ROBUST & ADAPTIVE CONTROL

ADAPTIVE MODEL PARAMETERS EVALUATION WITH P AND SP ESTIMATORS

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PARALLEL ESTIMATOR - THEORY BACKGROUND

PARALLEL MODEL: $\dot{\hat{x}} = -\hat{a}\,\hat{x} + \hat{b}\,u$

def: $\tilde{a} = \hat{a} - a$; $\qquad \tilde{b} = \hat{b} - b$

estimation error: $\epsilon_1 = x - \hat{x} \quad \rightarrow \dot{\epsilon}_1 = -a\,\epsilon_1 + \tilde{a}\,\hat{x} - \tilde{b}\,u$

Storage function:

$$S = \frac{\epsilon_1^2}{2} \quad \rightarrow \dot{S} = \epsilon_1\,\dot{\epsilon}_1 = \epsilon_1(-a\,\epsilon_1 + \tilde{a}\,\hat{x} - \tilde{b}\,u) = -a\,\epsilon_1^2 + \epsilon_1\,\tilde{a}\,\hat{x} - \epsilon_1\,\tilde{b}\,u = -a\,\epsilon_1^2 + [\,\epsilon_1\,\hat{x} \quad -\epsilon_1\,u\,]\begin{bmatrix}\tilde{a}\\ \tilde{b}\end{bmatrix}$$

PARALLEL ESTIMATOR - THEORY BACKGROUND

The time derivatives of the storage function reveal that the estimation error equation describes a passive system with an input: $\begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix}$ and output: $\begin{bmatrix} \epsilon_1 \hat{x} \\ -\epsilon_1 u \end{bmatrix}$. Interconnecting this to the passive system $\dot{\xi} = v$ we can obtain $\begin{cases} \dot{\tilde{a}} = -\epsilon_1 \hat{x} \\ \dot{\tilde{b}} = \epsilon_1 u \end{cases}$ these are not implementable, however, remembering that $(\dot{\tilde{a}} = \dot{\hat{a}}; \dot{\tilde{b}} = \dot{\hat{b}})$

the following estimator model has been derived, in which a multiplicative gain has been used:

$$\begin{cases} \dot{\hat{x}} = -\hat{a}\,\hat{x} + \hat{b}\,u \\ \dot{\hat{a}} = -\gamma_1 \epsilon_1 \hat{x} \\ \dot{\hat{b}} = \gamma_2 \epsilon_1 u \end{cases}$$

SERIES-PARALLEL ESTIMATOR - THEORY BACKGROUND

SERIES - PARALLEL MODEL: $\dot{\hat{x}} = -a_m \hat{x} + (a_m - \hat{a})\, x + \hat{b}\, u$

def: $\tilde{a} = \hat{a} - a;$ $\qquad \tilde{b} = \hat{b} - b$

estimation error: $\epsilon_1 = x - \hat{x} \quad \rightarrow \dot{\epsilon}_1 = -a_m \epsilon_1 + \tilde{a}\, x - \tilde{b}\, u$

Storage function:

$$S = \frac{\epsilon_1^2}{2} \quad \rightarrow \dot{S} = \epsilon_1 \dot{\epsilon}_1 = \epsilon_1(-a_m \epsilon_1 + \tilde{a}\, x - \tilde{b}\, u) = -a_m \epsilon_1^2 + \epsilon_1 \tilde{a}\, \hat{x} - \epsilon_1 \tilde{b}\, u = -a_m\, \epsilon_1^2 + \begin{bmatrix} \epsilon_1 x & -\epsilon_1 u \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix}$$

SERIES-PARALLEL ESTIMATOR - THEORY BACKGROUND

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the following estimator model has been derived, in which a multiplicative gain has been used:

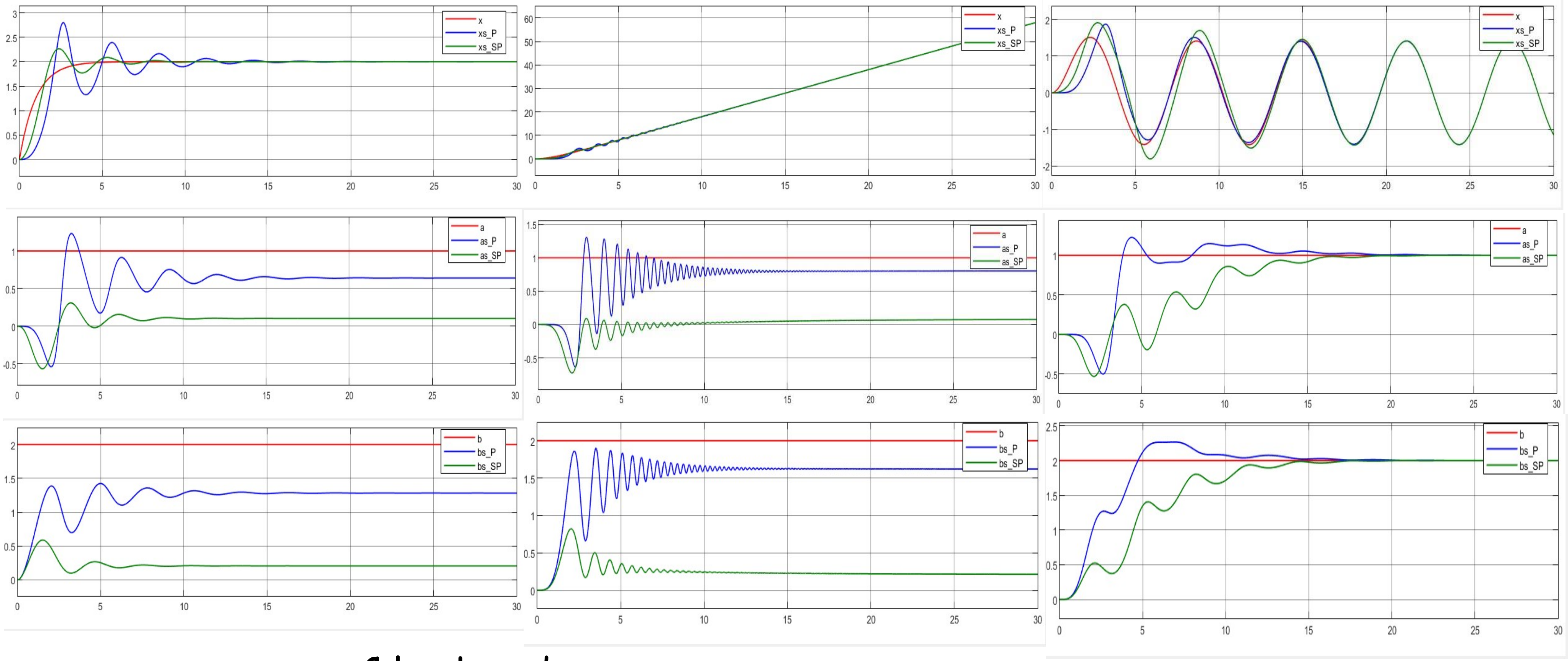
$$\begin{cases} \dot{\hat{x}} = -\hat{a}_m \, \hat{x} + (a_m - \hat{a}) \, x + \hat{b} \, u \\ \dot{\hat{a}} = -\gamma_1 \, \epsilon_1 \, x \\ \dot{\hat{b}} = \gamma_2 \, \epsilon_1 \, u \end{cases}$$

Estimated state and parameters with different input

Simulation Time: 30s

$$(a_m = 1, \gamma_1 = 1, \gamma_2 = 1)$$

$u(t) = 1$ $u(t) = t$ $u(t) = \sin(t)$



x

a

b

Colour Legend:

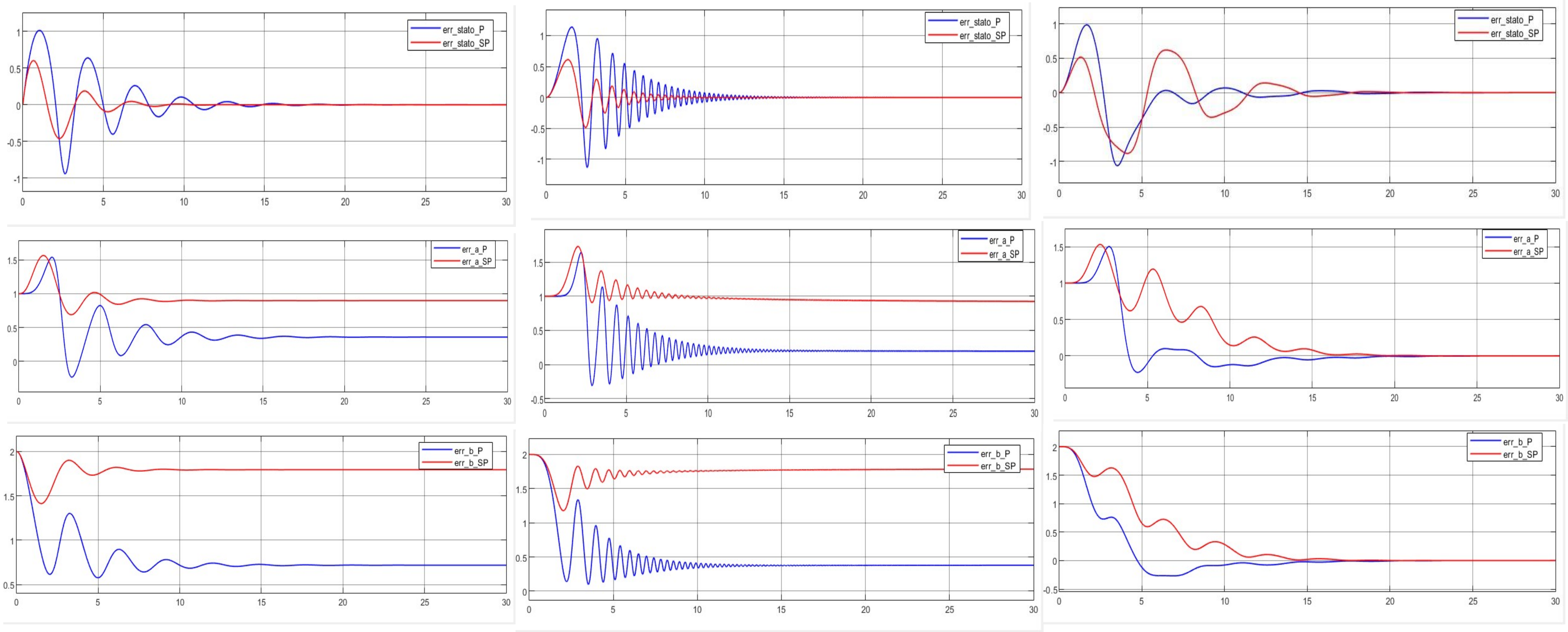
NOMINAL MODEL PARALLEL MODEL SERIES-PARALLEL MODEL

ERRORS of Estimated state and parameters with different input

$(a_m = 1, \gamma_1 = 1, \gamma_2 = 1)$

Simulation Time: 30s

$u(t) = 1$ | $u(t) = t$ | $u(t) = \sin(t)$



x

a

b

Colour Legend: PARALLEL MODEL SERIES-PARALLEL MODEL

OBSERVATIONS AND REMARKS

Estimated state and parameters with different input:

STEP and RAMP INPUT:
If the input signal is not rich, as in the case of step or ramp, both estimator P and SP do not reach convergence of parameters values. A premature convergence of the estimated state to the real state can be observed, which causes a stop in parameter evaluation. P estimator brings parameter values closer to the real ones. While SP estimator shows a quicker settling time and a smaller overshoot of the estimated state.

SINE INPUT:
Both estimators reach convergence of state and parameter.

It can be observed that **P estimator has a better performance in the parameter estimation**, with a smaller rise time, while settling time does not differ in a significant manner from SP estimator.

If the input is rich enough, we have a convergence of estimated parameters to the real ones occurs, as a matter of fact a sine wave belongs to the order 2, as the number of parameter that has to be estimated.

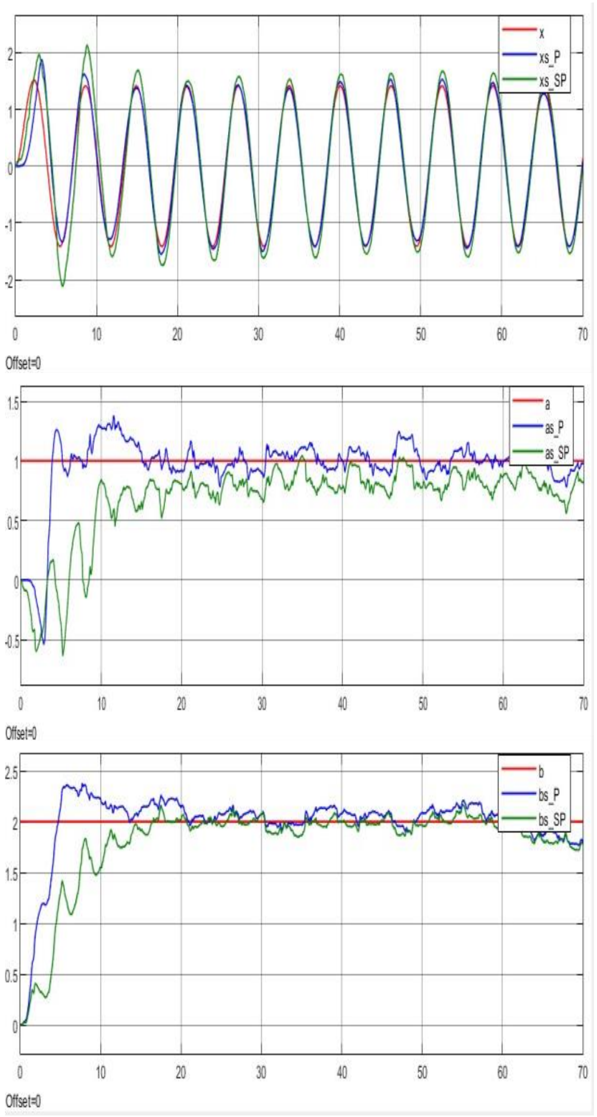
Simulation with noise

$(a_m = 1, \gamma_1 = 1, \gamma_2 = 1)$

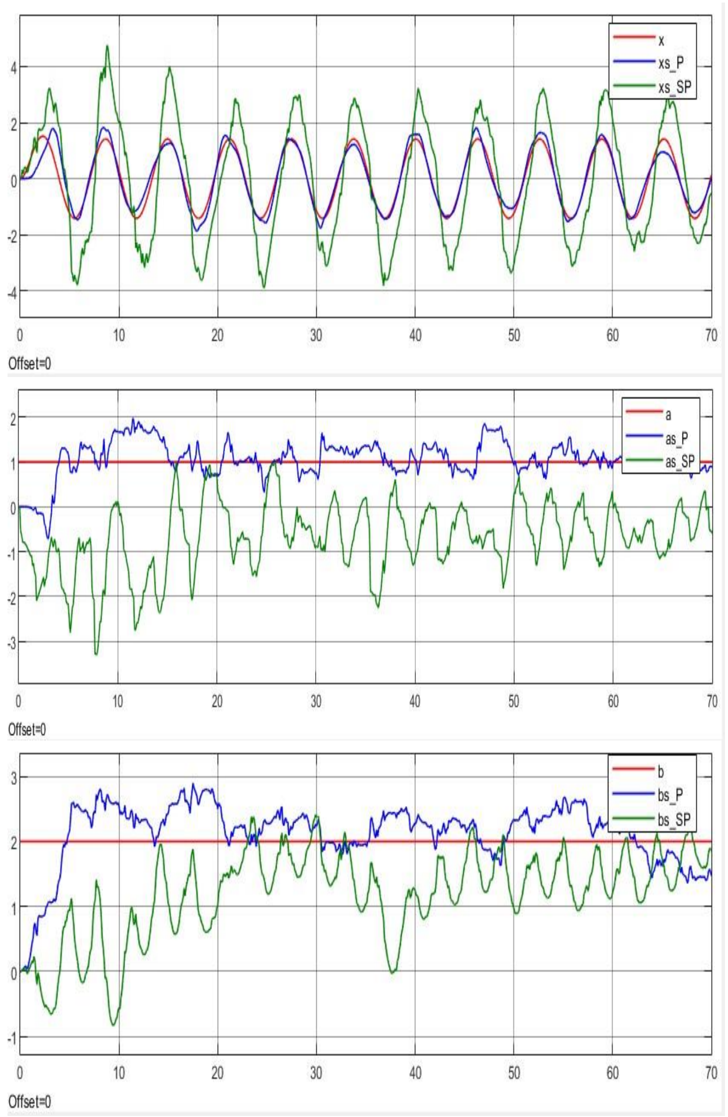
$$u(t) = \sin(t)$$

Simulation Time: 70s

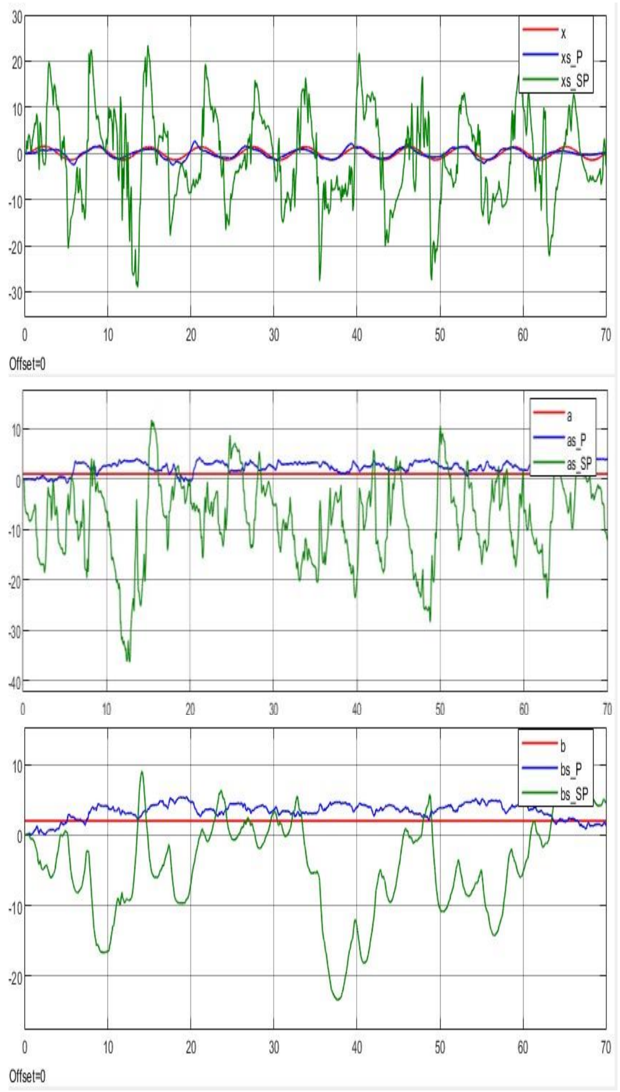
$noise_{pwr} = 0.01$



$noise_{pwr} = 0.1$

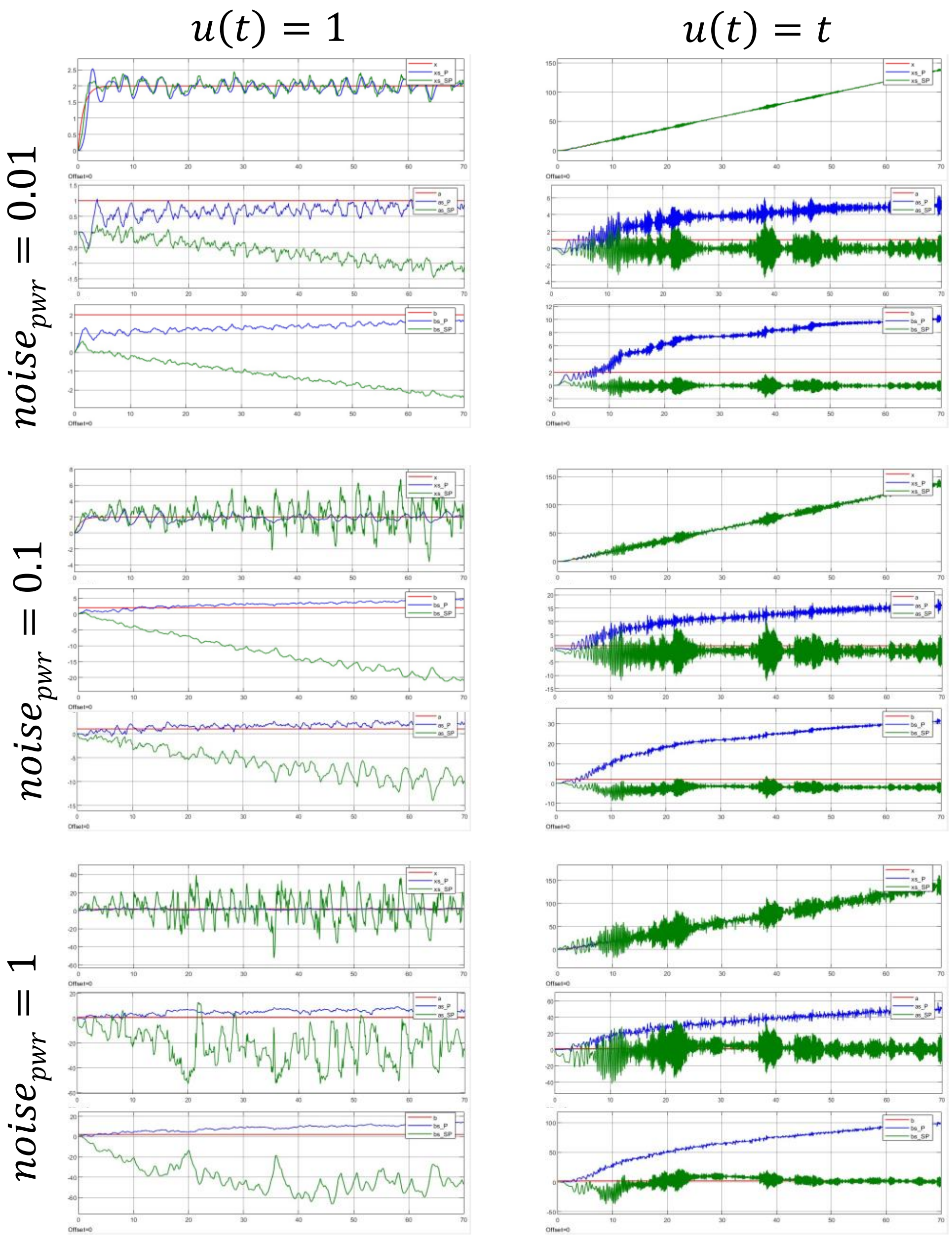


$noise_{pwr} = 1$



Simulation with noise

$(a_{\mathrm{m}} = 1, \gamma_1 = 1, \gamma_2 = 1)$



With a sine wave input it can be observed that the Parallel system performs a better rejection to a noise added to the real system output, this difference increases with noise power.

Even if it's not that meaningful to assert something on performance without having the estimator convergence it can be obesreved that:

- With a steady input and esepcially with high noise power, he SP estimator is not able to reject the noise and diverge in a sesible manner.
- With a ramp input it can be observed that P estimator not only does not converge to the correct parameter but rather it diverges.

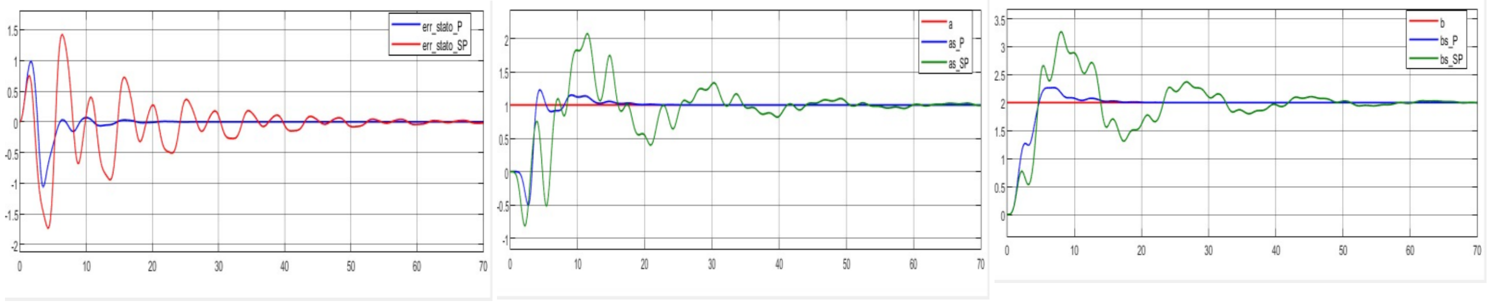
What can be inferred is that since SP estimator directly depends on the state input, it is affect not only by the estimator dynamics but also directly by the real system, this causes a general delay in tracking a signal and a stronger dependence to real system noises or meausurement errors.

Simulation with different a_m values

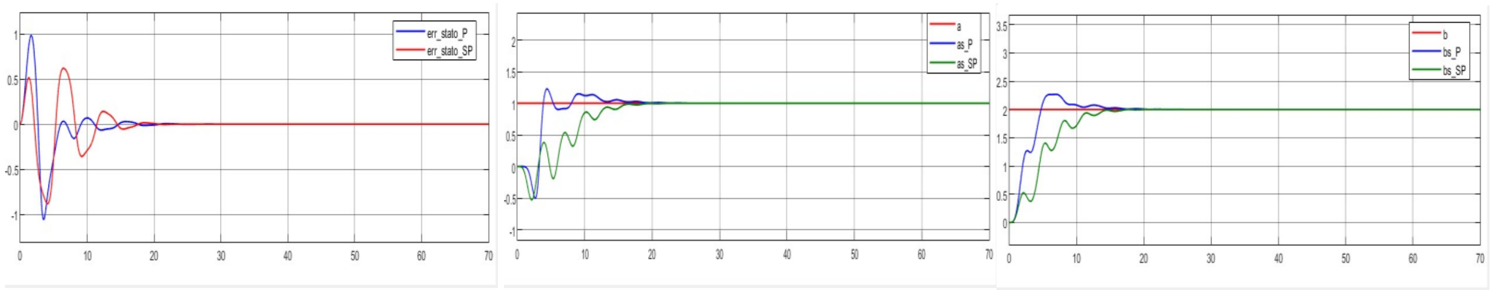
$(\gamma_1 = 1, \gamma_2 = 1)$

$$u(t) = \sin(t)$$

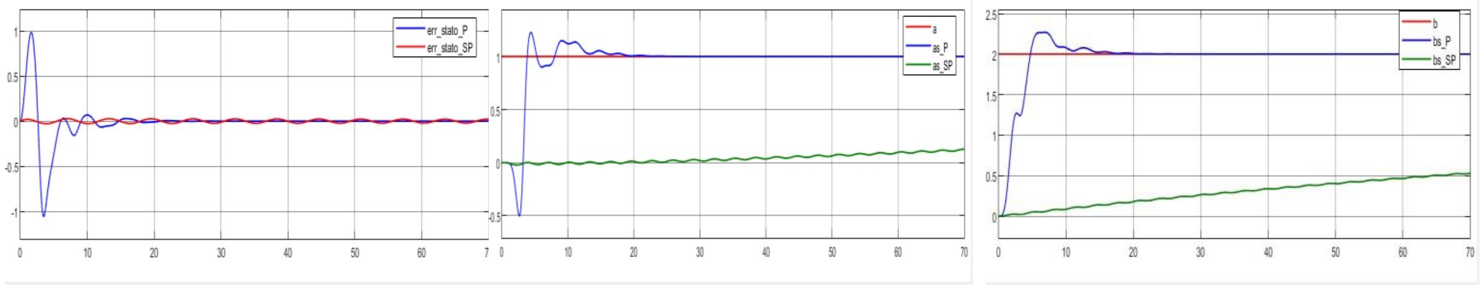
$a_m = 0.2$



$a_m = 1$



$a_m = 50$



Changing a_m causes a variation in SP estimator performance. It can be observed that with a higher a_m value the state error gets to zero quicker, however, this higher a_m value also causes a slower parameter convergence.

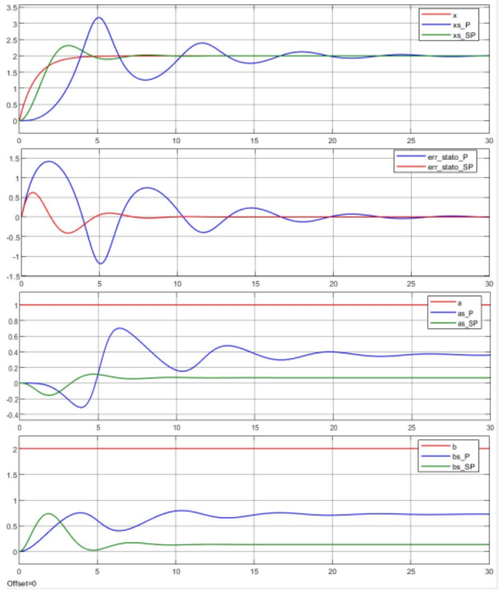
Simulation with γ values

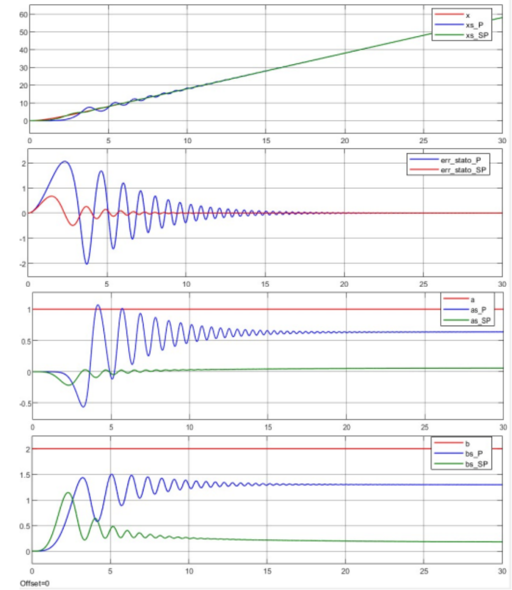
$(a_m = 1)$

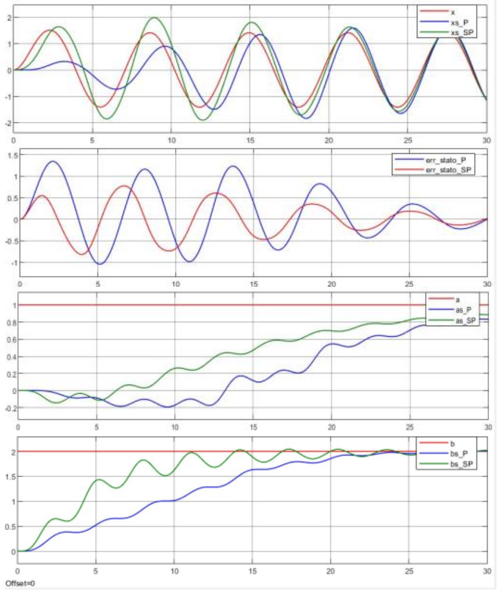
Simulation Time: 30s

$u(t) = 1$ | $u(t) = t$ | $u(t) = \sin(t)$

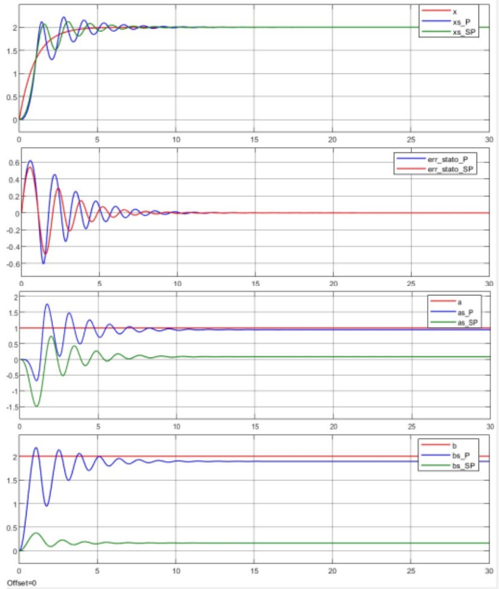
$\gamma_1 = 0.2,\ \gamma_2 = 0.2$

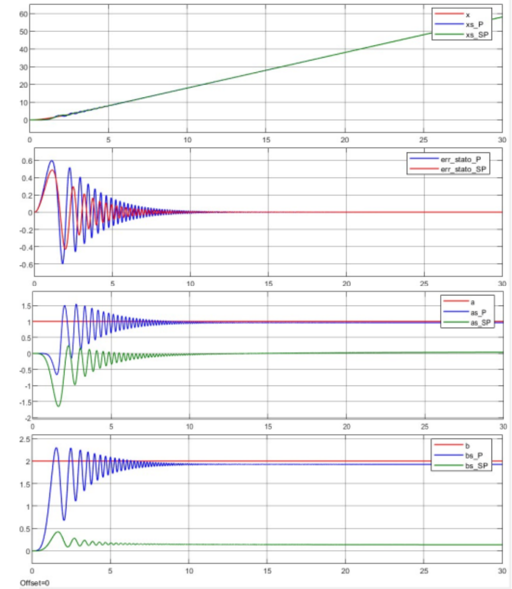


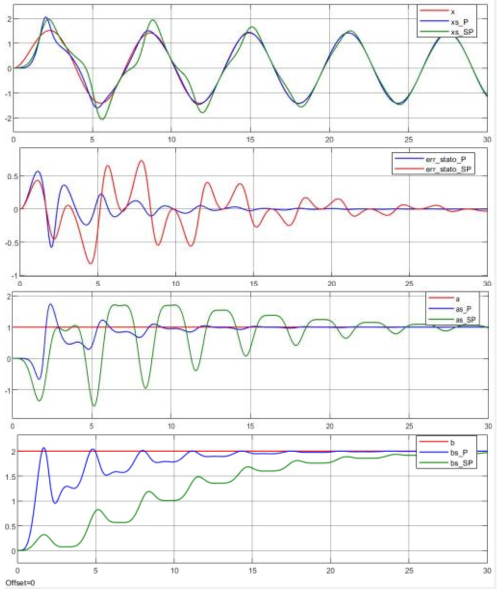




$\gamma_1 = 5,\ \gamma_2 = 5$





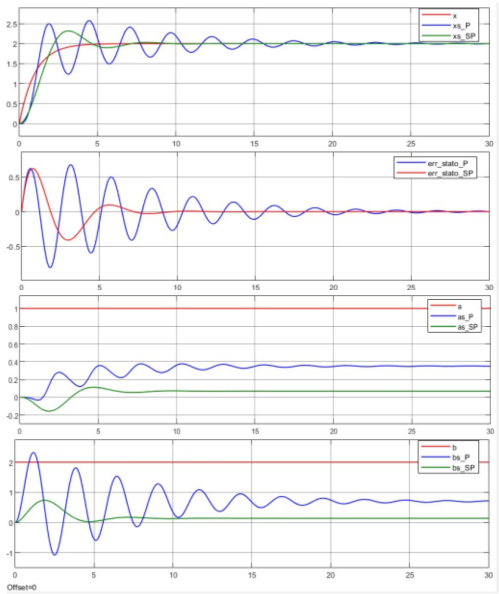
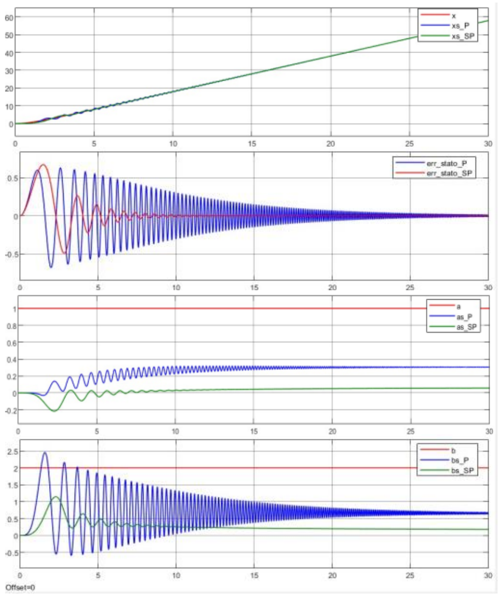
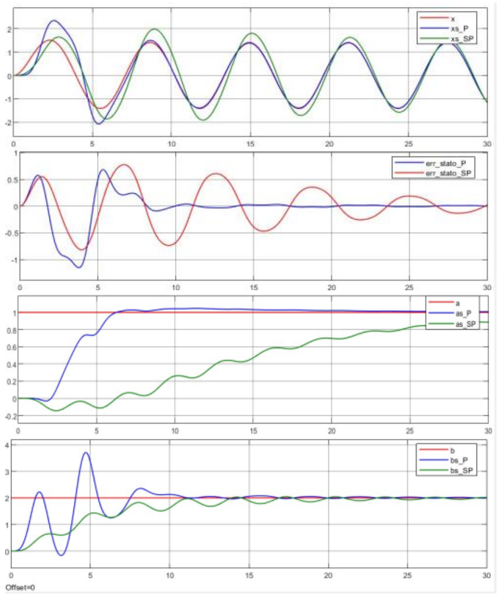
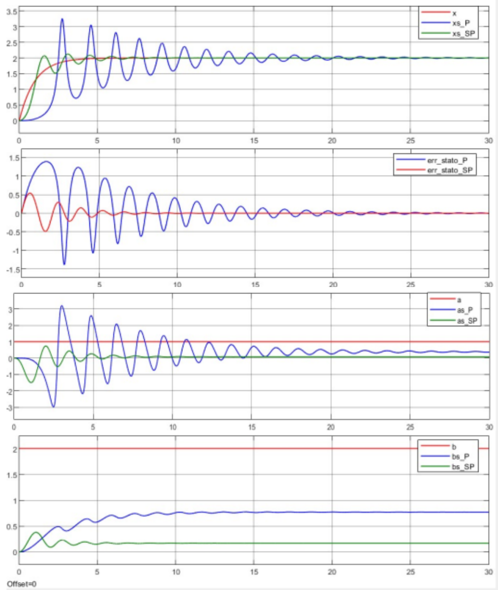
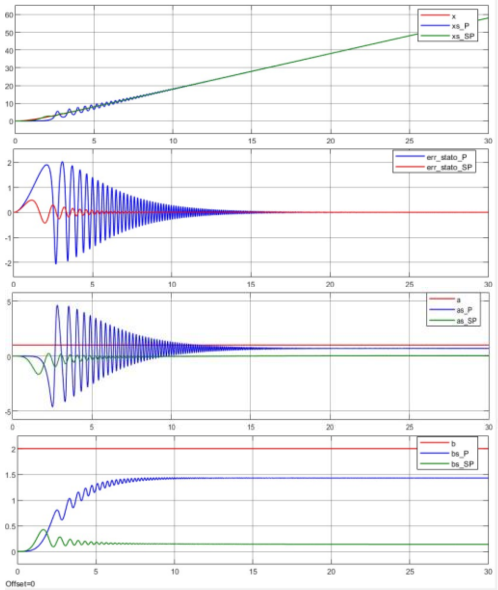
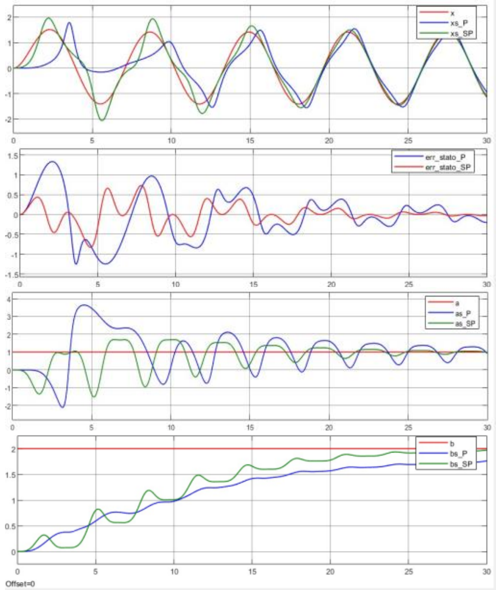


A systematic variation of γ has been performed.
If both $\gamma 1$ and γ_2 are modified together it can be observed that in the case of a steady or ramp input, raising gamma values will cause a quicker converge of the estimated state. While in the case of a sine input this improvement is not so marked.
What can be observed on the other side is that with low gamma ($\gamma 1 = \gamma 2 = 0.2$) SP estimator has a quicker convergence time, while with gamma = 5 the P estimator improves its performance considerably, overtaking the SP estimator.

Simulation with different γ: $\gamma_1 \neq \gamma_2$

$(a_m = 1)$

Simulation Time: 30s

	$u(t) = 1$	$u(t) = t$	$u(t) = \sin(t)$
$\gamma_1 = 0.2,\ \gamma_2 = 5$			
$\gamma_1 = 5,\ \gamma_2 = 0.2$			

In this group of simulations $\gamma 1$ and $\boldsymbol{\gamma_2}$ have been modified in such a manner that their sum is constant but their values differ.

For both estimators, convergence time is not affected in a significant way, while paremeter convergence time shows a stronger dependance.

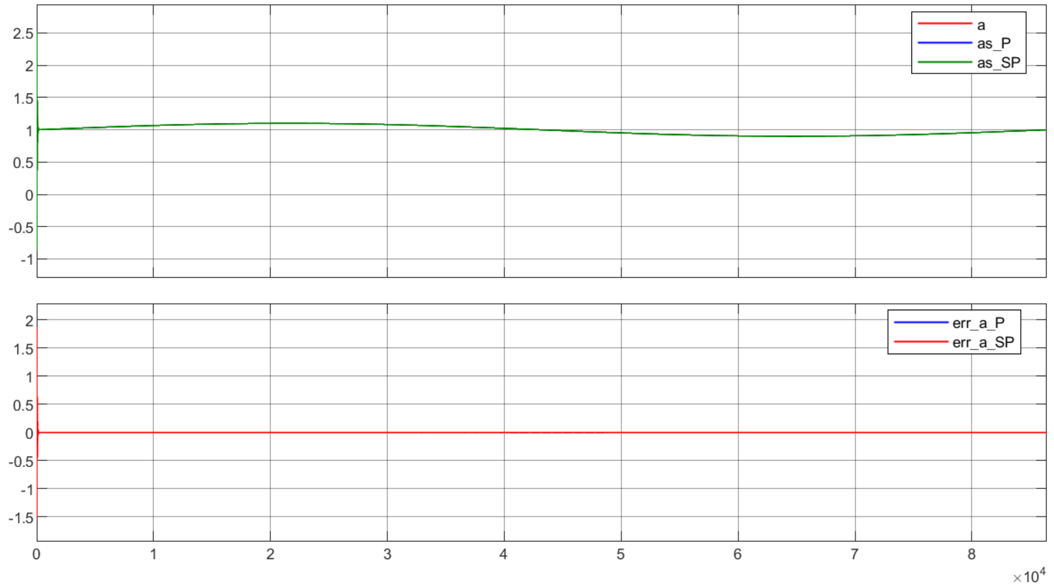
Especially for the SP estimator, a smaller γ_1 results in a slower convergence of parameter *a*. This result can also be observed in both P and SP estimators for b parameter in the case of a smaller γ_2.

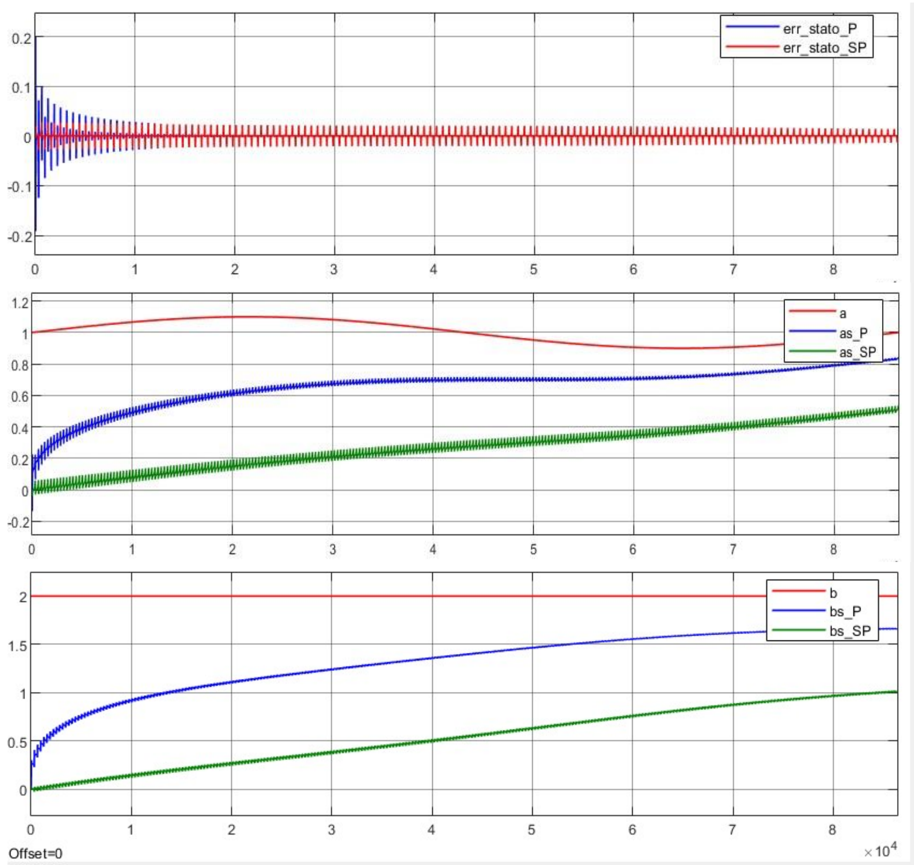
Simulation with a parameter time dependant

$(a_m = 1, \gamma_1 = 1, \gamma_2 = 1)$

Simulation Time: 1gg

$$u(t) = \sin(0.1\ t)$$





$$a(t) = \sin\left(\frac{2\pi t}{24 * 3600}\right)$$

It can be observed that a slow variation in the parameter does not affect the estimators effectiveness. Also, in this case P estimator shows better performances. If the variation speed of the parameter is close to the model and estimator time is constant, then the evaluation fails.

FINAL OBSERVATIONS AND REMARKS

- Accuracy:
  Both parallel and series-parallel estimators can provide accurate estimates of the system state if the signal is rich enough.
- Convergence:
  Both parallel and series-parallel estimators can converge to the true system state, but the convergence rate may be different. Parallel estimators converge more quickly, as it is operating independently and does not rely on the output of the model. Series-parallel estimators may converge more slowly.
- Robustness:
  Parallel estimators may be more robust than series-parallel estimators, as they do not rely on the output of a single estimator as their input. However, both types of estimators can be designed to be robust to measurement noise and uncertainty