TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

ROBUST & ADAPTIVE CONTROL

ADAPTIVE PARAMETER ESTIMATION WITH GRADIENT AND D.R.E.M METHOD

Valerio Auriemma (0334683)

Andrea Marchesi (0311590)

GRADIENT & DREM ESTIMATOR, TASK:

Consider the system $\dot{x} = -a\,x + b\,u$, with a > 0, b $\neq 0$ and a and b unknown.

Determine a parameterization of the considered system and design a DREM parameter identifier and a gradient identifier. Run simulations for the DREM and gradient estimators assuming

a $= 0.4$, b $= 0.4$, and selecting, for the DREM estimator, $H_1(s) = \frac{1}{s+1}$; $H_2(s) = \frac{1}{s+2}$.

Consider the cases $u(t) = 10$; $u(t) = 10\sin\left(\frac{5}{2}t\right)$.

Compare the performance of the estimators in terms of transient response, speed of response and overall performance. Plot the phase portraits of the parameter estimation errors and comment on the results.

GRADIENT ESTIMATOR, THEORY BACKGROUND:

$$y(s) = \frac{Z(s)}{R(s)} u(s) \rightarrow R(s) y(s) = Z(s)\, u(s) \rightarrow y^{(n)} = b_{n-1} u^{(n-1)} + \cdots + b_0 u - a_{n-1}\, y^{(n-1)} - \cdots - a_0 + y \rightarrow$$

$$y^{(n)} = \underbrace{\left[\begin{array}{cccccc} b_{n-1} \ldots & b_0 & a_{n-1} & \ldots & a_0 \end{array}\right]}_{\underbrace{[\,\theta_1 \;\; \theta_2\,]}_{\Theta^T}} \underbrace{\begin{bmatrix} u^{(n-1)} \\ \ldots \\ u \\ -y^{(n-1)} \\ \ldots \\ -y \end{bmatrix}}_{\Psi} = \Theta^T \, \Psi$$

This is unfeasible, we need to add $\Lambda(s)$, with a degree equal to n, to avoid derivation issues:

$$y(s) = \frac{Z(s)}{R(s)} \frac{\Lambda(s)}{\Lambda(s)} u(s) \rightarrow \frac{R(s)}{\Lambda(s)} y(s) = \frac{Z(s)}{\Lambda(s)}\, u(s) \rightarrow$$

GRADIENT ESTIMATOR, THEORY BACKGROUND:

$$z = \frac{y^{(n)}}{\Lambda(s)} = \frac{1}{\Lambda(s)}\left(b_{n-1}u^{(n-1)} + \cdots + b_0 u - a_{n-1}\, y^{(n-1)} - \cdots - a_0 + y\right) \rightarrow$$

$$z = \frac{y^n}{\Lambda(s)} = \frac{1}{\Lambda(s)} \underbrace{\left[\, b_{n-1} \ldots \quad b_0 \quad a_{n-1} \quad \ldots \quad a_0 \,\right]}_{\underbrace{[\,\theta_1 \ \ \theta_2\,]}_{\Theta^T}} \underbrace{\begin{bmatrix} u^{(n-1)} \\ \ldots \\ u \\ -y^{(n-1)} \\ \ldots \\ -y \end{bmatrix}}_{\Psi} = \Theta^T \frac{\Psi}{\Lambda(s)} = \Theta^T \Phi$$

$$z = W(s)\,\theta^T \psi \qquad \text{if } \phi = W(s)\,\psi \quad \rightarrow z = \theta^T \phi$$

define: $\hat{z} = \hat{\theta}^T \phi \qquad \epsilon = z - \hat{z} = \theta^T \phi - \hat{\theta}^T \phi = \tilde{\theta}^T \phi$ and the cost function: $J(\hat{\theta}) = \frac{\epsilon^2}{2} = \frac{(z - \hat{\theta}^T \phi)^2}{2}$

to minimize J the trajectory can be: $\dot{\hat{\theta}} = -\Gamma\, \nabla J(\hat{\theta}) = \Gamma\,(z - \hat{\theta}^T \phi)\phi = \Gamma\, \phi\, \epsilon \quad$ with $\Gamma = \Gamma^T > 0$

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \qquad z(s) = \underbrace{\frac{s}{s+1}}_{\Lambda(s)} y(s); \quad u_f(s) = \underbrace{\frac{1}{s+1}}_{\Lambda(s)} u(s); \quad \phi = \begin{bmatrix} u_f \\ -z \end{bmatrix}; \ [\,\theta_1 \ \ \theta_2\,] = [\, b \ \ a \,]$$

DREM ESTIMATOR, THEORY BACKGROUND:

$z = \theta^T \phi \qquad \theta \in \mathbb{R}^q$ and H linear filter one input and q output such as $H = \frac{b_i}{s + \tau_i}$ or $H = e^{-\tau_i s}$

$$Z = z\,H \qquad \Phi = H\,\phi^T \qquad \Phi \in \mathbb{R}^{\mathrm{qxp}}$$

$$\tilde{Z} = \mathrm{adj}(\Phi) Z = \mathrm{adj}(\Phi)\Phi\theta = \det(\Phi)\theta \quad \rightarrow \tilde{Z}_i = \det(\Phi)\theta_i$$

$$M^{-1} = \mathrm{adj}(M)\,/\det(M) \rightarrow \det(M)\,I = \mathrm{adj}(M)\,M$$

From which the update law can be defined as follows:

$$\dot{\hat{\theta}}_i = \gamma_i \det(\Phi)\,(\tilde{Z}_i - \det(\Phi)\hat{\theta}_i)$$

GRADIENT ESTIMATOR, THEORY BACKGROUND:

$$z = \frac{y^{(n)}}{\Lambda(s)} = \frac{1}{\Lambda(s)}\left(b_{n-1}u^{(n-1)} + \cdots + b_0 u - a_{n-1}\,y^{(n-1)} - \cdots - a_0 + y\right) \rightarrow$$

$$z = \frac{y^{n}}{\Lambda(s)} = \frac{1}{\Lambda(s)} \underbrace{\left[\, b_{n-1} \ldots \quad b_0 \quad a_{n-1} \quad \ldots \quad a_0 \,\right]}_{\underbrace{[\,\theta_1 \quad \theta_2\,]}_{\Theta^T}} \underbrace{\begin{bmatrix} u^{(n-1)} \\ \ldots \\ u \\ -y^{(n-1)} \\ \ldots \\ -y \end{bmatrix}}_{\Psi} = \Theta^T \frac{\Psi}{\Lambda(s)} = \Theta^T \Phi$$

$$z = W(s)\,\theta^T \psi \qquad \text{if } \phi = W(s)\,\psi \quad \rightarrow z = \theta^T \phi$$

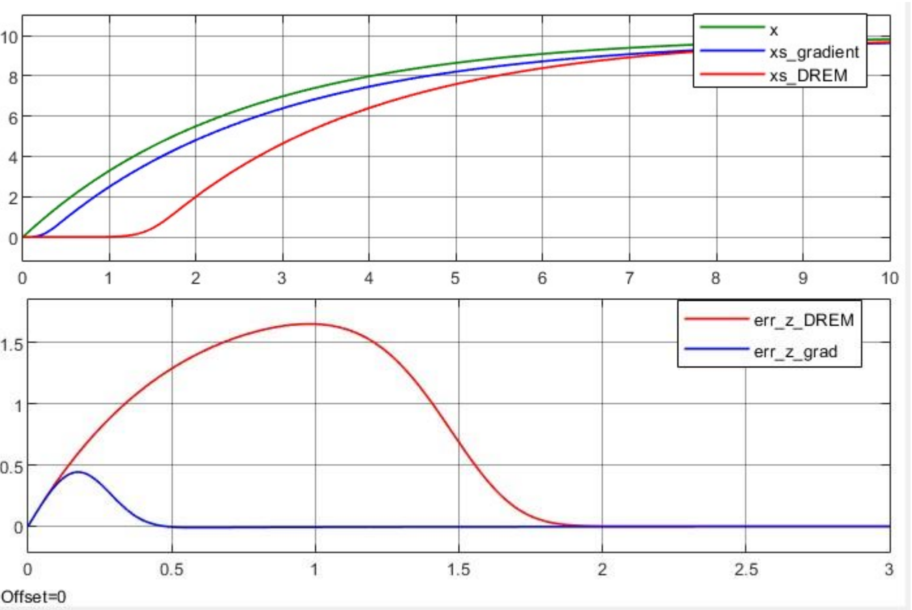
define: $\hat{z} = \hat{\theta}^T \phi \qquad \epsilon = z - \hat{z} = \theta^T \phi - \hat{\theta}^T \phi = \tilde{\theta}^T \phi$ and the cost function: $J(\hat{\theta}) = \frac{\epsilon^2}{2} = \frac{(z - \hat{\theta}^T \phi)^2}{2}$

to minimize J the trajectory can be: $\dot{\hat{\theta}} = -\Gamma\,\nabla J(\hat{\theta}) = \Gamma\,(z - \hat{\theta}^T \phi)\phi = \Gamma\,\phi\,\epsilon \quad$ with $\Gamma = \Gamma^T > 0$

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \qquad z(s) = \underbrace{\frac{s}{s+1}}_{\Lambda(s)}\,y(s); \quad u_f(s) = \underbrace{\frac{1}{s+1}}_{\Lambda(s)}\,u(s); \quad \phi = \begin{bmatrix} u_f \\ -z \end{bmatrix}; [\,\theta_1 \quad \theta_2\,] = [\,b \quad a\,]$$

$\gamma_{1,2} = 10;\ \Gamma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};\ W(s) = \frac{1}{(s+1)}$

Estimation analysis with different input

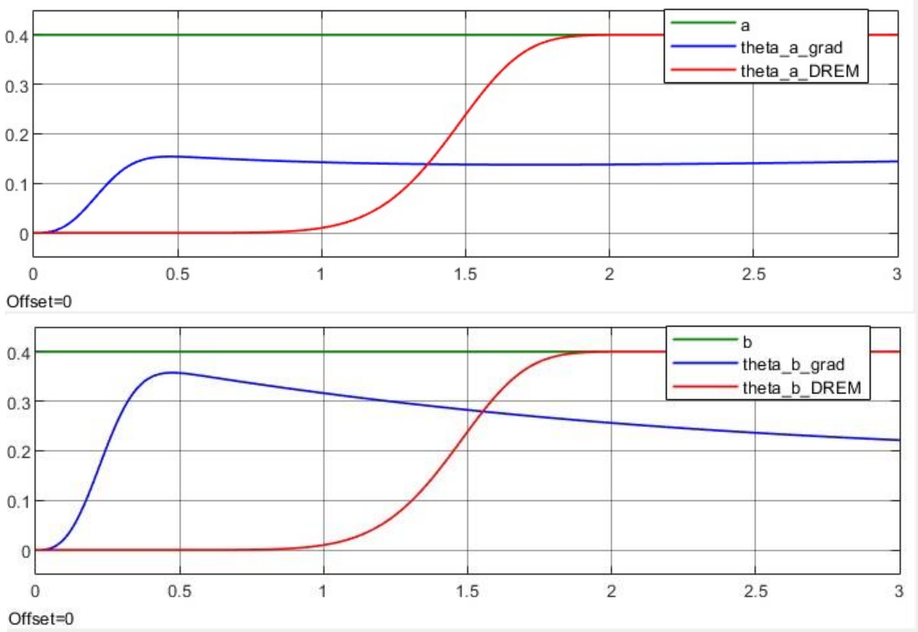


COLOR LEGEND:

NOMINAL MODEL

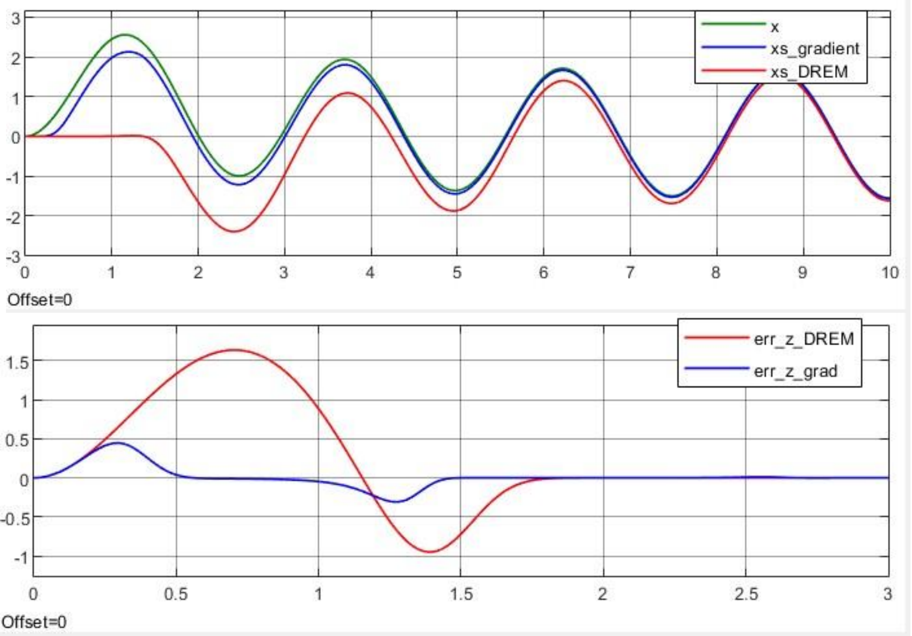
DREM

GRADIENT



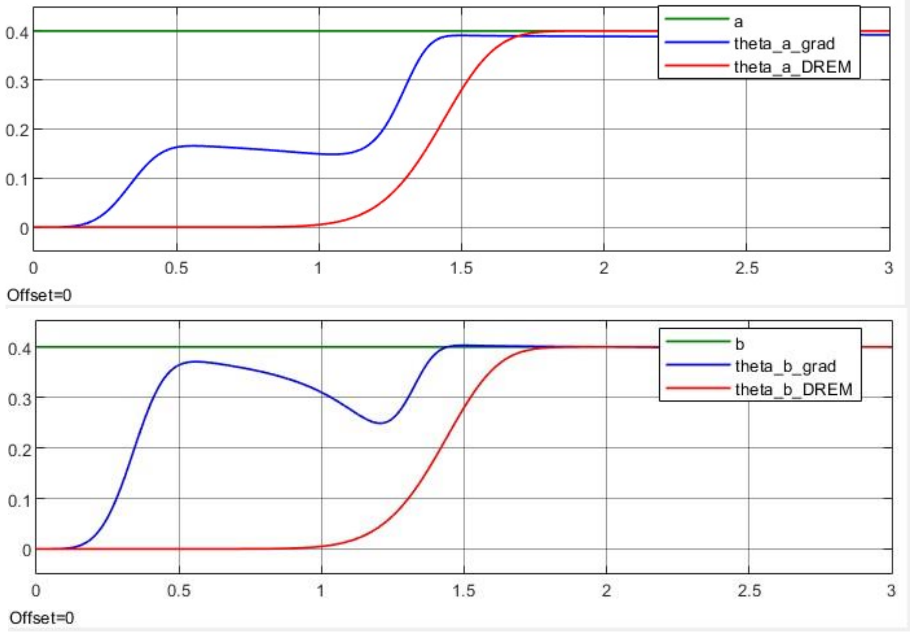
INPUT

$u(t) = 10$



INPUT

$u(t) = 10 \sin(\frac{5}{2} t)$



OBESRVATIONS AND REMARKS

With a step input it can be observed that in the DREM method brings parameter to convergence while the Gradient Method does not. This due to the richness of input, since there are 2 parameters to be estimated Gradient method will need an input of richness 2, while the DREM method estimating parameters unties one from the other and only requires a richness of 1.

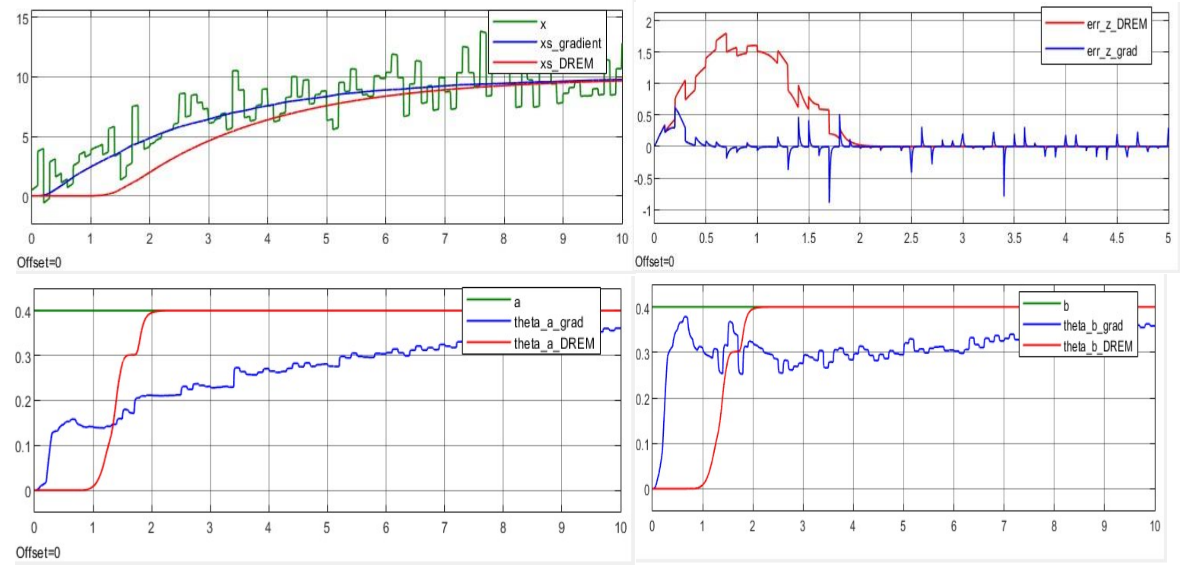
In the case of a sine input both methods converge. It can be observed that Gradient method has a better rise time in both parameter and state estimation, giving a smaller estimation error compared to DREM.

$\gamma_{1,2} = 10;\ \Gamma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};\ W(s) = \dfrac{1}{(s+1)}$

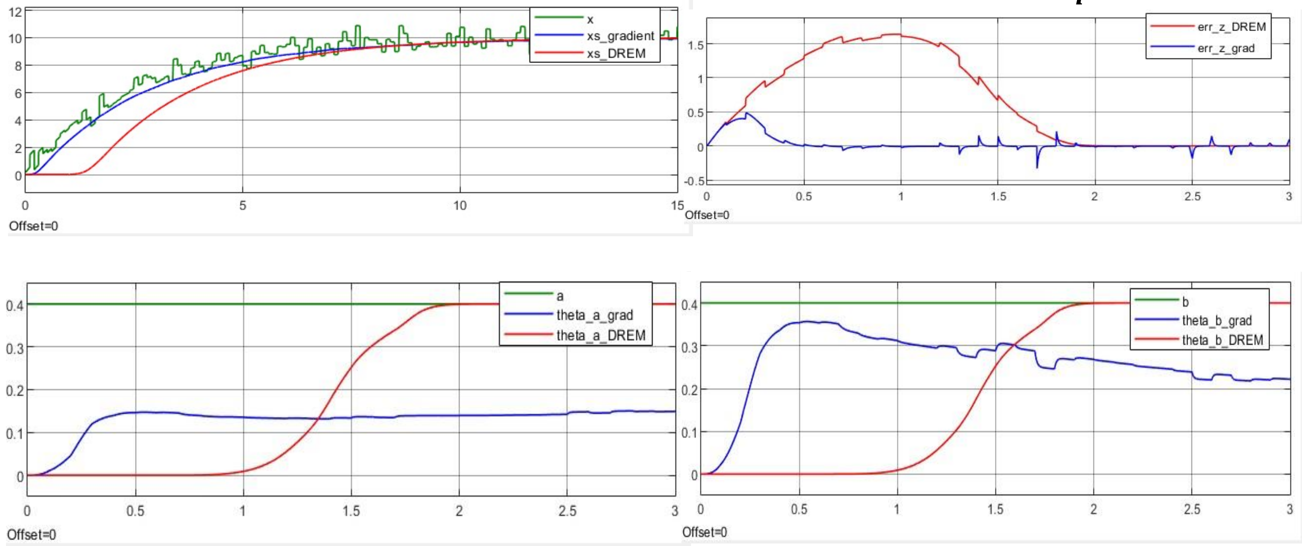
Estimation Analysis with noise on the nominale state

$u(t) = 10$

$noise_{pwr} = 0,2$

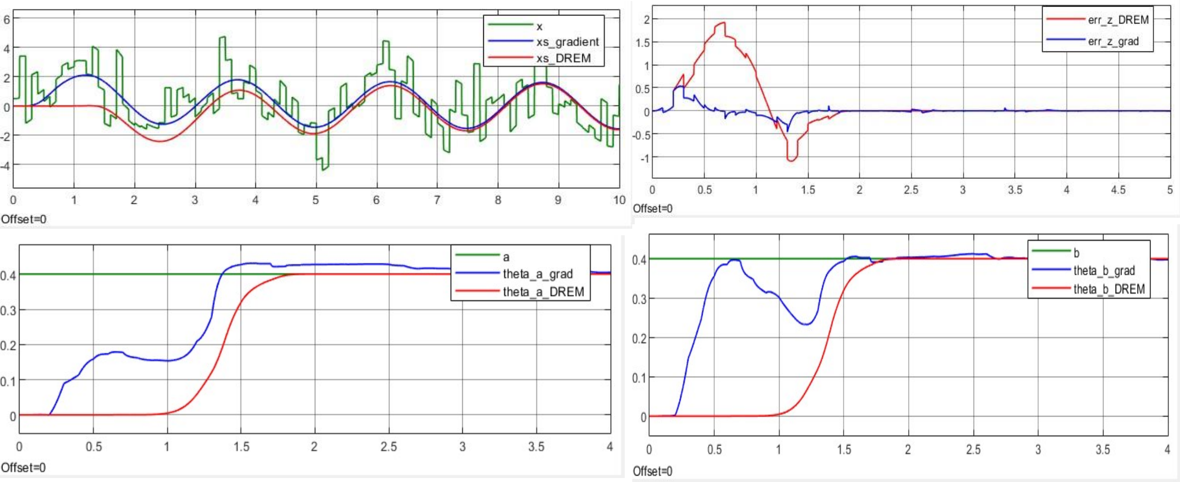


$noise_{pwr} = 0,02$

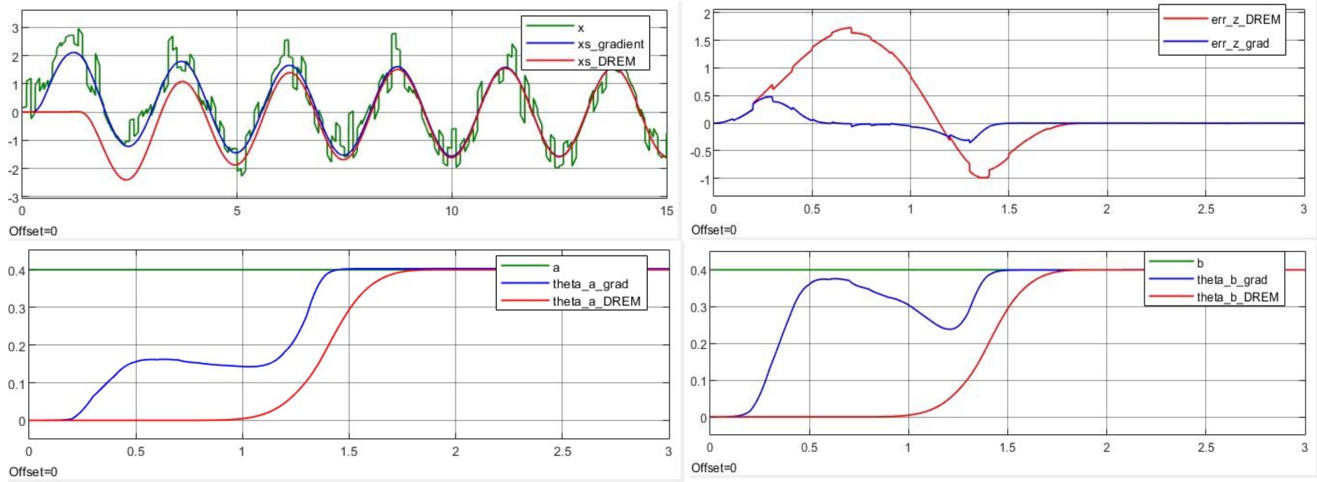


$u(t) = 10\sin(\frac{5}{2}t)$

$noise_{pwr} = 0,2$



$noise_{pwr} = 0,02$



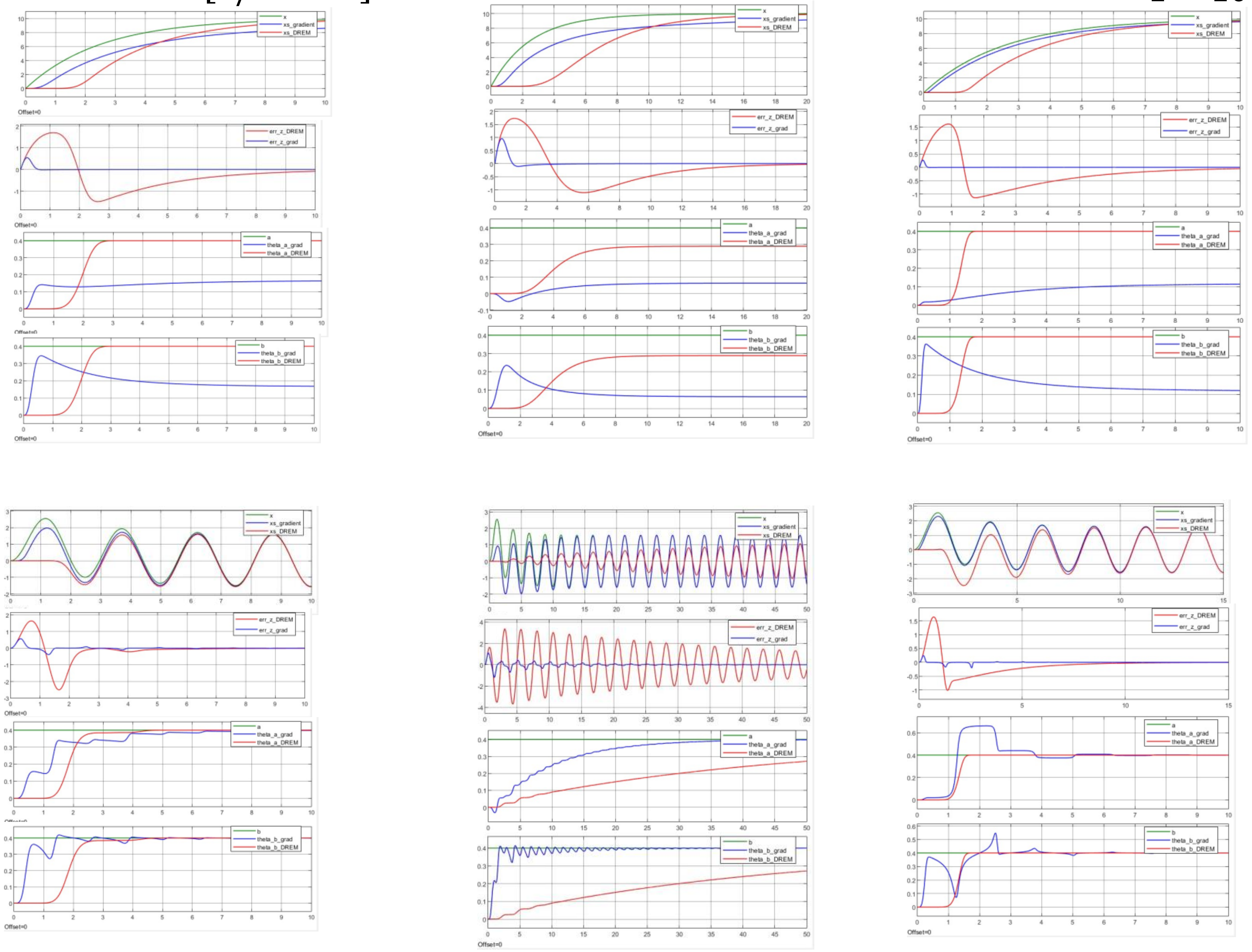
Gains effect on estimation Γ / γ

$$W(s) = \frac{1}{(s+1)}$$

$\gamma_{1,2} = 2;\ \Gamma = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$ | $\gamma_{1,2} = 0{,}02;\ \Gamma = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$ | $\gamma_{1,2} = 20;\ \Gamma = \begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix}$

$\boldsymbol{u(t) = 10}$

$\boldsymbol{u(t) = 10\sin(\frac{5}{2}t)}$



Noise rejection:
DREM estimator performs a better noise rejection on both state and parameters, this ability is lightly affected by noise power. This behaviour is due to the filtering action that on the one side, slows the estimation dynamic, but on the other side, helps noise rejection.

Gain effects:
It can be observed that for both estimators an increase in gains values will generate a quicker state and parameters estimation. While if the gain is decreased DREM method will loose its performance faster than Gradient method.

In transient phase DREM method shows a delay in estimation and this delay is not affected by gain variation.

COMPARISON

GRADIENT:
PRO:
- Quicker estimation
- Smaller estimation error
- Easily tunable through gains

CON:
- It needs a signal richness at least equal to the number of parameters
- It hardly rejects noises, this can be improved by placing a filter before it. However this leads to a trade off between noise rejection and response time.

DREM
PRO:
- It can evaluate all parameters even if signal richness is not equal or higher than the number of parameters
- It has a good noise rejection

CON:
- Slower in parameters estimation.
- High estimation error during initial transient, this can be smoothed by speeding up filter pole.