TOR VERGATA
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ROBUST & ADAPTIVE CONTROL

MRAC ADAPTIVE CONTROLLER FOR SISO SYSTEM

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TASK:

Consider the second order system with transfer function

$$G(s) = k\,\frac{s + b_0}{s^2 + a_1\,s + a_0}$$

where k, b0 > 0, a1 and a0 are unknown constants, and the reference model

$$y_m = \frac{1}{s+1}\,r$$

Let $\Lambda(s) = s + 2$

Verify that all assumptions for the design of a MRAC are met.

Perform simulations of the system in closed-loop with the MRAC assuming that $b_0 = 2, a_0 = 5$, $a_1 = -10$ and k=1 and $r = E_1 \sin(\omega_1\,t) + E_2 \sin(\omega_2\,t)$ with$\mathrm{E}_1 \neq 0, E_2 \neq 0, \omega_1 \neq \omega_2$

MRAC DESIGN CONDITION CHECK:

Plant assumptions:

$$G(s) = \frac{y(s)}{u(s)} = k\frac{Z(s)}{R(s)} = \frac{s+2}{s^2 - 10s + 5}$$

- Z(s) is a monic Hurwitz polynomial equation, in our case the degree is $m = 1$
- Known Upper bound N since R(s) degree is known: $R(s)\; n = N = 2.$
- Knowing the two polynomial equations degrees the relative degree is known $G(s),\; rd = n - m = 1.$
- The sign of the high frequency gain is known, $G(s)$ is known and its gain is $k = 1.$

Reference model assumptions:

$$W_m(s) = \frac{y_m(s)}{r(s)} = k_m\frac{Z_m(s)}{R_m(s)} = \frac{1}{s+1}$$

- *Zm*(*s*) and *Rm*(*s*) are monic Hurwitz polynomials of degree $m_m = 1$ and $n_m = 0$, respectively and $n_m \leq N$
- The relative degree of the model, that is $rd_m = n_m - m_m = 1$, is such that $rd_m = rd$.

CALCULATION:

Calculation to determine $\theta_1, \theta_2, \theta_3$ in order to compare them with the ones detrmined by the estimator:

$$G(s) = \frac{s+2}{s^2 - 10\,s + 5} \qquad \Lambda(s) = s + 2 \quad \rightarrow u = \theta_1 \frac{1}{s+2} u + \theta_2 \frac{1}{s+2} y + \theta_3\, y + c\, r$$

$$\rightarrow u \left(1 - \frac{\theta_1}{s+2}\right) = \theta_2 \frac{1}{s+2} y + \theta_3\, y + c\, r \quad \rightarrow u \left(\frac{s+2-\theta_1}{s+2}\right) = \theta_2 \frac{1}{s+2} y + \theta_3\, y + c\, r$$

$$\rightarrow u\,(s + 2 - \theta_1) = \theta_2\, y + (s+2)\,\theta_3\, y + (s+2)\, c\, r$$

$$\rightarrow u = \frac{(\theta_2 + (s+2)\,\theta_3)\; y + (s+2)\; c\, r}{(s + 2 - \theta_1)}$$ now from $u = (s)$ i can determine $G(s)\, u(s) = y(s)$

$$y(s) = \frac{(s+2)\;(\theta_2 + (s+2)\,\theta_3)\; y + (s+2)^2\, c\, r}{(s^2 - 10\,s + 5)(s + 2 - \theta_1)} = y(s)\,\frac{(s+2)\;(\theta_2 + (s+2)\,\theta_3)}{(s^2 - 10\,s + 5)(s + 2 - \theta_1)} + r(s)\frac{(s+2)^2\, c}{(s^2 - 10\,s + 5)(s + 2 - \theta_1)}$$

$$y(s)\,[(s^2 - 10\,s + 5)(s + 2 - \theta_1) - (s+2)\;(\theta_2 + (s+2)\,\theta_3)\,] = r(s)(s+2)^2\, c$$

It is then determined the complementary Sensitivity function:

$$G_{\mathrm{CL}}(s) = \frac{y(s)}{r(s)} = \frac{(s+2)^2\, c}{[(s^2 - 10\,s + 5)(s + 2 - \theta_1) - (s+2)\;(\theta_2 + (s+2)\,\theta_3)\,]}$$

CALCULATION:

Our Reference Model is: $y_m = \frac{1}{s+1} r$ so the afterward equation shall be respected:

assuming $c = 1$

$(s+2)^2(s+1) = (s^2 - 10\,s + 5)(s + 2 - \theta_1) - (s+2)\,(\theta_2 + (s+2)\,\theta_3)$

$s^3 + 5\,s^2 + 8\,s + 4 = s^3 + (-\vartheta_3 - \vartheta_1 - 8)\,s^2 + (-4\,\vartheta_3 - \vartheta_2 + 10\,\vartheta_1 - 15)\,s - 4\,\vartheta_3 - 2\,\vartheta_2 - 5\,\vartheta_1 + 10]$

by comparison $\theta_1, \theta_2, \theta_3$ are determined:

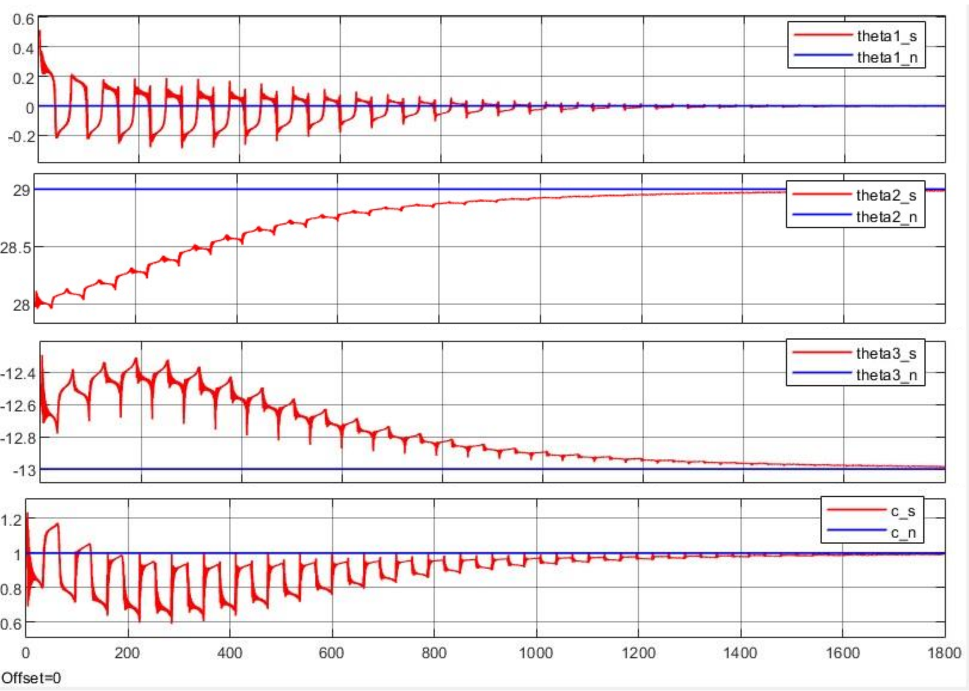
$$\begin{cases} 5 = -\vartheta_3 - \vartheta_1 - 8 \\ 8 = -4\,\vartheta_3 - \vartheta_2 + 10\,\vartheta_1 - 15 \\ 4 = -4\,\vartheta_3 - 2\,\vartheta_2 - 5\,\vartheta_1 + 10 \end{cases} \rightarrow [\vartheta_2 = 29, \vartheta_3 = -13, \vartheta_1 = 0]$$

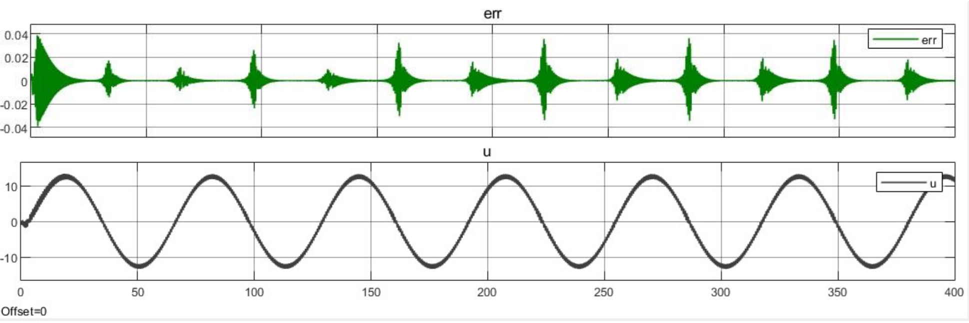
System dynamic with different reference signals

$$r = E_1 \sin(\omega_1 t) + E_2 \sin(\omega_2 t)$$

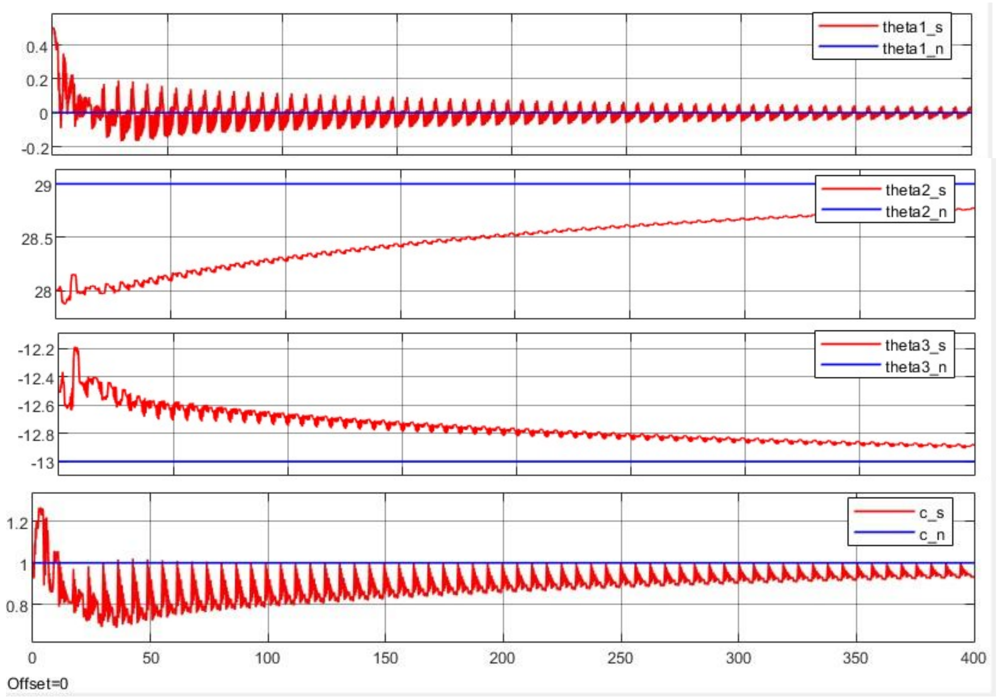
$$\Gamma = \begin{bmatrix} 50 & & & \\ & 50 & & \\ & & 50 & \\ & & & 50 \end{bmatrix}$$

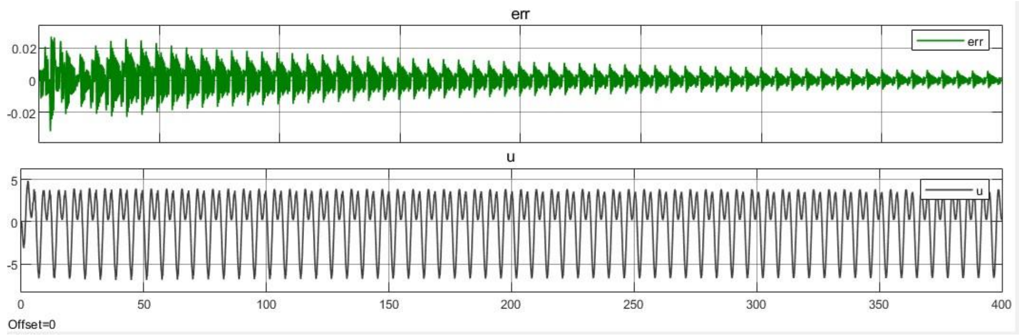
$E_1 = 0.5, \omega_1 = 10 \qquad E_2 = 5, \omega_2 = 0.1$





$E_1 = 0.2, \omega_1 = 2 \qquad E_2 = 5, \omega_2 = 0.05$

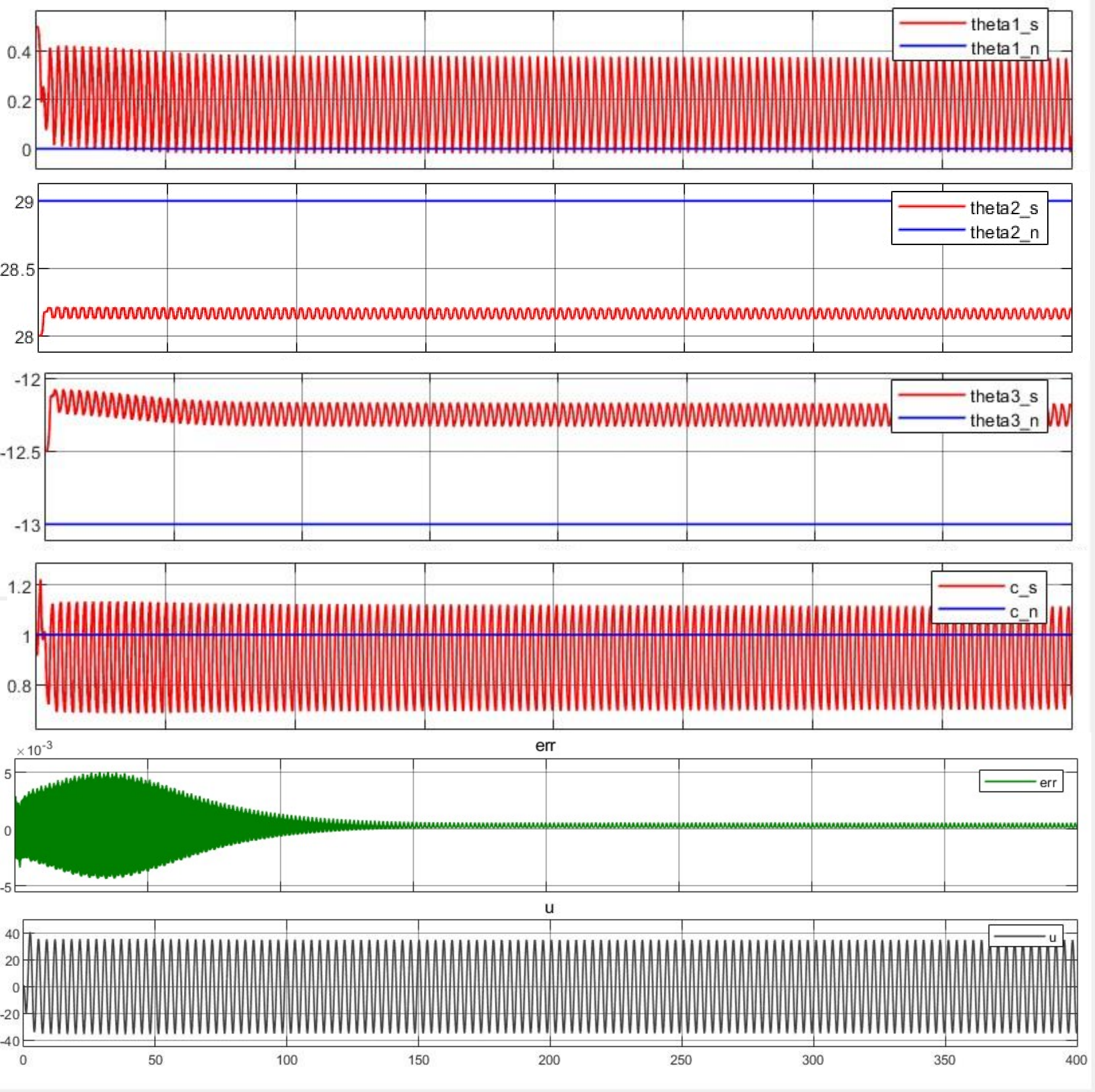




System dynamic with different reference signals

$$E_1 = 6, \omega_1 = 2 \qquad E_2 = 5, \omega_2 = 2$$

$$\Gamma = \begin{bmatrix} 50 & & & \\ & 50 & & \\ & & 50 & \\ & & & 50 \end{bmatrix}$$



MRAC control simulation with 4 parameter is influenced by the differential equations solver and its relative tolarance. In this simlulations it has been used ode with a relative tolerance of 10^{-7}

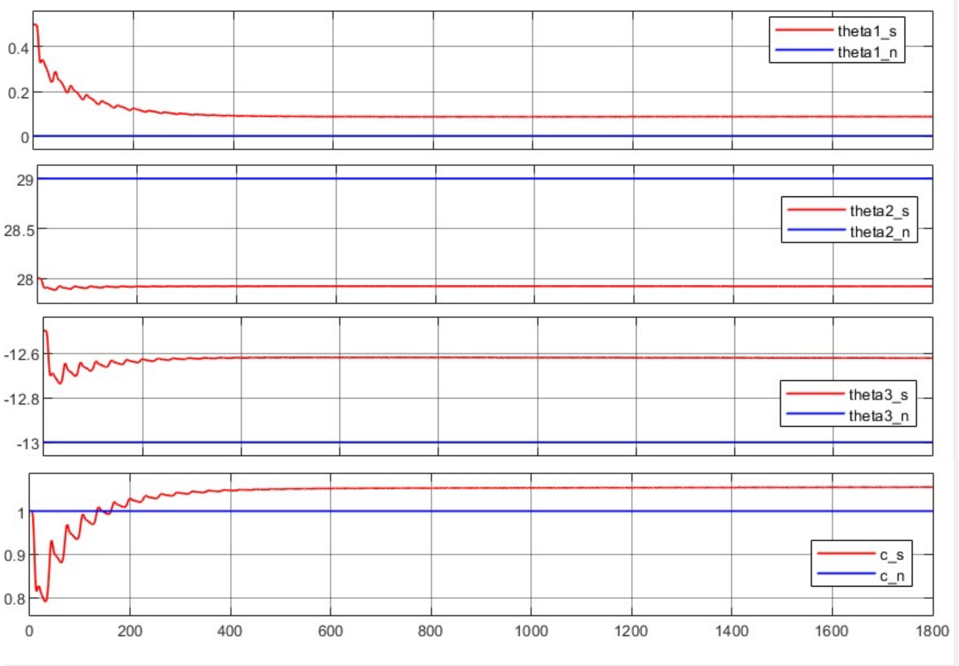
It can be observed that, if the two frequencies of the input signal are different, MRAC control leads to parameter and error convergence. With higher frequencies of the reference signal the parameter estimation get much slower, in this case measure error has a periodic irregular behaviour.

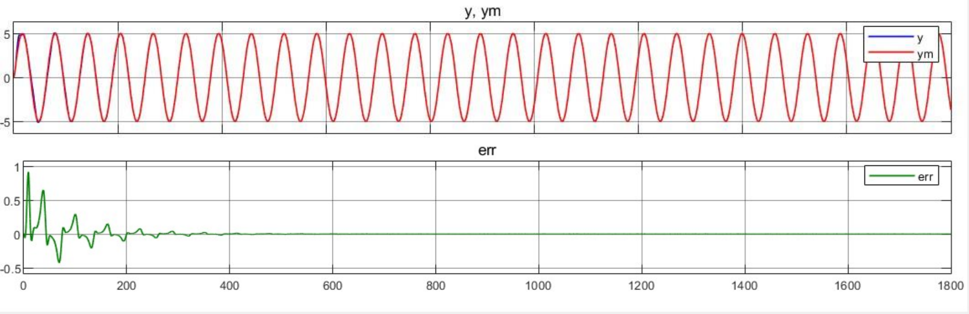
While if input signal is composed of two sine wave with the same frequency, it can be observed that MRAC does not reach parameter convergence. This happen because the input signal is not rich enough, i.e. it has rihcness = 2 with 4 parameter to estimate. On the other side error convergence to zero is reached, because this does not depend on signal richness.

Gamma variations effect on system dynamic

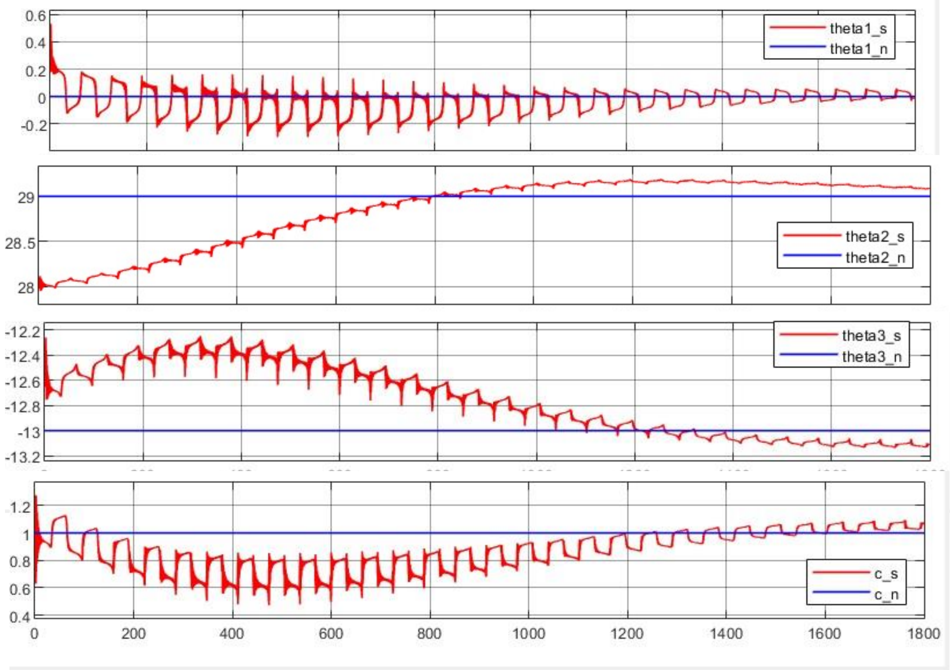
$E_1 = 0.5, \omega_1 = 10 \qquad E_2 = 5\ \omega_2 = 0.1$

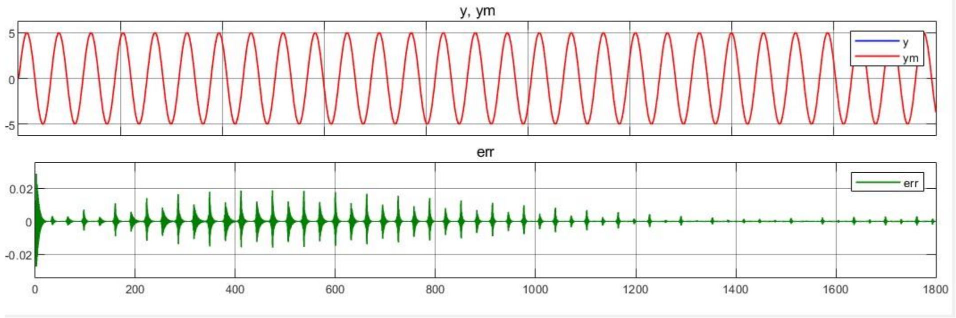
$$\Gamma = \begin{bmatrix} 0.01 & & & \\ & 0.01 & & \\ & & 0.01 & \\ & & & 0.01 \end{bmatrix}$$





$$\Gamma = \begin{bmatrix} 150 & & & \\ & 150 & & \\ & & 150 & \\ & & & 150 \end{bmatrix}$$





It can be obeserved that higher gammas lead to quicker dynamic and to the presence of overshoot. But very high values will generate an unstable behaviour.