# ROBUST & ADAPTIVE CONTROL

## IMMERSION AND INVARIANCE FREQUENCIES ESTIMATION

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# 1 TASK

To study the performance of the I&I frequency estimator consider the two signals:

$y_1(t) = \sin(100\,t)$; $y_2(t) = \sin(t)$

Implement the I&I frequency estimator and run simulations showing the effectiveness of the approach. Experiment with different values of the gain $k_1$ and discuss what is a good selection for such a gain.

Suppose that the measured signals are perturbed by a periodic disturbance, that is one measures $y_1 + d$, with $d = 0.001\sin(50\,t)$. Run simulations for this scenario and discuss the impact that the disturbance has on the convergence of the frequency estimates.

## 2 I&I frequency estimator

$y(t) = E\sin(\omega t + \phi)$, the task is to identify the values $E, \omega$ and $\phi$. The estimator has the afterward form:

$$\begin{cases} \dot{y} = x \\ \dot{x} = -\theta_1\, y \\ \dot{\theta}_1 = 0 \\ \theta_1 = \omega^2 \end{cases}$$

instead of estimating the state error, parameter error and output error with the I&I approach we consider only state and parameter errors adding an injection term.

$$\begin{cases} Z_x \triangleq \underbrace{k_1\, y}_{\text{inj. term}} + \hat{x}\,(k_1^2 + \theta_1) - x \qquad \text{with}\, k_1 > 0 \qquad \leftarrow \text{error on state evaluation} \\ Z_\theta \triangleq \underbrace{y\,\hat{x}}_{\text{inj. term}} + \hat{\theta}_1 - \theta_1 \qquad \leftarrow \text{error on parameter evaluation} \end{cases}$$

the system is derived on time and substitution with the estimator form is performed:

$$\begin{cases} \dot{Z}_x = k_1\, \dot{y} + \dot{\hat{x}}\,(k_1^2 + \theta_1) - \dot{x} = k_1\, x + \dot{\hat{x}}\,(k_1^2 + \theta_1) + \theta_1\, y \\ \dot{Z}_\theta = \dot{y}\,\hat{x} + y\,\dot{\hat{x}} + \dot{\hat{\theta}}_1 = x\,\hat{x} + y\,\dot{\hat{x}} + \dot{\hat{\theta}}_1 \end{cases}$$

the system is solved on $x$ and $\theta_1$:

$$\begin{cases} x = k_1\, y + \hat{x}\,(k_1^2 + \theta_1) - Z_x \\ \theta_1 = y\,\hat{x} + \hat{\theta}_1 - Z_\theta \end{cases}$$

and substituded in $\dot{Z}_\theta$:

$$\dot{Z}_\theta = (k_1\, y + \hat{x}\,(k_1^2 + \theta_1) - Z_x)\,\hat{x} + y\,\dot{\hat{x}} + \dot{\hat{\theta}}_1 = k_1\,\hat{x} y + \hat{x}^2\,(k_1^2 + \theta_1) - Z_x\,\hat{x} + y\,\dot{\hat{x}} + \dot{\hat{\theta}}_1$$

it can be observed that $\dot{Z}_\theta = -\hat{x}\,Z_x - \hat{x}^2\,Z_\theta - \Delta + \dot{\hat{\theta}}_1$

$$k_1\,\hat{x} y + \hat{x}^2\,(k_1^2 + \theta_1) - Z_x\,\hat{x} + y\,\dot{\hat{x}} + \dot{\hat{\theta}}_1 = -\hat{x}\,Z_x - \hat{x}^2\,Z_\theta - \Delta + \dot{\hat{\theta}}_1$$

$$k_1\,\hat{x} y + \hat{x}^2\,(k_1^2 + \theta_1) + y\,\dot{\hat{x}} + \hat{x}^2\,Z_\theta = -\Delta$$

$$\Delta = -k_1\,\hat{x} y - \hat{x}^2\,(k_1^2 + \theta_1) - y\,\dot{\hat{x}} - \hat{x}^2\,Z_\theta$$

$Z_\theta$ definition is then substituted:

$$\Delta = -k_1\,\hat{x} y - \hat{x}^2\,(k_1^2 + \theta_1) - y\,\dot{\hat{x}} - \hat{x}^2 \underbrace{(y\,\hat{x} + \hat{\theta}_1 - \theta_1)}_{Z_\theta}$$

$$\Delta = -k_1\,\hat{x} y - \hat{x}^2\,k_1^2 - \hat{x}^2\,\theta_1 - y\,\dot{\hat{x}} - \hat{x}^3\,y - \hat{x}^2\,\hat{\theta}_1 + \hat{x}^2\,\theta_1$$

$$\Delta = -k_1\,\hat{x} y - \hat{x}^2\,k_1^2 - y\,\dot{\hat{x}} - \hat{x}^3\,y - \hat{x}^2\,\hat{\theta}_1$$

by imposing the following estimator update law: $\dot{\hat{x}} = -k_1\,\hat{x} - y$ and substituting in $\Delta$:

$$\Delta = -k_1\,\hat{x}y - \hat{x}^2\,k_1^2 - y\,\underbrace{(-k_1\,\hat{x} - y)}_{\dot{\hat{x}}} - \hat{x}^3\,y - \hat{x}^2\,\hat{\theta}_1 = -k_1\,\hat{x}y - \hat{x}^2\,k_1^2 + y\,k_1\,\hat{x} + y^2 - \hat{x}^3\,y - \hat{x}^2\,\hat{\theta}_1$$

$$\Delta = \hat{x}^2\,(-\hat{x}\,y - \hat{\theta}_1 - k_1^2) + y^2$$

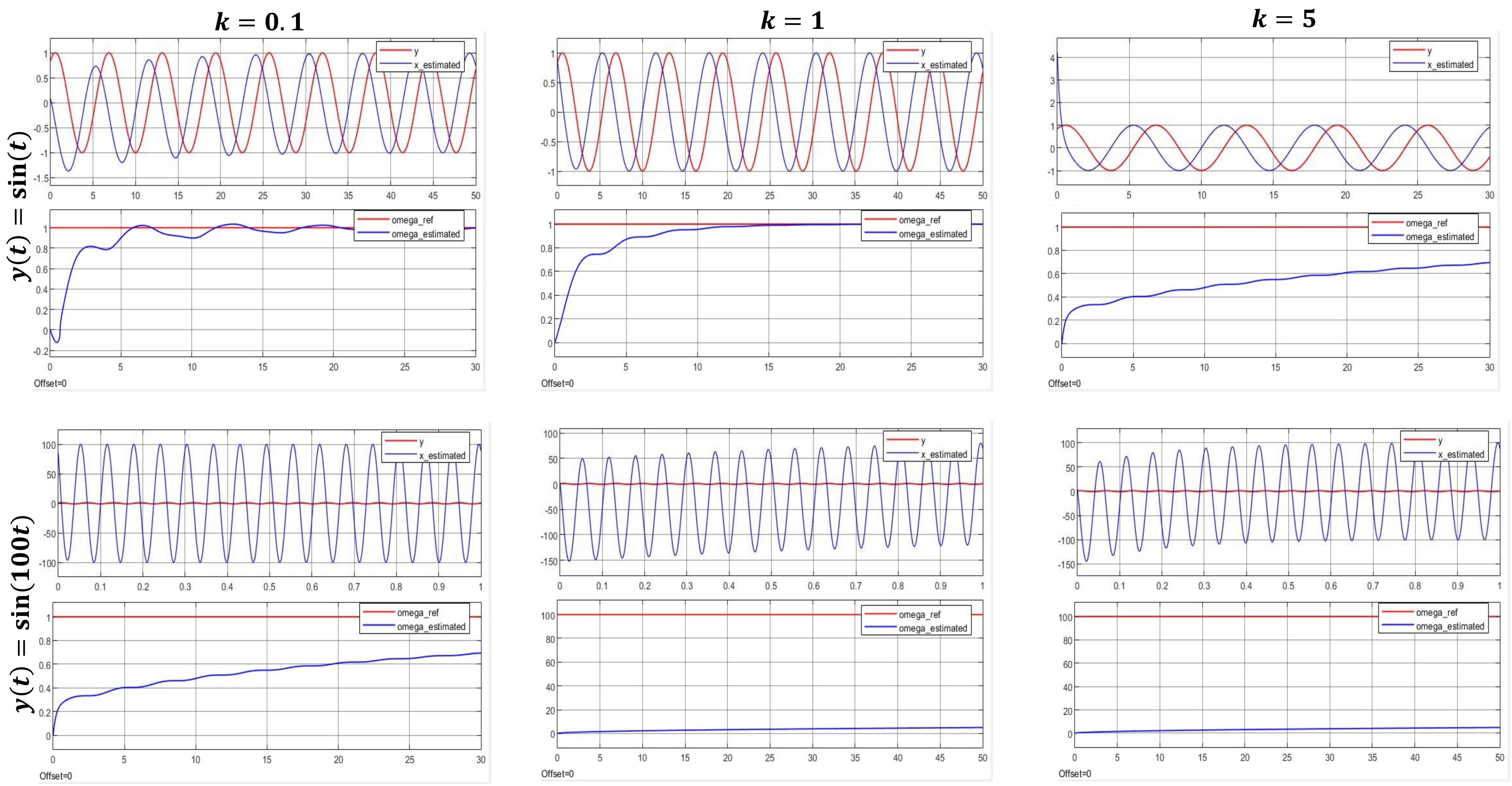
from which the $\hat{\vartheta}$ update law can be written:

$$\dot{\hat{\theta}}_1 = \hat{x}^2\,(-\hat{x}\,y - \hat{\theta}_1 - k_1^2) + y^2$$

To reduce the estimator size the injection term has been used, but to evaluate the real parameter it is necessary to add it to $\hat{\vartheta}$.

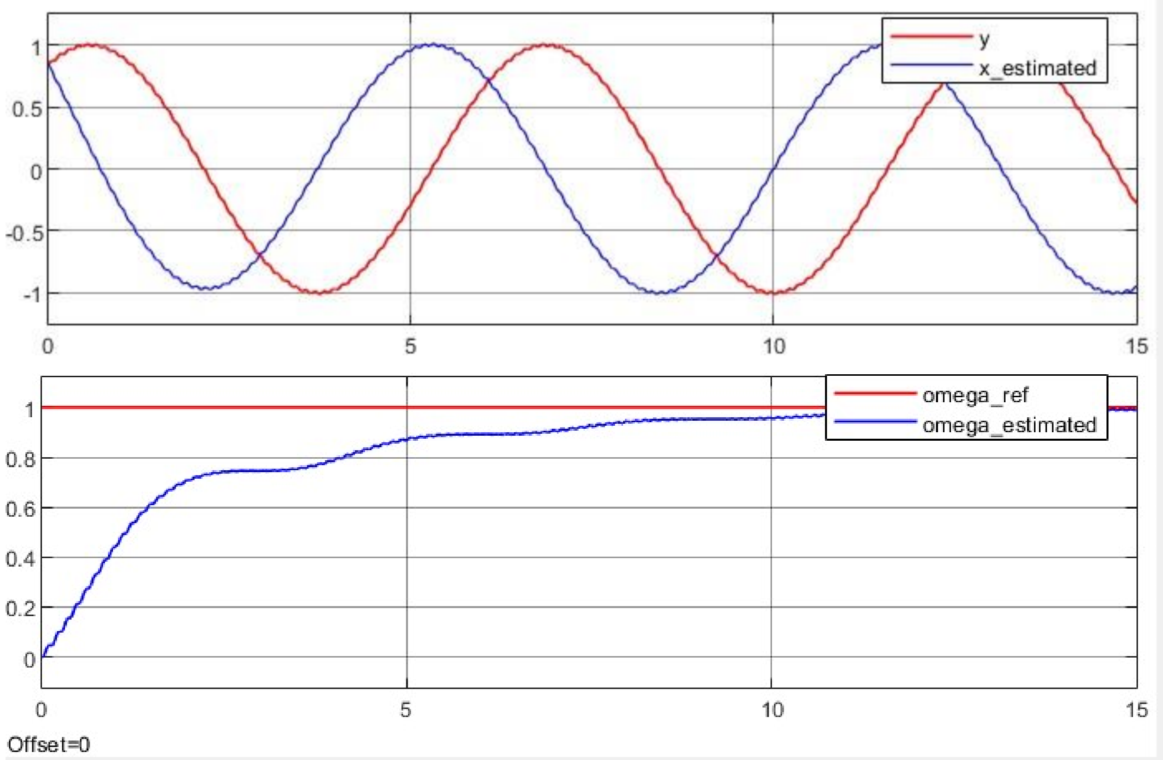
$\theta_{1\,\mathrm{estimated}} = y\,\hat{x} + \hat{\theta}_1$ in such a way that: $\lim_{t\to\infty}\theta_{1\,\mathrm{est}} = \omega$

# Etimation analysis with different k

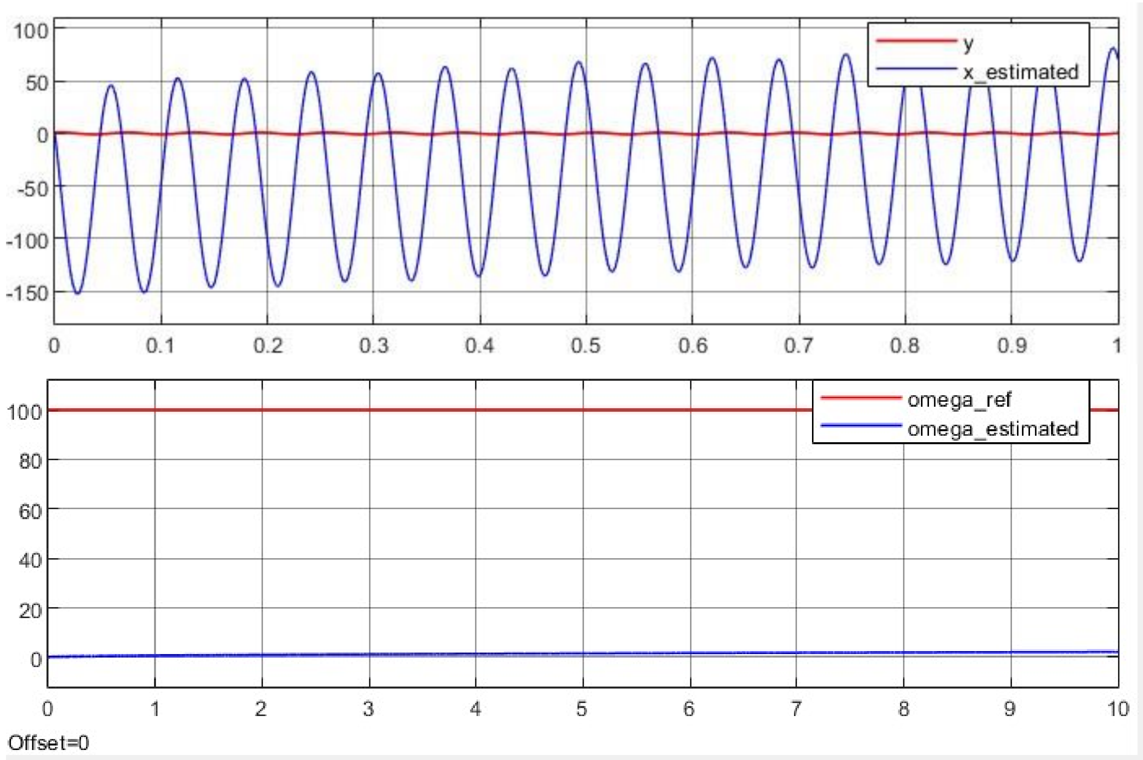


# Estimation Analysis with a noise = 0.01sin(50t)

$$y(t) = \sin(t)$$




$$y(t) = \sin(100t)$$



# OBSERVATIONS AND REMARKS

I&I technique allows signal frequency estimation, it can be observed that increasing frequency will delay the estimation convergence speed in a super linear way.

The effect of the update constant modification is as follows: in the case of an increase of k, estimation error will converge quicker but parameter will converge slower. Conversely, a strong k reduction will not cause the parameter estimation to speed up in a significant manner, but will cause an irregular transient.

If a sine noise is applied to input signal it can be observed that estimation performances are not altered, especially in the case of high frequency inputs. On the other hand in the case of low frequencies, estimation error will be slightly affected by the noise, generating a sine behaviour around zero.