TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

ROBUST & ADAPTIVE CONTROL

MRAC AND I&I ADAPTIVE CONTROLLER

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TASK:

Consider the system:

$$\dot{x} = a\,x + b\,u$$

with a constant and unknown. Use, in the simulations, $a = 1$ and $a_m = 1$.

Design a (classical) MRAC adaptive controller and an I&I adaptive controller to achieve adaptive regulation (with at least two selections of the function β for the I&I design). Compare the performance of the resulting closed-loop systems in terms of speed of response, control amplitude, and in the presence of measurement noise, that is the measured variable is $x + d$, with $d(t) = 0.1\sin\left(\frac{t}{5}\right)$.

Finally, compare the performance of the resulting controllers in the case in which the parameter is given by

$$a = 1 + \frac{1}{10}\sin(10\,t) \qquad a = 1 + 10\sin\left(\frac{t}{10}\right)$$

FORMULAS:

model reference: $\dot{x} = -a_m x$

MRAC:

(Model Reference Adaptive Controller)

control law: $u = -\hat{k}\, x$

estimator model: $\dot{\hat{k}} = \gamma_1 \,(x - x_m)\, x$

I&I:

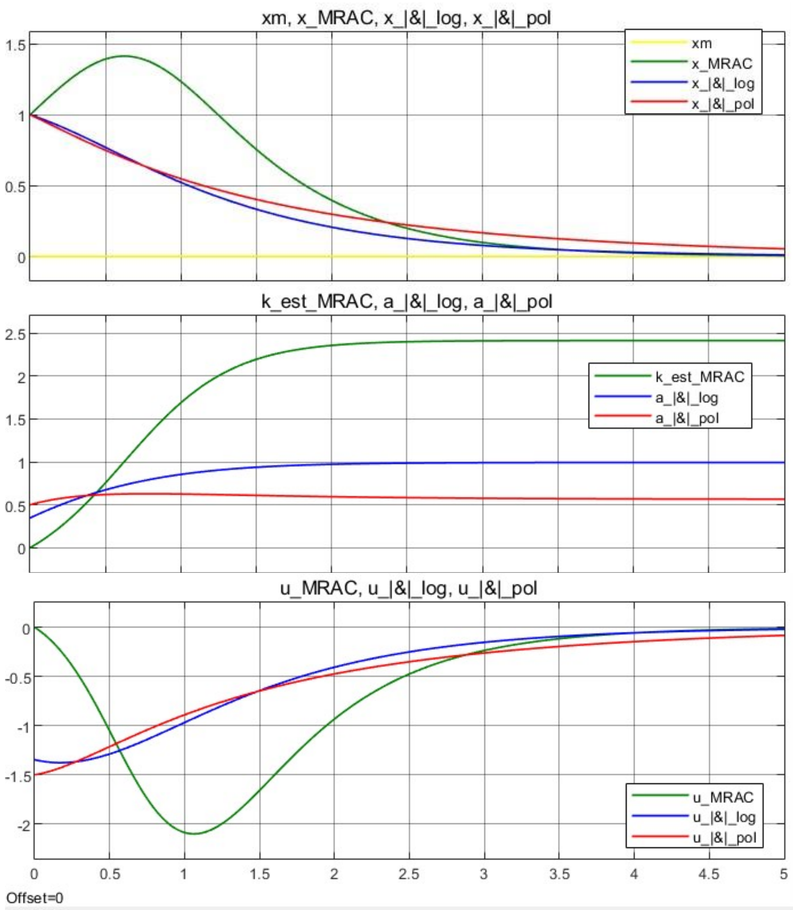
control law: $u = -a_m x - a_{\text{est}}\, x$

$$\text{estimator: } \begin{cases} a_{\text{est}} = \hat{a} + \beta(x) \\ \hat{a} = a_m \frac{\partial \beta(x)}{\partial x}\, x \end{cases}$$

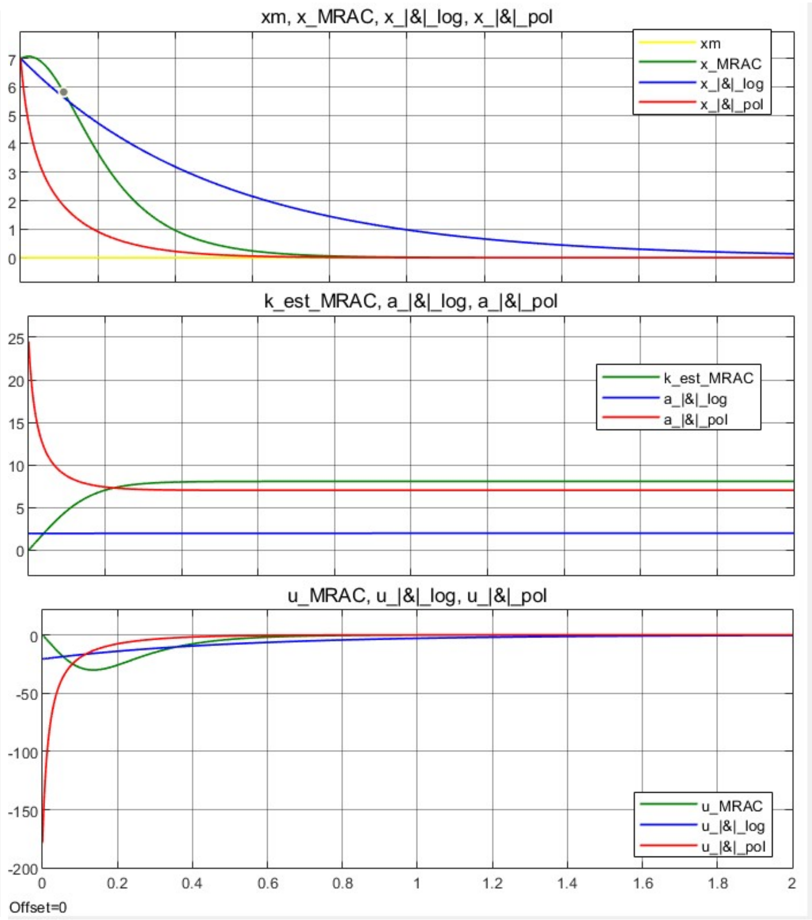
$$\begin{cases} \beta(x) = \frac{x^2}{2} \rightarrow \frac{\partial \beta(x)}{\partial x} = x \leftarrow \text{non normalized} \\ \beta(x) = \frac{1}{2}\ln(1 + x^2) \rightarrow \frac{\partial \beta(x)}{\partial x} = \frac{x^2}{1 + x^2} \leftarrow \text{normalized} \end{cases}$$

Analisi dei modelli

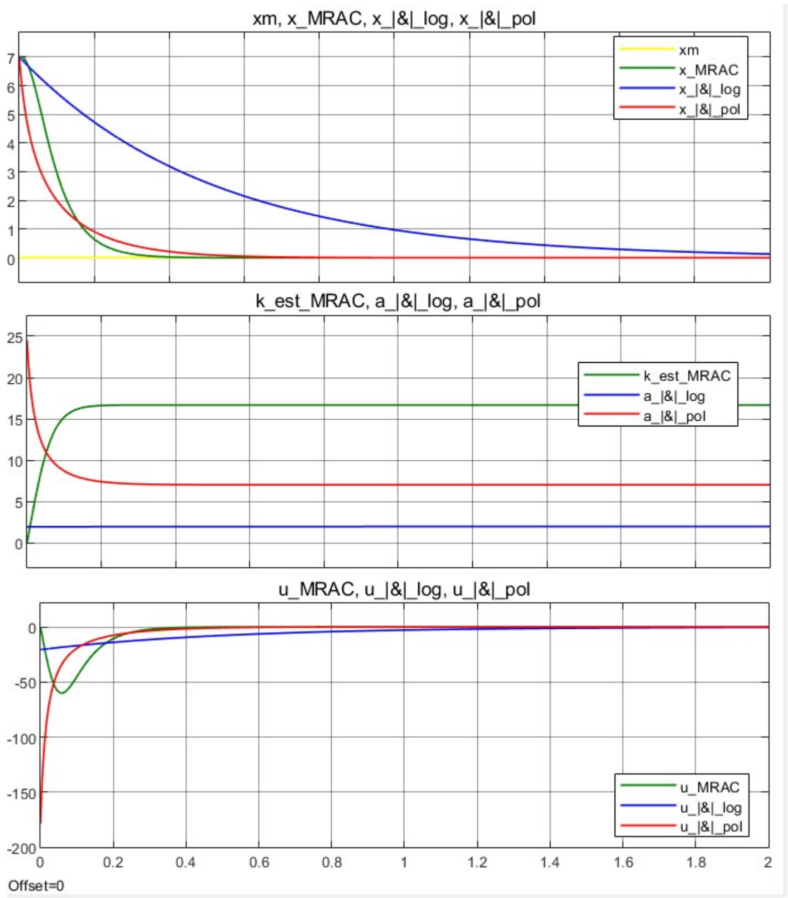
$x_0 = 1, \gamma = 1$



$x_0 = 7, \gamma = 1$



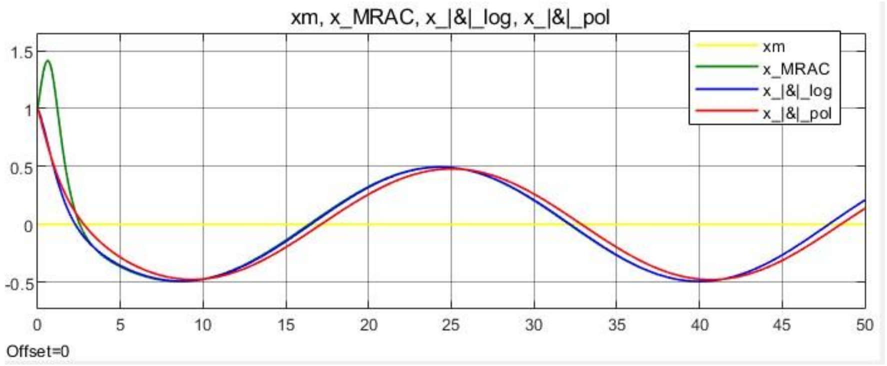
$x_0 = 7, \gamma = 5$

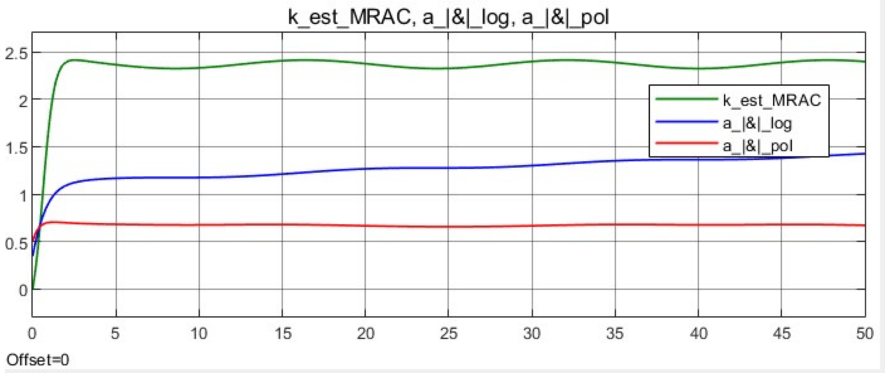


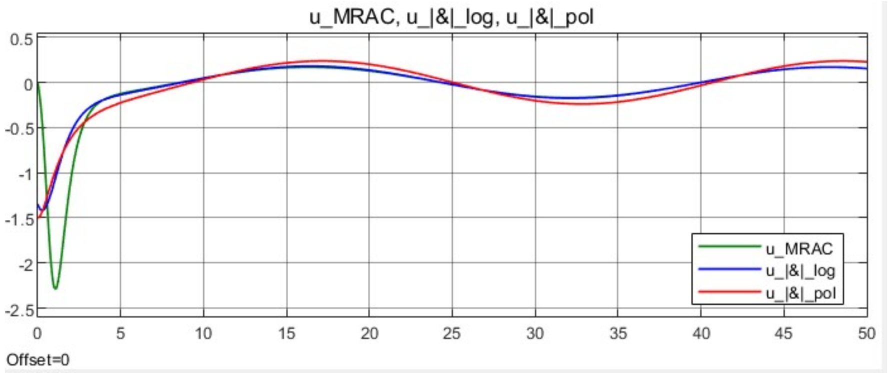
It can be observed in MRAC control that both estimated state and control signal are affected by an overshoot in the initial transient, this can be reduced rising γ value.
In the case of small initial conditions I&I controllers, polynomial and logaritmic, generate a smooth control signal which derive from a smooth estimated state.
It can be observed that in the case of high value of the initial condition, the polinomial I&I generates an input that has a peak one order of magnitude higher than the other controls, this could stress the actuator.

Analisi dei modelli con disturbo d(t) = 0.1 sin(0.2 t)

$\gamma = 1$







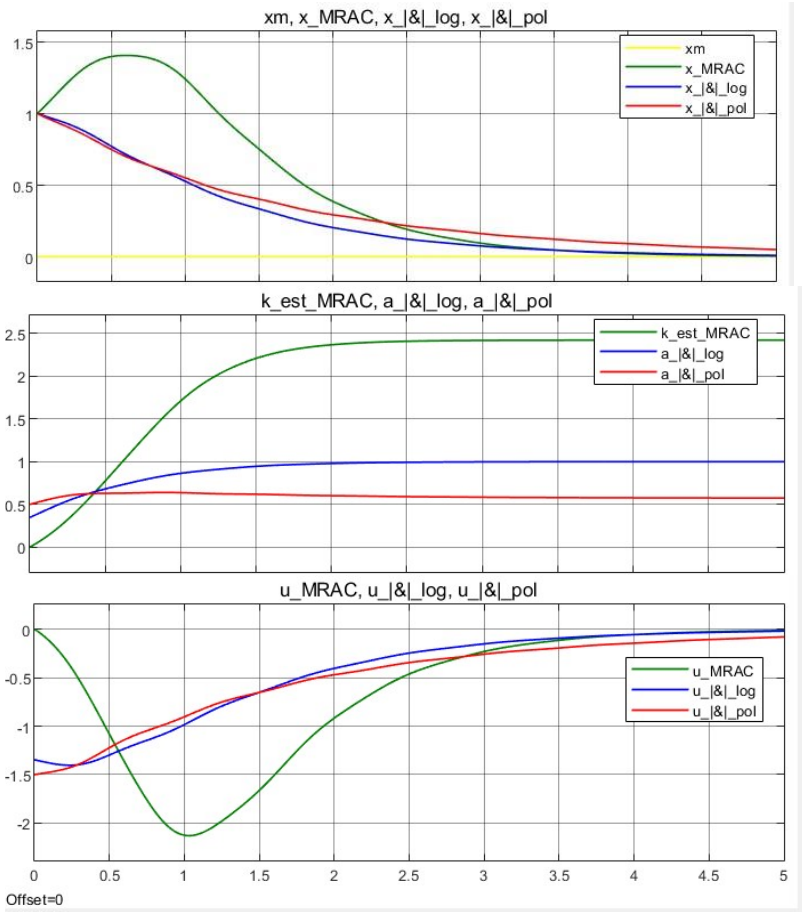
If there is a sine noise on the measured state then MRAC and I&I poly and log will bring a sine error on system regulation, as they try to follow the input signal received.

It can also be observed that the control parameter k is affected by the noise on the measure, this is more noticeable in MRAC control.
As a consequence of this also the control signal will have a sine behaviour.

Analysis of time dependant system parameter effect

$\gamma = 1$

$a = 0.1\sin(10t)$



If the real system parameter has a time dependance, MRAC, I&I poly and I&I log controls are all able to stabilize the system and track the error to zero.
In the case of a high amplitude of the parameter variation the controls show worse performance than the case of an high frequency of variation.

The MRAC control is more affected by the variation than the I&Is, leading to an almost unstable behaviour in the initial transient, on the other side it has a faster settling time.

$a = 10\sin(0.1t)$