“Macroeconometrics - PS 4”

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Vector Autoregression (VAR)

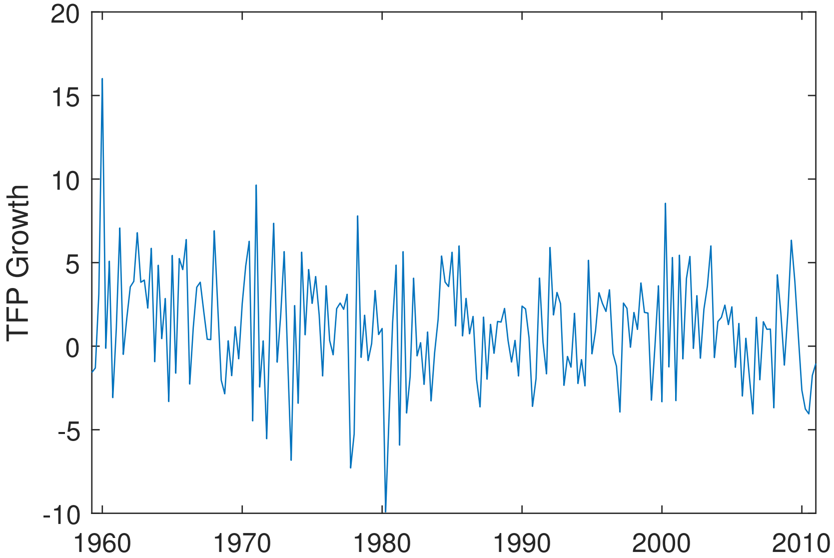
Consider the VAR(4)

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + A_3 Y_{t-3} + A_4 Y_{t-4} + \epsilon_t,$$

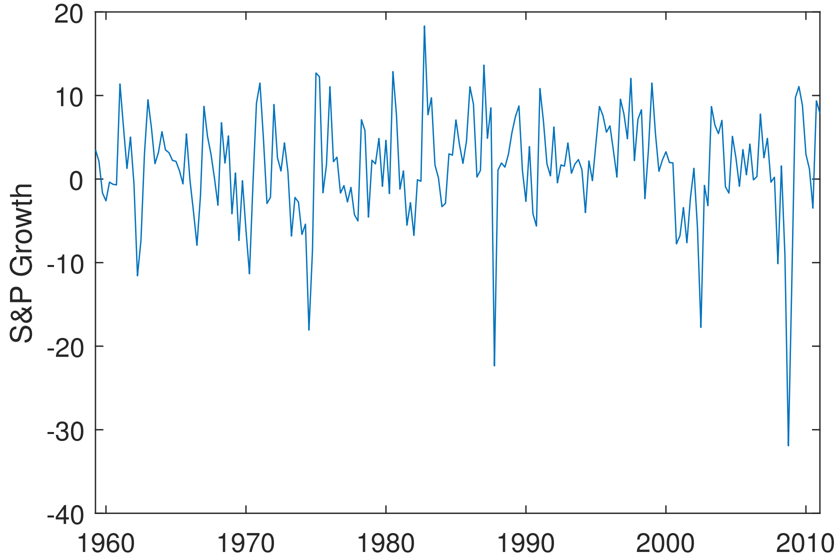
where Y_t is a $n \times 1$ vector and A_p are $n \times n$ matrices.

- Growth rates of TFP (adjusted for capacity utilization)
- Level of real stock prices (S& P500)
- Quarterly data from 1959 to 2011 (BP 2006 data span from 1948 to 2000)
- $Y_t = [TFP_t \;\; SP_t]'$

US data



TFP Growth
20
15
10
5
0
-5
-10
1960
1970
1980
1990
2000
2010



S&P Growth
20
10
0
-10
-20
-30
-40
1960
1970
1980
1990
2000
2010

VAR Estimation

- We have same regressor in each of the $n = 2$ equations.
- Use OLS equation by equation.
- Use OLS on the stacked representation.

Consider the VAR(4)

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + A_3 Y_{t-3} + A_4 Y_{t-4} + \epsilon_t,$$

where Y_t is a $n \times 1$ vector and A_p are $n \times n$ matrices.

$$Y_{T\times n} = X_{T\times np} A_{np\times n} + \varepsilon_{T\times n}$$

VAR Estimation

```
function [beta, residuals] = VAR(y,p,c)

% Function to estimate a VAR(p) with or without constant using OLS
% Inputs:    y = T x N matrix of endogeneous variables
%            p = VAR lag order
%            c = 1 if constant required
% Outputs:   beta = (Np+1 x N) matrix of estimated coefficients (Np x N) if
%                   no constant is included
%            residuals = (T-p x N) matrix of OLS residuals

[T, ~] = size(y);
yfinal = y(p+1:T,:);
if c == 1
    X = [ones(T-p,1), lagmakerMatrix(y,p)];
else
    X = lagmakerMatrix(y,p);
end

beta = (X'*X)\X'*yfinal;
residuals = yfinal - X*beta;

end
```

VAR Estimation

```
function x = lagmakerMatrix(y,p)
% Function to create the matrix of regressors according to the
% SUR representaiton for a VAR
% Inputs:    y = T x N matrix of endogeneous variables
%            p = VAR lag order
% Outputs:   x = T-p x Np matrix of lagged dependent variables as regressors

[T, N] = size(y);
x = zeros(T-p, N*p);
counter = 0;
for i=1:p
    for j=1:N
        counter = counter + 1;
        x(:,counter) = y(p+1-i:T-i,j);
    end
end
end
```

VAR Estimation

$$Y_{jt} = c_j + A_{j,1}Y_{t-1} + A_{j,2}Y_{t-2} + A_{j,3}Y_{t-3} + A_{j,4}Y_{t-4} + \epsilon_{jt}$$

```

% OLS equation by equation
beta = zeros(n*p+1,n);
for j = 1:n
Y = finaldata(5:end,j); % T x 1
X = [ones(t-p,1) TFP1 SP1 TFP2 SP2 TFP3 SP3 TFP4 SP4 ]; % T x (p+1)
beta(:,j) = (X'*X)\X'*Y;
end
```

Identify the news shock with SRR

Assume that the news shock has no contemporaneous effect on TFP growth.

$$\Sigma_\epsilon = PP'$$

Consider the MA representation to the reduced form VAR

$$Y_t = \nu + \Theta(L)PP^{-1}\epsilon_t = \nu + \Theta(L)Pw_t = \nu + C(L)w_t.$$

Using lower triangluar factorization

$$\begin{bmatrix} TFP_t \\ SP_t \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix}$$

Moreover, $E_t(w_t w_t') = P^{-1}\Sigma_\epsilon(P^{-1})' = I_n$

Identify news shock with SRR

```

% check stationarity
F=companionMatrix(beta, c, p); % BigA companion form, np x np matrix
ev1 = abs(eig(F)); disp(max(ev1))

hor=40;
% Wold IRFs:
T=zeros(n,n,hor+1);
for j=1:hor+1
    T=F^(j-1);
    T(:,:,j)=T(1:n,1:n); % Impulse response functions of the Wold
       representation
end

% Short-run restrictions - Cholesky:
omega=(err'*err)./(t-n*p-1-p); % Estimate of omega
P=chol(omega,'lower'); % Cholesky factorization, lower triangular matrix
% Structural shock (second - news):
eps=(P\err')';
% Cholesky IRFs:
C=zeros(n,n,hor+1);
for i=1:hor+1
    C(:,:,i)=T(:,:,i)*P; % Cholesky wold respesentation
end
```

Log-level IRFs

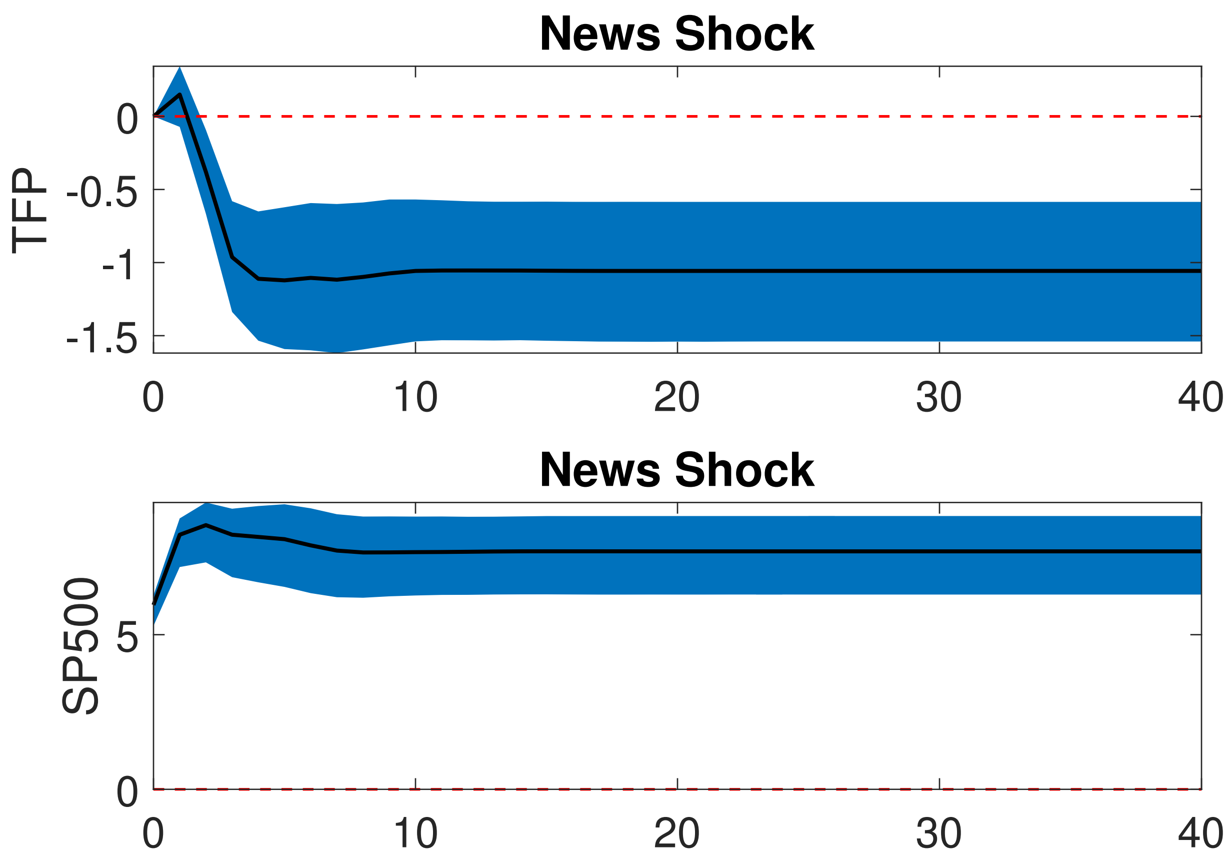
$$\frac{\partial Y_t^g}{\partial w_t} = \frac{\partial}{\partial w_t}(\ln Y_t - \ln Y_{t-1}) = \frac{\partial \ln Y_t}{\partial w_t} = C_0$$

$$\frac{\partial \ln Y_{t+1}}{\partial w_t} = \frac{\partial}{\partial w_t} \ln\left(\frac{Y_{t+1}}{Y_t}\frac{Y_t}{Y_{t-1}}\right) = \frac{\partial}{\partial w_t}(\ln Y_{t+1} - \ln Y_t) + \frac{\partial}{\partial w_t}(\ln Y_t - \ln Y_{t-1}) = C_0 + C_1$$

$$\dots$$

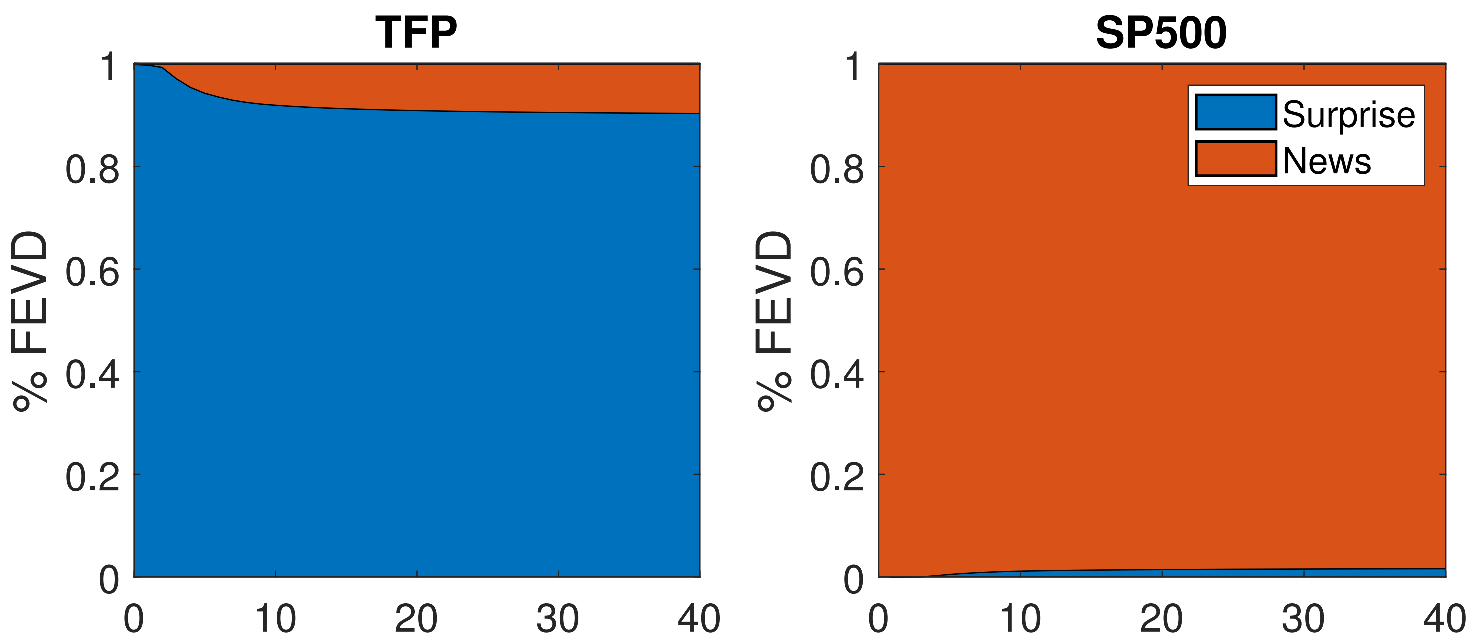
```

cumulate = [1,2];
C(cumulate,:,:) = cumsum(C(cumulate,:,:),3);

nboot1 = 1000;
prc = 68;
nboot2 = 2000;

[bootchol, upper, lower, boot beta] =...
 bootstrapChol corrected(finaldata,p,c,beta,err,nboot1,hor,prc,cumulate,nboot2
    );
```

IRFs to news shock



- The news shock is not about TFP...
- or it materializes in a different way than expected.

FEVD news shock



Identify technology shock with LRR

The TFP shock is the only one with long run effects on TFP log-level.

As before we need to work with the cumulated IRFs to get the level IRF.

$$\Theta_0 = I_n, \Theta_1 = F_n, \Theta_2 = F_n^2, ...$$

$$\Theta_0^l = I_n, \Theta_1^l = I_n + F_n, \Theta_2^l = I_n + F_n + F_n^2, ...$$

$$C_0^l = P, C_1^l = (I_n + F_n)P, C_2^l = (I_n + F_n + F_n^2)P, ...$$

So now

$$D := C^l(1) = \sum_{h=0}^{\infty} C_h^l \neq (I_n - F_n)^{-1} P.$$

Identify technology shock with LRR

The TFP shock is the only one with long run effects on TFP log-level.

We replace $D := C^l(1)$ with $C^l_H = (I_n + F_n + F_n^2 + ... + F_n^H)P = \Theta^l_H P$.

Then we can apply the Choleski factorization to

$$QQ' = \Theta^l_H \Sigma_\epsilon (\Theta^l_H)',$$

with $P = (\Theta^l_H)^{-1} Q$ we can isolate structural shocks on the levels.

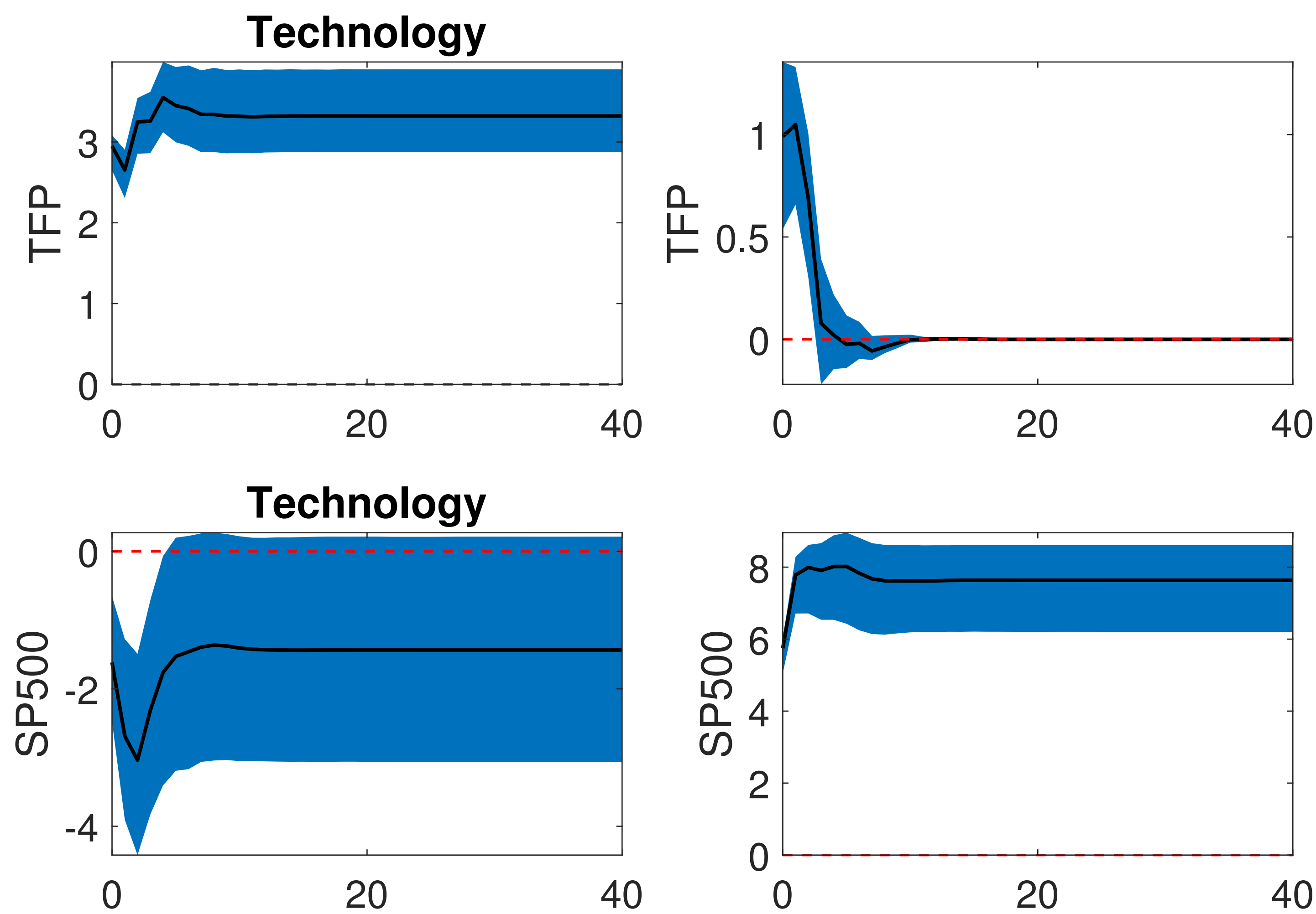
We use lower triangluar factorization as before.

$$\begin{bmatrix} \sum_{h=0}^H c^h_{11} & 0 \\ \sum_{h=0}^H c^h_{21} & \sum_{h=0}^H c^h_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix}$$

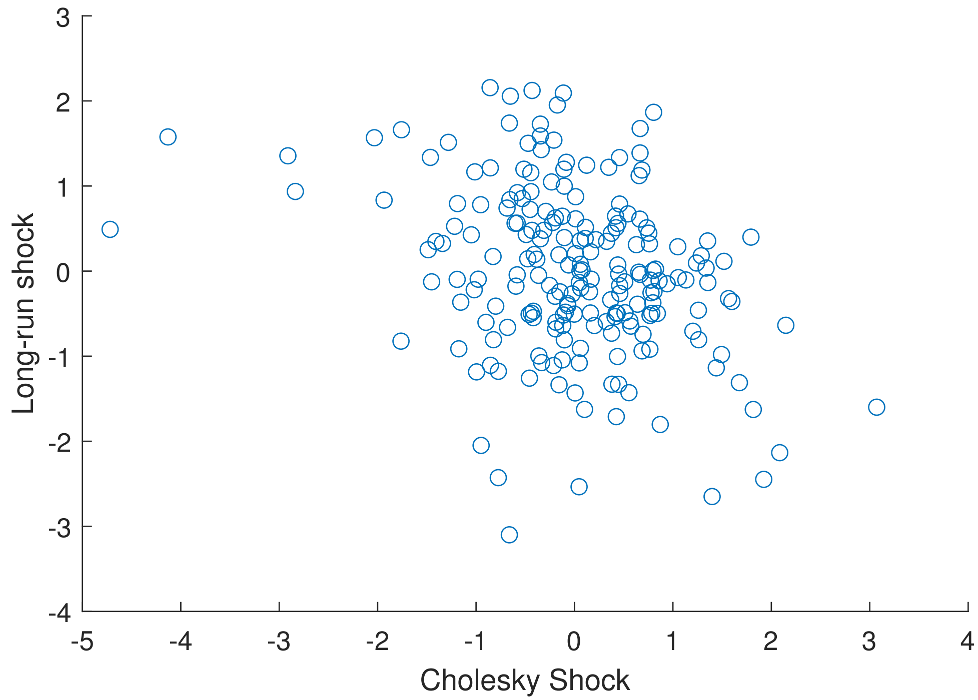
Identify technology shock with LRR

```
TH = sum(T,3); % sum the IRFs across the horizons
%C1 = inv(eye(n) - BigA(1:n,1:n));
Q=chol(TH*omega*TH','lower');
P=TH\Q;
C=zeros(n,n,hor+1);

% Structural Shock (first - technology):
eta=(P\err')';

% IRFs:
for i=1:hor+1
    C(:,:,i)=T(:,:,i)*P;
end
```

Identify technology shock with LRR



Correlation between the shocks



- In BP (2006) the correlation is very high at 0.97 (same shock)
- Correlation between technology shocks and news shocks for us is -0.31.

Correlation between the shocks

```
figure;
scatter(eps(:,2),eta(:,1));hold on;
xlabel('Cholesky Shock')
ylabel('Long-run shock')

% Compute correlation
correlation=corr(eps(:,2),eta(:,1));
```