“Macroeconometrics - PS 2”

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Full Sample Forecast (a)

- Download the time series for US quarterly real GDP (FRED website).
- Consider an AR(1) process for GDP growth.

$$y_t = c + \phi y_{t-1} + \epsilon_t,$$

where $\epsilon_t \sim iid(0, \sigma^2)$.

- Transform the series in growth rates using either
  - $100(\ln(y_t) - \ln(y_{t-1}))$.
  - $100(y_t - y_{t-1})/y_{t-1}$.
- Multiply by 4 to get annualized data for comparison with SPF (point e).

Full Sample Forecast (a)

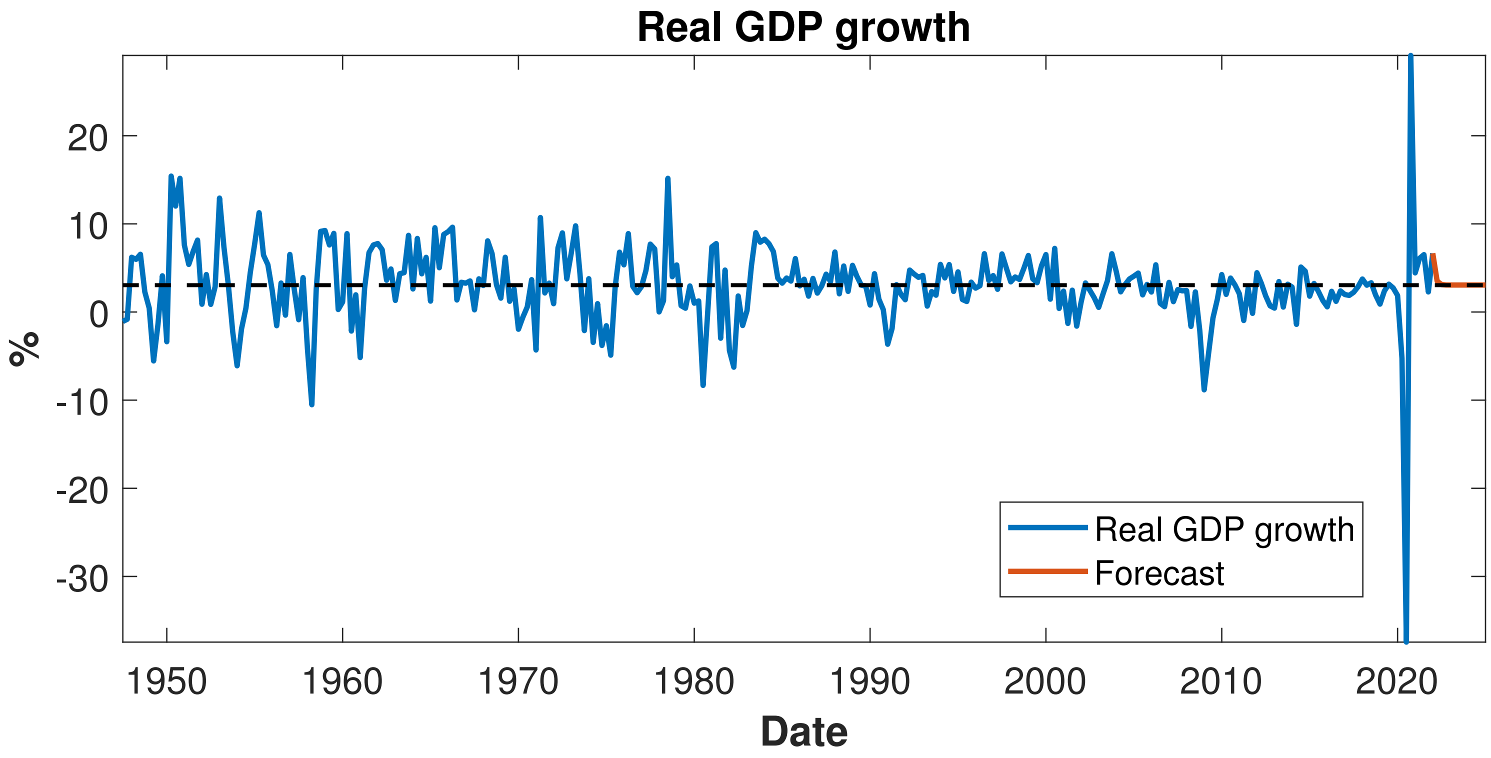
Using a recursive procedure

$$y_{t+1} = c + \phi y_t + \epsilon_{t+1}$$
$$y_{t+2} = c + \phi y_{t+1} + \epsilon_{t+2}$$
$$\ldots$$
$$\hat{y}_{t+j|t} = \hat{c} + \hat{\phi} \hat{y}_{t+j-1|t}$$

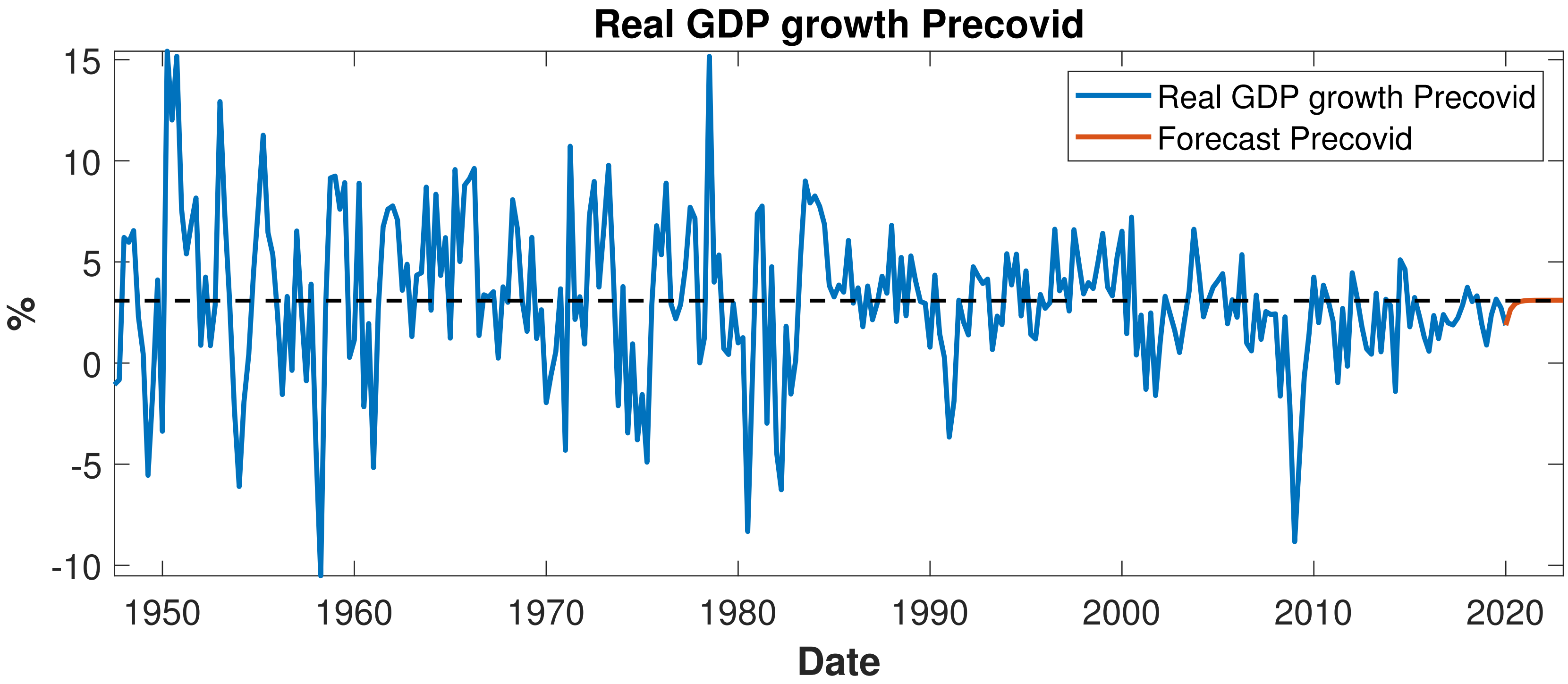
Full Sample Forecast (a)

```
GDPC1 = csvread('GDPC1.csv',1,1); % Q1:1947 - Q4:2021

y=diff(log(GDPC1))*400;

p=1; % number of lags
c=1; % include the constant

beta_hat=AR(y,p,c);

% 12 period ahead forecasts
% Then the forecasts for T+1,...,T+12 are given by:

forecast=zeros(12,1);
forecast(1)=beta_hat(1)+beta_hat(2)*y(end); % one period ahead forecast

for j=2:12 % 12 periods ahead
    forecast(j)=beta_hat(1)+beta_hat(2)*forecast(j-1);
end
```

Full Sample Forecast (a)



Full Sample Forecast (a)



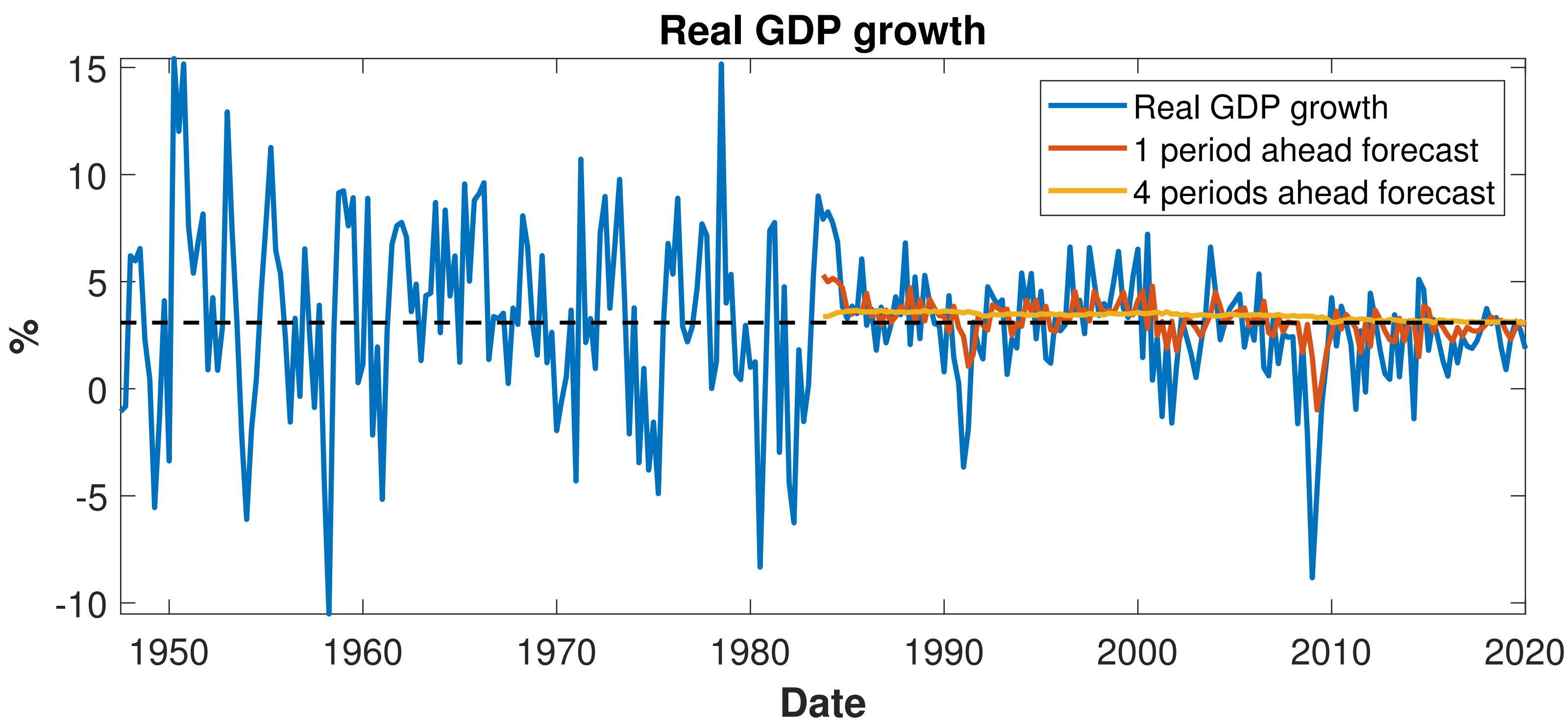
Forecasts (b)

```
y=y_precovid;
T=size(y,1);

T_est = 145; % length of the estimation sample
T_eval = T-T_est;

forecasts1=zeros(T_eval,1); % 1 period ahead forecasts
forecasts4=zeros(T_eval,4); % 1,2,3,4 periods ahead forecasts

% One period ahead:

for k=1:T_eval

    beta_hat1=AR(y(1:T_est+k-1),p,c);
    forecasts1(k,1)=beta_hat1(1)+beta_hat1(2)*y(T_est+k-1);

end

% Compute the forecast errors
w1=(y(T_est+1:end)-forecasts1(:,1));
```

Forecasts (b)

```

% Four periods ahead:

for k=1:T_eval

    beta_hat1=AR(y(1:T_est+k-4),p,c);
    forecasts4(k,1)=beta_hat1(1)+beta_hat1(2)*y(T_est+k-4); % 1 period ahead

    for j=2:4
    forecasts4(k,j)=beta_hat1(1)+beta_hat1(2)*forecasts4(k,j-1); % for j=4, 4
       periods ahead
    end

end

% Compute the forecast errors
w4=(y(T_est+1:end)-forecasts4(:,4));
```

Forecasts (b)



- $MSE_1 = 4.5$
- $MSE_4 = 5.8$

Higher order process (c)

Now suppose that GDP admits an AR(2) representation

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t,$$

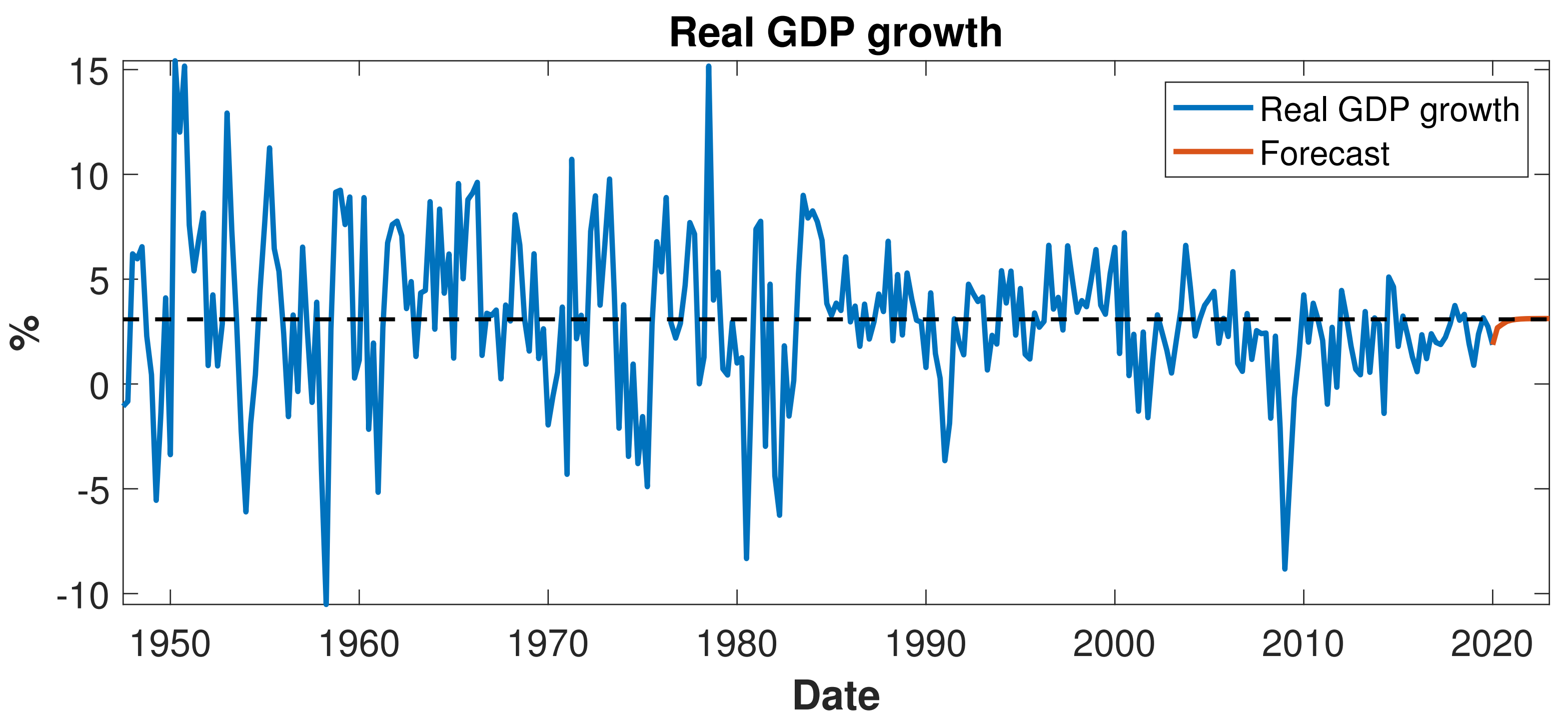
where $\epsilon_t \sim iid(0, \sigma^2)$.

- Use companion form.
- Same recursive procedure as before.

Higher order process (c)

```

p=2; % number of lags

beta_hat2=AR(y,p,c);

% Companion form matrices:

F_hat=[beta_hat2(2) beta_hat2(3);1 0];
C_hat=[beta_hat2(1); 0];

% Forecasts:

forecastAR2=zeros(2,12);
forecastAR2(:,1)=C_hat+F_hat*[y(end); y(end-1)];

for i=2:12
    forecastAR2(:,i)=C_hat+F_hat*forecastAR2(:,i-1);
end

forecast2=forecastAR2(1,:)';
```

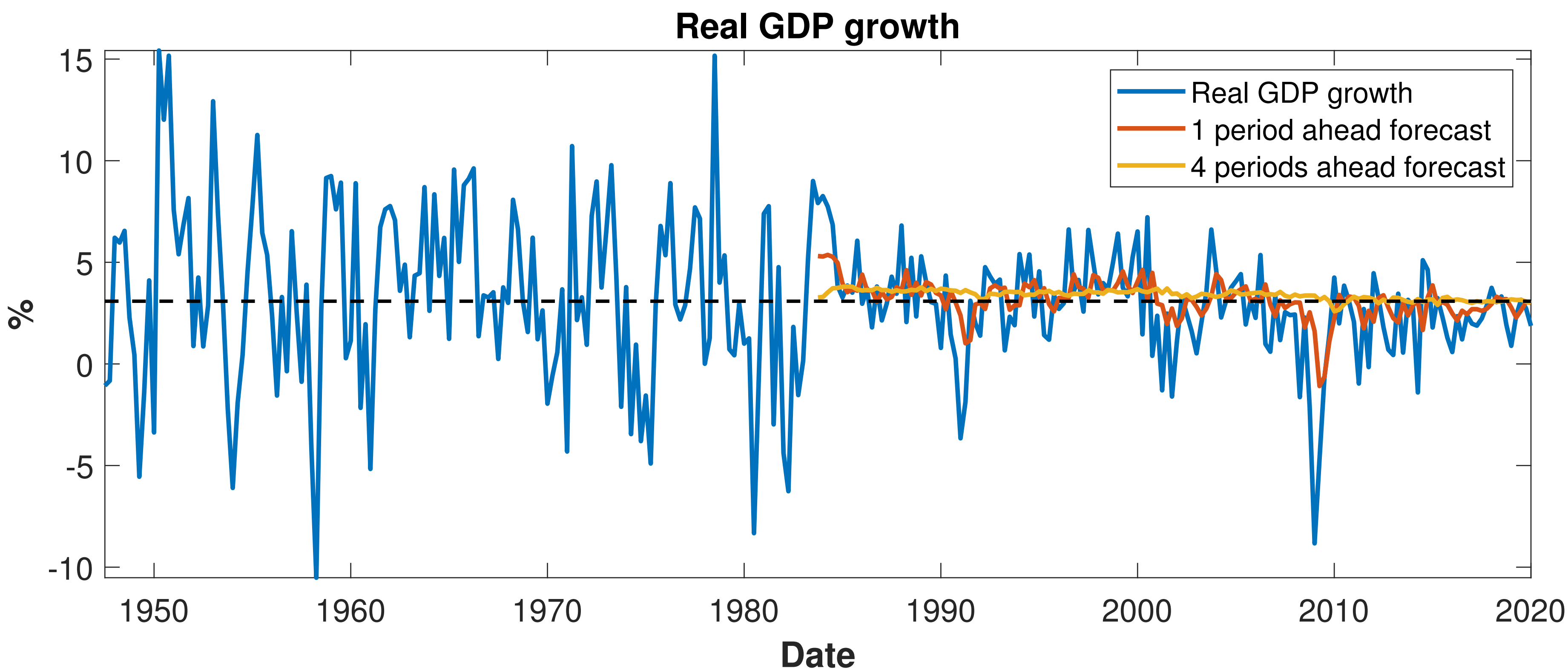
Higher order process (c)



Higher order process (d)

```

forecasts1b=zeros(p,T_eval); % 1 period ahead forecast
forecasts4b=zeros(p,T_eval,4); % 1,2,3,4 periods ahead forecast

for k=1:T_eval

    beta_hat2b=AR(y(1:T_est+k-1),p,c);

    F_hat=[beta_hat2b(2) beta_hat2b(3);1 0]; %companion form
    C_hat=[beta_hat2b(1); 0];

    forecasts1b(:,k)=C_hat+F_hat*[y(T_est+k-1);y(T_est+k-2)]; % 1 period ahead

end

% Compute the forecast errors
w1b=y(T_est+1:end)-forecasts1b(1,:)';
```

Higher order process (d)



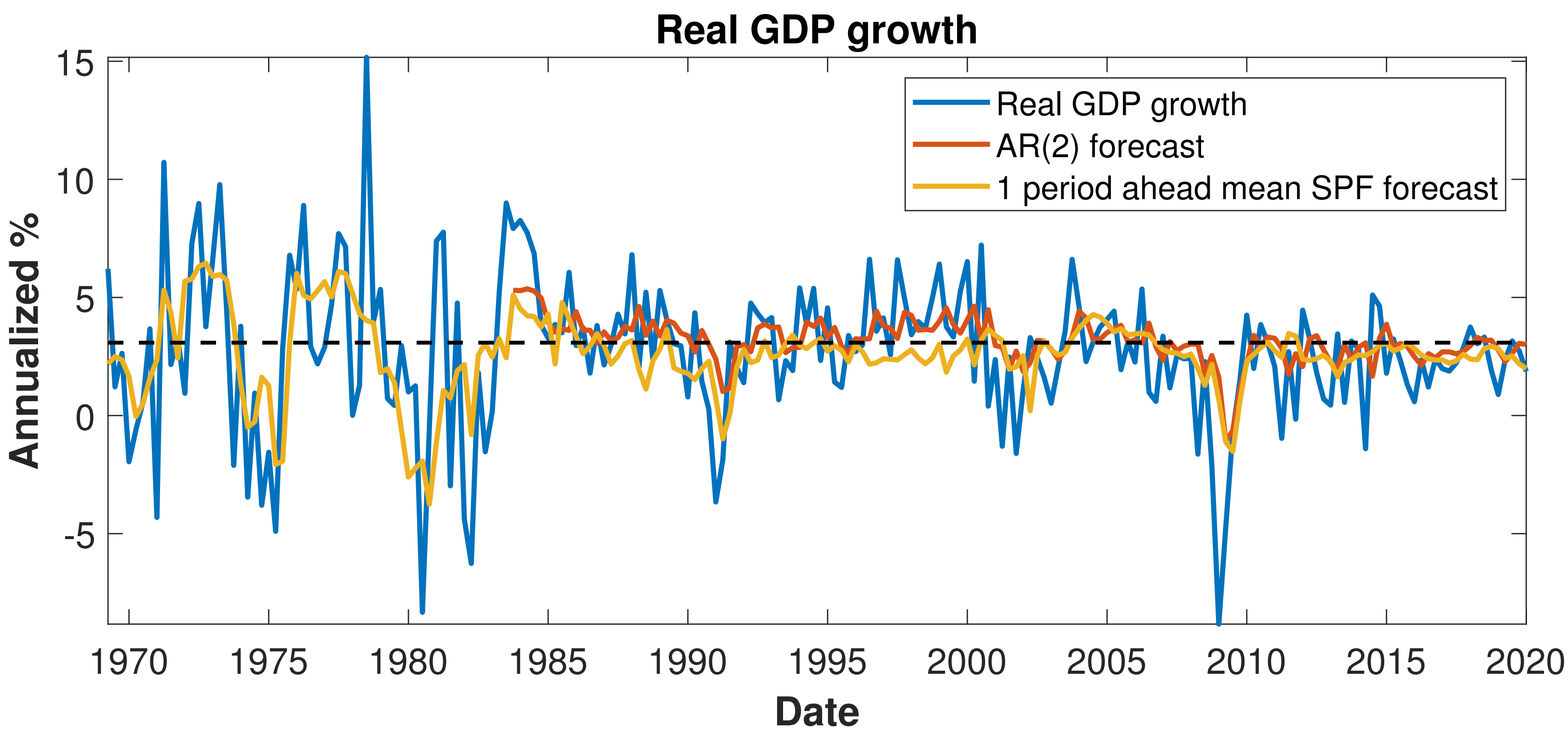
- $MSE_1 = 4.3$
- $MSE_4 = 5.7$

Survey of Professional Forecaster (e)

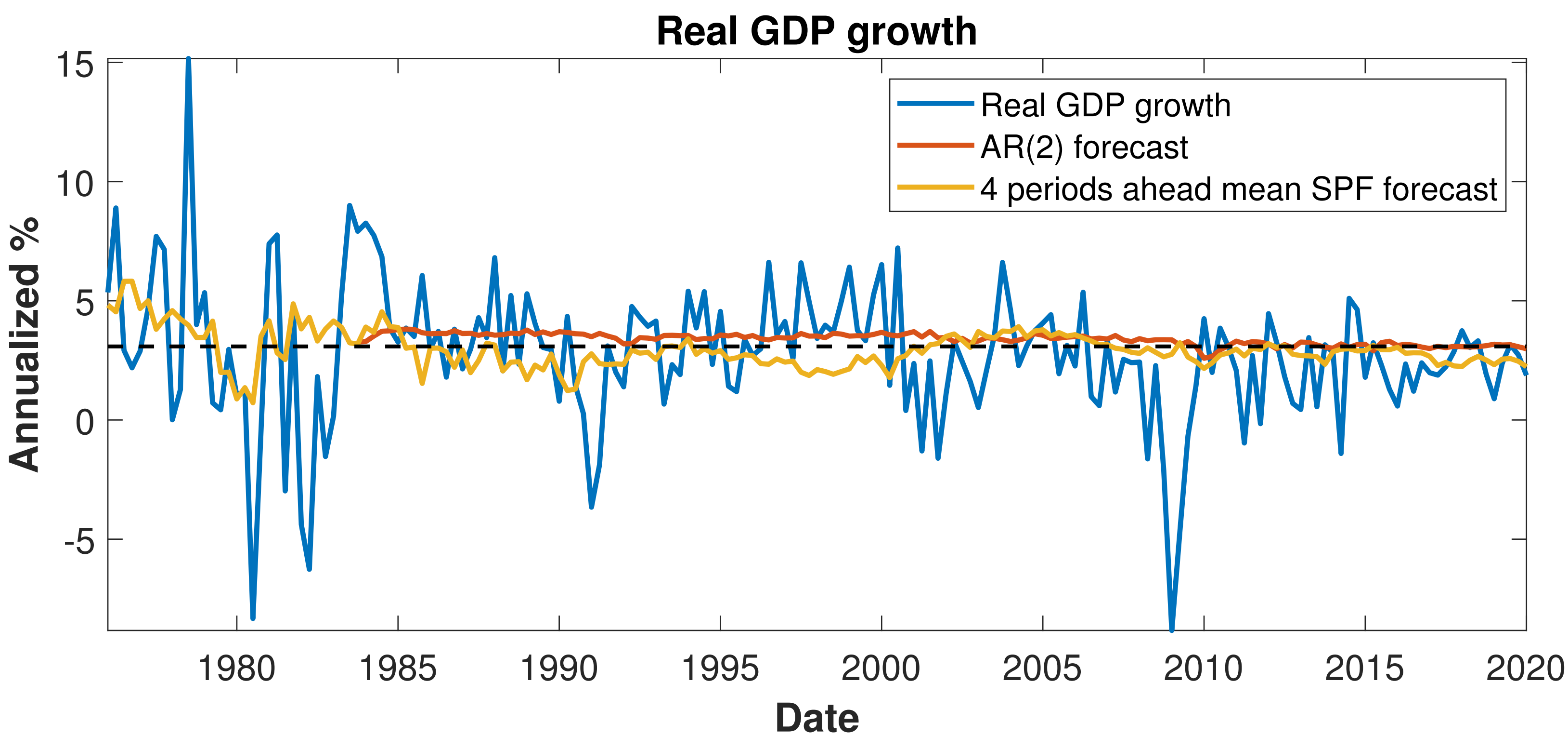
- Read in data from the Survey of Professional Forecasters.
- Mean forecast, which sometimes is also called consensus forecast.
- Annualized growth series of GDP.

- $MSE_1 = 7.4$
- $MSE_4 = 8.4$

Survey of Professional Forecaster (e)



Survey of Professional Forecaster (e)



Accuracy Test (f)

- Test for equal forecast performance using the Diebold-Mariano test.
- $H_0 = MSE^j_{AR(2)} - MSE^j_{SPF} = 0$
- Compute the critical values and compare to a standard normal at 5% signi cance (1.96 critical value).

```
S1=dmtest(w1b,w1new(Fdiff1:end),1);
S4=dmtest(w4b,w4new(Fdiff4:end),4);

table(S1,S4)

if abs(S1)>1.96
    disp('Since S1>1.96, we reject the null hypothesis.')
else, disp('Since S1<1.96, we do not reject the null hypothesis.')
end

if abs(S4)>1.96
    disp('Since S4>1.96, we reject the null hypothesis.')
else, disp('Since S4<1.96, we do not reject the null hypothesis.')
end
```