INFINITE SERIES AND SEQUENCES

NUMBER OF QUESTIONS—45

NO CALCULATOR

1. What is the difference between a sequence and a series?

(A) A sequence is the sum of a set of numbers, whereas a series is a set of numbers.

(B) A sequence is a set of numbers arranged in ascending order, whereas a series is the sum of a set of numbers arranged in ascending order.

(C) A sequence is a set of numbers, whereas a series is the sum of a set of numbers.

(D) A sequence always diverges, whereas a series may converge or diverge.

(E) A sequence always converges, whereas a series may converge or diverge.

2. Which sequence converges?

(A) $\{-1, 1, -1, 1, -1, \dots\}$

(B) $\{-2, -1, 0, 1, 2, \dots\}$

(C) $\{1, 2, 4, 8, 16, \dots\}$

(D) $\left\{1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \frac{81}{256}, \dots\right\}$

(E) $\left\{\frac{1}{2}, -\frac{3}{2}, \frac{5}{2}, -\frac{7}{2}, \frac{9}{2}, \dots\right\}$

3. $\dfrac{\pi}{4}-\dfrac{\pi^2}{16}+\dfrac{\pi^3}{64}-\dfrac{\pi^4}{256}+\cdots+(-1)^{n+1}\left(\dfrac{\pi}{4}\right)^n+\cdots$ is

(A) $\dfrac{-4}{4+\pi}$ (B) $\dfrac{\pi}{4-\pi}$ (C) $\dfrac{\pi}{4+\pi}$ (D) $\dfrac{4}{4-\pi}$ (E) divergent

4. By the Binomial Theorem, $(3x-5)^4$ is

(A) $(3x)^4+\binom{4}{1}(3x)^3(-5)+\binom{4}{2}(3x)^2(-5)^2+\binom{4}{3}(3x)(-5)^3+(-5)^4$

(B) $-(3x)^4-\binom{4}{1}(3x)^3(-5)-\binom{4}{2}(3x)^2(-5)^2-\binom{4}{3}(3x)(-5)^3-(-5)^4$

(C) $(3x)^4+\binom{4}{1}(3x)^3(5)+\binom{4}{2}(3x)^2(5)^2+\binom{4}{3}(3x)(5)^3+(5)^4$

(D) $(3x)^4+\binom{4}{1}-(3x)^3(5)-\binom{4}{2}(3x)^2(5)^2-\binom{4}{3}(3x)(5)^3-(5)^4$

(E) $(3x)^4+\binom{4}{1}-(3x)^3(-5)-\binom{4}{2}(3x)^2(-5)^2-\binom{4}{3}(3x)(-5)^3-(-5)^4$

5. The Maclaurin series of $\cos x$ is

(A) $1+\dfrac{x^2}{2}+\dfrac{x^4}{4!}+\dfrac{x^6}{6!}+\cdots+\dfrac{x^{2n}}{(2n)!}+\cdots$

(B) $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\cdots+\dfrac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$

(C) $x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\dfrac{x^7}{7!}+\cdots+\dfrac{x^{2n+1}}{(2n+1)!}+\cdots$

(D) $1+x+\dfrac{x^2}{2}+\dfrac{x^3}{3!}+\cdots+\dfrac{x^n}{n!}+\cdots$

(E) $1-\dfrac{x^2}{2}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots+\dfrac{(-1)^n x^{2n}}{(2n)!}+\cdots$

6. Let $a_n = \dfrac{1}{\sqrt[3]{n^2}}$. Which option is true?

(A) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(B) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because it is a p-series with $p \leqslant 1$.

(C) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(D) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because it is a p-series with $p \leqslant 1$.

(E) It cannot be determined from the given information.

7. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}}$ (B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ (C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ (D) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ (E) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2\sqrt{n}}$

8. Suppose that $a_n \geqslant b_n > 0$ and $\{b_n\}$ diverges. Which of the following must be true?

I. $\{a_n\}$ diverges.

II. $\lim_{n\to\infty} a_n = \infty$.

III. $\lim_{n\to\infty} b_n = \infty$.

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

9. Which series is alternating?

(A) $\sum_{n=1}^{\infty} (-1)^{2n}$

(B) $\sum_{n=1}^{\infty} (-1)^n \cos(\pi n)$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^{2n+3}}{n}$

(D) $\sum_{n=1}^{\infty} \frac{\sin(\pi n)}{\sqrt{n}}$

(E) All of the above

10. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Which of the following must be true?

I. $\lim_{n\to\infty} a_n = 0$

II. The sequence $\{a_n\}$ converges.

III. $\lim_{N\to\infty} \sum_{n=1}^{N} a_n = \pm\infty$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

11. The Maclaurin series $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots$ converges to which function?

(A) $\sin x$ (B) $\cos x$ (C) $x^2 e^x$ (D) $x^2 \sin x$ (E) $x \sin x$

12. Which series converges?

(A) $\sum_{n=1}^{\infty} (-1)^n$ (B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ (C) $\sum_{n=1}^{\infty} (-1)^n n$ (D) $\sum_{n=1}^{\infty} (-1)^{2n}$ (E) $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n}$

13. Suppose that $\sum_{n=1}^{\infty} b_n$ converges and $a_n \leqslant b_n$ for $n \geqslant 1$. Which choice must be true?

(A) $\sum_{n=1}^{\infty} a_n$ converges.

(B) $\sum_{n=1}^{\infty} a_n$ diverges.

(C) $\lim_{n\to\infty} a_n = 0$.

(D) $\lim_{n\to\infty} b_n \neq 0$.

(E) None of the above

14. Function g has derivatives of all orders. The table below shows selected values of the derivatives of $g(x)$ at $x = 3$, with $g(3) = 1$. What is the fourth-degree Taylor series of $g(x)$ centered at $x = 3$?

n	1	2	3	4
$g^{(n)}(3)$	-2	2	$3/2$	$-4/3$

(A) $1-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(B) $1-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(C) $-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(D) $-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(E) $1-2(x+3)+(x+3)^2+\frac{1}{4}(x+3)^3-\frac{1}{18}(x+3)^4$

15. For what values of k does $\sum_{n=1}^{\infty} \frac{1}{n^{2k-3}}$ diverge?

(A) $k \leqslant 2$

(B) $k < \frac{3}{2}$

(C) $k > \frac{3}{2}$

(D) $k > 2$

(E) $k \geqslant 2$

16. Which choice correctly describes the convergence or divergence of $S = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}}$?

(A) S converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(B) S diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(C) S converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) S diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(E) It cannot be determined from the given information.

17. For all x, $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$ converges to

(A) $\sin x$ (B) $\cosh x$ (C) $\sinh x$ (D) $\cos x$ (E) e^{2x}

18. Let f be a function with derivatives of all orders. The fourth-degree Taylor series of $f(x)$ centered at $x=2$ is $2+(x-2)-\frac{2}{5}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{1}{7}(x-2)^4+\cdots$. What is the value of $f'''(2)$?

(A) -9 (B) $-\frac{9}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{1}{4}$ (E) 0

19. The coefficient of x^3 in the Maclaurin series of $e^{-x/3}$ is

(A) $-\frac{1}{27}$ (B) $-\frac{1}{162}$ (C) $\frac{1}{162}$ (D) $\frac{1}{27}$ (E) $\frac{1}{6}$

20. Which series converges conditionally?

(A) $\sum_{n=1}^{\infty} \frac{1}{n+1}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+1}}$

(D) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(E) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+2}}$

21. Which choice about $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+3}}$ is correct?

(A) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,\mathrm{d}x$ converges.

(B) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,\mathrm{d}x$ diverges.

(C) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,\mathrm{d}x > 0$.

(D) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,\mathrm{d}x$ converges.

(E) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,\mathrm{d}x$ diverges.

22. The binomial series expansion of $\frac{1}{2}\sqrt{4-x}$ is

(A) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(B) $1-\frac{1}{2}\left(-\frac{x}{4}\right)-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(C) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

(D) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(E) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

23. For which series is the Root Test inconclusive?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{8^n+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+3}{n+2}\right)^n$

(C) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+7}{3n-1}\right)^n$

(D) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6n^2+n-2}{4n^2+9}\right)^n$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{5^{3n+1}}$

24. Suppose that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges and c is a finite constant. All the following series must converge *except*

(A) $\displaystyle c+\sum_{n=1}^{\infty} a_n$

(B) $\displaystyle c\sum_{n=1}^{\infty} a_n$

(C) $\displaystyle\sum_{n=1+|c|}^{\infty} a_n$

(D) $\displaystyle\sum_{n=1}^{\infty} [a_n]^c$

(E) $\displaystyle\sum_{n=1}^{\infty} [a_n+c\,a_n]$

25. The fourth-degree Taylor series of e^x centered at $x=2$ is

(A) $1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^3$

(B) $1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^3$

(C) $e^2\left[1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^3\right]$

(D) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^3\right]$

(E) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3}(x-2)^3+\frac{1}{4}(x-2)^3\right]$

26. Which function does not have a Taylor series at the given center?

(A) e^x centered at $x=e$

(B) $\tan x$ centered at $x=\frac{\pi}{3}$

(C) $\sqrt{x}$ centered at $x=1$

(D) $\ln x$ centered at $x=1$

(E) $\sqrt{x-1}$ centered at $x=0$

27. The radius of convergence of $\sum_{n=1}^{\infty}\frac{x^n n^2}{n!}$ is

(A) 0 (B) 1 (C) 2 (D) e (E) ∞

28. Suppose that $\sum_{n=1}^{\infty} a_n \geqslant \sum_{n=1}^{\infty} b_n > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges. Which of the following must be true?

I. $\sum_{n=1}^{\infty} a_n$ diverges.

II. $\sum_{n=1}^{\infty} a_n$ must diverge to $+\infty$.

III. $\sum_{n=1}^{\infty} b_n$ must diverge to $+\infty$.

(A) I only

(B) I and II only

(C) II and III only

(D) I and III only

(E) I, II, and III

29. For $-1 < x < 1$, which series is equivalent to $\dfrac{1}{1-x^2}$?

(A) $x^2 - x^4 + x^6 - x^8 + \cdots$

(B) $x^2 + x^4 + x^6 + x^8 + \cdots$

(C) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(D) $1 - x^2 + x^4 + -x^6 + x^8 + \cdots$

(E) $-1 - x^2 - x^4 - x^6 - x^8 - \cdots$

30. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^3+9}}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5}}{\sqrt{n^6+9}}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{3n^4+4}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+2}}$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{\sqrt{5n^4+3}}$

31. Let $a_n = \dfrac{n^2\sqrt{n^2+4}}{n^3}$. Which choice is correct?

(A) Because $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$, $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(B) Because $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(C) Because $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, $\displaystyle\sum_{n=1}^{\infty} a_n$ converges by the Ratio Test.

(D) Because $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(E) Because $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test for $\displaystyle\sum_{n=1}^{\infty} a_n$ is inconclusive.

32. Function f has derivatives of all orders. At $x = 0$, $f(x)$ is decreasing and concave up. Which choice could be the third-degree Maclaurin series for f ?

(A) $2+x-\frac{1}{3}x^2+4x^3$

(B) $2+x+\frac{1}{3}x^2-4x^3$

(C) $-2-x-\frac{1}{3}x^2-4x^3$

(D) $-2-x+\frac{1}{3}x^2-4x^3$

(E) $2+x+\frac{1}{3}x^2+4x^3$

33. Let $a_n = \dfrac{1}{\sqrt{n+2}}$ for $n \geq 0$. Let f be a continuous function such that $f(n) = a_n$. Which choice must be true?

(A) $\sum_{n=2}^{\infty} a_n \leqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(B) $\sum_{n=2}^{\infty} a_n \geqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(C) $\sum_{n=2}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(D) $\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(E) It cannot be determined from the given information.

34. $\dfrac{d}{dx}\displaystyle\sum_{n=1}^{\infty}\frac{x^{3n+1}}{n!}=$

(A) $\dfrac{x^5}{5}+\dfrac{x^8}{8(2!)}+\dfrac{x^{11}}{11(3!)}+\cdots+\dfrac{x^{3n+2}}{(3n+2)n!}+\cdots$

(B) $4x+\dfrac{7}{2!}x^4+\dfrac{10}{3!}x^7+\cdots+(3n+1)\dfrac{3^{3n-2}}{n!}+\cdots$

(C) $\dfrac{x^7}{7}+\dfrac{x^{10}}{10(2!)}+\dfrac{x^{13}}{13(3!)}+\cdots+\dfrac{x^{3n+4}}{(3n+4)n!}+\cdots$

(D) $4x^3+\dfrac{1}{2!}x^6+\dfrac{1}{3!}x^9+\cdots+\dfrac{x^{3n}}{n!}+\cdots$

(E) $4x^3+\dfrac{7}{2!}x^6+\dfrac{10}{3!}x^9+\cdots+(3n+1)\dfrac{x^{3n}}{n!}+\cdots$

35. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2+n}\right)$ is

(A) 0 (B) 1 (C) $\dfrac{3}{2}$ (D) 2 (E) divergent

36. Let $S = \sum_{n=1}^{\infty} \frac{\sin n}{n^2}$. Which of the following must be true?

I. S converges absolutely.

II. S converges conditionally.

III. S converges.

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

37. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!}$. What is the coefficient of the x^6 term in $\int_0^x f(t)\,dt$?

(A) $-\frac{7}{6}$ (B) $-\frac{1}{12}$ (C) $-\frac{1}{48}$ (D) $\frac{1}{12}$ (E) $\frac{7}{6}$

38. What is the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$?

(A) The series diverges for all x.

(B) $-1 < x < 1$

(C) $-1 \leqslant x \leqslant 1$

(D) $0 < x < 1$

(E) $0 \leqslant x \leqslant 1$

39. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+4}}{n^2}$. The coefficient of the x^6 term in $f'(x)$ is

(A) $-\frac{7}{9}$ (B) $-\frac{1}{4}$ (C) $-\frac{1}{24}$ (D) $\frac{1}{6}$ (E) $\frac{7}{9}$

40. Function g has derivatives of all orders. The fourth-degree Maclaurin series of $g(x)$ is $m(x)$. It is known that $|g^{(5)}(x)| \leqslant 0.7$ for $0 \leqslant x \leqslant 0.4$. Which choice must be true?

(A) $|g(0.4) - m(0.4)| \leqslant 0.7$

(B) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{4!}(0.4)^4$

(C) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{4!}(0.4)^4$

(D) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{5!}(0.4)^5$

(E) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{5!}(0.4)^5$

41. The power series $S = \sum_{n=0}^{\infty} c_n(x-5)^n$ converges at $x = 7$ and diverges at $x = 8$. Which of the following must be true?

I. S converges at $x = 4$.

II. S converges at $x = 3$.

III. S diverges at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

42. The power series $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ has a radius of convergence of R. The power series of which of the following must also have a radius of convergence of R ?

I. $f'(x)$

II. $\int f(x)\,\mathrm{d}x$

III. $2f(x)$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

43. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+8}}$. Using the Alternating Series Error Bound, what is the least amount of terms must be summed to guarantee a partial sum that is within $\frac{1}{30}$ of S ?

(A) 22 (B) 30 (C) 891 (D) 900 (E) 908

44. The partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^3}$ is used to estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Based on the Integral Test, what is the smallest value of k such that the error in S_k is no more than $\frac{1}{200}$?

(A) 2 (B) 10 (C) 50 (D) 100 (E) 200

45. Let $f(x) = \frac{x}{1+x^6}$. Let $g(x) = \int f(x)\,\mathrm{d}x$ and $g(0) = 1$. What is the Maclaurin series expansion of g ?

(A) $\frac{x^2}{2} - \frac{x^8}{8} + \frac{x^{14}}{14} - \frac{x^{20}}{20} + \cdots + \frac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(B) $x - x^7 + x^{13} - x^{19} + \cdots + (-1)^n x^{6n+1} + \cdots$

(C) $1 + x - x^7 + x^{13} + \cdots + (-1)^n x^{6n+1} + \cdots$

(D) $1 + \frac{x^2}{2} - \frac{x^8}{8} + \frac{x^{14}}{14} + \cdots + \frac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(E) $1 - 7x^6 + 13x^{12} - 19x^{18} + \cdots + (-1)^n (6n+1) x^{6n} + \cdots$

1. C
2. D
3. C
4. A
5. E
6. D
7. C
8. A
9. D
10. C
11. E
12. B
13. E
14. A
15. A
16. D
17. B
18. A
19. B
20. C
21. E
22. A
23. B
24. D
25. D
26. E
27. E
28. E
29. C
30. C
31. E
32. D
33. A
34. E
35. B
36. C
37. D
38. C
39. E
40. E
41. A
42. D
43. C
44. B
45. D