# Exploring Buoyancy: Why Objects Float or Sink

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## 1 Introduction to Archimedes' Principle

Have you ever dropped a paper clip in water and observed it rise? Have you ever thrown a rock into a pond and watched it sink to the bottom? These interactions are explained by the fact that lower-density objects rise and higher-density objects sink. We'll justify this fact using *Archimedes' Principle*, a fundamental law in the study of fluids. Archimedes' Principle states that the weight of fluid an object displaces is equal to the buoyant force acting on that object. Specifically, the buoyant force that acts on a submerged object is

$$F_b = m_D g\,, \tag{1}$$

where $m_D$ is the mass of the volume of fluid displaced by the object and $g$ is the acceleration due to gravity, 9.8 m/s$^2$.

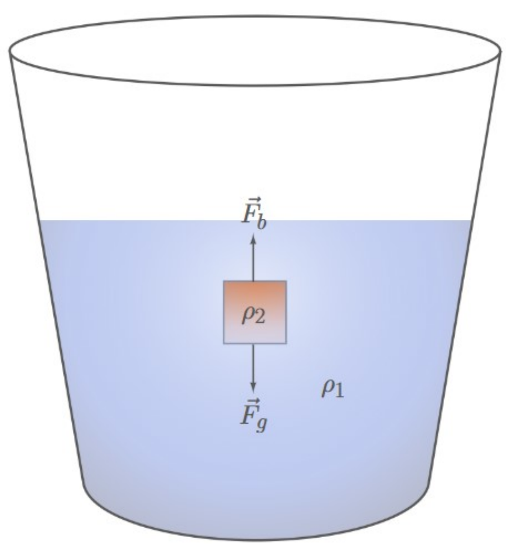


Figure 1: An object of density $\rho_2$ is placed in a cup of fluid of density $\rho_1$.

Consider Figure 1, in which an object of density $\rho_2$, volume $V$ and mass $m$ is submerged in a cup containing fluid of density $\rho_1$. Let's recall that

$$\text{density} = \frac{\text{mass}}{\text{volume}},$$

or $\rho = m/V$. We can then re-express $m_D$ as $\rho_1 V$, where $\rho_1$ is the density of the fluid and $V$ is the object's volume. (Note that the object's volume equals the volume of fluid displaced!) Substituting $\rho_1 V$ for $m_D$ in equation (1), we get

$$F_b = \rho_1 g V \tag{2}$$

for the buoyant force. In Figure 1, two forces act on the submerged object: the buoyant force ($\vec{F_b}$) and the weight of the object,

$$F_g = mg = \rho_2 g V\,.$$

## 2 Physical Analysis

Applying Newton's Second Law ($F_{\text{net}} = ma$), the net force on the object (choosing the positive direction to be up) is

$$\begin{aligned} F_b - F_g &= ma \\ \rho_1 g V - \rho_2 g V &= ma \\ gV(\rho_1 - \rho_2) &= ma\,. \end{aligned} \tag{3}$$

From equation (3), we have two cases:

$$\underbrace{\rho_1 > \rho_2}_{\text{Case 1}} \qquad \text{and} \qquad \underbrace{\rho_1 < \rho_2}_{\text{Case 2}}\,.$$

Let's analyze each case: In case 1, the density of the fluid is greater than the density of the object. Therefore, the acceleration, $\vec{a}$, is positive, meaning the object floats upward. Conversely, in case 2, the density of the object is greater than the density of the fluid. It then follows that $a < 0$, indicating that the object sinks down. And if $\rho_1 = \rho_2$, then the net force and, as a result, the acceleration equal zero.

## 3 Conclusion

Our analysis proves that (a) an object with lower density floats and that (b) an object placed with higher density sinks. These principles apply not only to water but to systems of any fluid, such as a balloon flying in the wind. So next time you're sipping that drink on a hot summer day, think of Archimedes' Principle pushing the ice up to the drink's surface.