VALCALC

Chapter 6 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 6: Integration Techniques.

6.1: Integration by Parts
6.2: Trigonometric Integrals
6.3: Trigonometric Substitution
6.4: Integration by Partial Fractions
6.5: Improper Integrals

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Integration Techniques

Number of Questions—50

NO CALCULATOR

1. $\displaystyle\int_{-4}^{5} \frac{1}{(x+2)^2}\,\mathrm{d}x$ is

(A) $-\dfrac{9}{14}$ (B) $-\dfrac{5}{14}$ (C) $\dfrac{5}{14}$ (D) $\dfrac{9}{14}$ (E) divergent

2. The best substitution for evaluating $\displaystyle\int \frac{1}{x^2\sqrt{9x^2-25}}\,\mathrm{d}x$ is

(A) $x = \dfrac{3}{5}\sin\theta$

(B) $x = \dfrac{3}{5}\sec\theta$

(C) $x = \dfrac{5}{3}\sin\theta$

(D) $x = \dfrac{5}{3}\tan\theta$

(E) $x = \dfrac{5}{3}\sec\theta$

3. When the rational function $f(x) = \dfrac{x^3+6}{x^2\left(5x^2-7x+4\right)\left(x^2-8x-9\right)}$ is decomposed into partial fractions, there are

(A) 4 fractions and 4 unknown constants

(B) 4 fractions and 5 unknown constants

(C) 5 fractions and 5 unknown constants

(D) 5 fractions and 6 unknown constants

(E) 6 fractions and 6 unknown constants

4. $\displaystyle\int \sec^4 x \tan^{12} x \, \mathrm{d}x =$

(A) $\dfrac{1}{13}\tan^{13} x + C$

(B) $\dfrac{1}{15}\tan^{15} x + C$

(C) $\dfrac{1}{15}\sec^{15} x + C$

(D) $\dfrac{1}{15}\tan^{15} x + \dfrac{1}{13}\tan^{13} x + C$

(E) $\dfrac{1}{15}\sec^{15} x + \dfrac{1}{13}\sec^{13} x + C$

5. $\int_1^3 xe^x \, dx =$

(A) $4e^3 - 2e$ (B) $3e^3 - e - 2$ (C) $2e^3 - 2$ (D) $2e^3$ (E) $4e^3$

6. Consider the family of functions $f(x) = \dfrac{c}{x^2}$ where c is a positive constant. Which of the following integrals are improper?

I. $\int_1^{\infty} f(x) \, dx$

II. $\int_{-\infty}^{-1} f(x) \, dx$

III. $\int_{-1}^{1} f(x) \, dx$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

7. Which integration technique is most appropriate for evaluating $\int x^3 \sqrt[5]{x}\,\mathrm{d}x$?

(A) Trigonometric substitution

(B) Integration by Parts

(C) Partial fraction decomposition

(D) The Substitution Rule

(E) None of the above

8. $\int \cos^5\theta\,\mathrm{d}\theta =$

(A) $-\frac{1}{5}\sin^5\theta + \frac{2}{3}\sin^3\theta - \sin\theta + C$

(B) $-\frac{1}{5}\cos^5\theta + \frac{2}{3}\cos^3\theta - \cos\theta + C$

(C) $\frac{1}{5}\sin^5\theta - \frac{2}{3}\sin^3\theta + \sin\theta + C$

(D) $\frac{1}{5}\cos^5\theta - \frac{2}{3}\cos^3\theta + \cos\theta + C$

(E) $\frac{1}{6}\sin^6\theta\,\mathrm{d}\theta + C$

9. $\int \sin^3\theta \cos^2\theta \, d\theta =$

(A) $-\frac{1}{5}\sin^5\theta + \frac{1}{3}\sin^3\theta + C$

(B) $-\frac{1}{5}\cos^5\theta + \frac{1}{3}\cos^3\theta + C$

(C) $\frac{1}{5}\sin^5\theta - \frac{1}{3}\sin^3\theta + C$

(D) $\frac{1}{5}\cos^5\theta - \frac{1}{3}\cos^3\theta + C$

(E) $\frac{1}{5}\cos^5\theta + \frac{1}{3}\cos^3\theta + C$

10. $\int_{-\infty}^{-4} \frac{2}{x^3}\, dx$ is

(A) $-\frac{1}{32}$ (B) $-\frac{1}{16}$ (C) $-\frac{1}{8}$ (D) $-\frac{1}{4}$ (E) divergent

11. $\displaystyle\int \frac{1}{x^2-4}\,\mathrm{d}x =$

(A) $\dfrac{1}{4}\ln|(x+2)(x-2)|+C$

(B) $\dfrac{1}{4}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(C) $\dfrac{1}{4}\ln\left|\dfrac{x+2}{x-2}\right|+C$

(D) $\dfrac{1}{2}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(E) $\dfrac{1}{2}\ln\left|\dfrac{x+2}{x-2}\right|+C$

12. $\int \sin^2 4t \, dt =$

(A) $-\frac{1}{16}\cos 8t + C$

(B) $\frac{t}{2} - \frac{1}{16}\sin 8t + C$

(C) $\frac{t}{2} + \frac{1}{16}\sin 8t + C$

(D) $\frac{t}{2} - \frac{1}{16}\cos 8t + C$

(E) $\frac{t}{2} + \frac{1}{16}\cos 8t + C$

13. $\int x^6 \ln x \, dx =$

(A) $x^6 \ln x + \frac{x^7}{49} + C$

(B) $x^6 \ln x - \frac{x^7}{49} + C$

(C) $\frac{x^7}{7} \ln x - \frac{x^6}{6} + C$

(D) $\frac{x^7}{7} \ln x - \frac{x^7}{49} + C$

(E) $\frac{x^7}{7} \ln x + \frac{x^7}{49} + C$

14. $\int_0^\infty \cos x \, dx$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{2}$ (E) divergent

15. $\int x \sec x \tan x \, dx =$

(A) $x \sec x + \sec x \tan x + C$

(B) $x \sec x - \sec x \tan x + C$

(C) $x \sec x + \ln|\sec x + \tan x| + C$

(D) $x \sec x - \ln|\sec x + \tan x| + C$

(E) $x \sec x \tan x - \ln|\sec x + \tan x| + C$

16. $\int \tan^6 x \cos^9 x \, dx =$

(A) $-\frac{1}{9}\sin^9 x + C$

(B) $-\frac{1}{9}\sin^9 x - \frac{1}{7}\sin^7 x + C$

(C) $-\frac{1}{9}\sin^9 x + \frac{1}{7}\sin^7 x + C$

(D) $\frac{1}{9}\sin^9 x + \frac{1}{7}\sin^7 x + C$

(E) $\frac{1}{9}\sin^9 x - \frac{1}{7}\sin^7 x + C$

17. $\int_0^1 \frac{1}{\sqrt[3]{x}}\,\mathrm{d}x$ is

(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$ (E) divergent

18. Two continuous functions f and g satisfy $0 < g(x) \leqslant f(x)$. Which statements must be true?

I. If $\int_1^\infty f(x)\,\mathrm{d}x$ diverges, then $\lim\limits_{k\to\infty}\int_1^k f(x)\,\mathrm{d}x = \infty$.

II. If $\int_1^\infty g(x)\,\mathrm{d}x$ diverges, then $\int_1^\infty f(x)\,\mathrm{d}x$ also diverges.

III. If $\int_1^\infty g(x)\,\mathrm{d}x$ converges, then $\int_1^\infty f(x)\,\mathrm{d}x$ also converges.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

19. $\int \sin 7x \sin 3x \, \mathrm{d}x =$

(A) $-\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + C$

(B) $-\frac{1}{8}\cos 4x + \frac{1}{20}\cos 10x + C$

(C) $\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + C$

(D) $\frac{1}{8}\sin 4x - \frac{1}{20}\sin 10x + C$

(E) $\frac{1}{8}\sin 4x + \frac{1}{20}\sin 10x + C$

20. $\dfrac{1}{x^3+4x^2+8x} =$

(A) $-\dfrac{1}{8x} - \dfrac{5}{8\left(x^2+4x+8\right)}$

(B) $\dfrac{1}{8x} - \dfrac{5}{8\left(x^2+4x+8\right)}$

(C) $\dfrac{1}{8x} - \dfrac{x-4}{8\left(x^2+4x+8\right)}$

(D) $\dfrac{1}{8x} - \dfrac{x+4}{8\left(x^2+4x+8\right)}$

(E) $\dfrac{1}{8x} + \dfrac{x+4}{8\left(x^2+4x+8\right)}$

21. $\displaystyle\int \sin 3x \cos 2x \, \mathrm{d}x =$

(A) $-\dfrac{1}{10}\cos 5x - \dfrac{1}{2}\cos x + C$

(B) $-\dfrac{1}{10}\sin 5x - \dfrac{1}{2}\sin x + C$

(C) $\dfrac{1}{10}\cos 5x + \dfrac{1}{2}\cos x + C$

(D) $\dfrac{1}{10}\sin 5x + \dfrac{1}{2}\sin x + C$

(E) $\dfrac{1}{10}\sin 5x - \dfrac{1}{2}\sin x + C$

22. $\int_0^{\infty} \frac{1}{x^2+81}\,\mathrm{d}x$ is

(A) $\frac{\pi}{162}$ (B) $\frac{\pi}{81}$ (C) $\frac{\pi}{18}$ (D) $\frac{\pi}{9}$ (E) divergent

23. $\int \frac{2x+1}{x^2+8x+16}\,\mathrm{d}x =$

(A) $-7\ln|x+4| - \frac{2}{x+4} + C$

(B) $-2\ln|x+4| - \frac{7}{x+4} + C$

(C) $2\ln|x+4| - \frac{7}{x+4} + C$

(D) $2\ln|x+4| + \frac{7}{x+4} + C$

(E) $7\ln|x+4| + \frac{2}{x+4} + C$

24. $\int_{-1}^{\infty} e^{-6x}\,\mathrm{d}x$ is

(A) $\frac{1}{6}$ (B) 1 (C) $\frac{e^6}{6}$ (D) e^6 (E) divergent

25. $\int_0^4 \frac{1}{2x^2+7x+5}\,\mathrm{d}x =$

(A) $\frac{1}{3}\ln\left(\frac{5}{13}\right)$

(B) $\frac{1}{3}\ln\left(\frac{13}{25}\right)$

(C) $\frac{1}{3}\ln\left(\frac{25}{13}\right)$

(D) $\frac{1}{3}\ln 5+\frac{2}{3}\ln\left(\frac{5}{13}\right)$

(E) $\frac{1}{3}\ln 5+\frac{2}{3}\ln\left(\frac{13}{5}\right)$

26. $\int \cos 12x \cos 14x \, \mathrm{d}x =$

(A) $-\frac{1}{52}\sin 26x - \frac{1}{4}\sin 2x + C$

(B) $-\frac{1}{52}\cos 26x - \frac{1}{4}\cos 2x + C$

(C) $\frac{1}{52}\sin 26x - \frac{1}{4}\sin 2x + C$

(D) $\frac{1}{52}\sin 26x + \frac{1}{4}\sin 2x + C$

(E) $\frac{1}{52}\cos 26x + \frac{1}{4}\cos 2x + C$

27. $\int x\cos 9x \, \mathrm{d}x =$

(A) $\frac{1}{9}x\sin 9x - \frac{1}{81}\cos 9x + C$

(B) $\frac{1}{9}x\sin 9x + \frac{1}{81}\cos 9x + C$

(C) $\frac{1}{9}x\sin 9x + \frac{1}{9}\cos 9x + C$

(D) $x\cos 9x + \frac{1}{81}\cos 9x + C$

(E) $x\cos 9x - \frac{1}{81}\cos 9x + C$

28. $\int \sec^3 x \tan^5 x \, dx =$

(A) $-\frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C$

(B) $-\frac{1}{7}\sec^7 x + \frac{2}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

(C) $-\frac{1}{7}\tan^7 x + \frac{2}{5}\tan^5 x - \frac{1}{3}\tan^3 x + C$

(D) $\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$

(E) $\frac{1}{7}\tan^7 x - \frac{2}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$

29. $\displaystyle\int \frac{1}{x^3+8x^2}\,\mathrm{d}x =$

(A) $-\dfrac{1}{8x}+\dfrac{1}{64}\ln\left|x^2+8x\right|+C$

(B) $-\dfrac{1}{8x}+\dfrac{1}{64}\ln\left|\dfrac{x+8}{x}\right|+C$

(C) $-\dfrac{1}{8x}+\dfrac{1}{64}\ln\left|\dfrac{x}{x+8}\right|+C$

(D) $\dfrac{1}{8x}+\dfrac{1}{64}\ln\left|\dfrac{x+8}{x}\right|+C$

(E) $\dfrac{1}{8x}+\dfrac{1}{64}\ln\left|\dfrac{x}{x+8}\right|+C$

30. $\displaystyle\int_1^\infty \frac{1}{x^{6s+2}}\,\mathrm{d}x$ diverges for

(A) $s > -\dfrac{1}{6}$ (B) $s < -\dfrac{1}{6}$ (C) $s \leqslant -\dfrac{1}{6}$ (D) $s < -\dfrac{1}{3}$ (E) $s \leqslant -\dfrac{1}{3}$

31. Let f and g be twice-differentiable functions such that $f(2) = 5$, $f(7) = 8$, $g(2) = 4$, and $g(7) = -3$. If $\int_2^7 f'(x)g(x)\,\mathrm{d}x = 12$, then $\int_2^7 f(x)g'(x)\,\mathrm{d}x =$

(A) -56 (B) -32 (C) -16 (D) 0 (E) 8

32. $\int \csc^{11} x \cot^3 x \,\mathrm{d}x =$

(A) $-\frac{1}{13}\cot^{13} x - \frac{1}{11}\cot^{11} x + C$

(B) $-\frac{1}{13}\cot^{13} x + \frac{1}{11}\cot^{11} x + C$

(C) $-\frac{1}{13}\csc^{13} x + \frac{1}{11}\csc^{11} x + C$

(D) $\frac{1}{13}\cot^{13} x - \frac{1}{11}\cot^{11} x + C$

(E) $\frac{1}{13}\csc^{13} x - \frac{1}{11}\csc^{11} x + C$

33. $\dfrac{x^3}{x^2-4x-5} =$

(A) $x+4-\dfrac{125}{6(x+1)}+\dfrac{1}{6(x-5)}$

(B) $x+4+\dfrac{125}{6(x+1)}+\dfrac{1}{6(x-5)}$

(C) $x+4-\dfrac{1}{6(x+1)}+\dfrac{125}{6(x-5)}$

(D) $x+4-\dfrac{1}{6(x+1)}-\dfrac{125}{6(x-5)}$

(E) $x+4+\dfrac{1}{6(x+1)}+\dfrac{125}{6(x-5)}$

34. $\displaystyle\int \csc^4\theta\cot^9\theta\,\mathrm{d}\theta =$

(A) $-\dfrac{1}{12}\cot^{12}\theta - \dfrac{1}{10}\csc^{10}\theta + C$

(B) $-\dfrac{1}{12}\csc^{12}\theta - \dfrac{1}{10}\csc^{10}\theta + C$

(C) $\dfrac{1}{12}\cot^{12}\theta + \dfrac{1}{10}\csc^{10}\theta + C$

(D) $\dfrac{1}{12}\cot^{12}\theta - \dfrac{1}{10}\csc^{10}\theta + C$

(E) $\dfrac{1}{12}\csc^{12}\theta + \dfrac{1}{10}\csc^{10}\theta + C$

35. $\displaystyle\int \frac{5}{x^3-9x}\,\mathrm{d}x =$

(A) $-\dfrac{5}{18}\ln|x| - \dfrac{5}{9}\ln\left|x^2-9\right| + C$

(B) $-\dfrac{5}{18}\ln|x| + \dfrac{5}{9}\ln\left|x^2-9\right| + C$

(C) $-\dfrac{5}{9}\ln|x| + \dfrac{5}{18}\ln\left|x^2-9\right| + C$

(D) $\dfrac{5}{18}\ln|x| + \dfrac{5}{9}\ln\left|x^2-9\right| + C$

(E) $\dfrac{5}{9}\ln|x| - \dfrac{5}{18}\ln\left|x^2-9\right| + C$

36. $\int \sin^{-1} 8x \, dx =$

(A) $x\sin^{-1} 8x + \frac{1}{8}\sin^{-1} 8x + C$

(B) $x\sin^{-1} 8x + \frac{\sqrt{1-64x^2}}{8} + C$

(C) $x\sin^{-1} 8x - \frac{\sqrt{1-64x^2}}{8} + C$

(D) $\frac{x\sqrt{1-64x^2}}{8} + \frac{\sqrt{1-64x^2}}{8} + C$

(E) $\frac{x\sqrt{1-64x^2}}{8} - \frac{\sqrt{1-64x^2}}{8} + C$

37. $\displaystyle\int \frac{1}{x\sqrt{x^2+81}}\,\mathrm{d}x =$

(A) $-\dfrac{1}{9}\ln\left|\dfrac{\sqrt{x^2+81}}{x}+\dfrac{9}{x}\right|+C$

(B) $-\dfrac{1}{9}\ln\left|\dfrac{x}{\sqrt{x^2+81}}+\dfrac{x}{9}\right|+C$

(C) $-\dfrac{1}{81}\ln\left|\dfrac{\sqrt{x^2+81}}{x}+\dfrac{9}{x}\right|+\dfrac{1}{81}\left(\dfrac{9}{\sqrt{x^2+81}}\right)+C$

(D) $-\dfrac{1}{81}\ln\left|\dfrac{x}{\sqrt{x^2+81}}+\dfrac{x}{9}\right|+\dfrac{1}{81}\left(\dfrac{\sqrt{x^2+81}}{9}\right)+C$

(E) $\ln\left|\dfrac{\sqrt{x^2+81}}{9}+\dfrac{x}{9}\right|+C$

38. $\displaystyle\int \frac{x^3}{\sqrt{x^2-4}}\,\mathrm{d}x =$

(A) $\ln\left|\dfrac{x}{2}+\dfrac{\sqrt{x^2-4}}{2}\right|+C$

(B) $\dfrac{64}{3\left(x^2-4\right)^{3/2}}+\dfrac{16}{\sqrt{x^2-4}}+C$

(C) $\dfrac{\left(x^2-4\right)^{3/2}}{3}+4\sqrt{x^2-4}+C$

(D) $\dfrac{\left(x^2-4\right)^{3/2}}{3}+\dfrac{4}{\sqrt{x^2-4}}+C$

(E) $\dfrac{3\left(x^2-4\right)^{3/2}}{64}+\dfrac{\sqrt{x^2-4}}{16}+C$

39. $\displaystyle\int \ln\left(x^9\right)\,\mathrm{d}x =$

(A) $9\ln x-9x+C$

(B) $9x\ln x+9x+C$

(C) $9x\ln x-9x+C$

(D) $9x\ln x+9\ln x+C$

(E) $9x\ln x-9\ln x+C$

40. $\displaystyle\int \frac{1}{\sin^3\theta\cos\theta}\,\mathrm{d}\theta =$

(A) $-\csc 2\theta\cot 2\theta - \frac{1}{2}\csc^2\theta + C$

(B) $-\csc 2\theta\cot 2\theta + \frac{1}{2}\csc^2\theta + C$

(C) $-\ln|\csc 2\theta + \cot 2\theta| + \frac{1}{2}\csc^2\theta + C$

(D) $-\ln|\csc 2\theta + \cot 2\theta| - \frac{1}{2}\csc^2\theta + C$

(E) $\ln|\csc 2\theta + \cot 2\theta| + \frac{1}{2}\csc^2\theta + C$

41. $\displaystyle\int \sec^6 5x\,\mathrm{d}x =$

(A) $-\frac{1}{25}\tan^5 5x - \frac{2}{15}\tan^3 5x - \frac{1}{5}\tan 5x + C$

(B) $\frac{1}{25}\tan^5 5x + \frac{2}{15}\tan^3 5x + \frac{1}{5}\tan 5x + C$

(C) $\frac{1}{25}\sec^5 5x + \frac{2}{15}\sec^3 5x + \frac{1}{5}\sec 5x + C$

(D) $\frac{1}{5}\sec^5 5x + \frac{2}{3}\sec^3 5x + \sec 5x + C$

(E) $\frac{1}{5}\tan^5 5x + \frac{2}{3}\tan^3 5x + \tan 5x + C$

42. $\int (3x+7)(x-2)^6\,\mathrm{d}x =$

(A) $\frac{1}{7}(3x+7)(x-2)^7 + \frac{3}{49}(x-2)^7 + C$

(B) $\frac{1}{7}(3x+7)(x-2)^7 - \frac{3}{49}(x-2)^7 + C$

(C) $\frac{1}{7}(3x+7)(x-2)^7 - \frac{3}{49}(x-2)^8 + C$

(D) $\frac{1}{7}(3x+7)(x-2)^7 + \frac{3}{56}(x-2)^8 + C$

(E) $\frac{1}{7}(3x+7)(x-2)^7 - \frac{3}{56}(x-2)^8 + C$

43. $\displaystyle\int \frac{x}{\sqrt{x^2-8x+20}}\,dx =$

(A) $\dfrac{x^2}{8}+x-6+C$

(B) $2\sqrt{x^2-8x+20}+C$

(C) $\dfrac{2x-6}{\sqrt{x^2-8x+20}}+C$

(D) $4\ln\left|\dfrac{\sqrt{x^2-8x+20}}{2}+\dfrac{x-4}{2}\right|+C$

(E) $4\ln\left|\dfrac{\sqrt{x^2-8x+20}}{2}+\dfrac{x-4}{2}\right|+\sqrt{x^2-8x+20}+C$

44. $\displaystyle\int_2^{\infty} \frac{1}{x\ln 6x}\,dx$ is

(A) $-\ln(\ln 12)$ (B) $-\dfrac{1}{\ln 12}$ (C) $\dfrac{1}{\ln 12}$ (D) $\ln(\ln 12)$ (E) divergent

45. Which choice is true about the convergence or divergence of $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$?

(A) Since $0 \leqslant \dfrac{\sin^2 x}{x^4} \leqslant \dfrac{1}{x^4}$ and $\displaystyle\int_1^{\infty} \frac{1}{x^4}\,\mathrm{d}x$ converges, $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$ converges.

(B) Since $0 \leqslant \dfrac{\sin^2 x}{x^4} \leqslant \dfrac{1}{x^4}$ and $\displaystyle\int_1^{\infty} \frac{1}{x^4}\,\mathrm{d}x$ diverges, $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$ converges.

(C) Since $0 \leqslant \dfrac{\sin^2 x}{x^4} \leqslant \dfrac{1}{x^4}$ and $\displaystyle\int_1^{\infty} \frac{1}{x^4}\,\mathrm{d}x$ diverges, $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$ diverges.

(D) Since $\dfrac{\sin^2 x}{x^4} \geqslant \dfrac{1}{x^4} \geqslant 0$ and $\displaystyle\int_1^{\infty} \frac{1}{x^4}\,\mathrm{d}x$ converges, $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$ converges.

(E) Since $\dfrac{\sin^2 x}{x^4} \geqslant \dfrac{1}{x^4} \geqslant 0$ and $\displaystyle\int_1^{\infty} \frac{1}{x^4}\,\mathrm{d}x$ diverges, $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x^4}\,\mathrm{d}x$ diverges.

46. $\int x^2 e^{-4x}\,\mathrm{d}x =$

(A) $-\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} - \frac{1}{32}e^{-4x} + C$

(B) $-\frac{1}{4}x^2e^{-4x} + \frac{1}{8}xe^{-4x} - \frac{1}{32}e^{-4x} + C$

(C) $-\frac{1}{4}x^2e^{-4x} + \frac{1}{8}xe^{-4x} + \frac{1}{32}e^{-4x} + C$

(D) $\frac{1}{4}x^2e^{-4x} + \frac{1}{8}xe^{-4x} + \frac{1}{32}e^{-4x} + C$

(E) $\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} + \frac{1}{32}e^{-4x} + C$

47. $\displaystyle\int \frac{1}{x^4-16}\,\mathrm{d}x =$

(A) $-\dfrac{1}{16}\tan^{-1}\left(\dfrac{x}{2}\right)+\dfrac{1}{32}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(B) $-\dfrac{1}{16}\tan^{-1}\left(\dfrac{x}{2}\right)-\dfrac{1}{32}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(C) $-\dfrac{1}{8}\tan^{-1}\left(\dfrac{x}{2}\right)+\dfrac{1}{32}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(D) $\dfrac{1}{16}\tan^{-1}\left(\dfrac{x}{2}\right)+\dfrac{1}{32}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(E) $\dfrac{1}{16}\tan^{-1}\left(\dfrac{x}{2}\right)-\dfrac{1}{32}\ln\left|\dfrac{x-2}{x+2}\right|+C$

48. $\int e^{3x} \cos x \, dx =$

(A) $-\frac{1}{10}e^{3x}\sin x - \frac{3}{10}e^{3x}\cos x + C$

(B) $\frac{1}{10}e^{3x}\sin x + \frac{3}{10}e^{3x}\cos x + C$

(C) $\frac{1}{10}e^{3x}\sin x - \frac{3}{10}e^{3x}\cos x + C$

(D) $\frac{1}{10}e^{3x}\sin x + \frac{3}{10}e^{3x}\sin x + C$

(E) $\frac{1}{10}e^{3x}\sin x - \frac{3}{10}e^{3x}\sin x + C$

49. $\displaystyle\int \frac{e^x}{e^{2x}+7e^x-8}\,\mathrm{d}x =$

(A) $\dfrac{1}{9}\ln\left|\dfrac{x-1}{x+8}\right|+C$

(B) $\dfrac{1}{9}\ln\left|\dfrac{x+8}{x-1}\right|+C$

(C) $\dfrac{1}{9}\ln\left|\dfrac{e^x-1}{e^x+8}\right|+C$

(D) $\dfrac{1}{9}\ln\left|\dfrac{e^x+8}{e^x-1}\right|+C$

(E) $\dfrac{1}{9}\ln\left|(e^x+8)(e^x-1)\right|+C$

50. $\displaystyle\int x^7 e^{x^4}\,\mathrm{d}x =$

(A) $x^4e^{x^4}-e^{x^4}+C$

(B) $x^4e^{x^4}+e^{x^4}+C$

(C) $\dfrac{x^4}{4}e^{x^4}+\dfrac{e^{x^4}}{4}+C$

(D) $\dfrac{x^4}{4}e^{x^4}-\dfrac{e^{x^4}}{4}+C$

(E) $\dfrac{x^4}{4}e^{x^4}-e^{x^4}+C$

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. E
2. E
3. D
4. D
5. D
6. E
7. E
8. C
9. D
10. B
11. B
12. B
13. D
14. E
15. D
16. C
17. D
18. C
19. D
20. D
21. A
22. C
23. D
24. C
25. C
26. D
27. B
28. D
29. B
30. C
31. A
32. C
33. E
34. A
36. B
37. A
38. C
39. C
40. D
41. B
42. E
43. E
44. E
45. A
46. A
47. A
48. B
49. C
50. D