LIMITS AND CONTINUITY

NUMBER OF QUESTIONS—45

NO CALCULATOR

1. If $\lim_{x\to 3} f(x) = 8$, then which option is true?

(A) $f(3) = 8$.

(B) As x approaches 3, $f(x)$ approaches 8.

(C) $f(8) = 3$.

(D) As x approaches 8, $f(x)$ approaches 3.

(E) f is continuous at $x = 3$.

2. $\lim_{x\to 0} \dfrac{\sin 3x}{x}$ is

(A) 0 (B) $\dfrac{1}{3}$ (C) 1 (D) 3 (E) nonexistent

3. $\lim_{x\to -7} \dfrac{x+7}{49-x^2}$ is

(A) -7 (B) $-\dfrac{1}{14}$ (C) $-\dfrac{1}{49}$ (D) $\dfrac{1}{14}$ (E) 49

4. $\lim\limits_{x\to 0} \dfrac{x}{\tan x}$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

5. $\lim\limits_{x\to \infty} \dfrac{3x^4 - 8x^3 + x^2 - 10}{2x^2 - 5x^3 - 10x^4}$ is

(A) $-\dfrac{3}{10}$ (B) 0 (C) $\dfrac{3}{10}$ (D) $\dfrac{3}{2}$ (E) nonexistent

6. The horizontal asymptote of $f(x) = \dfrac{3x^3 + x^2 - 4}{8 - 5x^3}$ is

(A) $y = -\dfrac{5}{3}$ (B) $y = -\dfrac{3}{5}$ (C) $y = \dfrac{3}{8}$ (D) $y = \dfrac{3}{5}$ (E) $y = \dfrac{5}{3}$

7. $\lim\limits_{x\to \pi/2} \tan 2x$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) 0 (D) 1 (E) π

8. Function g is discontinuous at $x = 5$. Selected values of g are shown in the table below.

x	4.99	4.999	5	5.0001	5.001
$f(x)$	2.99	2.999	-4	3.001	3.01

A reasonable estimate for $\lim\limits_{x\to 5} g(x)$ is

(A) -4 (B) -3 (C) 2 (D) 3 (E) 5

9. If $f(x) = \begin{cases} 3x - 2\cos x & x < \pi \\ x^2 & x \geqslant \pi, \end{cases}$ then $\lim\limits_{x\to \pi^-} f(x)$ is

(A) $-\pi^2$ (B) 3π (C) $3\pi - 2$ (D) $3\pi + 2$ (E) π^2

10. $\lim\limits_{x\to 3} \dfrac{\sqrt{x-2} - 1}{9 - 3x}$ is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{6}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{2}$ (E) nonexistent

11. Given that $\lim_{x \to a} f(x)$ exists, which statements must be true?

I. $f(x)$ is continuous at $x = a$.

II. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

III. $f(a)$ is defined.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

12. If $\lim_{k \to 0} \dfrac{e^k - 1}{k} = 1$, then $\lim_{k \to 0} \dfrac{e^2 - e^{2+k}}{k}$ is

(A) $-e^2$ (B) $-e^{-2}$ (C) e^{-2} (D) 1 (E) e^2

13. $g(x) = \dfrac{x^2 - 5x + 6}{x - 3}$ has a removable discontinuity at

(A) $x = -3$ (B) $x = -2$ (C) $x = 0$ (D) $x = 2$ (E) $x = 3$

14. $\lim\limits_{x\to\infty} \dfrac{\sin(2x)}{x+2}$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) ∞

15. Let $f(x) = \begin{cases} 2-kx & x \leqslant 3 \\ kx^2-22 & x > 3. \end{cases}$ For what value of k is f continuous at $x=3$?

(A) -4 (B) 0 (C) 1 (D) 2 (E) 3

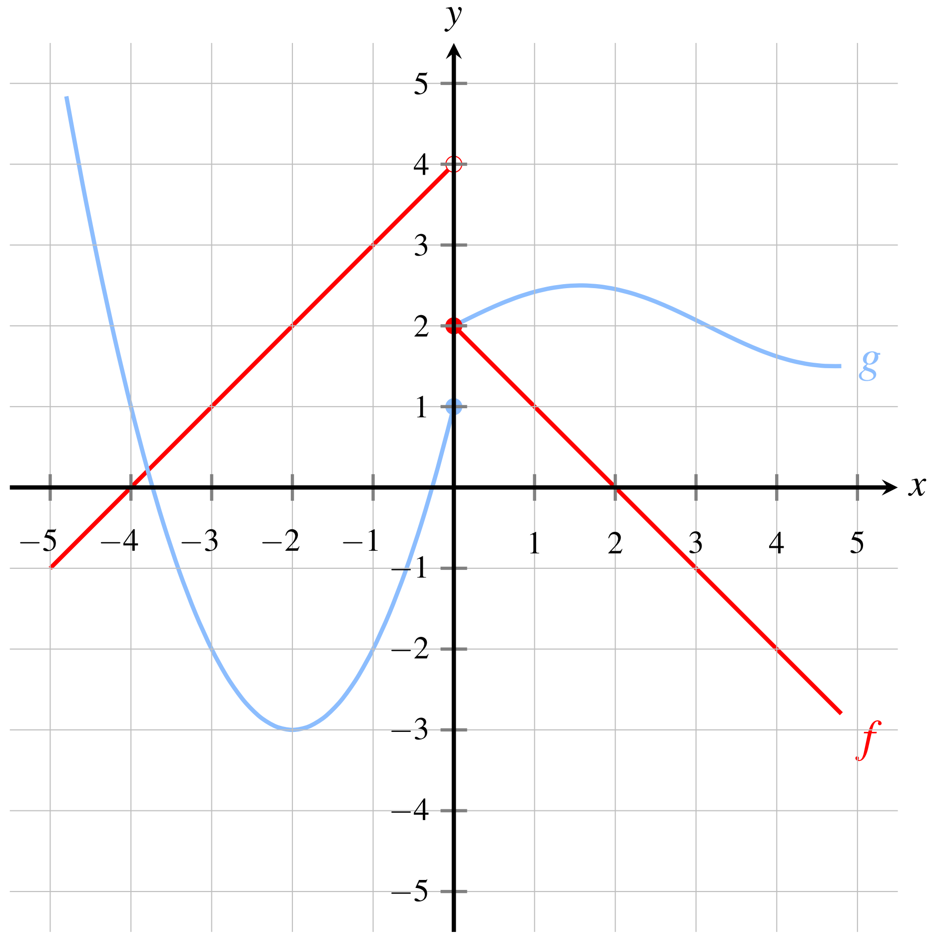
16. The oblique asymptote of $f(x) = \dfrac{x^2+7x+1}{x-2}$ is

(A) $y=9$ (B) $y=x-9$ (C) $y=x$ (D) $y=x+5$ (E) $y=x+9$

17. $\lim\limits_{x\to\infty} \dfrac{\sqrt{4x^2-1}}{x+3}$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) nonexistent

Questions 18–23 refer to the following graph.



18. $\lim_{x \to 0} f(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

19. $\lim_{x \to 0^+} g(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

20. $\lim\limits_{x\to -2}[f(x)-g(x)]$ is

(A) -5 (B) -1 (C) 1 (D) 5 (E) nonexistent

21. $\lim\limits_{x\to -2}[g(x)]^2$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) 9

22. $\lim\limits_{x\to 0}[f(x)g(x)]$ is

(A) 1 (B) 2 (C) 4 (D) 8 (E) nonexistent

23. $\lim\limits_{x\to 0} f(-x^2)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

24. On what interval of x is $f(x)=\dfrac{\ln(x-2)}{x^2-9}$ continuous?

(A) $(-\infty,-3)\cup(3,\infty)$

(B) $(-\infty,-2)$

(C) $(2,\infty)$

(D) $(3,\infty)$

(E) $(2,3)\cup(3,\infty)$

25. $\displaystyle\lim_{x\to-3}\frac{5\sin(x+3)}{6+2x}$ is

(A) 0 (B) $\dfrac{5}{6}$ (C) 1 (D) $\dfrac{5}{2}$ (E) nonexistent

26. Function f is continuous and satisfies $f(4)=8$. If $\displaystyle\lim_{x\to2}g(x)=4$, then $\displaystyle\lim_{x\to2}f(g(x))$ is

(A) -8 (B) -4 (C) 2 (D) 4 (E) 8

27. $\displaystyle\lim_{x\to2}\frac{\frac{1}{2}-\frac{1}{x}}{2-x}$ is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) nonexistent

28. $\lim\limits_{x\to 0} x\sin\left(\dfrac{1}{x^2}\right)$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E) nonexistent

29. Let $f(x) = \begin{cases} x^2+1 & x<p \\ 2x & x \geqslant p. \end{cases}$ If $f(x)$ is continuous at $x=p$, then p is

(A) -1 (B) 0 (C) 1 (D) 2 (E) 4

30. $\lim\limits_{t\to\infty} \sin t$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

31. If $\lim\limits_{x\to 2} f(x) = -3$, then $\lim\limits_{x\to 2} \left([f(x)]^2 - 2x\right)$ is

(A) -7 (B) -3 (C) 4 (D) 5 (E) 9

32. Function f is continuous. Selected values of $f(x)$ are shown in the table below.

x	-1	2	3	6	11
$f(x)$	2	1	1	1	2

Following the Intermediate Value Theorem, which value of $f(x)$ is guaranteed to exist for $-1 \leqslant x \leqslant 11$?

(A) 0 (B) 3 (C) 5 (D) 6 (E) 11

33. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^6 - 4x^2 + 1}}{3x^3 + 2}$ is

(A) $-\infty$ (B) $-\dfrac{2}{3}$ (C) $\dfrac{2}{3}$ (D) ∞ (E) nonexistent

34. $\lim\limits_{x \to \infty} x \sin\left(\dfrac{1}{x}\right)$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

35. At $x = 4$, which choice about $g(x) = \dfrac{12 + x - x^2}{x - 4}$ is true?

(A) $g(x)$ has a vertical asymptote at $x = 4$.

(B) $g(x)$ has a jump discontinuity at $x = 4$.

(C) $g(x)$ has a removeable discontinuity at $x = 4$.

(D) $\lim\limits_{x \to 4} g(x)$ does not exist.

(E) $g(x)$ is continuous at $x = 4$.

36. $\lim\limits_{x \to 0^+} \ln(\sin x)$ is

(A) $-\infty$ (B) 0 (C) 1 (D) e (E) ∞

37. $\lim\limits_{x \to 0} \dfrac{x^2}{\sin^2 2x}$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) 1 (D) 4 (E) nonexistent

38. $\lim\limits_{x \to 0} \dfrac{x - x\cos(x)}{x^2}$ is

(A) $-\pi$ (B) 0 (C) 1 (D) π (E) nonexistent

39. $\displaystyle\lim_{x\to 0^+} \arctan\left(\frac{1}{x}\right)$ is

(A) $-\infty$ (B) $-\dfrac{\pi}{2}$ (C) 0 (D) $\dfrac{\pi}{2}$ (E) ∞

40. $\displaystyle\lim_{x\to\infty} \frac{5e^x - x}{8e^x + 9}$ is

(A) $-\dfrac{5}{8}$ (B) 0 (C) $\dfrac{5}{8}$ (D) 1 (E) ∞

41. $\displaystyle\lim_{x\to\pi/4} \frac{\cos 2x}{\cos x - \sin x}$ is

(A) 0 (B) $\dfrac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) π (E) nonexistent

42. Functions g and h are continuous and satisfy $g(1) = h(1) = 3$. Function f satisfies $g(x) \leqslant f(x) \leqslant h(x)$ for $0 \leqslant x \leqslant 2$. Which statements must be true?

I. $\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1} h(x) = 3$.

II. $\lim\limits_{x\to 1} f(x) = 3$.

III. $f(x)$ is continuous at $x = 1$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

43. If $f(1) = 2$, $\lim\limits_{x\to 1^-} f(x) = 4$, and $\lim\limits_{x\to 1^+} f(x) = -1$, then $\lim\limits_{x\to 1} f(\cos(x-1))$ is

(A) -1 (B) 0 (C) 2 (D) 4 (E) nonexistent

44. If $\lim\limits_{x\to 3}(2x+4) = 10$, then $|(2x+4)-10| < \varepsilon$ and $|x-3| < \delta$, where $\delta =$

(A) $\dfrac{\varepsilon}{4}$ (B) $\dfrac{\varepsilon}{2}$ (C) ε (D) 2ε (E) 4ε

45. If $\lim_{x \to a} f(x) = \infty$, then which option is true?

(A) For positive M, there exists a positive δ such that $f(x) > M$ for $|x-a| > 0$.

(B) For positive M, there exists a positive δ such that $f(x) < M$ for $|x-a| > \delta$.

(C) For positive M, there exists a positive δ such that $f(x) > M$ for $|x-a| > \delta$.

(D) For positive M, there exists a positive δ such that $f(x) < M$ for $0 < |x-a| < \delta$.

(E) For positive M, there exists a positive δ such that $f(x) > M$ for $0 < |x-a| < \delta$.

1. B
2. D
3. B
4. C
5. A
6. B
7. C
8. D
9. D
10. B
11. B
12. A
13. E
14. B
15. D
16. E
17. C
18. E
19. C
20. D
21. E
22. C
23. D
24. E
25. D
26. E
27. B
28. A
29. A
30. E
31. D
32. A
33. A
34. C
35. C
36. A
37. B
38. B
39. D
40. C
41. C
42. E
43. D
44. B
45. E