Chapter 7 Free-Response Review Exercises

Directions: These review exercises are free-response questions based on the content in Chapter 7: Further Applications of Integration.

7.1: Arc Length
7.2: Surface Areas of Revolution
7.3: Consumer Surplus and Producer Surplus
7.4: Moments and Centers of Mass
7.5: Hydrostatics
7.6: Probability

For each question, show all your work. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Further Applications of Integration

Number of Questions—16

Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points Earned	Points Available
Short Questions		70
Question 15		15
Question 16		15
TOTAL		**100**

Short Questions

1. A supply curve is $y = 4\sqrt[3]{x+8}$, and the market price is \$12. Calculate the producer surplus. *(5 pts.)*

2. Calculate the surface area generated by rotating the arc given by $y = \sqrt{25-x^2}$, $-3 \leqslant x \leqslant 3$, about the x-axis. *(5 pts.)*

3. Calculate the arc length of $x = \frac{1}{3}(y+1)^{3/2}$ from $(0,-1)$ to $(\frac{8}{3},3)$. *(5 pts.)*

4. A randomly selected number follows the following probability density function. *(5 pts.)*

$$\begin{cases} 3x^2e^{-x^3} & x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the number is greater than 1 ?

5. The portion of the graph of $y = \frac{1}{2}x^2$ from $(0,0)$ to $(2,1)$ is rotated about the y-axis. Calculate the surface area of the revolution. *(5 pts.)*

6. Find the mean of the following probability density function. *(5 pts.)*

$$\begin{cases} \frac{1}{3}e^x & 0 \leqslant x \leqslant \ln 3 \\ 0 & \text{otherwise} \end{cases}$$

7. A rectangle of width 4 meters and height 2 meters is submerged in water such that the top face is 3 meters below the water's surface. Calculate the hydrostatic force acting on the rectangle. *(5 pts.)*

8. At a call center, 60% of calls are answered within 3 minutes. Assuming an exponential probability density function, calculate the exact probability that a customer is helped within 5 minutes. *(5 pts.)*

9. Calculate the hydrostatic force acting on an equilateral triangle of side lengths 2 meters whose top vertex lies 1 meter beneath the water's surface.

10. What is the centroid of the region bounded by the graphs of $y = 4\cos x$, $y = 1$, $x = -\dfrac{\pi}{3}$, and $x = \dfrac{\pi}{3}$? *(5 pts.)*

11. Determine an arc length function for the graph of $y = 8 + 4t^{3/2}$ beginning at $(0, 8)$. *(5 pts.)*

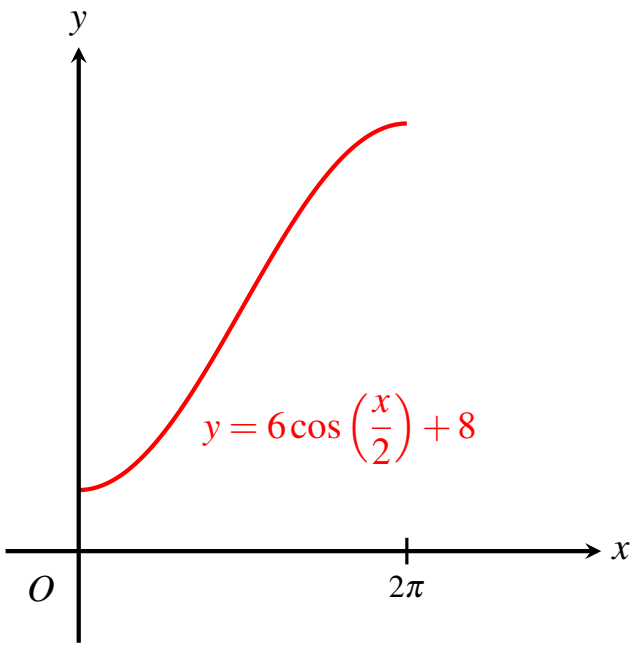
12. Find the centroid of the region bounded by the graphs of $x = 10 - y^2$ and $x = 1$. *(5 pts.)*

13. Determine the centroid of the region bounded by $y = 7 - x^2$, $y = x + 1$, and the y-axis. *(5 pts.)*

14. A circular metal disk of radius 2 feet is submerged such that the disk's center sits 6 feet beneath the water's surface. Set up, but do not evaluate, an integral that equals the hydrostatic force acting on the disk. *(5 pts.)*

Long Questions

15. Consider the graph of $y = -6\cos\left(\frac{x}{2}\right) + 8$ for $0 \leqslant x \leqslant 2\pi$.



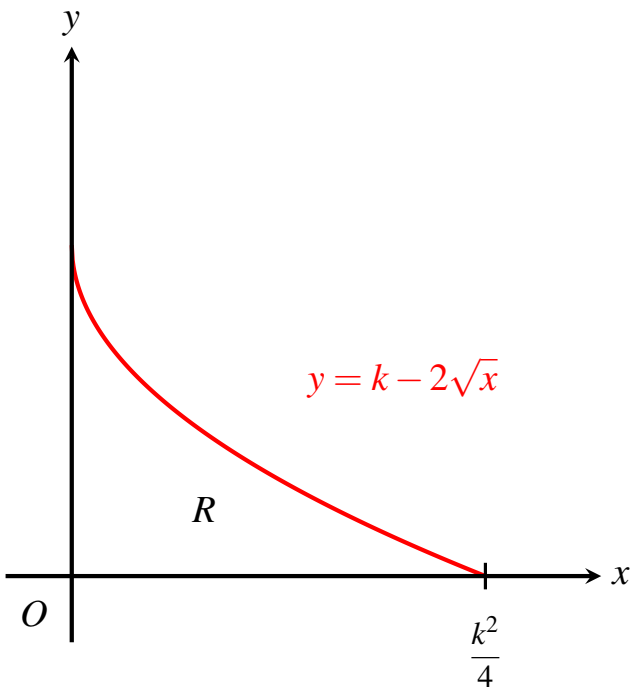
(a) Write, but do not evaluate, an integral that equals the curve's arc length. *(3 pts.)*

(b) Write, but do not evaluate, an integral that equals the surface area generated by rotating the curve about the x-axis. *(4 pts.)*

(c) Write, but do not evaluate, an integral that equals the surface area generated by rotating the curve about the y-axis. *(4 pts.)*

(d) A lamina is the shape bounded by the curve, the x-axis, the y-axis, and the line $x = 2\pi$. Set up integrals for the lamina's centroid $(\overline{x}, \overline{y})$. *(4 pts.)*

16. For $k > 0$, the following figure shows the graph of $y = k - 2\sqrt{x}$. Region R is bounded by this curve in the first quadrant.



(a) For $k = 6$, region R is a lamina of density $\rho = 2$. Calculate the lamina's moment about the y-axis. *(3 pts.)*

(b) Calculate the perimeter of R if $k = 8$. *(4 pts.)*

(c) The graph represents a demand curve for $k = 14$. Calculate the consumer surplus if the market price is \$6. *(4 pts.)*

(d) For what value of k is the following function a probability density function? *(5 pts.)*

$$f(x) = \begin{cases} k - 2\sqrt{x} & 0 \leqslant x \leqslant \dfrac{k^2}{4} \\ 0 & \text{otherwise} \end{cases}$$

This marks the end of the review exercises.