C
31. E
32. D
33. A
34. E
35. B
36. C
37. D
38. C
39. E
40. E
41. A
42. D
43. C
44. B
45. D