PRELIMINARIES

NUMBER OF QUESTIONS—60

NO CALCULATOR

1. A line connects the points $(0,1)$ and $(1,5)$. Its slope is

(A) -4 (B) -1 (C) 1 (D) 4 (E) 5

2. The function ax^2+2x+c is rewritten in the form $a(x-h)^2+k$, where h and k are constants. What is the value of k ?

(A) $\dfrac{1}{2a^2}$ (B) $\dfrac{1}{a^2}$ (C) $\dfrac{ac-1}{a}$ (D) $\dfrac{a^2c-1}{a}$ (E) $\dfrac{1-ac}{a}$

3. $\dfrac{x-2}{x+4}>0$ if

(A) $x<-4$

(B) $-4<x<2$

(C) $-4<x\leqslant 2$

(D) $x>2$

(E) $x<-4, x>2$

4. The domain of $e^{\sqrt{x-2}} - 6$ is

(A) $x \geqslant -6$

(B) $x \geqslant 0$

(C) $x \leqslant 2$

(D) $x \geqslant 2$

(E) all real numbers

5. For $\pi < \theta < \dfrac{3\pi}{2}$, $\cos\theta = -\dfrac{1}{5}$. Then $\sin\theta =$

(A) $-\dfrac{\sqrt{24}}{5}$ (B) $-\dfrac{4}{5}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{4}{5}$ (E) $\dfrac{\sqrt{24}}{5}$

6. What is the solution set of x in the following system of equations?

$$y = 5x - 3$$
$$y = x^2 + 1$$

(A) $\{-17, -2\}$

(B) $\{-4, -1\}$

(C) $\{1, 4\}$

(D) $\{2, 17\}$

(E) $\{4\}$

7. If $f(x) = x^3 - 4$, then $f^{-1}(x) =$

(A) $f^{-1}(x) = \sqrt[3]{x+4}$

(B) $f^{-1}(x) = \dfrac{1}{x^3-4}$

(C) $f^{-1}(x) = \sqrt[3]{x-4}$

(D) $f^{-1}(x) = \sqrt[3]{x^3-8}$

(E) $f^{-1}(x) = \dfrac{1}{\sqrt[3]{x+4}}$

8. A man stands 50 feet from a building that is 200 feet tall. The angle between the man and the top of the building is

(A) $\tan^{-1}(4)$ (B) $\tan^{-1}\left(\dfrac{1}{4}\right)$ (C) $\sin^{-1}(4)$ (D) $\sin^{-1}\left(\dfrac{1}{4}\right)$ (E) $\cos^{-1}(4)$

9. The horizontal asymptote of $f(x) = \dfrac{2x^3+8x^2-6x}{4+8x^3} + 1$ is

(A) $y = 0$ (B) $y = \dfrac{1}{4}$ (C) $y = 1$ (D) $y = \dfrac{5}{4}$ (E) $y = 2$

10. Which transformation must be performed to $f(x)$ to obtain the new function $g(x) = 3f(2x-4)+7$?

(A) $f(x)$ must be shifted 2 units to the left.

(B) $f(x)$ must be shifted 2 units to the right.

(C) $f(x)$ must be shifted 4 units to the left.

(D) $f(x)$ must be shifted 4 units to the right.

(E) $f(x)$ must be shifted down 7 units.

11. $\cos\left(\frac{3\pi}{4}\right)$ is

(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{2}}{2}$

12. The slope of $x-2y=3$ is

(A) -2 (B) -1 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

13. Which statement is true about $f(x) = x^2 - 4x + 5$?

(A) $f(x)$ has a minimum at $(-2, 1)$.

(B) $f(x)$ has a maximum at $(-2, 1)$.

(C) $f(x)$ has a minimum at $(2, 1)$.

(D) $f(x)$ has a maximum at $(2, 1)$.

(E) $f(x)$ has a minimum at $(4, 5)$.

14. Which function has a vertical asymptote at $x = 1$?

(A) $f(x) = \dfrac{1}{x^2 - 3x + 2}$

(B) $f(x) = \sqrt{x - 1}$

(C) $f(x) = \dfrac{x^2 - 1}{x - 1}$

(D) $f(x) = \dfrac{x - 1}{x + 1}$

(E) $f(x) = \dfrac{x^2}{x}$

15. Which expression is equivalent to $\left(4w^2x^8y^{-2}\sqrt{z}\right)^{-1/2}$?

(A) $\dfrac{2wx^4\sqrt[4]{z}}{y}$ (B) $\dfrac{y}{2wx^4\sqrt[4]{z}}$ (C) $\dfrac{2y}{wx^4\sqrt[4]{z}}$ (D) $\dfrac{wx^4\sqrt[4]{z}}{2y}$ (E) $\dfrac{2wx^4}{y^2\sqrt{z}}$

16. $\dfrac{2x^2-5x+9}{x-1}=$

(A) $2x^2-3x+6$

(B) $2x-3+\dfrac{6}{x-1}$

(C) $2x^2-3x+\dfrac{6}{x-1}$

(D) $2x-3$

(E) $2x-7+\dfrac{2}{x-1}$

17. Assuming that the domains are restricted to avoid division by zero, $\dfrac{x^2-10x+24}{x-3}\cdot\dfrac{2(x-6)^{-1}}{x-4}=$

(A) $\dfrac{2(x-4)}{x-3}$

(B) $\dfrac{2}{x-3}$

(C) $\dfrac{2}{(x-3)(x-6)}$

(D) $\dfrac{2(x-6)(x-4)}{x-3}$

(E) $\dfrac{2}{x-4}$

18. The domain of $g(x) = \dfrac{\log_2(x)}{x-5}$ is

(A) $x \neq 0$

(B) $x > 0$

(C) $x \neq 5$

(D) $x > 0, x \neq 5$

(E) all real numbers

19. Which option best describes the end behavior of $f(x) = e^{-2x} - 2$?

(A) As $x \to \infty, f(x) \to -\infty$.

(B) As $x \to \infty, f(x) \to -2$.

(C) As $x \to \infty, f(x) \to 0$.

(D) As $x \to \infty, f(x) \to 1$.

(E) As $x \to \infty, f(x) \to \infty$.

20. $\dfrac{\sin^8(x) - \cos^8(x)}{\sin^4(x) + \cos^4(x)} =$

(A) $\dfrac{1}{2}$

(B) 1

(C) $\sin^2(x) - \cos^2(x)$

(D) $\sin^4(x) + \cos^4(x)$

(E) $\cos^2(x) - \sin^2(x)$

21. The period of $7\sin\left(3x-\frac{\pi}{4}\right)-2$ is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) π (D) 2π (E) 4π

22. $\log(12)=$

(A) $2\log 6$

(B) $3\log 4$

(C) $\log(4)+\log(3)$

(D) $\log(12)\log(1)$

(E) $\log(6)\log(2)$

23. If $a=be^{cd}$, then $d=$

(A) $\frac{\ln(b)-\ln(a)}{c}$

(B) $\frac{\ln a}{c\ln b}$

(C) $\sqrt[c]{\frac{a}{b}}$

(D) $\frac{1}{c}e^{a/b}$

(E) $\frac{\ln(a)-\ln(b)}{c}$

24. $\sqrt{a}+\sqrt{b}=$

(A) $\sqrt{a+b}$

(B) $\sqrt{a-b}$

(C) $\sqrt{ab}$

(D) $a\sqrt{b}$

(E) None of the above

25. What are the solutions to the equation $\dfrac{x^2-5x-24}{x-8}=2x-5$?

(A) $x=-3$

(B) $x=\dfrac{5}{2}$

(C) $x=8$

(D) $x=-3, x=8$

(E) The equation has no solution.

26. $4^a\dfrac{2^{3b-1}}{8^{1-c}}=$

(A) $2^{a+3b+c-2}$

(B) 2^{a+3b-c}

(C) $2^{2a+3b-3c+2}$

(D) $2^{2a+3b+3c-4}$

(E) $2^{a(3b-1)/(1-c)}$

27. Which function does *not* have all real numbers as its range?

(A) $y = 3x$

(B) $y = 4x^3 - 8x^2 + x - 17$

(C) $y = \log_4(x+2)$

(D) $y = \dfrac{x^2 - 3x - 10}{x+2}$

(E) $y = \dfrac{e^x}{x-3}$

28. What values of x satisfy $\ln(x-3) + \ln(x+1) = \ln(5)$?

(A) $x = -4, x = 2$

(B) $x = -2, x = 4$

(C) $x = -2$

(D) $x = 2, x = 4$

(E) $x = 4$

29. What is an equation of the line that passes through the point $(-2,1)$ and has a slope of $-\frac{1}{2}$?

(A) $y+1=-\frac{1}{2}(x-2)$

(B) $y+1=-\frac{1}{2}(x+2)$

(C) $y-1=-\frac{1}{2}(x+2)$

(D) $y+1=\frac{1}{2}(x-2)$

(E) $y-1=\frac{1}{2}(x+2)$

30. The solution set of x in $\frac{3}{x-1}=\frac{x-2}{2}$ is

(A) $\{-4,1\}$ (B) $\left\{-\frac{3}{2},1\right\}$ (C) $\{-1,4\}$ (D) $\{-1\}$ (E) $\{4\}$

31. The range of $-3\cos\left(4x-\frac{\pi}{3}\right)+7$ is

(A) $[-4,10]$ (B) $[-3,3]$ (C) $[4,10]$ (D) $[5,11]$ (E) $[11,15]$

32. A downward-opening parabola intersects the y-axis at $(0,30)$ and has zeros of $x=-2$ and $x=5$. The parabola's equation is

(A) $y=(x-2)(x-5)$

(B) $y=(x+2)(x+5)$

(C) $y=3(x-2)(x-5)$

(D) $y=-3(x+2)(x+5)$

(E) $y=-3(x-2)(x-5)$

33. $\cos\left(\tan^{-1}x\right)=$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) x (C) $\dfrac{1}{\sqrt{1+x^2}}$ (D) $\dfrac{1}{\sqrt{1-x^2}}$ (E) $\dfrac{\sqrt{1+x^2}}{x}$

34. Which function does *not* have all real numbers as its domain?

(A) $y=\dfrac{x^2}{x}$

(B) $y=2^x$

(C) $y=-3\sin(x-4)+9$

(D) $y=|x-4|-2$

(E) $y=-7$

35. If $f(x) = x^2 - 4$ and $g(x) = \sin^2(x-3)$, then $g(f(x)) =$

(A) $\sin^4(x-3) - 4$

(B) $\sin^2(x^2-7)$

(C) $\sin^4(x-3) - 4\sin^2(x-3)$

(D) $\sin^2(x^2-x-7)$

(E) $\sin^2(x^2-4)$

36. What value of x satisfies $2^{3x-1} = 8^{4x-2}$?

(A) $\log_2\left(\frac{1}{5}\right)$ (B) $\log_2\left(\frac{5}{9}\right)$ (C) $\frac{1}{5}$ (D) $\frac{5}{9}$ (E) 1

37. An object's kinetic energy varies directly as the square of the object's speed. A cannonball traveling at 5 meters per second has a kinetic energy of 100 joules. How much kinetic energy would the cannonball have if it traveled at 10 meters per second?

(A) 40 joules (B) 100 joules (C) 200 joules (D) 400 joules (E) 2000 joules

38. Polynomial $f(x)$ contains the distinct factor $(x-p)^2$. Which statement describes the behavior of $f(x)$ at $x=p$?

(A) $f(x)$ intersects the x-axis at $x=p$.

(B) $f(x)$ curves through the x-axis at $x=p$.

(C) $f(x)$ bounces off the x-axis at $x=p$.

(D) $f(x)$ does not touch the x-axis at $x=p$.

(E) $f(x)$ is undefined at $x=p$.

39. $f(x)=\log_3(\sqrt[3]{x-p})$ is undefined for

(A) $p \leqslant 0$ (B) $p \geqslant 0$ (C) $p=x$ (D) $p \leqslant x$ (E) $p \geqslant x$

40. Which statement describes the end behavior of $f(x)=x^6+2x^5-x^2+4$?

(A) As $x \to -\infty$, $f(x) \to 0$; as $x \to \infty$, $f(x) \to 0$.

(B) As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to -\infty$.

(C) As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to \infty$.

(D) As $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x)- \to \infty$.

(E) As $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to \infty$.

41. $\sum_{i=1}^{5}(2i-1) =$

(A) 8 (B) 20 (C) 25 (D) 30 (E) 125

42. The range of $\dfrac{2}{5-x}-1$ is

(A) all real numbers

(B) $x > -1$

(C) $x > -\dfrac{3}{5}$

(D) $x \neq 0$

(E) $x \neq 5$

43. What are the solutions to $|x-4|+2=5$?

(A) $x=-1$

(B) $x=3$

(C) $x=7$

(D) $x=-1, x=7$

(E) $x=1, x=7$

44. $\sec\left(\frac{\pi}{2}\right)$ is

(A) -1 (B) 0 (C) 1 (D) π (E) undefined

45. Which expression is equivalent to $\frac{1}{a}+\frac{1}{b}$?

(A) $\frac{a+b}{ab}$ (B) $\frac{1}{a+b}$ (C) $\frac{1}{ab}$ (D) $\frac{ab}{a+b}$ (E) $\frac{2}{a+b}$

46. Which expression is equivalent to $\left(4a^4b^3c^{-7}d^{-5}\right)^2$?

(A) $16a^8b^6c^{-14}d^{-10}$

(B) $\frac{a^4b^3}{16c^7d^5}$

(C) $\frac{c^{14}}{16a^8b^6}$

(D) $\frac{a^2b}{16c^9d^7}$

(E) $\frac{16c^9d^7}{a^2b}$

47. A sphere with radius r has a volume of $\frac{4}{3}\pi r^3$ and a surface area of $4\pi r^2$. A spherical ball has a surface area of 400π square meters. Its volume, in cubic meters, is

(A) 10 (B) $\frac{40\pi}{3}$ (C) 1000 (D) $\frac{400\pi}{3}$ (E) $\frac{4000\pi}{3}$

48. $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right) =$

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3}$ (C) $\dfrac{2\pi}{3}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{\sqrt{3}}{2}$

49. Which expression is equivalent to $e^{f(x)}$? (Note: $\log x = \log_{10} x$.)

(A) $10^{f(x)\log e}$

(B) $10^{f(x)+\log e}$

(C) $10^{f(x)\ln 10}$

(D) $10^{f(x)-\log e}$

(E) $10^{f(x)+\ln 10}$

50. $2x^2 - kx + 8$ has no real solutions for

(A) $-8 < k, k > 8$

(B) $k < -\sqrt{2}, k > \sqrt{2}$

(C) $-8 < k < 8$

(D) $-2\sqrt{2} < k < 2\sqrt{2}$

(E) $k < 64$

51. If $f(x)=x^2+4$, then $\dfrac{f(x+h)-f(x)}{h}=$

(A) $2x+h$ (B) $2x+2h$ (C) x^2+h (D) $2x+h+4$ (E) x^2+h+4

52. For $0<x<1$, which inequality is true?

(A) $\sqrt{x}<x^2<x$

(B) $x^2<x<\sqrt{x}$

(C) $\sqrt{x}<x<x^2$

(D) $x^2<\sqrt{x}<1$

(E) $x<\sqrt{x}<x^2$

53. Which set of x satisfies the following system of equations?

$$y=2\sin^2 x$$
$$y-1=\sin x$$

(A) $\{-1,2\}$

(B) $\left\{-\dfrac{1}{2},1\right\}$

(C) $\left\{\dfrac{3\pi}{2}\right\}$

(D) $\left\{\dfrac{\pi}{2},\dfrac{7\pi}{6}\right\}$

(E) No solution exists.

54. If $f(x) = \cos(2x+4)$ and $g(x) = \sqrt{x-2}$, then the domain of $f(g(-2x))$ is

(A) $x \leqslant -2$

(B) $x \leqslant -1$

(C) $x \geqslant 1$

(D) $x \geqslant 2$

(E) all real numbers

55. What values of x satisfy the inequality $x^2 - 4x + 3 > 0$?

(A) $-\infty < x < \infty$

(B) $1 < x < 3$

(C) $1 \leqslant x \leqslant 3$

(D) $x < 1, x > 3$

(E) $x < 1, x \geqslant 3$

56. Which set of x satisfies $e^{2x} - 5e^x + 6 = 0$?

(A) $\{-\ln(3), -\ln(2)\}$

(B) $\{-\ln(2), \ln(3)\}$

(C) $\left\{-\ln\left(\frac{3}{2}\right), -\ln\left(\frac{2}{3}\right)\right\}$

(D) $\{\ln(2), \ln(3)\}$

(E) $\{2, 3\}$

57. Which expression is equivalent to $\sec(x) - \csc(x)$?

(A) $\dfrac{\cos(x) - \sin(x)}{\sin(x)\cos(x)}$

(B) $\dfrac{\sin(x)\cos(x)}{\sin(x) + \cos(x)}$

(C) $\dfrac{\sin(x) + \cos(x)}{\sin(x)\cos(x)}$

(D) $\dfrac{\sin(x) - \cos(x)}{\sin(x)\cos(x)}$

(E) $\dfrac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)}$

58. What values of x satisfy the equation $\dfrac{2}{x+4} - \dfrac{2}{x-1} = 3$?

(A) $x = \dfrac{9 \pm \sqrt{9^2 - 4(3)(2)}}{2(3)}$

(B) $x = \dfrac{9 \pm \sqrt{9^2 - 4(3)(2)}}{2(3)}$

(C) $x = \dfrac{9 \pm \sqrt{9^2 - 4(3)(2)}}{3}$

(D) $x = \dfrac{-9 \pm \sqrt{9^2 - 4(3)(2)}}{3}$

(E) $x = \dfrac{9 \pm \sqrt{9^2 - (3)(2)}}{3}$

59. Let f be an invertible function. The graph of $y = g(x)$ is obtained by performing the following transformations to the graph of $y = f(x)$:

- The graph is reflected across the line $y = x$.
- The graph is then shifted 4 units to the left.
- The graph is then translated 3 units down.

Which function is $g(x)$?

(A) $g(x) = f(x+4) - 3$

(B) $g(x) = f(x-4) - 3$

(C) $g(x) = f^{-1}(x+4) - 3$

(D) $g(x) = f^{-1}(x-4) - 3$

(E) $g(x) = f^{-1}(x-4) + 3$

60. Which expression is equivalent to $\log(2w) + 3\log(x) - \frac{1}{2}\log(4y) + 3\log(z)$?

(A) $\log\left(\frac{\sqrt{y}}{wx^3z^3}\right)$

(B) $\log\left(\frac{wx^3z^3}{2\sqrt{y}}\right)$

(C) $\log\left(\frac{w\sqrt{y}}{4x^3z^3}\right)$

(D) $\log\left(\frac{wz^3}{2x^3\sqrt{y}}\right)$

(E) $\log\left(\frac{wx^3z^3}{\sqrt{y}}\right)$

1. D
2. C
3. E
4. D
5. A
6. C
7. A
8. A
9. D
10. B
11. B
12. D
13. C
14. A
15. B
16. B
17. B
18. D
19. B
20. C
21. B
22. C
23. E
24. E
25. E
26. D
27. D
28. E
29. C
30. C
31. C
32. E
33. C
34. A
35. B
36. D
37. D
38. C
39. E
40. E
41. C
42. A
43. E
44. E
45. A
46. C
47. E
48. B
49. A
50. C
51. A
52. B
53. D
54. B
55. D
56. D
57. D
58. B
59. C
60. E