TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

ROBUST & ADAPTIVE CONTROL

ROBUST STABILITY AND ROBUST PERFORMANCE

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TASK:

Consider the family of plants:

$\tilde{P} = P(1 + \Delta W_2)$

with:

$P(s) = \frac{1}{s-1}$ $\qquad W_2(s) = \frac{2}{s+10}$ $\qquad C(s) = k$ $\qquad W_1(s) = \frac{1}{s+1}$

Assume that Δ is such that $\|\Delta\|_\infty \leqslant 2$.

Determine the range of values of k for which robust stability is achieved.

Determine the value of k which gives robust stability and minimizes the robust performance level α.

ROBUST STABILITY:

Robust stability with uncertainty β:

$\|\Delta\|_\infty \leqslant \beta \rightarrow \beta = 2, \text{from wich } \|W_2 T\|_\infty < \frac{1}{\beta}$

$$T = \frac{L}{1+L} = \frac{PC}{1+PC} = \frac{\frac{k}{s-1}}{1+\frac{k}{s-1}} = \frac{k}{s+(k-1)}$$

$$W_2 T = \frac{2}{(s+10)} \frac{k}{[s+(k-1)]} = \frac{2\,k}{(s+k-1)\,(s+10)}$$

but since in Bode Diagram the maximum gain will be located at $\omega = 0$ $s = j\,\omega|_{\omega=0} = 0$ then:

$$\|W_2 T\|_\infty = |W_2 T||_{\omega=0} = \frac{2\,k}{10\,(k-1)} = \frac{k}{5\,k-5}$$

to obtain the requested robust stability the condition $\|W_2 T\|_\infty < \frac{1}{2}$ has to be respected:

$$\frac{k}{5\,k-5} < \frac{1}{2} \rightarrow 2\,k < 5\,k - 5 \rightarrow -3\,k < -5 \rightarrow k > \frac{5}{3} \longrightarrow k \in \left(\frac{5}{3}; +\infty\right) \qquad (E.C. \quad k \neq 1 \, \text{is respected})$$

ROBUST PERFORMANCE

The task is to guarantee robust perfomences at an α level and robust stability of β level.

$$\|W_2 T\|_\infty < \frac{1}{2} \rightarrow k \in \left(\tfrac{5}{3}; +\infty\right) \qquad \wedge \left\|\frac{W_1 S}{1+\Delta W_2 T}\right\|_\infty < \alpha \qquad \forall \Delta : \|\Delta\| \leqslant 2$$

considering the maximum perturbation:

$$\max_{|\Delta|<2}\left|\frac{W_1 S}{1+\Delta W_2 T}\right| = \frac{|W_1 S|}{1-2|W_2 T|} < \alpha \qquad \rightarrow \qquad \alpha_{\min} = \max_\omega \frac{|W_1 S|}{1-2|W_2 T|} = \left\|\frac{|W_1 S|}{1-2|W_2 T|}\right\|_\infty$$

but also for $|W_1 S|$ the maximum gain in Bode Diagram will be located at $\omega = 0$,

then since: $S = \dfrac{1}{1+PC} = \dfrac{1}{1+\frac{k}{s-1}} = \dfrac{s-1}{s+k-1}$

and: $W_1 S = \dfrac{1}{s+1}\,\dfrac{s-1}{s+k-1}\Big|_{s=j\omega} = \dfrac{1}{j\omega+1}\,\dfrac{j\omega-1}{j\omega+k-1}\Big|_{\omega=0} = -\dfrac{1}{k-1}$

so knowing: $\begin{cases} |W_2 T(j\omega)|_{\omega=0} = \dfrac{k}{5k-5} \\ |W_1 S(j\omega)|_{\omega=0} = \dfrac{1}{k-1} \end{cases}$ and substituting: $\alpha_{\min} = \dfrac{\frac{1}{(k-1)}}{1-2\frac{k}{5(k-1)}} = \dfrac{5}{5k-5-2k} = \dfrac{5}{3k-5}$

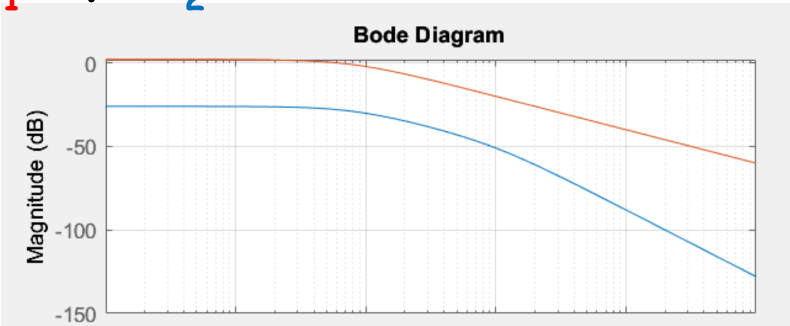
to determine the perfomance robustness varying k derivation is performed:

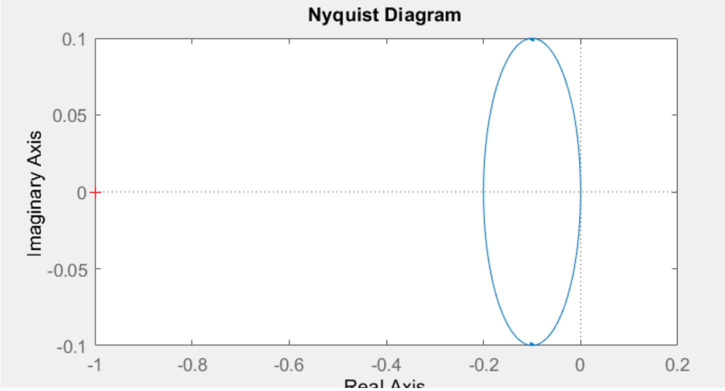
$\dfrac{d\alpha_{\min}}{dk} = -\dfrac{15}{(3k-5)^2} < 0, \forall k > \frac{5}{3}$, since k is at the denominator an increase will determine an α reduction

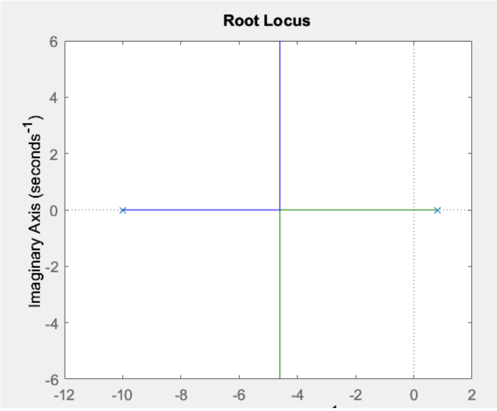
Robust stability analysis K : [0.2 1.1 5/3 2.2 3]

Legend: $W_1\ S$; $W_2\ T$

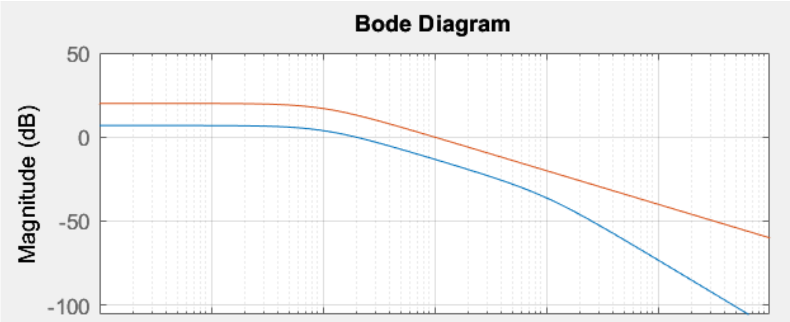
K = 0.2

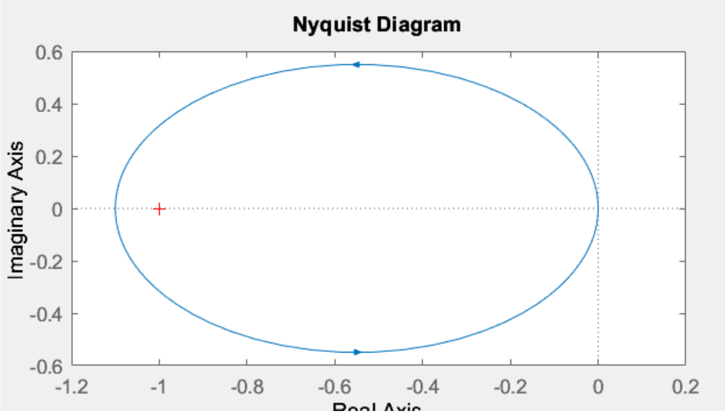


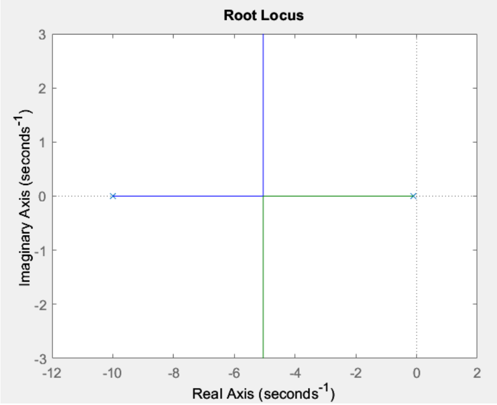




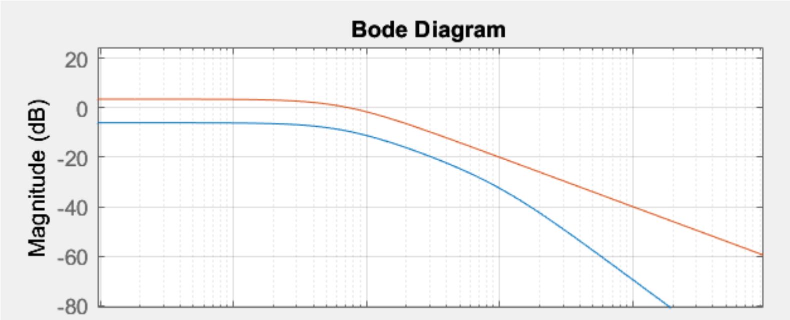
K = 1.1

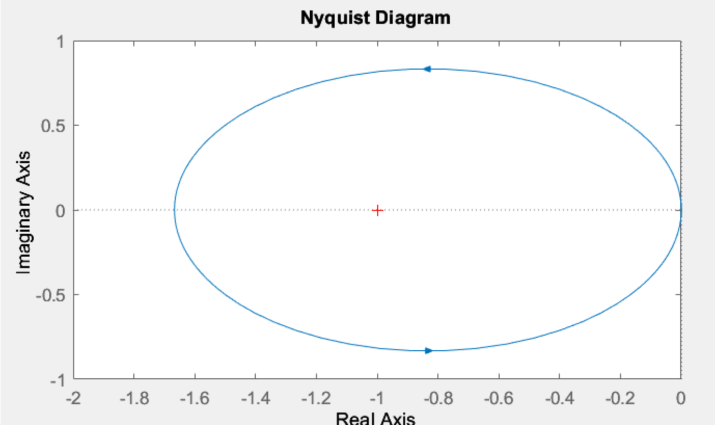


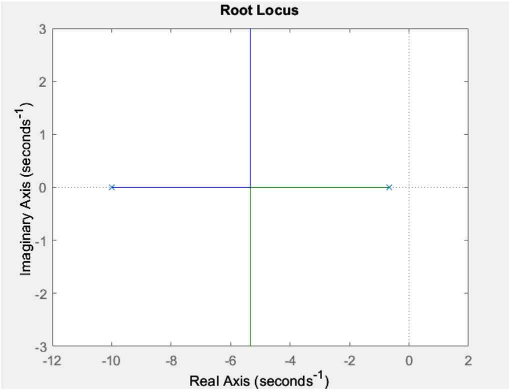




K = 5/3







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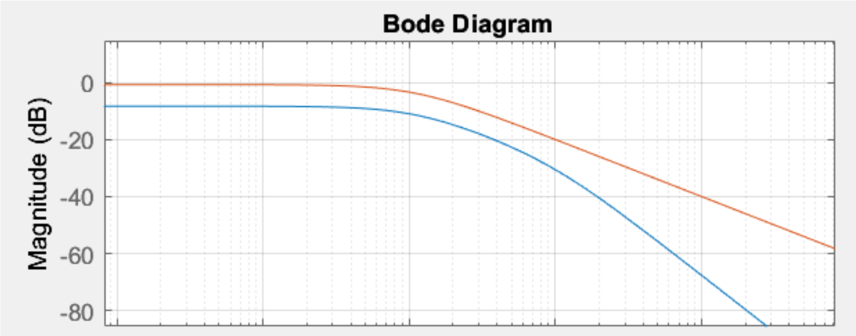
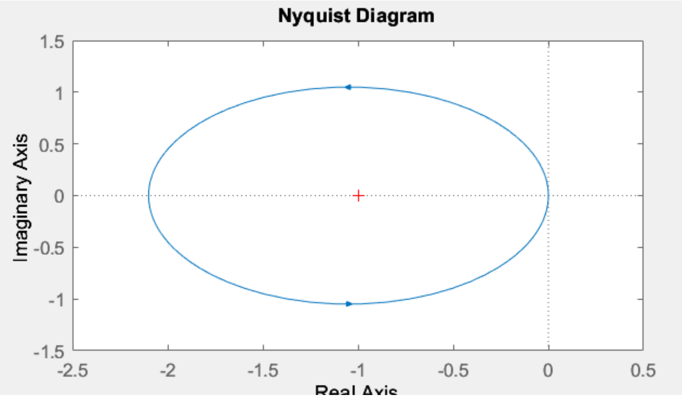
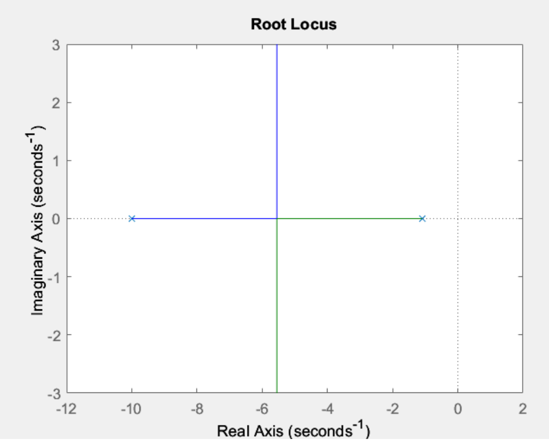
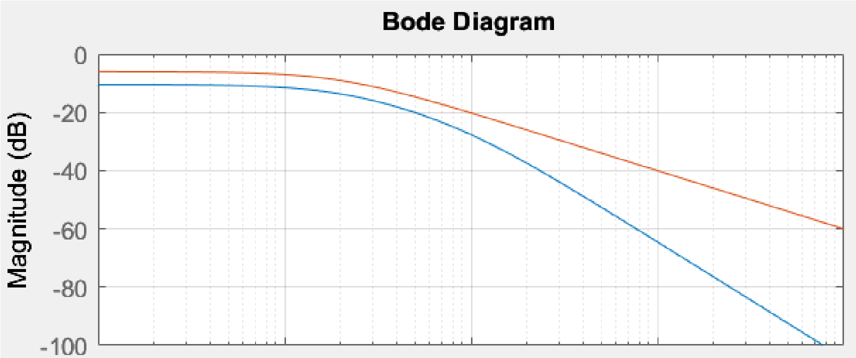
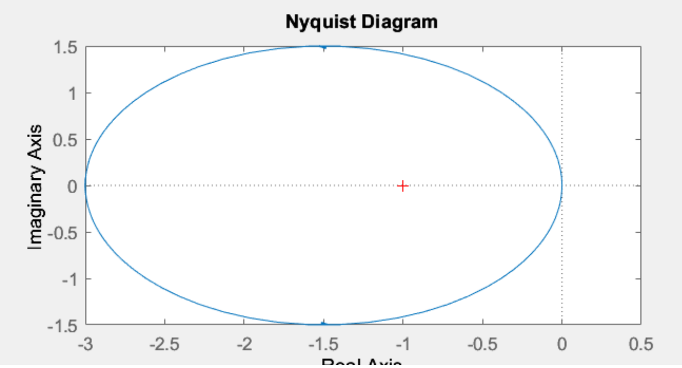

Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80

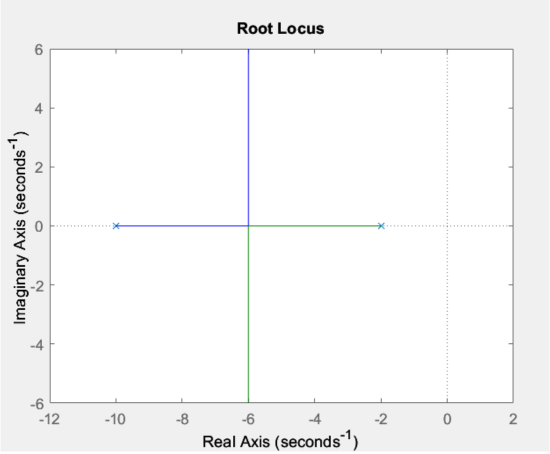

Nyquist Diagram
Imaginary Axis
Real Axis
1.5
1
0.5
0
-0.5
-1
-1.5
-2.5
-2
-1.5
-1
-0.5
0
0.5


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
3
2
1
0
-1
-2
-3
-12
-10
-8
-6
-4
-2
0
2

K = 3


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
-100


Nyquist Diagram
Imaginary Axis
1.5
1
0.5
0
-0.5
-1
-1.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
6
4
2
0
-2
-4
-6
-12
-10
-8
-6
-4
-2
0
2

OBSERVATIONS AND REMARKS:

A variation of k parameter has been performed to evaluate the results calculated.

From Nyquist diagrams it can be observed that an higher k value will result in a shift of the curve to the left of the complex plan and the the complex point (-1,0) will remains inside the curve.

From Bode Diagram it can be observed that robustness condition is respected.

From Root Locus diagram it can be observed that for $k > 5/3$ both poles belong to negative reals.

Those observations confirm the result calculated.