Chapter 4 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 4: Integration.

- **4.1**: Antidifferentiation
- **4.2**: Definite Integrals
- **4.3**: Fundamental Theorem of Calculus
- **4.4**: Integration by Substitution
- **4.5**: Numerical Integration

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Integration

Number of Questions—50

NO CALCULATOR

1. How many antiderivatives does $f(x) = x^3 + 5x - 7$ have?

(A) None

(B) One

(C) Two

(D) Three

(E) Infinitely many

2. $\dfrac{d}{dx}\displaystyle\int_{\pi/3}^{x} \cos t \, dt =$

(A) $\dfrac{1}{2}$

(B) $\sin x$

(C) $\cos x$

(D) $\sin x - \dfrac{\sqrt{3}}{2}$

(E) $\cos x - \dfrac{1}{2}$

3. For which of the following mathematical concepts is the result expected to include a constant of integration, C ?

I. An indefinite integral

II. A definite integral

III. A derivative

IV. A particular solution to a differential equation

(A) I only

(B) I and II only

(C) I and IV only

(D) I, II, and IV only

(E) I, II, III, and IV

4. Ducks enter a pond at a rate given by $f(t)$, where t is the number of hours after midnight. What is the best interpretation of the equation $\int_0^5 f(t)\,\mathrm{d}t = 60$?

(A) At 5 AM, there are 60 ducks in the pond.

(B) From 12 AM to 5 AM, 60 ducks enter the pond every hour.

(C) From 12 AM to 5 AM, the rate at which ducks enter the pond increases by 60 ducks per hour.

(D) From 12 AM to 5 AM, the pond always has 60 ducks.

(E) From 12 AM to 5 AM, 60 ducks enter the pond.

5. $\int_{-\pi/3}^{\pi/6} \cos x\,\mathrm{d}x =$

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1-\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{3}}{2}$ (E) $\frac{\sqrt{3}-1}{2}$

6. $\int \left(\sqrt[3]{x} - \cos x + \frac{1}{x} \right) \mathrm{d}x =$

(A) $\frac{1}{3}x^{-2/3} + \sin x + \ln|x| + C$

(B) $\frac{1}{3}x^{-2/3} + \sin x - \ln|x| + C$

(C) $\frac{1}{3}x^{-2/3} - \sin x + \ln|x| + C$

(D) $\frac{3}{4}x^{4/3} + \sin x + \ln|x| + C$

(E) $\frac{3}{4}x^{4/3} - \sin x + \ln|x| + C$

7. $\int_1^2 \left(3x^2 + 6x + 4\right) \mathrm{d}x =$

(A) 6 (B) 18 (C) 20 (D) 28 (E) 30

8. Which definite integral is equivalent to $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{1}{1+\left(4+\frac{2i}{n}\right)^3}\right)\frac{2}{n}$?

(A) $\int_0^2 \frac{1}{4+x^3}\,\mathrm{d}x$

(B) $\int_0^4 \frac{1}{1+(4+x)^3}\,\mathrm{d}x$

(C) $\int_4^6 \frac{1}{x^3}\,\mathrm{d}x$

(D) $\int_4^6 \frac{1}{1+x^3}\,\mathrm{d}x$

(E) $\int_4^6 \frac{1}{1+(4+x)^3}\,\mathrm{d}x$

9. $\int_{-4}^{3} 5\,\mathrm{d}x =$

(A) -35 (B) -7 (C) -5 (D) 7 (E) 35

10. If $\dfrac{\mathrm{d}y}{\mathrm{d}t} = t^4 + 3t^2 - 8t + 9$, then

(A) $y = 12t^2 + 6 + C$

(B) $y = 4t^3 + 6t - 8 + C$

(C) $y = t^4 + 3t^2 - 8t + 9 + C$

(D) $y = \dfrac{1}{5}t^5 + t^3 - 4t^2 + 9t + C$

(E) $y = \dfrac{1}{30} + \dfrac{1}{4}t^4 - \dfrac{4}{3}t^3 + \dfrac{9}{2}t^2 + C$

11. $\displaystyle\int_4^{16} \frac{1}{x}\,\mathrm{d}x =$

(A) $-\dfrac{3}{16}$ (B) $\dfrac{3}{16}$ (C) $\ln 4$ (D) $\ln 12$ (E) $\ln 64$

12. $\int_{-7}^{7} \sqrt{49 - x^2}\,\mathrm{d}x =$

(A) $\frac{49\pi}{8}$ (B) $\frac{49\pi}{4}$ (C) $\frac{49\pi}{2}$ (D) 49π (E) 98π

13. $\int \frac{x-4}{x}\,\mathrm{d}x =$

(A) $-4\ln|x| + C$

(B) $x - \ln|x| + C$

(C) $x - 4\ln|x| + C$

(D) $x + 4\ln|x| + C$

(E) $\frac{x^2 - 8x}{x^2} + C$

14. $\int x^3 e^{x^4}\,\mathrm{d}x =$

(A) $\frac{1}{4}e^{x^4}+C$ (B) $e^{x^4}+C$ (C) $4e^{x^4}+C$ (D) $x^3e^{x^4}+C$ (E) $\frac{e^{x^4}}{x^3}+C$

15. Let f be a twice-differentiable function. If $f(0)=4$, $f(1)=7$, and $f(4)=8$, then $\int_0^1 f'(3x+1)\,\mathrm{d}x =$

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) 7 (E) 11

16. $\int x\sqrt[3]{x}\,\mathrm{d}x =$

(A) $\frac{4}{3}\sqrt[3]{x}+C$

(B) $\frac{3}{4}\sqrt[3]{x^4}+C$

(C) $\sqrt[3]{x^4}+C$

(D) $\frac{3}{7}\sqrt[3]{x^7}+C$

(E) $\frac{3}{8}\sqrt[3]{x^{10}}+C$

17. If $g(1)=8$ and $\int_4^1 g'(x)\,\mathrm{d}x = 7$, where g' is continuous on $[1,4]$, then what is the value of $g(4)$?

(A) -15

(B) -1

(C) 1

(D) 15

(E) It cannot be determined.

18. $\displaystyle\int \frac{1}{(x^2+1)\tan^{-1}x}\,\mathrm{d}x =$

(A) $\ln\left(x^2+1\right)+C$

(B) $\ln\left|\tan^{-1}x\right|+C$

(C) $\dfrac{-1}{(\tan^{-1}x)^2}+C$

(D) $\tan^{-1}x+C$

(E) $\tan^{-1}\left(x^2+1\right)+C$

19. A stone is thrown downward with an initial speed of 20 meters per second from a position 10 meters above the ground. The stone's height above the ground as a function of time t, in seconds, is

(A) $-4.9t^2+20t+10$

(B) $-4.9t^2-20t+10$

(C) $-4.9t^2+20t-10$

(D) $-4.9t^2-20t-10$

(E) $4.9t^2+20t-10$

20. $\displaystyle\int \frac{\tan^2 x}{\sec x \sin^2 x}\,\mathrm{d}x =$

(A) $\ln|\cos x| + C$

(B) $\ln|\sec x| + C$

(C) $-\ln|\csc x| + C$

(D) $\ln|\sec x + \tan x| + C$

(E) $-\ln|\csc x + \cot x| + C$

21. What is the value of $\displaystyle\int_{-1}^{1} \frac{x^4 \sin 3x}{\cos x}\,\mathrm{d}x$?

(A) 0

(B) 2

(C) 4

(D) 8

(E) It cannot be determined analytically.

22. Let f be a continuous function such that $f(0) = 1$, $f(2) = 3$, and $f(4) = 6$. A left Riemann sum with two equal-width subintervals estimates $\int_0^4 f(x)\,\mathrm{d}x$ to be

(A) 4 (B) 8 (C) 9 (D) 18 (E) 20

23. If $2 \leqslant f(x) \leqslant 8$ and f is integrable on $[1,6]$, then

(A) $2 \leqslant \int_1^6 f(x)\,\mathrm{d}x \leqslant 8$

(B) $10 \leqslant \int_1^6 f(x)\,\mathrm{d}x \leqslant 40$

(C) $12 \leqslant \int_1^6 f(x)\,\mathrm{d}x \leqslant 48$

(D) $14 \leqslant \int_1^6 f(x)\,\mathrm{d}x \leqslant 56$

(E) $20 \leqslant \int_1^6 f(x)\,\mathrm{d}x \leqslant 80$

24. $\int_0^1 (2x-1)^4 \, \mathrm{d}x =$

(A) $\frac{1}{40}$ (B) $\frac{1}{20}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$ (E) $\frac{2}{5}$

25. $\frac{\mathrm{d}}{\mathrm{d}x} \int_2^{x^4-3x+2} \frac{1}{t} \, \mathrm{d}t =$

(A) $\frac{1}{x^4-3x+2}$

(B) $\frac{4x^3-3}{x^4-3x+2}$

(C) $\ln\left|x^4-3x+2\right|$

(D) $\ln\left|x^4-3x+2\right| - \ln 2$

(E) $\left(4x^3-3\right)\ln\left|x^4-3x+2\right|$

26. Selected values of the continuous function $f(x)$ are shown in the table below.

x	0	1	2	3	4
$f(x)$	1	5	7	2	-3

A midpoint approximation with two equal-width subintervals approximates $\int_0^4 f(x)\,\mathrm{d}x$ to be

(A) 7 (B) 8 (C) 12 (D) 14 (E) 16

27. Water begins entering a tank at a rate given by $f(t) = 3t^2 + t + 1$, where $f(t)$ is measured in gallons per minute and t is measured in minutes. If the tank initially has 50 gallons, then how many gallons of water are in the tank after 2 minutes?

(A) 12 (B) 15 (C) 62 (D) 64 (E) 65

28. If $\int_2^4 g(x)\,\mathrm{d}x = 7$ and $\int_2^6 g(x)\,\mathrm{d}x = 4$, then $\int_4^6 g(x)\,\mathrm{d}x =$

(A) -11 (B) -3 (C) 3 (D) 11 (E) 28

29. Which function solves the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6x + 2$, $y(0) = 7$?

(A) $y = 6x - 6$

(B) $y = 6x + 13$

(C) $y = x^3 - 3x^2 + 2x$

(D) $y = x^3 - 3x^2 + 5$

(E) $y = x^3 - 3x^2 + 2x + 7$

30. By Simpson's Rule with $n=4$, $\displaystyle\int_2^{10} \frac{1}{x}\,\mathrm{d}x \approx$

(A) $\dfrac{73}{45}$ (B) $\dfrac{73}{15}$ (C) $\dfrac{121}{90}$ (D) $\dfrac{121}{30}$ (E) $\dfrac{137}{180}$

31. Which set contains all the critical numbers of $f(x) = \displaystyle\int_1^x \left(t^3 - 8t^2 - 20t\right)\,\mathrm{d}t$?

(A) $\left\{-\dfrac{16-\sqrt{496}}{6}, -\dfrac{16+\sqrt{496}}{6}\right\}$

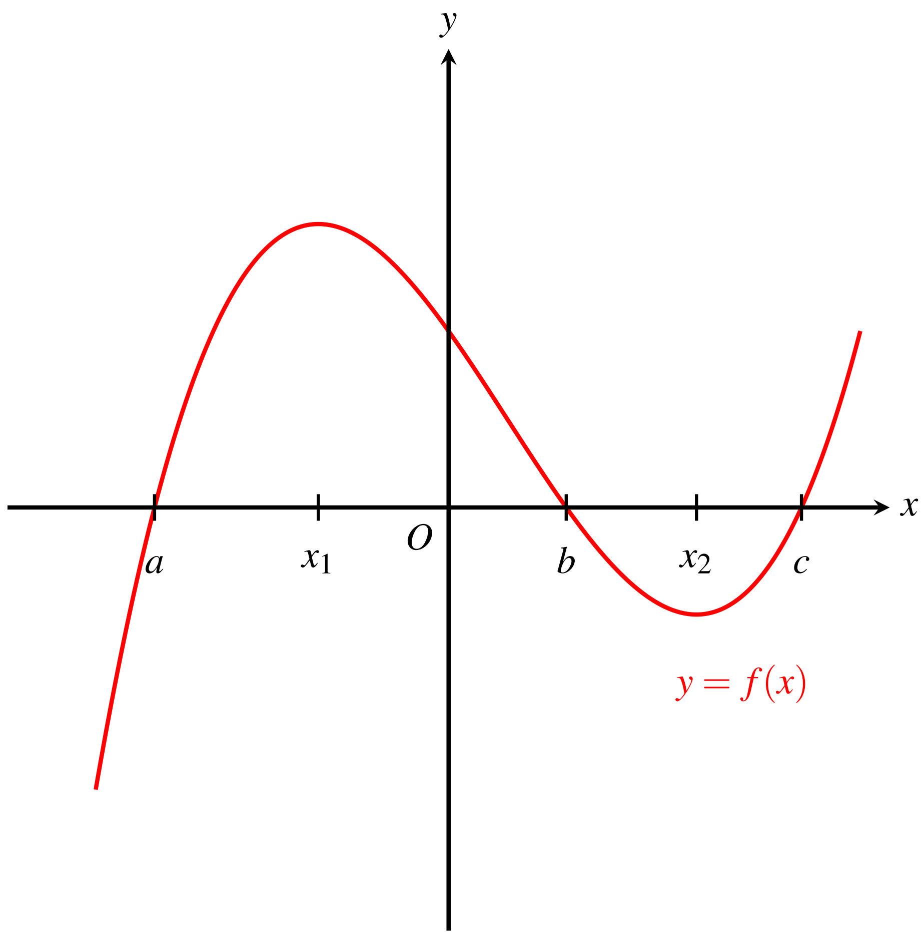
(B) $\left\{\dfrac{16-\sqrt{496}}{6}, \dfrac{16+\sqrt{496}}{6}\right\}$

(C) $\{-2, 0, 10\}$

(D) $\{-10, 0, 2\}$

(E) $\emptyset$

Questions 32–34 refer to the following graph of f. Let $g(x) = \int_0^x f(t)\,\mathrm{d}t$.



32. Which statement is true?

(A) $\int_a^b f(x)\,\mathrm{d}x < \int_a^c f(x)\,\mathrm{d}x < \int_b^c f(x)\,\mathrm{d}x$

(B) $\int_a^b f(x)\,\mathrm{d}x < \int_b^c f(x)\,\mathrm{d}x < \int_a^c f(x)\,\mathrm{d}x$

(C) $\int_a^c f(x)\,\mathrm{d}x < \int_b^c f(x)\,\mathrm{d}x < \int_a^b f(x)\,\mathrm{d}x$

(D) $\int_b^c f(x)\,\mathrm{d}x < \int_a^b f(x)\,\mathrm{d}x < \int_a^c f(x)\,\mathrm{d}x$

(E) $\int_b^c f(x)\,\mathrm{d}x < \int_a^c f(x)\,\mathrm{d}x < \int_a^b f(x)\,\mathrm{d}x$

33. g has a relative maximum at

(A) $x = a$ (B) $x = x_1$ (C) $x = b$ (D) $x = x_2$ (E) $x = c$

34. On the interval $a \leqslant x \leqslant c$, the graph of g is concave up for x in

(A) (a, b)

(B) (b, c)

(C) (x_1, x_2)

(D) (x_2, c)

(E) $(a, x_1) \cup (x_2, c)$

35. By Simpson's Rule with $n = 6$, $\int_0^\pi \sin x \, dx \approx$

(A) $\frac{\pi}{18}\left(2+\sqrt{3}\right)$

(B) $\frac{\pi}{9}\left(4+\sqrt{3}\right)$

(C) $\frac{\pi}{3}\left(4+\sqrt{3}\right)$

(D) $\frac{2\pi}{9}\left(1+\sqrt{3}\right)$

(E) $\frac{2\pi}{3}\left(1+\sqrt{3}\right)$

36. Which of the following statements are true about endpoint approximations of $I = \int_0^1 x^8 \sin x \, \mathrm{d}x$?

I. A left-endpoint approximation overestimates I.

II. A right-endpoint approximation underestimates I.

III. The Midpoint Rule is more effective than endpoint approximations in approximating I.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

37. The function $f(x) = \int_0^x \left(t^2 - 6t + 8\right) \mathrm{d}t$ is decreasing on

(A) $(-\infty, \infty)$

(B) $(-\infty, 2] \cup [4, \infty)$

(C) $(-\infty, 3]$

(D) $[2, 4]$

(E) $[3, \infty)$

38. $\int x\sqrt{3x+5}\,\mathrm{d}x =$

(A) $\frac{1}{9}(3x+5)^{5/2} - \frac{5}{9}(3x+5)^{3/2} + C$

(B) $\frac{2}{45}(3x+5)^{5/2} - \frac{10}{27}(3x+5)^{3/2} + C$

(C) $\frac{2}{15}(3x+5)^{5/2} - \frac{10}{9}(3x+5)^{3/2} + C$

(D) $\frac{2}{5}(3x+5)^{5/2} - \frac{10}{3}(3x+5)^{3/2} + C$

(E) $\frac{6}{5}(3x+5)^{5/2} - 10(3x+5)^{3/2} + C$

39. Starting at $x = 2$, a particle travels along the x-axis with a velocity function given by $v(t) = 6t - 4$. When $t = 3$, the particle is located at

(A) $x = 13$ (B) $x = 14$ (C) $x = 15$ (D) $x = 16$ (E) $x = 17$

40. $\displaystyle\int \frac{1}{x^2 - 6x + 9}\,\mathrm{d}x =$

(A) $-\dfrac{1}{x-3} + C$

(B) $\dfrac{1}{(x-3)^2} + C$

(C) $-\dfrac{1}{3(x-3)^3} + C$

(D) $-\ln|x-3| + C$

(E) $\ln\left[(x-3)^2\right] + C$

41. $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_{x^2}^{\sin x} e^{4t^2}\,\mathrm{d}t =$

(A) $e^{4\sin^2 x} - e^{4x^4}$

(B) $8xe^{4\sin^2 x} - 8xe^{4x^4}$

(C) $e^{4\sin^2 x}\sin x - x^2 e^{4x^4}$

(D) $e^{4\sin^2 x}\cos x - 2xe^{4x^4}$

(E) $8xe^{4\sin^2 x}\cos x - 16x^2 e^{4x^4}$

42. $\displaystyle\int x^3\sqrt{x^2+4}\,\mathrm{d}x =$

(A) $\dfrac{1}{6}\left(x^2+4\right)^3 - \left(x^2+4\right)^2 + C$

(B) $\dfrac{1}{5}\left(x^2+4\right)^{5/2} + \dfrac{4}{3}\left(x^2+4\right)^{3/2} + C$

(C) $\dfrac{1}{5}\left(x^2+4\right)^{5/2} - \dfrac{4}{3}\left(x^2+4\right)^{3/2} + C$

(D) $\dfrac{2}{5}\left(x^2+4\right)^{5/2} + \dfrac{8}{3}\left(x^2+4\right)^{3/2} + C$

(E) $\dfrac{2}{5}\left(x^2+4\right)^{5/2} - \dfrac{8}{3}\left(x^2+4\right)^{3/2} + C$

43. $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(2+\frac{3i}{n}\right)\frac{3}{n}$ is

(A) $\frac{7}{2}$ (B) 7 (C) $\frac{21}{2}$ (D) 21 (E) nonexistent

44. The Trapezoidal Rule with $n=4$ approximates $\int_0^{\pi}\cos^2 x\,\mathrm{d}x$ to be

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π

45. $\displaystyle\int \frac{1}{\sqrt{-x^2-12x}}\,\mathrm{d}x =$

(A) $-6\sin^{-1}\left(\dfrac{x+6}{6}\right)+C$

(B) $-\dfrac{1}{6}\sin^{-1}\left(\dfrac{x+6}{6}\right)+C$

(C) $\dfrac{1}{6}\sin^{-1}\left(\dfrac{x+6}{6}\right)+C$

(D) $\sin^{-1}\left(\dfrac{x+6}{6}\right)+C$

(E) $6\sin^{-1}\left(\dfrac{x+6}{6}\right)+C$

46. The Trapezoidal Rule with $n=4$ is used to approximate $\displaystyle\int_1^4 \ln x\,\mathrm{d}x$. The error bound for the estimate is

(A) $\dfrac{9}{8}$ (B) $\dfrac{9}{16}$ (C) $\dfrac{9}{32}$ (D) $\dfrac{9}{64}$ (E) $\dfrac{9}{128}$

47. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,\mathrm{d}x =$

(A) $\dfrac{1}{2}\sin^{-1}\left(x^2\right)+C$

(B) $\dfrac{1}{2}\tan^{-1}\left(x^2\right)+C$

(C) $\sin^{-1}\left(x^2\right)+C$

(D) $\tan^{-1}\left(x^2\right)+C$

(E) $2\sin^{-1}\left(x^2\right)+C$

48. Simpson's Rule with $n=8$ approximates $\displaystyle\int_0^{\pi} \cos 8x\,\mathrm{d}x$ with an error bound given by

(A) $\dfrac{\pi^3}{180}$ (B) $\dfrac{\pi^3}{90}$ (C) $\dfrac{\pi^5}{360}$ (D) $\dfrac{\pi^5}{180}$ (E) $\dfrac{\pi^5}{90}$

49. A left-endpoint approximation with $n = 32$ is used to approximate $\int_0^{\pi/4} \cos\left(32x^4\right)\,\mathrm{d}x$. The error bound in the estimate is

(A) $\dfrac{\pi^5}{32}\sin\left(\dfrac{\pi^4}{16}\right)$

(B) $\dfrac{\pi^5}{64}\sin\left(\dfrac{\pi^4}{16}\right)$

(C) $\dfrac{\pi^5}{128}\sin\left(\dfrac{\pi^4}{16}\right)$

(D) $\dfrac{\pi^5}{256}\sin\left(\dfrac{\pi^4}{16}\right)$

(E) $\dfrac{\pi^5}{512}\sin\left(\dfrac{\pi^4}{16}\right)$

50. $\lim\limits_{x\to 0} \dfrac{1}{\sin x}\int_1^{\cos x} 2t\,\mathrm{d}t$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) nonexistent

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. E
2. C
3. A
4. E
5. D
6. E
7. C
8. D
9. E
10. D
11. C
12. C
13. C
14. A
15. A
16. D
17. C
18. B
19. B
20. D
21. A
22. B
23. B
24. D
25. B
26. D
27. C
28. B
29. E
30. A
31. C
32. E
33. C
34. E
35. B
36. C
37. D
38. B
39. E
40. A
41. D
42. C
43. C
44. C
45. D
46. D
47. A
48. D
49. B
50. C