Chapter 1 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 1: Limits and Continuity.

1.1: Defining a Limit
1.2: Evaluating Limits Analytically
1.3: Squeeze Theorem and Trigonometric Limits
1.4: Continuity
1.5: Formal Definition of a Limit
1.6: Limits with Infinity

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Limits and Continuity

Number of Questions—45

NO CALCULATOR

1. If $\lim_{x\to 3} f(x) = 8$, then which option is true?

(A) $f(3) = 8$.

(B) As x approaches 3, $f(x)$ approaches 8.

(C) $f(8) = 3$.

(D) As x approaches 8, $f(x)$ approaches 3.

(E) f is continuous at $x = 3$.

2. $\lim_{x\to 0} \frac{\sin 3x}{x}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) nonexistent

3. $\lim_{x \to -7} \dfrac{x+7}{49-x^2}$ is

(A) -7 (B) $-\dfrac{1}{14}$ (C) $-\dfrac{1}{49}$ (D) $\dfrac{1}{14}$ (E) 49

4. $\lim_{x \to 0} \dfrac{x}{\tan x}$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

5. $\lim\limits_{x\to\infty} \dfrac{3x^4-8x^3+x^2-10}{2x^2-5x^3-10x^4}$ is

(A) $-\dfrac{3}{10}$ (B) 0 (C) $\dfrac{3}{10}$ (D) $\dfrac{3}{2}$ (E) nonexistent

6. The horizontal asymptote of $f(x)=\dfrac{3x^3+x^2-4}{8-5x^3}$ is

(A) $y=-\dfrac{5}{3}$ (B) $y=-\dfrac{3}{5}$ (C) $y=\dfrac{3}{8}$ (D) $y=\dfrac{3}{5}$ (E) $y=\dfrac{5}{3}$

7. $\lim\limits_{x\to\pi/2} \tan 2x$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) 1 (E) π

8. Function g is discontinuous at $x = 5$. Selected values of g are shown in the table below.

x	4.99	4.999	5	5.0001	5.001
$f(x)$	2.99	2.999	-4	3.001	3.01

A reasonable estimate for $\lim\limits_{x\to 5} g(x)$ is

(A) -4 (B) -3 (C) 2 (D) 3 (E) 5

9. If $f(x) = \begin{cases} 3x - 2\cos x & x < \pi \\ x^2 & x \geqslant \pi, \end{cases}$ then $\lim\limits_{x \to \pi^-} f(x)$ is

(A) $-\pi^2$ (B) 3π (C) $3\pi - 2$ (D) $3\pi + 2$ (E) π^2

10. $\lim\limits_{x \to 3} \dfrac{\sqrt{x-2} - 1}{9 - 3x}$ is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{6}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{2}$ (E) nonexistent

11. Given that $\lim_{x \to a} f(x)$ exists, which statements must be true?

I. $f(x)$ is continuous at $x = a$.

II. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

III. $f(a)$ is defined.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

12. If $\lim_{k \to 0} \dfrac{e^k - 1}{k} = 1$, then $\lim_{k \to 0} \dfrac{e^2 - e^{2+k}}{k}$ is

(A) $-e^2$ (B) $-e^{-2}$ (C) e^{-2} (D) 1 (E) e^2

13. $g(x) = \dfrac{x^2 - 5x + 6}{x - 3}$ has a removable discontinuity at

(A) $x = -3$ (B) $x = -2$ (C) $x = 0$ (D) $x = 2$ (E) $x = 3$

14. $\lim\limits_{x \to \infty} \dfrac{\sin(2x)}{x + 2}$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) ∞

15. Let $f(x)=\begin{cases} 2-kx & x \leqslant 3 \\ kx^2-22 & x>3. \end{cases}$ For what value of k is f continuous at $x=3$?

(A) -4 (B) 0 (C) 1 (D) 2 (E) 3

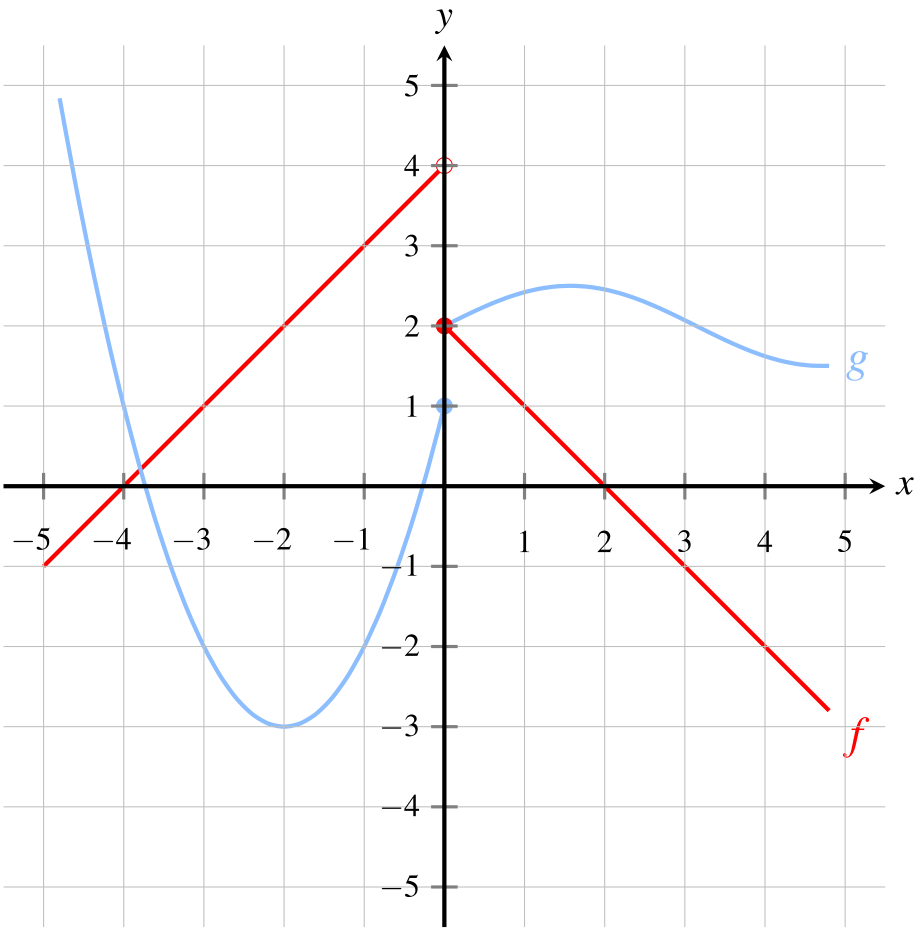
16. The oblique asymptote of $f(x)=\dfrac{x^2+7x+1}{x-2}$ is

(A) $y=9$ (B) $y=x-9$ (C) $y=x$ (D) $y=x+5$ (E) $y=x+9$

17. $\lim\limits_{x\to\infty} \dfrac{\sqrt{4x^2-1}}{x+3}$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) nonexistent

Questions 18–23 refer to the following graph.



18. $\lim_{x \to 0} f(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

19. $\lim_{x \to 0^+} g(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

20. $\lim_{x \to -2} [f(x) - g(x)]$ is

(A) -5 (B) -1 (C) 1 (D) 5 (E) nonexistent

21. $\lim\limits_{x\to -2}[g(x)]^2$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) 9

22. $\lim\limits_{x\to 0}[f(x)g(x)]$ is

(A) 1 (B) 2 (C) 4 (D) 8 (E) nonexistent

23. $\lim\limits_{x\to 0} f(-x^2)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

24. On what interval of x is $f(x) = \dfrac{\ln(x-2)}{x^2-9}$ continuous?

(A) $(-\infty,-3)\cup(3,\infty)$

(B) $(-\infty,-2)$

(C) $(2,\infty)$

(D) $(3,\infty)$

(E) $(2,3)\cup(3,\infty)$

25. $\displaystyle\lim_{x\to-3}\frac{5\sin(x+3)}{6+2x}$ is

(A) 0 (B) $\dfrac{5}{6}$ (C) 1 (D) $\dfrac{5}{2}$ (E) nonexistent

26. Function f is continuous and satisfies $f(4) = 8$. If $\lim_{x\to 2} g(x) = 4$, then $\lim_{x\to 2} f(g(x))$ is

(A) -8 (B) -4 (C) 2 (D) 4 (E) 8

27. $\lim_{x\to 2} \dfrac{\frac{1}{2} - \frac{1}{x}}{2 - x}$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) nonexistent

28. $\lim\limits_{x \to 0} x \sin\left(\dfrac{1}{x^2}\right)$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E) nonexistent

29. Let $f(x) = \begin{cases} x^2+1 & x < p \\ 2x & x \geqslant p. \end{cases}$ If $f(x)$ is continuous at $x = p$, then p is

(A) -1 (B) 0 (C) 1 (D) 2 (E) 4

30. $\lim_{t\to\infty} \sin t$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

31. If $\lim_{x\to 2} f(x) = -3$, then $\lim_{x\to 2} \left([f(x)]^2 - 2x\right)$ is

(A) -7 (B) -3 (C) 4 (D) 5 (E) 9

32. Function f is continuous. Selected values of $f(x)$ are shown in the table below.

x	-1	2	3	6	11
$f(x)$	2	1	1	1	2

Following the Intermediate Value Theorem, which value of $f(x)$ is guaranteed to exist for $-1 \leqslant x \leqslant 11$?

(A) 0 (B) 3 (C) 5 (D) 6 (E) 11

33. $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{4x^6 - 4x^2 + 1}}{3x^3 + 2}$ is

(A) $-\infty$ (B) $-\dfrac{2}{3}$ (C) $\dfrac{2}{3}$ (D) ∞ (E) nonexistent

34. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

35. At $x=4$, which choice about $g(x)=\dfrac{12+x-x^2}{x-4}$ is true?

(A) $g(x)$ has a vertical asymptote at $x=4$.

(B) $g(x)$ has a jump discontinuity at $x=4$.

(C) $g(x)$ has a removeable discontinuity at $x=4$.

(D) $\lim_{x\to 4} g(x)$ does not exist.

(E) $g(x)$ is continuous at $x=4$.

36. $\lim\limits_{x \to 0^+} \ln(\sin x)$ is

(A) $-\infty$ (B) 0 (C) 1 (D) e (E) ∞

37. $\lim\limits_{x \to 0} \dfrac{x^2}{\sin^2 2x}$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) 1 (D) 4 (E) nonexistent

38. $\lim\limits_{x \to 0} \dfrac{x - x\cos(x)}{x^2}$ is

(A) $-\pi$ (B) 0 (C) 1 (D) π (E) nonexistent

39. $\lim\limits_{x \to 0^+} \arctan\left(\dfrac{1}{x}\right)$ is

(A) $-\infty$ (B) $-\dfrac{\pi}{2}$ (C) 0 (D) $\dfrac{\pi}{2}$ (E) ∞

40. $\lim_{x\to\infty} \dfrac{5e^x - x}{8e^x + 9}$ is

(A) $-\dfrac{5}{8}$ (B) 0 (C) $\dfrac{5}{8}$ (D) 1 (E) ∞

41. $\lim_{x\to\pi/4} \dfrac{\cos 2x}{\cos x - \sin x}$ is

(A) 0 (B) $\dfrac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) π (E) nonexistent

42. Functions g and h are continuous and satisfy $g(1) = h(1) = 3$. Function f satisfies $g(x) \leqslant f(x) \leqslant h(x)$ for $0 \leqslant x \leqslant 2$. Which statements must be true?

I. $\lim_{x \to 1} g(x) = \lim_{x \to 1} h(x) = 3$.

II. $\lim_{x \to 1} f(x) = 3$.

III. $f(x)$ is continuous at $x = 1$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

43. If $f(1) = 2$, $\lim_{x \to 1^-} f(x) = 4$, and $\lim_{x \to 1^+} f(x) = -1$, then $\lim_{x \to 1} f(\cos(x - 1))$ is

(A) -1 (B) 0 (C) 2 (D) 4 (E) nonexistent

44. If $\lim_{x \to 3} (2x+4) = 10$, then $|(2x+4)-10| < \varepsilon$ and $|x-3| < \delta$, where $\delta =$

(A) $\frac{\varepsilon}{4}$ (B) $\frac{\varepsilon}{2}$ (C) ε (D) 2ε (E) 4ε

45. If $\lim_{x \to a} f(x) = \infty$, then which option is true?

(A) For positive M, there exists a positive δ such that $f(x) > M$ for $|x-a| > 0$.

(B) For positive M, there exists a positive δ such that $f(x) < M$ for $|x-a| > \delta$.

(C) For positive M, there exists a positive δ such that $f(x) > M$ for $|x-a| > \delta$.

(D) For positive M, there exists a positive δ such that $f(x) < M$ for $0 < |x-a| < \delta$.

(E) For positive M, there exists a positive δ such that $f(x) > M$ for $0 < |x-a| < \delta$.

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. B
2. D
3. B
4. C
5. A
6. B
7. C
8. D
9. D
10. B
11. B
12. A
13. E
14. B
15. D
16. E
17. C
18. E
19. C
20. D
21. E
22. C
23. D
24. E
25. D
26. E
27. B
28. A
29. A
30. E
31. D
32. A
33. A
34. C
35. C
36. A
37. B
38. B
39. D
40. C
41. C
42. E
43. D
44. B
45. E