ValCalc

Chapter 9 Review Exercises

Directions: These review exercises are multiple-choice questions based on the content in Chapter 9: Parametric Equations and Polar Coordinates.

9.1: Parametric Equations
9.2: Differentiating and Integrating Parametric Functions
9.3: Polar Coordinates and Functions
9.4: Differentiating Polar Functions
9.5: Areas with Polar Curves
9.6: Additional Calculus with Parametric and Polar

For each question, select the best answer provided. To make the best use of these review exercises, follow these guidelines:

- Print out this document and work through the questions as if this paper were an exam.
- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Try to spend no more than three minutes on each question. Work as quickly as possible without sacrificing accuracy.
- Do your figuring in the margins provided. If you encounter difficulties with a question, then move on and return to it later.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Parametric Equations and Polar Coordinates

Number of Questions—50

NO CALCULATOR

1. A point given in polar coordinates by $(6, 300^\circ)$ is expressed in Cartesian coordinates by

(A) $\left(-3, 3\sqrt{3}\right)$

(B) $\left(3, 3\sqrt{3}\right)$

(C) $\left(3, -3\sqrt{3}\right)$

(D) $\left(3\sqrt{3}, 3\right)$

(E) $\left(3\sqrt{3}, -3\right)$

2. The polar equation for the line $y = 12$ is

(A) $r = 12$

(B) $r = 12\cos\theta$

(C) $r = 12\sin\theta$

(D) $r = 12\sec\theta$

(E) $r = 12\csc\theta$

3. The polar curve $r = \cos 5\theta$ is

(A) a limacon with one loop

(B) a circle of radius 1

(C) a circle of radius 4

(D) a rose with five petals

(E) a rose with 10 petals

4. If $\omega \neq 0$, then which set of equations parameterizes the ellipse $\dfrac{x^2}{64} + \dfrac{y^2}{81} = 1$?

(A) $x = 8\sin\omega t, \qquad y = 8\sin\omega t$

(B) $x = 8\sin\omega t, \qquad y = 8\cos\omega t$

(C) $x = 8\sin\omega t, \qquad y = 9\cos\omega t$

(D) $x = 9\sin\omega t, \qquad y = 8\sin\omega t$

(E) $x = 9\sin\omega t, \qquad y = 9\cos\omega t$

5. For the polar curve $r = \cos^3\theta$, $\dfrac{dx}{d\theta} =$

(A) $-4\sin\theta\cos^2\theta$

(B) $-4\sin\theta\cos^3\theta$

(C) $4\sin\theta\cos^3\theta$

(D) $\cos^4\theta - 3\sin^2\theta\cos^2\theta$

(E) $\cos^4\theta + 3\sin^2\theta\cos^2\theta$

6. If $x = t^2 + 4$ and $y = 3t^3 - 6t$, then $\dfrac{dy}{dx} =$

(A) $9t^2 - 6$ (B) $\dfrac{9t^2-6}{2t}$ (C) $\dfrac{2t}{9t^2-6}$ (D) $\dfrac{3t^3-6t}{t^2+4}$ (E) $\dfrac{t^2+4}{3t^3-6t}$

7. The line $y = 4x$ is represented in polar coordinates by

(A) $r = 4$

(B) $r = \tan^{-1} 4$

(C) $r = 4\theta$

(D) $\theta = 4$

(E) $\theta = \tan^{-1} 4$

8. If $r = \cot\theta$, then $\dfrac{d^2y}{d\theta^2} =$

(A) $-\sin\theta$

(B) $-\cos\theta$

(C) $\sin\theta$

(D) $\cos\theta$

(E) $2\csc^2\theta\cot\theta$

9. If $x = \cos 2t$ and $y = t - 6$, then

(A) $x = \cos(y+6)$

(B) $x = \cos(y-6)$

(C) $x = \cos(y-12)$

(D) $x = \cos(2y+12)$

(E) $x = \cos(2y-12)$

10. If $f(\theta) = 1 + 2\cos\theta$, then which statement is true about the polar curve $r = f(\theta)$?

(A) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving toward the pole when $\theta = \frac{\pi}{4}$.

(B) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

(C) Since $f\left(\frac{\pi}{4}\right) > 0$ and $f'\left(\frac{\pi}{4}\right) > 0$, the graph is moving toward the pole when $\theta = \frac{\pi}{4}$.

(D) Since $f\left(\frac{\pi}{4}\right) < 0$ and $f'\left(\frac{\pi}{4}\right) < 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

(E) Since $f\left(\frac{\pi}{4}\right) < 0$ and $f'\left(\frac{\pi}{4}\right) > 0$, the graph is moving away from the pole when $\theta = \frac{\pi}{4}$.

11. On a platform 5 feet above the ground, a ball is thrown upward at an angle of 30° above the horizontal with a speed of 40 feet per second. If x is the ball's horizontal position from the platform and y is its height above the ground, then which set of parametric equations represents the ball's motion?

(A) $x = 20t$, $\quad y = -16t^2 + 20t\sqrt{3} + 5$

(B) $x = 20t$, $\quad y = -16t^2 - 20t\sqrt{3} + 5$

(C) $x = 20t\sqrt{3}$, $\quad y = -16t^2 - 20t + 5$

(D) $x = 20t\sqrt{3}$, $\quad y = -16t^2 + 20t + 5$

(E) $x = 40t$, $\quad y = -16t^2 + 40t + 5$

12. Which limacon has an inner loop?

(A) $r = 2 - \sin\theta$

(B) $r = 1 - \cos\theta$

(C) $r = 6 + 6\cos\theta$

(D) $r = 8 + 4\sin\theta$

(E) $r = 3 - 8\cos\theta$

13. If $x = 3t + 7$ and $y = \sqrt{t^3 - t - 5}$, then

(A) $y = \sqrt{\dfrac{x^3}{27} - \dfrac{x}{3} - 5}$

(B) $y = \sqrt{(x-7)^3 - (x-7) - 5}$

(C) $y = \sqrt{(3x+7)^3 - (3x+7) - 5}$

(D) $y = \sqrt{\dfrac{(x+7)^3}{27} - \dfrac{x+7}{3} - 5}$

(E) $y = \sqrt{\dfrac{(x-7)^3}{27} - \dfrac{x-7}{3} - 5}$

14. For the polar curve $r = 3 + \cos\theta$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

(A) $\dfrac{-3\cos\theta - \cos 2\theta}{3\sin\theta + \sin 2\theta}$

(B) $\dfrac{-3\cos\theta - \cos 2\theta}{3\cos\theta + \cos^2\theta}$

(C) $\dfrac{3\cos\theta + \cos^2\theta}{3\sin\theta + \sin\theta\cos\theta}$

(D) $\dfrac{3\sin\theta + \sin 2\theta}{-3\cos\theta - \cos 2\theta}$

(E) $\dfrac{3\sin\theta + \sin\theta\cos\theta}{3\cos\theta + \cos^2\theta}$

15. A curve is parameterized by the equations $x = \sqrt{t+3}$ and $y = 3t^2 - 8t + 2$. When $x = 2$, $\frac{dy}{dx} =$

(A) -8 (B) $-\frac{3}{2}$ (C) $-\frac{2}{\sqrt{5}}$ (D) 4 (E) $8\sqrt{5}$

16. What is the length of the curve parameterized by $x = \frac{2}{3}t^{3/2} + 8$ and $y = 2t^{3/2} - 4$ on the interval $0 \leqslant t \leqslant 2$?

(A) $\frac{2}{3}\sqrt{10}$ (B) $\frac{2}{3}\sqrt{80}$ (C) $\frac{8}{3}\sqrt{8}$ (D) $\frac{8}{3}\sqrt{10}$ (E) $\frac{8}{3}\sqrt{80}$

17. Which forms of symmetry does the polar curve $r = 2 - 3\sin\theta$ exhibit?

I. Symmetry about the x-axis

II. Symmetry about the y-axis

III. Symmetry about the origin

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

18. The area enclosed by $r = 2 + \cos\theta$ is

(A) 2π (B) 4π (C) $\dfrac{9\pi}{4}$ (D) $\dfrac{9\pi}{2}$ (E) 9π

19. Which equation represents the tangent line to the curve parameterized by $x = \sqrt[3]{t}$ and $y = t^3 - 4t^2 + 8$ at $t = 1$?

(A) $y - 5 = -15(x - 1)$

(B) $y - 5 = -5(x - 1)$

(C) $y - 5 = -\frac{1}{15}(x - 1)$

(D) $y - 5 = \frac{1}{3}(x - 1)$

(E) $y - 5 = 5(x - 1)$

20. A particle travels counterclockwise along a circle of radius 4 centered at $(5, -2)$. The particle begins at the top of the circle and completes one revolution over $0 \leqslant t \leqslant 3$. Which parametric equations describe the particle's motion?

(A) $x = 5 + 4\cos\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\sin\left(\frac{2\pi}{3}t\right)$

(B) $x = 5 - 4\cos\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\sin\left(\frac{2\pi}{3}t\right)$

(C) $x = 5 + 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\cos\left(\frac{2\pi}{3}t\right)$

(D) $x = 5 - 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\cos\left(\frac{2\pi}{3}t\right)$

(E) $x = 5 - 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 - 4\cos\left(\frac{2\pi}{3}t\right)$

21. An equation of the line tangent to the polar curve $r = 4\sin\theta$ at $\theta = \frac{\pi}{6}$ is

(A) $y - 1 = 2\left(x - \sqrt{3}\right)$

(B) $y - 1 = \sqrt{3}\left(x - \sqrt{3}\right)$

(C) $y - 1 = \frac{1}{\sqrt{3}}\left(x - \sqrt{3}\right)$

(D) $y - \sqrt{3} = \sqrt{3}\,(x - 1)$

(E) $y - \sqrt{3} = \frac{1}{\sqrt{3}}\,(x - 1)$

22. When $\theta = \frac{\pi}{3}$, the distance between the polar curves $r = 5\cos\theta$ and $r = 1 + 2\cos\theta$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) $\frac{3\sqrt{3}}{2}$

23. The area of the region enclosed by the curve $x = \sin t$, $y = 2t$ for $0 \leqslant t \leqslant \pi$ is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

24. If $x = 3\cos 2t$ and $y = 2\sin 2t$, then $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} =$

(A) $-\frac{2}{9}\csc^3 2t$

(B) $-\frac{2}{3}\cot 2t$

(C) $\frac{2}{3}\tan 2t$

(D) $\frac{4}{3}\csc^2 2t$

(E) $\frac{1}{3}\csc^2 2t \sec 2t$

25. The arc length of the curve parameterized by $x = 6t + 5$ and $y = 8 - \dfrac{16}{3}t^{3/2}$ for $0 \leqslant t \leqslant 1$ is

(A) $\dfrac{1}{16}$ (B) 4 (C) $\dfrac{49}{6}$ (D) 10 (E) $\dfrac{1568}{3}$

26. The area of the region that is outside the cardioid $r = 4 - 4\cos\theta$ and also inside the circle $r = 4$ is

(A) 4π (B) 8π (C) $16 - 2\pi$ (D) $32 - 4\pi$ (E) $64 - 8\pi$

27. If $x = 6 - e^{-8t}$ and $\frac{\mathrm{d}^5y}{\mathrm{d}x^5} = 3\ln t$, then $\frac{\mathrm{d}^6y}{\mathrm{d}x^6} =$

(A) $\frac{3}{8te^{-8t}}$ (B) $\frac{3}{6t - te^{-8t}}$ (C) $\frac{3\ln t}{6 - e^{-8t}}$ (D) $\frac{8te^{-8t}}{3}$ (E) $\frac{6 - e^{-8t}}{3\ln t}$

28. The area enclosed by $r^2 = 8\cos 2\theta$ is

(A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) 2 (D) 4 (E) 8

29. A cycloid has a peak at $(10\pi, 20)$. Parametric equations for the cycloid are

(A) $x = 10(1 - \cos\theta), \qquad y = 10(\theta - \sin\theta)$

(B) $x = 10(\theta - \sin\theta), \qquad y = 10(1 - \cos\theta)$

(C) $x = 20(1 - \cos\theta), \qquad y = 20(1 - \sin\theta)$

(D) $x = 20(1 - \cos\theta), \qquad y = 20(\theta - \sin\theta)$

(E) $x = 20(\theta - \sin\theta), \qquad y = 20(1 - \cos\theta)$

30. The graph parameterized by $x = 6 - 4t^2$ and $y = t^4 - 2$ is concave up for

(A) $t < 0$

(B) $t < \frac{\sqrt{6}}{2}$

(C) $t > 0$

(D) $t > \sqrt[4]{2}$

(E) $-\infty < t < \infty$

31. Which integral equals the length of the polar curve $r = \cos^2\theta$ over $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$?

(A) $\displaystyle\int_0^{\pi/2} \sqrt{1 + 4\sin^2\theta\cos^2\theta}\,d\theta$

(B) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^2\theta + 2\sin\theta\cos\theta}\,d\theta$

(C) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^2\theta - 2\sin\theta\cos\theta}\,d\theta$

(D) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^4\theta + 4\sin^2\theta\cos^2\theta}\,d\theta$

(E) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^4\theta + 4\sin^2\theta\cos^2\theta}\,d\theta$

32. Half of a petal on the curve $r = \cos 2\theta$ is revolved around the x-axis to generate a solid whose surface area is given by

(A) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{1 + 4\sin^2 2\theta}\, d\theta$

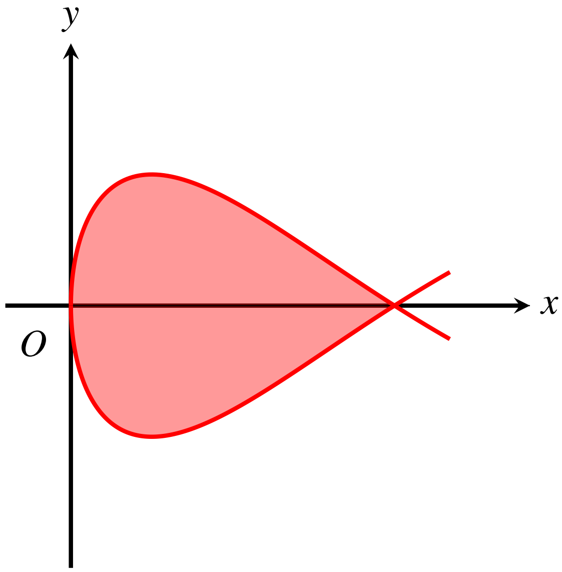
(B) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{\cos^2 2\theta - 4\sin^2 2\theta}\, d\theta$

(C) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta$

(D) $2\pi \int_0^{\pi/4} \cos\theta \cos 2\theta \sqrt{\cos^2 2\theta - 4\sin^2 2\theta}\, d\theta$

(E) $2\pi \int_0^{\pi/4} \cos\theta \cos 2\theta \sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta$

33. The graph parameterized by $x = t^2$ and $y = \sin 2t$ traces out a teardrop, as shown below.



The area of the teardrop is

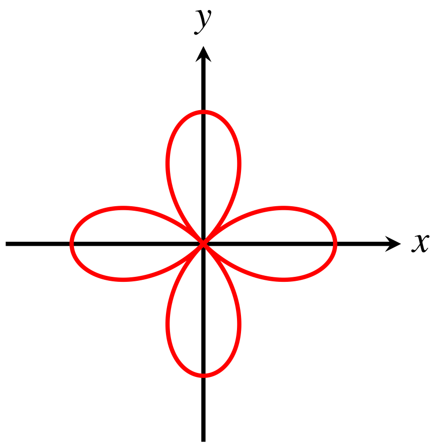
(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π

34. The area of the region enclosed by $r = 4\cos\theta$ and $r = 4\sin\theta$ is

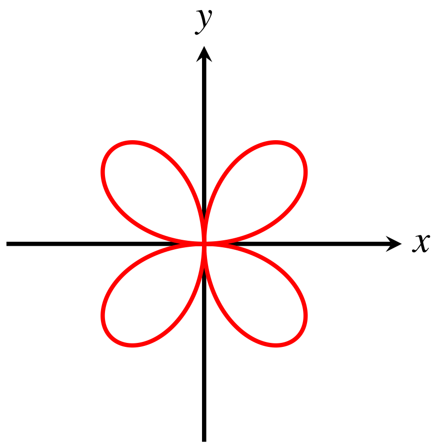
(A) 4 (B) $\pi + 4$ (C) $2\pi - 2$ (D) $2\pi - 4$ (E) $2\pi + 4$

35. Which choice is the graph of $r = \sin 4\theta$?

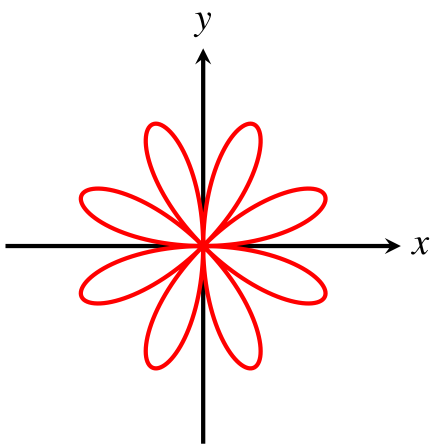
(A)



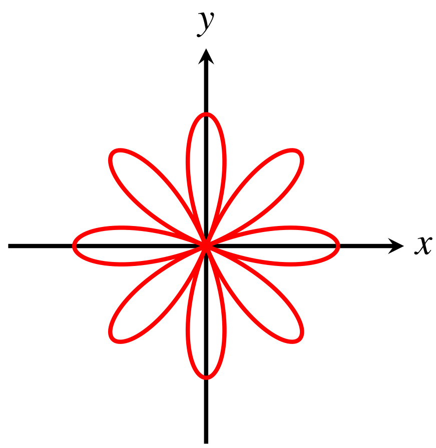
(B)



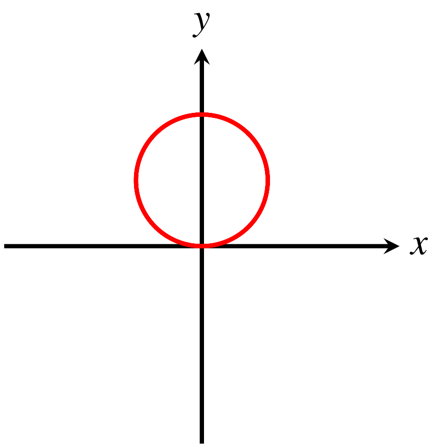
(C)



(D)



(E)



36. The area enclosed by the inner loop of the limacon $r = 2 - 4\cos\theta$ is

(A) $2\pi + 3\sqrt{3}$

(B) $2\pi - 3\sqrt{3}$

(C) $4\pi + 6\sqrt{3}$

(D) $4\pi - 6\sqrt{3}$

(E) $8\pi - 12\sqrt{3}$

37. The graph of $\left(x^2 + y^2\right)^3 = 4x^2y^2$ is identical to which polar curve?

(A) $r = \sin\theta$

(B) $r = \sin 2\theta$

(C) $r = \cos 2\theta$

(D) $r = 2\sin 2\theta$

(E) $r = 2\cos 2\theta$

38. The area of the region bounded by $r = 4\theta$, where $0 \leqslant \theta \leqslant \pi$, and the x-axis is

(A) $\dfrac{\pi^2}{2}$ (B) π^2 (C) $\dfrac{4\pi^3}{3}$ (D) $\dfrac{8\pi^3}{3}$ (E) $\dfrac{16\pi^3}{3}$

39. For $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$, the tangent to the polar curve $r = 4\sin\theta$ is 1 when

(A) $\theta = \dfrac{\pi}{16}$ (B) $\theta = \dfrac{\pi}{8}$ (C) $\theta = \dfrac{\pi}{6}$ (D) $\theta = \dfrac{\pi}{4}$ (E) $\theta = \dfrac{\pi}{2}$

40. Curve C is parameterized by $x = 5 + 3\cos t$ and $y = 3\sin t$ for $0 \leqslant t \leqslant \frac{\pi}{2}$. The surface area of revolution in rotating C about the x-axis is

(A) $6\pi\sqrt{18}$ (B) $12\pi\sqrt{18}$ (C) 9π (D) 18π (E) $15\pi^2$

41. For $0 \leqslant \theta \leqslant \pi$, the polar curve $r = 9\cos\theta$ has a horizontal tangent when

(A) $\theta = 0$ (B) $\theta = \frac{\pi}{4}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{3\pi}{4}$ (E) $\theta = \pi$

42. When $\theta = \dfrac{3\pi}{4}$, the polar curve $r = 9\cos 3\theta$ is getting

(A) closer to the x-axis and closer to the y-axis

(B) closer to the x-axis and farther away from the y-axis

(C) farther away from the x-axis and closer to the y-axis

(D) farther away from the x-axis and farther away from the y-axis

(E) farther away from the x-axis, and neither closer to nor farther away from the y-axis

43. The area of *one* petal of the rose $r = \cos 8\theta$ is

(A) $\dfrac{\pi}{64}$ (B) $\dfrac{\pi}{32}$ (C) $\dfrac{\pi}{16}$ (D) $\dfrac{\pi}{8}$ (E) $\dfrac{\pi}{4}$

44. A woman standing on a balcony 10 feet above the ground throws a flying disk upward with a speed of 20 meters per second at an angle of 45° to the horizontal. If x is the disk's horizontal distance from the woman and y is its height above the ground, then the disk's trajectory is modeled by

(A) $y = 10 + x + \dfrac{9.8x^2}{200}$

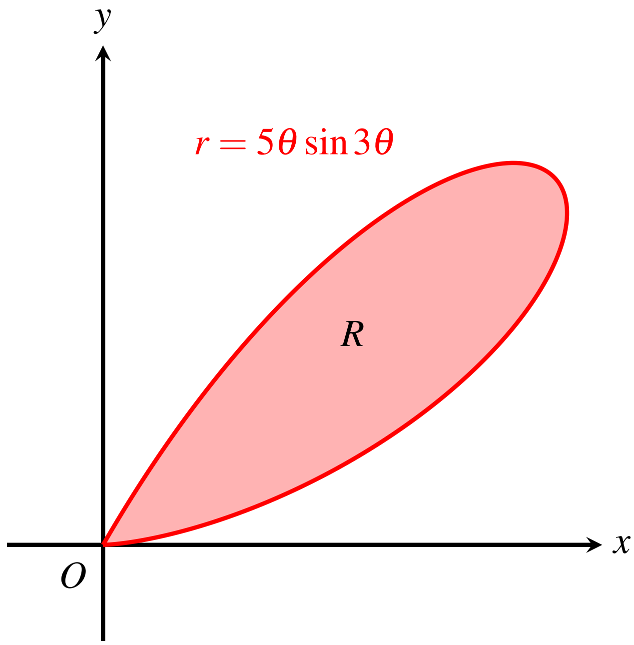
(B) $y = 10 + x + \dfrac{9.8x^2}{400}$

(C) $y = 10 + x + \dfrac{9.8x^2}{800}$

(D) $y = 10 + x - \dfrac{9.8x^2}{400}$

(E) $y = 10 + x - \dfrac{9.8x^2}{800}$

Questions 45 and 46 refer to the following region.



45. Which integral equals the area of R ?

(A) $\dfrac{1}{2}\displaystyle\int_{-\pi/2}^{0} 25\theta^2 \sin^2 3\theta \, d\theta$

(B) $\dfrac{1}{2}\displaystyle\int_{0}^{\pi/6} 25\theta^2 \sin^2 3\theta \, d\theta$

(C) $\dfrac{1}{2}\displaystyle\int_{0}^{\pi/3} 25\theta^2 \sin^2 3\theta \, d\theta$

(D) $\dfrac{1}{2}\displaystyle\int_{0}^{\pi/2} 25\theta^2 \sin^2 3\theta \, d\theta$

(E) $\dfrac{1}{2}\displaystyle\int_{0}^{\pi} 25\theta^2 \sin^2 3\theta \, d\theta$

46. The average distance from O to any point on the border of R is

(A) $\frac{5}{9}$ (B) $\frac{5}{6}$ (C) $\frac{5}{3}$ (D) $\frac{5\pi}{9}$ (E) $\frac{5\pi}{3}$

47. The region S is bounded by the line $\theta = \frac{\pi}{2}$ and the polar curve $r = 7\theta$. Which integral equals the surface area of the solid generated by rotating S about the line $\theta = \frac{\pi}{2}$?

(A) $49\pi \int_0^{\pi/2} \theta \sin\theta \sqrt{\theta^2+1}\,\mathrm{d}\theta$

(B) $49\pi \int_0^{\pi/2} \theta \cos\theta \sqrt{\theta^2+1}\,\mathrm{d}\theta$

(C) $98\pi \int_0^{\pi/2} \sin\theta \sqrt{\theta^2+1}\,\mathrm{d}\theta$

(D) $98\pi \int_0^{\pi/2} \theta \sin\theta \sqrt{\theta^2+1}\,\mathrm{d}\theta$

(E) $98\pi \int_0^{\pi/2} \theta \cos\theta \sqrt{\theta^2+1}\,\mathrm{d}\theta$

48. Which expression equals the area of the region that is inside $r = 6 - 4\cos\theta$ and also inside $r = 4$?

(A) $\displaystyle\int_0^{\pi} (2 - 4\cos\theta)^2 \,\mathrm{d}\theta$

(B) $\displaystyle\int_0^{\pi/3} 16 \,\mathrm{d}\theta + \int_{\pi/3}^{\pi} (6 - 4\cos\theta)^2 \,\mathrm{d}\theta$

(C) $\displaystyle\int_0^{\pi/3} (6 - 4\cos\theta)^2 \,\mathrm{d}\theta + \int_{\pi/3}^{\pi} 16 \,\mathrm{d}\theta$

(D) $\displaystyle\int_0^{\pi} \left[(6 - 4\cos\theta)^2 - 16\right] \mathrm{d}\theta$

(E) $\displaystyle\int_0^{\pi} \left[16 - (6 - 4\cos\theta)^2\right] \mathrm{d}\theta$

49. If C is the curve parameterized by $x = \sin t$ and $y = e^t$ for $0 \leqslant t \leqslant 1$, then the area between C and the x-axis is

(A) $\dfrac{e\sin 1 + e\cos 1 - 1}{2}$

(B) $\dfrac{1 + e\sin 1 + e\cos 1}{2}$

(C) $\dfrac{1 + e\sin 1 - e\cos 1}{2}$

(D) $\dfrac{1 - e\sin 1 + e\cos 1}{2}$

(E) $\dfrac{1 - e\sin 1 - e\cos 1}{2}$

50. Let $A(\theta)$ be the area that is enclosed by the polar curves $r = 6\cos\theta$ and $r = k\sin\theta$, where k is a positive constant. Then $\lim_{k\to\infty} A(\theta)$ is

(A) $\dfrac{9\pi}{2}$ (B) 9π (C) 18π (D) 36π (E) nonexistent

This marks the end of the review exercises. The following page contains the answers to all the questions.

1. C
2. E
3. D
4. C
5. B
6. B
7. E
8. B
9. D
10. A
11. D
12. E
13. E
14. A
15. A
16. B
17. B
18. D
19. A
20. D
21. B
22. B
23. D
24. A
25. C
26. D
27. A
28. E
29. B
30. E
31. D
32. C
33. C
34. D
35. C
36. D
37. B
38. D
39. B
40. D
41. B
42. A
43. B
44. D
45. C
46. C
47. E
48. C
49. A
50. A